Physics I : Oscillations and Waves
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Lecture - 22
Diffraction – III

Good morning. In the last class, we were discussing the resolving power of telescopes let me resolve the resume the discussion where we had left it.

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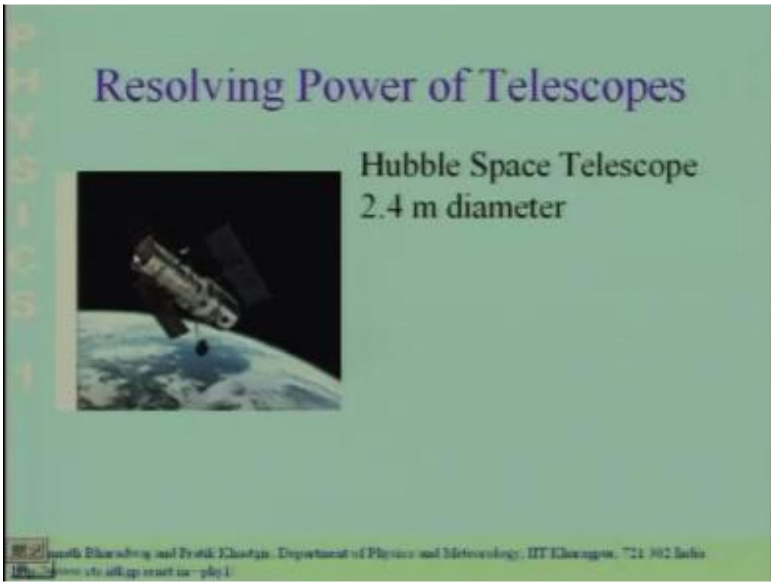

So, I had told you that telescopes have an aperture through which it lets light in the Aperture will always have a finite size so, here I show you the Hubble Space telescope it has an aperture of diameter 2.4 meters this shows you the aperture.

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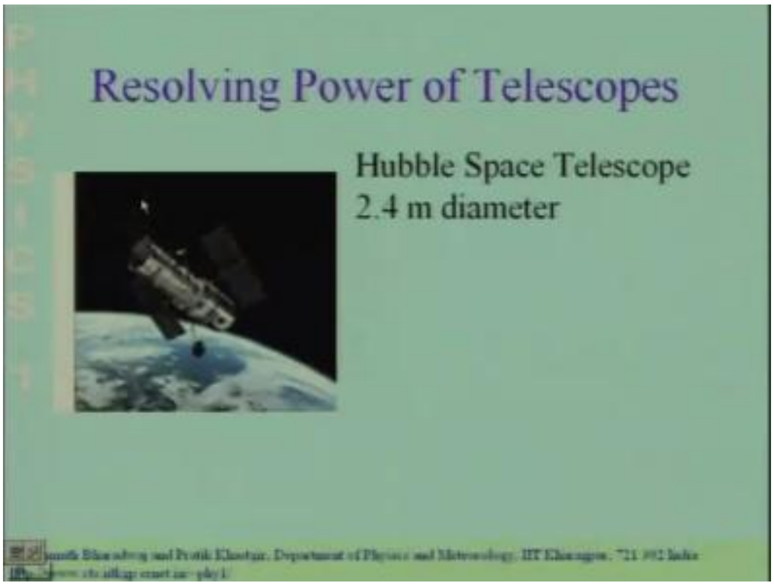

This shows you the joint meter wave radio telescope this has an aperture of diameter 45 meters this works at radio wavelengths now.

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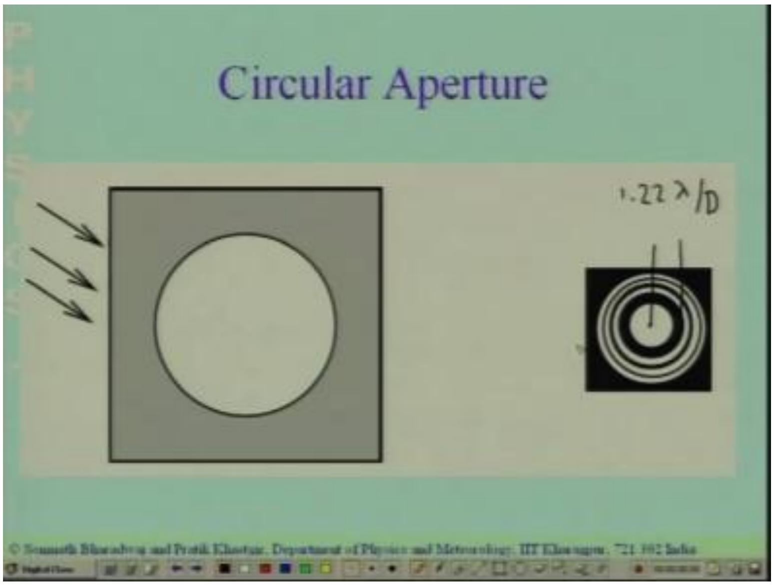

The point was that if you use such a telescope to image a star which is a point source star being located far away is a point source. So, if you use the telescope to image a star the image is a, the diffraction pattern of the aperture of the telescope. So, assuming that the telescope has a circular aperture I had told you that the diffraction pattern is going to look like this. There is going to be a central bright spot the radius of the central bright spot is lambda by D 1.22 lambda by D. So, I had told you that the, a star is going to produce a diffraction pattern the radius of the central bright spot is going to be 1.22 lambda by D in radians.

And then we have going to have set of dark and bright rings around it. And the question which we were addressing is how close can, 2 stars be distinguished. So, there are 2 stars in the sky if the 2 stars are very close together their diffraction patterns would over lap. And I would not be able to make out that they the time actually seeing. The diffraction pattern of 2 stars not 1 where as if the stars were far apart the diffraction patterns would be distinct. And I could be able to make out that they are 2 stars. So, the question that we were trying to address is how close can, the stars be. So, that what is the closest that the smallest angular separation between the stars at which we could distinguish. And say that there really are 2 stars there is a criteria which has been given for this purpose.

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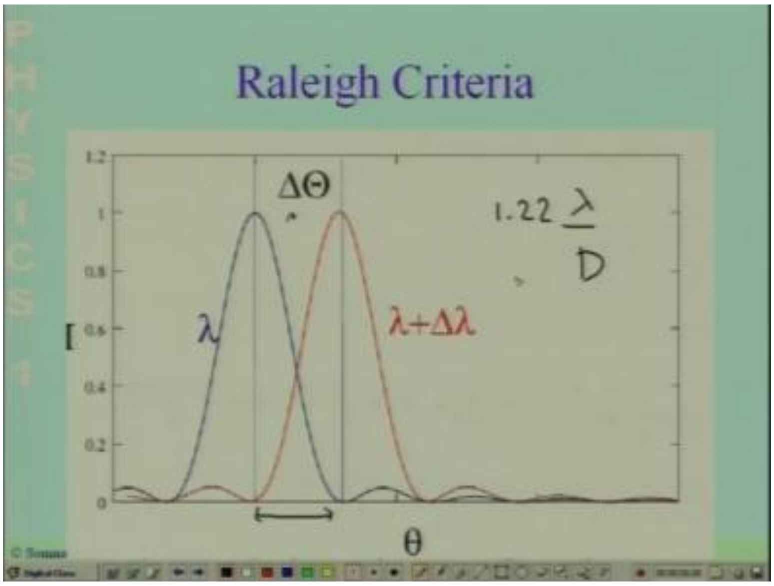

The criteria was a proposed by Lord Raleigh so, it is called the Raleigh's resolution criteria criteria for resolving 2 2 stars or 2 such images. So, the criteria, is as follows the blue curve over here shows you the diffraction pattern. That the intensity pattern that would be produce by one of the stars it has a maxima at theta is equal to 0. Let us say now, we have another star at different angle so, it would produce its own diffraction pattern which is the red curve showed over here. The 2 diffraction pattern would be displace by a by an amount corresponding to the angular separation of the star. And what

Lord Raleigh proposed was that we shall be able to distinguish to resolve these 2 diffraction patterns.

We shall be able to say that there are actually 2 diffraction 2 different sources which are producing 2 diffraction patterns if the maxima of one of them is at least at least coincides with the minima of the other one. So, if the maxima of this coincides with the minima of this this is the smallest angular separation at which we shall be able to distinguish. We shall be able to resolve the 2 sources this is Lord Raleigh's criteria. So, it is call the Raleigh criteria if this, the 2 stars are so, close that this diffraction pattern has a peak somewhere here. Then we will not be able to distinguish them the peak of one has to coincide at least with the minima of the other that.

So, we can just resolve them when the peak of 1 coincides with the minimum of the other if this peak were even further out you could easily resolve them. The place this angular separation where you can just resolve them is when the peak of one coincides with the minima of the other. And we have learnt that this for a circular aperture the minima is 1.22 lambda by D this angle away from the maxima. So, what we see here is that for a circular aperture you would be able to distinguish an 2 stars.

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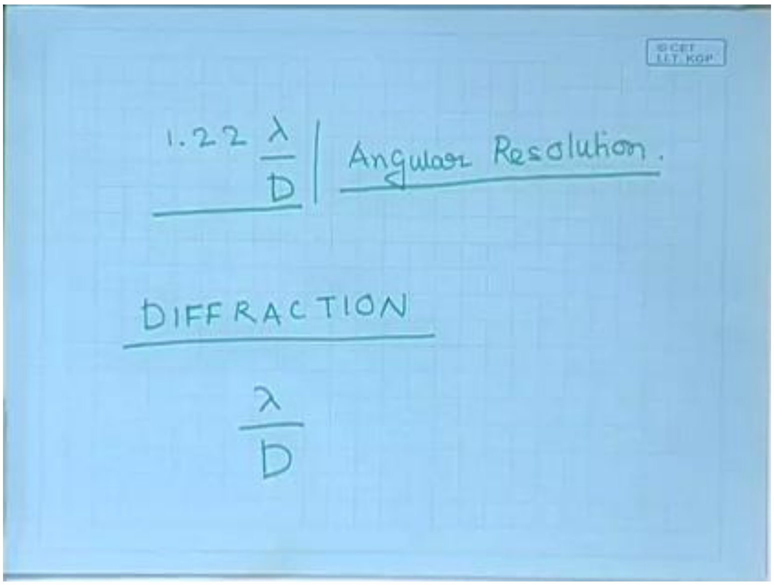

The smallest angle at angular separation at which you would be able to make out that there are 2 stars this is given by 1.22 lambda by D and this is what is call the angular resolution of a telescope so, for a circular telescope with a circular aperture. The angular resolution is 1.22 lambda by D this is the smallest angular separation at which I can distinguish 2 objects if the 2 objects. The stars were at an angular separation which is smaller than this I would not be able to make out the fact that. They are actually 2 and not 1 source whereas if the angular separation is more than this or equal to this I would be able to make out that. They are actually 2 sources not 1 so; this is what is referred to as the angular resolution.

So, for all optical instruments there is always a finite angular resolution. And the angular resolution of optical instruments is set by diffraction is decided by the diffraction. And it is typically of the order of lambda by D where lambda is the wavelength of the wave that. We are using so, if it is optical it could be 0.5 micron it your working in radio it could be meters or millimeters or centimeters. And D is the size of the aperture of your instrument so; let us calculate a few examples of this. So, let us go back to let us go back to our, to the 2 telescopes which we had discussed.

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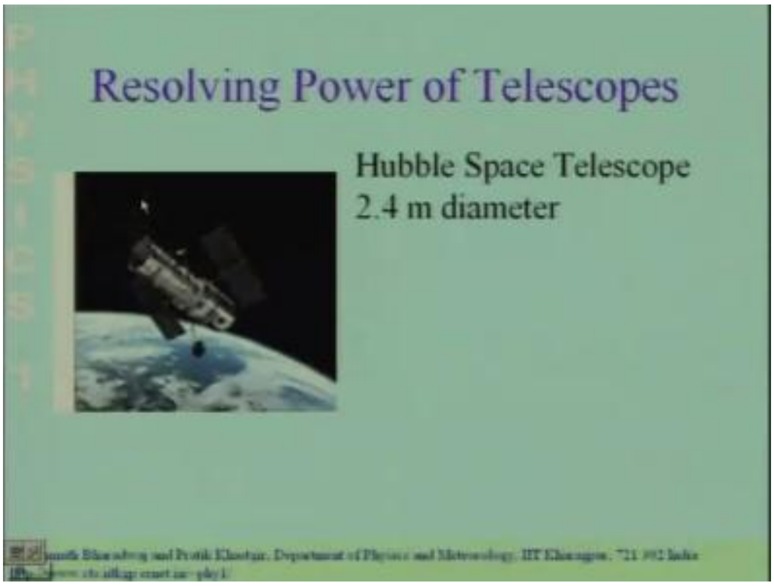

So, let us first calculate the angular resolution for the Hubble space telescope so, the angular resolution for any optical instrument of the order of lambda by D.

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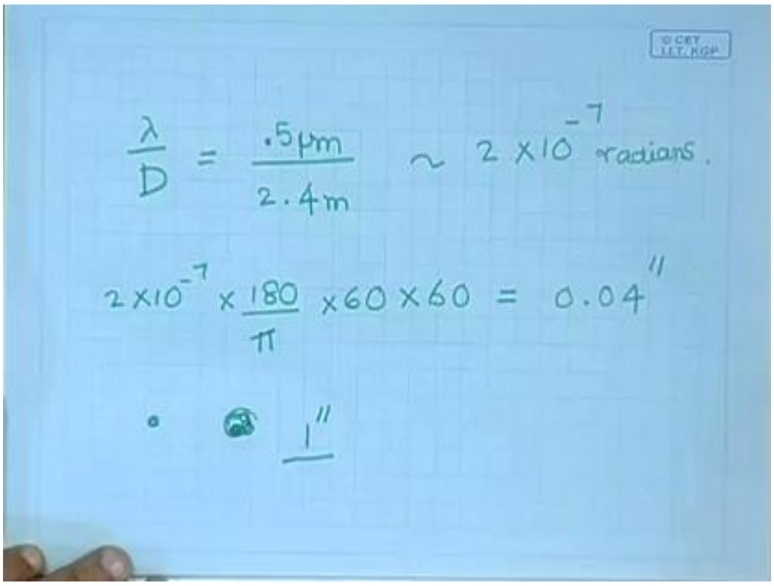

And in the optical so, let us assume that the, we are using that the Hubble telescope to make an optical image. So, optical wavelength is of the order 0.5 micrometers and the diameter of the Hubble telespace telescope is 2.4 meters. This is a approximately equal to 2 into 10 to the power minus 7 radians so, the angle. So, the so, with the Hubble space telescope I would be able to distinguish I would able resolve 2 sources if the angular separation of these 2 sources were equal or more than 2 into 10 to the power minus 7 radians. If the angular separation was less than this I would not be able to resolve the 2 sources. It is convenient and customary to talk about angles in terms of degrees and each degree if the angle is smaller than a degree then each degree is divided into 1 arc minute to 60 arc minutes. If it is even smaller than 1 arc minute then each arc minute is again further sub divided into 60 arc seconds.

So, let us just estimate what this value of this angular separation which we have calculated. This is the resolving the angular resolution of Hubble's space telescopes let us estimate. This in degrees minutes or arc seconds whichever is appropriate. So, to it converts this into degrees what we have do is multiplied by 2 into 10 to the power minus 7 into 180 divided by pi you can see that. This number is much less than 1 and then we multiply by sixty to get into arc minutes and then into 60 to get it into arc seconds. And this if you do the multiplication comes out to be 0.04 arc seconds. So, the angular resolution of Hubble's space telescope is around 0.04 arc seconds if use this lambda by diffraction limit is 0.04 arc seconds. What is the angular resolution of the, I for example, right you could estimate. This state lambda by D for the, I the pupil of I the, is the aperture. That you which is there in the, I and lambda is the wavelength of light.

The angular resolution of the, I is the order of the arc minutes it could be vary from individual to individual, but it is of the order of arc minutes. So, you see that this the Hubble’s space telescopes indeed as a very a high angular resolution it is of the order of 0.04 arc seconds. Now, the question may arise in your mind why is it necessary to send a telescope into space why not put a 2.4 meter telescope on the surface of the earth you would have the same angular resolution well. There is a problem here the earth’s atmosphere is a refracting medium and this refracting medium is actually changing with time. So, as a consequence of this if I were to make an image for a star at 1 instant of time. The image of star would appear somewhere here at the next instant of time the refractive index of the atmosphere would have changed. And the causing the image to move around so, image would be somewhere shifted and shifted.

