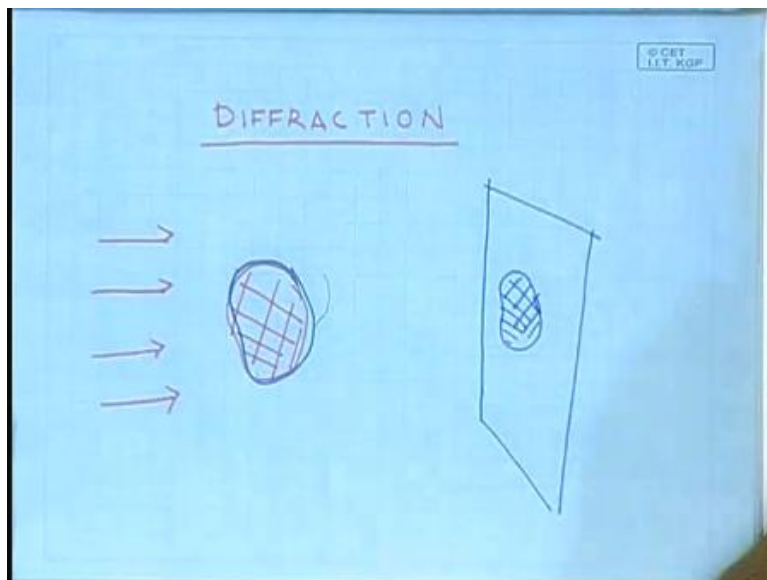


**Physics I : Oscillations and Waves**  
**Prof. S Bharadwaj**  
**Department of Physics & Meteorology**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 20**  
**Diffraction - I**

We have been discussing interference, the past few lectures and in interference we had a situation where we had superposed 2 waves. And the 2 waves could have different phase relation and if we found that if the phases are the same than the 2 or a multiple even multiple of  $2\pi$  then the 2 waves add up constructively. And you have an increase in the intensity, where as if the 2 waves had opposite phases. The 2 waves add up destructively and you had a diminishing reduction in the resultant intensity. Now, you could generalize this situation and consider 3 waves, 4 waves 5 waves many waves. Now, we usually refer to a situation where we have many waves interfering, many waves being superposed, such a situation is usually referred to as diffraction.

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So, in today's lecture and in the next few lectures is, we are going to discuss diffraction. Let us consider a situation, where we have a plane wave incident like this; and we place an obstruction in the path of the wave. So, let me repeat we are going to start our discussion of diffraction by considering a situation where we have a plane wave, incident like this and there is an obstruction. So, this is the obstruction,

which is placed in the path of the wave. And we are interested in the resultant image. The image of this on a screen placed far away.

Now, we know from our experience that, we expect to see a shadow on the screen. So, on the screen over here; we expect to see a shadow and from our experience we know that the shadow. So, the whole the screen would be illuminated except the place where the light would be blocked by this obstruction and that is where you would see the shadow. So, on the screen you would see a shadow and the shadow would be, would have the same shape. As the object over here, if you were to see the object from this side the shadow would have exactly the same shape. This is what you expect and this picture would be precisely true if light work corpuscular in nature.

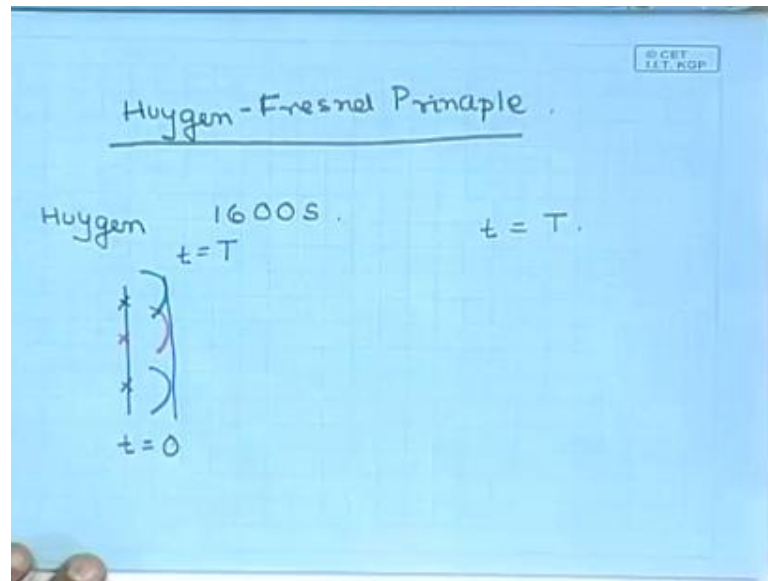
But if it turns out that if you look closely at the edges of the shadow. These regions you will see that, there are dark and bright lines which occur over there you will find a pattern of dark and bright lines in the region near the edge of the shadow. These are refer to as fringes we have already discussed what fringes are, these are refer to as fringes and this is an example of the phenomena of diffraction where we have the superposition of many waves. 1 of the first people to notice and record the phenomena of diffraction fringes of these fringes, near the edges of shadow was Leonardo da Vinci. People might have noticed this phenomenon earlier, but 1 of the first records written, that we have of somebody noticing these fringes is in the records of Leonardo da Vinci.

So, in today's class, we are going to focus on a particular situation. Before coming to the particular situation, let me discuss the problem in more generality. So, we are going to discuss the situation where we have a wave. In general, we are going to discuss in a situation where we have a wave and the wave is incident on an obstruction in the path. So, there is an obstruction placed in the path of the wave and we would like to consider the image form, in such a situation. The question is how does 1 analyze this situation? Now, light we know is a wave and there is an equation governing this wave. So, there is an equation call the wave equations which govern this wave. We shall discuss the wave equation a little later in this course, not right now, but let me just tell you that there is an equation a partial differential equation which governs the evolution of the wave.

This equation is called the wave equation and the wave equation can be solved in many situations, the wave equation can be solved. So, the plane wave that we have which is incident of this obstruction is 1 such solution of the wave equation. Now, the movement you put an obstruction in the path of the plane wave, the situation becomes considerably more complex. Now, if you wish to analyze this situation where we have an obstruction in the path of the wave, we have a plane wave and we have an obstruction over here. You now, have to solve the plane wave with appropriate boundary conditions corresponding to this obstruction. Inside the obstruction the plane wave should vanish. So, you have to impose appropriate boundary conditions, which represent the obstruction and you have to solve the plane wave.

The wave equation, solving such a wave equation is actually very difficult mathematical exercise. So, if I have some obstruction in the path of light of some light of some wave. Then this produces some very difficult boundary conditions and solving in general, solving the wave equation for such boundary conditions is impossible in most situation. So, in most situations, it is not possible to analytically solve this problem. So, you are in you this is a difficult situation, you have a wave equation governing the wave and you have a you have some obstruction in the path of the wave. So, you can set up the wave equation with appropriate boundary conditions, but in most situations it is not possible to analytically solve it.

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So, it is a complicated situation, but there is a method there is a heuristic method, call which goes by the name of the Huygen-Fresnel principle which allows us to handle such situation. So, it is we shall refer to it as the Huygen-Fresnel principle it is a heuristic approach it tells you a prescription how to solve such problems. And this can be done in most situations and this gives us a handle to tackle these problems. So, let me discuss the Huygen-Fresnel principle the Dutch astronomer Huygen in the 1600s introduced the Huygen principle.

The Huygen principle itself is not directly applicable to studying diffraction, but let me first tell you what the Huygens principle is. The Huygens principle, tells us as follows, the problem is as follows; there is a wave front so, at let me consider the simplest possible situation I have a plane wave. So, at  $t$  equal to 0 there is a wave front, a plane wave  $t$  equal to 0 and we would like to determine study the evolution of this wave. So, we would like to find out what happens to this wave front at a later time  $t$  equal to  $T$ . So, let me state the problem again, there is a wave we would like to study the propagation of this wave. And you are given the information that at certain time, where  $t$  is equal to 0. I know the wave front; I have drawn the simplest possible situation here when the wave front is the plane wave. We would like to study the evolution of this wave.

So, the question is at a later time  $t$  equal to  $T$ , what this wave's front looks like. And what Huygen proposed is as follows; Huygen proposed that you handle this problem as follows, his principle is this follows to follow the evolution this wave front consider each point on this wave front as a source. So, you should consider each point on this wave front at  $t$  equal to 0 as a source. So, let us take 1 point first. So, you take this particular point here. So, this point is a source for a secondary wavelet and this secondary wavelet travels at the same speed as the wave. So, the secondary wavelet travels at the speed  $c$  if a considering light in vacuum.

So, after a time  $T$ , at a time  $T$  the secondary wave let emitted from this point will the secondary each source emits a second spherical secondary wavelet. So, at a later time  $T$ , the wave which is emitted from here, at a time  $t$  equal to 0 will look like this. Now, each point on this wave front will like act like a secondary source. So, let me consider another point, this point here. So, at that so, we have a source which emits a secondary wave wavelet at a time  $t$  equal to 0. Let we draw the wave front at the time  $t$  equal to  $T$  at a time  $t$  equal to  $T$  is going to travel at distance  $c$  into  $T$ . So, it is going to be a sphere of radius  $ct$  which will again look like this; the centre of this sphere has shifted we are considering the propagation of wave in vacuum.

I could do the similar exercise for a third point, a third source on this wave front each point on this wave front acts like a source. I am only going to draw it for a few of them. So, let we consider a third source over here, this is also going to emit a secondary wavelet, which again travels at the same speed  $c$ . So, at a later time it is going to be a spherical wave front like this. Now, what Huygen said is you do this and then you consider the envelop of all these wave fronts. So, the envelop of all these wave fronts in this case, looks something like this and this tells me the wave front at a time  $t$  equal to later time  $t$  equal to  $T$ .

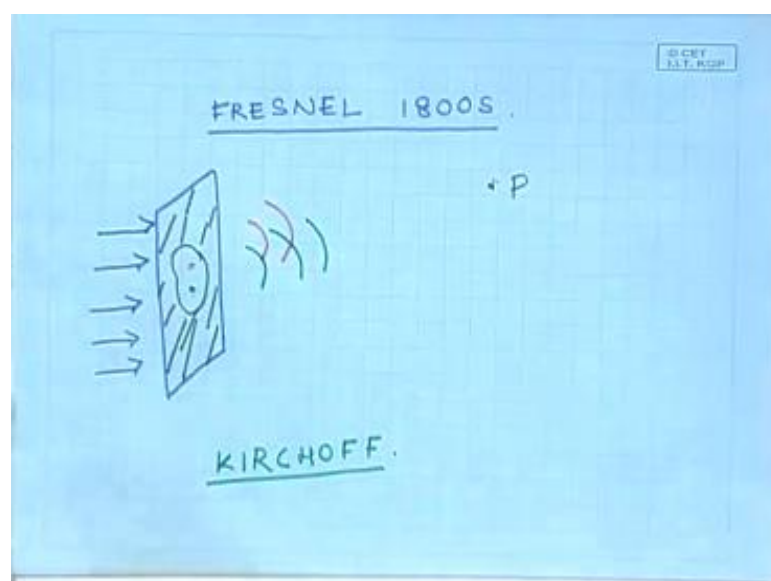
So, this is the prescription which allows you to follow the propagation of a wave in a medium because you can now use this to determine, what the wave front will look like a at a later time  $T$ . And you can do it for different values of the later time and you will get, to see how the wave front actually propagates inside in this medium. Here I have taken the medium to be vacuum. So, a plane wave will propagate onto the other plane wave and the so, forth and the whole thing will move forward with

the speed  $c$ . Which is what you have just seen takes place if I apply the Huygen's principle.

