Physics I : Oscillations and Waves
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Lecture No-18
Coherence

Good morning in today's lecture, we shall discuss coherence. So, let us start by revisiting the main formula that we use in interference. So, the main formula let me show you the main formula that we use and interference is shown here.

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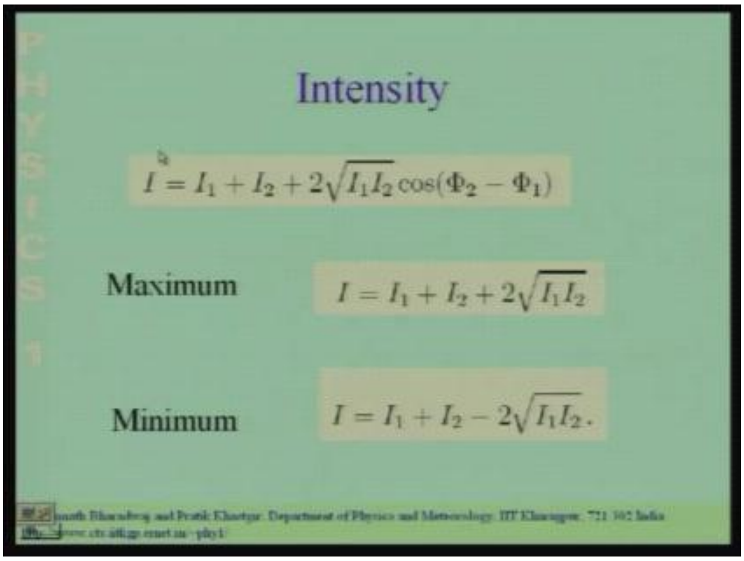

The intensity. So, we have superposed 2 waves E 1 and E 2.

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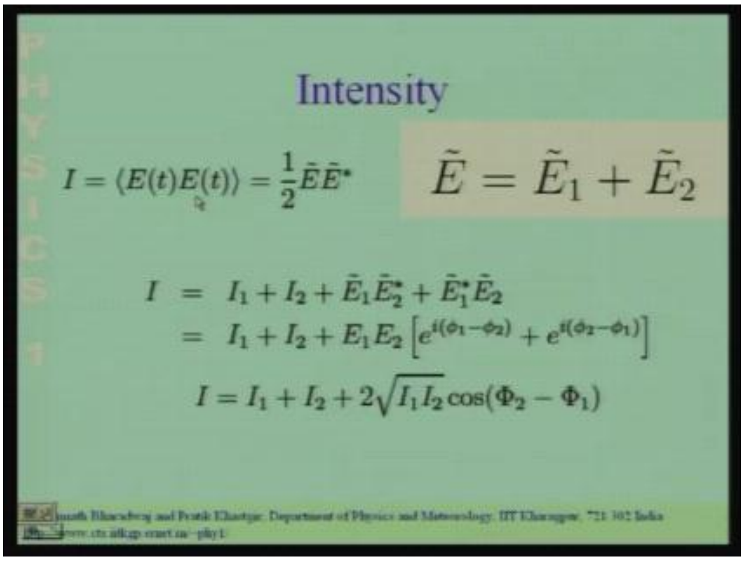

We have superposed 2 waves E 1 and E 2 to obtain a resulted wave E. and then we have calculated the intensity of the resultant wave. So, the intensity is the time average of E square right. So, we have calculated we have superposed 2 wave E 1 E 2 and we than calculated the intensity of the resultant.

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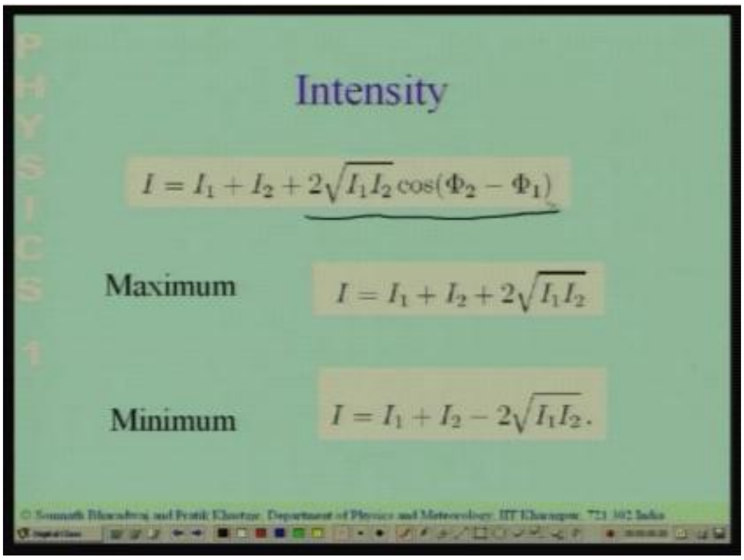

So, which is the time average of the resultant E and the intensity of the resultant is the intensity of the individual waves I 1 plus I 2 plus this extra term over here the extra term which, which over here is 2 root I 1 I 2 cos 5 2 minus 5 1. It is this extra term it is this term over here which is responsible for interference. And it is this term which is of interest. So, let us just again recapitulate how this particular term comes about.