And finally, the image which you get would actually be a little quite bit larger than that which is that. So, the bright spot which you get corresponding to a star would be significantly larger. Than the bright spot which credited by the diffraction and the size of this bright spot which you get when you do optical observations from inside the earth atmosphere would be decided more by the earth atmosphere. And less by the diffraction and typically you could achieve 1 arc second resolution from a good observing site. So, if you want to achieve angular resolution below 1 arc second like 0.04 for the Hubble’s space telescope you have to send your telescope outside. The earth’s atmosphere let us now make another estimate we have seen that the joint meter wave radio telescope has a diameter of 45 meters.

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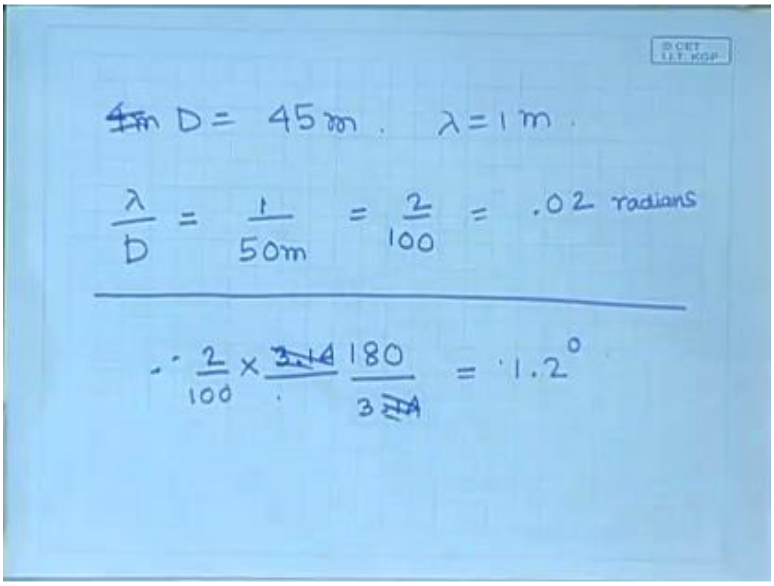

And it works in the meter wavelength so, let us assume that the lambda is 1 meter and make an estimate of the angular resolution of the joint of a single dish of the joint meter wave radio telescope. This figure lambda by D now comes out to be 1 and I can take is to be approximately 50 meters so, 1 divided by 50 is a 2 divided by 100. So, this is equal to 0.02 radians and to convert this into degrees minutes or arc seconds whichever is appropriate point. We can just work with 2 divided by 100 into pi pi I can take to be 3.14 divided by sorry divided by into 180 divide by 3.14 let me take this to be 3 to just get a rough number. So, this is going to be of the order of 1.2 degrees so, you see that with a single joint meter wave radio telescope with a single telescope of a diameters 45 meters I get an angular resolution of only 1.2 degrees.

This is a very poor angular resolution the sun or the moon, subtend half a degree. So, the, if the moon and sun were close together the joint meter in radio telescope could not be able to say that. There are actually 2 objects not 1, because the angular resolution of the joint meter wave radio telescope of a single dish of the joint meter wave radio telescope is 1.2 degrees. So, I hope I think the problem is clear to you at large wavelengths for example, radio wavelengths. It is very difficult to achieve angular resolutions which are comparable to the optical. And optical you can go up to arc seconds where as in radio if you wish to achieve an angular resolution of 1 arc second you would have to let us just estimate this.

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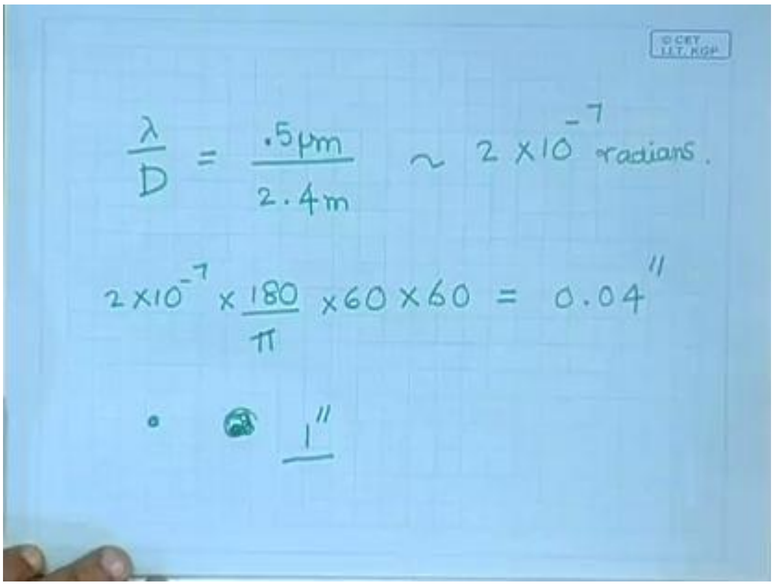

So, if you want achieve the kind of resolution let us say which you achieve with the Hs D with a Hs D you had angular resolution of 2 into 10 to the power of minus 7.

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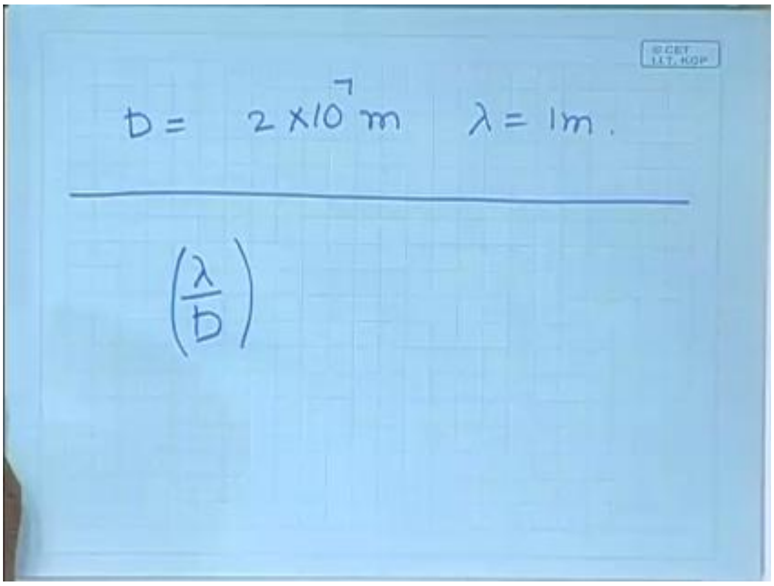

So, if you wish to achieve this kind of resolution you should have an antenna of diameter 2 into 10 the power 7 meters 20 million meters which is not possible you cannot make a single dish of a 20 million meters. The joint meter wave radio telescope so, at lambda equal to 1 if you working at 1 meter you will have to build a telescope of a diameter 2 into 10 to the power 7 meters. If you want to achieve a angular resolution comparable to the Hs D; obviously, this is not practical. So, the question is how do you achieve high angular resolution at low wavelengths large wavelengths or low frequencies right.

Optical frequencies have small wavelength lambda by D is small even for modest size aperture like few meters in radio the wavelength is large. So, lambda by D is going to be a large number if you want to achieve a small lambda by D comparable to optical you will have to build a really a enormous radio telescope which is not practical. So, there is a technique called aperture synthesis which I will not discuss.

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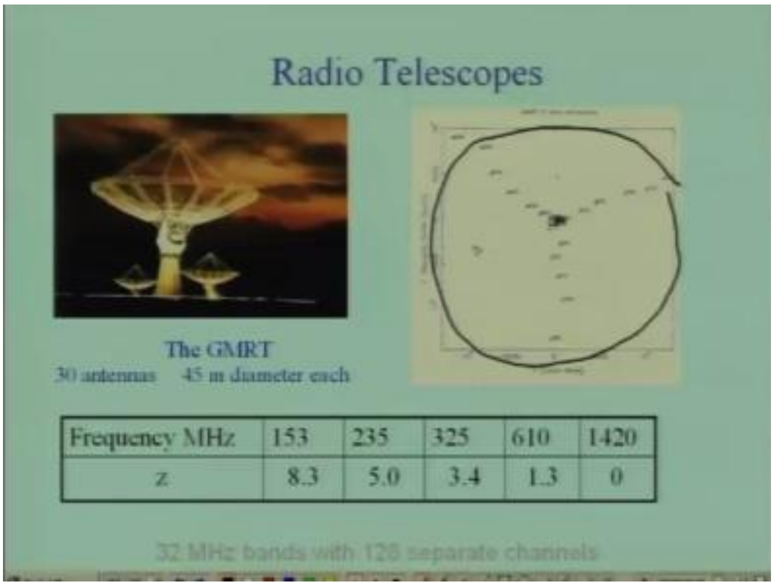

So in the GMRT for example, you do not use 1 antenna you have a have an array of antennas. There are actually 30 antennas distributed in a Y shaped configuration like this the length of each arm over here is 40 in meters. So, by using these few antennas you are actually able to achieve the angular resolution as if you had a single telescope of this size telescope which is this being. So, as if you had a single telescope of diameter around 25 kilometers so, there is a technique call aperture synthesis which allows you to achieve. The angular resolution as if you had a telescope whose diameter was so, big it is not particle to build the 1 telescope of such a large diameter.

But there is technique called aperture synthesis which allows you to build to put together many small telescopes spread out over this. And recover the resolution that you would achieve if you had a single dish of the size. It is call aperture synthesis it is a very interesting technique we shall not going to this the bottom line of this discussion let me just the recollect the bottom line of this discussion. The bottom line of this discussion is that there is a fundamental restriction on the angular resolution of any optical instrument. This fundamental restriction comes, because light or any kind of electromagnetic radiation it is a wave as a consequence of this. The image of a point source is not going to be a spot it is going to be have a finite size and this imposes the angular the restriction that.

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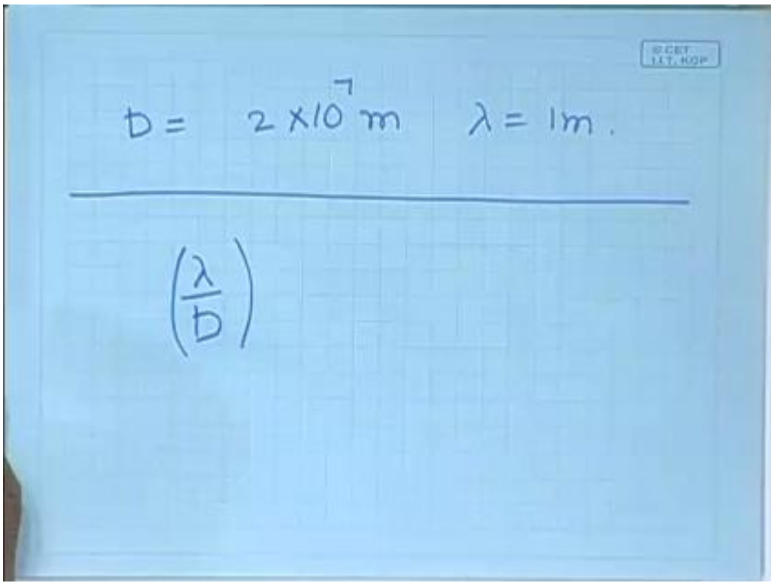

I cannot distinguish 2 sources I will not be able to resolve 2 sources if they are if they subtend an angle which is smaller than this. This is the resolving the angular resolution of any optical instrument it is a fundamental restriction. That comes from wave nature of light let me now move ahead let me now take up something else let me.

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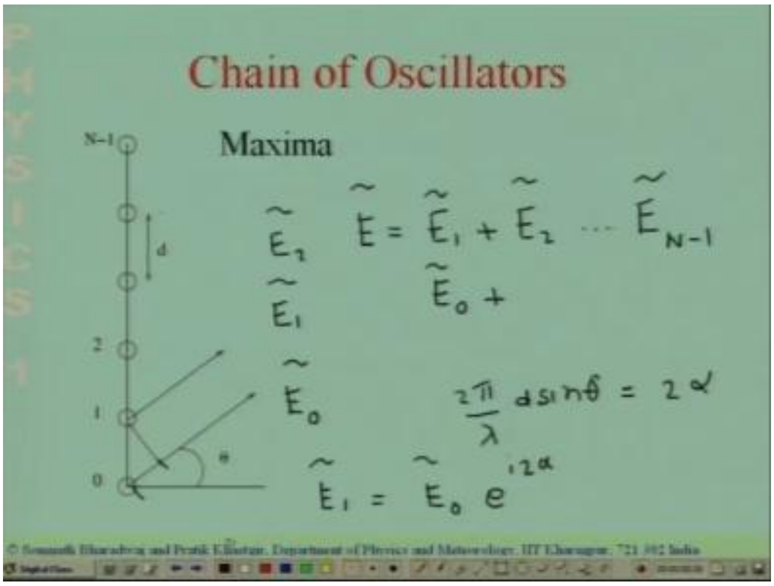

So, now, we will have a quick switch switch in the topic of discussion we are still going to continue discussing diffraction. But we going to start we going to take up another topic for discussion so, the topic which we are going to shift over to let me introduce it so, the problem that. We are discussing now is as follows we have a chain of oscillators you can think of each arrange like this each circle over here is an oscillator each oscillator is emitting radiations. So, you can think of each of this circle as been a dipole oscillator which we have discussed which emits radiations. If you send an oscillating current in a dipole it emits radiations you can think of each of this circle has a dipole antenna which is emitting radiation. There are in such dipoles we have labeled them 0 1 2 3 4 5 6 all the way up to N minus 1.

So, there are N such dipoles arranged in a linear a chain as shown over here the separation between 2 of the oscillators is distance d. The oscillators are all oscillating with the same frequency and same phase so; they all emit the same radiation radiation of the same frequency and same phase. The question that we are interested in is what is the radiation that will be received by a distant observer at an angle theta? So, in this direction what is the radiation that will be received by a distant observer at an angle theta? So, a distant observer is going to receive the sum of the radiation from each of these oscillators. So, if you call if you use E 0. So, I am going to use E 0 to denote the radiation that is going to be received from the zeroth oscillator. And then I am going to call the radiation that will we received from the first oscillated E 1. And similarly the radiation that will be received from second oscillator is going to be E 2.

So, the total radiation that is going to be received by a distant observer in at an angle theta is going to or at any other angle for that matter is going to be the sum of all of these. So, it is going to be E 1 plus E 2 oh plus E 0 sorry this plus E 0 plus E 1 plus E 2 all the way up to EN minus 1. So, let me remind you again what we wish to calculate there is a distant of observer the distant observer is going to receive. The super position of the radiation from all of these oscillators and this is going to be E 0 plus E 1 plus E 2 all the way to EN plus 1 N minus 1. Now, each of these oscillator is oscillating at the same phase so, they are remitting the radiation of the same phase.

But the radiation will not be received at the same phase this is, because there is a path difference between the radiations from any 2 oscillators. So, let us look at E 0 and E 1 the radiation from E 0 has to travel slightly longer distance than the radiation from E 1. So, the path difference is this bit over here the path difference is the bit over here and this will introduce a phase difference. So, the path difference between E 0 and E 1 the radiation from E 0 and from radiation E 1 is d sin theta. And this will introduce a phase difference 2 pi by lambda d sin theta and this phase difference I am going to call 2 alpha. So, what we can say is that E 1 is going to be E 0 e to the power I. It is going to come with a phase difference of 2 alpha e to the power i 2 alpha right. And similarly E 1 E 2 is going to be so, let me write it down here so.