Now, Huygen's principle does not tell me anything very interesting. If I applied in vacuum, but Huygen's principle is very useful and it gives us deep inside into a situation where we have light propagating in a medium whose refractive index is different, in different places, different directions variation. So, the refractive index essentially manifests itself as a change in the speed of light. So, the spherical wavelets which should be emitted will, if the medium is such that the speed of light is different in different in directions.

For example, then the secondary wavelets are not going to be spheres there will be a ellipses or some other such thing. And you can then use it to the Huygen construction the Huygen principle to determine. How the wave front is going to evolve, as a function of time in vacuum it does not tell us anything very interesting as i have already told you, but if you dealing with refractive medium the Huygen's principle is very useful. Now, the Huygen principle as originally stated by Huygen 1600s does not tell us anything about diffraction. It allows us to follow the propagation of the wave front in a refracting medium. But it does not tell us anything about diffraction. This was generalized later by Fresnel in the 1800s.

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So, Fresnel in the 1800s generalized Huygens principle. So, that you could apply it to explain the phenomena of diffraction. So, let me explain to you what the Huygen-Fresnel principle is, so, we will consider a situation where we have a wave incident let us say it is incident on a screen like this. And the screen is opaque except for an aperture where through which light can pass through. So, the screen is opaque this part of the screen is opaque and except for an aperture through which the wave can pass the rest of the screen is opaque. And we would like to study the intensity of the radiation that passes through the wave that passes through at some point P over here.

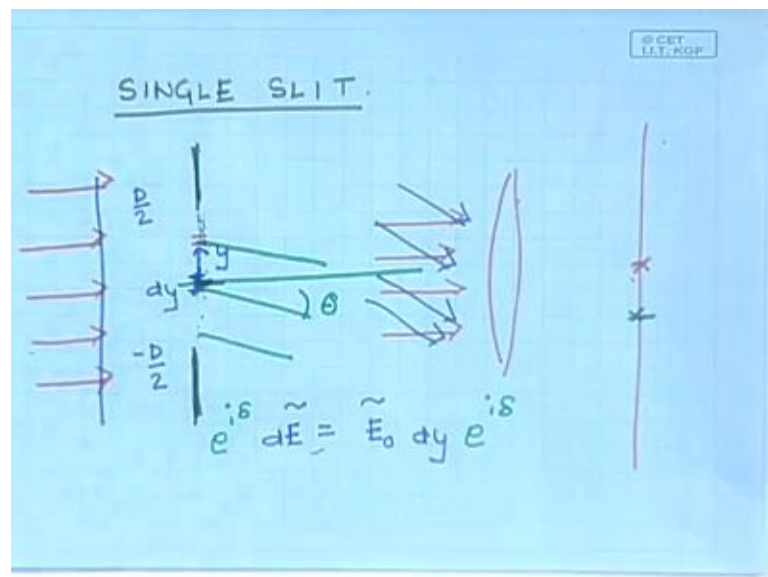
So, what the Huygen-Fresnel principle, Huygen-Fresnel construction tells us is, that each point on the aperture. So, each point on this aperture is going to act like a source for a secondary spherical wavelet. So, each point here is going to act like source for a secondary spherical wavelet. So, this is 1 point it is going to act like a secondary a source, for a secondary wavelet and if I wish to calculate the resultant intensity here. I should superpose the waves, the secondary waves emitted from all of the points on this aperture. Each point on this aperture is going to act like a separate source, to find the resultant here. I should superpose the waves emitted the secondary waves emitted by all of these sources. So, each point so, this point over here is going to emit a secondary wave and another point on this is also going to act like a source which is going emit a secondary wave and to find the intensity at this point due to this aperture.

I have to superpose the secondary waves from all of these sources and then use that to calculate the intensity that is the Fresnel the Huygen-Fresnel construction. The Huygen-Fresnel principle which is the heuristic approach to describe the phenomena of diffraction. Later on, a Russian physicist Kirchhoff, he showed that this prescription by the Huygen-Fresnel principle. He showed that this prescription is indeed a solution of the wave equation. So, we shall discuss the wave equation later, but let me just tell you right now, that Huygen and Fresnel gave a prescription by which we can calculate you can handle the phenomena of you can describe you can calculate quantities for the describing the phenomena of diffraction. We know those waves are governed by an equation called the wave equation.

Kirchhoff later on showed that the prescription in terms of secondary wavelet etcetera, given by Huygen and Fresnel together, the combination of the modification

of Huygen's principle proposed by Fresnel. So, it is a prescription, this how Kirchhoff showed that this prescription is actually a solution of the wave equation. So, let us now, apply this prescription let us apply the Huygen-Fresnel principle to a particular situation. The situation that we are going to consider is as follows we are going to consider a single slit.

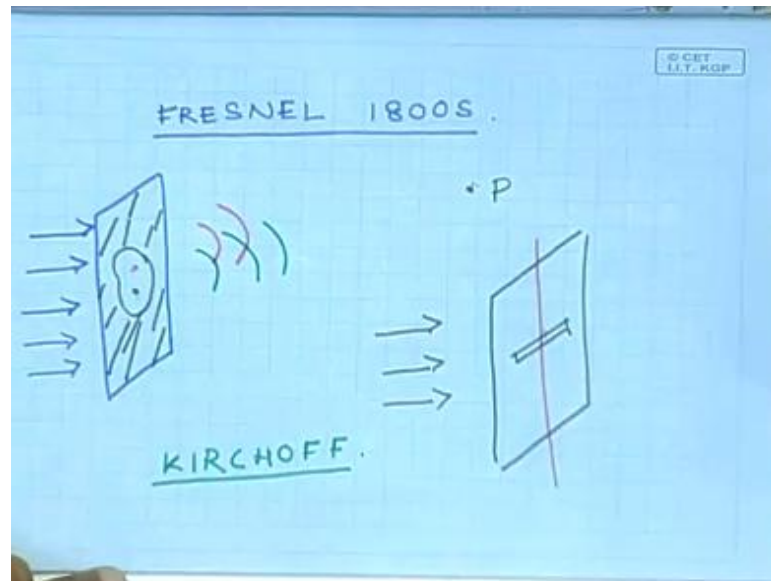
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So, we have a slit we are going to consider the diffraction pattern due to a single slit. So, the situation is as follows let me draw the situation here. We have the screen over here, on that screen we have a slit. So, there is a there an opaque screen and in that screen we have a slit. So, I have drawn an enlarged diagram the slit is actually quite small and it.

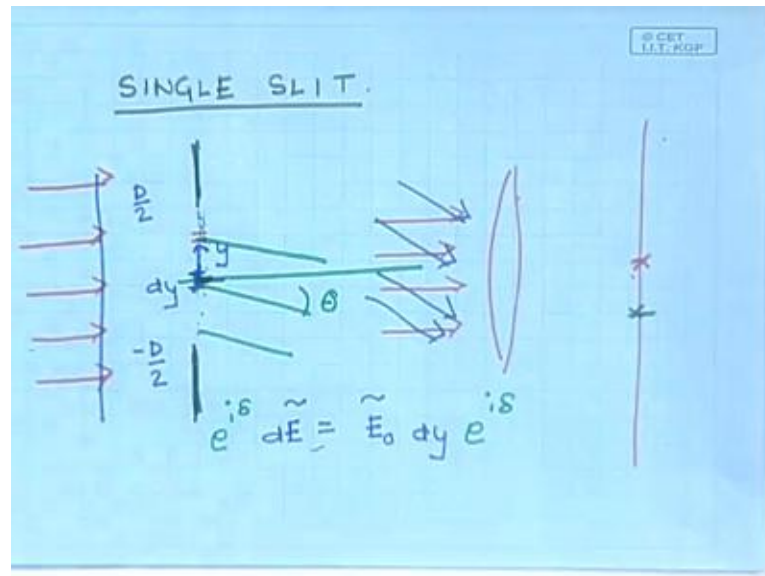


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So, give you a picture of the slit, let me draw it for you here. We have a we have an opaque screen like this, in this screen I make a small slit like this. And we have a plane wave incident on this we are interested in the intensity pattern on a screen far away. So, we will draw a section, I will draw a section through this whole thing. So, the section is going to be like this; for the time being I will ignore the other dimension of the slit which is larger i will focus only on the smaller dimension of the slit and I will draw, I have drawn it for you here.

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So, this is the centre of the slit and there is a plane wave which is incident on this whole thing. And we are interested in the intensity pattern on a screen which is placed far away. So, we are interested in the intensity pattern on a screen which is placed far away or alternatively we could place a screen over here, put a lens over here, the screen is at the focal length of this lens. So, what the lens does is that it takes all the waves, all the light which is incident in a particular direction. Let us say, in this direction and focuses it to a single point. So, all the waves coming in this direction, it will get focus to the point on the centre of the screen. All the waves in this direction in some other direction will be brought to a different point on the focal plane.

So, we would like to calculate the intensity pattern on the screen placed over here. The screen could either we taken to be very far away or you could put a lens make a lens arrangement which will focus the light on to the screen. Which will focus parallel waves on to the screen? So, we would like to consider this situation. Now, how do we handle this situation? So, we will apply the Huygen-Fresnel principle and analyze this particular situation. So, what is the Huygen-Fresnel principle tells us. The Huygen-Fresnel principle tells us that each point on this aperture on the slit is going to act like a source for a secondary wave.

So, let me now focus on this small area with a point at the centre. Now, you see here, let us first ask the question. How many secondary sources are there in the slit? You see, if you take this aperture, the aperture has a length of  $d$ . So, it goes from  $d/2$  to  $d/2$ , I will align the  $y$  axis in this direction. So, I will call this the  $y$  direction. This is the centre  $y = 0$  as at the horizon and this is a distance  $y$  away. This  $y$  goes from  $-d/2$  to  $d/2$  this is my aperture this is my slit, each point in this slit is going to act like a secondary source.