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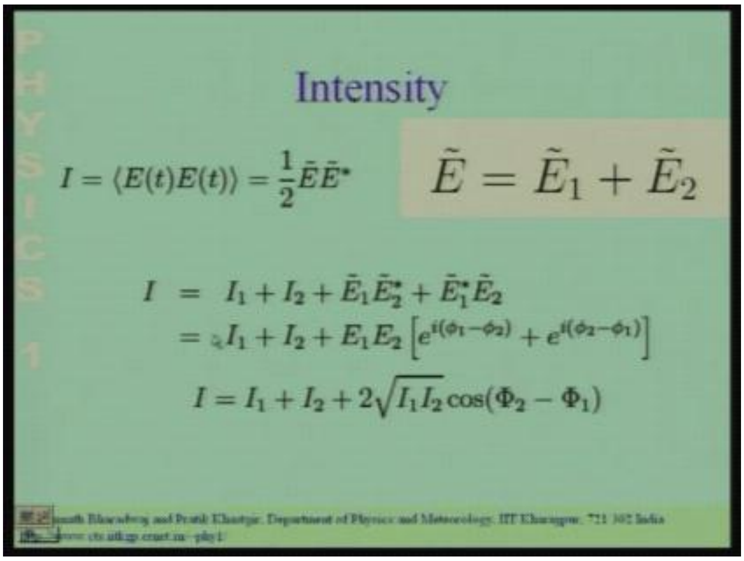

So, if I go back 1 step. So, we are calculating the in the time average of the square of this and this note. So, we have we have this I 1 which is the square of this the time average of the square of this then we have I 2 the time average of the square of this. And then we have the cross terms E 1 E 2 which in complex notation is return over here. And it is this term that gives rise to the term responsible for interference.

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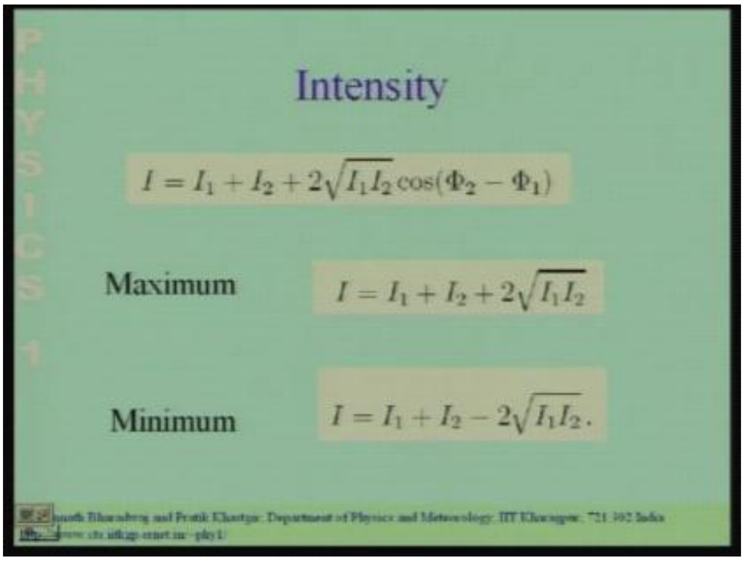

So, let me again let me just remind you how we which is the term that is of interest; the term responsible for interference arises.

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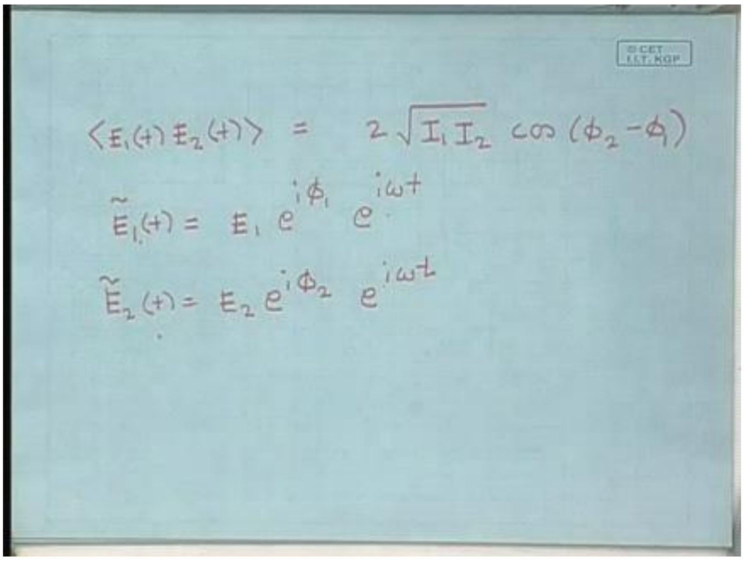

From this for that time average of the product of E 1 and E 2 and we had assumed that both E 1 and E 2. So, E 1 t we had assume that E 1 t in the complex notation could be written as a real amplitude a phase phi 1 into e to the power i omega t. E 2 could be written as E 2 the power i phi 2 e to the power i omega t. So, there was an assumption that both E 1 and E 2 are of exactly the same frequencies they have the same frequency or they have only 1 frequency. So, both way we have resumed that both the waves.

That we have superposing are may have a sinusoidal oscillations of a single frequency and these 2 frequencies are exactly the same. Let me recapitulate again what we had assume we had assume that E 1 and E 2 i would both perfectly sinusoidal oscillations of the same frequency omega which allows us to write E 1 and E 2 in this form. And then we had we can calculate the time average of this product E 1 E 2 and this is 2 root I 1 I 2 cos phi 2 minus phi 1 where I 1 is the intensity of this wave I 2 is the intensity of this wave phi 2 is the phase of this wave phi 1 is the phase of this wave.

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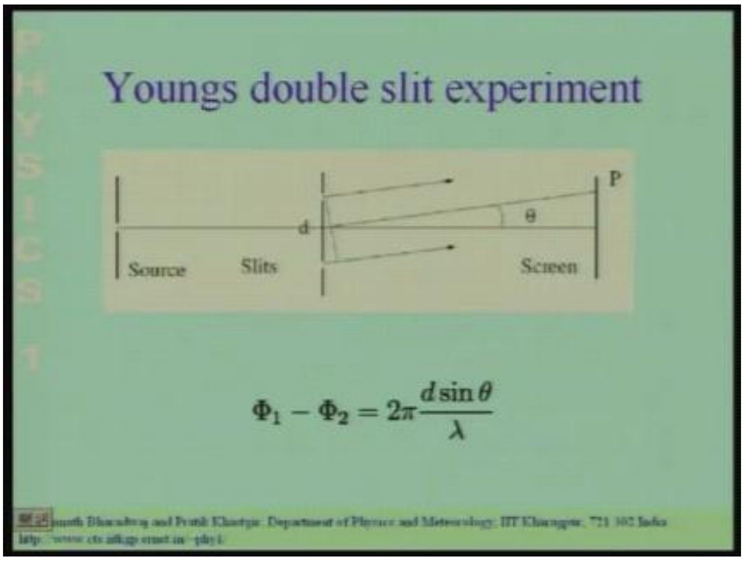

So, we have focusing, in the expression for the intensity.

