**Physics I : Oscillations and Waves**
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**Lecture - 21**
**Diffraction-II**

Good morning. In the last class, we had introduced diffraction interference refers to a situation where we have the superposition of 2 waves or few waves. And we use the term diffraction in a situation where we have the superposition of many waves. So, let me in today is class let me first recapitulate what we had been discussing in the last class. And I shall continue from there so, diffraction occurs one of the situation where diffraction occurs is when I have a wave.

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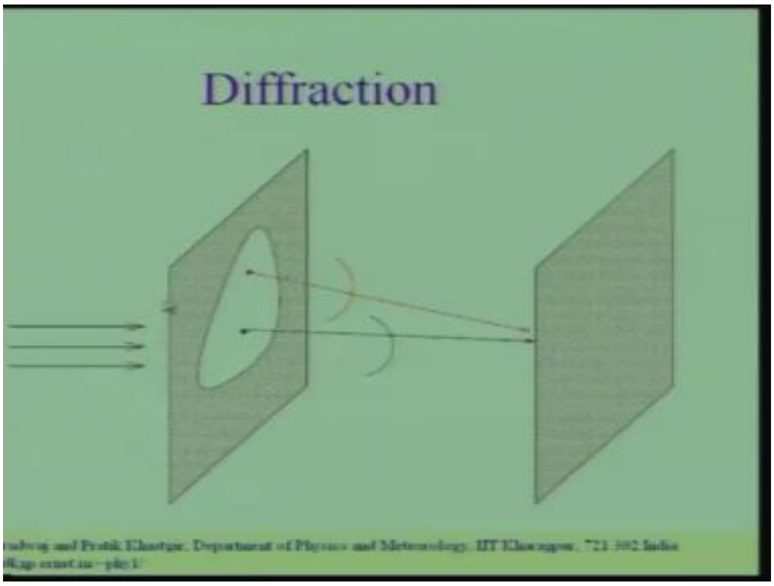


Incident on an aperture like the one shown over here so, in this situation we have a plane wave the plane wave is incident on an opaque screen in that opaque screen. We have an aperture so; the wave can pass through the opaque screen only through the aperture. And we are interested in the intensity pattern that this aperture would produce on a screen on other screen which is placed far away from the aperture. So, the question is how do, we handle such problems? And in the last class I had told you that we can apply the Huygen-Fresnel principle to handle such problems. So, what is the Huygens-Fresnel principle? Let me just first recapitulate this.

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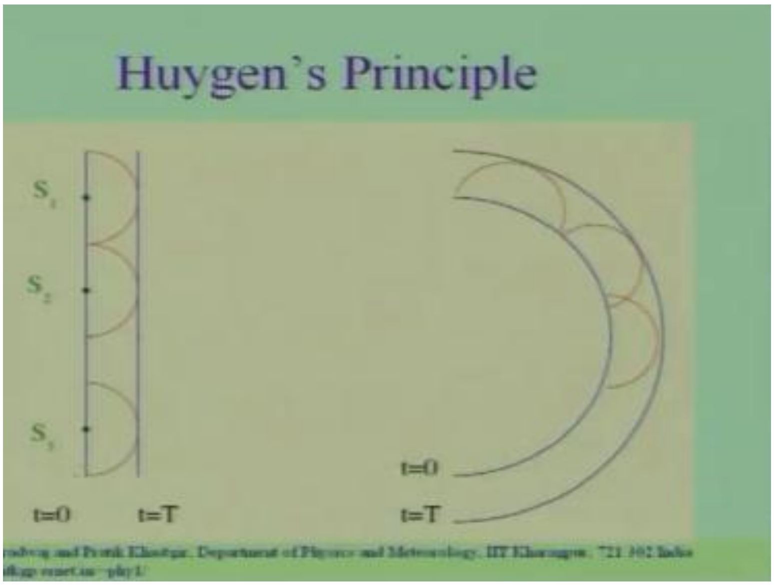


Huygen had introduced a principle to follow the evolution of waves in arbitrary media. So, Huygens's principle can be applied to follow the propagation of light for example in any in an arbitrary medium. So, this is the wavefront at a time t equal to 0 and we would like to calculate, determine what where is the wavefront at a time where is the same wavefront at a time t equal to capital T. And Huygens's principle tells us that each point on this wavefront at the time t equal to 0 acts like a source for a secondary wavelet. So, should draw a secondary wavelet originating from sources. And each point on the wavefront acts like a source so, at a later time capital T t equal to capital T.

The secondary wavelet emitted from this particular source looks like this this source looks like this. We are suppose to do it for all the sources every point of this wavefront. And then to find the wavefront at a later time we should considered the, envelop of all of these wavelets. So, plane wave evolves into another plane wave which is shifted a spherical wave evolves into another spherical wave of larger radius. And the Huygen's principle is not very very useful if you are dealing with the propagation in vacuum. But if you have if you wish to study the propagation of light in an antistrophic medium for example, where the speed of light is dependent on the direction in the, which the light is going the Huygens's principle is very useful.

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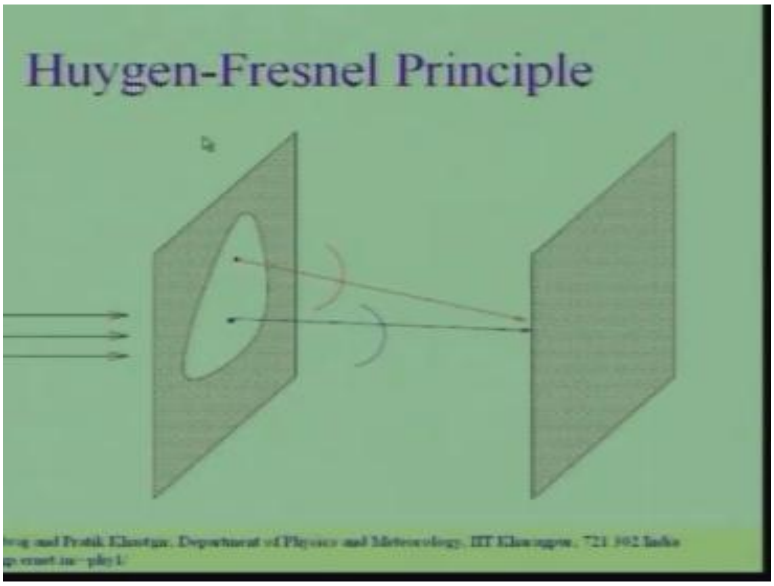


Now, this does not tell us anything about diffraction this was generalize by Fresnel. So, the Huygens-Fresnel principle is as follows when we have a wave incident on this aperture then every point on this aperture acts like a source for secondary wavelets. So, every point here acts like a source for secondary wavelets. It emits secondary wavelets which are spherical waves if I wish to calculate the resultant. The wave the intensity or the wave, produce by this aperture at some point here on the screen. We have to superpose the secondary wavelets emitted by all of these sources.

So, every point in this aperture acts like a source to find the resultant wave at a point here I have to superpose. The contribution from all of these points here I have shown you the contribution from only 2 points. So, this point emits a secondary wavelet and the secondary wavelet, propagate so, to this point over here. So, does this point this also act like a source this emits the secondary wavelet which again propagates here. And every point on this aperture does exactly the same thing. So, to find the resultant wave over here I have to add up the contributions from all the points on this aperture this is the Huygens-Fresnel principle.

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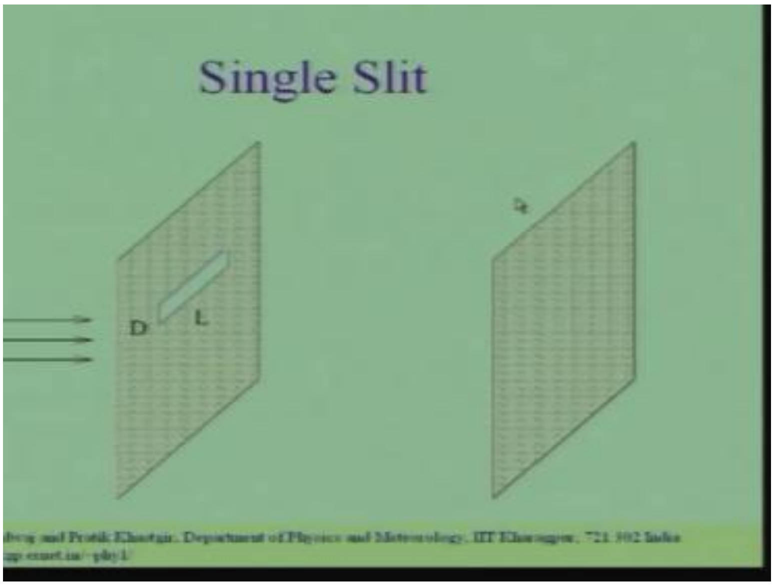


And then we had considered a particular problem the problem was to determine the diffraction pattern of a single slit. And the slit that we have considered we had considered was rectangular. So, we have a plane wave incident on a opaque screen in which there is a rectangular slit which has been cut out. The rectangular slit 1 dimension of the rectangular slit is D and the other dimension is L. And we had assume that D is much smaller than L. And we would like to calculate the intensity pattern produced by the slit on another screen located far away. Now, we had assumed that D is much smaller than L and if you take the limit of L going to infinity where this length is arbitrary large. This dimension is arbitrary large this essentially reduces to a 1 dimensional problem we have to be concerned with only the dimension over here so, through a section like this.

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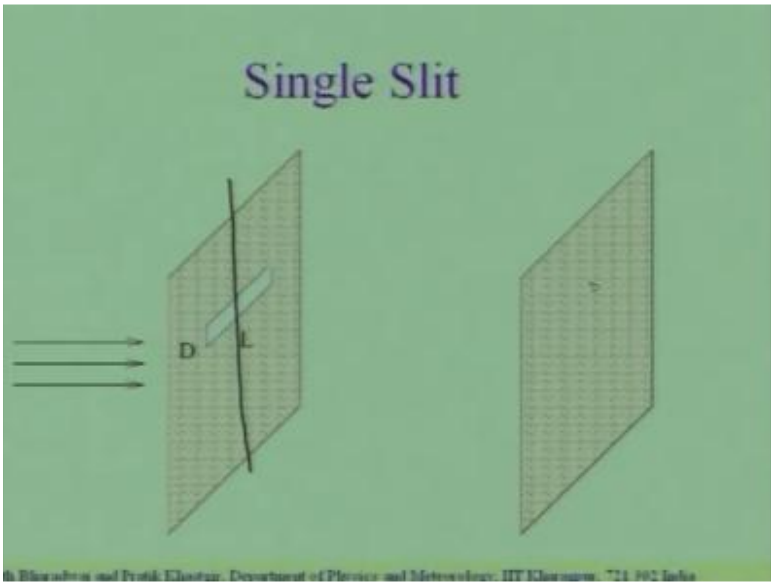


Another point which I should mention that in this particular situation we are dealing we have a plane wave which is incident on the slit. Now, every point on the slit acts like a secondary source like a source for a secondary wavelet. So, these spherical waves come out we are interested in the intensity on a screen which is sufficiency far away. So, that the secondary wavelets that come out from the sources in the slit may be to a good approximation treated as plane waves by the time they reach the screen. So, we have a plane wave coming into the screen into the slit. We have plane waves going out this situation where you can deal with plane waves. So, where you can the whole diffraction problem can be thought of in terms of only plane, waves, is refer to as.

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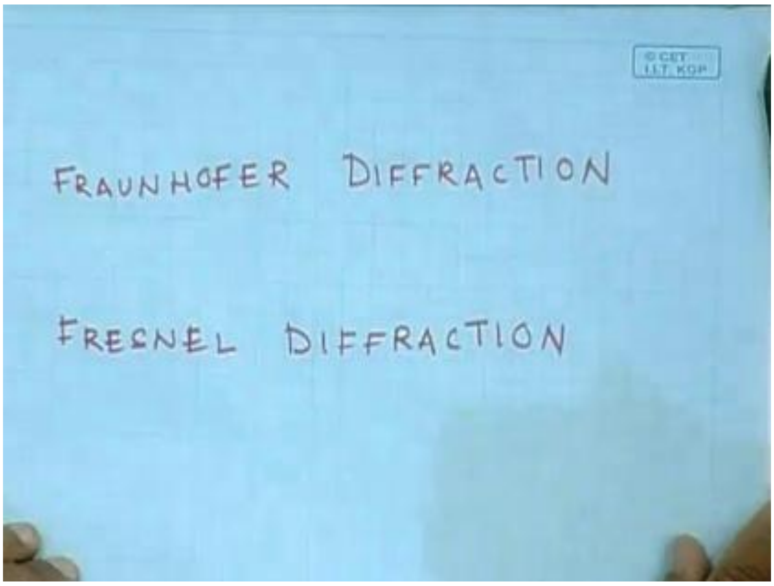


Fraunhofer diffraction and we shall we restricting our analysis to only this situation so, let me just remind you once more. What we mean by Fraunhofer diffraction in this refers to a situation where the incident wave is a plane wave. And the wave that goes out is a plane wave this is a suitable description provided. The source which illuminates the slit is sufficiently far away and the screen where we want to study. The intensity pattern is also sufficiently far away from the slit in case any of these, assumption. They are not valid if the screen located near or if the source is near then you have to take into account. The spherical nature of the wave and that is that situation is refer to as Fresnel diffraction we shall not be considering this in our lectures. It is a little more complication and we shall not be dealing with this. So, if you have a situation where the spherical nature of the wave is important that situation is referred to us Fresnel diffraction. We shall be considering only the Fraunhofer diffraction and we have the made the assumption here that.

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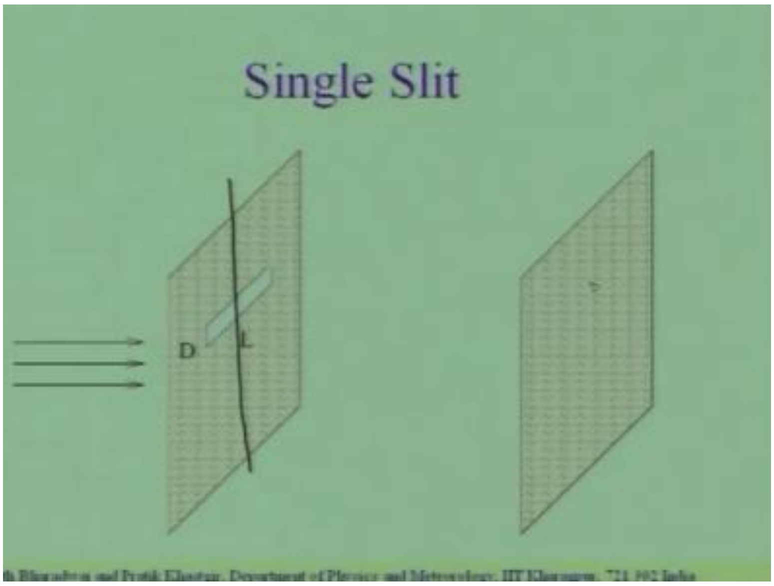


In the rectangular slit one of the dimensions D is much smaller than L so, we will take the limit where L becomes the infinitely large. And the whole problem reduces to a 1 dimensional problem where it only. This direction which is important the 1 along the smaller dimension of the slit so, the whole problem now reduces to a 1 dimensional problem which I have shown over here.