So, we have infinite number of points in this range  $-d/2$  to  $d/2$ . Let us focus on a small element  $dy$  at the origin. So, this is the small element  $dy$  placed at the origin and ask the question. What is the amplitude of the wave which comes out from here? So, the wave which comes out from here is going to be the amplitude of the wave, we will write that as  $dE$  the amplitude in complex notation is a complex number  $E$ . So, we will write down the amplitude of the wave as  $E dy$ .

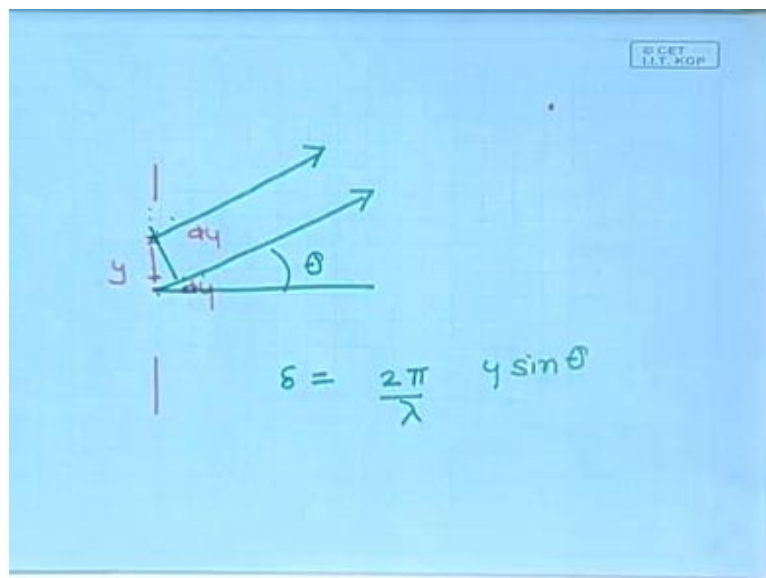
So, let me repeat what we are trying to do is, we are trying to calculate we have a plane wave which is incident on a slit. So, we have a plane wave incident on a screen which has a slit. We want to calculate the intensity pattern on a screen far away; each point on the slit acts like a secondary, like a source for a secondary wave each point on the slit acts like a source for a secondary wave. Let us take a line element  $dy$  on the slit located near the centre on the origin. And ask the question, what is the secondary wave emitted by the sources in this element  $dy$ . And I have written the secondary wave emitted from the sources in the small element  $dy$  as  $dE$  is equal to  $E dy$ .

So, it is proportional to the length of this because the number of sources is going to be proportional to the length of this and it is related to length through some constant  $E$ . See notice that, the plane wave is incident on this screen such that the wave fronts are parallel to the slit. Since the wave front is parallel to the slit, each of these sources which emit secondary waves is going to be oscillating with the same phase. If the wave front, incident wave front were at an angle then the sources would be emitting secondary waves with different phases, but in this case, all the sources inside this element  $dy$  are oscillating with the same phase. Because wave front is parallel to this and the secondary wave produced by them is proportional to the

number of sources, that I have which is  $dy$  and the through a constant of proportionality  $E_0$  tilde.

So, this is the amplitude of the wave produced by the small element  $dy$  at the centre of the slit. So, when I go and on the sit on the screen, I have to add up the contributions from all such elements  $dy$  located at different places not only at the centre, but a different distances. So, let me now consider a small line element  $dy$  located at a distance  $y$  over here. .Let me draw this picture a little more enlarged.

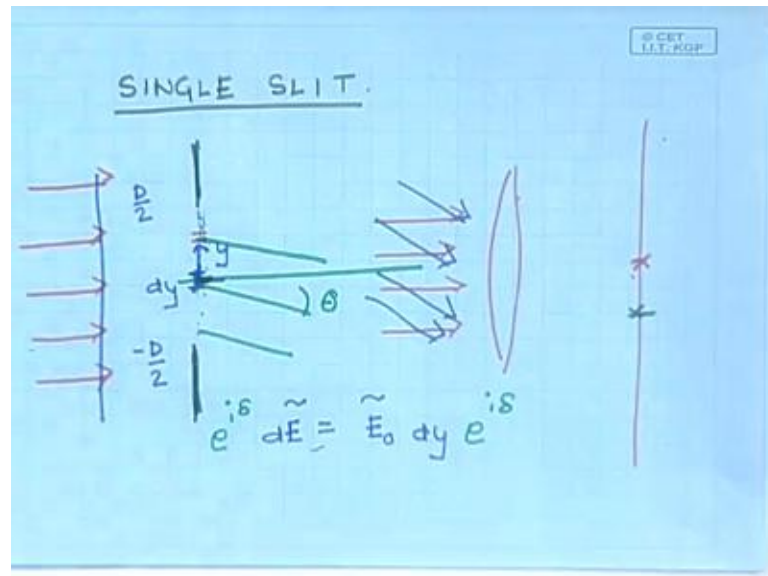
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This is my slit, this is the line element  $dy$  at the origin; this is the line element  $dy$  at the centre of the slit and this is the line element  $dy$  the distance  $y$  away from the centre. When I wish to calculate the intensity over here, I have to add up the contributions from all of these points. So, let me considered the contributions so, this is going to contribute and this 2 is going to contribute, but when I consider the super position at some point at an angle  $\theta$  away, these 2 secondary waves are going to contribute with the phase difference. And this phase difference occurs because of the path difference, the path difference is  $y \sin \theta$ .

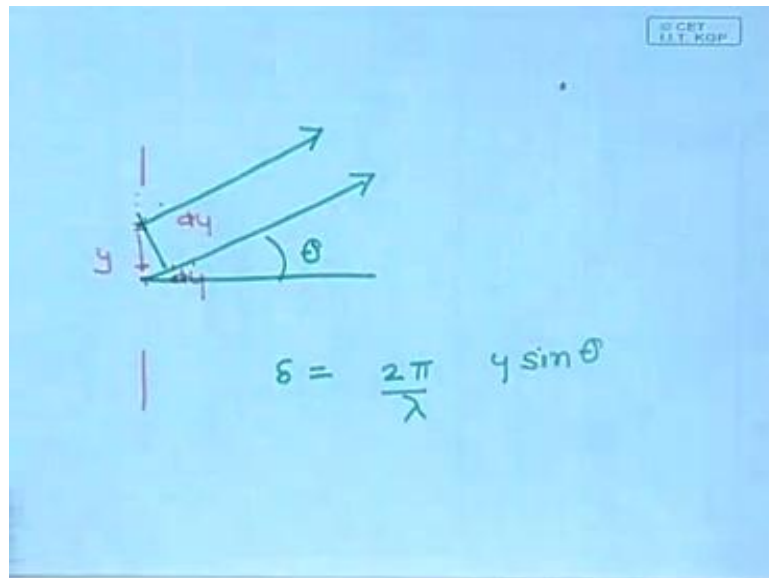
So, these 2 waves secondary waves emitted 1 emitted from the centre and 1 emitted at the from a distance  $y$  away are going to have phase difference. The phase difference  $\delta$  is  $2\pi$  by  $\lambda$  into the path difference  $y \sin \theta$ .

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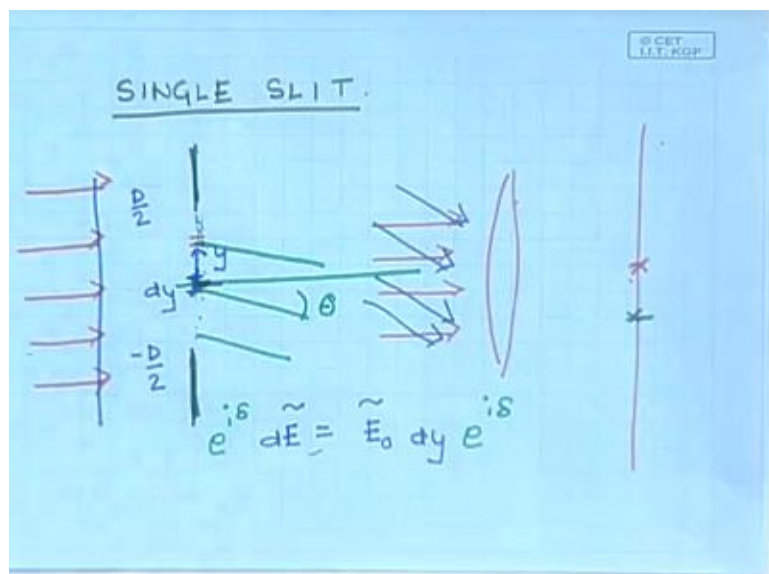


So, the point is this that if I wish to calculate the intensity at any point over here on the screen. I should be looking at the superposition from the waves, secondary waves from all of these sources at a particular angle theta. The role of the lens is that it focuses all the waves which arrive at a angle theta to this particular point. So, I should be considering all the waves emitted at an angle theta from all the source at an angle theta. And when they reach this screen these different waves are not going to arrive at the same phase they will be a phase difference. So, the wave emitted from a line element  $dy$  some distance  $y$  away from the origin is going to be at a phase difference. And the phase difference is  $\delta$  which I have calculated just now, the phase difference is  $\delta$  which I have calculated just now, over here.

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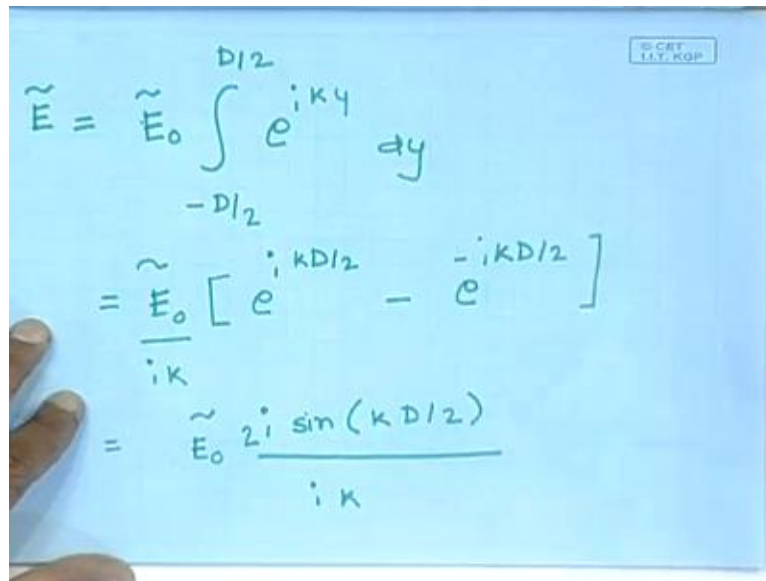


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So, the contribute form any element  $dy$  on the slit is going to be this, if the element were at the centre it would be just this much. But if the element is shifted a distance  $y$  from the centre this is going to be a phase difference, because it has travel different path to this point. Now, when I want to calculate the resultant over here, I have to add up the contribute from all sources on the slit, which in this case is an integral from over  $y$  from minus  $d$  by 2 to plus  $d$  by 2.