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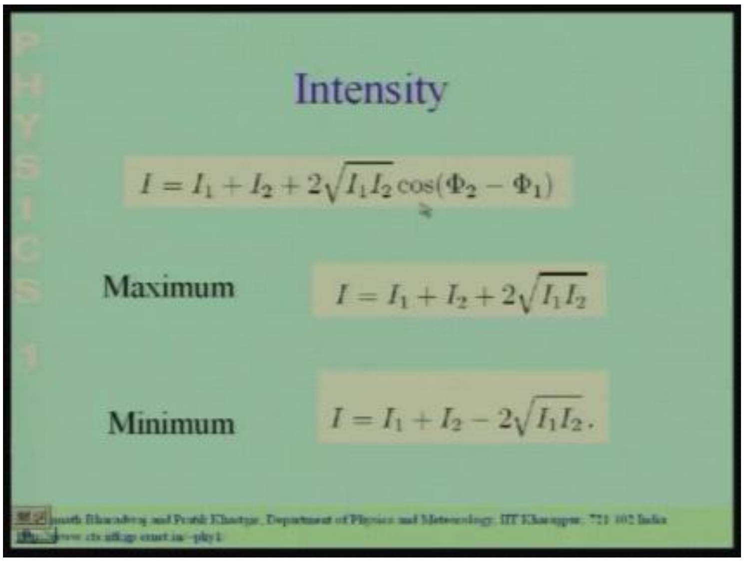

When I superpose to waves we intensity expression for the intensity. I told you that it is this term which is responsible for interference and we are focusing on the origin of this term how did this term come about. And you told you that this term comes about when we calculate this time average and this involves the assumption that both E 1 and E 2 both the waves are monochromatic they have a single frequency which is same for both of them. Now, there is a very important fact which I should tell you that you really do not have waves of a single frequency such things do not really exits.

You always have a spread in the frequencies. So, when in reality it is not possible to produce waves of a single frequency there is always speared. So, when we deal with real waves in some real practical situation there is always a spread in the frequencies. The analysis which we have been do in till lowed is a mathematical idealization which makes it simple to handle the situation. So, it makes the calculation simpler if you assume that there is only a single frequency in the 2 waves at we are superposing or in any other situation where we have a wave. It is mathematically easy to handle a situation where that is only a single frequency.

But in reality there never are waves of a single frequency. So, we have to we have to at a certain inside for certain purposes we have to take into account the factor there is a spread in frequencies. So, in today's lecture, we are going to ask the question what is the consequence of the factor of the that, there is always a spread in the frequency how is this going to affect my results. So, in this particular case we are looking at in interference. So, how is the fact that there is going to be a spread in frequencies how is this going to affect the basic formula that we use in interference the basic formula is the 1.

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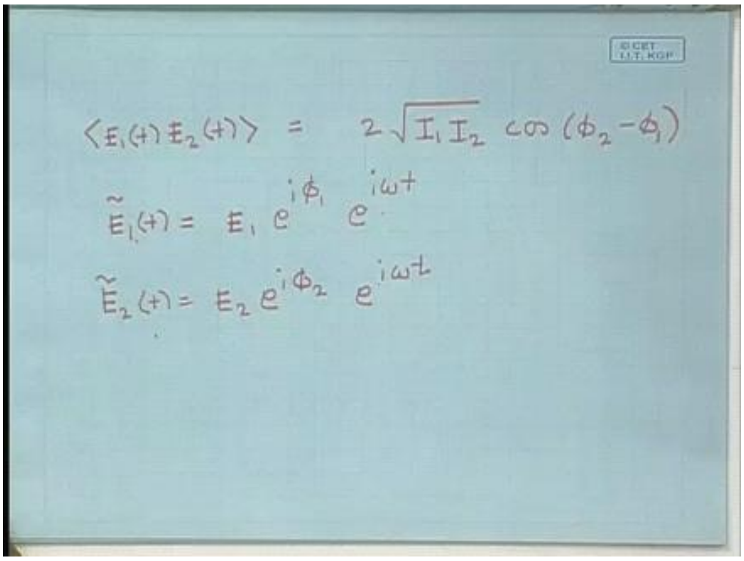

This is what is shown over here and the.

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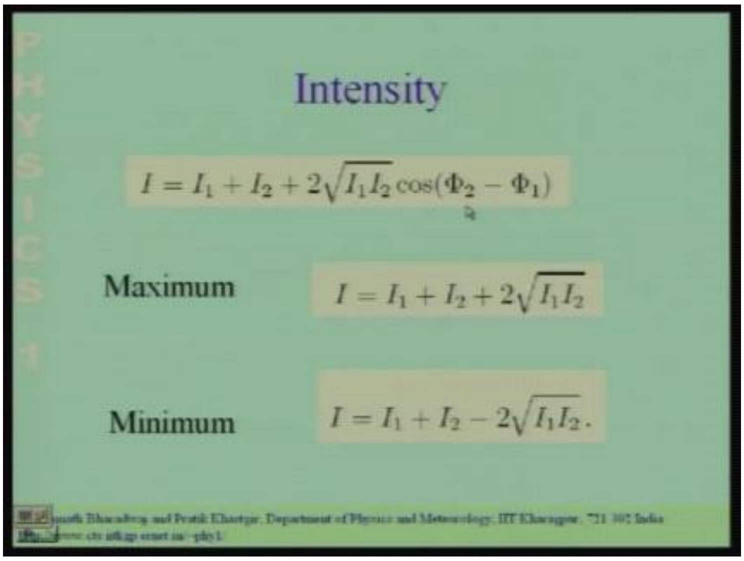

The term arising the term which is responsible for interference is what I have shown over here. So, we are going to discuss.