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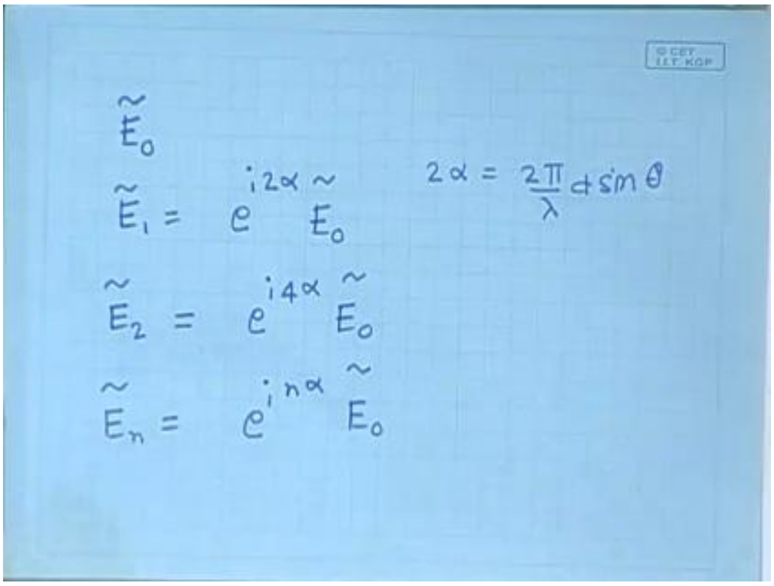

The radiation from E 0 from zeroth oscillator we are calling E 0 the radiation from the first oscillator E 1 is going to be e to the power i 2 alpha into E 0. The radiation from the second oscillator has an extra phase of 2 alpha with respect to E 1. So, there is an extra phase of e to the power i 4 alpha with respect to E 0. And the nth oscillator is going to be at a phase difference e to power i n alpha with respect to E 0 where alpha 2 alpha is 2 pi by lambda d sin theta right. So, the basic point over here is that a distant observer is going to receive the radiation from these different oscillators with a phase difference.

So, the there is going to be phase difference between the radiation from the zeroth oscillator. And the first oscillator there will be a phase difference between radiation from the first oscillator and the second oscillator. So, the radiation from all of this oscillator are going to arrive at different phases these this phase difference is, because of the path difference. The path from the different oscillators to the observer is, are not the same they are different. So, when I combine all of these I have to put in this phase so, the resultant radiation that will be received by a.

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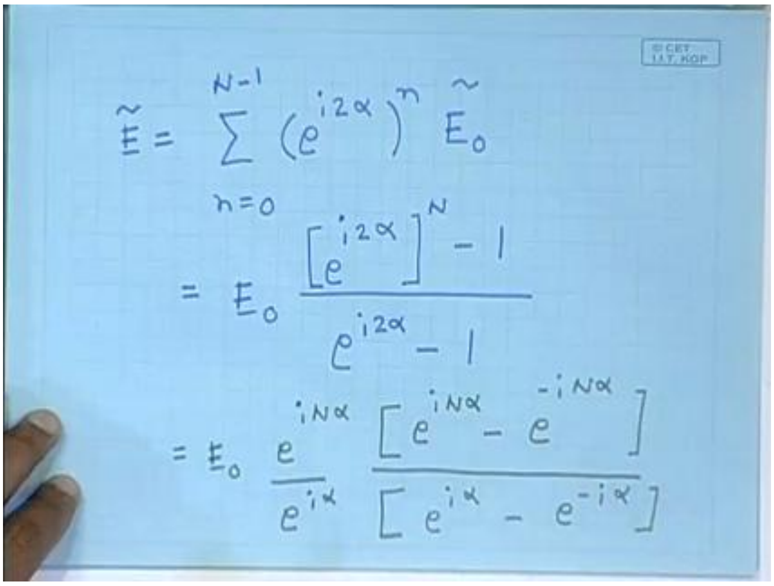

Distant observer is going to be a sum of all of these and this sum we can write has n going from 0 to N minus 1. There are N of these oscillators e to the power i 2 alpha to the power of n into E 0 right and this so, this is just restatement of this.

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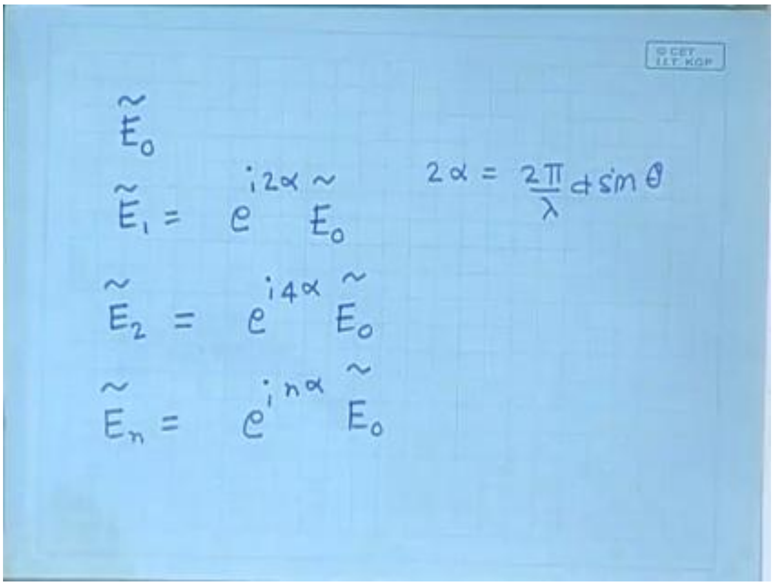

So, I have added up the 0 E 1 E 2 E 3 this going to contribute e to the power I 2 alpha time E 0. This is going to contribute e to the power i 2 alpha square i 2 alpha to the power n E 0 which is what I have done over I have written up written down the sum now this is a geometrical progression I can take the E 0 outside..

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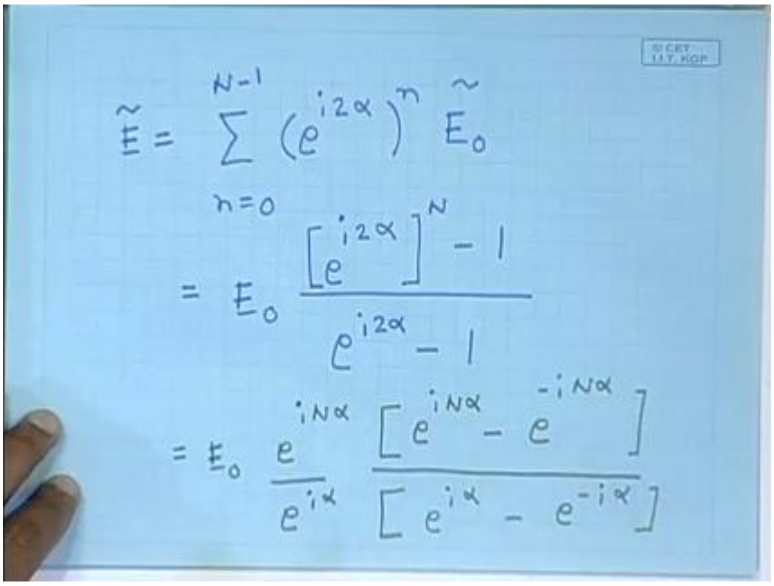

And this is a geometrical progression if I sum this geometrical progression what I have is E 0 into the ratio of 2 consecutive terms to the power of N N being the number of terms. So, the ratio of 2 consecutive terms here is E to the power i 2 alpha this to the power N minus 1 divided by the ratio i 2 alpha minus 1. Tell me what to do now? This is the resultant radiation that will be received by the observer. We want to do we have summed up all the contribution from all the oscillators we want to simplify this expression a little bit.

And this can be done by writing it as E 0 i will take e to the power i N alpha common over here. So, I have e to the power i N alpha common and then what I have over here is e to the power i N alpha minus e to the power i minus i N alpha divided by in the denominator I will take e to the power i alpha common. So, this I will take e to the power i alpha common so, in the denominator i will have e to the power i alpha minus e to the power minus i alpha. And you can see that this is going to be sin N alpha this is going to be sin alpha so, the resultant electric field.

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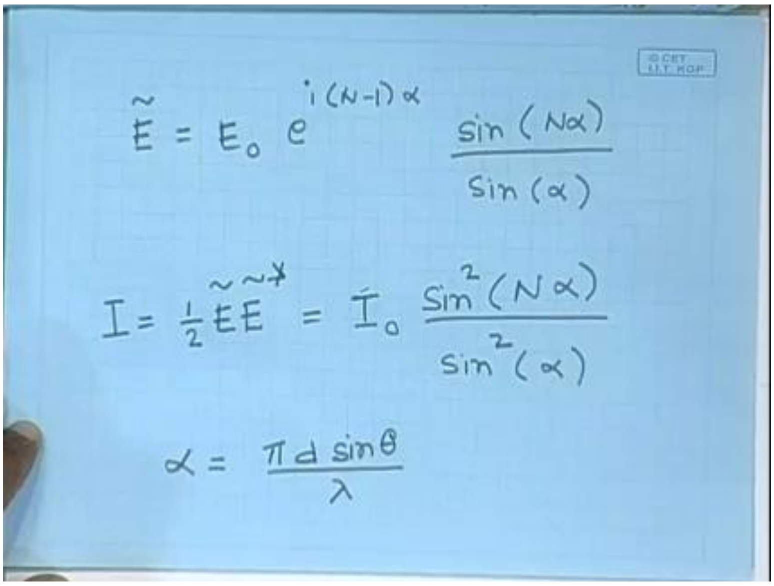

The resultant radiation received by a distant observer is going to be E 0 e to the power i N minus 1 alpha into sin N alpha divided by sin alpha.

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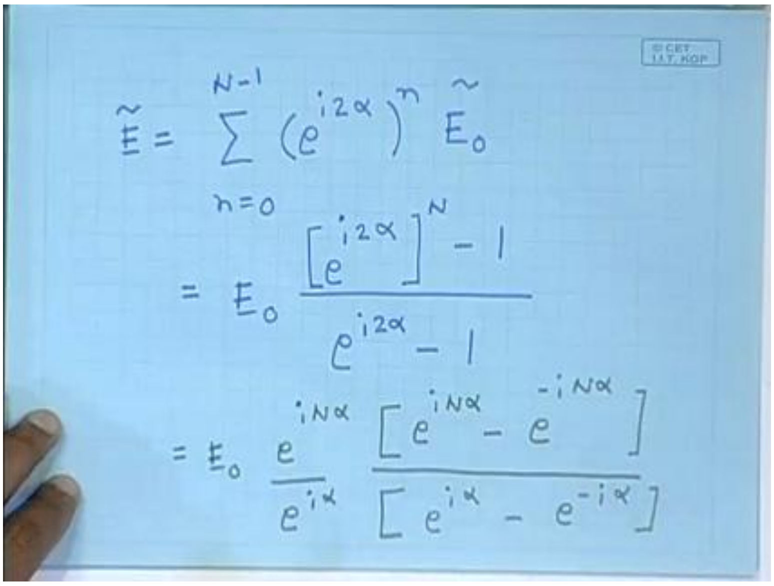

The sin comes sin N alpha comes from this the sin alpha comes from this this difference the there factors of 2. And I which will cancel out form both of these and e to the power i N minus alpha is this ratio over here.

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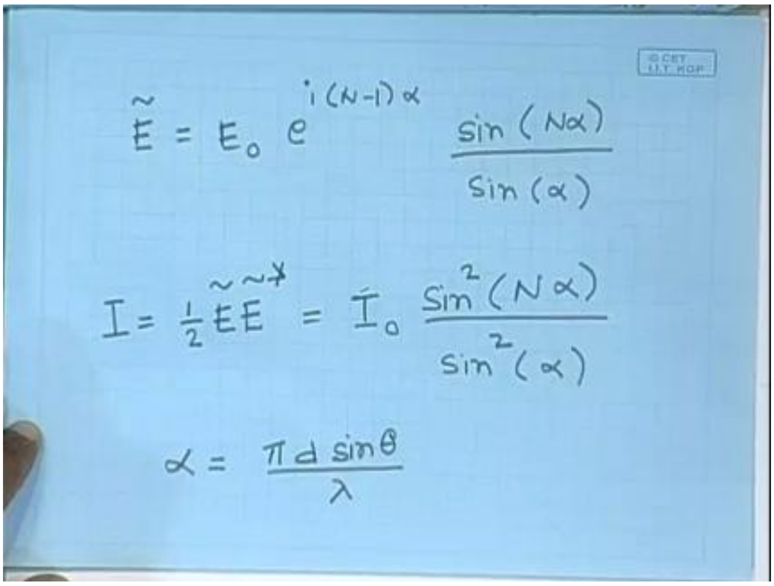

Now, we want to calculate the intensity so, the intensity as we have seen many a times is half E into E star and this can be written as. So, I have multiply this with the it is complex conjugate if I multiplied with this complex conjugate I will get is E to naught square multiplying. This with its complex conjugate gives 1 and I will have sin square and sin square here. So, here I can write it as minus I naught sin square N alpha divided by sin alpha sin square alpha.

So, this is the intensity pattern that will be produced by this chain of oscillator so, this is the intensity pattern. That we observed by an distant observer produced by this chain of oscillators let me also write down here what alpha is? So, alpha is pi d sin theta by lambda so, this is the intensity pattern now let us analyze for a few minutes. Let us analyze what this intensity pattern looks like to start with let us consider the point where theta is equal to 0. What is going to be the value of intensity where theta is equal to 0 theta equal to 0 implies that alpha is also 0. So, this I can write this I want to evaluate this at alpha equal to 0.

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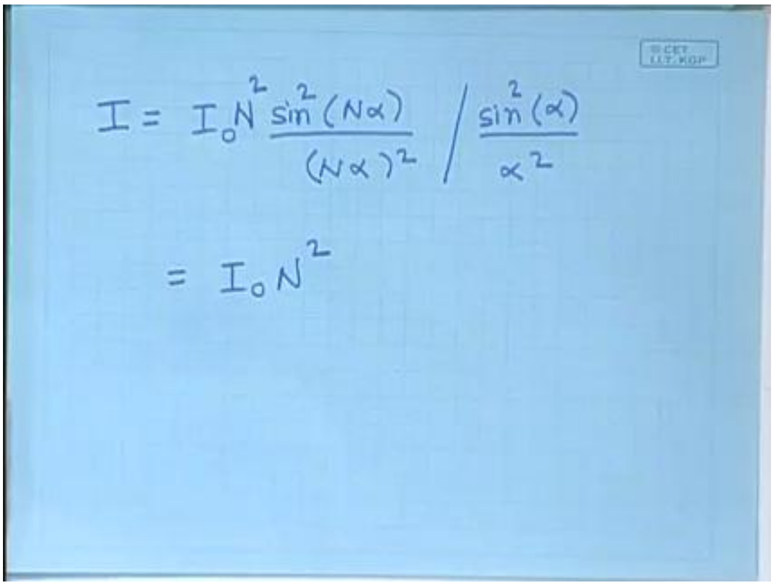

So, I can write this let me do it a little elaborately for you I can write it as I naught sin square N alpha divided by N alpha square. This divided by sin square alpha divided by alpha square I have to put a factor of N square here.

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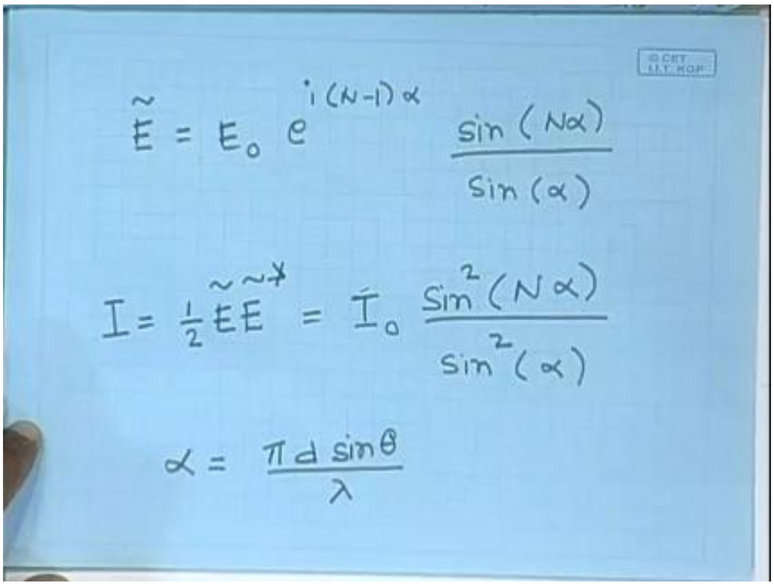

So, this it should be clear is same as this expression for the intensity all that I have done is I have divided by N square alpha, square divided. This by alpha, square there is an extra factor of N square which I have to put over here.

