Physics - I Oscillations and Waves
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Lecture - 19
Coherence (Contd.)

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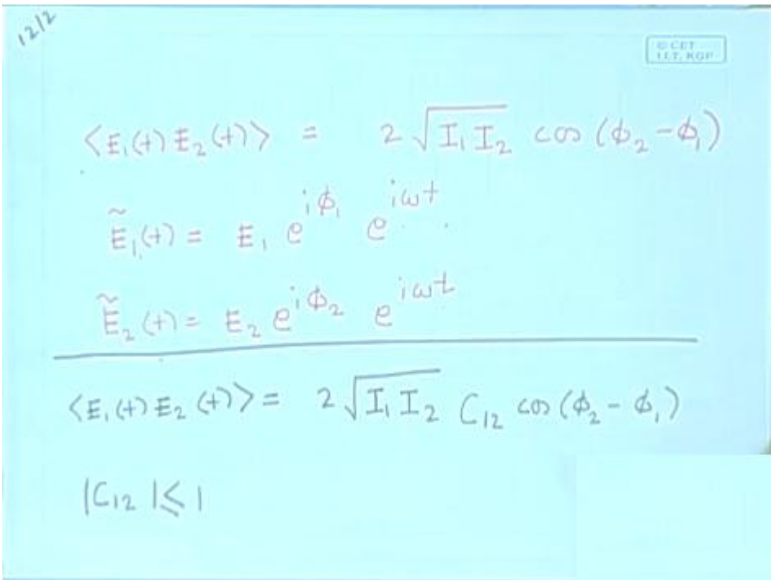

Good morning. In the last class we had started discussing coherence. Let me first recapitulate what we had discussed in the last class and I shall then, continue from there.

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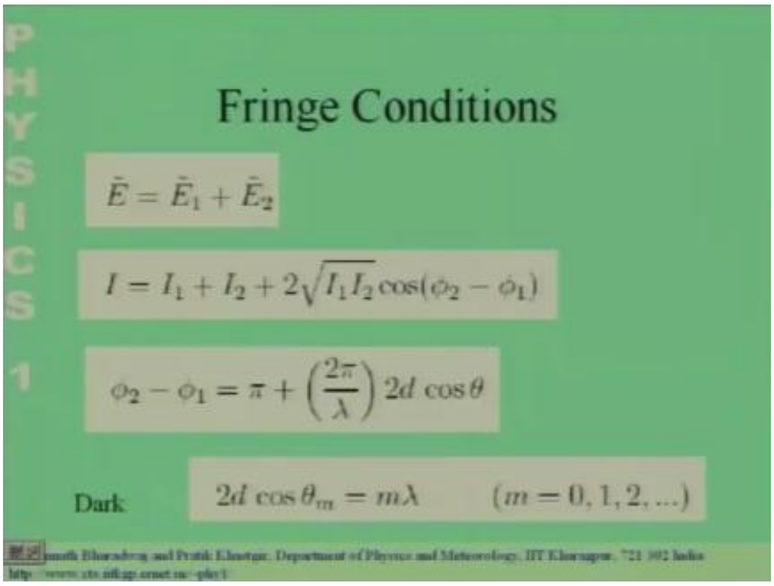

So, we had started off by taking a look at the basic for the fundamental formula, which we use when discussing interference. So, in interference we have the super position of 2 waves and the intensity of the resultant is I 1; the intensity of the first wave plus I 2 the intensity of the second wave plus 2 square root of I 1 I 2 cos phi 2 minus phi 1, where phi 2 minus phi 1 represents the phase difference between the 2 waves. And it is the second term, this particular term which is responsible for interference. And we were taking a closer look at the origin of this particular term.

Now, let me also remind you that, when we had derived this particular expression for the resultant intensity, we had assumed that we have 2 waves which are precisely monochromatic, in the sense that, they have they are sinusoidal waves of a single frequency, which is exactly the same for both the waves. And with this assumption, we had arrived at the term, responsible for interference by from this product.

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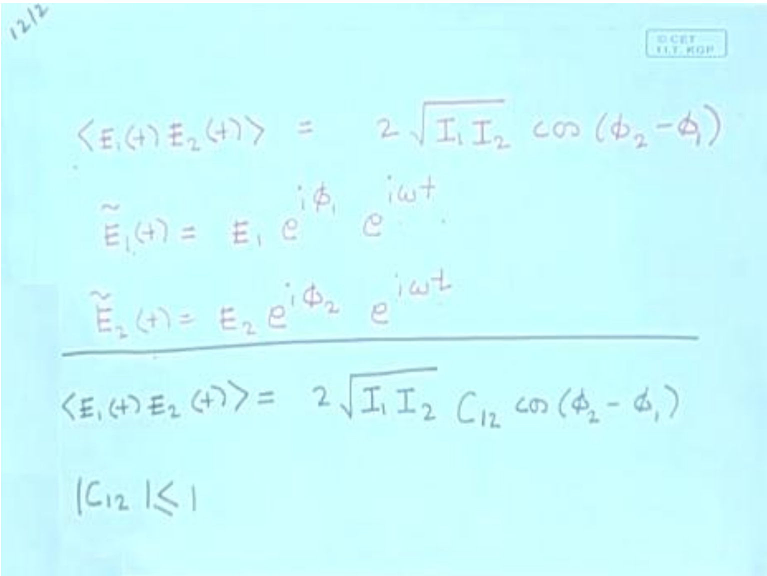

From looking at from that the term responsible for interference arises from this term E 1 into E 2. So, it arises from the product of the 2 waves, the time average of this and in the situation where these 2 waves are purely monochromatic, pure sinusoidal waves of the same frequency. This time average gives us 2 root I 1 I 2 cos phi 2 minus phi 1. And this is the term which is responsible for interference.

Now, it this assumption that we have made that, each wave is a pure sinusoidal of a single frequency is never true. So, it is not possible actually to have waves of a single frequency, there will always be a spread in frequencies. And in the last class, we were studying a consequence of this; we were studying the consequence of the fact that, there will always be a spread in the frequencies. And we had seen that, if you incorporate this if you incorporate the spread in frequencies, there will be a modification of this time average.

So, this time average is no longer going to be equal to this, it will be different and it will be less than this. And the difference between this monochromatic situation; the situation where I have only a single frequency, we quantified through this factor C 1 2, which quantifies the degree of coherence. So, now the time average of the 2 product of the 2 waves is; we write it as 2 times root of I 1 I 2 with the same factor of cos phi 2 minus phi 1 the phase difference between the 2 waves.

But, we now have introduced an extra factor C 1 2 which takes into account, the differences that arise the differences with respect to this that arise because, of the fact that you now have a spread in frequencies. And this is we referred to this as the, coherence the the degree of coherence are the coefficient of coherence. So, this quantifies how much coherence you have between these 2 waves. And this number is going to be less than or equal to 1 and it is going to be less than 1. The modulus of this number is going to be less than 1 when, the 2 when you put into when you put in the fact that both of these waves have some spread. And it is going to be equal to 1, when both these waves are precisely sinusoidal waves of the same frequency.

So, this degree of coherence is going to fall, the moment you incorporate the fact that you have a spread in frequencies.

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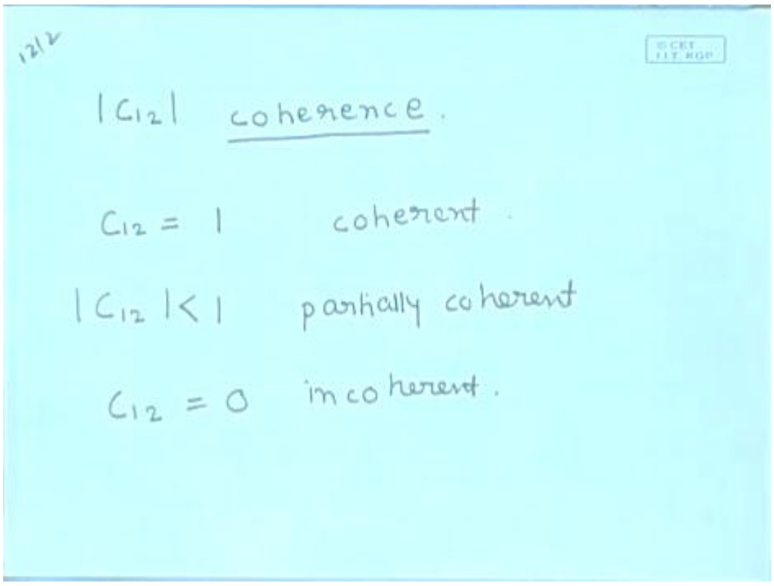

And this coherence is precisely equal to 1 when the 2 waves are perfectly coherent, which occurs when they have exactly they are exactly sinusoidal waves. The 2 waves are partially coherent when this is less than 1, which is the situation that you usually encounter and the 2 waves are said to be incoherent if this coefficient is 0. Say if the 2 waves are incoherent the

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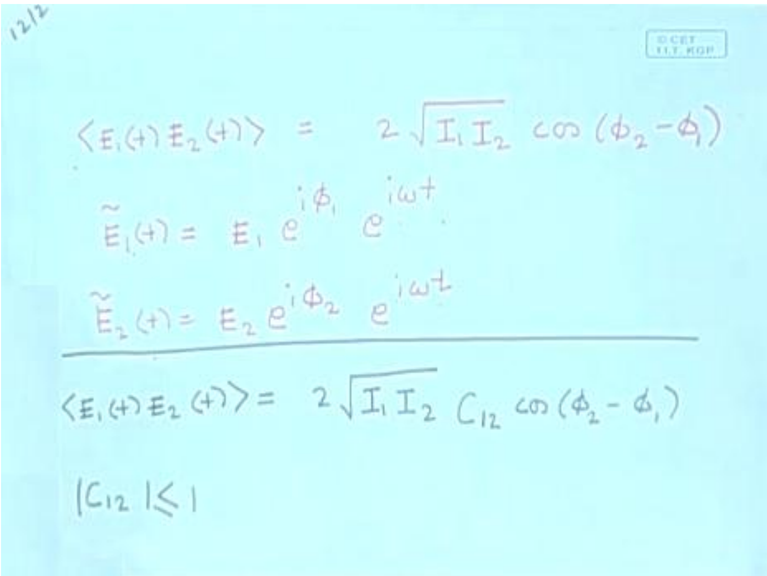