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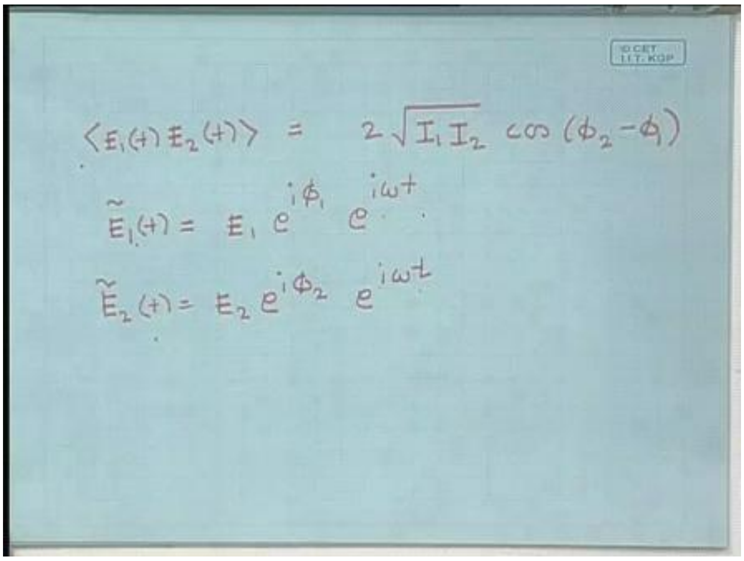

The consequences of the fact that both the 2 waves have a spread in frequency we are going to discuss the consequences of this fact on this particular expression over here, on this particular expression over here; which is the term responsible for interference. So, before we we address this very important question. Let us first discuss, what happens if there is a spread in frequency how does the how does the the wave look as a function of time is if there is a spread in frequencies.

So, we have already discussed this when we were discussing coupled oscillations. When we were discussing coupled oscillations i if you remember we in countered a superposition of 2 oscillations 1 slow mode and a fast mode right. So, we have already in countered a situation where we have a superposition of oscillations of 2 different frequencies.

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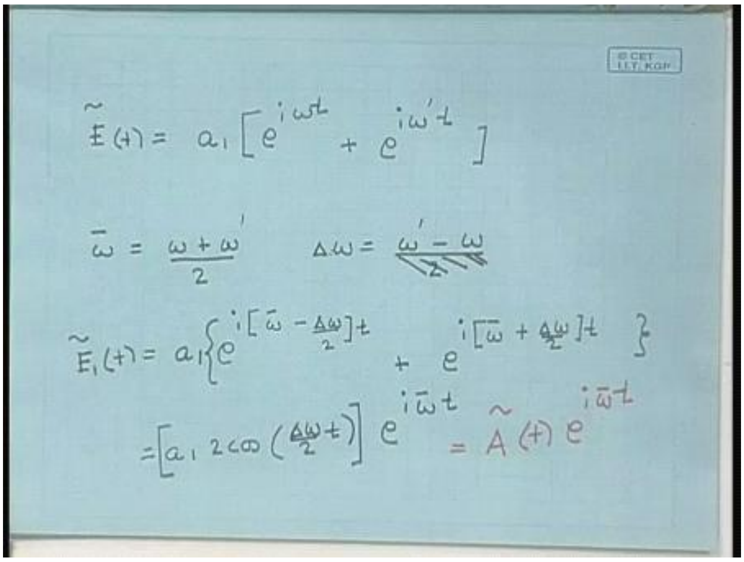

So, let us look at that again. So, we will considered a situation where E t and let us work in the complex notation. So, the wave which we are looking at at at of a fix point is an oscillation of is an oscillation that is a superposition of 2 different frequencies. So, we will write it like this a 1 e to the power i omega t plus e to the power i omega prime t. Now, we can define the mean frequency omega bar to be omega plus omega prime by 2 and the difference is difference in frequencies delta omega omega prime minus omega by 2.

Then this wave can be written as E 1 and we will write omega in terms of the average frequency and the difference in the frequencies. So, if i write omega in terms of this it is going to be e to the power i then I will have omega bar minus delta omega by sorry this delta omega does not have this 1 by 2 over here. The difference in frequency is omega bar minus omega prime minus omega. So, we can write omega omega as this minus half of this. So, the first term over here can be written like this and the second term can be written as e to the power i omega bar plus delta omega by 2 into t.

So, we have this so, the same electric field the same wave which I had which over here which was a superposition of 2 different frequencies can also be written like this. And this can be written as a 1 now, I can take e to the power i delta omega by 2 t common outside. If i take it common outside then I will get a term which is 2 cos delta omega by 2 t e to the power i omega bar t. So, what we have done here is just a little bit of algebra by means of which we had written the wave which was a superposition of 2 different frequencies in this form. I have noticed that, here if we assume that omega and omega prime are very close over here.

We have 2 oscillations which we can interpret as follows we have already discussed this interpretation in the context of the coupled harmonic oscillators we have a fast oscillation at the average frequency omega bar. So, the average frequency omega bar which is omega plus omega prime by 2. So, we have a fast oscillation at the average frequency omega bar with some amplitude with the which is the term within the square brackets. And the amplitude also changes with time it is a slow fashion.

So, there is an amplitude which changes on a slower time scale the the angular frequency corresponding to the time scale at which the amplitude change is delta omega by 2 it is a difference in the frequencies divided by 2. And we have assumed that these 2 angular frequencies are very close. So, this delta omega is a small number and delta omega by 2 is a smaller number. So, we can interpret this as A tilde t e to the power i omega bar t.

So, we can interpret this as a complex as a as an oscillation at a frequency omega bar fast oscillation at a frequency omega bar with complex amplitude A tilde t. The complex amplitude itself changes slowly with time. So, the amplitude itself changes slowly with time and in this particular case the amplitude is a 1 2 cos delta omega by 2 t. So, the amplitude itself does oscillation with the angular frequency delta omega by 2 which is very small. So, it does slow it changes very slowly with time.