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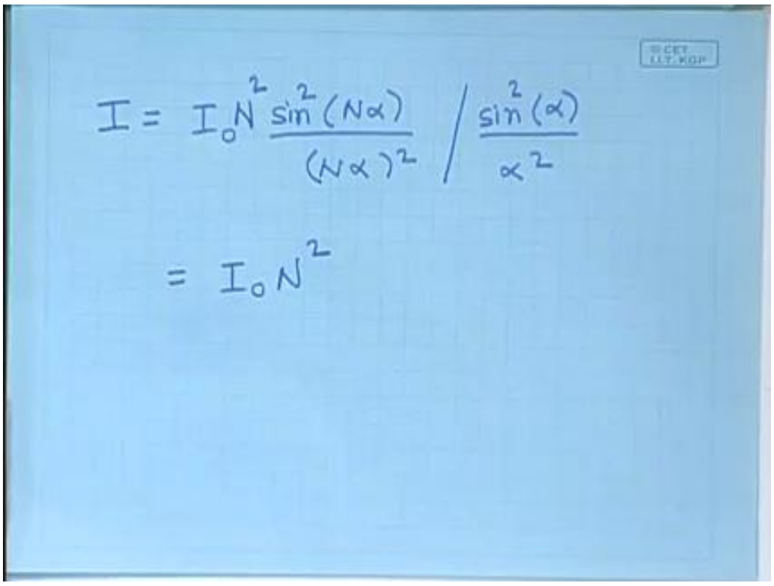

So, this is what I get now if I take limit of alpha going to 0 this ratio we know is 1 this ratio is also 1 so, we know that this is going to have a value I naught N square.

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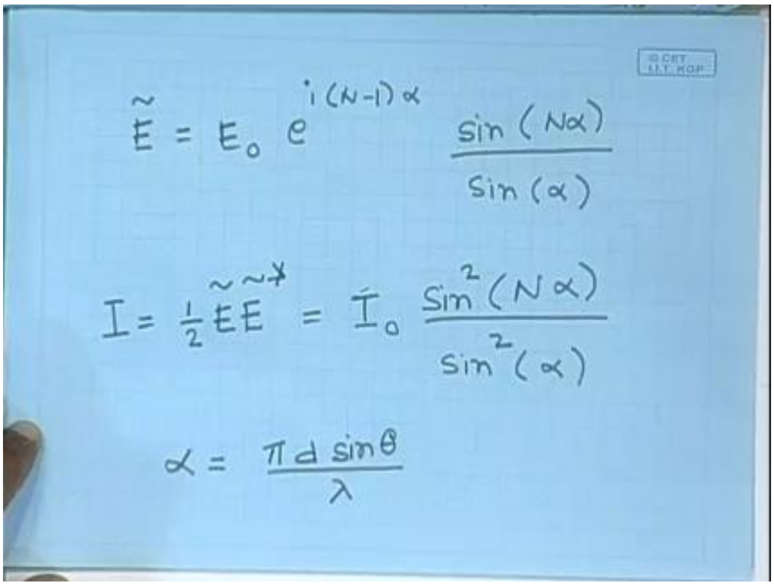

So, what we have seen is that alpha going to 0 the denominator and the numerator both both vanish. And we have the ratio of the both vanish, but the ratio is finite it assumes a value N square. And this is the maximum value that the intensity is going to have now, you increase alpha from if you increase theta from 0. So, the value of alpha is also going to increase sin theta is going to increase this is also going to increase. But this ratio is going to fall but there will be maxima again whenever the value of alpha disappears. So, the value of alpha is again going to disappear when sin theta vanishes.

So, sin theta is going to vanish whenever it is when theta is a multiple of a when sorry. It is not that the, this ratio you see when if alpha changes so, an alpha equal to 0 this ratio is N square. Now, sin the denominator vanishes the denominator as you can see is going vanish whenever sin alpha is 0. We know that sin alpha is 0 okay let me make this point clear the intensity is going to have the maximum value whenever. The denominator becomes 0 so, the intensity is going to have a maximum value whenever the denominator become 0 it should be cleat so, this happens whenever.

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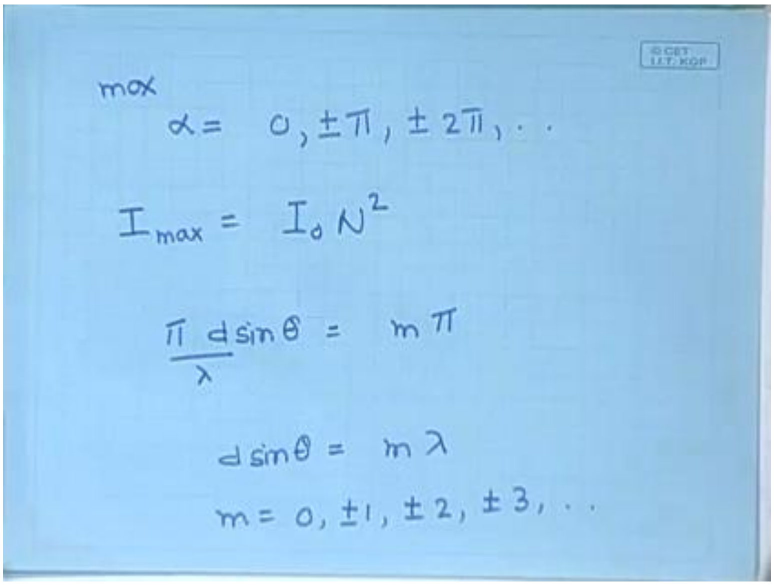

Alpha is equal to either 0 or plus minus pi or plus minus 2 pi etcetera.

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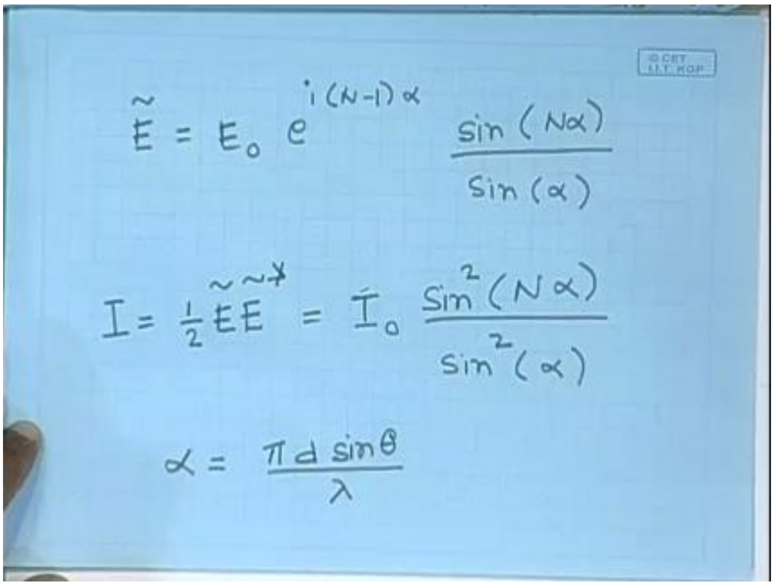

It is also clear that whenever the denominator is 0 the numerator is also 0 so, this ratio becomes N square and this is these are the points where the intensity is maximum.

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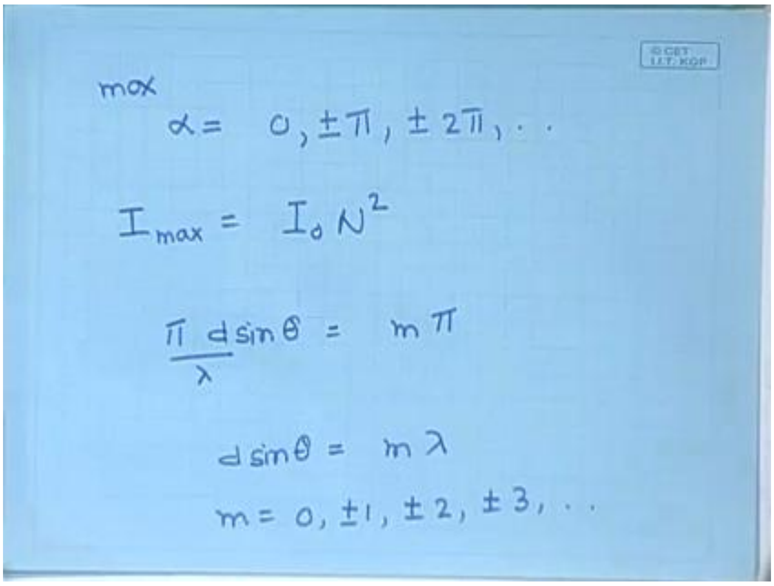

So, the intensity is maximum whenever alpha is equal to plus minus pi 2 pi etcetera the maximum value of the intensity is N square I naught. So, the maximum value of the intensity is I naught into N square. And there will be a many of them whenever alpha is equal to 0 or plus minus pi are 2 pi etcetera.

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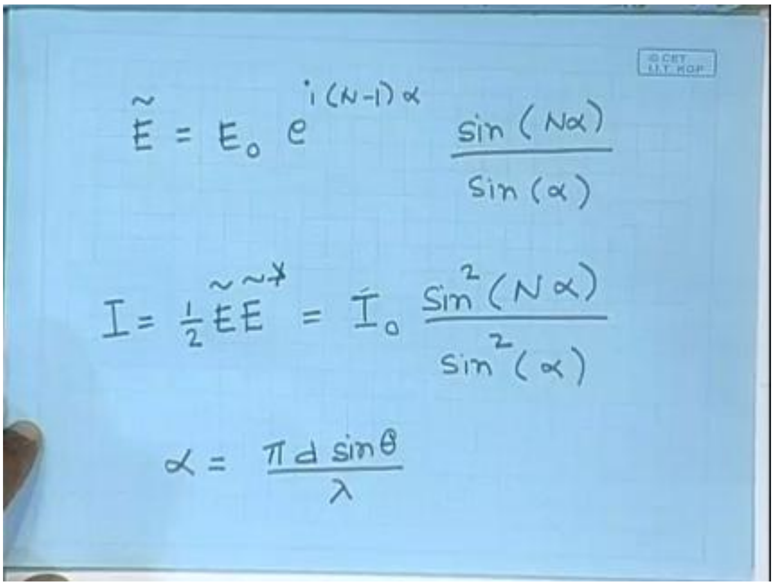

Writing this out in terms of the angle theta we see that.

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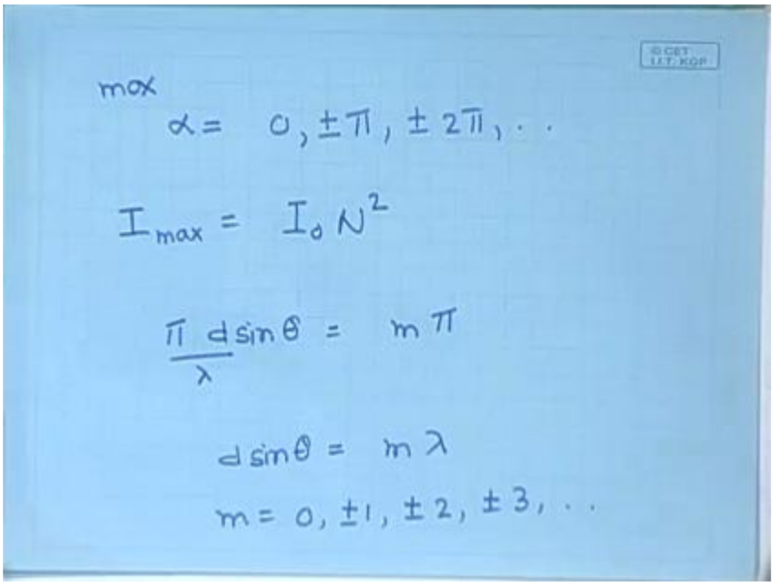

The maxima is going to occur whenever d sin theta pi by lambda is equal to m pi where m can be any integer 0 plus minus 1 2 3 etcetera. Or it tells us that d sin theta is equal m lambda where m can values 0 plus minus 1 plus minus 2 plus minus 3 etcetera let me repeat this again we are looking.

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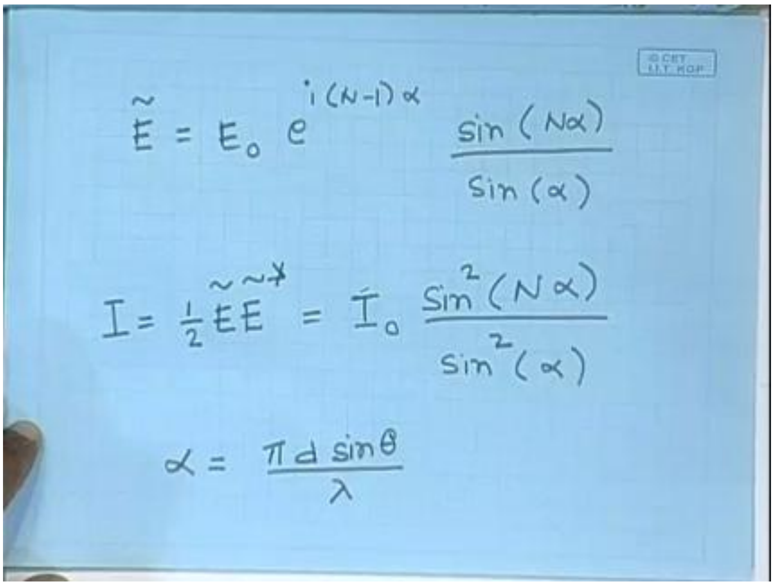

At the intensity that is going to produce by a chain of oscillators and I have we have derived. And found the intensity pattern is given by I is equal to I naught sin square N alpha divided by sin square alpha where alpha is pi d sin theta by lambda let me just get things clear let me remind you.

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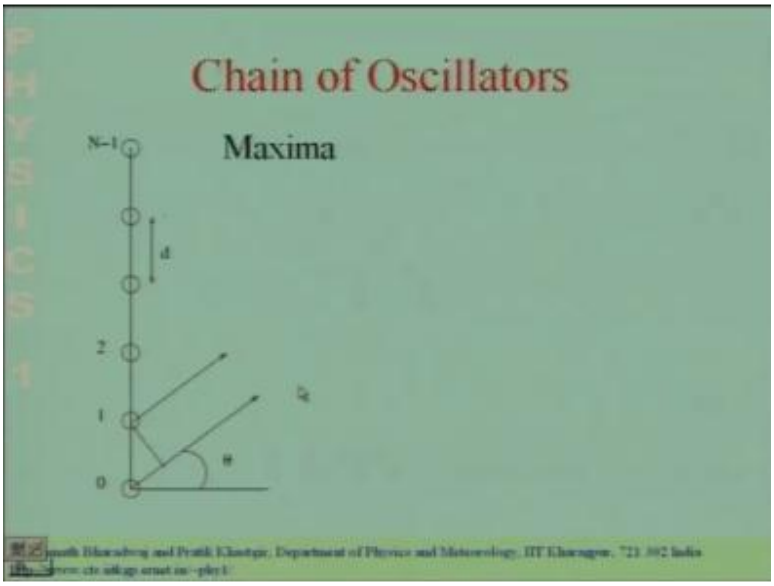

Theta is the angle at which the observer is located so; this tells us how the intensity is going to vary if I change the angle theta in different directions. And what we find is that the intensity is going to have maximum value whenever the denominator, become 0.

Now, it is ensured that whenever the denominator become 0 the numerator is also 0. Because the denominator, become 0 whenever alpha is multiple of pi and if alpha is a multiple of pi alpha 0 plus minus pi to pi etcetera.

So, this is when the denominator vanishes it is guaranteed that the numerator also vanishes and this has a value N squared these are the maxima of this intensity. So, there will be certain directions at which the intensity will be maxima these directions these values of theta correspond to.

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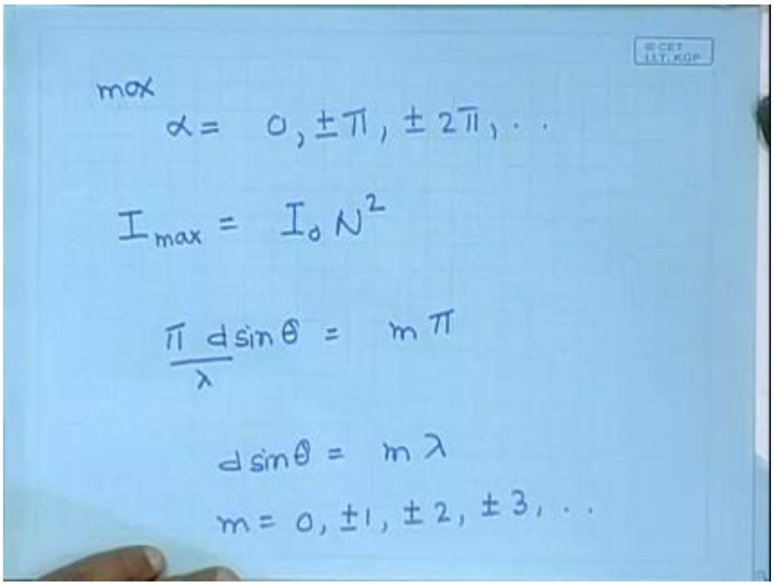

Alpha equal to 0 plus minus pi plus minus 2 pi etcetera in these directions the intensity is going to maximum the maximum value is I naught N squared. So, to determine the direction where we are going to have the maxima set alpha equal to this which in terms of the angle theta. Now, becomes d sin theta is equal to m lambda so, for different integer m I will have different solutions theta or.