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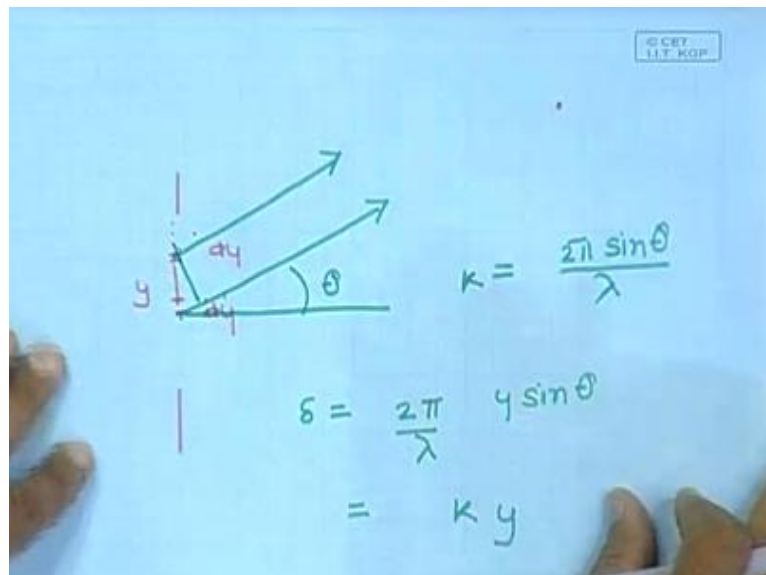


The image shows a handwritten derivation on a blue background. It starts with the expression for the electric field  $\tilde{E}$  as an integral of  $\tilde{E}_0 e^{iky}$  from  $-D/2$  to  $D/2$ . The next step shows the integral evaluated as  $\tilde{E}_0 [e^{iKD/2} - e^{-iKD/2}] / iK$ . The final result is  $\tilde{E}_0 \frac{2i \sin(KD/2)}{iK}$ .

$$\begin{aligned}\tilde{E} &= \tilde{E}_0 \int_{-D/2}^{D/2} e^{iky} dy \\ &= \tilde{E}_0 \left[ \frac{e^{iKD/2}}{iK} - \frac{e^{-iKD/2}}{iK} \right] \\ &= \tilde{E}_0 \frac{2i \sin(KD/2)}{iK}\end{aligned}$$

So, the resultant electric field the resultant wave at any angle theta is the superposition of all of those contributions which is an integral from minus D by 2 to D by 2 E nought that is the amplitude over all amplitude e to the power i delta and delta,

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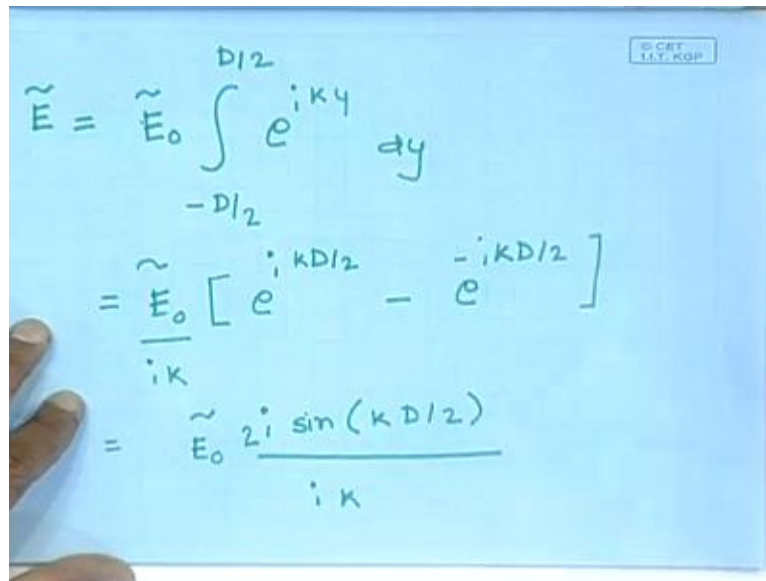


The image shows a diagram and two equations. The diagram illustrates a vertical line of length  $y$  with a horizontal line extending from its base. An angle  $\theta$  is shown between the horizontal line and a line from the top of the vertical line. The path difference  $\delta$  is indicated as the vertical distance between the horizontal line and the line from the top. The equations are  $k = \frac{2\pi \sin \theta}{\lambda}$  and  $\delta = \frac{2\pi}{\lambda} y \sin \theta = ky$ .

$$k = \frac{2\pi \sin \theta}{\lambda}$$
$$\delta = \frac{2\pi}{\lambda} y \sin \theta = ky$$

We see can be written as, some K into y, where k you can see from here, K is  $2\pi \sin \theta$  by lambda.

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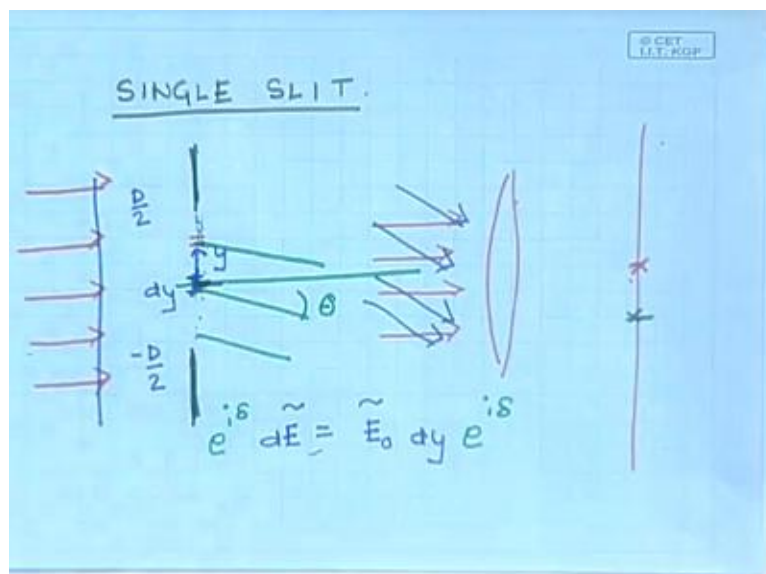


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$$\begin{aligned}\tilde{E} &= \tilde{E}_0 \int_{-D/2}^{D/2} e^{iKy} dy \\ &= \tilde{E}_0 \left[ \frac{e^{iKy}}{iK} \right]_{-D/2}^{D/2} \\ &= \tilde{E}_0 \frac{2i \sin(KD/2)}{iK}\end{aligned}$$

So, we can write this as e to the power i Ky dy.

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Let me remind you again, what I am trying to do what we are doing is we want to calculate the resultant of the super position of all the waves from the secondary sources; as they would be received at an angle theta. So, to do this I have to do add up the contributions from all these waves. Now, this is  $y$ ; so, the ways the different sources are located at different  $y$ 's values of  $y$  have to superpose them which in this case is not a sum, but an integral.



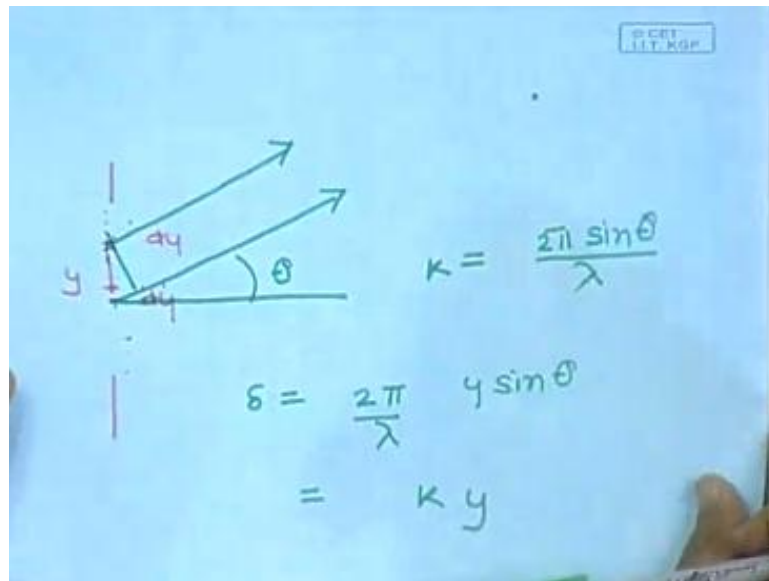
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$$\begin{aligned}\tilde{E} &= \tilde{E}_0 \int_{-D/2}^{D/2} e^{iky} dy \\ &= \tilde{E}_0 \left[ \frac{e^{ikD/2}}{ik} - \frac{e^{-ikD/2}}{ik} \right] \\ &= \tilde{E}_0 \frac{2i \sin(kD/2)}{ik}\end{aligned}$$

I have basically, I am superposing an infinite number of waves, in the slit I have an infinite number of sources; these infinite sources each point in the interval minus  $D/2$  to plus  $D/2$  is a source for a secondary wavelet. I have to superpose the contribution from each of these secondary sources. So, I have an integral in this case, I have to integrate  $y$  from minus  $D/2$  to plus  $D/2$ ; which is what I have written over here. So, this shows me the resultant, the result of superposing the secondary wavelets emitted from all of those sources on the slit located at different points on the slit.

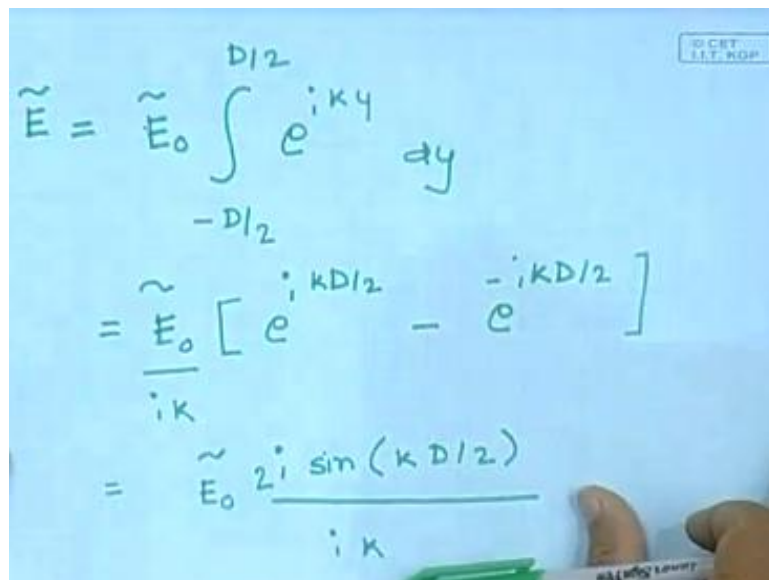
Now, this integral we know how much it gives us. So, let us do this integral it gives us  $\tilde{E}_0$  and here, we have  $e$  to the power  $ikD/2$  minus  $e$  to the power  $-ikD/2$  this divided by  $e$  to the power  $ik$ . So, I can write it here  $ik$  and  $e$  to the power  $ikD/2$  minus  $e$  to the power  $-ikD/2$ . We know that, this is  $E_0$  tilde, this is going to be  $2i \sin(kD/2)$  divided by  $ik$  and there will be a factor of 2. Which let me write this we can write putting in the value of  $k$ ?