So, this is what we learnt if we have a superposition of waves of 2 frequency. So, if my wave I have a wave if that wave has 2 frequency components. Than we see that, we can think of it as doing fast oscillations at the average frequency and the amplitude of that fast oscillation changing slowly with time. With and the amplitude changes with angular frequency delta omega by to the difference in the 2 angular frequencies. So, from this situation let us directly now jump to a situation where we have many many waves which are superposed.

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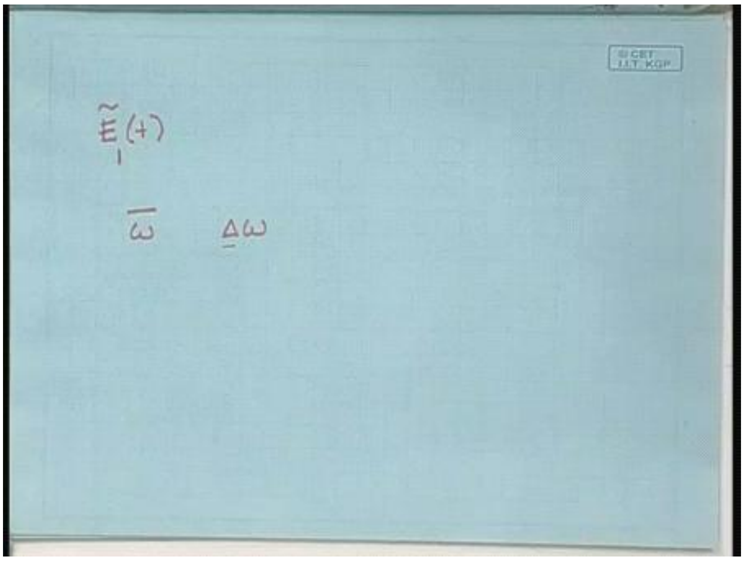

So, we have a wave, many frequencies which are superposed we have a wave which I represent as E tilde the t E 1 tilde the let us say I have a wave. And this wave is the superposition of many frequencies centred around the average frequency omega bar and the frequencies are spread out in a small interval delta omega around the average frequency omega bar. So, let me again tell you what E 1 this oscillating wave what this wave E 1 t is E 1 t is the is a superposition of oscillations of different frequencies. And these different frequencies are in the range of angular frequency delta omega centred around the value omega bar.

Right and the question that that arises is no,w how does the wave look like as a function of time and our analysis with 2 frequencies?

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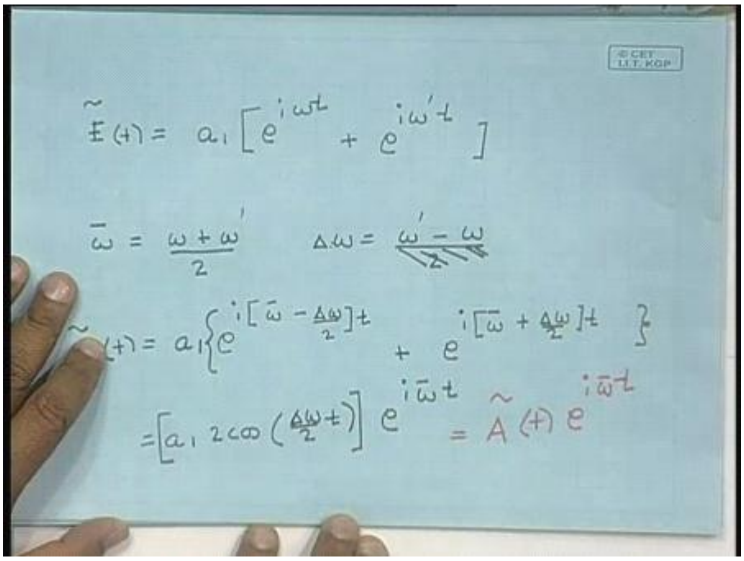

Here we guess consider 2 frequencies tells us at the resultant is an is the fast oscillation with the frequency omega bar, angular frequency omega bar. And the amplitude changes slowly with angular frequency delta omega by 2.

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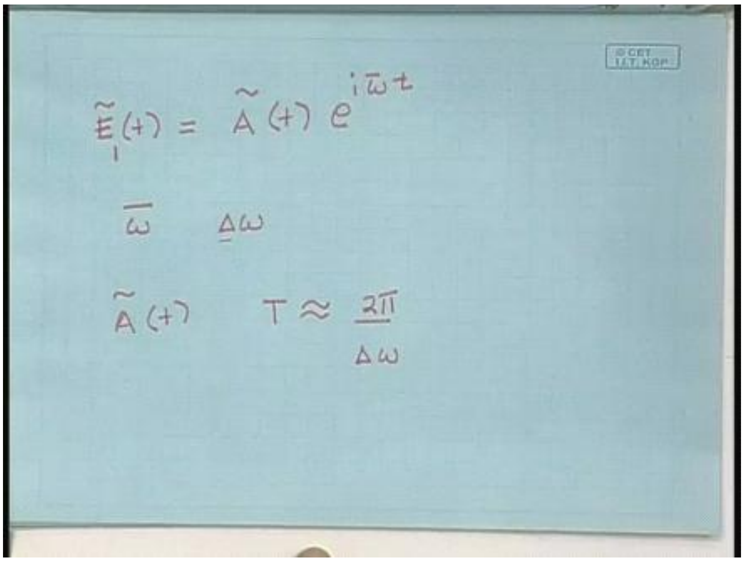

So, let me tell you without not, any proof, but with sufficient motivation which I have already given you that we will expect this to have a very similar behaviour. This the resultant oscillation is going to be a product of 2 terms 1 fast oscillation at omega bar which is the average frequency of all the different frequencies that have been superposed. So, the wave is going to execute fast oscillations at the average value omega bar the amplitude of the wave is not going to be a fix number the amplitude of the wave is also going to change in time.