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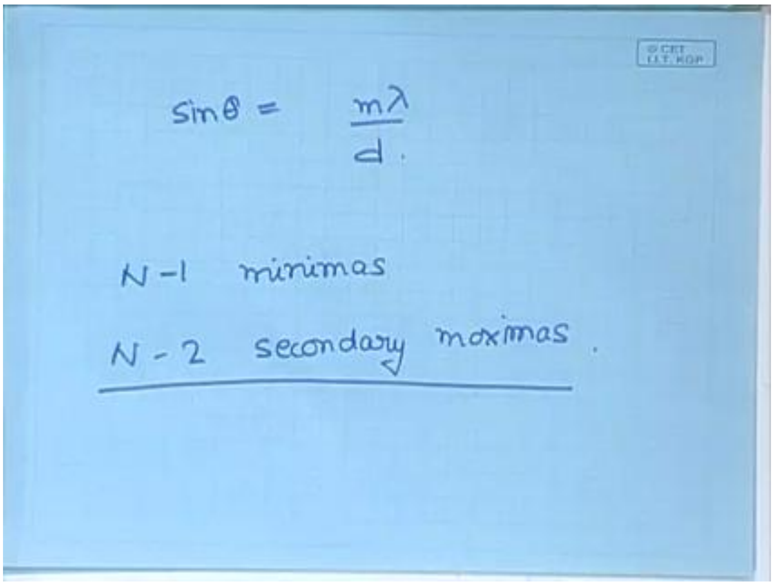

Sin theta is equal to m lambda by d so, if lambda by d is more than 1 if lambda by d is more than 1 you will have only 1 solution m equal to 0 which is theta is equal to 0.

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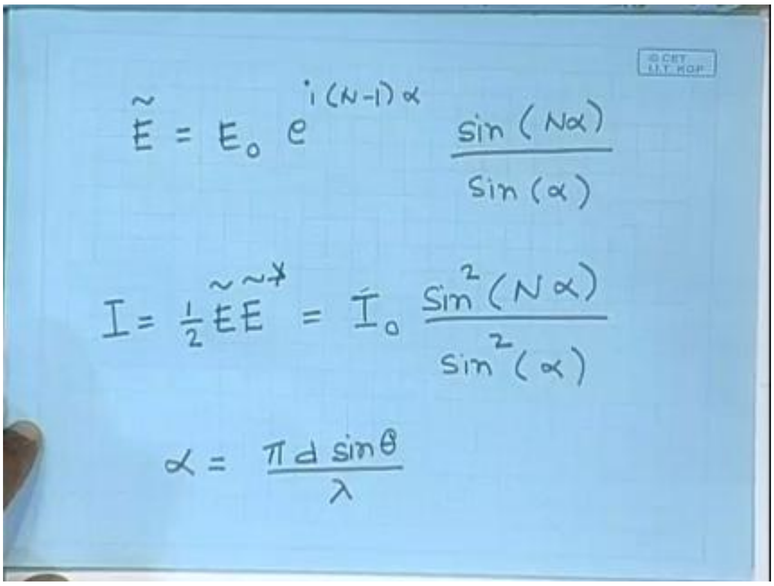

If lambda by d is less than 1 you can have more than 1 solution for different integers n m right and these are these solutions correspond to these values of alpha where the denominator and numerator both becomes 0 now, let me plot this intensity pattern and show it to you as a function of alpha.

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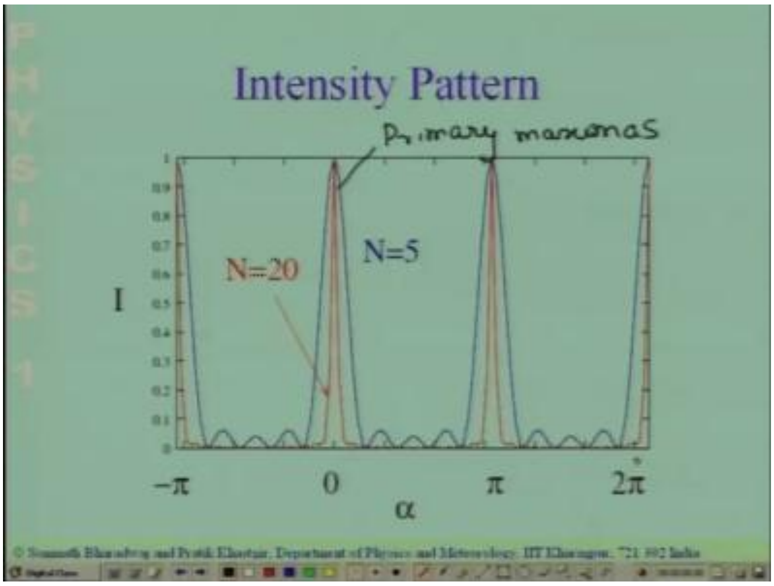

So, if I plot the intensity pattern as a function of alpha it looks like this. Whenever alpha is 0 or alpha is pi 2 pi minus pi I have these maximas the numerator and the denominator both vanish. Now, if let me just consider a situation where we are looking at alpha equal to 0 so, I am looking at this point. And I increase alpha if I increase alpha the value of the intensity is going to fall and it is going to go to 0.

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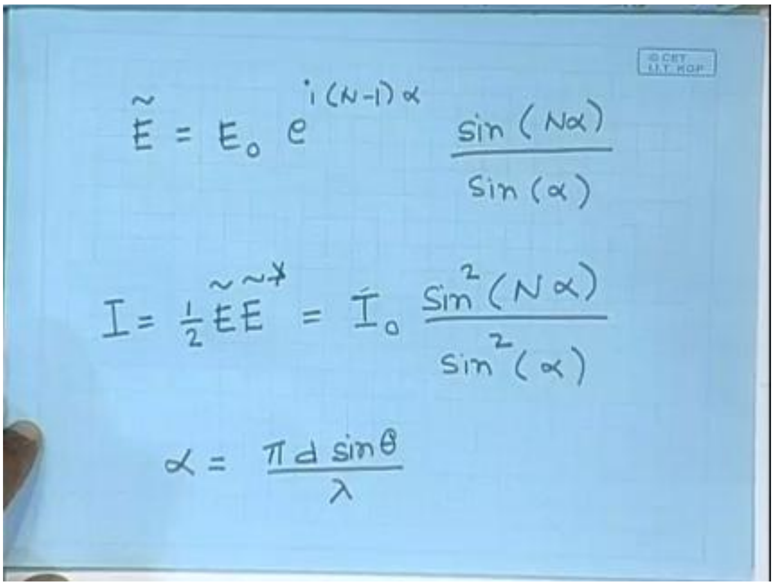

The numerator see look at the denominator first not the numerator look at the denominator first. The denominator starts from 0 it increases and then it becomes 0 again when alpha goes to pi.

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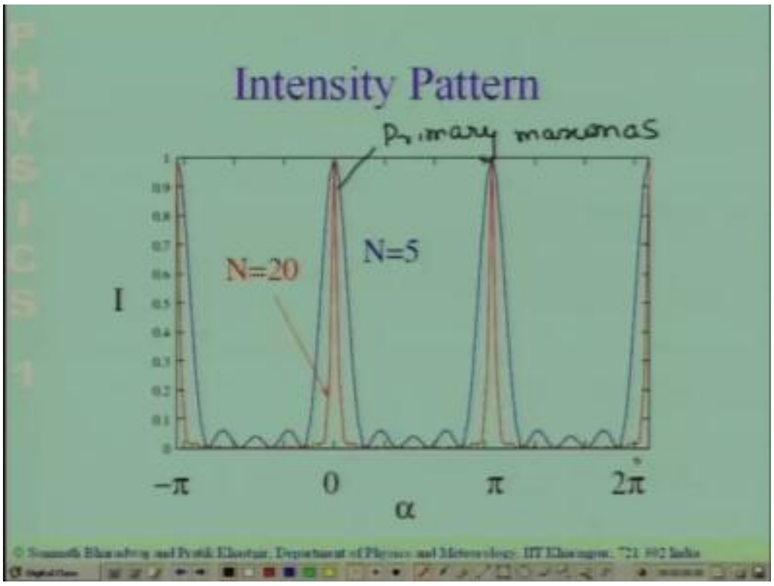

So, we are looking at alpha starting from value 0 where we have a, maxima in intensity as you increase alpha the denominator increases. And then it falls again goes to 0 at pi that is the other maxima over here. These maximas are called the primary maximas these are call the primary maximas. So, we shall refer to these maximas as the primary maximas and the primary maximas occur whenever the denominator becomes 0 which is alpha is equal to 0 pi etcetera now, let us look at the numerator as alpha is increased.

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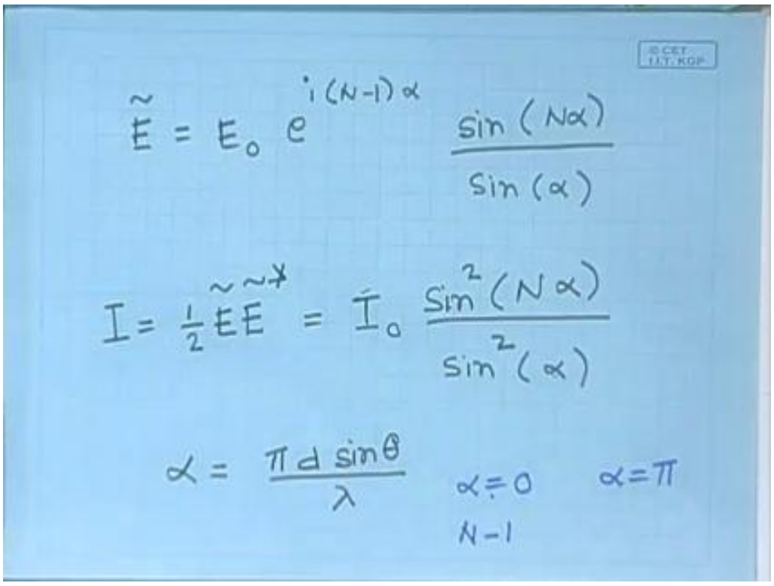

The numerator goes to 0 N minus 1 times between so, we alpha starts from alpha equal to 0. And then alpha goes to pi the denominator has gone from 1 0 to other how many times does the numerator becomes 0 in between. It should be quite obvious that the numerator becomes 0 N minus 1 times in between this is what you see over here.

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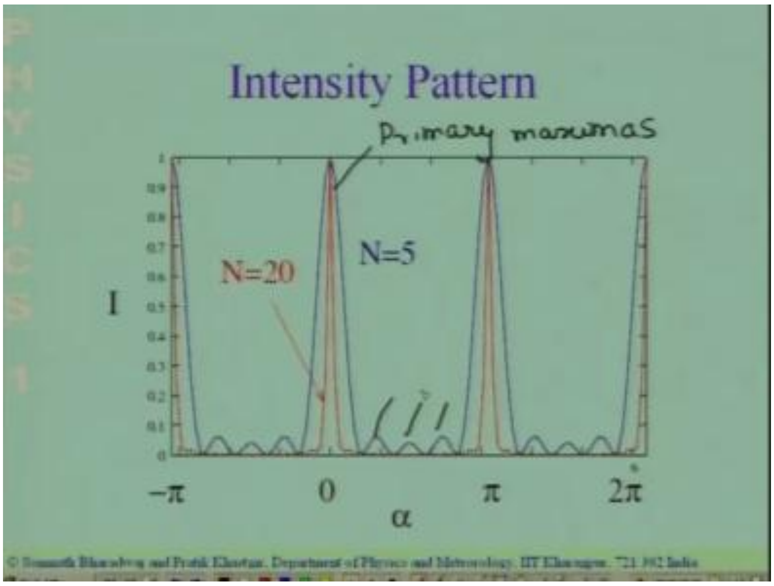

So, the blue curve shows you the intensity pattern if I had 5 oscillators and you can see that it become 0 once twice 3 times. And 4 times in between the value 0 and pi the numerator becomes 0 exactly 4 times. So, in general it is going to become 0 N minus 1 times in between 2 primary maximas. And between the zeros of the numerator there is there are going to be secondary maximas over here here and here so, between 2 primary maximas.

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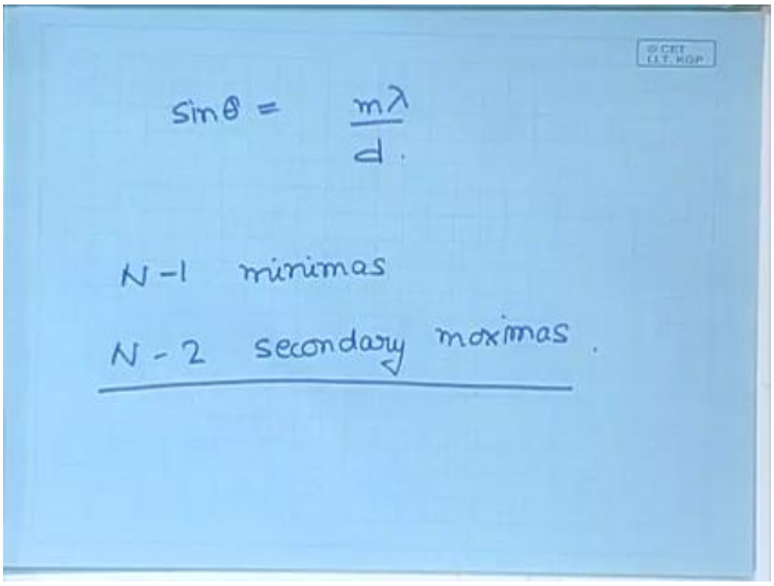

We will have N minus 1 minimas and we will have N minus 2 secondary maximas so, has you can see here.

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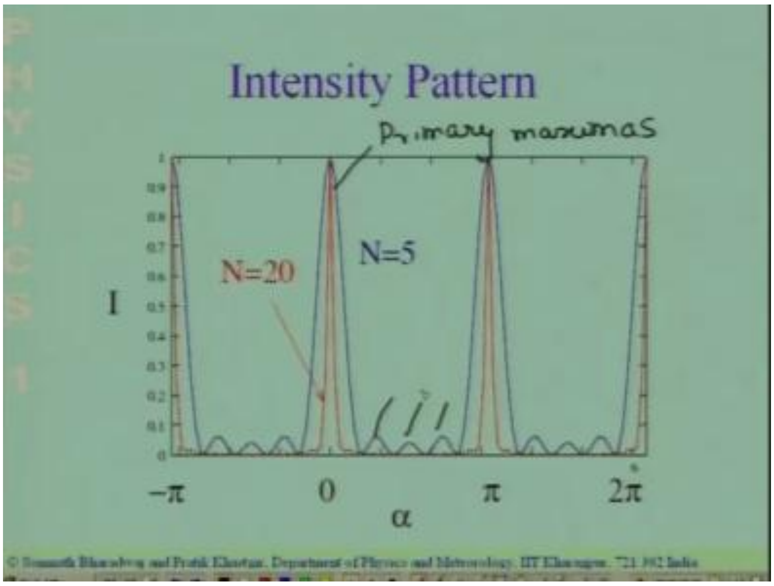

There are two things the secondary maximas have a much lesser intensity as compared to the primary maximas. The other point to note is has I increase a number of oscillators the number of zeros in between between. The 2 primary maximas increases the number of zeros not only does the number of zeros increase. But, there intensity of these secondary maximas also goes down you cannot you can just you cannot see it here actually you can only possibly see the first 1. And you cannot see any of them after that also the width of the primary maxima decreases as you increase the number of oscillators. So, the primary maximas get sharper and sharper let us. Now, estimate the width of the primary maxima so, the primary maxima occurs whenever.