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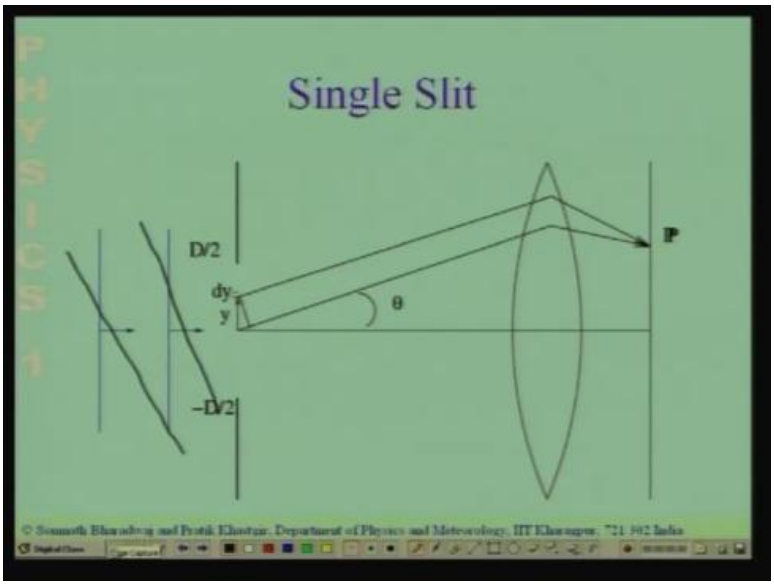


So, this is the smaller dimension of the slit D this is the plane wave. That is incident on this slit each point on the slit in the slit acts like a secondary source that is what the

Huygens's Fresnel principle tells us. Now, the point to note here is that the wave which is incident on the slit is such that. The wavefronts are parallel to the slit has the consequence of this all the sources for the secondary wavelet emit. The secondary wavelet at exactly the same phase we should note that this would not be true in the situation where the wave the incident wave was at an angle. And the wave fronts were like this in such, situations. The secondary sources in the slit would not be emitting the secondary wave the sources on this slit would not emitting. The secondary wavelets with exactly the same phase there would be a phase difference. This would be ahead this part would lack in case the wavefronts were at an angle.

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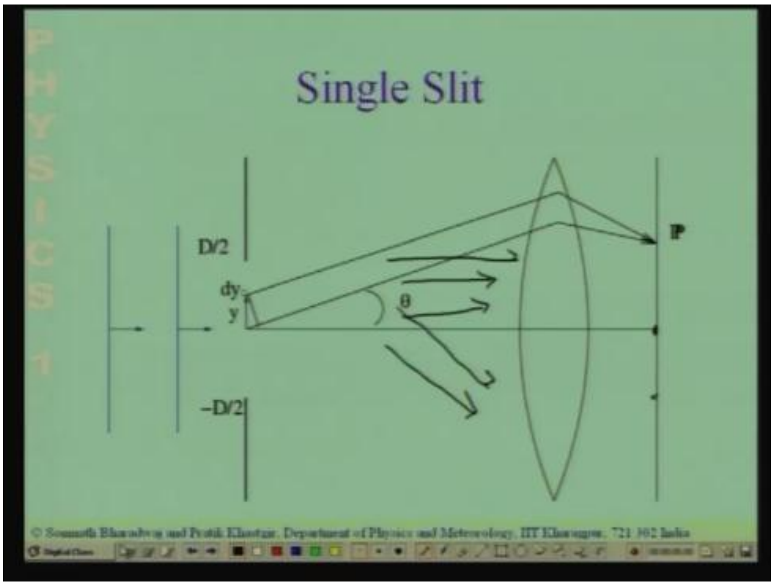


But we are considering the situation where the wave front is parallel to the slit so, all of these points are oscillating with the same phase. The wave over here is oscillating with the same phase and each of these point acts like a source which emits secondary wavelets at the same phase. Now, we have shown the screen over here you could either place the screen very very far away. Or what you could do is you could put a lens in front of the screen put the screen at a finite distance. The, what the lens does is that all the waves incident at an angle theta the wave the lens focuses brings all of these waves to a single point. So, if I had so, for the angle theta all the waves' incident on this lens at an angle theta. They are brought to same point P over here all the waves emitted from the slit let us say like this these are all focused to a point over here. And all waves which are emitted like this from the slit would be focus to another point some over here.

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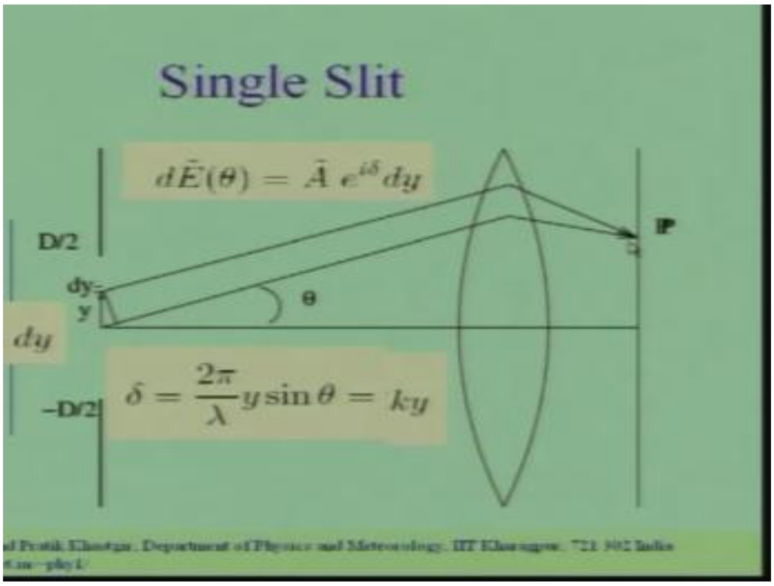


So, what the lens does is it brings together all the waves' incident on it that are at a particular angle theta it brings them together to a point. And to find the resultant wave at this point we have to add up the contributions from all the sources located at each point along the slit. So, we add up the confusion from all of these points all of these points acts like a source. So, let us first look at point a small element dy located near the origin so, we shall considered the contribution from the small element dy. So, this length line element we shall call dy and it is located at the origin. So, we will use the y axis we shall align the y axis along along with the slit so, the slit.

So, we have the placed the y axis in the direction of the slit .And the origin of the y axis is located at the center of the slit. Let us ask the question what is the contribution to the wave over here from this from the sources within the small line element dy located at the center of the slit at the origin. And we will call this d E theta, because we are only interested in the waves emitted at an angle theta that is what the lens brings to this, points. So, d theta is proportional to this line element, because the number sources increases if I increase this.

And there will be some over all amplitude A tilde so, the contribution from this small line element dy to the waves reaching here is d E is equal to A tilde dy. Now, let us ask the same question for a line element for the for line element dy which is displaced from the center by a amount y. So, how much is the contribution from a line element dy which is located distance y from the center. And what you will realize is that the contribution from this line element dy which is displaced from the center it leaves source at exactly. The same phase as the contribution from the line element dy at origin, but while the time is reaches the point P it picks up of a phase difference. There is a phase difference because of the path difference between the path from the center. And the path from this point you see there a path difference between these 2 waves to this point. And the path difference is this much so, this path difference will cause a phase difference. So, the contribution from the line element dy a distance y displaced by a distance y can be written as the same thing as the contribution from the origin.

But with the extra phase difference and the extra phase difference arises due to the path difference. So, the phase difference delta arises due to the path difference. And it is 2 pi the lambda into the path difference which is why sin theta where theta is the angle over here and we will write this as k into y. So, the point to note over here is that each point on the slit acts like a secondary as a source for a secondary wavelet. All of these secondary wavelets emitted at angle theta arrive at the point P at the same point p with different phases. And the phase difference depends on the displacement from the origin from the center of the slit so, to.

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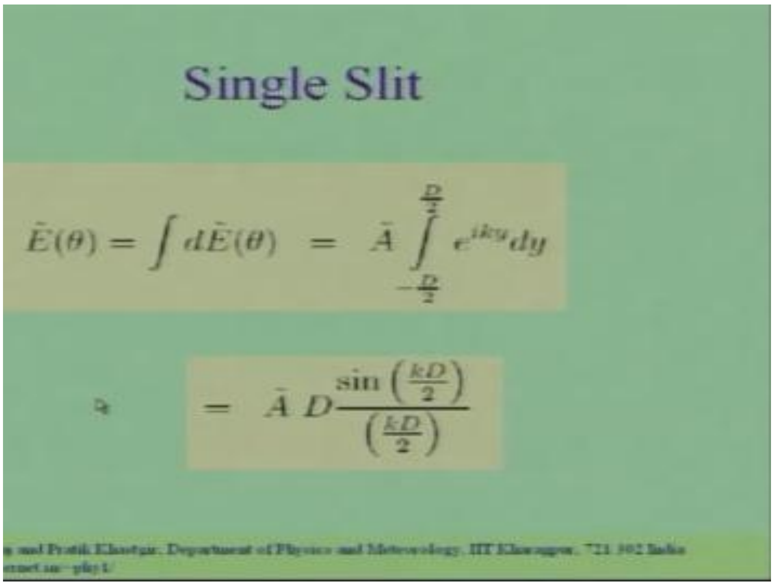

Calculate the resultant you have to add up the contribution from all of the sources this is what we have done in the last class. We had added up the contribution so, this superposition of the contributions from all of these sources.

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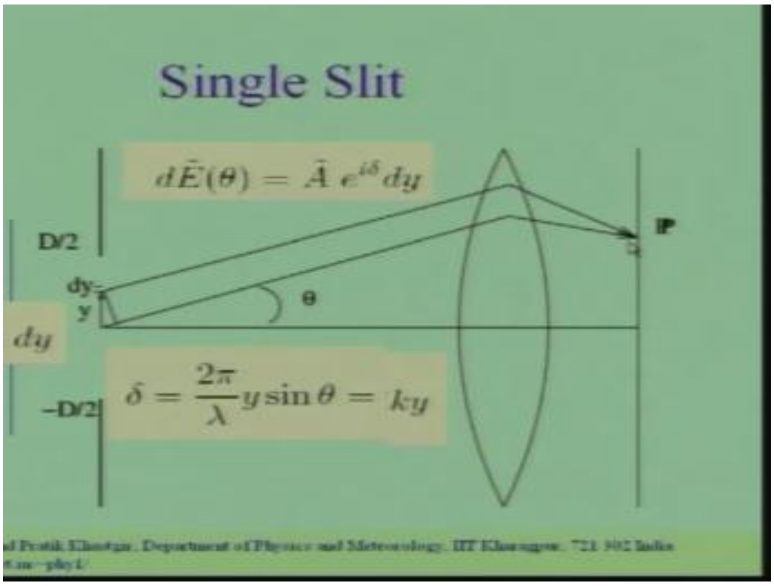


Located at different points so, there are source all along the slit you have to add up these contributions. And this is an infinite number of sources, because this is continues variable y and this continues some.

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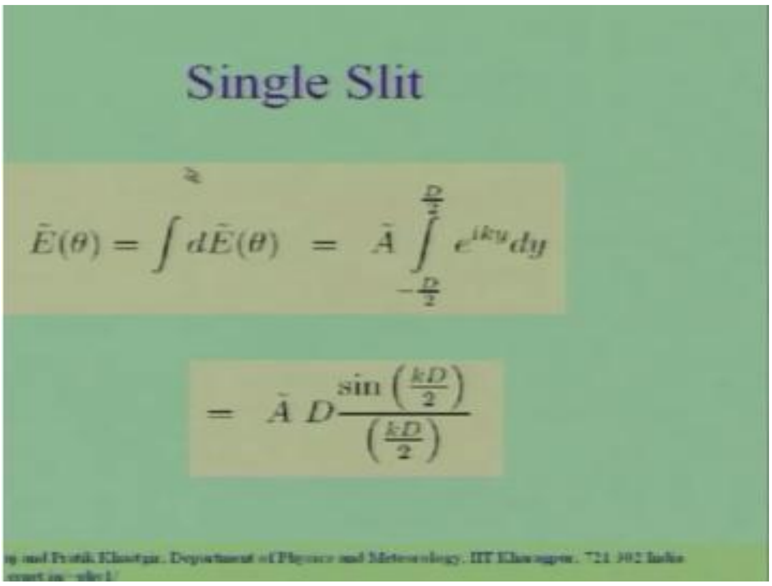

Is represented by an integral so, you have to add up the contribution from all of the sources. And this integral can be written as A tilde e to the power of iky dy from will and the integral limits are minus D by 2 to plus D by 2. We had done this integral in the last lecture and it gave us A tilde into D into sin k D by 2 divided by k D by 2 so, we had calculated this to get an idea of.

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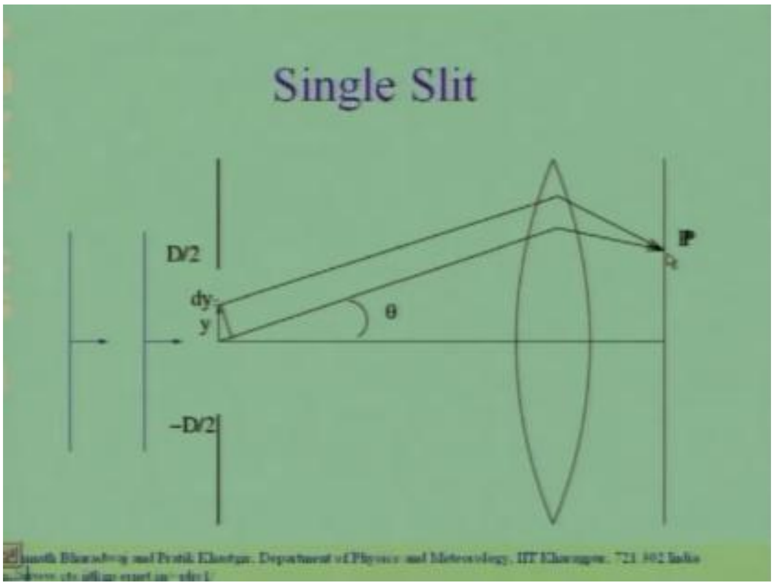


Get a physical picture of what this this all about we are essentially adding up the contributions. The waves emitted from all from sources located all, the wave from here to here. And they are all emitted with the same phase, but by the time reach here they arrive with different phases. Let us ask the question in which direction theta do we expect to have the maximum intensity without doing the mathematics. Now, notice that if we look at theta equal to 0 all the waves would arrive at the exactly. The same phase there would be no phase difference so; all the sources would contribute at exactly the same phase.

And they would all add up all the waves the waves some all of the sources would add up constructively. And we expect to get the maximum intensity let us now ask the question where do we expect to get the minimum intensity. The minimum intensity you can see will occur when the phase difference between the wave coming from the center. And the wave coming from the tip is pi the phase between the center and 1 tip is pi so, the phase between the center and the other tip is also going to be pi. So, you run from the phase changes from minus pi to plus pi. So, each of these sources contribute the same amplitude, but with the different phase and the phase runs from minus pi to plus pi.

Now, we all know that if you integrate e to the power I delta with running from minus pi to plus pi e to the power I delta is cos delta. And sin delta with a factor of i in front of the sin and if I integrate this from minus pi to plus pi. Then we get 0 so, the intensity the intensity is going to be minimum. When the phase difference between this tip and this tip is exactly 2 pi or the phase difference between the center. And one of the tips is pi the contribution from all of the sources. Now, cancel out the, a cancel out at some other point where this phase condition is satisfied let us now go back to the exercise that we are doing so.