And the amplitude A tilde t is going to change on the time scale T which is approximately of the order of 2 pi by delta omega. So, if we have a wave E 1 t which is a superposition of many frequencies all of these frequencies which we have which are superposed into this spread out. These frequencies are spread out in the range delta omega centred around the mean value omega bar. If in this situation then the resultant wave can be written as can be thought of as a fast oscillation at a frequency omega bar. With the amplitude of this oscillation changing slowly all a time scale T which is 2 pi by delta omega.

So, I have not prove I have not given any proof why this should be this should be so. But, I have given you sufficient motivation by considering 2 2 superposition of 2 frequencies and showed you that there is sufficient motivation to believe why this should be. So, and you can actually do a little more of mathematics considerably more mathematics and actually arrive at some at a confusion like this.

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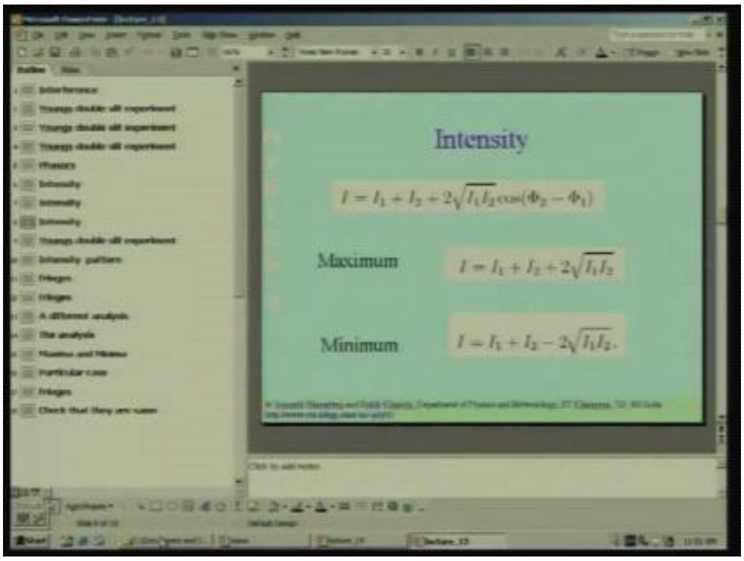

So, let me now show you a picture of what, the a wave form will look like.

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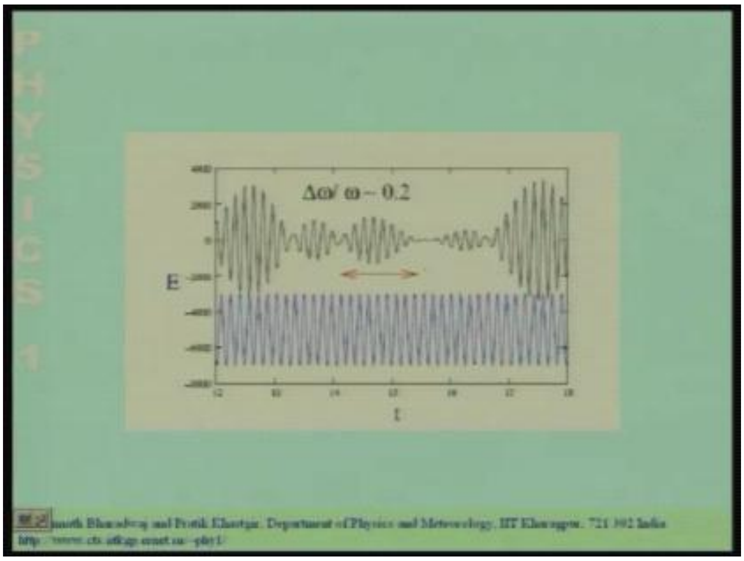

So,what is shown over here is as follows. So, let me explain to you what I what I have shown you what what is being shown over here. So, what is being shown over here is as is as is follows we have superposed . So, before that, so we the bottom curve over here the 1 in blue shows you a wave which is oscillating at some angular frequency omega bar. So, the bottom curve over here shows you a wave which is oscillating at a particular angular frequency omega were.

The value of omega bar here is not very crucial the discussion does not crucially depend on critically depend on the value of omega bar you can determine the value by just looking at the curve and using the values along the x axis. The main point is that this is a pure sinusoidal wave with the fixed angular frequency omega nought. The upper curve over here has been produced by superposing 2 thousand different frequencies in the small range of angular frequency delta omega. Centred around this value same value omega not which has been use to plot this.

So, we have superposed 2 thousand different frequency components in the frequency range delta omega centred around the same value use for this curve such that delta omega by omega bar is point 2. So, those spread in frequencies is twenty percent of the central frequency around which the values are spread. So, we have superposed 2 thousand different frequency components and plotted the resultant.

So, the first thing that you should note is that the resultant shows fast oscillations at the same frequency as this at the central at the average value which corresponds to this. So, the fast oscillations over here have the same frequencies; they have nearly the same frequency as the oscillation in this particular wave. Because, here we have superposed 2000 different frequency components centred around this same value.

So, the fast oscillations here are at the mean frequency mean angular frequency which is the same. So, you can see that the fast oscillations here look very much like the oscillations in the lower curve. But, the amplitude of the this oscillation change is with time. Whereas, this amplitude this is a pure sinusoidal amplitude of this remains fixed the amplitude of this oscillation changes slowly with time the time scale of this change in the amplitude. We have just seen the I have just rather I have just told you that the time scale.