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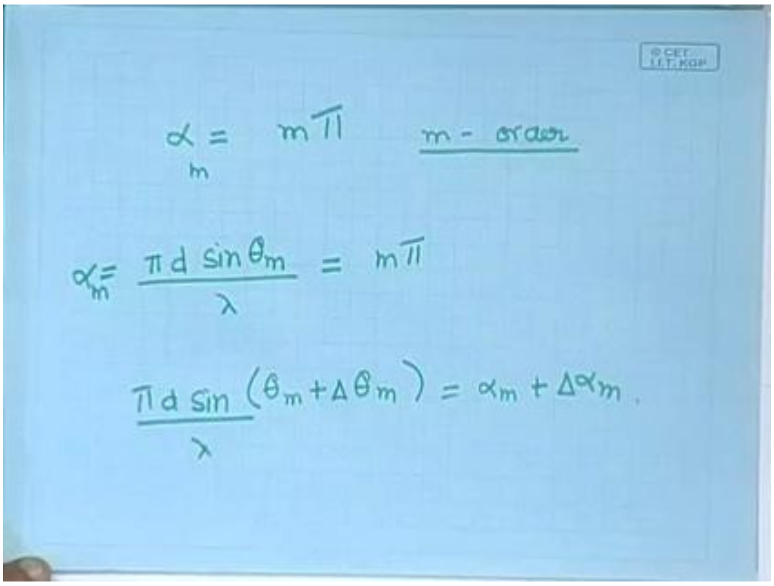

Alpha is equal to a multiply of pi so, let us say that it is equal to m pi m is called the order of the maximas so; m is the order of the primary maxima. So, we are looking at the mth order primary maxima alpha equal to 0 is the zeroth order maxima alpha equal to 1 sorry alpha is equal to pi is the first order maxima are pi to 2 pi is second order maxima.

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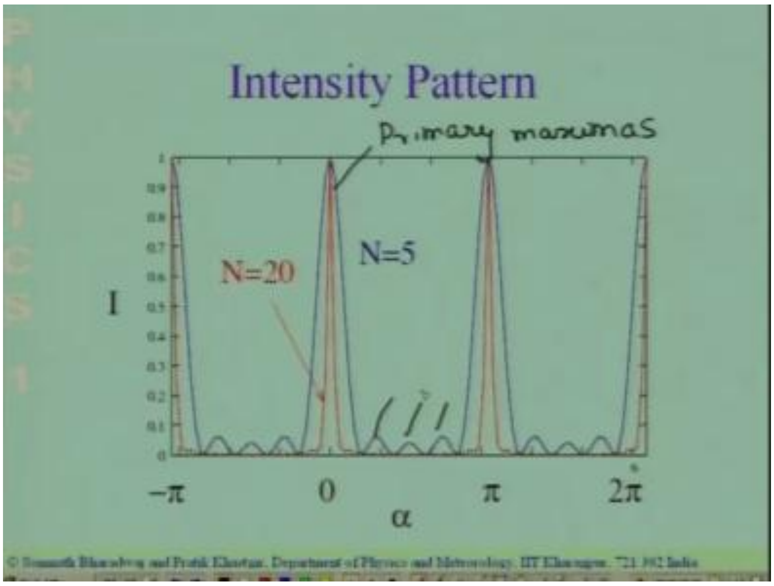

In this picture this is the zeroth order maxima first order maxima, second order maxima second order primary maxima. So, this is the value of alpha corresponding to the mth order primary maxima let me call it alpha m.

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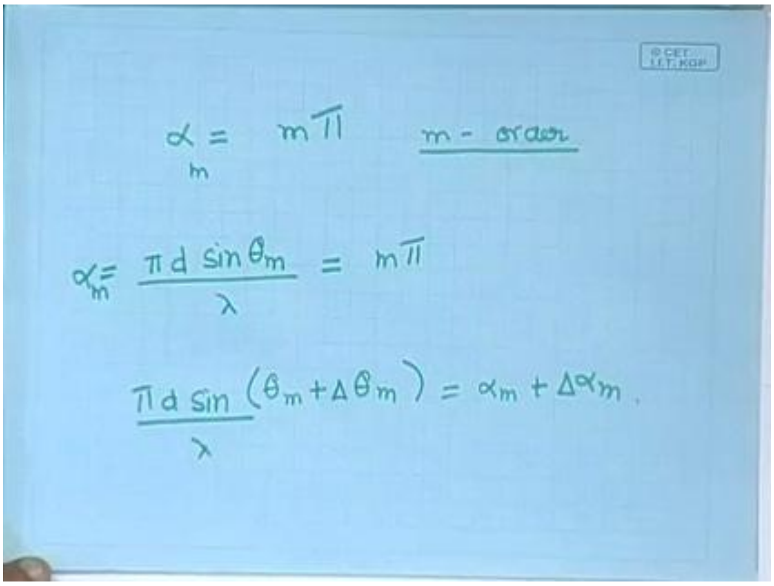

And alpha m alpha we know is pi d sin theta so, the corresponding angle would be theta m m equal to 0 corresponds to theta is equal to 0 let me just show you this right.

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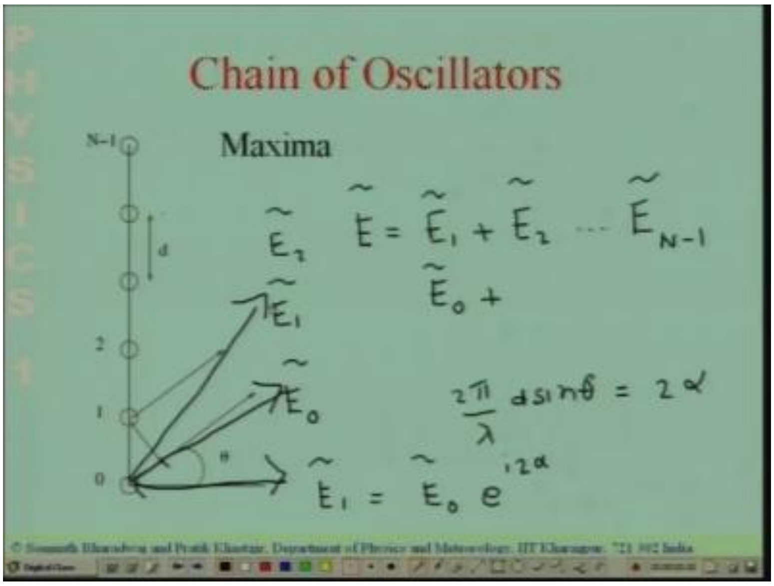

So, theta equal to 0 means that in this direction theta equal to 0 is this direction m equal to 1 theta would be larger. So, it could be somewhere possibly here m equal to 2 would be even larger so, higher order maximas occur at a larger angle.

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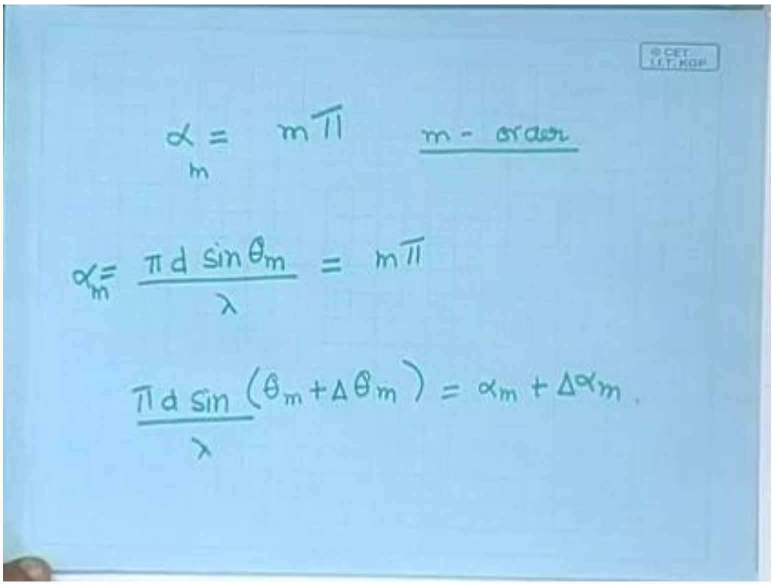

So, let us focus our attention on a particular order of the max primary maxima.

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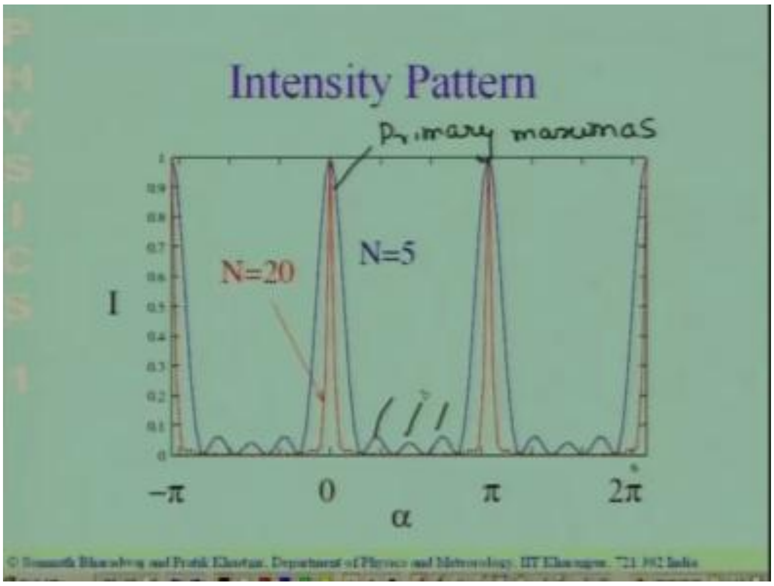

So, we have a particular order mth order primary maxima.

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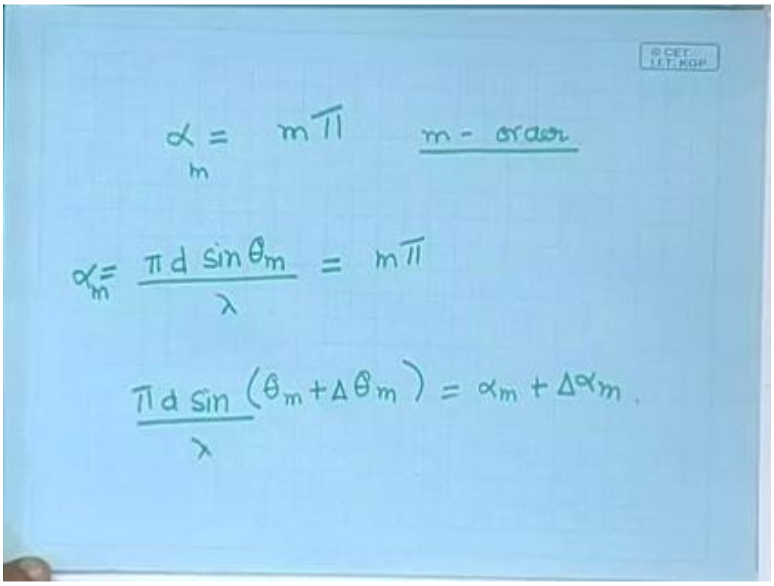

Now, has you increase theta away from that so, if you change theta away from theta m the value of alpha is also going to increase this is alpha. So, let us say that we have changed theta to theta m plus delta theta m. And alpha would have changed correspondingly pi d sin theta by lambda. This is going to be alpha m plus delta alpha m and a change in alpha away from m pi means at the intensity is going to fall.

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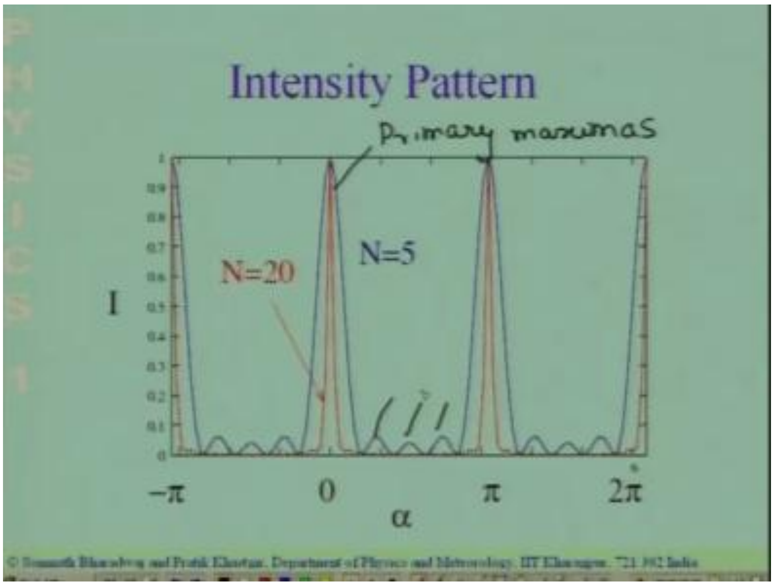

And we want to estimate the width of this, maxima so; we should with question we ask is how much should we change theta so, that the intensity falls to 0. So, we start from, a maxima the question is how much should we change theta away from the maxima so, that the intensity falls to 0.

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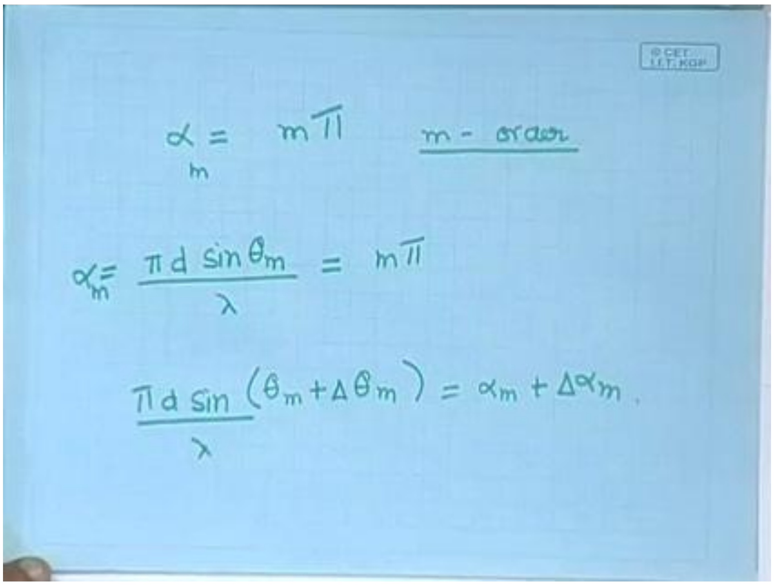

Or how much should we change alpha. So, that we intensity falls to 0 now at the maxima alpha is equal to m pi. So, let us look at the numerator of the expression for the intensity the numerator of the expression for the intensity.

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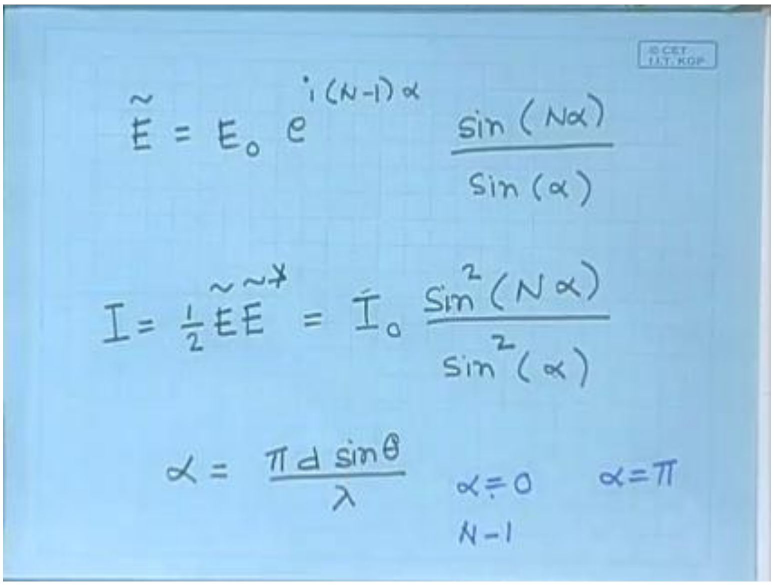

So, this is the expression for the intensity the numerator for the expression for the intensity is this let us look at just the numerator term.