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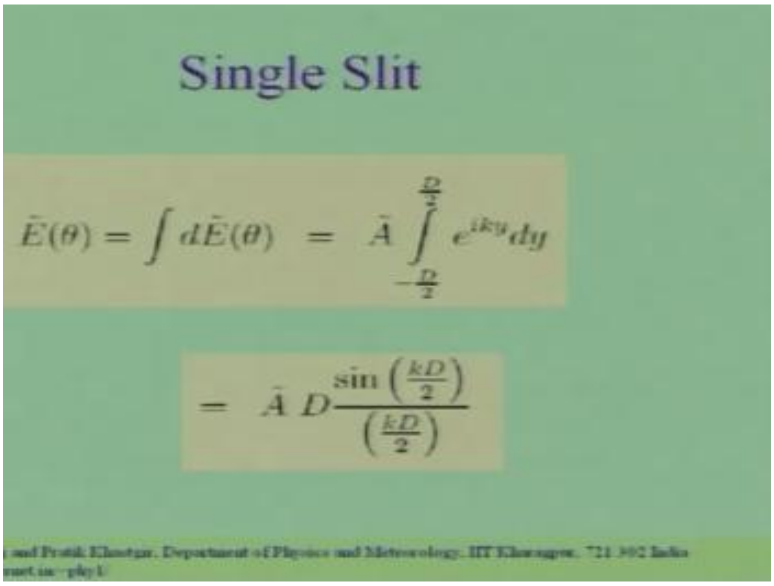


We had calculated the mathematical expression for the superposition of waves emitted from the different points on the source on the slit.

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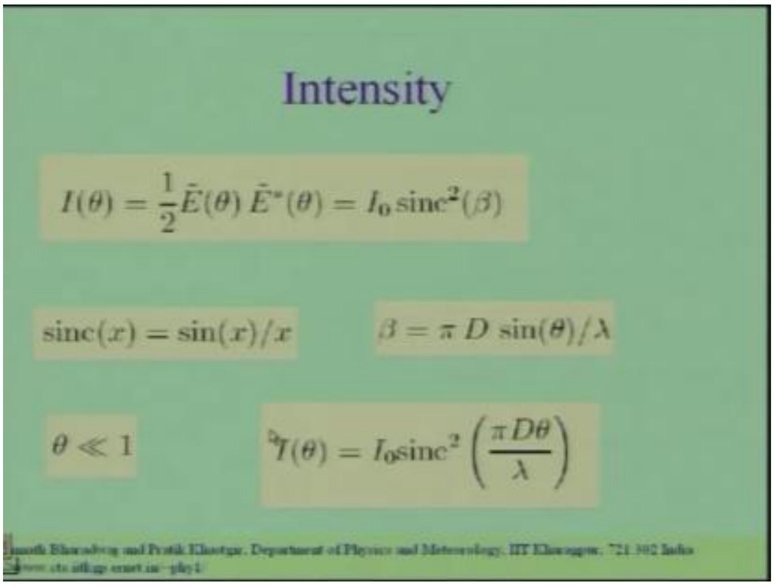


And now we use this calculate the intensity the intensity is half E into E star and we have I naught we had done. This calculation in the last class where the intensity is I naught some constant into sinc square beta where the sinc sinc function which we have encountered. The earlier is sin by x and beta is pi D sin theta by lambda so, this gives us the intensity pattern on the screen in the situation where theta is much smaller. Than 1 the intensity pattern is little simplified in the situation where theta. We should remember that theta is radians so, when theta is much smaller. Than 1 beta is approximately beta D theta by lambda and the expression for the intensity is I naught sinc square beta D theta by lambda instead of sin theta you can now write theta. So, this gives the intensity pattern on the screen in the situation with theta is small. We shall be considering this situation in the rest of this lecture so, let me now plot the intensity pattern.

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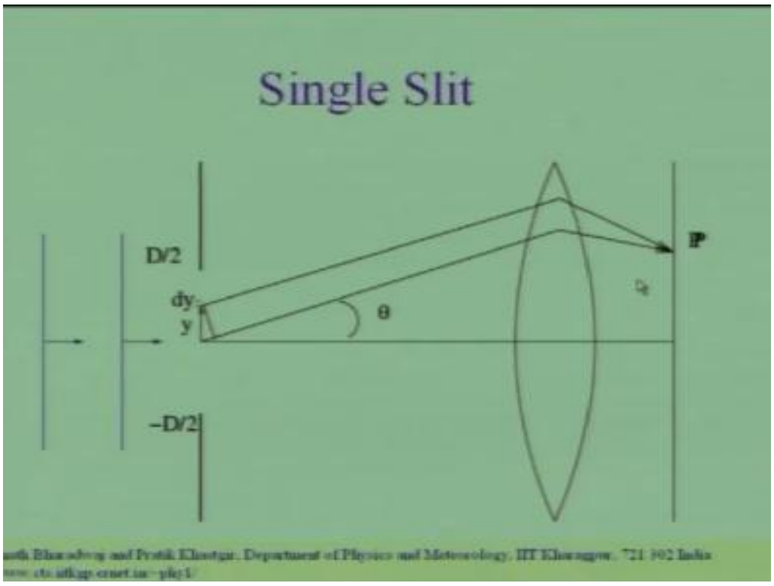


As a function of theta let before the let me remind you what we are doing so, we have a plane wave incident on the slit. And each point here emits in different emits secondary wavelets if I wish to calculate emit secondary wavelets. And this is incident on the lens what the lens does is all the waves emitted at a particular angle theta it focuses to a single point. So, the intensity corresponding to a certain value of theta you will get a particular point over here theta equal to 0 would correspond to the center as you increase theta the point would move up. And you would get that intensity pattern which you which I had just which I had just worked out on the screen over there.

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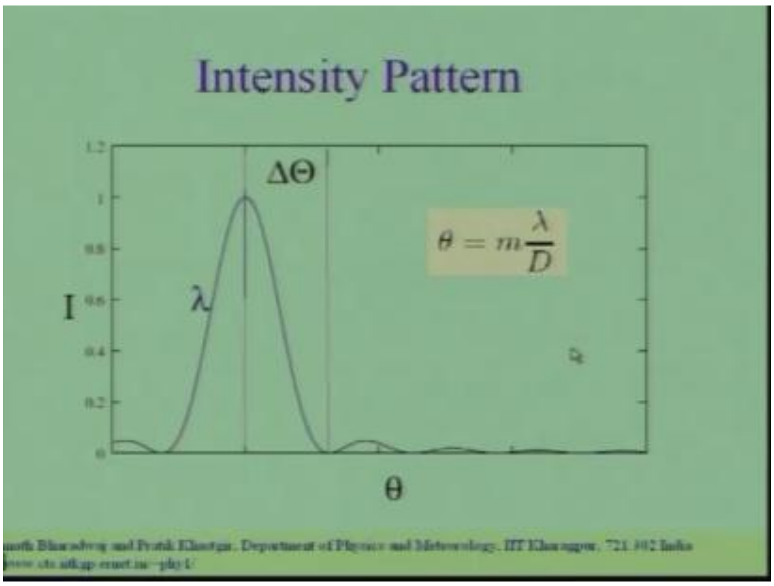

So, the intensity pattern, look like this so, before discussing let me show you.

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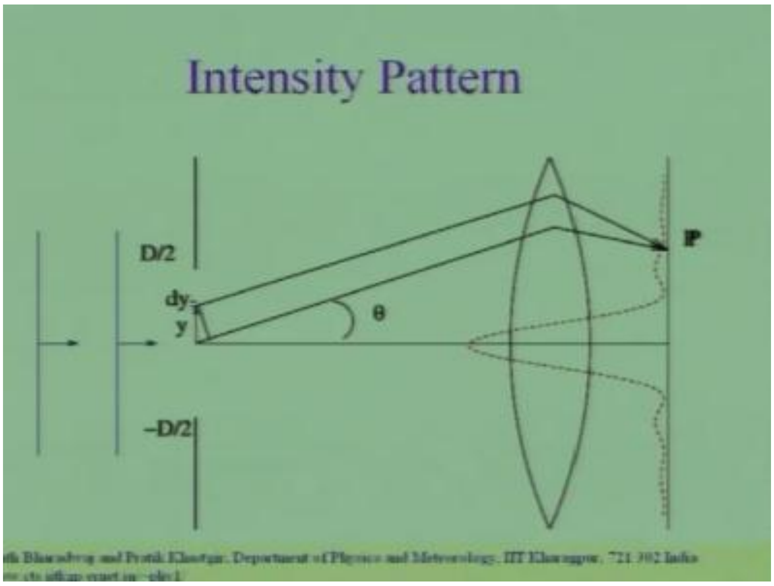


What the intensity pattern actually looks like so, this shows you the intensity pattern the red dash curve over here shows you. The intensity of light that you expect to receive on the screen theta equals to 0 corresponds to the center. And that is where you have the maximum of the sink function.

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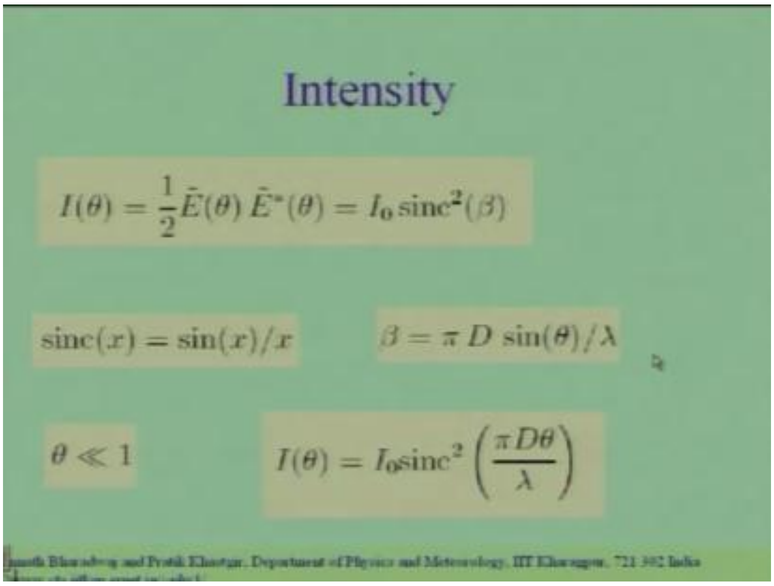


The intensity pattern is given by the sinc function over here you have I naught sinc square pi D theta by lambda. So, as theta when theta is equal to 0 the function sin x by x has a value 1 so, the intensity I naught. So, this constant I naught is the value of the intensity at the center as you move away from the center the sinc function falls it oscillates and falls.

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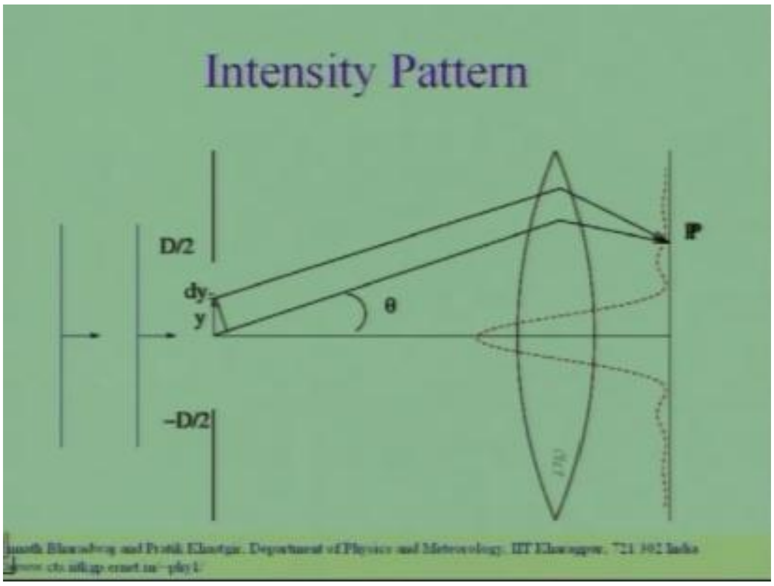


So, as you move away from the center the intensity falls and then the intensity goes to 0 and it rises again and then again it goes to 0 and you have these oscillations. So, what you will see on the screen is you will see a bright spot at the center. Then beyond this bright spot it will be dark and then there will be another bright spot at some larger value of theta or a larger distance from the center. And then you will have a, another bright spot somewhere over here in between it will be dark. So, you have these bright spots separated by dark regions on the screen over here as you can see. So, this line this the height of this curve shows you the intensity now, to put things in perspective.

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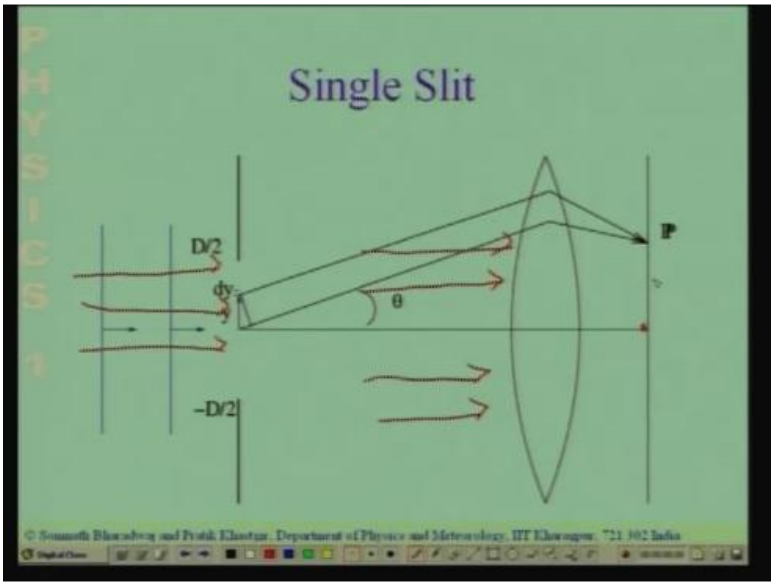


Let us just get a feel for what you would expect in case you were doing a geometrical optics. So, if you were doing a geometrical optics let us just consider a situation where you are doing geometrical optics in geometrical optics you would think of this a. So, if you were interpret this in terms of geometrical optics you would think of this in terms of rays coming like you. Would think of this, waves in terms of rays coming like this all of the rays are in the same direction I had made a mistake in drawing it. But all of the rays are in the same direction and if you have rays in the same direction incident on the lens over here.

And the direction is parallel to the axis of the lens what the lens would do is it would focus all of these rays to a single point at the origin. So, if you were to interpret this situation where you have light falling on a slit using geometrical optics what you would expect to see on the screen is a bright spot at the center. And if all the waves if all the rays in the incident wave had only a single wave vector. Then all the rays incident would be in the same direction and they would be focus to the same point. So, in geometrical optics you would get a point a very shot point which has no size. It would be very bright sharp point located at the center of the screen you do not expect to get any size for it.

So, in a geometrical optics interpretation of this experiment if you go to interpret this whole thing and predict what you expect to see using geometrical optics you would predict. That you expect to see a very bright spot at the center of the screen and nothing else why a very bright spot let me again go through it. This wave would be interpreted in terms of rays all the rays are parallel to the axis of the lens. And they would all be focus to the same point at the center of the screen. But in reality this is not true light is a wave so, in reality.