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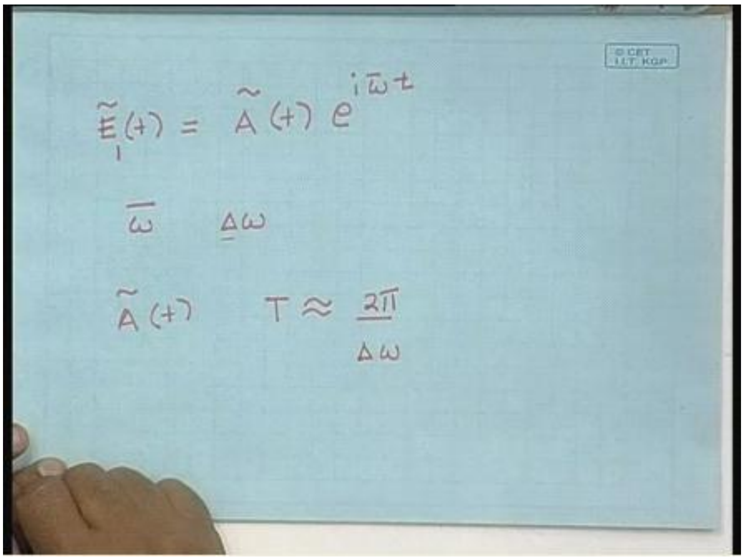

At which you expect the amplitude of the oscillations to change is 2 pi by delta omega.

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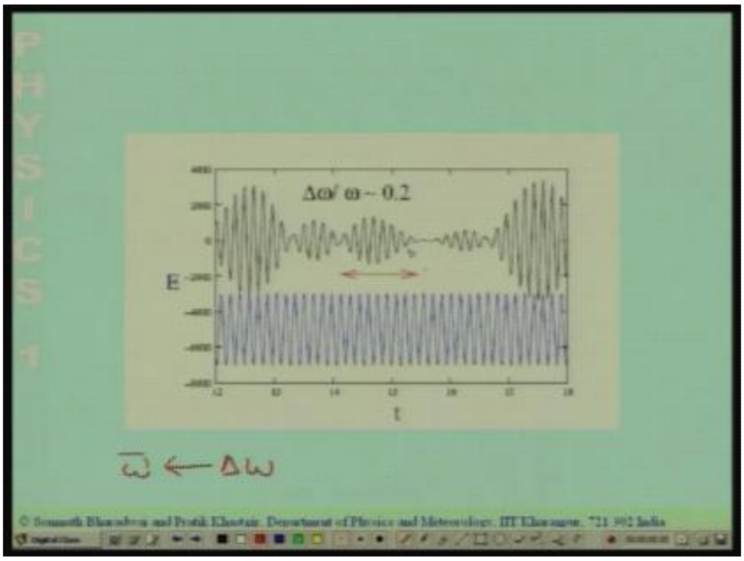

So, you see here that, the amplitude is changing and the amplitude is changing at a scale which is quite large compare to the fast oscillations. Let us ask the question on the time scale which is which I have shown now. So, this you would call the time scale of the change of the amplitude at this time scale over which, the amplitude is changing is much faster that the time scale at which you have the fast oscillations. Let us know ask the question how many fast oscillations do we expect to get in the time in the time over which the amplitude changes right.

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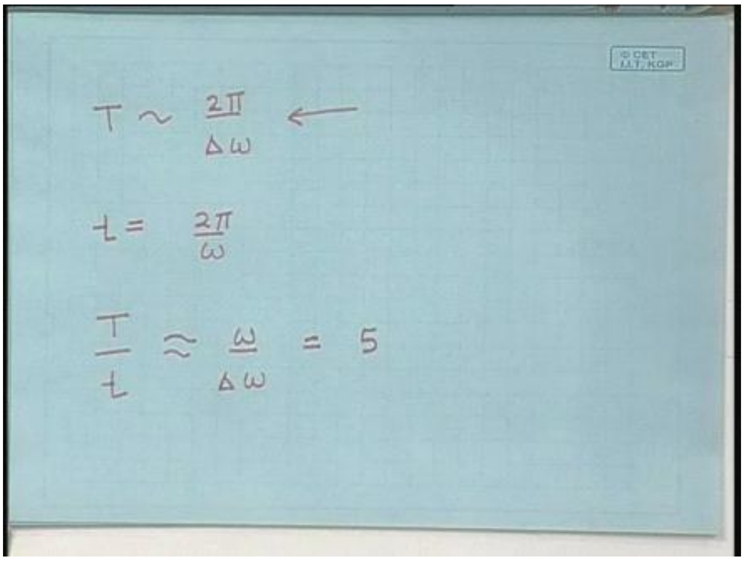

So, the amplitude changes on the time scale approximately 2 pi by delta omega the fast oscillations t occur on that time scale 2 pi by omega. The question we are asking is, in this time scale how many oscillations do we expect to get. So, this will be T by t is T by t would tell you the number of oscillations you would get on the time scale at which the amplitude changes. And this is approximately this is of the order of omega by delta omega which is in this case 1 by point 2 which is of the order of 5.

So, you would expect in this particular situation, if what we have what I have told you is correct. You would expect to get 5 fast oscillations in that on the time scale at which the amplitude changes slowly. So, let us check if that is indeed correct.

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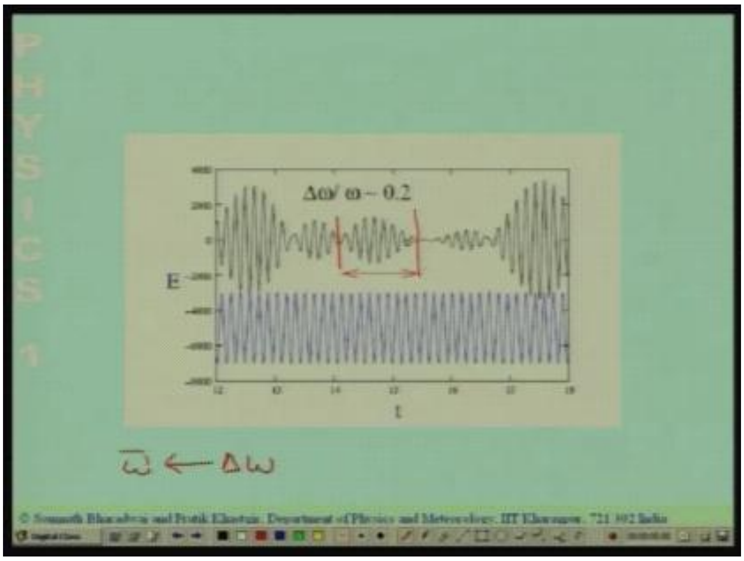

So, you can look at the graph. The upper curve and see that, the time scale over which the amplitude changes so, 1 time scale over which the amplitude changes would correspond to something from here to here. And you have 1 2 3 4 5 6 7 8 around 8 which is of the order of 5. We have 8 fast oscillations on the time scale at which on the over the time scale in which the amplitude changes. So, the crucial point over here is that when you superpose a large number of frequencies spread in a range delta omega around a central value omega bar.