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So, remember that  $k$  is  $2\pi$  lambda the  $2\pi \sin \theta$  by lambda let me also put in the value of  $k$  And write this.

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Handwritten equations on a blue background:

$$\tilde{E} = E_0 D \frac{\sin\left[\frac{\pi D \sin\theta}{\lambda}\right]}{\frac{\pi D \sin\theta}{\lambda}}$$
$$I = \frac{1}{2} \tilde{E} \tilde{E}^*$$
$$= I_0 \text{sinc}^2\left[\frac{\pi D \sin\theta}{\lambda}\right]$$

$\text{sinc}(x) = \sin(x)/x$

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Handwritten equations on a blue background:

$$\tilde{E} = \tilde{E}_0 \int_{-D/2}^{D/2} e^{iky} dy$$
$$= \frac{\tilde{E}_0}{ik} \left[ e^{iKD/2} - e^{-iKD/2} \right]$$
$$= \tilde{E}_0 \frac{2 \sin(KD/2)}{k}$$

So, the resultant superposition of all of these waves from this infinite number of sources which are on the slit the resultant super position is E is equal to E nought then this i cancels out. And I put in the value of k, k is 2pi sin theta by lambda the factor of 2 cancels out.

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The image shows a handwritten derivation on a blue background. At the top right, there is a small logo that reads "© CBT I.I.T. KGP". The main derivation consists of three lines of equations:

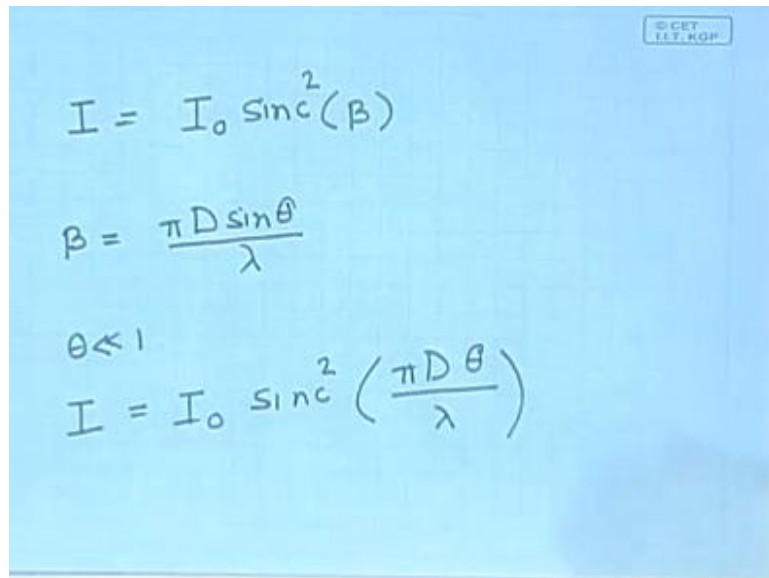
$$\vec{E} = E_0 D \frac{\sin \left[ \frac{\pi D \sin \theta}{\lambda} \right]}{\frac{\pi D \sin \theta}{\lambda}}$$
$$I = \frac{1}{2} \vec{E} \vec{E}^*$$
$$= I_0 \text{sinc}^2 \left[ \frac{\pi D \sin \theta}{\lambda} \right]$$

Below the equations, the definition of the sinc function is given as:

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

And I can write this as  $\sin \pi D \sin \theta$  by  $\lambda$  divided by  $\pi D \sin \theta$  by  $\lambda$  and there is an extra factor of  $D$  which I have introduced. So, I can multiply this thing by  $D$  over here. So, this is the superposition of all the waves and if I wish to calculate the intensity is half  $E$  into its complex conjugate. And this gives me sinc square, I will write like this sinc square  $\pi D \sin \theta$  by  $\lambda$ . Where remember that  $\text{sinc } x$  is equal to  $\sin x$  divided by  $x$ . We can also write this; so, the usually we write this as follows; we write it like this.

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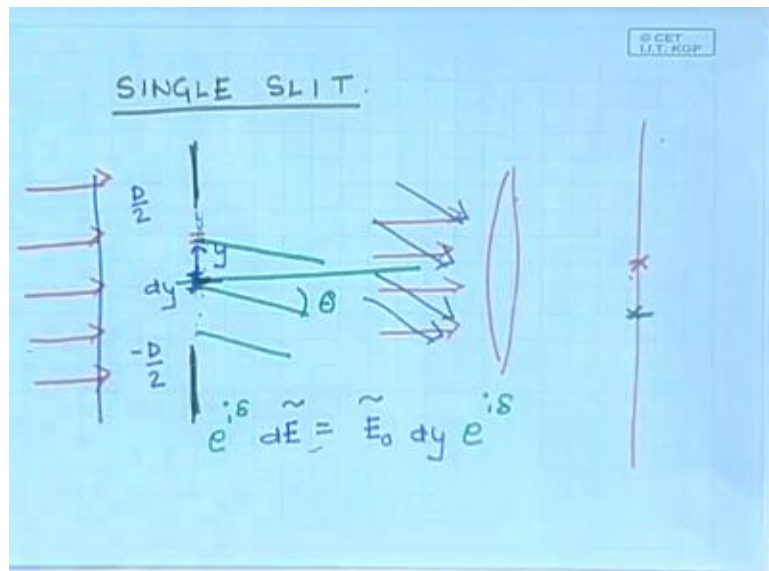
Handwritten equations on a blue grid background:

$$I = I_0 \text{sinc}^2(\beta)$$
$$\beta = \frac{\pi D \sin \theta}{\lambda}$$
$$\theta \ll 1$$
$$I = I_0 \text{sinc}^2\left(\frac{\pi D \theta}{\lambda}\right)$$

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I is equal to  $I_0 \text{sinc}^2 \beta$  where  $\beta$  is  $\pi D \sin \theta / \lambda$  and if  $\theta$  is much less than 1. Then  $I$  is equal to  $I_0 \text{sinc}^2 \left( \frac{\pi D \theta}{\lambda} \right)$ . So, let me rewind you now, what we have let rewind you again what we have calculated what we have calculated is as follows.

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We have a plane wave, which is incident on a screen. So, we have a screen this, is the screen in that screen we have a slit the slit has a width  $D$  the length of slit in slit

in the other direction is very large. It has width  $D$  the plane wave is incident on this. We want to calculate the intensity that would be observed on a screen which is placed very far away or you have a screen you have placed a lens over here and the screen is at the focal length of this lens. We would like to calculate the intensity pattern on the screen over here and we have calculated it and it turns out to be.

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The image shows handwritten mathematical derivations on a blue background. At the top right, there is a small logo for '© CET IIT, KGP'. The main equations are:

$$\tilde{E} = E_0 D \frac{\sin\left[\frac{\pi D \sin\theta}{\lambda}\right]}{\frac{\pi D \sin\theta}{\lambda}}$$

$$I = \frac{1}{2} \tilde{E} \tilde{E}^*$$

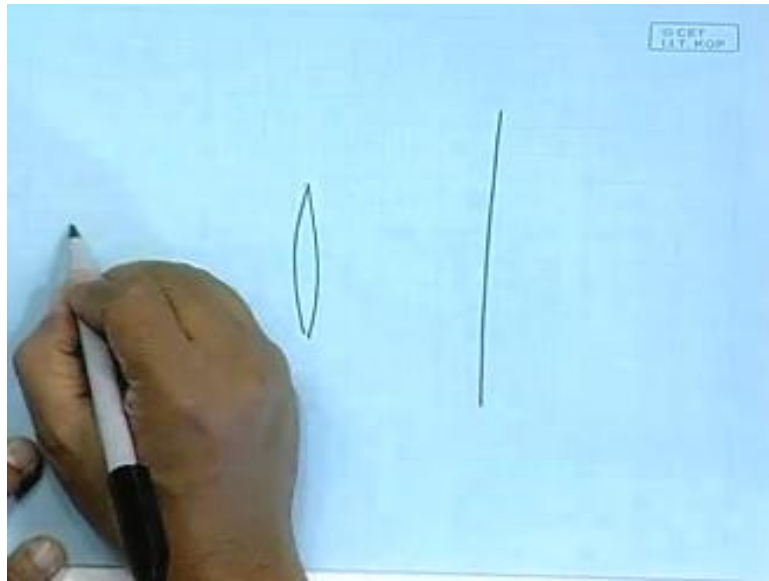
$$= I_0 \text{sinc}^2\left[\frac{\pi D \sin\theta}{\lambda}\right]$$

Below these equations, the definition of the sinc function is given as:

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

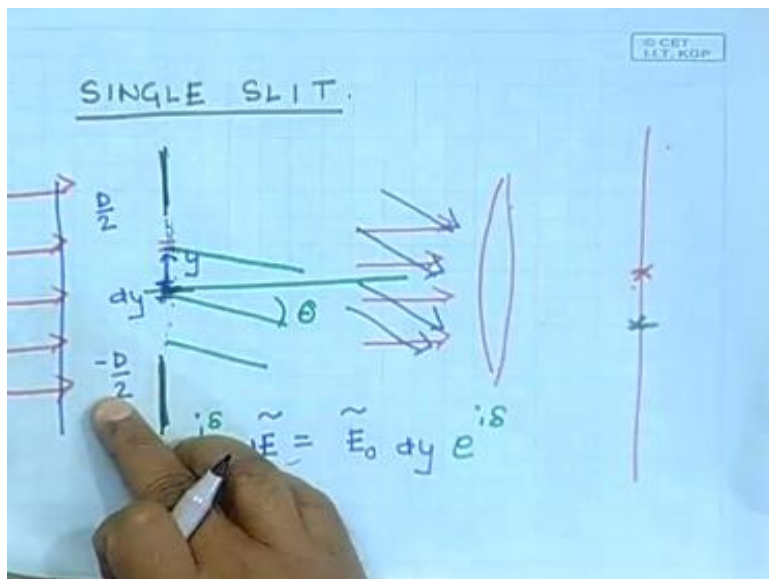
I which is the function of theta is  $I_0 \text{sinc}^2(\pi D \sin\theta / \lambda)$ . Let us, just forget about this expression and rethink about the problem.