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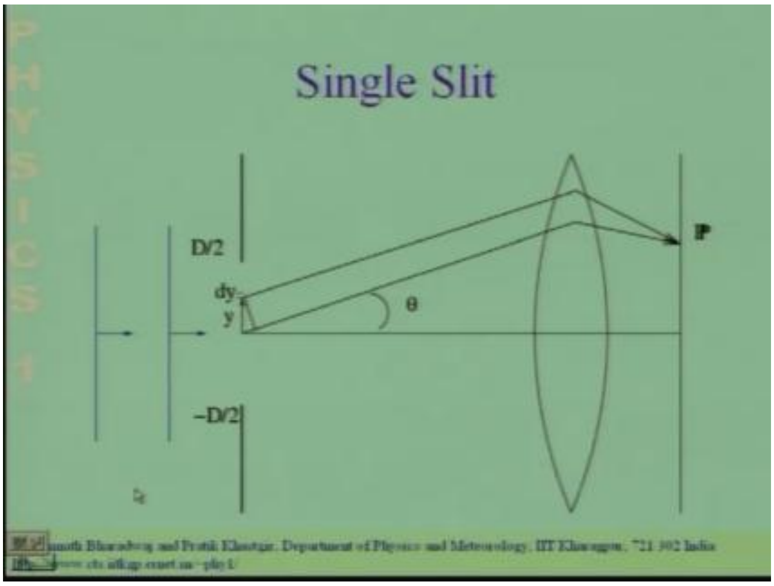


You do not get a single bright spot at the center in reality what you get is a extended bright spot at the center. And there are other spots also located above this and below this bright spot you have this fringe pattern. So, let us analyze this fringe pattern so, this shows you.

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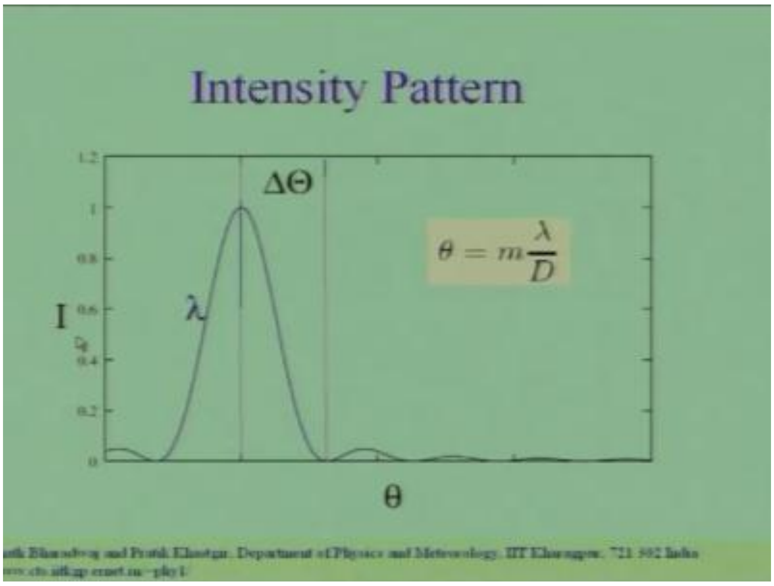


The intensity as a function of theta this is the same thing.

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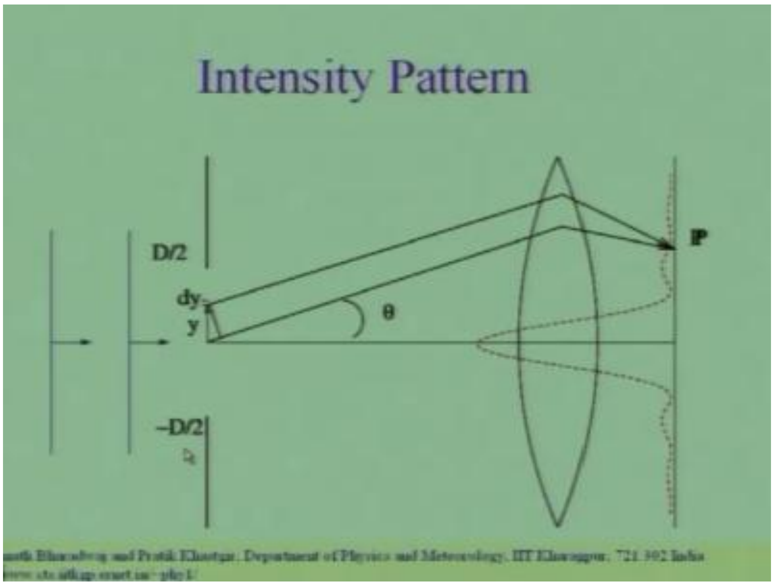


As this I have now rotated this and plotted this along the x axis theta varying theta means moving in this direction.

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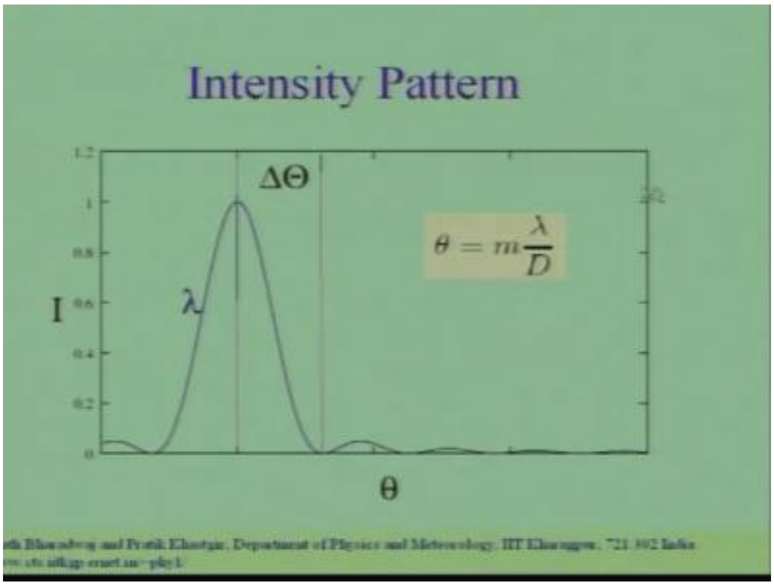


That is what is shown over here so, the intensity pattern is search that at theta equal to 0 which is at the center I expect to get the maximum intensity. But there as I move away from the center the intensity does not immediately fall to 0. There is a finite extent of the central bright spot the intensity falls to 0 let us ask the question where does, the intensity fall to 0.

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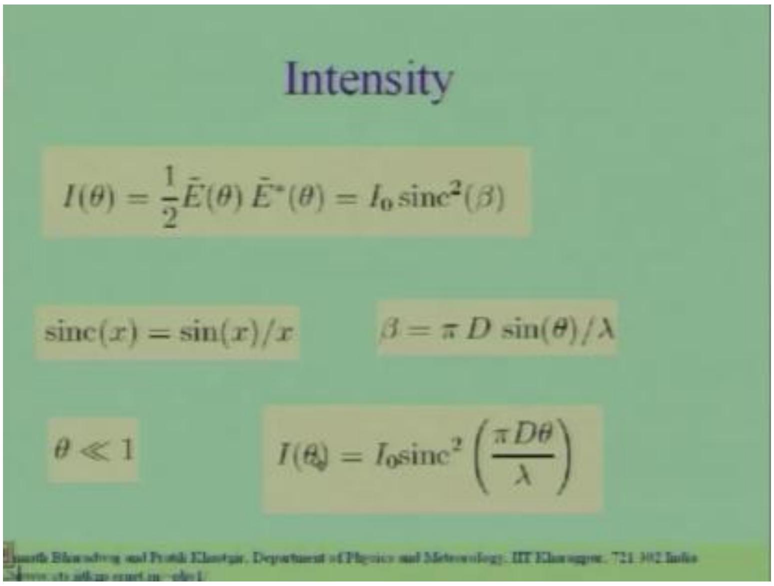

What is the width of the central bright spot so, you have to look at the value to theta where the sinc for which the sinc functions sinc function is 0 and we know that so, the function to look at is.

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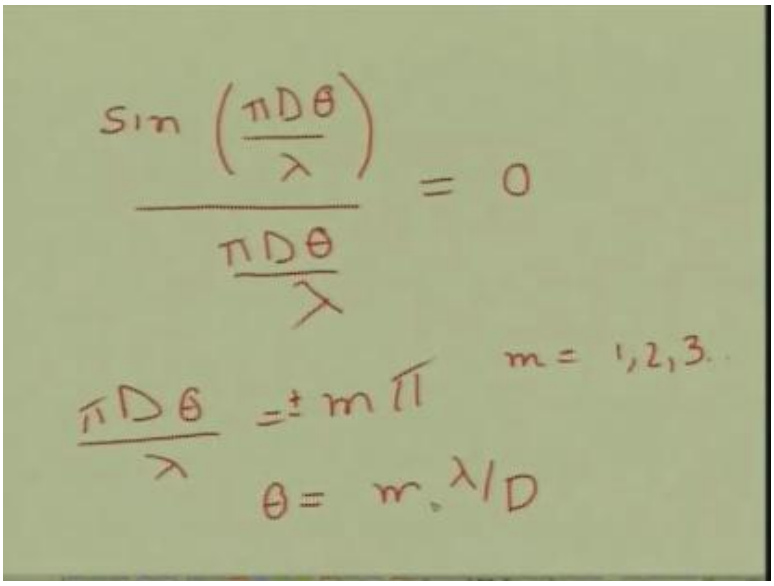


Sin pi d theta D theta by lambda divided by pi D theta by lambda and we want to see where it assumes the value 0. This will assume a value 0 wherever pi D theta by lambda is equal to m pi plus minus m pi where m could be 1 2 3. And any integer or what it tell that is that it will be 0 the intensity will be 0 whenever theta is equal to m lambda by D. So, these are the zeros of the intensity so, let us go back to the intensity pattern.

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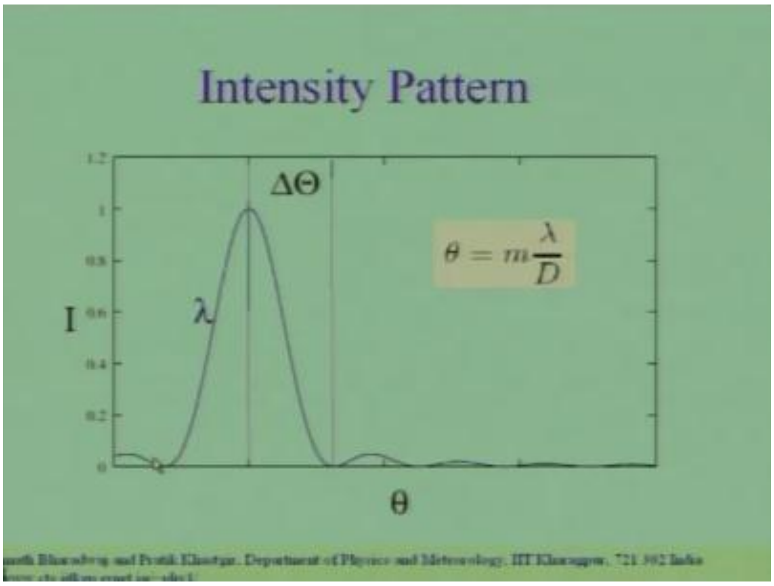


So, the first so, the, we at theta equal to 0 the intensity maximum and then as you increase theta the intensity falls. And it becomes 0 for the first time at an angle delta theta away from the center. And the corresponding value of theta where ever you have 0 is m lambda by D and delta delta theta is just lambda by D. So, the if you move an angle lambda by D from the center you have the first 0 same thing happens on the other side also so, let us ask the question that we have this.

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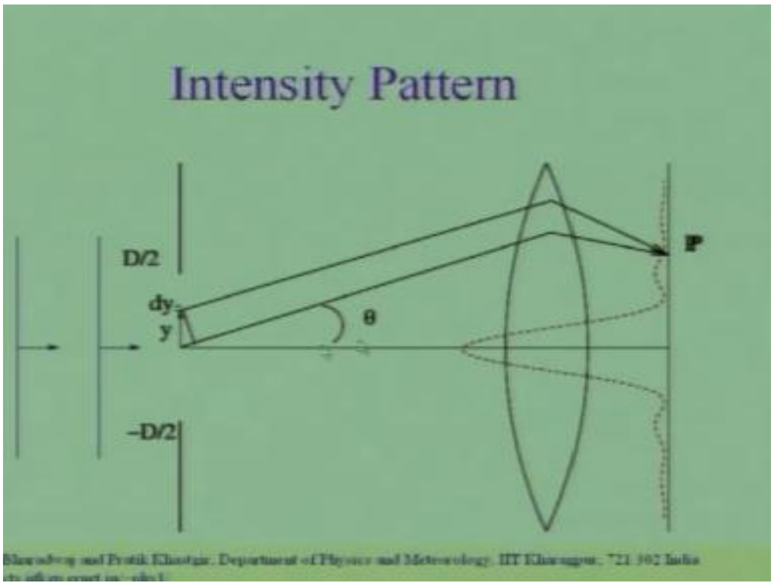

Central bright spot what is the width of the central bright spot by width we mean the distance between the 2 zeros.

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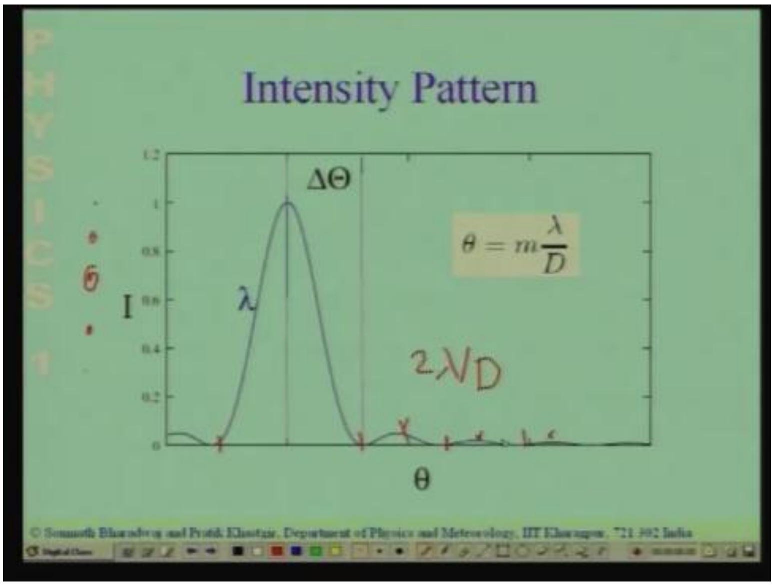


What we mean is the distance between these 2 minimas so, we see that this distance distance between here and here is 2 lambda by D. So, this is one of the big implications of the wave nature of light if you had used geometrical optics to analyze this situation as I have told you you expect to see only a bright spot at the center. But light is really a wave and, because of this you have the phenomena of diffraction instead of seeing a bright spot at the center you get a finite size spot. The width of this bright at the center is not 0, but it is has a finite width the finite width is 2 lambda by D. So, the narrower the slit the smaller the slit width the larger is the width of the central bright spot if you were to increase. The width of the slit the central bright spot would get smaller and smaller if you were to decrease the width of the slit the central bright spot would get broader.

And broader it would occupy a larger spread in the central bright spot would be more and more. This is what we see the central bright spot has a width in angle which is 2 lambda by D. Now, beyond the first minima you again have a, maxima and then you have the second minima. The second minima, occurs at 2 lambda by D then you have a, third minima somewhere over here. The third minima, occurs at 3 lambda by D you have a maxima somewhere between the intensity of the second maxima. This is the first maxima the intensity of the second maxima is considerably fainter than the first 1. So, the so, you have a sequence of bright spots you will get 1 bright spot at the center. And then you have a somewhat fainter as bright spot here and here and you will have even fainter spot over here. And here, because the intensity of this, third, maxima gets even smaller and may beyond that you will have you might be able to see more. And then gets to faint so, the intensity pattern look like this.