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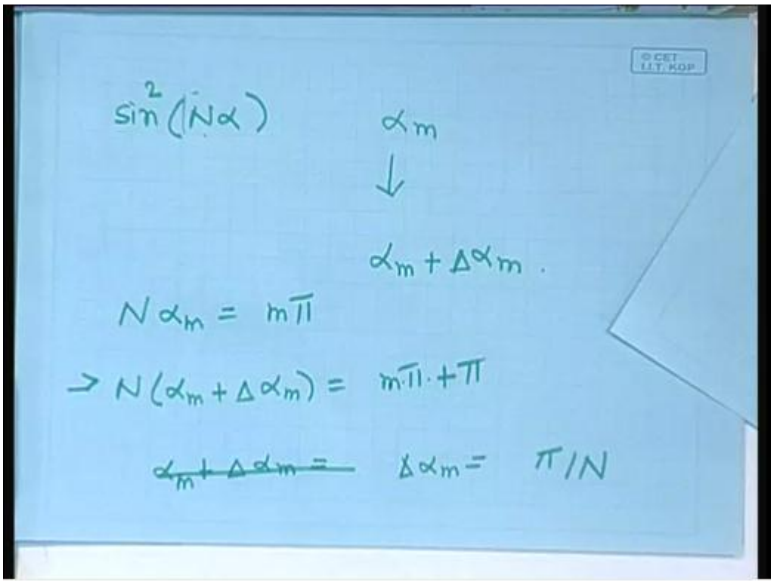

The numerator term is a sin square N alpha so, it will start with we are at alpha m where the numerator is 0. But, the denominator is also 0 and then we increase alpha to alpha m plus delta alpha m.

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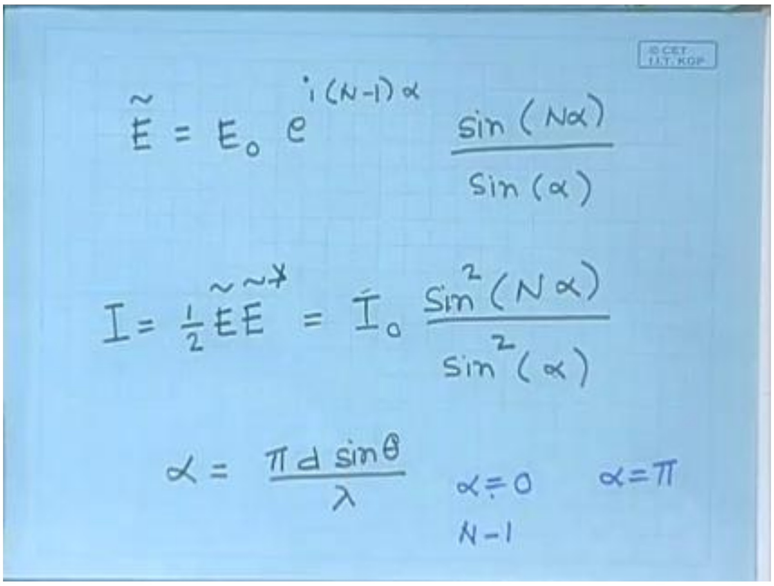

And ask the question what how much should the increase be so, that they intensity vanishes.

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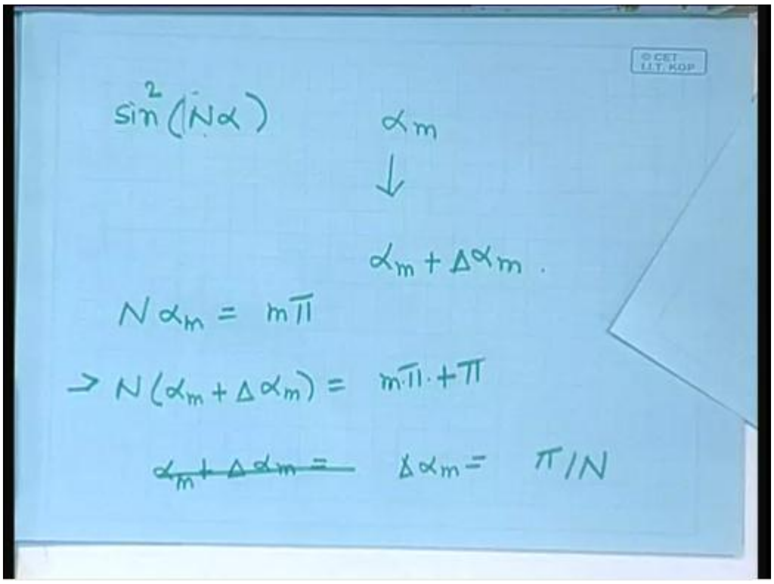

And it is very clear that the change so, we have start from N alpha m is equal to m pi and the change should be such that. That N alpha m plus delta alpha m should be m pi plus pi let we start from 1 0, but there the denominator is also 0. And then we increase alpha and ask the question when does the numerator becomes 0 again. That is point which will correspond to this when the intensity vanishes. So, this is the condition that the change in alpha should exactly occur the change in numerator this. This argument of the numerator should be exactly 1 should have increase by exactly pi. So, this basically tells us that delta alpha m or alpha plus delta alpha m should be equal to this is the condition. And alpha m we know is already equal to m pi so, this basically tells that delta alpha m should be equal to pi by N that which you can see straight away from here also that.

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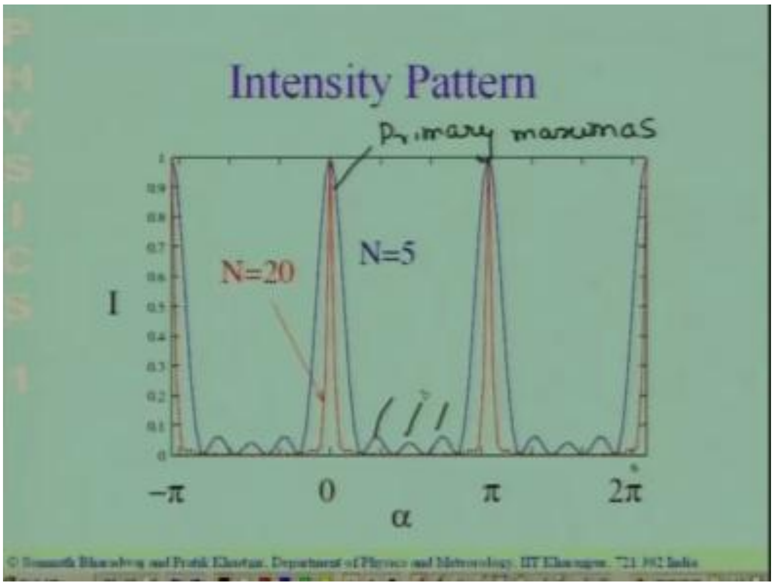

There are N minus 1 zeros in between so, delta alpha N should be pi this is pi divided by the number of zeros which is an number of intervals in between.

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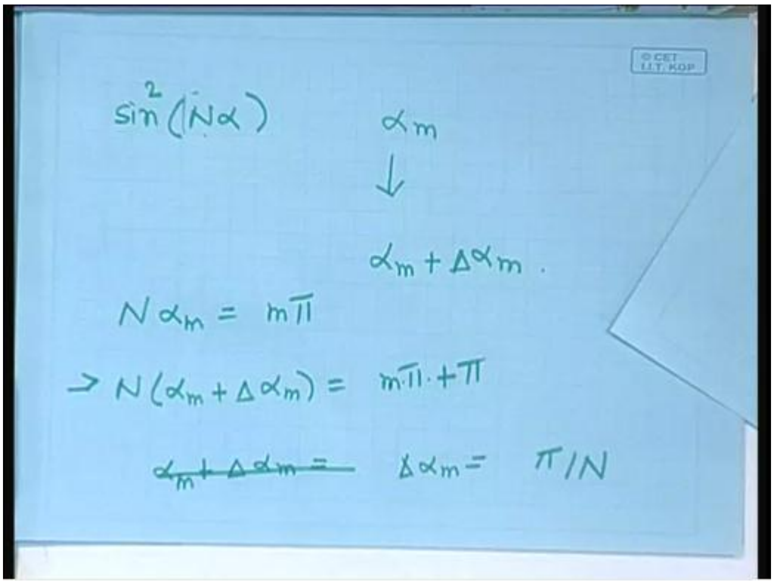

And how much is delta alpha m we can estimate that from here so, delta alpha m we can assume that delta theta change in theta is small. So, if the change in theta is small we can expand this and we can write it as.

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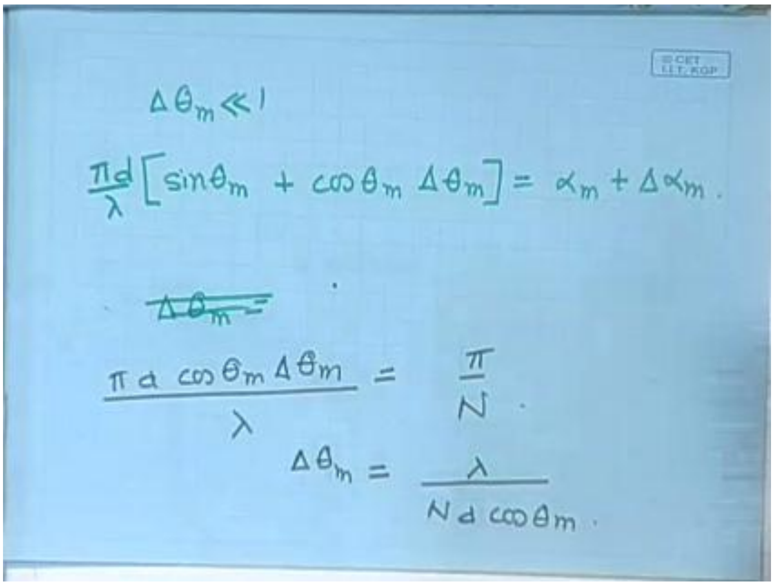

Pi d by lambda sin theta plus delta theta can be written as sin theta m cos delta theta we can take to be 1 assuming that delta theta m is much less than 1. So, this plus cos theta m into delta theta m this is equal to alpha m plus delta alpha m. And alpha m is exactly equal to this first term over here so, this can be drop. So, what we get is delta theta m is equal to delta alpha m then delta alpha m for the numerator to vanish we have worked out should be pi into pi divided by N. So, the condition that we get is that this delta delta theta into sorry I can write it out like this pi d cos theta m delta theta m by lambda is equal to pi by N which gives us. That delta theta m is equal to lambda by N d cos theta m okay so, what we have worked out let me just a remind you what we have worked out.

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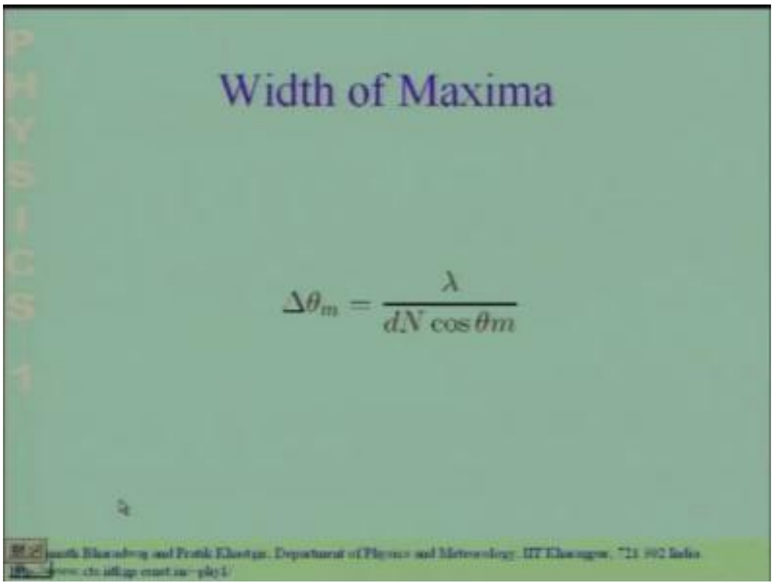

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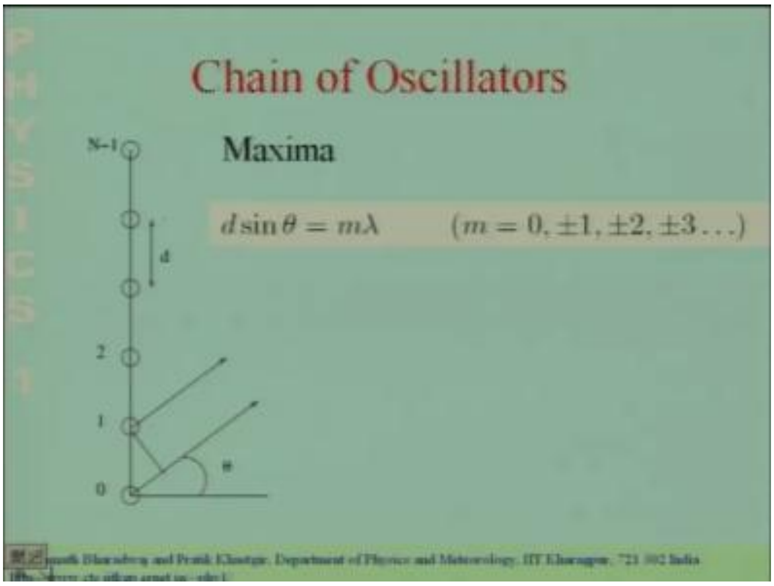

We have worked the width of the maxima so when I have the chain of oscillators the, what we see is that the intensity pattern will have maximas in different directions wherever this condition is satisfied. And has I change theta away from these directions where these, this condition is satisfied. The intensity is going to fall and the question. We ask is what the width of the intensity peak and we have calculated the width of a intensity peak.

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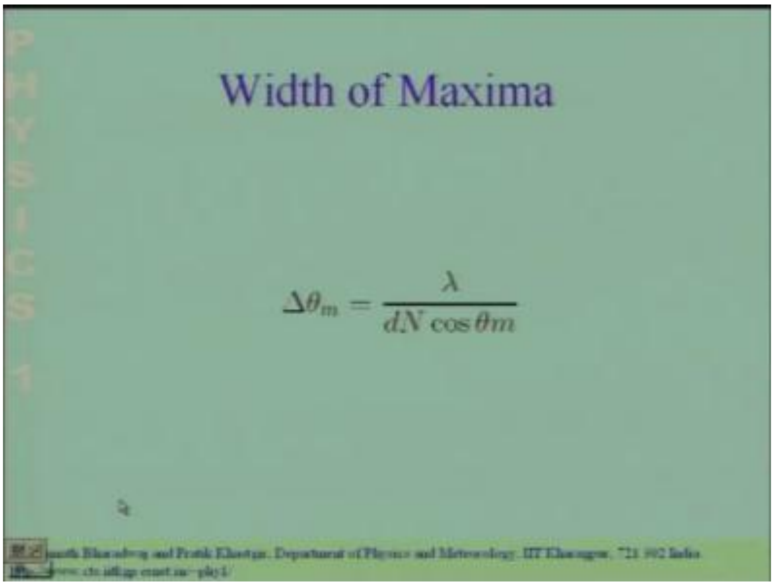

So, the width of the mth order principle maxima is lambda by d N cos theta m. D is the distance between oscillators. So, has I increase the distance between the oscillators the primary maximas become sharper. If I increase a number of oscillators the primary the width of the primary maximum becomes sharper it gets smaller the primary maxima itself gets sharper. And if I look at this for example, if I look at the zeroth order maxima.