Time average of the product of these 2 waves is going to be 0, they are incoherent. So, there will be no interference in a situation where there is, when which is when the 2 waves are incoherent. So, we could write the resultant expression for the intensity in a situation where

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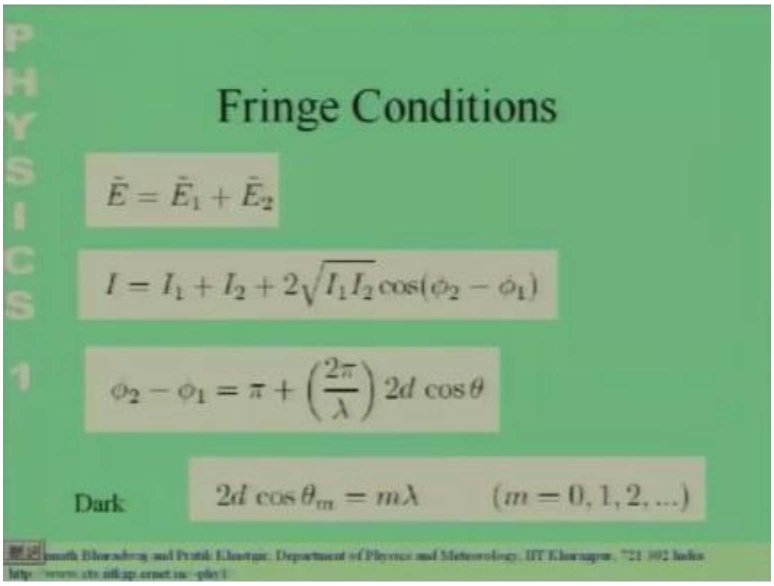

The 2 waves have the same intensity. So, we will write down the expression now for the resultant intensity.

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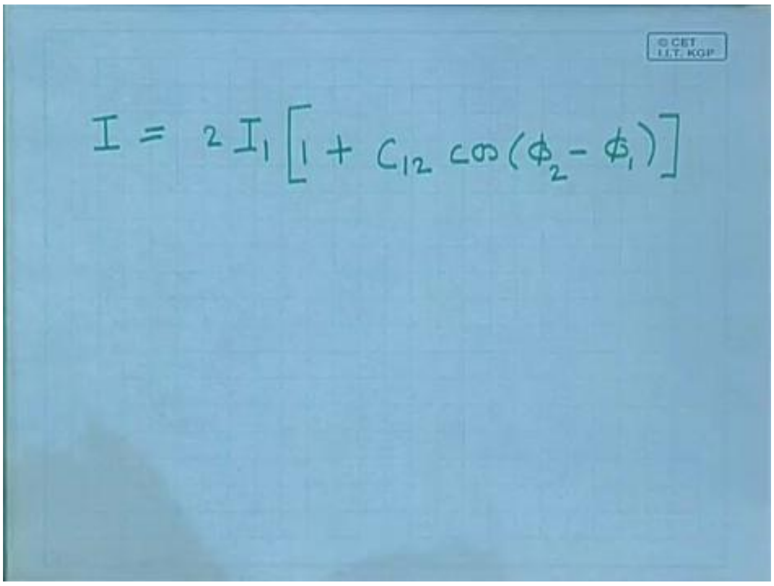

So, we are going we are basically considering this expression over here.

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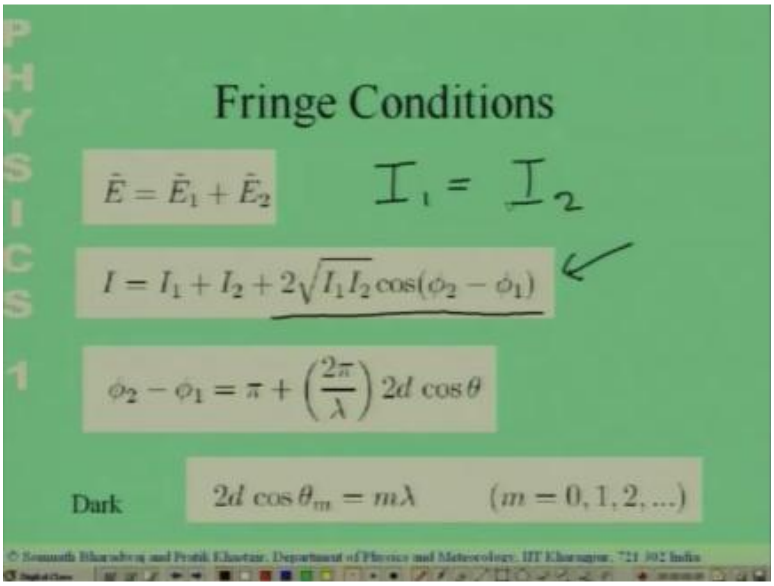

We are considering this expression for the resultant intensity, when I have superposed 2 waves. We will now incorporate the fact that, the 2 waves can have a spread in frequency, but we will assume for simplicity which is and the assumption which we make is valid in the 2 situations, in for both the Michelson and the Young's double slit experiment, for is approximately valid.

So, we will assume that I 1 is equal to I 2 and write down the expression for the resultant intensity, incorporating the fact that the 2 waves are going to be partially coherent, they are not going to be exactly coherent. So, let me write it down like this; so, I is equal to, so, I 1 and I 2 are same, I have a factor of 2 I 1. And I will have a factor of 2 I 1 from the other expression also. So, this is also going to give me a factor of 2 I 1. So, I can take this common, I have 1 plus now, when I put in the fact that the 2 waves are going to be partially coherent, there will be a factor of C 1 2 and cos phi 2 minus phi 1.

So, this is the intensity pattern, this gives me the resultant intensity. Well I have superposed 2 waves whose amplitudes are the same 2 waves, which could have a phase difference phi 2 minus phi 1 and both these waves have a spread in frequency. The waves have now water spread in frequency. The consequence of the spread in frequency is now there in this coherence. So, this is the general expression for the intensity when I superpose 2 waves, which are partially coherent. You'll now have the coherence the degree of coherence over here C 1 2.

With this background in the last lecture, we were discussing the Young's double slit experiment, we were now discussing the consequence of this for the

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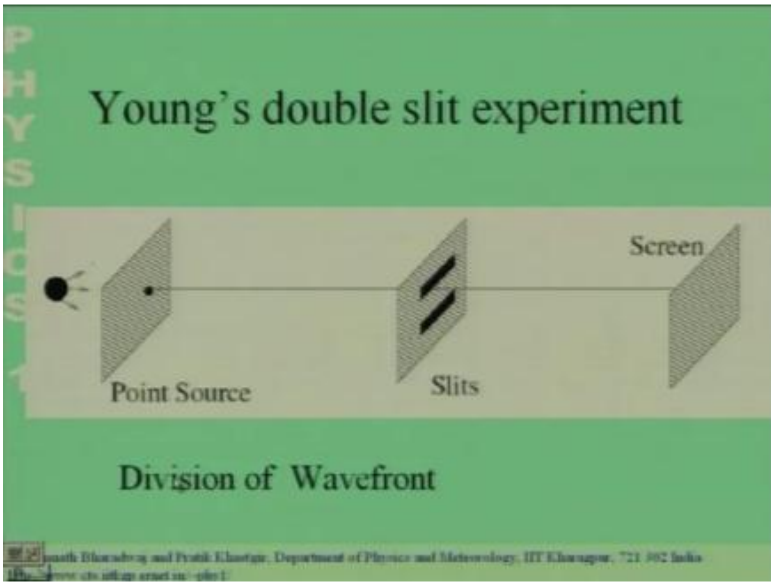

Young's double slit experiment. So, let me recapitulate that, the discussion where it just before it had ended in the last lecture. So, I had told you that the Young's double slit experiment, works by the division of wave front. So, the wave front, incident over here on this obstruction over here is divided into 2 by the 2 slits and the contribution from these 2 slits. So, let me draw the picture for you here.

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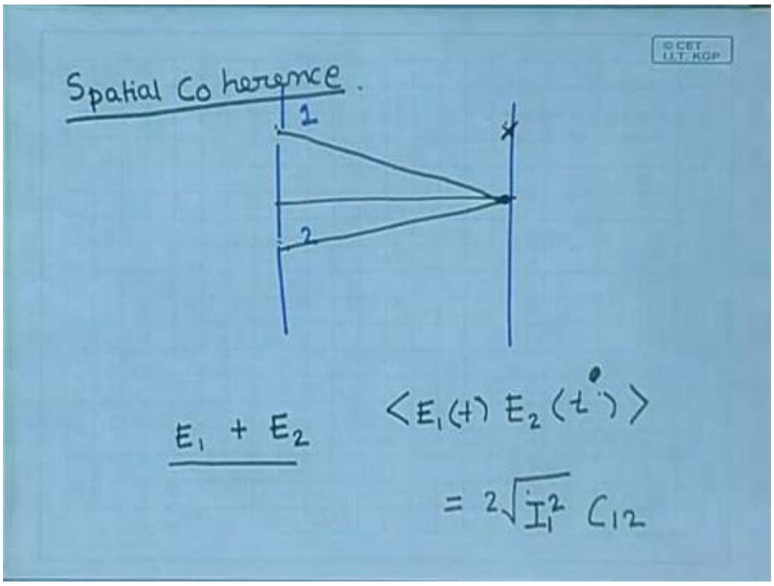

I have the 2 slits, which I call 1 and 2 and this is the screen. Now, if you wish to calculate the intensity at any point on the screen over here let us say at this point; it is going to be the sum of the wave from 1. So, it is going to be E 1 plus E 2, where E 2 is the contribution from here. And if you look at a point which is of the centre, so if you look at for example this point, these 2 waves will arrive with different time delays So, the essentially the term that the quantity which is responsible for interference over here, arises from a term from this particular term E 1 t E 2 t at some other time t prime. These 2 times are different because, the path lengths to this point are different.

Now, if you consider the point at the centre, the quantity responsible for interference over here, is essentially E 1 t into E 2 t the time average of this. The phase difference and the path difference time at the centre is 0. So, what you can say is that, this is equal to 2 root of I 1 and I 2 are assumed to be same, so I 1 I 1 square into C 1 2. So, the Young's double slit experiment, through the intensity pattern that you see on the screen measures the coherence, between the wave at this point and this point and different points on the screen measure this. So, measure the coherence between the wave here and at the at this point and this point with different time dealings. And at the centre, you essentially measure the coherence between this point and this point with no time delay.