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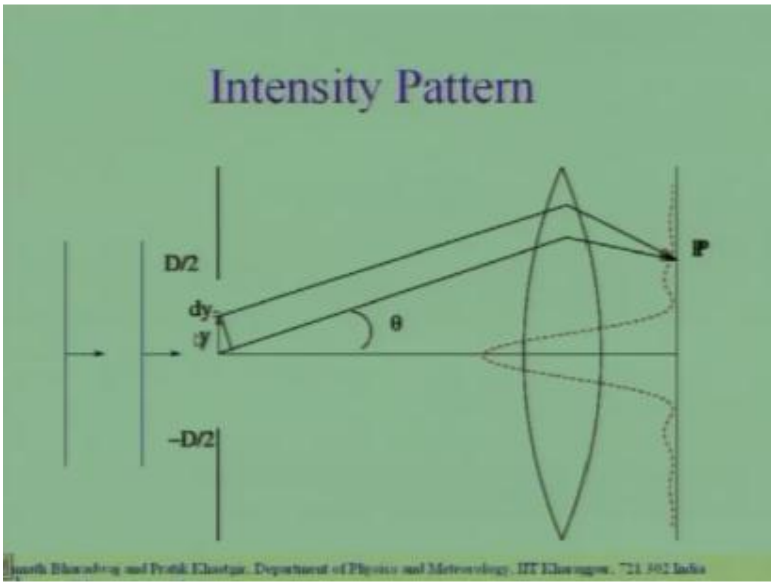


So, if this is the direction of the slit you will get the fringe pattern in this direction the same direction as the slit. And you will have the brightest bright spot at the center then you will have a dark place where the intensity becomes 0. And again another bright spot which is fainted than the central 1 and another one another one and so, far then slowly to get 2 faint to be observed. So, this is the intensity pattern a produce by a single rectangular slit we see that there are 2 main consequences of the wave nature which we encounter over here.

he first thing is at the central bright spot is going have a finite size the size is 2 lambda by D. And the second consequence is that in addition to the central bright spot we shall have higher order maxima. We shall have second order maxima the third order maxima. So, beyond the central bright spot you are going to have other bright spots other because of the phenomena of diffraction let me. Now, take up a small example to get a clear picture of the, of what is happening.

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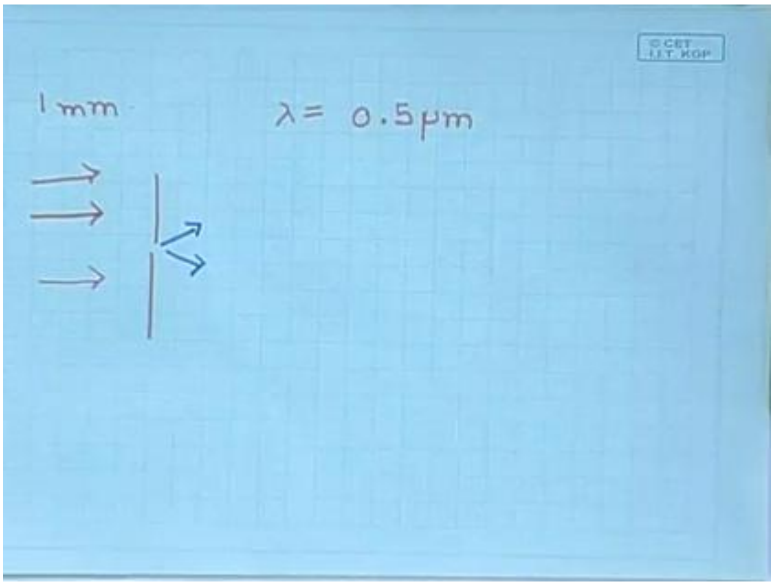


So, we will consider a situation where we have a slit whose of size 1 millimeter so, there is a slit the slit looks like this it has there is plane wave incident on it. And the slit is of size 1 millimeter the wave which is incident on it has a wavelength lambda. It is in the optical range. So, we it has a wavelength 0.5 let us say 0.5 micrometers. So, what we have learnt just now is that all thought the incident wave is a plane is a single plane wave. It has a unique wave vector it is traveling in a unique direction like this when the wave is sent through a slit. The wave that immerges is not going to be in a single direction that. It is going to have a spread how do we know that it is going to have a spread it is quite clear from the Intensity pattern over here just come back to this intensity pattern what the lens does is that all the waves in a particular angle theta it brings to a point.

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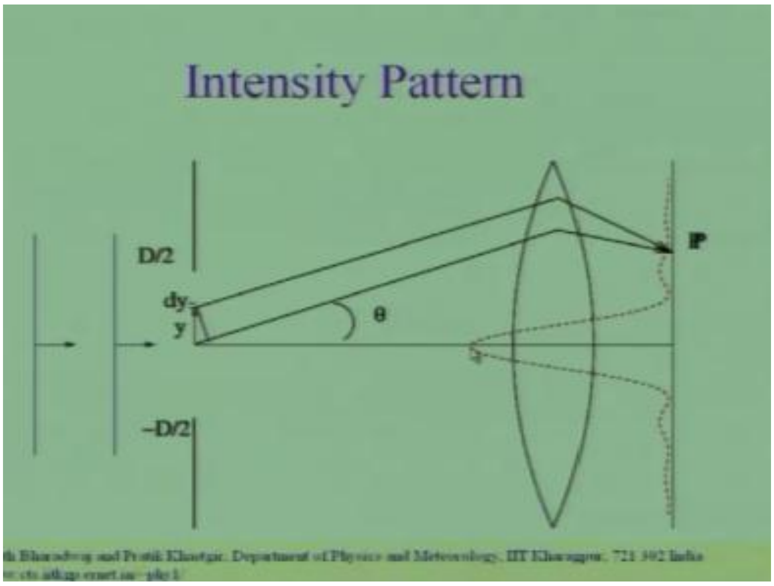


So, you see that the incident wave was only along theta equal to a 0 see you would have expected only a bright spot here. But what you see is that there is an intensity of light at other angle thetas also which essentially means that when the wave has come out of the slit. There is some contribution there is some wave propagating at other values of theta.

So, for this value for example, you would have some contribution in the wave. So, the wave gets spread out and we can say that the approximately the spread in the wave that comes out. The wave that comes in is in one direction the wave comes out is spread over an angle which is this much. So, we can say that the wave is spread over this angle over from here to here which is twice lambda by D. This gives an estimate of the spread in the wave after it comes out through the slit so, the question is that for a slit which is 1 millimeter in size into which we have sent plane wave with wavelength 0.5 micrometer.

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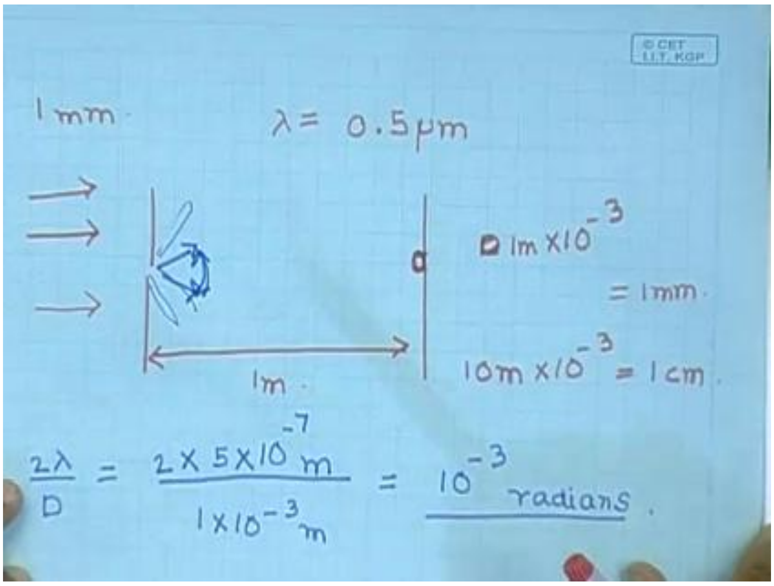


What is the spread in angle of the wave that comes out from the slit? So, this we have seen is 2 lambda by D which in this case will be 2 into 5 into ten to the power minus 7 meters divided by D which is 1 millimeter 1 into 10 2. The power minus 3 meters so, this gives us ten to the power minus 3 radians. So, waves that comes in is all going in a single direction the wave that comes out is now spread over an angle ten to the power minus 5 minus 3 radium. So, this wave will be spread over an angle the first maxima the maxima at the center itself will be spread over ten to the power minus 3 radians.

And then there will be other maximas also which will much smaller in intensity there will be small amount of wave coming out in other direction also which will correspond to the second maxima. And third maxima, but they have a much smaller intensity the central bright spot itself will be spread over this angle. Now, the next part of the question is as follow suppose I put a screen which is 1 meter away. So, I have a screen and the screen is 1 meter away. The question is what is going to be the size of the central spot that is going to be produced on the screen.

So, that is quite easy to estimate so, we will get a central bright central spot over here. And the size of the central spot is going to be L the distance which is L in this case it is 1 meter. So, 1 meter into ten to the power minus 3 that is the angle the angular extent of the, of the wave that comes out and this. So, this is going to be 1 millimeter let us now ask the question if I move the screen to a distance which is ten meters instead on 1 meter what is the size of the central bright spot. So, again we can do the same exercise multiply this with the angle minus 3 and this is going to give us 1 centimeter.

So, what we see that this wave travelling in a unique direction when you send it through finite slit when it emerges which is spread out over a range of directions approximately of the order of 2 lambda by D. And if you put a screen far away and or you put a lens under screen not may be not necessary far away. Then you will get a spot arising from this finite spread in the wave that comes out. And we have estimated the size of this spot for this particular slit let us now continue our discussion where we had left it so.

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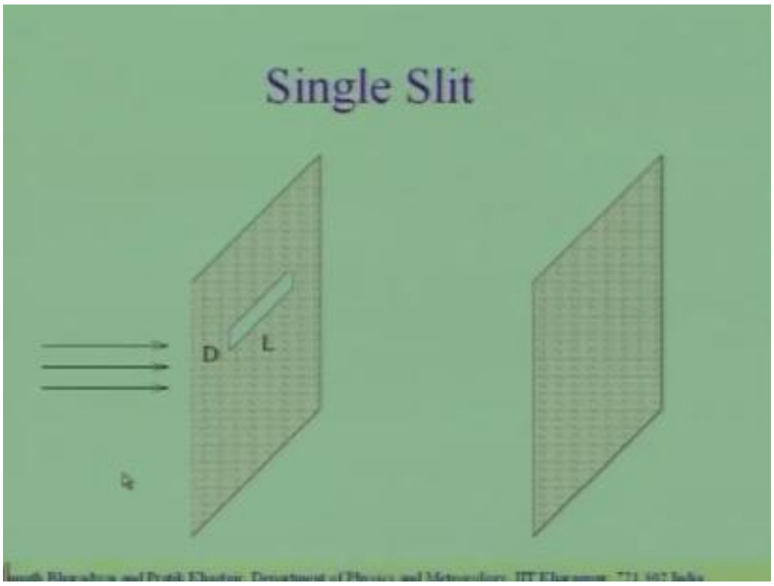


So, let us go back to the single slit problem we had considered the situation where 1 dimension of the slit was much smaller than the other dimension L. So, D was assumed to be much smaller then L and we had considered the situation where L is infinitely large. And the whole problem, are reduce to a 1 dimensional problem in this direction. Now, let us ask the question what happens when we also take into account this, this other extent of the slit L. Now, again we have to go back to the Huygens principle each point on the slit will contribute to the intensity at any point on the screen if I want to calculate. The intensity at any point on the screen I have to add up the contribution from all of these points. But now you see it becomes a surface integral I have to integrate over the surface of the slit it is a 2 dimensional surface integral well in this particular case. The integral can be done it is not very difficult I will not go through the mathematics.

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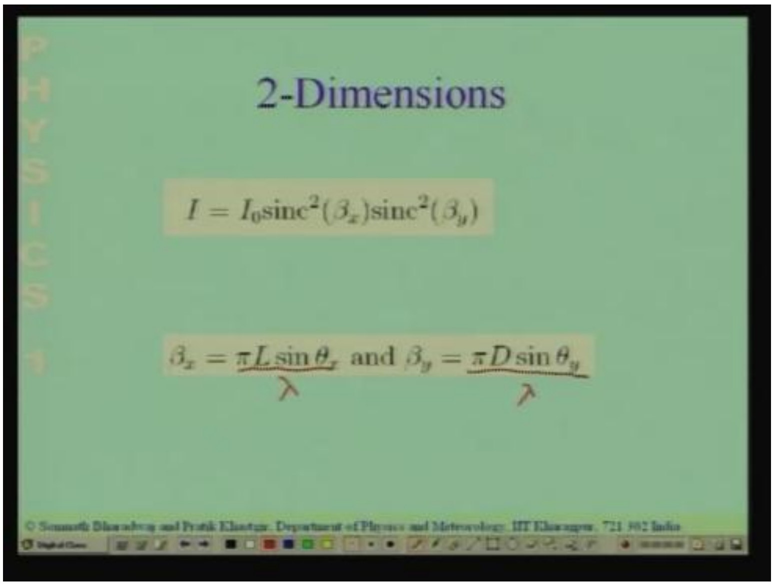


And i will just show a result which I had shown you in the last class also so; in this case you have a more general formula for the intensity pattern. It is now given by I naught sinc square beta x into sinc square beta y where beta x. And beta y are defined as beta x being pi D L the length along the x axis is L. So, it will be pi L sin theta x where theta x is the angle it makes to the x axis in the x direction. So, theta x and there should be a factor of 1 by lambda so, this whole thing divided by lambda is beta x. And this whole thing divided by lambda is beta y so, remember. The D is the extent along the y direction theta y is the angle it makes with the y axis in that along that direction. And the theta theta y is angle it makes with a y axis theta x is the angle it makes with the x axis. And we have a more generalized formula which holds in a situation where I have to take into account where I have taken into account. The both the dimensions of this slit so, let me show you the intensity pattern that you expect to see in such a situation.

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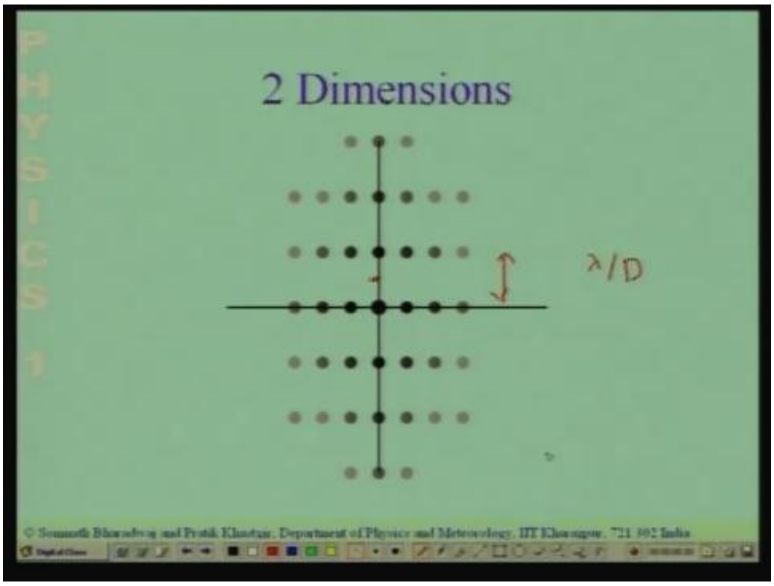


The intensity pattern that you expect to see in such a situation will look like this.