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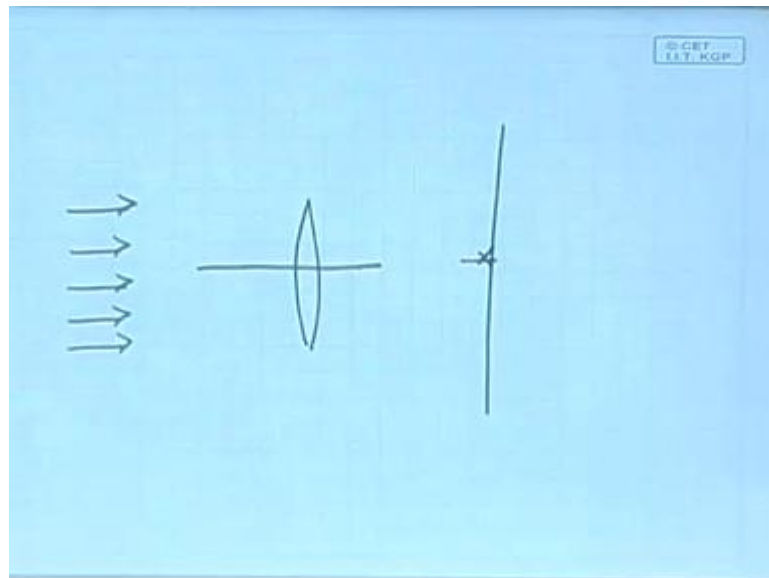


So, I have a screen, I have a lens, I have a plane wave in this case.

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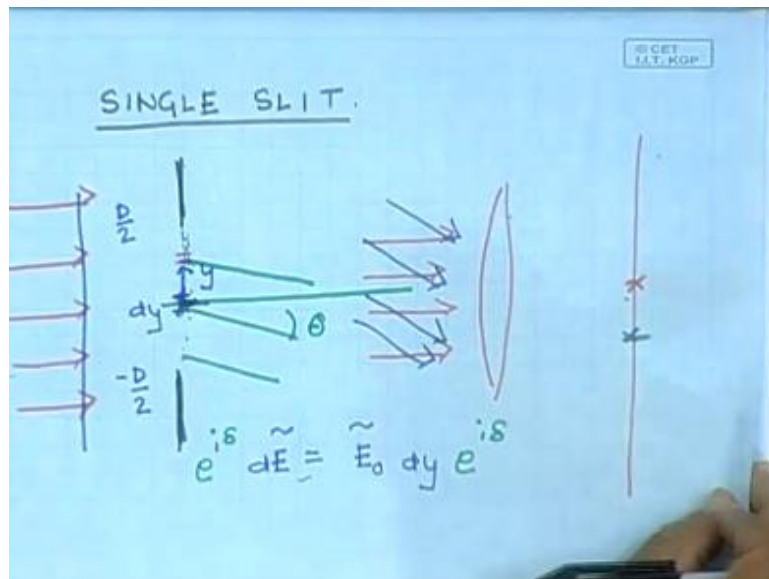
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The plane wave is incident like this, horizontally normal to the screen. So, I have a plane wave which is incident like this. Question is, what do you expect the lens to do? What kind of an image do you expect to see on the screen over here or equivalently the screen is located very far away. Now, what the lens does in this case, we will discuss the situation with the lens. What the lens does it the focuses the plane wave to a point and if the wave is parallel with the optic axis of the lens, the wave gets focused to a point at the centre.

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So, in this case, what you expect to see is that you will get a spot at the centre of the screen. This is exactly the same situation, you have a plane wave which is incident and naively you would think that a part of the plane wave would go through fall on this lens and you would get a spot at the centre. But when you take into account, the wave nature of the light, when you take into account the wave nature of light, what you find is that the intensity pattern on the screen is actually given by this.

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$$I = I_0 \text{sinc}^2(\beta)$$

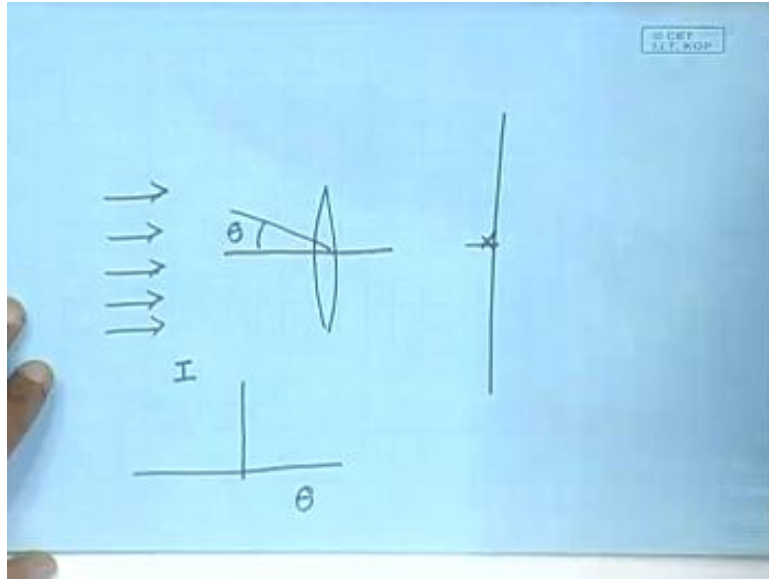
$$\beta = \frac{\pi D \sin \theta}{\lambda}$$

$\theta \ll 1$

$$I = I_0 \text{sinc}^2\left(\frac{\pi D \theta}{\lambda}\right)$$

Let me plot the intensity patter for you on the screen.

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So, as would be seen on the screen. So, if I plot the intensity pattern as a function of theta, theta is the angle over here, each different value theta corresponds to different point of the screen. So, I am plotting the intensity as a function of theta.

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$$\beta = \frac{\pi D \sin \theta}{\lambda}$$

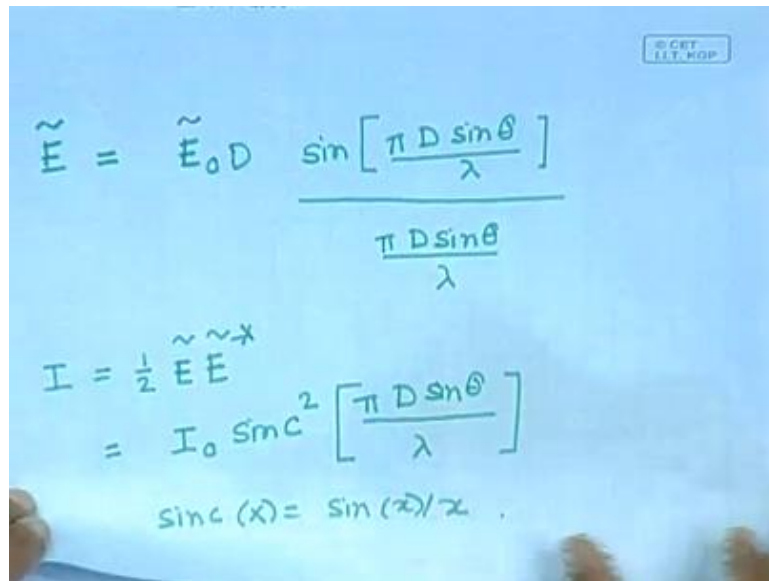
$$\theta \ll 1$$

$$I = I_0 \operatorname{sinc}^2 \left( \frac{\pi D \theta}{\lambda} \right)$$

The image shows handwritten equations and a partial graph. The equations are  $\beta = \frac{\pi D \sin \theta}{\lambda}$  and  $I = I_0 \operatorname{sinc}^2 \left( \frac{\pi D \theta}{\lambda} \right)$ , with the condition  $\theta \ll 1$ . Below the equations, a hand is drawing a graph with a vertical axis labeled  $I$  and  $I_0$ .

As given by this formula and will considered the situation where theta is small. So, for small theta you see that the intensity pattern for is a sinc function the square of the sinc function  $\sin x$  by  $x$  at theta equal to 0. We know that, the sinc function will have is  $\sin x$  by  $x$  at  $x$  equal to 0 we know has a value 1.

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The image shows handwritten mathematical derivations on a blue background. At the top right, there is a small logo that reads "© CBT IIT, KGP". The main equations are:

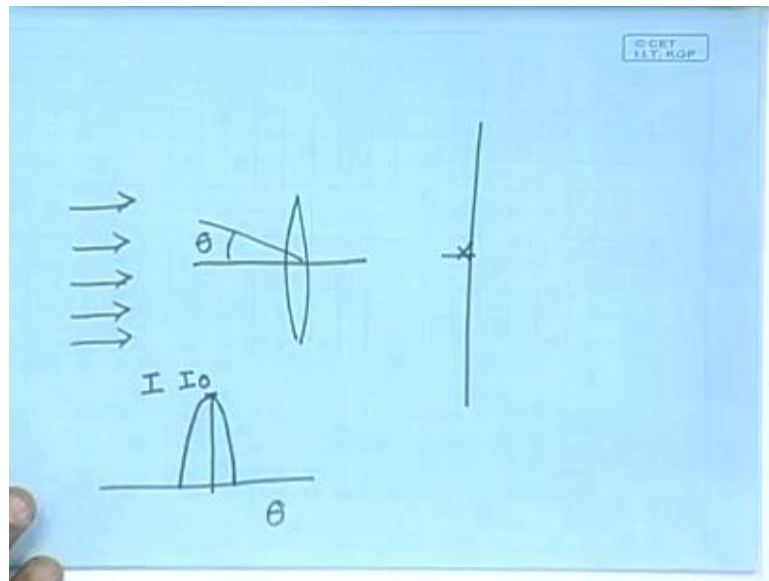
$$\tilde{E} = \tilde{E}_0 D \frac{\sin\left[\frac{\pi D \sin\theta}{\lambda}\right]}{\frac{\pi D \sin\theta}{\lambda}}$$
$$I = \frac{1}{2} \tilde{E} \tilde{E}^*$$
$$= I_0 \text{sinc}^2\left[\frac{\pi D \sin\theta}{\lambda}\right]$$

Below these equations, the definition of the sinc function is given as:

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

So, at theta equal to 0 the intensity is going to be I nought, remember what I nought is I nought is just the, see when I have calculated the intensity, I have to square this formula take its complex conjugate and multiplied with itself this is real except for this term E0 here.