So, what you measure here is E 1 t into E 2 t, where E 1 is the wave the value of the wave here and E 2 t is the wave over here at the same instant of time. So, it measures the coherence, between the wave at these 2 different points at the same time. This is what is called spatial coherence. So, the first point which I wish to make over here is that, the Young's double slit experiment measures the spatial coherence of the wave. So, it takes the value of the wave at 2 different points, multiplies them and measures the time average of this.

So, the term responsible for interference in the Young's double slit experiment, arises from the spatial coherence of the wave. And if you look at the point at the centre of the screen, the point right between the 2 slits on the screen, this particular point measures the coherence between the wave at 2 different points at the same instant of time. So, it is the spatial coherence which is measured over there.

Now, let us now look at, what how much is the spatial coherence of this of in the Young's, how much is the spatial coherence in different situations. So, recollect the expression that we had calculated, for the intensity in the Young's double slit apparatus.

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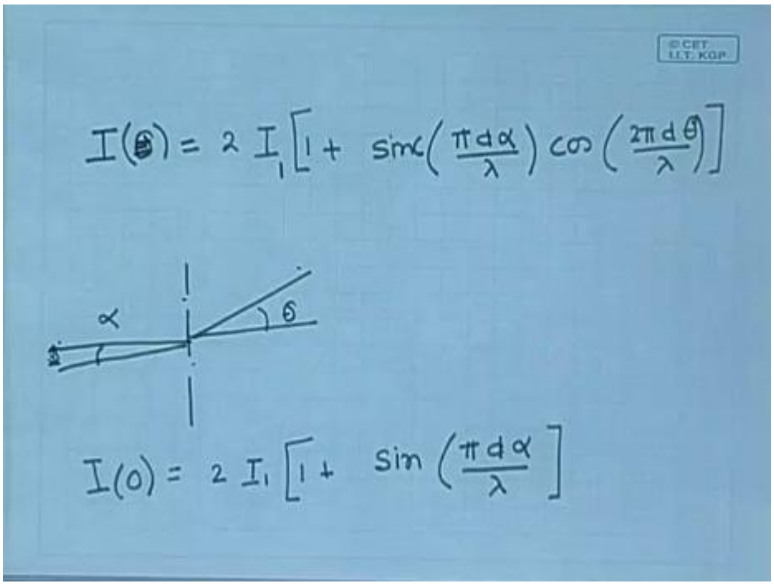

We had, we have calculated the expression for the intensity in the Young's double slit apparatus, I is a function of theta where is the angle on the screen. So, I is a function of theta we had calculated this, it was 2 I 1 plus sinc, this is going to be sinc pi d alpha by lambda into cos 2 pi d theta by lambda. So, this was the expression for the intensity in the Young's double slit experiment, this was the expression for the intensity in the direction theta, intensity on the screen at a direction theta.

So, the intensity varies as a function of theta, the dependence is given over here; it is 2 times the intensity of these individual slits into 1 plus sinc pi d alpha by lambda, where alpha is the angle subtended by the source. So, the whole thing is illuminated by a source an aperture, alpha is the angle subtended by the source, this angle at the slit, so this angle is alpha. If, the source is a point source then, this factor becomes 1 sinc remember sin x by x sinc x is sin x by x.

So, if the source is a point source alpha equal to 0, this factor is 1 and you have 1 plus cos 2 pi d theta by lambda, 2 pi d theta by lambda is the phase difference between these 2 waves at an angle theta. So, at theta equal to 0, you have 2 I 1 1 plus sinc pi d alpha by lambda. This is the intensity at theta equal to 0.

And I have told you that at theta equal to zero that is,

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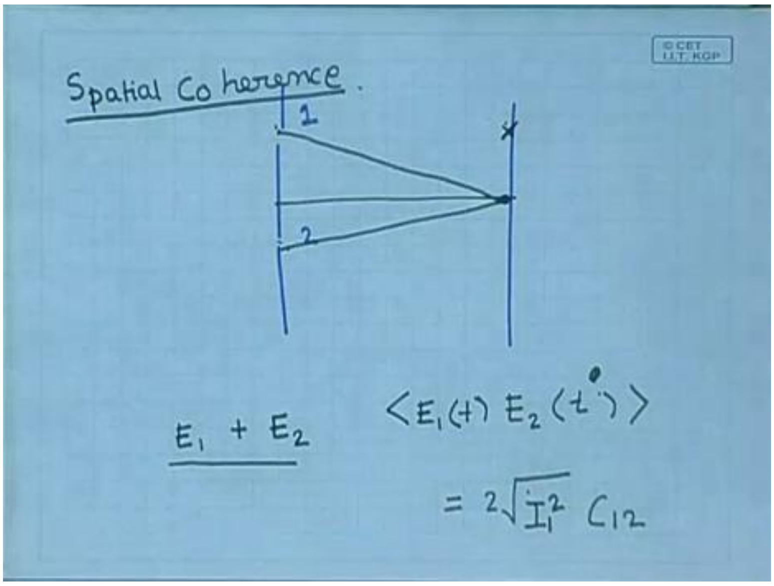

At the central point, the term responsible for the interference in this Young's double slit experiment, is essentially measuring the spatial coherence.

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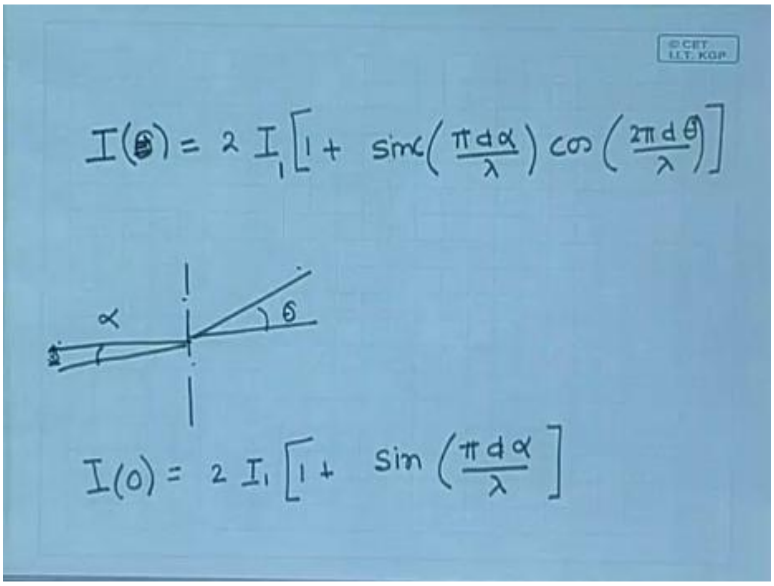

So, if i, so for partially coherent light, let me go back to the expression.

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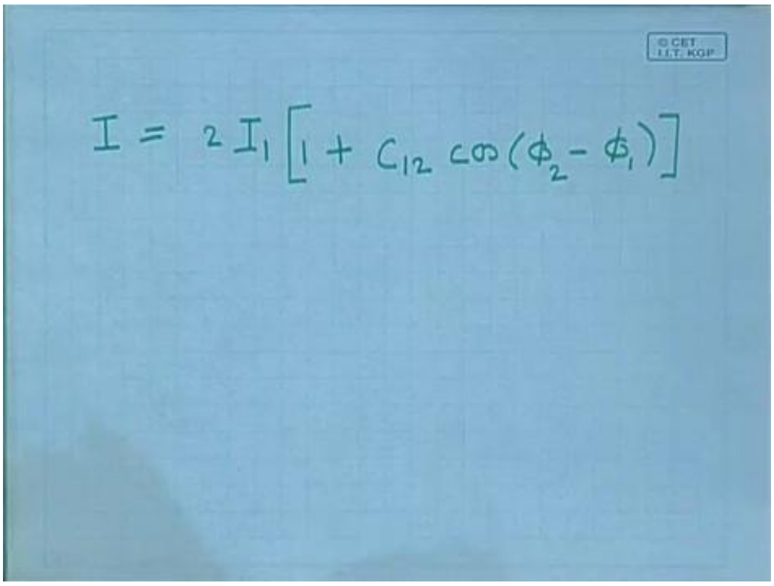

So, for partially coherent light, this is the expression for the intensity, phi 2 minus phi 1 is the phase difference between the 2 waves. At the central point the phase different is 0. So, if you now match this expression, which is valid for partially coherent light, partially coherent waves

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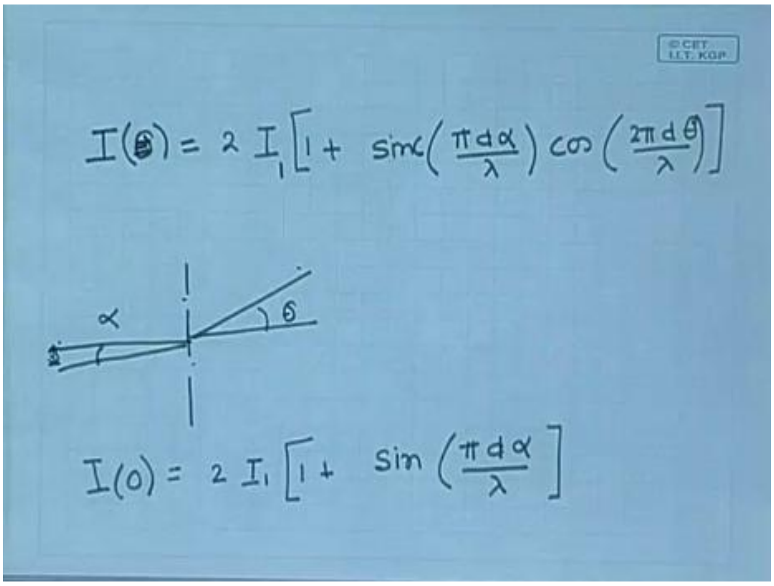

With the result expression for the intensity at the centre, in the Young's double slit experiment, you can see that the spatial coherence