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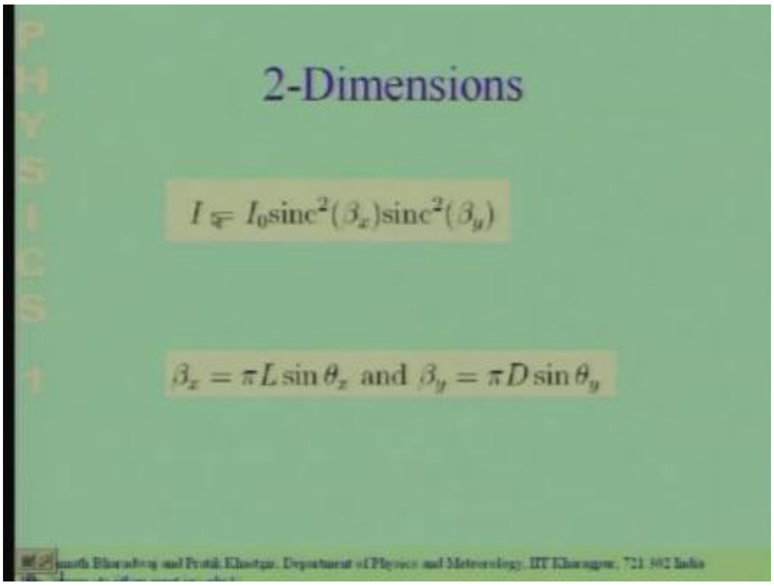


Let me go through this in a little detail so, first point to notice where is the intensity go to brightest it is quite obvious that intensity is going to be brightest when theta x a theta y are both 0 sinc function is 1 when theta x is 0. And the sinc function is 1 when theta y 0 so, intensity at the center is I naught.

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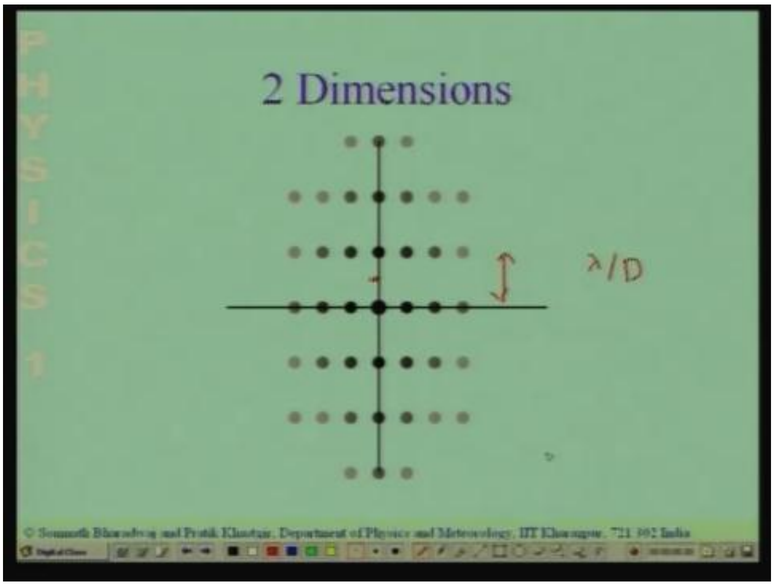


So, the brightest region at the center now, if I move only along the y axis.

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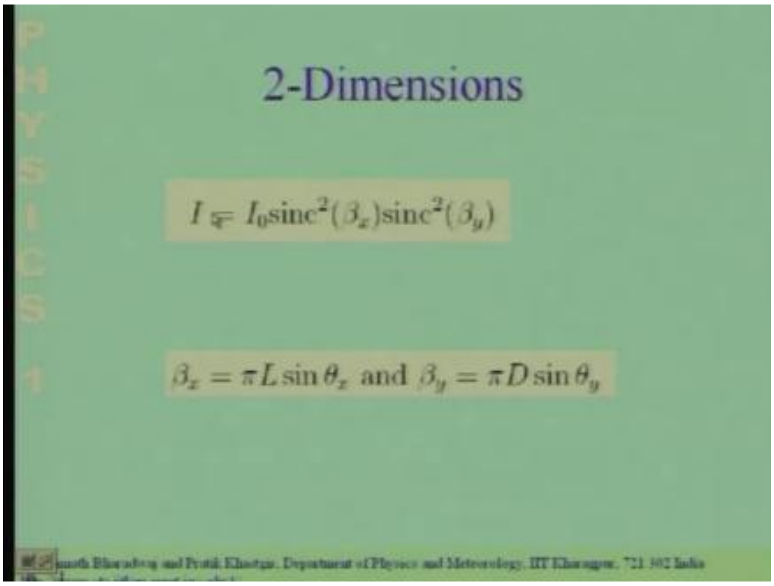


So, if I move only along the y axis theta x remains 0 so, this sinc function remains 1 I do not have to bother about it I only change move along the y axis. Then I have the fringe pattern a 1 dimensional fringe pattern essentially because this, factor become 0. It is just like a 1 dimensional 1 dimensional fringe which we had just discussed and the fringe spacing. The dark the positions of the dark of the dark lines of where it become dark is decided by this D.

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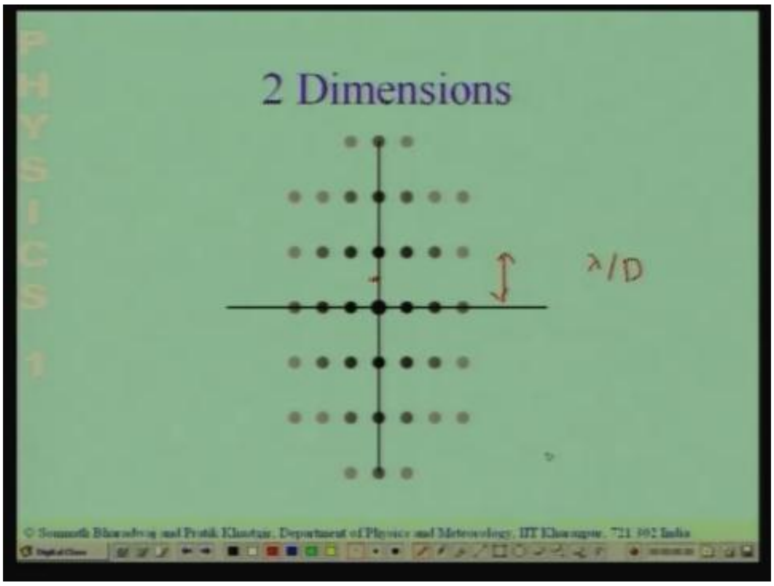


So, in this along the y axis along the y axis the fringes are going to be separated this separation is going to be lambda of the order. So, the dark the dark is going to this is going to be decided by lambda by D the. So, the spot where the fringe become the darkest this is going to be lambda by D. So, the spacing is going to be decided by the inverse of D.

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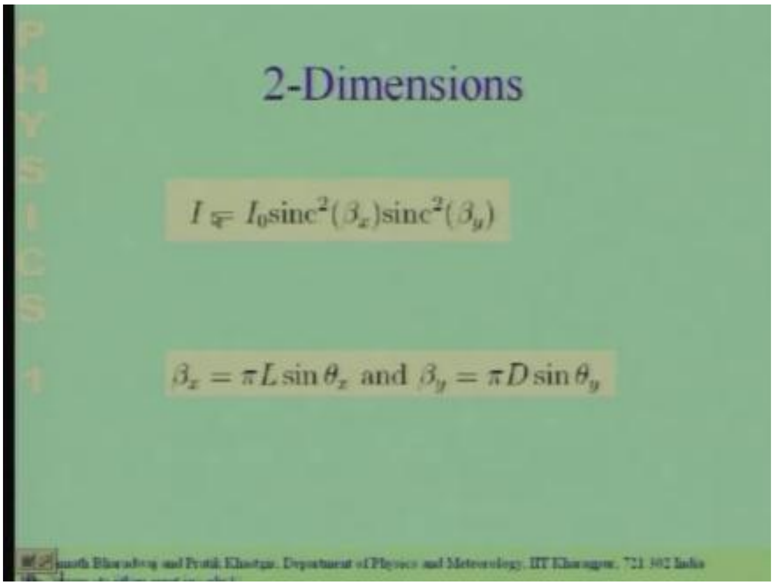


Similarly, if I were to move only along the x axis remaining at y holding theta y 0 then it again would be another 1 dimensional fringe system this factor would continue to be 1.

So, the, I would have these fringes the bright and dark spots along the x axis and the in this case.

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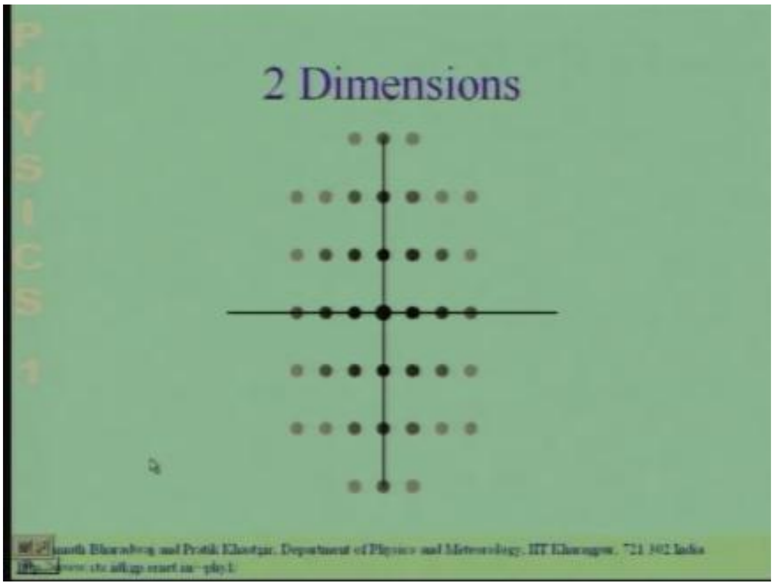


The spacing would be 1 by L.

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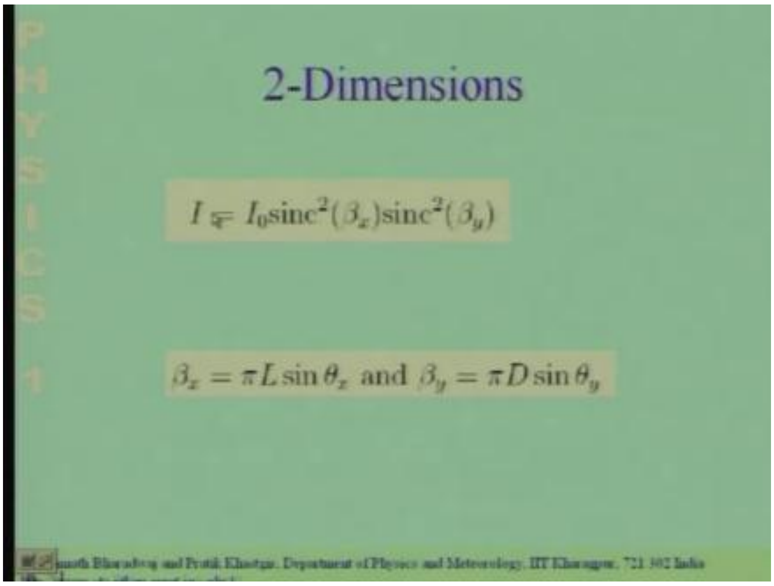


And if I were change both theta x and theta y then both of these factors would coming the intensity would fall of quite fast, because the sinc function falls of both this and this.

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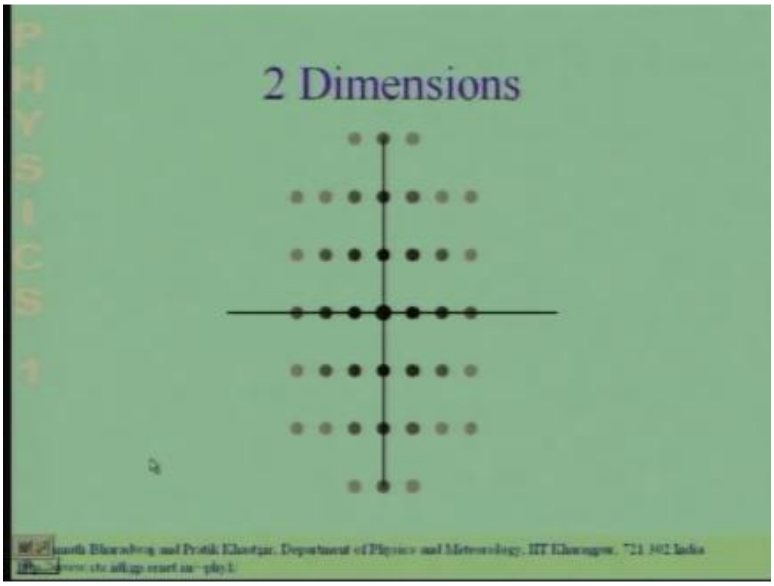


L

And we have in now having understanding of this picture so, as we move in this direction. The intensity falls up as move in this direction should also does the intensity fall of and when you move both along theta x theta why the intensity falls of even faster. So, you have the, you will see of few a fingers which bright spots which will look like this. And then they will slowly fade away as you go further I have shown only the brightest parts of the fringe now, the point to note over is what happens.

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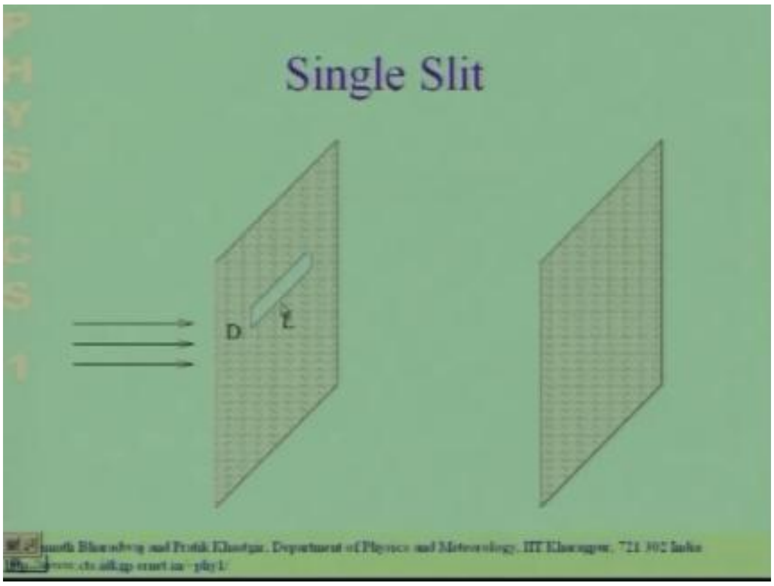


If you change D and L so, if when D and L are comparable.