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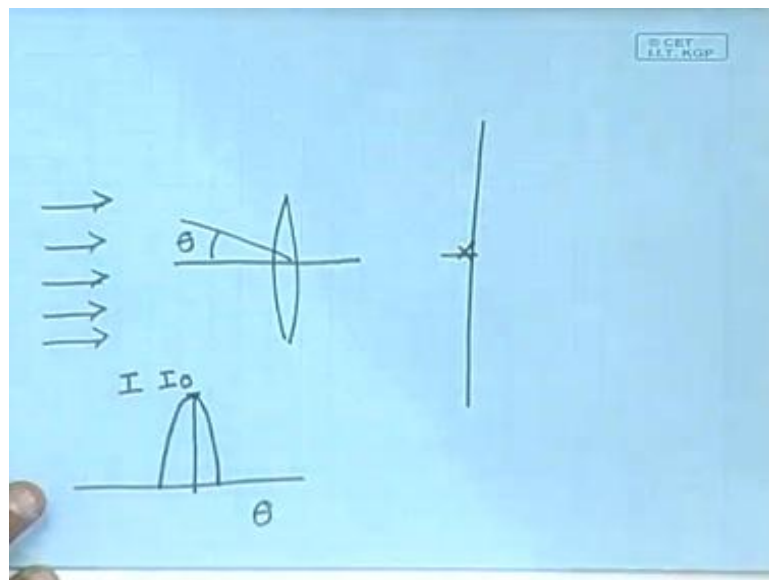
So, whatever constant term I get outside, I call that  $I_0$  it is an overall constant at the centre the intensity that you are going to get is this constant  $I_0$ . So, conversely the  $I_0$  is the value of the intensity that you are going to get at the centre of the screen over here. Now, naively you would expect that is all that they would be just a spot at the centre, but know our calculation tells us that the intensity does not fall to 0 as I move away on the screen. The intensity drops as the sinc square. So, the intensity drops on both sides and it goes to 0.

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$$I = I_0 \text{sinc}^2(\beta)$$
$$\beta = \frac{\pi D \sin \theta}{\lambda} \quad \beta = \pm \pi$$
$$\theta \ll 1$$
$$I = I_0 \text{sinc}^2\left(\frac{\pi D \theta}{\lambda}\right)$$
$$\theta = \lambda / D$$

When beta is equal to plus minus pi sin pi 0; so, sinc pi also 0 or in this case, what you have is it goes to 0 when theta is equal to lambda by D.

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$$I = I_0 \text{sinc}^2(\beta)$$

$$\beta = \frac{\pi D \sin \theta}{\lambda} \quad \beta = \pm \pi$$

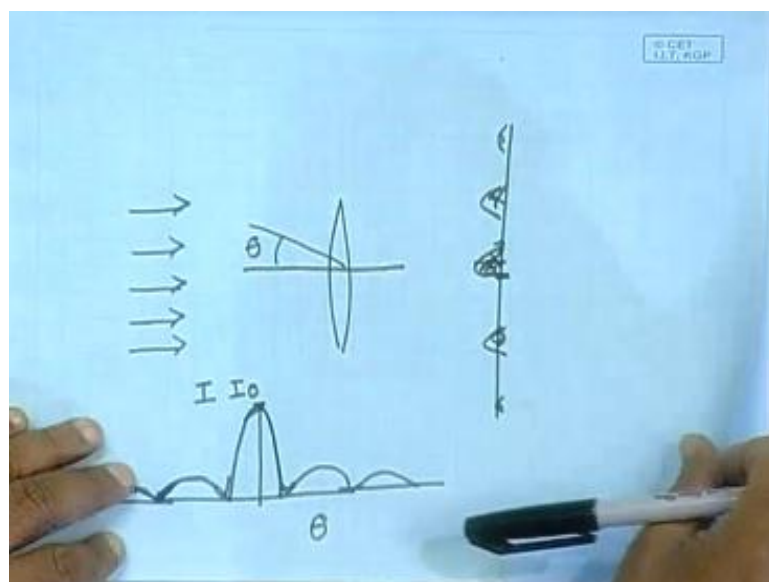
$$\theta \ll 1$$

$$I = I_0 \text{sinc}^2\left(\frac{\pi D \theta}{\lambda}\right)$$

$$\text{min } \theta = \lambda/D \quad \text{max } \theta = \frac{2\lambda}{D}$$

So, then further away as you increase theta; the value of the sinc function again rises it reaches a maxima when beta is 2pi plus minus 2 pi or in this case, this minimum. It will have a maxima again, first maxima when theta is equal to 2 lambda by D. And so, forth you will have a pattern of minima and maximas.

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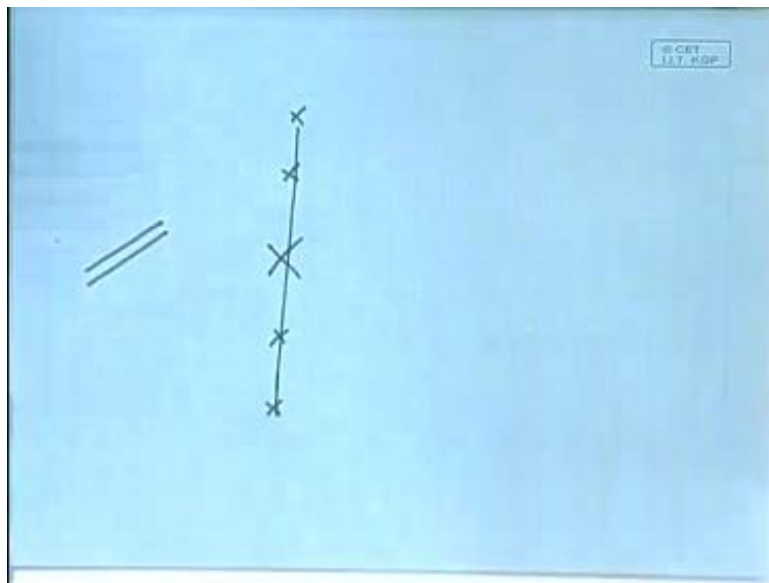


So, the intensity pattern will look and the screen is going to look something like this; the amplitude of the later maxima is going to fall. So, the sinc square function the intensity pattern on the screen over here is going to have a pattern which looks like this. You are going to have a finite size spot over here and then you will get some dark region and then again you will have a bright spot some over here. So, you will have a

bright spot here, which is going to be of finite size and you will have a bright spot here and then you will have a smaller intensity bright here.

Similarly, here and here, so, you are going to get a sequence of bright spots, the intensity of bright spot is going to diminish as go to larger and larger distances from the centre or as you go to larger and larger angles, but they will be there and the intensity of these bright spot is going fall. This is what is refer to as the diffraction pattern of a single slit. Let me draw it for you here.

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So, along the if this is the slit, in the direction along the smaller direction of the slit you are going to get bright spot here and then a small fainters a brighter bright spot not has bright as the central here. And then you are going to get a bright spot here.

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The image shows handwritten mathematical derivations on a blue background. The first equation is 
$$\tilde{E} = \frac{\tilde{E}_0 D \sin\left[\frac{\pi D \sin\theta}{\lambda}\right]}{\frac{\pi D \sin\theta}{\lambda}}$$
 The second equation is 
$$I = \frac{1}{2} \tilde{E} \tilde{E}^* = I_0 \text{sinc}^2\left[\frac{\pi D \sin\theta}{\lambda}\right]$$
 Below the second equation, the definition of the sinc function is given as 
$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

so, you going to get a sequence of bright spots this is whose intensity is given by this formula or for small theta it is given by The formula over here.

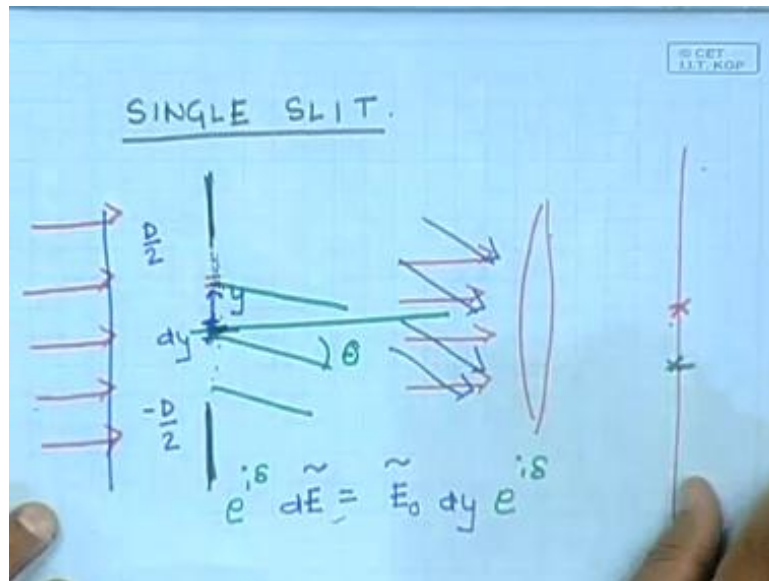
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The image shows handwritten mathematical derivations on a blue background. The first equation is 
$$I = I_0 \text{sinc}^2(\beta)$$
 The second equation is 
$$\beta = \frac{\pi D \sin\theta}{\lambda} \quad \beta = \pm \pi$$
 The third equation is 
$$\theta \ll 1$$
 The fourth equation is 
$$I = I_0 \text{sinc}^2\left(\frac{\pi D \theta}{\lambda}\right)$$
 The fifth equation is 
$$\text{min } \theta = \lambda/D \quad \text{max } \theta = \frac{2\lambda}{D}$$

You have minimas and you have maximas, the minimas will occur whenever beta is an all multiple of pi, the maximas will occur whenever theta is an even multiple of pi when sin theta becomes equal to 1. So, what you see here is that when you send a plane wave through an aperture of a finite size.

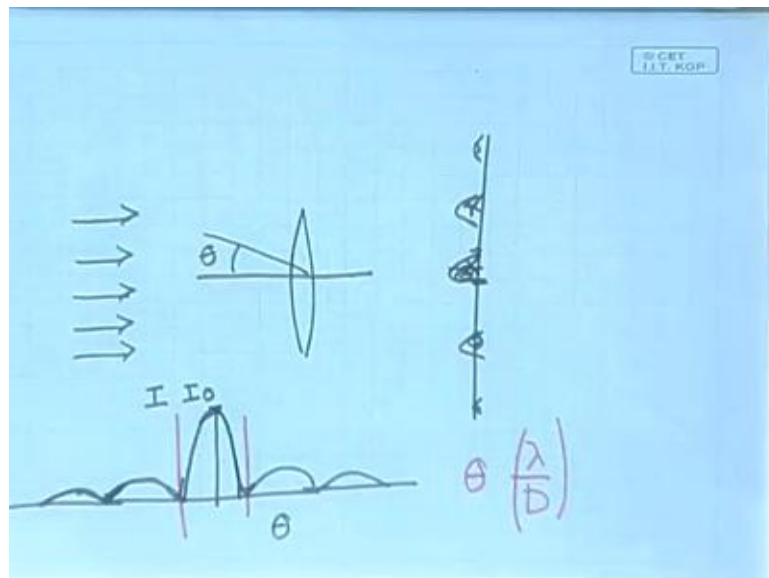


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So, in this case we have sent a plane wave, through an aperture of size  $D$ . The wave that comes out, the plane wave which is incident is all in 1 direction when the wave comes out there will be a spread in directions. Let us ask the question, what is going to be the spread in directions?