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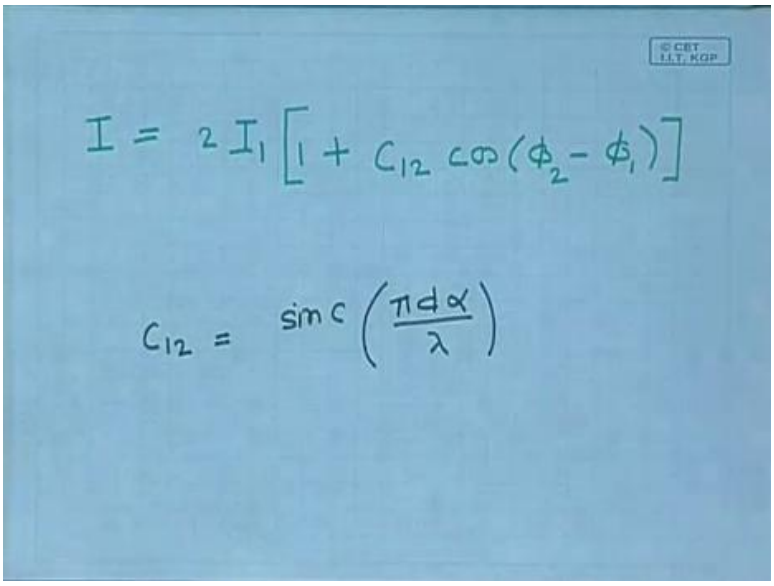

Of this wave C 1 2 is equal to sinc pi d alpha by lambda. So, let me now repeat, what I have just told you. Let me just repeat, to make sure that we all really followed what was the chest of the discussion. What I the first the first point which you should realise is this that, the Young's double slit experiment. If, you look at the if you restrict your attention to the point, at the on the screen which is aligned with the centre of the 2 slits, at this point the quantity being measured is the spatial coherence

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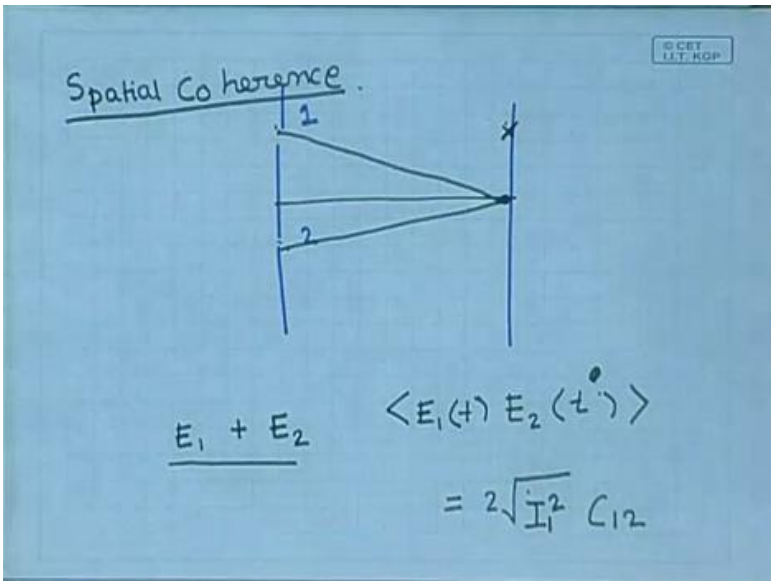

Which is the electric, the wave at this point 1 at the first slit 1 into the wave at the second slit E 2, the time average of this is, what is responsible for the interference at this point at the centre. In addition to this term, you will also have the intensity from this and the intensity from this. But, the term responsible for interference at the point to the centre is this, which we call the spatial coherence of the wave.

So, if you take the wave at these 2 different points at the same time, the time average of that is coherence and since these 2 waves arise from these 2 values, these 2 waves arised are basically the same wave measured at 2 different points. We refer to this as spatial coherence. So, it is the coherence between the oscillations in the wave at 2 different points; spatial coherence.

And this term if you look at the intensity.

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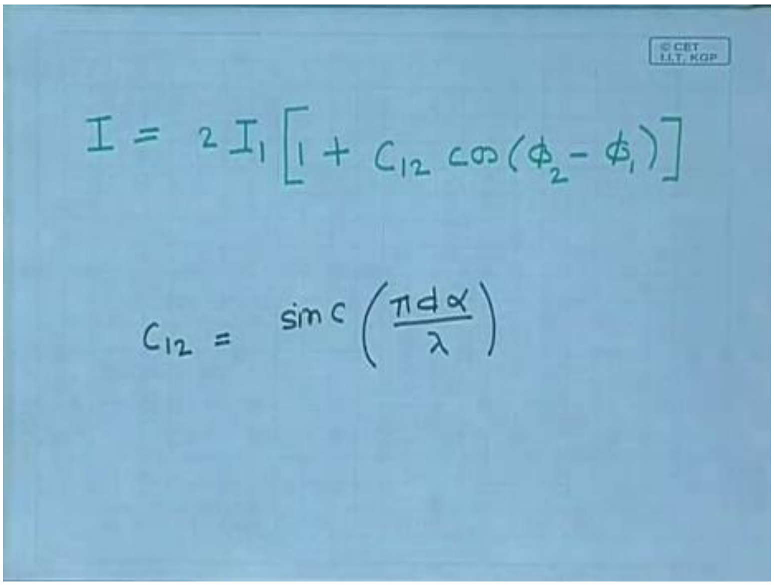

Then this term, essentially gives rise to this term over here; this into this. This is the sum of 2 individual intensities; this is the term responsible for interference. At the central point phi 2 minus phi 1 is 0. So, at the central point you have 2 I 1 C 1 2; this is the degree of coherence. Now we have already calculated

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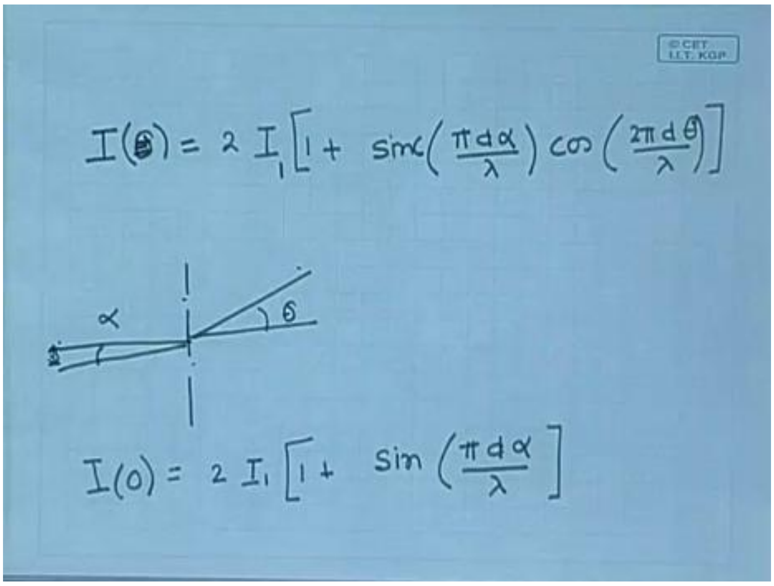

The intensity at at different points on the screen, for the Young's double slit experiment. So, if you take this expression and restrict it to the central point, make theta 0, this is the expression that you get. So, by comparing this.

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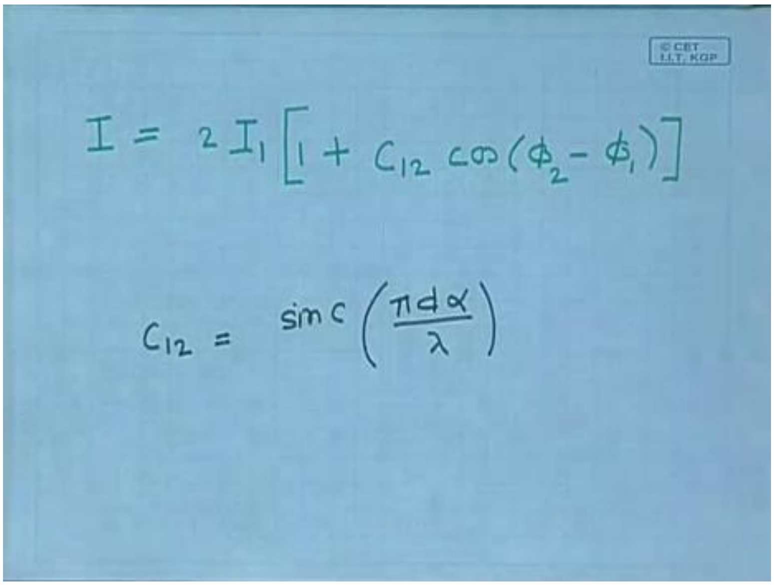

With this, you can now identify the expression for the spatial coherence in this particular case and it is; sinc alpha sinc into pi d alpha by lambda. So, what do we learn from this exercise? What we learned is that at the centre of this, at the central point you are measuring the spatial coherence of the wave. By spatial coherence of the wave you mean: the coherence between the wave at 2 different points, at the same instant of time. And what we find is that, the spatial coherence of the wave in this particular case, depends on the angle subtended by the aperture which is illuminating it, the angular extent of the source.

So, recollect that, we have assumed that the 2 slits

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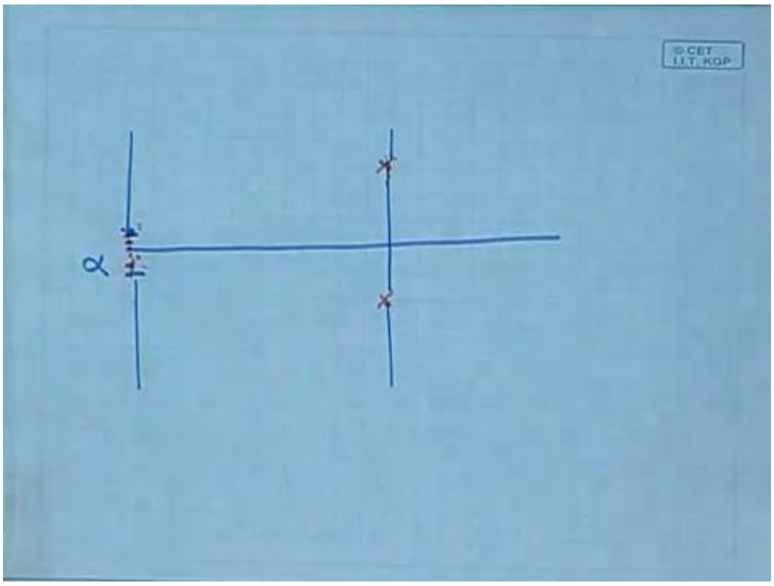

Are illuminated by a source and alpha is the angular extent of this source. And why did this particular source is, the less is the spatial coherence of the wave, why is this so? Let us just think a few seconds and try to figure out why is this so? Why does the spatial coherence of the wave decrease as we increase the extent of the aperture which is illuminating it? Why does the spatial coherence of the wave decrease if the source subtend the larger angle?