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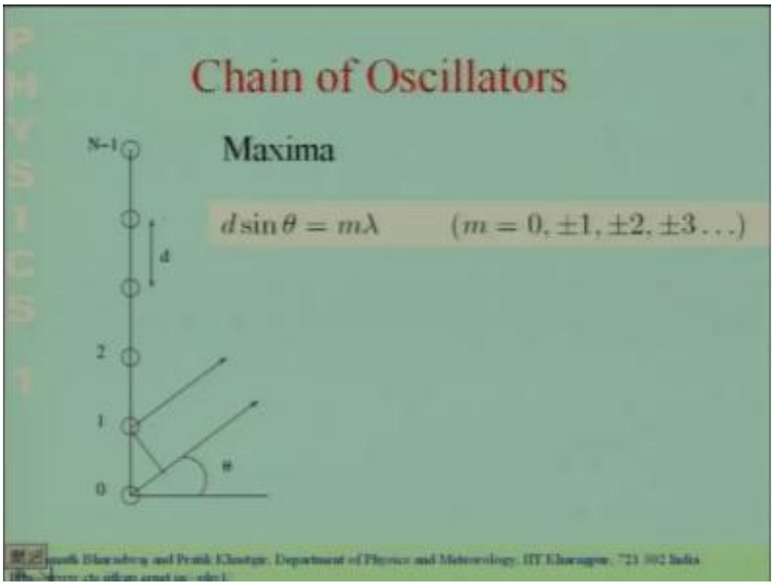

The zeroth order maxima occurs when theta is equal to 0 in this direction if i look at zeroth order maxima cos theta m is going be 1. If I look at the first order maxima first order maxima is going to occur at theta equal to sin inverse lambda by d

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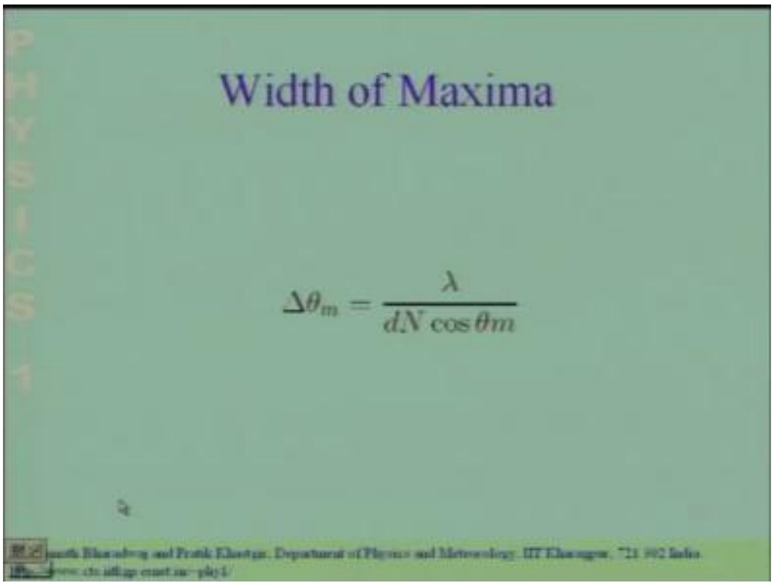

So, it is a larger value of theta so, cos theta is going to be smaller so, what we can say is that the zeroth order maxima is the sharpest. The first order maxima is somewhat broader the second order maxima is even more broader. So, what we have done till now is that we have considered the radiation that will be received by a distant of observer from a chain of linear chain of oscillators. Now, let me give you a problem and end the class in the next lecture be before give you a problem let me discuss one more thing. And then I shall give you the problem when we started our discursion of the linear chain of oscillators we had assumed that all the oscillators oscillating with exactly the same phase.

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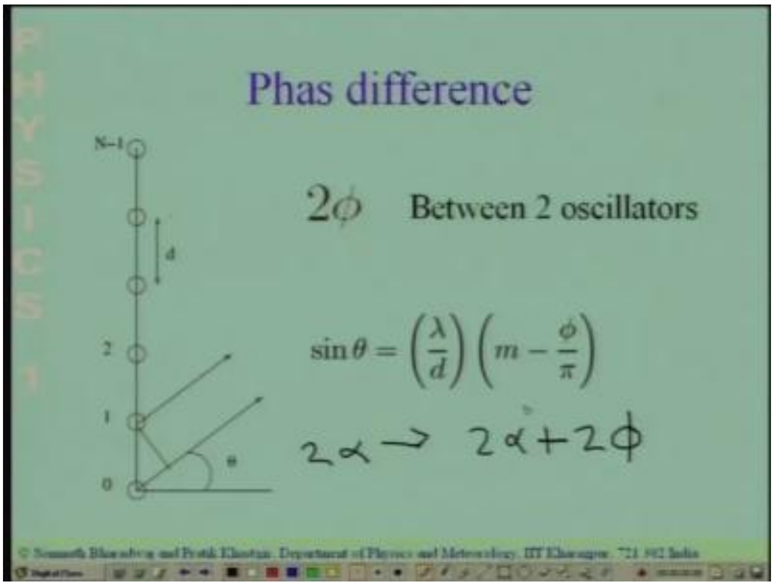

Now, we will consider a situation where the oscillators themselves are oscillating with a phase difference. What we will assume is that there is a phase difference of 2 phi between any 2 successive oscillators between oscillator 0 and oscillator 1. There is a phase difference of 2 phi between oscillator 1 and oscillator 2 there is a phase difference of 2 phi and so, forth. The question is how does the intensity pattern get modified? Because of this if you look at the way we have derived the intensity pattern the what we have done is we have added up the contribution from all of these oscillators with phase differences. The phase difference arising slowly due to the path difference and this gives rise to a phase difference of 2 alpha. So, the path difference introduce a extra phase differences of 2 alpha with occurs due to this path difference. Now, in this situation when the oscillators are themselves oscillating with a extra phase difference what we have to do is throughout all the expression you have to replace 2 alpha by 2 alpha plus 2 phi. That is all that has to be done in the in all the steps.

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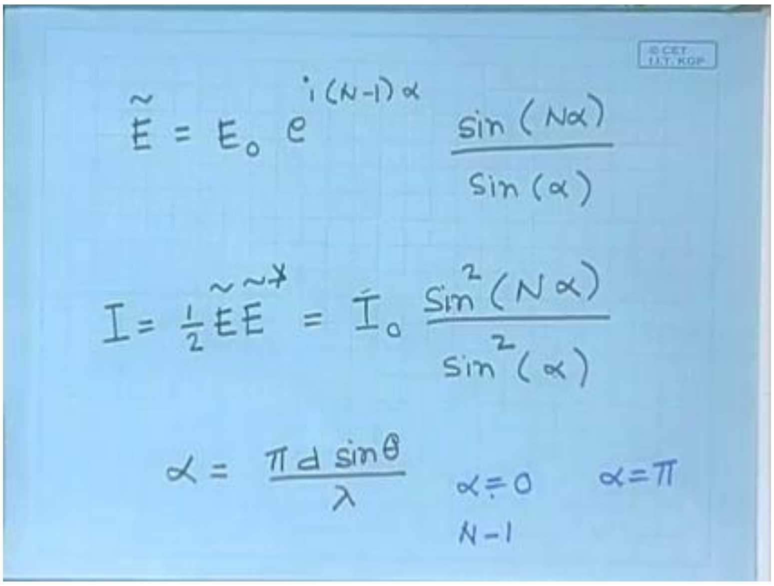

So, for example in the expression for the intensity what we have to do is you have replace alpha by alpha plus phi where phi is now the phase difference between the oscillators themselves.

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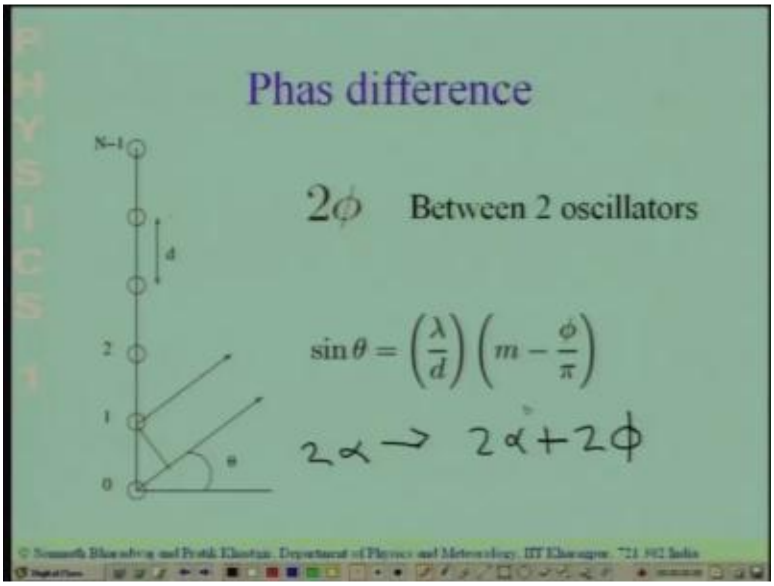

And if you do this replacement you will find that the condition for the maxima is what is given over here. So, we have now seen what how to also handle the situation where the oscillators are not oscillating with the same phase, but there is a phase difference of phi between any 2 2 phi between any 2 consecutive oscillator. Let me now give you a small problem and I will end the lecture over here. So, the problem is as follows.

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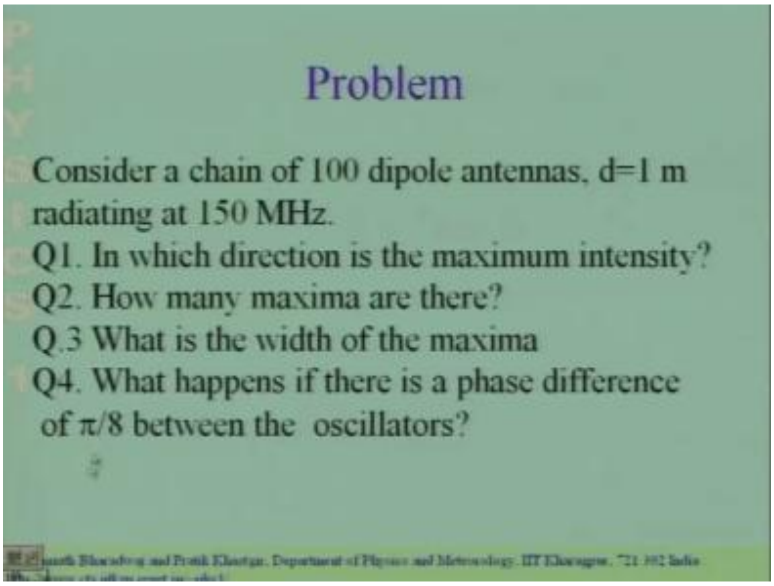

Consider a linear chain a chain of 100 dipole antennas. So, each oscillated over there is a dipole antenna. So, in the picture which I had in this picture.

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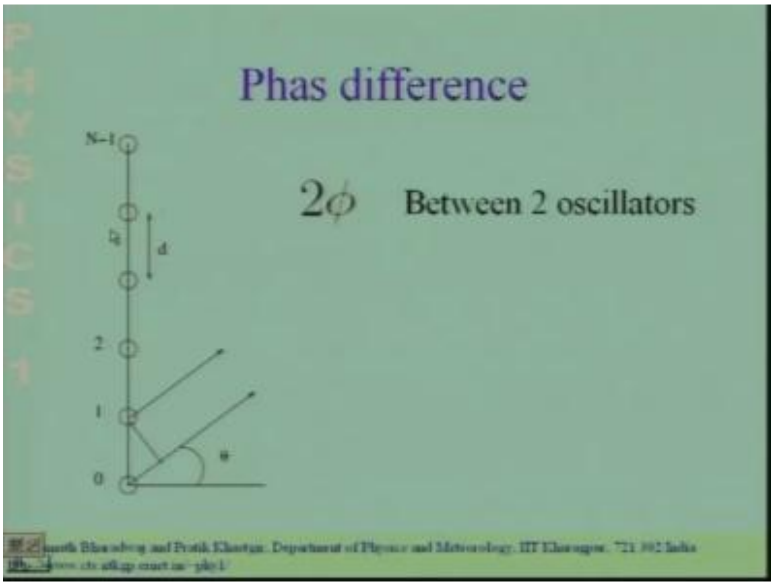

Each oscillator is a dipole antenna and we have 100 such oscillators.

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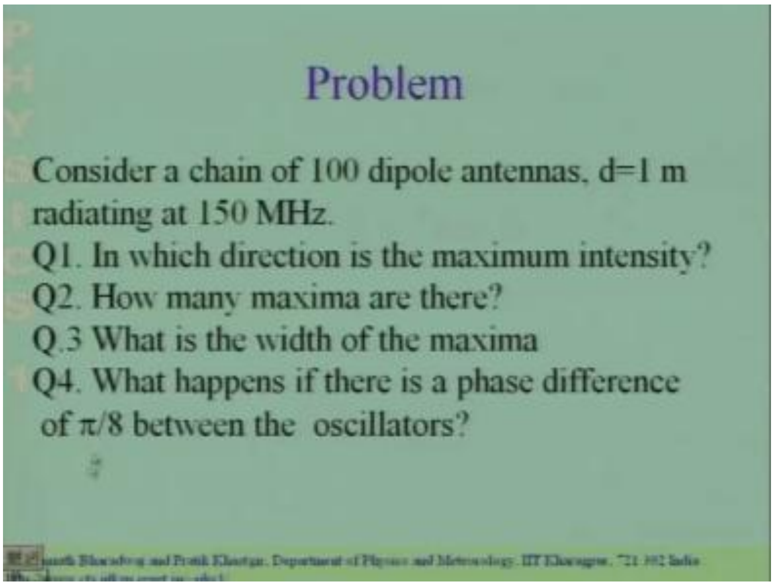

The distance d between 2 successive oscillators is 1 meter and all the oscillators are radiating at frequency of 150 mega hertz so, there. So, all the oscillators have been fed with the signal with the current oscillating current which is oscillating at 150 mega hertz same phase. So, all the oscillators dipole antenna are oscillating with the same phase. So, all the radiating with the same phase, the question is the first question is in which direction do we have the intensity maxima? So, in which direction is the maximum intensity? The second question is how many such maxima are there we have seen that there could be more than 1 maxima. So, how many such maxima are there? Then what is the width of the maxima? And finally, suppose we give what happens if there is a phase difference of pi by 8 between the oscillators.

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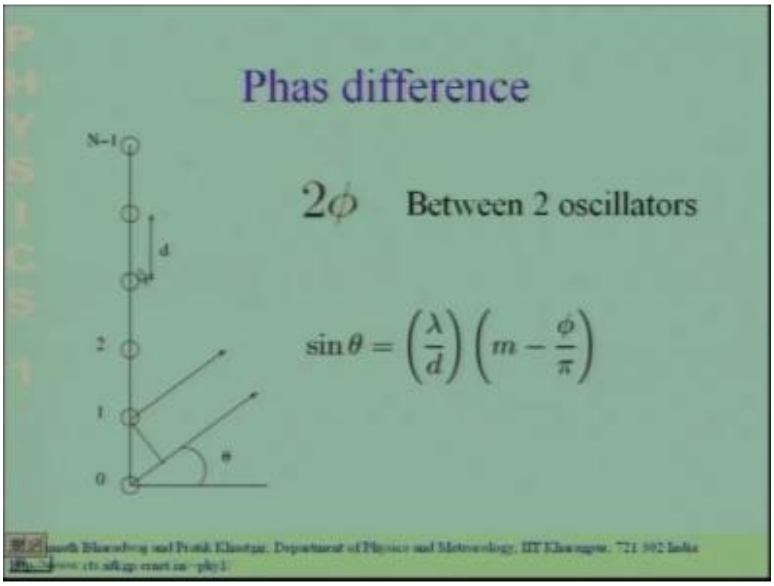

So, suppose I give A phase difference of pi by 8 between any 2 oscillator. So, pi by 8 between this and this pi by 8 between this and this. So, there is a phase difference between pi by 8 between each of these oscillators.

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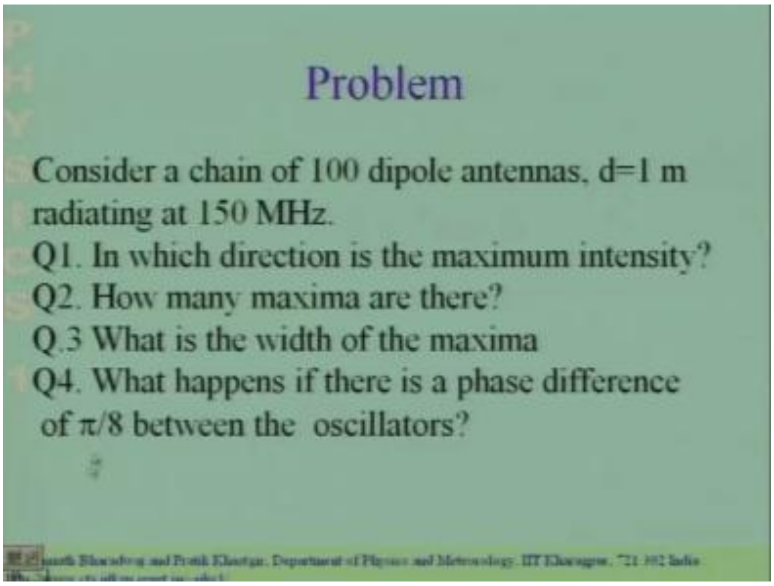

So, what happens to this, all of these questions if I introduce an extra phase of pi by 8 between any 2 oscillators? So, we shall continue our discussion from here we shall a take up the solution of these problems. And then discuss several implications of the various fformulas that we have derived today.