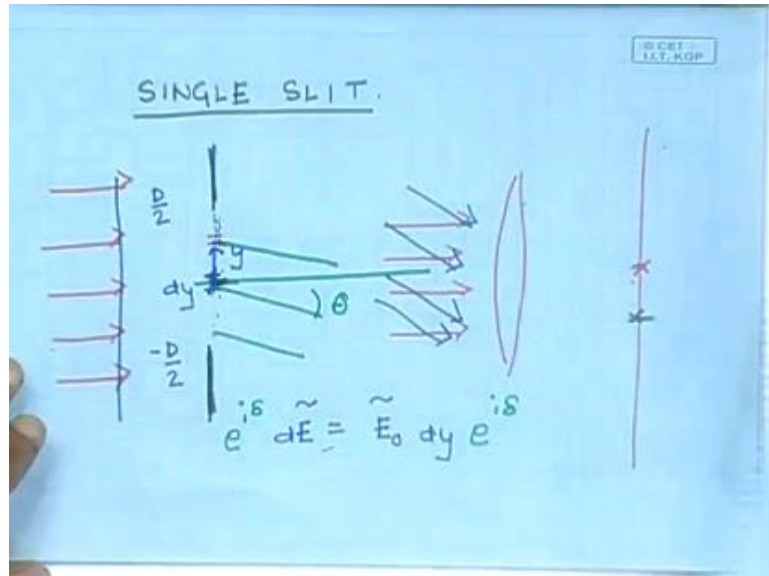
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The spread in directions you may think of as being this much, this first 0 occurs at  $\lambda = D \theta$ . So, this  $\lambda = D \theta$  the wave length

compared to the dimension of the slit gives you an idea of the spread in directions of the waves that come out.

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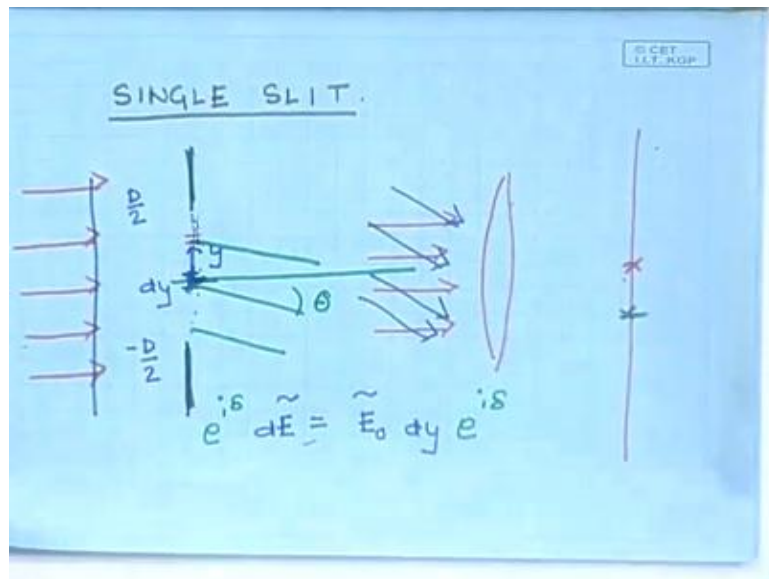
So, when the waves that come out from this aperture are going to be spread over a range of directions of the and the ray spread in angle is going to be of the order of  $\lambda$  by  $D$ . And this is going to produce, then you will this is the main bright spot that you have it is going to have a width of the twice  $\lambda$  by  $D$ . And then you are going to have a more bright spots located on the both sides the intensity of these bright spots is going to fall. And then you are going to have even further bright spots; even located, even further away they are going to be fainter we are going to have a sequence of bright spots like this. Which are going to be equally spaced and the intensities are going to be smaller and smaller.

So, this is what the phenomena of diffraction is all about you have these fringes produced whenever you place an abstraction in the path of light. When you whenever there is an abstraction in the path of light there will be fringe pattern as you have seen which is going to be produce. Each point in the aperture, in each path, each point which is not abstracted is going to act like a secondary is going to act like a source for a secondary wave. And if you wish to calculate the intensity at some of light at some other point, you have to superpose the contribution of all of these

secondary waves coming from each of the sources and these could add up the constructively or destructively just like interference.

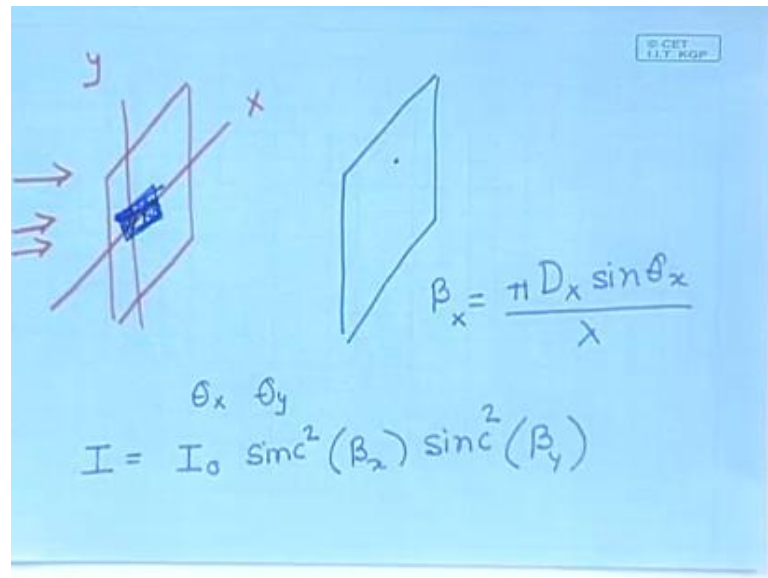
The only difference being that you now have, many waves not just 1 or 2; in this situation you have an infinite number of waves you have to add up all these waves; they could add up either constructively or destructively and this gives rise to the intensity pattern which we refer to as the diffraction fringes. Now, the phenomena diffraction is quite ubiquitous it occurs practically everywhere. So, let me now, discuss some of the implications, let me discuss the diffraction pattern in some more detail and then go into some of the implications of this diffraction pattern.

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So, the first thing which I should tell now is, as is that is follows; we have till now, we have only considered 1 the slit being of only 1 demand of having only 1 dimension. In reality you do not have a 1 dimensional slit; the slit always has a will always have 2 dimensions. So, what happens when I have a real slit which has 2 dimensions?

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So, let me consider a slit which is align with the xy plane. So, the light we are considering an aperture and this is the x axis this is the y axis and the light is incident along the z direction and I have a slit, the slit let us say looks something like this. This is the slit, the rest of the screen is opaque and we are interested in the diffraction pattern and the pattern on a screen which is placed far away. So, we now want to calculate the intensity at some point on the screen. So, now there will be 2 angles theta x and theta y the angle there will be 2 angles the theta x and theta y.

The intensity pattern can now, be written as I nought sinc square beta x sinc square beta y, where beta x is equal to pi Dx sin theta x by lambda. Where Dx is the width along the x direction, theta x is the angle along the x direction and beta y is the same as beta x, but I have to replace Dx with Dy sin theta x by sin theta y. So, is the product of the 2 diffraction pattern. So, the resultant intensity pattern is a product of sinc square beta x into sinc square beta y.

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$$I = I_0 \text{sinc}^2(\beta)$$

$$\beta = \frac{\pi D \sin \theta}{\lambda} \quad \beta = \pm \pi$$

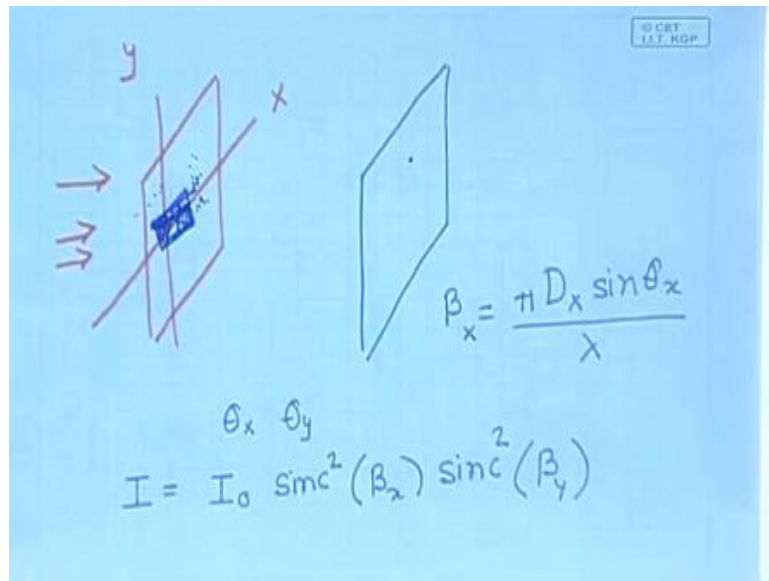
$$\theta \ll 1$$

$$I = I_0 \text{sinc}^2\left(\frac{\pi D \theta}{\lambda}\right)$$

$$\theta_{\min} = \lambda / D \quad | \quad \theta_{\max} = \frac{2\lambda}{D}$$

What's the consequence of this let us go back to our 1 dimensional situation? Notice, that the width of the first bright spot all the location of the first maxima, all of these increase if I decrease the width of the slit. The smaller the value of D the larger is going to be the width of the first maxima of the primary maxima that comes out the larger is going to be the spacing to the first order maxima. So, these are going to get larger and larger; the diffraction phenomena is important, the importance of the diffraction phenomena increases. When I make these slit width smaller and in such a situation.

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The diffraction is going to be more prominent along the direction, in which the slit is smaller. So, if I have a slit which looks like this then this direction is smaller. So, this is going to play a more important role in the diffraction. This is going to play a less important role in diffraction. The diffraction pattern of this is going to spread out more. This is going to be more concentrated. Let me stop our discussion over here and resume from this point in the next class.