Now, recollect that we had modelled our source, so we had modelled our source in the following fashion. We had assumed that, each point on the source each point on the over here in this source is an incoherent point source. So, each point acts like an incoherent source. So, the wave that is emitted from this point does not interfere with the wave that is emitted from a different point, it only interferes with the wave emitted from the same point.

So, when you consider the value of the wave at this point and ask what is it is coherence with the value of the wave here? You should remember that the value of the wave, the wave over here is essentially a sum of contributions from all these incoherent sources. Similarly, the wave over here is also a sum of contributions from all these incoherent sources. Now, when you ask what is the coherence between this and this? Only the contribution from, so when I calculate the coherence between this and this, this is a superposition of contributions from all these sources, so is this. Only the contributions from the same point are going to be coherent with each other and as you make this aperture wider and wider the this coherence is going to go down because, the fractional contribution from each point each point makes a smaller contribution now to the total.

So, the spatial coherence, reduces as you increase the size of the aperture. And as you as the spatial coherence goes down as the spatial coherence goes down, the visibility of the of the fringe pattern the visibility as you saw in the last lecture, is directly related to the coherence the degree of coherence of the 2 waves. So, in this case it is the spatial coherence. So, if I make the aperture, if I make the source broader and broader, the spatial coherence goes down, the visibility also goes down and the contrast the brightness of the fringe pattern will get reduced and the fringe pattern will slowly get washed out.

So, in this discussion I have tried to explain to you, how it is possible to measure the spatial coherence of a wave using the Young’s double slit experiment. The Young’s double slit experiment is essentially measures the spatial coherence of a wave. And you can get this by looking at the intensity at the central position. This has got several applications; let me explain 1 of the applications to you. This is an application which was first put to practice by Michelson.

The application is to determining, the angular diameter of stars the angular extent of stars. Planets we know have a finite angular extent. So, if you look at a planet; it is not a point it is a disk which has a sub finite angle.

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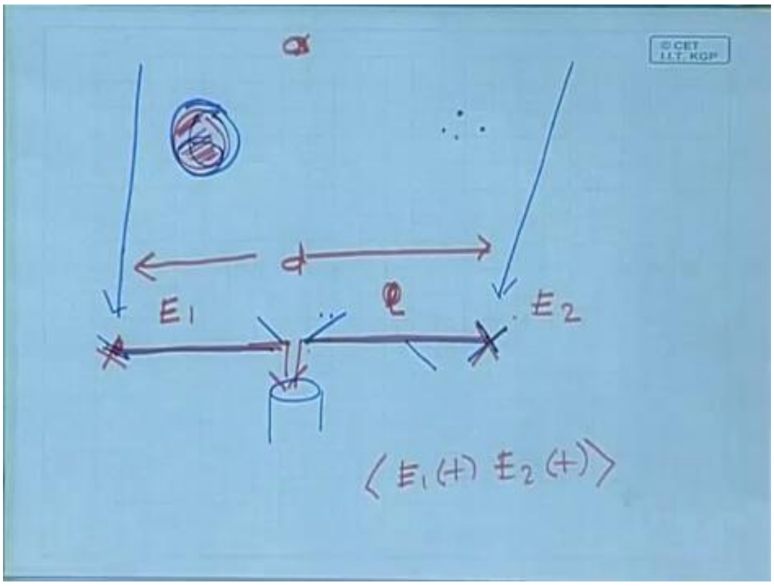

So, if you look at the a planet on the sky, it has a finite angle. Whereas, when you look at a star as far as our eye is concerned, a star is just a point; our eye cannot determine the angular diameter of stars. This is why planets do not twinkle, but stars twinkle. We are looking at planets and stars through the earth's atmosphere. The earth's atmosphere is constantly changing. And these changes in the earth's atmosphere also cause the images of planets and stars to move around on the sky.

So, an instant, the image of the planet looks like this, at a later instant it will look like this and at an neither rather instant, it will look something like this. So, the image of the planet that our eye sees through the atmosphere, keeps on moving around. But, as it moves around it has a overlap with itself. So, the only consequence of this is, it will make it look a little broader, it will make it look a little bigger.

Now, whereas, a star is a point as far as our eye is concerned. So, because of this variation in the earth's atmosphere the, this point is going to become a different point there is no overlap and a different point and a different point. So, it is going to move around in the sky and this is why we see the stars twinkle. So, if you take very fast image, you will see that the point corresponding to a star is actually moving around because, of the earth's atmosphere.

Whereas, so there is no overlap with the previous image and this causes the image of the star, to appear as if it is twinkling for us it to array it actually it twinkles, whereas; the planet does not. Now, the question is; suppose I had an instrument which was which had a better resolving power which could make out smaller angles, could I determine the angular diameter of stars? It so turns out that, the angular diameters of stars are so small that, for more stars you cannot determine it directly using telescopes from the earths.

So, Michelson proposed the following method, to determine the angular the angular diameter of stars. So, suppose you have a telescope like this and you have 2 mirrors over here. So, you have some optical arrangement; the lens should be same. So, you have some optical arrangement, by which you can so there is a star faraway. So, this lights of the star comes to both of these. So, these 2, there are 2 mirrors here which will and there is and there is an optical arrangement. So, you could have it like this and so you will have to I mean like this.

So, you have some kind of an optical arrangement, which will measure the wave, from this star at this point and at this point and bring these 2 waves together through equal path lengths so that, there is no time delay and superpose these. So, you have E 1 the wave from the star the same source incident at this point, E 2 the wave from the same star. So, both the waves arrive from the same star the same source. You have some optical arrangement, by which you can take the wave incident over here and incident over here to the same point through equal path lengths and put them into a something, which will measure the intensity of the resultant.

Now, as you vary the length l or you may say that d the distance k between these 2. So, these are effectively like a Young's this is effectively an Young's double slit apparatus. Instead of the slits, you have these 2 mirrors over here which will come collect the wave coming on the wave coming on it and bring them to the same point. So, it is light measuring the intensity in a Young's double slit experiment. It is like measuring the intensity at the centre of the 2 slits and on the screen. So, the waves arriving at these 2 points are brought together and made to interfere.

So, the term which is responsible for the interference here will be E 1 t into E 2 t the time average of this, which we had already discussed. So, you measure the spatial coherence of this wave and as we have seen, the spatial coherence of this wave depends on the angular size subtend by this source, the source here is a star. So, the spatial coherence of this wave, which you can measure by through by a superposing them and causing them looking at the intensity the fringes, this will depend on the angular diameter of the star. So, you look at the visibility of the fringes that you get; as you vary the separation between these 2 mirrors.

And as you vary the separation you except the visibility

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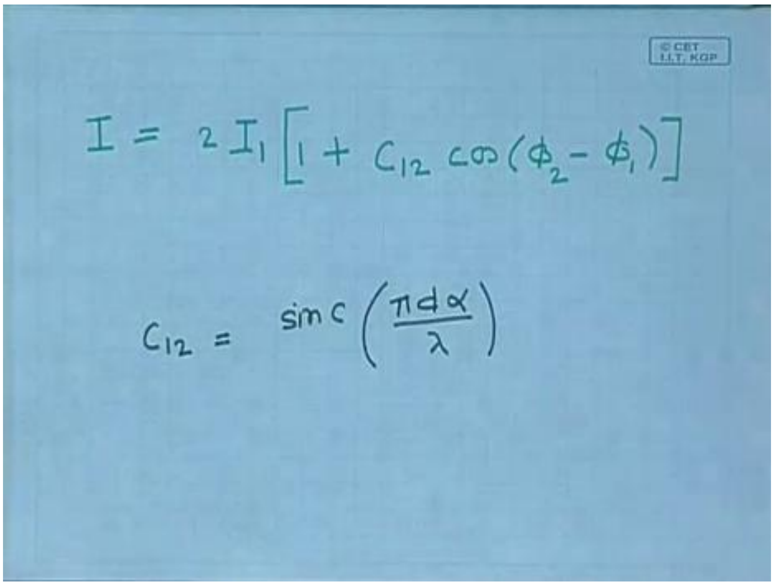

which is we have already seen that, the visibility is directly equal to the degree of coherence in this particular case. You expect this to vary as sinc pi d alpha by lambda, you want to you do not know this alpha the angle subtended by the source, you want to determine this. So, if you can measure this sinc type of behaviour with varying d, what you do is you vary this distance

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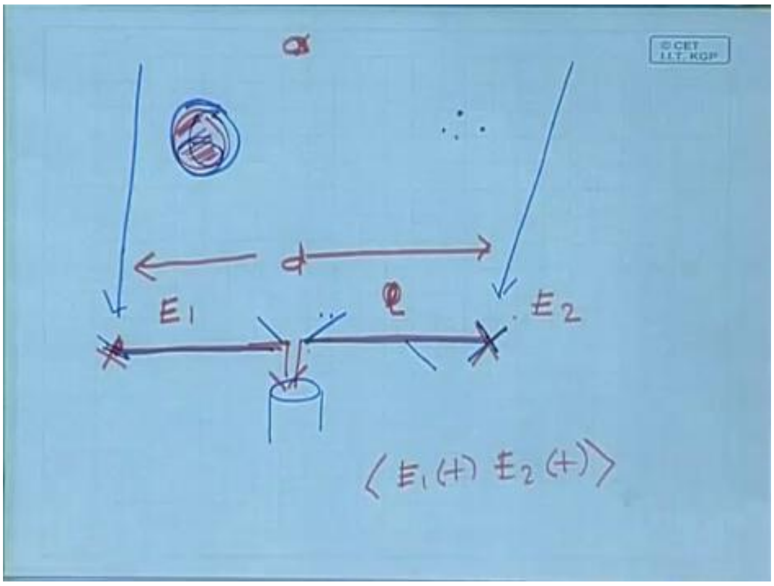

Between these 2 mirrors, which collect the wave coming from this distance source if, you can vary this d and measure the spatial coherence?