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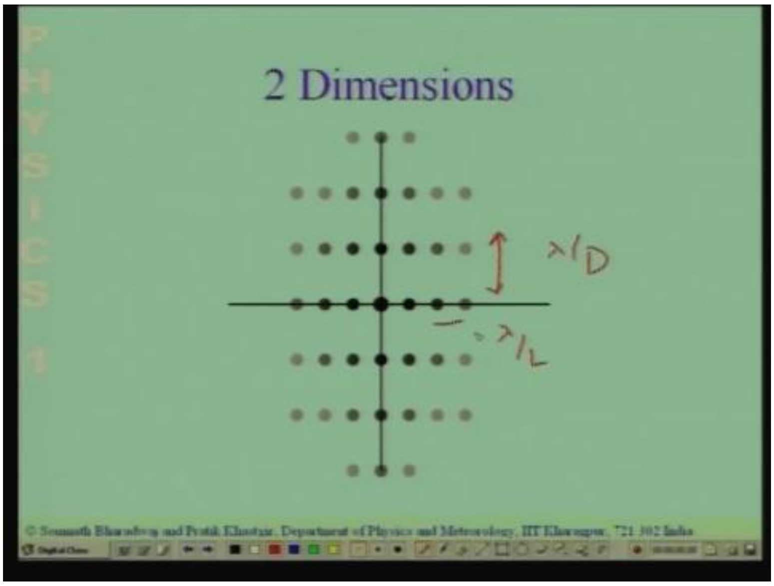


The fringe pattern will look like this in this picture this spacing is of the order of lambda by D and this spacing is of the order of lambda by L if I increase L keeping D fixed. So, if I make the slit longer and longer in 1 dimension. Then the whole fringe pattern is going to collapse this spacing is going to get smaller and smaller. And for very large L you possibly will not be able distinguish the fringes in this direction. They will all be very close together and will it will effectively the fringe pattern is only going to the seen along the y axis which justifies what we had.

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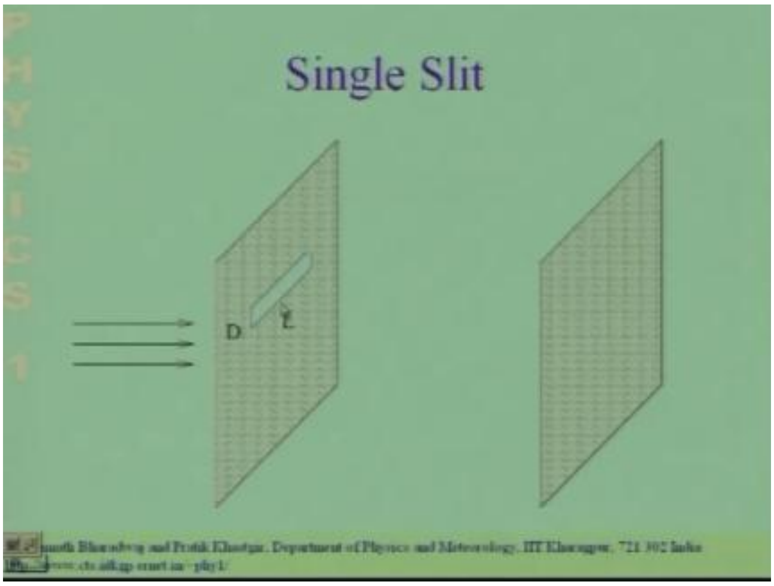

What I had told you earlier so, when you make L very large if you make L very large.

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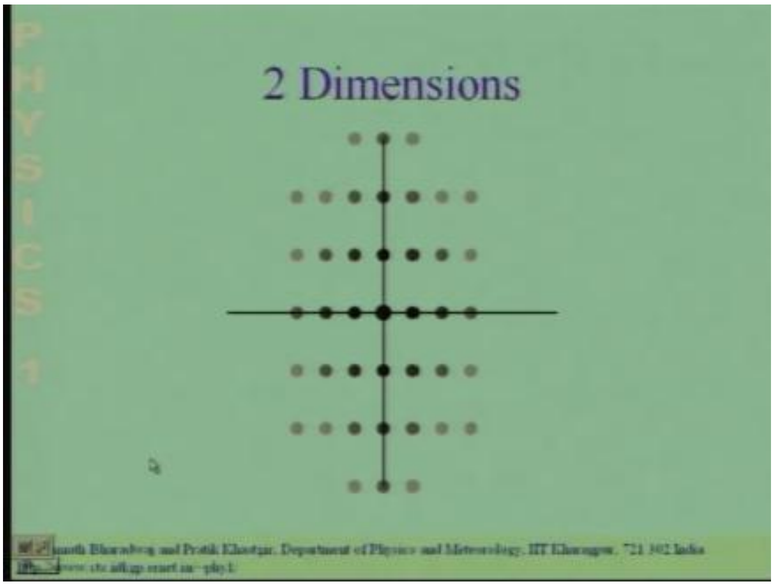


The entire fringe pattern is going to be very concentrated concentrated this spacing is going to get very small. So, the entire fringe pattern is going to have a very small extent in this direction and you will only see this extent of the fringes. Now, in yesterday's lecture and today's lecture I have been discussing the rectangular slit. The diffraction pattern produced by a rectangular slit now, a rectangular slit is not. So, common what we commonly encounter which the thing which is very important let me now move on to that what we commonly encounter.

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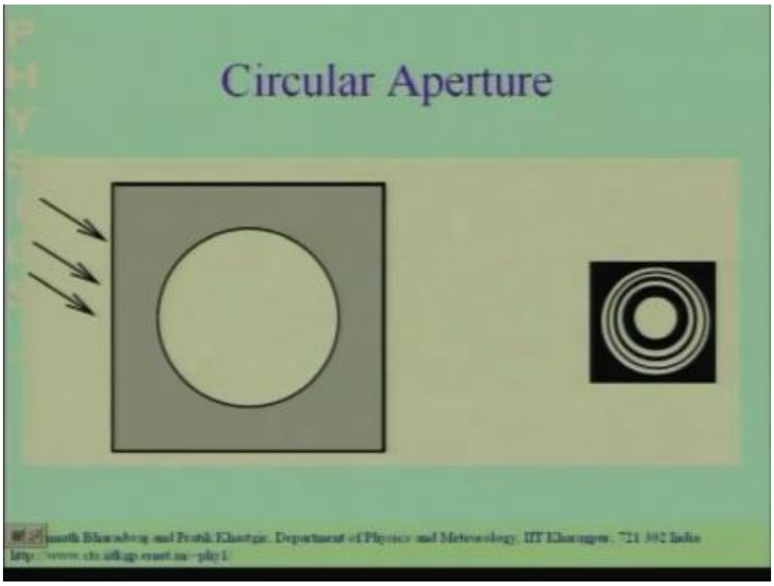


Is a circular aperture so, this picture shows you the circular aperture so, we have a screen over here in that screen we have a circular aperture. So, there is a part of the screen which is a circle which has been cutout and on the screen. We have a plane wave which is incident so, there is a plane wave which is incident now, if I put another screen very far away from the screen. And see the image or if put a lens under screen if we had only the plane wave we would get just a spot. But we have seen that because of this circular aperture you will have the diffraction pattern.

And for a rectangular slit we have worked out the diffraction pattern you have to do the same thing over here. So, when you want to calculate the diffraction pattern due to the circular aperture you have each point on the circular aperture will act like a secondary source secondary wavelets will be emitted. And if you wish to calculate the intensity of on a screen which is placed far away from this aperture you have to add up the contribution from each of these sources you. Now, have a surface integral the mathematics is a little more complicated.

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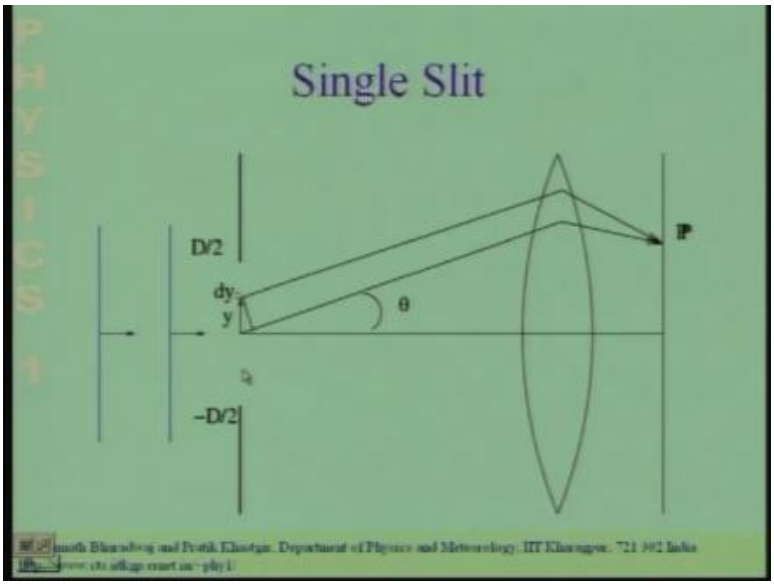


So, I will not go through it I will just show you the resulting diffraction pattern that you expect to see on a screen located far away from this aperture or if you put a screen and a lens as if you put a screen lens like this if you add a circular aperture here and you had this kind of a screen. And lens arrangement the intensity pattern that would seen on the screen over here.

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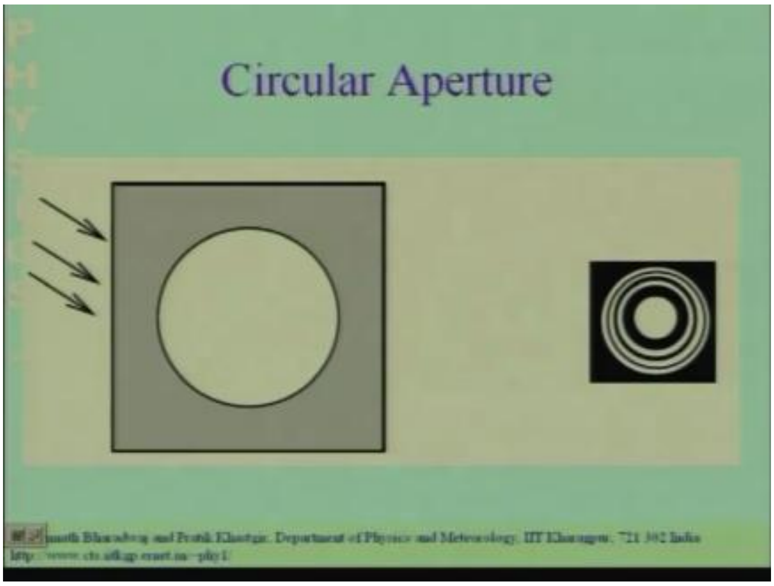


Would look like this you would have a extended bright spot at the center and then you would have a dark circular fringe around it. And then you would have the second maxima, which in this case is again circular, because of the symmetry circular symmetry of the aperture. And then again another dark line and again another circular fringe the intensity of these higher order fringe is the first the first central spot is bright. The circular rings the intensity slowly diminishes and beyond the certain level you would not be able to see it so, to explain this picture once more.

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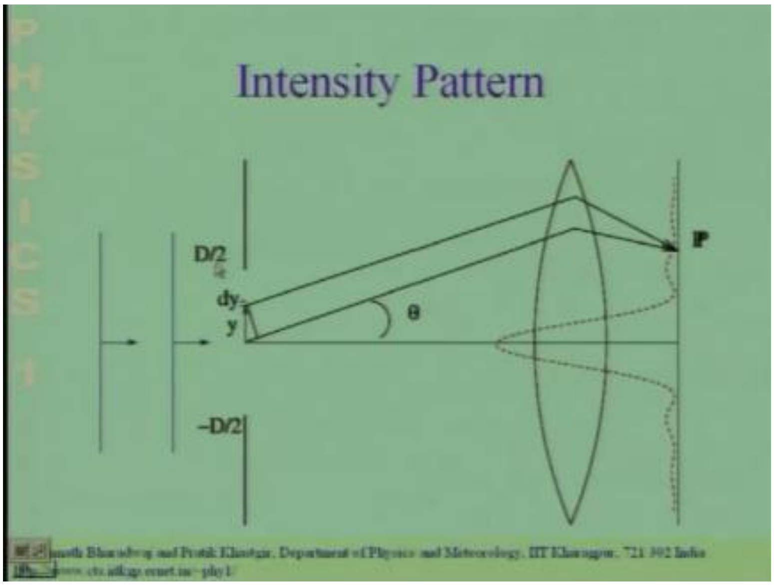


If I had the circular aperture over here this is so, if I had the circular aperture over here it could produce an intensity pattern on the screen which would like this. There would be a central bright spot and then there would be a dark ring. And again a bright ring the dark ring the bright ring and so, forth which is what I have shown over here in the situation when we had a rectangular slit we calculated the angular distance from the center to this.

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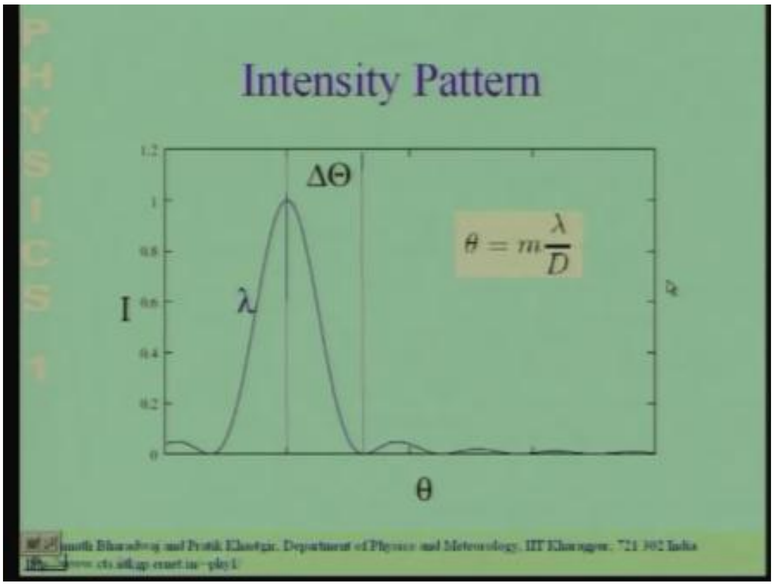


It was so, the angular distance between the center of the bright spot and where it becomes dark. The dark the point where it becomes dark is we had calculated this it was lambda by D.

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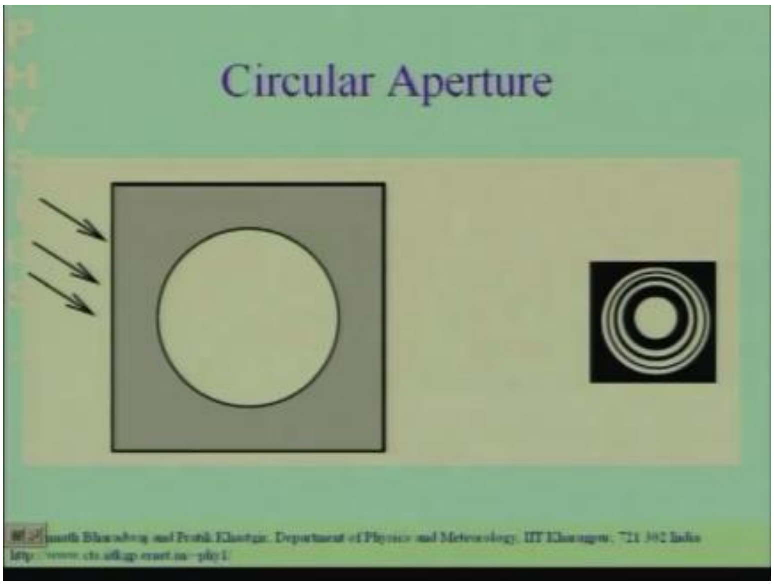


We will not do the calculation for a circular fringe I will just tell you the result there is a small modification. Now, when you do it for the circular aperture and instead of being lambda by D it is now.