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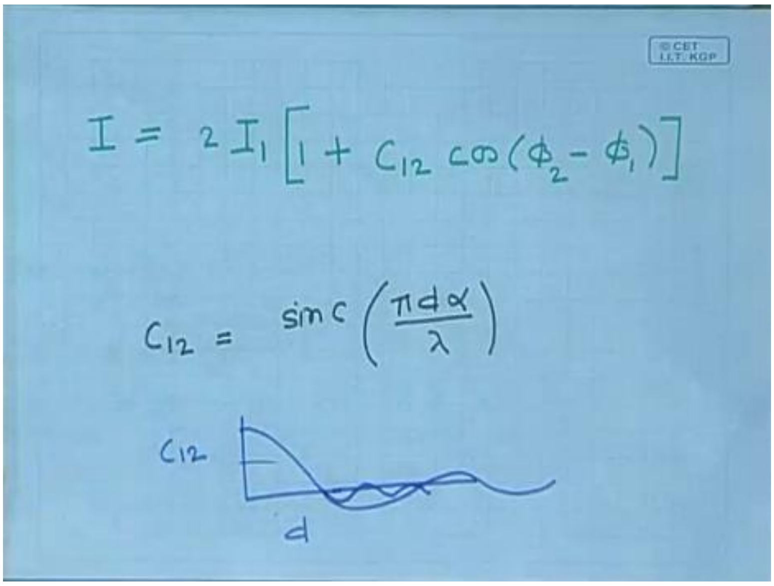

If, you can measure the spatial coherence varying d, you can then, if you can measure this. So, you measure the spatial coherence C 1 2 for different values of d and you except this to be a sinc function like this. If you can actually determine this sinc function, the sinc function is not going to be, is going to look like this actually. If you can experimentally determine this sinc function, then you could get the value of alpha from this. So, this is a method which has been applied, to measure the angular diameter of very bright stars.

So, we have discussed the consequence of this coherence on the fact that, you have partially coherent light; you do not have perfectly coherent light, you always are dealing with partially coherent light. So, we have discussed the consequence of this for the Young’s double slit experiment. I told you that the Young’s double slit experiment, essentially measures the spatial coherence and we also discussed an application of this.

Now, let me move on to the other situation of interference that we have been studying. So, the other situation that we have been studying is the Michelson Interferometer.

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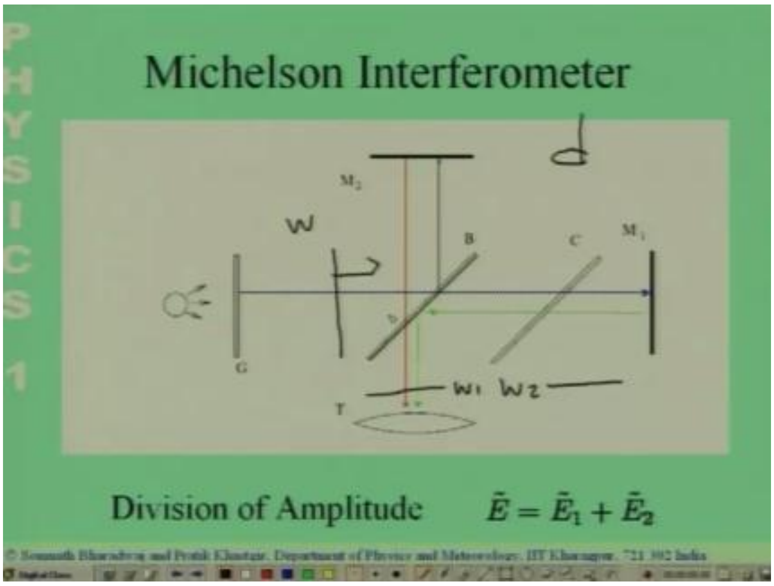

In the Michelson Interferometer, we have interference by the division of amplitude. So, in the Michelson Interferometer, we have a wave front arriving like this over here. And this wave front, the amplitude of this wave front gets split into 2. So, we have 2 wave fronts. And finally, we have 2 wave fronts W 1 and W 2, let me draw them like this. So, we have 2 wave fronts which arrive like this 1 W 1 goes through, let us reflect it from mirror 1 and then comes down, W 2 goes up reflected up and then gets reflected back and then comes down. So, we have waves coming over here, which were produced from the same wave W.

Now, the question is that, from a single wave W we have produced 2 waves. Now, when we superpose these waves over here, when we superpose these waves finally, when we superpose these waves over here, are we superposing the wave at the same time instant or is there a time delay? Now, we had introduced this quantity d, if you remember in the Michelson Interferometer d was the difference in the path length at the 2 arms.

Now, the wave W 1 and the wave W 2 if you remember, have to travel different distances. So, there will be a difference, there'll be a path difference which, if you look at the wave which is travelling straight, there is a path difference which is 2 d. So, the 2 waves

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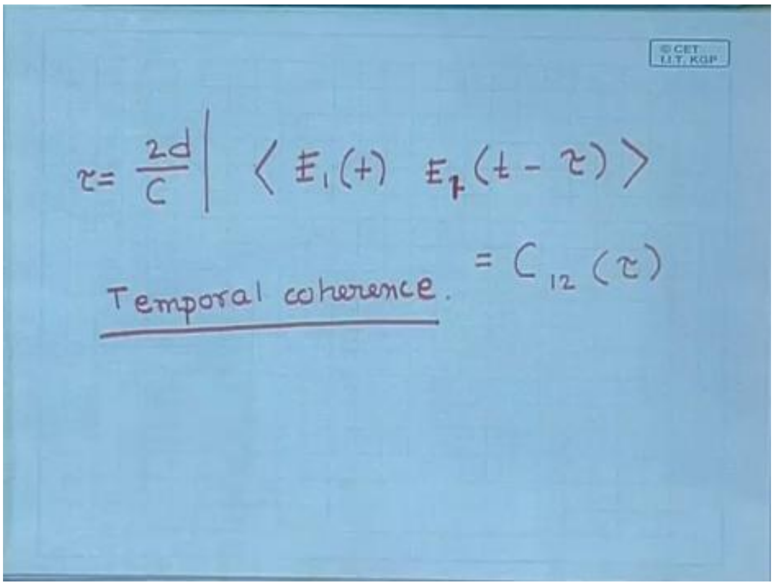

Have a path, have to travel different paths and they and there is a difference in the paths is 2 d. So, the 2 waves, which you are superposing E 1 and E 2, the 2 waves W 1 and W 2 arrive with a time delay of 2 d by C. So, let us call this time delay tau. The 2 waves arrive with the time delay of 2 d by C. So, the quantity let us assume that, the wave E 2 has to wave 2 has to travel a larger distance. So, the quantity which you measure over here, the quantity which you measure over here the quantity responsible for interference over here, is not the coherence between the wave at the same instant of time.

So, here in this particular case, you are producing E 1 and E 2 from the same wave E. So, they are the exactly the same, but there is a time delay. So, the quantity which you are measuring through the interference over here, is the coherence between the oscillations of the wave, at 2 different times which are separated by tau which is and tau is equal to 2 d by C. So, the Michelson Interferometer introduces an extra a time delay of 2 d by C for 1 of the waves.

So, you have 1 wave coming, you split into it into 2 parts, introduce a time delay of tau which is 2 d by C into 1 of the parts. Then, superpose the 2 again and look at the time average of the product of the 2 waves, which is, what I have shown over here. So, the Michelson Interferometer, essentially measures the temporal coherence. So, this is what we call temporal coherence.

So, by spatial coherence, we had meant the coherence between the same wave, but at 2 different points the wave the vibrations the at 2 different points. Temporal coherence, we mean that the same wave, you put a time delay and then look at the coherence between the waves. So, you are now correlating or you are now superposing a wave with it with itself, but with the time delay. So, the quantity the coherence this particular coherence is called temporal coherence. So, and this will be your function of the of the time delay that you have introduced. So, this will be your function of the time delay that you have introduced.

So, let me recapitulate again; the Michelson Interferometer measures the temporal coherence. So, it there is a there is a wave coming through, you divide the wave into 2 parts, put a time delay into 1 of them and then look at the coherence between these 2 waves that you have produced, that is the original wave and the same wave for the time delay, what’s the coherence between this. To get an understanding of what the coherence between this is going to be, let me now ah show you, so let me show you what you expect over here.

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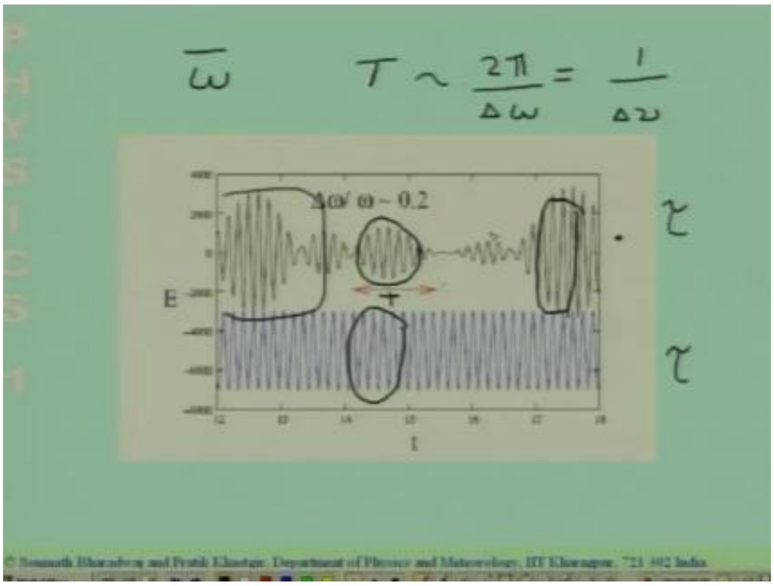

And to get a understanding or what you except let us look at this picture which we had introduced in the last lecture

So, remember what this shows us. We have superposed 2000 oscillations, oscillations of 2000 different frequencies. We have superposed oscillations of 2000 different frequencies, these frequencies are spread have. So, these frequencies at we have superposed are spread in the range delta omega, around a central value omega bar. And the spread in frequencies, in this particular case is such that delta omega by omega bar is 0.2, which is what I have shown over here.

So, delta omega by omega bar omega which I have just denoted as omega here. So, delta omega by omega is 0.2. So, we have superposed 2000 frequencies, different frequencies which are in the small range delta omega, centred around this value omega. And the resultant oscillation, so we have superposed sinusoidal oscillations many sinusoidal oscillations, the resultant oscillation as you can see, is not sinusoidal it is not a pure sinusoidal oscillation. You can think of the resultant oscillation, as being a sinusoidal oscillation of fast sinusoidal oscillation, with a frequency which is angular frequency which is equal to the central value.

So, you can think of the resultant oscillation, as being a fast oscillation at the mean angular frequency. So, we have a spread of angular frequencies centred around this and when you superpose all these oscillations, you can think of the resultant as being a fast oscillation at the central at the mean frequency, the amplitude of this fast oscillation. So, you have this fast oscillation. If, it were a pure sinusoidal it would look like this, but it is not the pure sinusoidal, it is a superposition of many frequencies centred around this value. So, you have still have the at this central value, but the amplitude of this oscillation changes slowly on a time scale delta on a time scale not delta but T, which is of the order of 2 pi by delta omega the spread in frequencies or you can write this as equal to the order of 1 by delta nu.

So, when you have a spread in frequencies, you have a fast oscillation at the mean frequency, but the amplitude and the phase, both changes slowly on a time scale T, which is of the order of 1 by the bandwidth 1 by the frequency spread or in terms of angular frequencies 2 pi by delta omega. And here I have shown you the time scale on which a time scale T, on which the amplitude is changing.

Now, when we are looking at the coherence, the temporal coherence we have a wave. So, we have this wave at, which is oscillating fast at a frequency omega bar oscillating slowly at a frequency at a time scale T which is of the order of 2 pi by delta omega. When we calculate the coherence, we take the wave, give it a time delay multiply it with the original wave take the time average.

So, what is the coherence between this wave, the same this wave over here and itself with the time delay tau? We know that if I had this wave a pure sinusoidal, the coherence of this wave with itself with the time delay tau, if there is a single frequency the coherence we have seen is 1, perfectly coherent. So, if I had a pure sinusoidal the coherence is it is perfectly coherent. So, if I give a time delay and then multiply it and ask the question what is the time average? It is going to be cos of phi 2 minus phi 1, the phase difference between the 2 waves.

So, the coherence the degree of the coherence is exactly 1. This is going to change because; we have a spread in frequencies. Now the point here is that, if I give a time delay you see over a time period which is small. So, over a time period which is small compared to this, I can think of this part of the wave as being a pure sinusoidal. But, then if I keep on following it for a longer time, I will see that there are deviations from the sinusoidal, but this part of the wave this small part of the wave looks, just like this as such the amplitude is a little smaller.

So, a small stretch of this wave looks like a pure sinusoidal. But, if I look at a larger stretch if I look at this entire large stretch over here, you see it looks it does not look anything like a pure sinusoidal because, the amplitude you can notice the change in the amplitude, over this small range, you cannot notice the change in the amplitude. So, from this, it should be clear from this discussion it should be clear that, if I displace the wave if I displace 1 of the wave if I put a time delay to 1 of the waves such that

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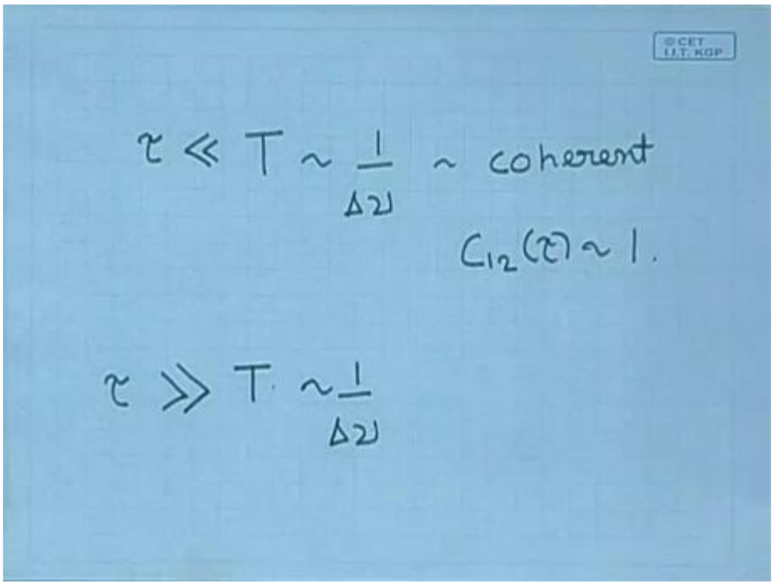

tau is less than T which is I have told you of the order of 1 by delta nu. If, the time delay which is I have give to 1 of the waves between the 2 waves is; less than this time scale on which the amplitude changes, the wave is going to be largely coherent. So, C 1 2 is going to be of the order of unit T, not exactly 1, but something of the order of unit T may be half, somewhere over there something quite a bit larger than 0.

Whereas, if tau is much greater than this time, on which on which the amplitude the slow variation on the amplitude occurs. So, if I give a time delay to 1 of the waves which is much larger than this, so let me show you what I mean over here.

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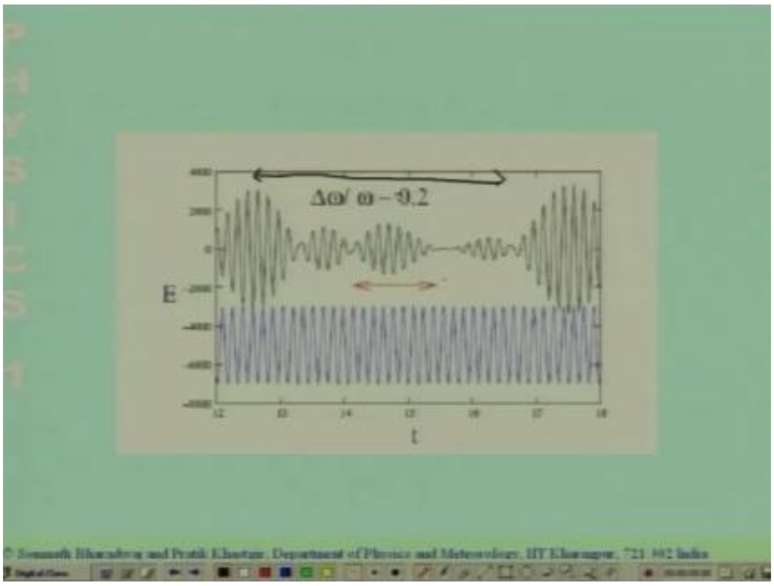

So, if I give a time delay which is let us say this much and multiply these 2 waves and then take the time average question is, what do we except? The amplitude and the phase have changed considerably by the time you reach from here to here. So, what it what you except essentially is that, this the 2 waves should be incoherent.

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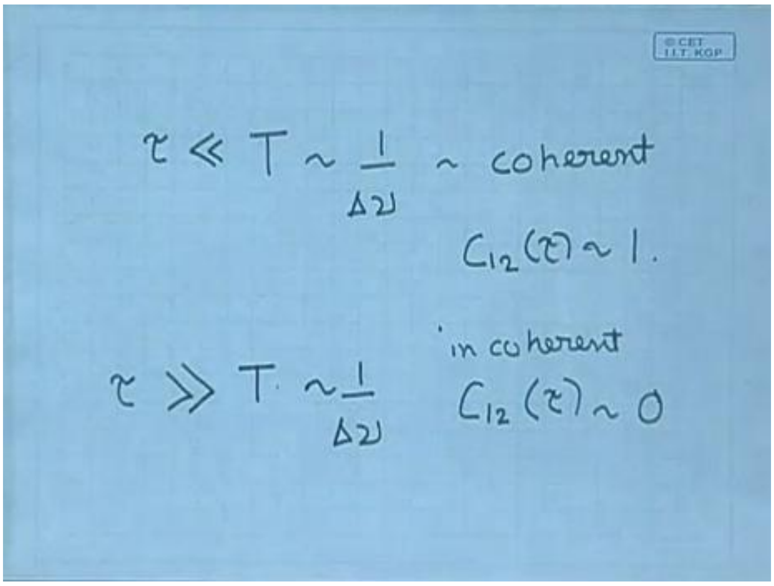

So, this is going to be incoherent and this number is going to be close to 0. So, I hope I have been able to convenes you that, if the time delay which I gave is smaller than the time scale on which the amplitude and phase of the wave changes then, I expect the waves 2 waves to be coherent. If, the time delay between the 2 between the 2 waves, so I give a time delay to wave and multiply for itself and then take the time average.

If the time delay is larger than the time scale on which the amplitude and phases are changing, I expect them to be incoherent. So, we have a time scale, which is the time scale on which the amplitude changes slowly, this time scale is called the coherence time of the wave.

So, the time scale.

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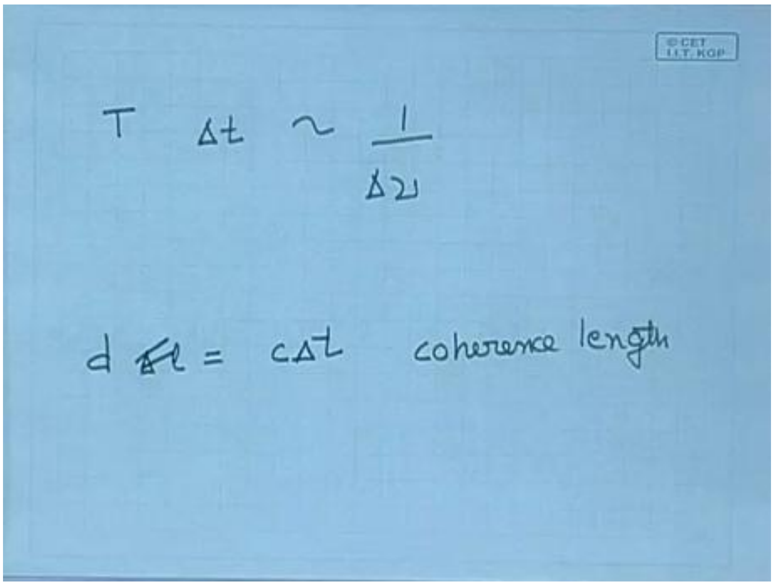

T which is also sometimes denoted as delta t the coherence time, this is of the order of 1 by delta nu. So, the coherence time of a wave is inversely proportional to the bandwidth of the wave. So, the larger the bandwidth, the smaller the coherence time the smaller, the bandwidth the larger the coherence time. And we can also convert this to the coherence lengths. So, the length l delta l which is c into delta t, the length that the displacement that you have to give to 1 of the mirrors. So, in this case it is going to be d.