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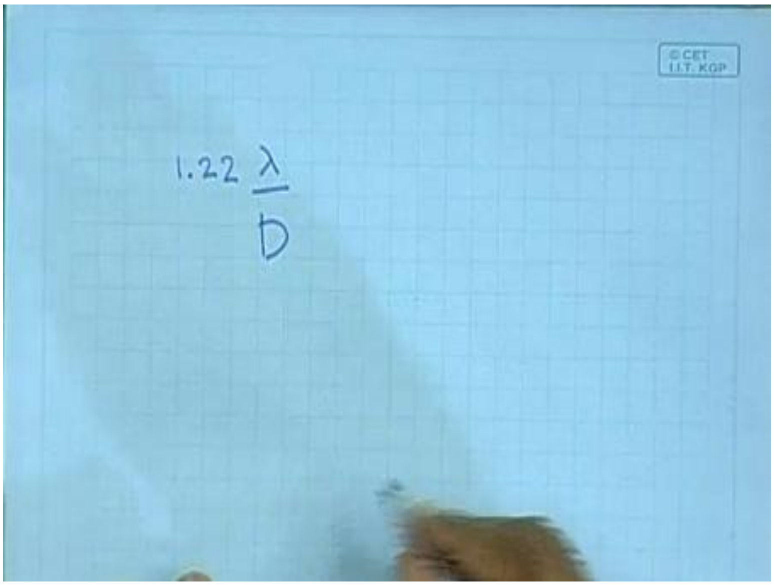


1.2 2 lambda by D so, we central bright spot is now of radius.

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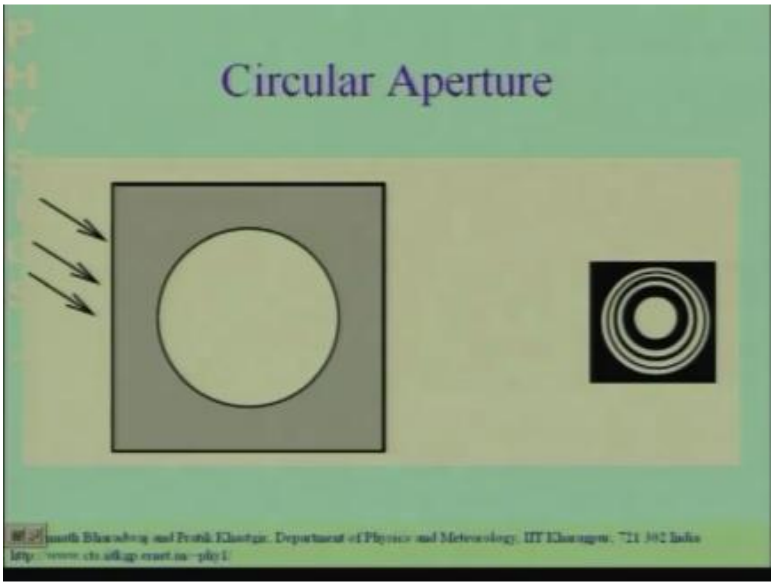


1.2 2 lambda by D or of diameter 2.4 4 lambda by D and then you have a bright dark ring around it. And then you have the second maxima and the third maxima and so, forth and then it gets fainter and fainter and you will not see it beyond a certain level. Let me now discuss this is a very important. So, what I have told you just now is something which is very important let me now discuss why this is so, important.

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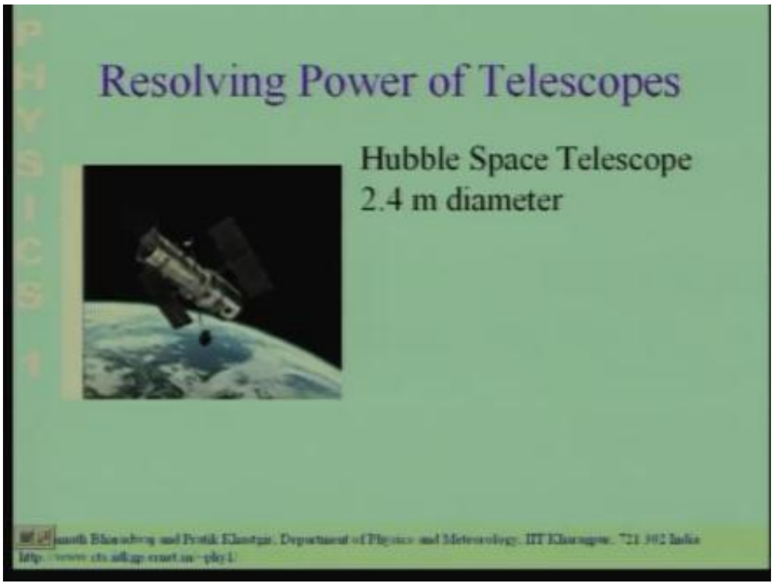


Let me just take up 1 particular situation and the question that we are going to discuss is the resolving power of telescopes the resolving power of telescopes. So, telescopes as we now usually have a circular aperture right. So, here I have shown you the Hubble space telescope you can see that there is a circular aperture over here through which light enters the telescope light which comes out this circular aperture does not enter this telescope for the. This is true for all telescopes. So far the Hubble space telescope the circular aperture is of diameter 2.4 meters.

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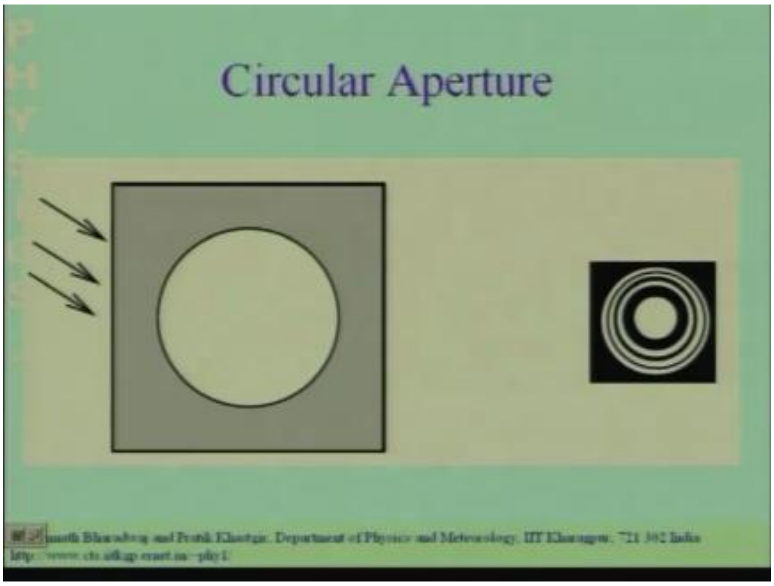

So, the haggles space telescope has a circular aperture all telescope not only the Haggles space telescope all telescopes have some kind of an aperture. Which lets light in usually this is circular that is why we are discussing that is the importance of the circular aperture. So, only light which comes in through this aperture is let in light which comes outside does not get into the telescope which is what I have shown over here.

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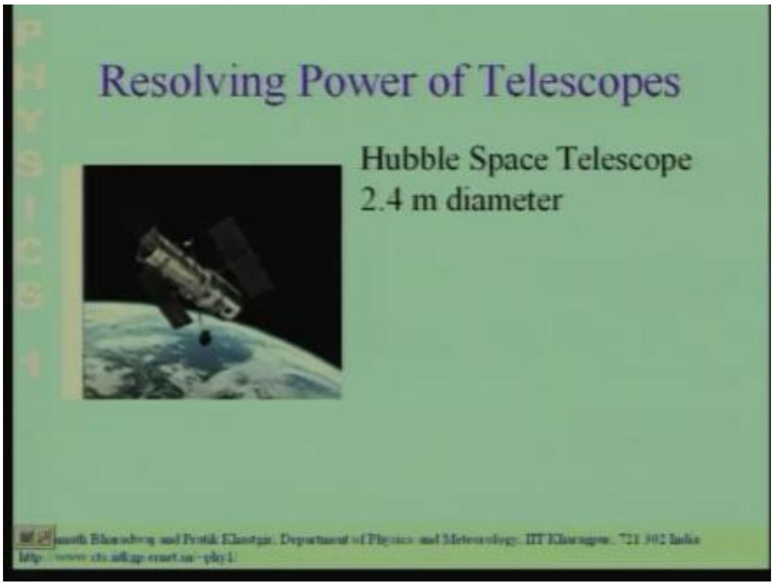


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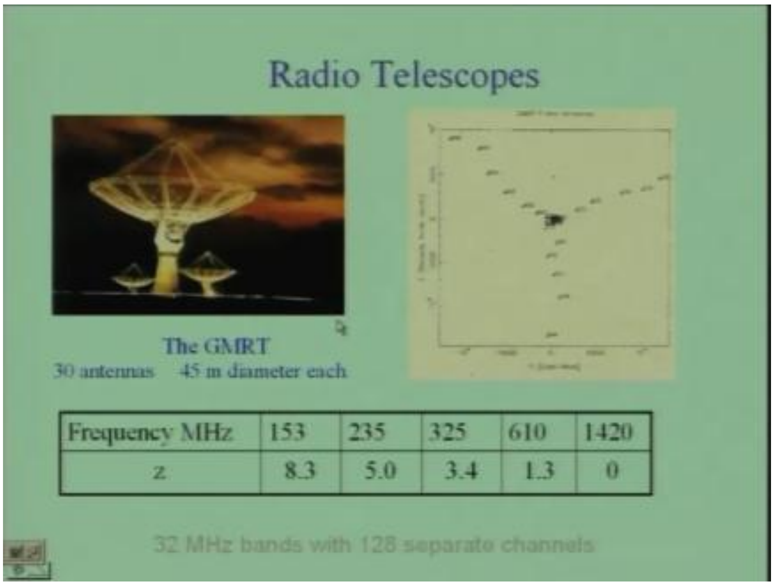


Similarly, this shows you the joint meter wave radio telescope GMRT about which I have told to earlier. This works at radio wavelengths the Hubble space telescope works at around optical length around optical wave length this works at radio wave lens. So, only the waves incident on this circular aperture gets collected the waves that come beyond this are not collected. And the joint meter wave radio telescope has an aperture of diameter 45 meters. What is the implication of this? So, I have told you that all telescopes have some aperture and it is usually circular. The question is what is the implication of this? The implication of this is as follows suppose I have the telescope.

And let me just discuss certain words suppose I have telescope and I point it toward the star. Now, a star is a source which is sufficiently far away so, that the light coming from the star can be thought of as a plane wave. So, the light coming from the star is the plane wave it is all travelling in 1 direction and if I think of it using geometrical optics I pass it through a telescope. So, the telescope I expect the telescope to focus all of this light to a single point. So, if I had 2 star in my field of view and I point my telescope in my image I expect to get 2 points unfortunately this is not exactly so.

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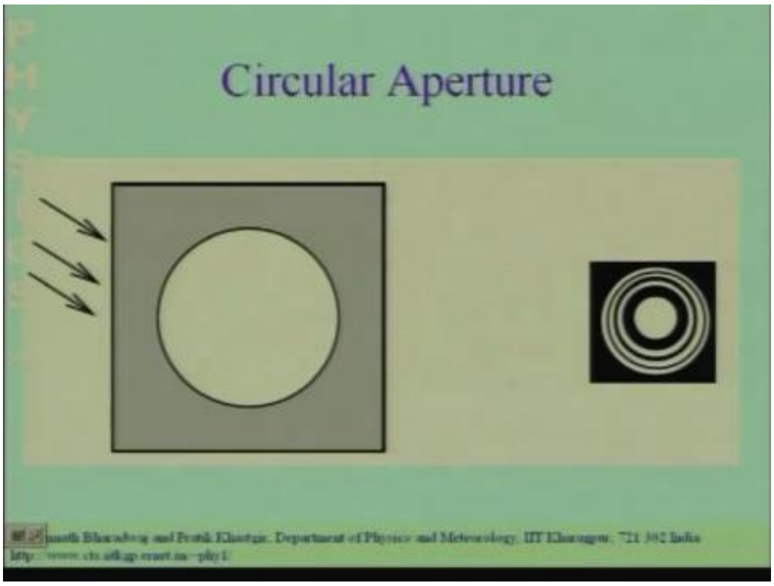


And the reason why it is not exactly so, is because the light which comes from the from the star is not is cannot be thought of in terms of geometrical optics it is actually a wave. So, when the light coming from the distance star passes through the circular aperture of the telescope the image that is produced is not going to be a point is going to be the diffraction. It is going to be the diffraction pattern produced by the circular aperture. So, the image of a star reduce by a telescope is actually going to be the diffraction pattern shown over here. You are going to have a central bright spot and then you’re have going to have brings rings around it. So, this is what you get what. So, when you observe any star through a telescope its image the image that you are going to get from the star is not going to be a spot which you expect from geometrical optics. The image that you are going to get is the diffraction pattern of the circular aperture. Now, suppose I had 2 stars in the sky which are very close.

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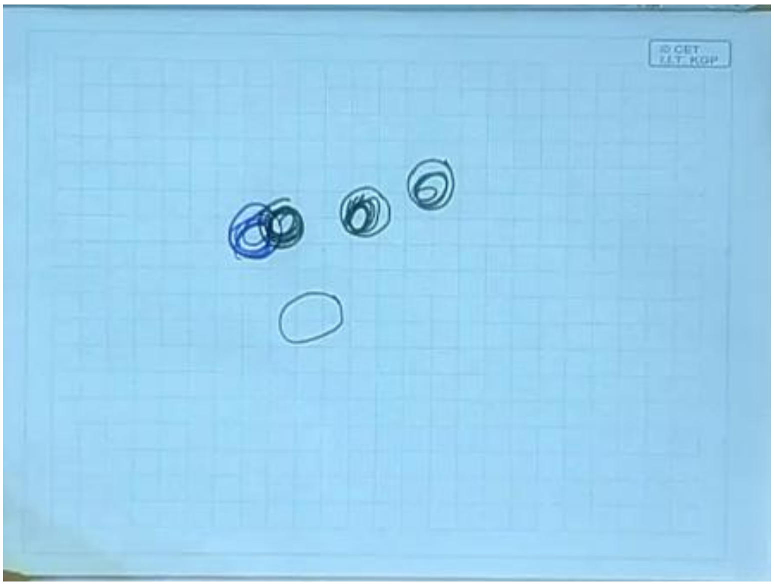

So, 1 star is going to produce a central bright spot like this and the other star is also going to produce its own central bright spot. And if the 2 bright spots overlap now other fringe is would also be there, but if the 2 bright spot overlap then I cannot distinguish I cannot say that there are actually 2 stars. Because what I would see will be some big thing like this and I would not be able to say that there are 2 stars. So, the question is i would be able to say that there are 2 stars only if I had some a situation where the 2 bright spots. Now, the diffraction pattern of the 2 stars did not overlap. So, the question is what is under what is the criteria for me to be able to say that there are actually 2 sources and not 1. So, I will stop this lecture here today in the next lecture is shall take up this criteria for discussion it is a very important and interesting issue. So, I shall take it up fresh in the next lecture instead of that instead of introducing it right now.