So, if the length so corresponding to the coherence time, I can I have the coherence length. Coming back to the Michelson Interferometer again, so let me go back to the Michelson Interferometer again.

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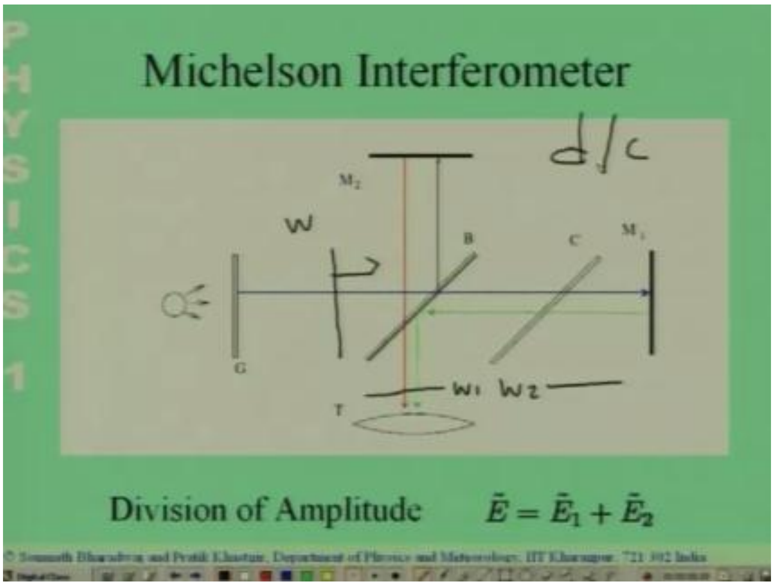

Coming back to the Michelson Interferometer, by moving the mirrors M 1 and M 2, I can change the time delay that I give to 2 waves. The time delay is exactly d by c and if this time delay is less than the coherence time, the degree of coherence between the 2 waves is going to be quite large; the visibility is going to be quite large. We will see good fringes very bright fringes with the good contrast.

If I increase d so that, it exceeds substantially exceeds, so that d by c substantially exceeds the coherence time or d substantially exceeds the coherence length. The degree of coherence between the 2 waves, which I am superposing will be extremely small. So, the visibility is also going to be very low, I will not get fringes. Even if I do get fringes they will be very week. So, the fringes are going to get washed away

So, the Michelson Interferometer, measures the temporal coherence of the wave. You can use the Michelson Interferometer, to measure the temporal coherence of the wave. What you need to do is, you increase the separation between the 2 mirrors slowly and as you and measure the visibility as a function of the separation. The visibility as you can see is directly the temporal coherence in this case. And as you keep on increasing, the separation between the 2 mirrors you will find that slowly will get washed away. And from this, you can determine the coherence time and the coherence length, what to determine directly is the coherence length. You can also determine the coherence time of the light source that you are using.

Now, for example, if you use a white light source, white light has a very broad range in has a very broad frequency spectrum. So, if you use white light, the coherence time is going to be very small. So, you will not typically you will not get fringes at all. It is to use a light source with the narrow, with the small spread in frequencies to get fringes for a range of displacement.

If, you use white you will get fringes only if the difference between the 2 arm lengths is very small. If you use for example, sodium vapour lamp a sodium lamp you'll get fringes, but you will not get fringes if you increase the path length say beyond some reasonable distances. The order of centimetres you will not get fringes. Lasers have got very small spread in frequency, the lasers the light produced by lasers have a very small spread in frequency. So, if you use laser light lasers have a large coherence length goes into meters.

So, if you use lasers as your source for the Michelson Interferometer, you can have large differences in the arm lengths and still get good fringes. So, this brings to an end our discussion of the Michelson Interferometer. Let me now discuss a problem before we finish this class.

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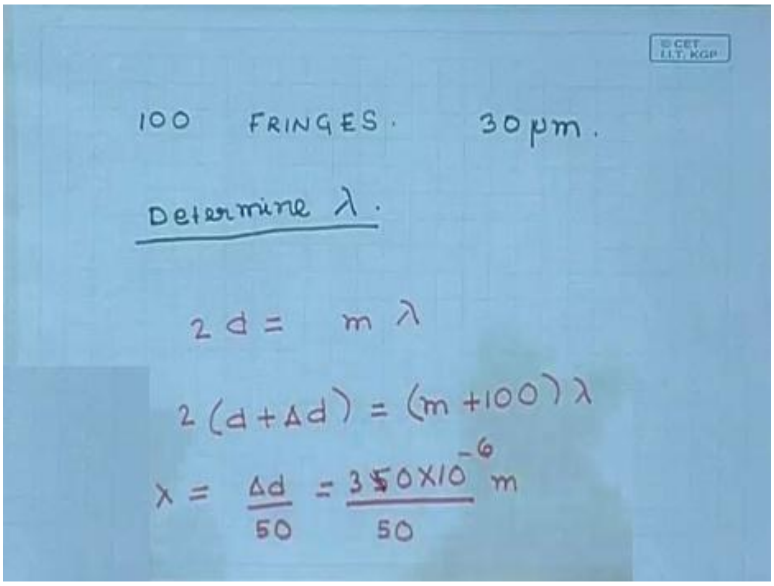

Been adjusted; so that, there is a dark fringe at the centre. We now move 1 of the mirrors so that, you move it further out. So, that you get 100 new fringes appearing at the centre. So, as you move 1 of the mirrors, you get newer and newer fringes at the centre. So, you move it keep on moving it, till you get 100 new fringes at the centre. And it is found that in a particular situation, the mirror has to be moved by 30 micrometers so that, you get 100 new fringes at the centre. The question is; determine the value of the wave length of the light.

Now, this problem is very simple, we have already discussed all that is required, to solve this problem. So, we know that, for the condition for a dark fringe at the centre of a Michelson Interferometer is 2 d should be equal to m lambda and now, when you move 1 of the mirrors so that, you get 100 new dark fringes, then by so 2 d plus delta d where, delta d is the distance you have moved the mirror, this should be equal to m plus 100 lambda. This tells us that is equal to 2 delta d, subtract these 2 so, you get 2 delta d by 100. So, this is equal to delta d by 50.

So, you have 30 into 10 to the power minus 6 meters, this should be 30 into 10 to the power minus 6 meters divided by 50, which we can write as 60 divided by 100.

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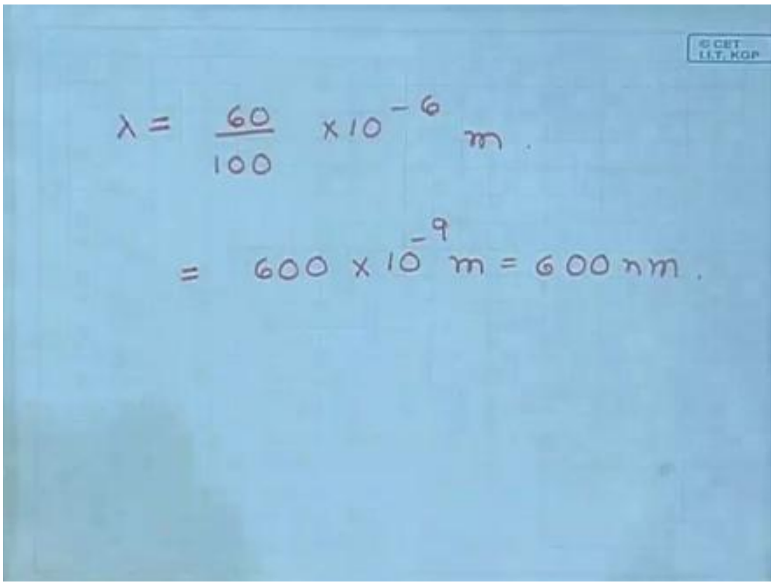

So, lambda is 60 divided by 100 10 to the power minus 6 meters and 60 divided by 100, we can write as 600 divided by 1000. So, this is equal to 600 and dividing by 1000 means 10 to the power minus 9 meters. So, this is 600 nanometers. So, we see that, the wave length of the light that you are using here is 600 nanometers. So, we have in this last part of this lecture

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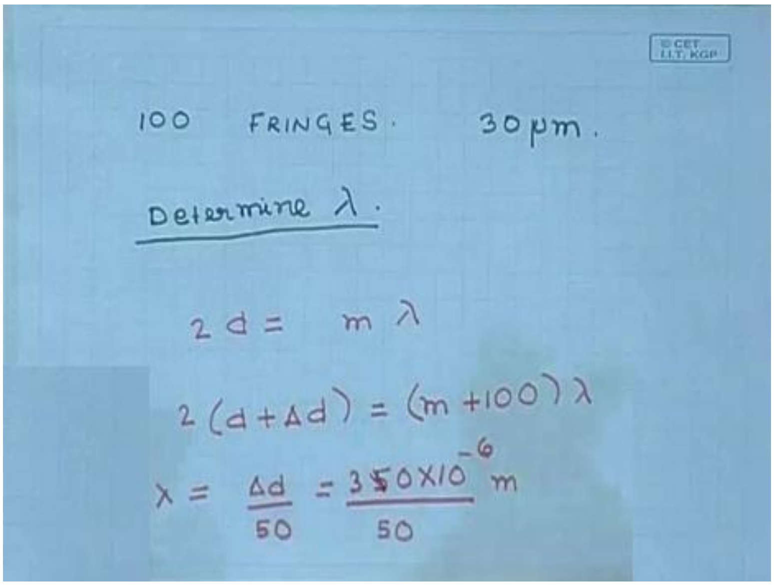

We have taken up a very simple problem, which essentially illustrates how the Michelson Interferometer, can be used to determine the wave length of light.