The resultant wave has a fast oscillation you can think of it as a fast oscillation at the mean angular frequency omega bar. The amplitude of this fast oscillation changes slowly with time and the time scale over which this amplitude changes capital T over here is decided by the spread.

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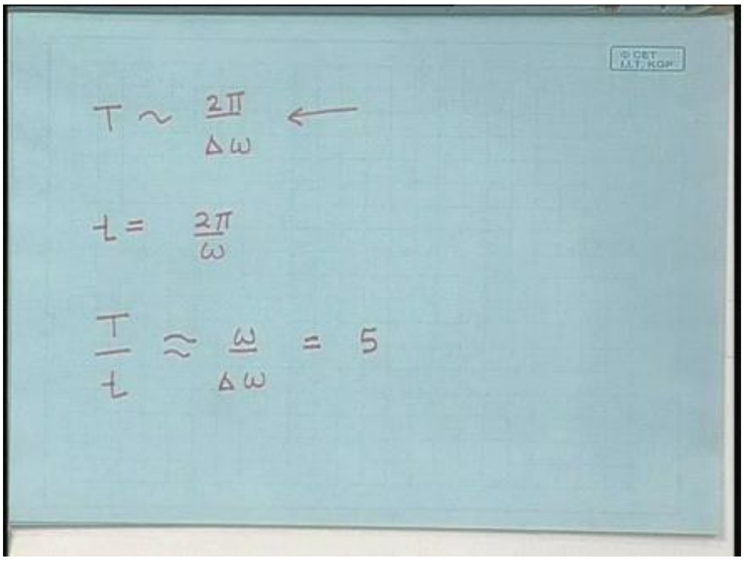

In the angular frequencies delta omega and this of the order of 2 pi by delta omega.

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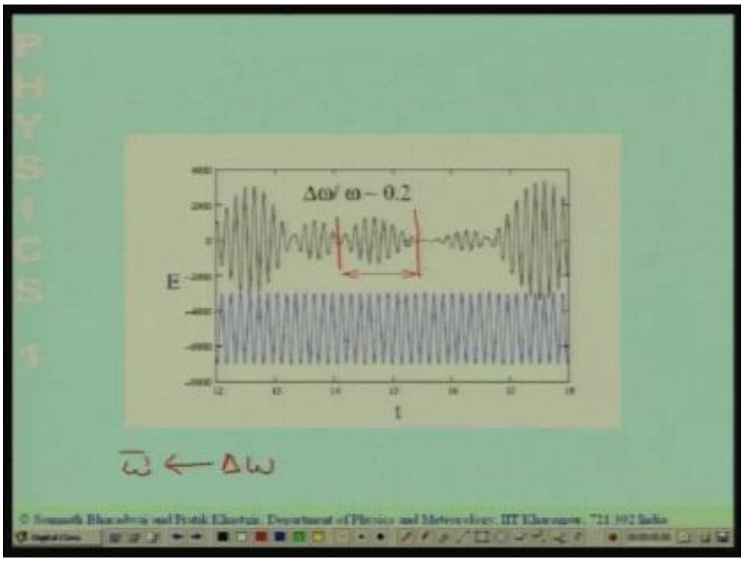

Not only does the magnitude of this amplitude change, you can see that the magnitude of the amplitude change is it is larger here it become smaller etcetera. But also, we phase of the oscillation changes slowly with time notice that to start with the upper curve and the lower curve are exactly out of phase. But if you look at the 2 over here you will see that the 2 are oscillating more or less in phase. So, the phase of this particular curve has also change slowly over time.

So, the I have been so what i have been trying to do over here.

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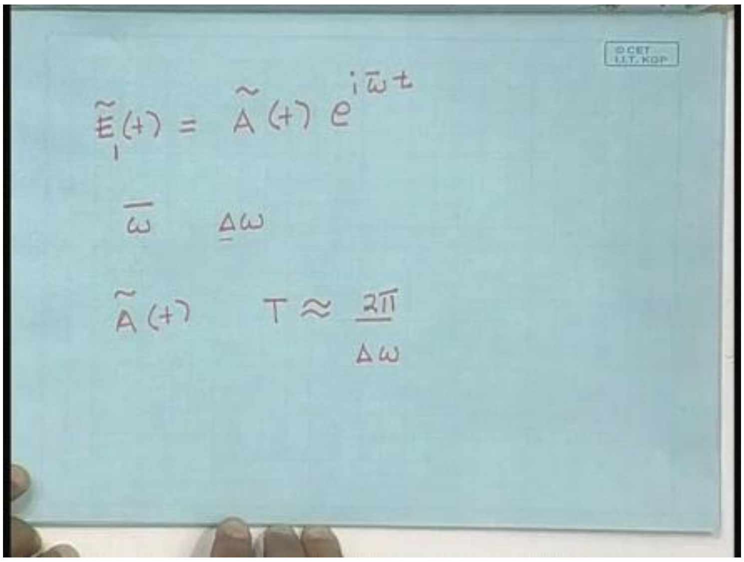

In the past a few minutes is to convince you that when I have a wave which is a superposition of many frequencies in a small band delta omega around a central value the resultant wave is a fast oscillation, at the central value omega bar, at the mean value omega bar. And amplitude now also change with time all a time scale 2 pi by delta omega. Now, the question is what is the consequence? The question that we were interested in to start with, let us get back to that question now.

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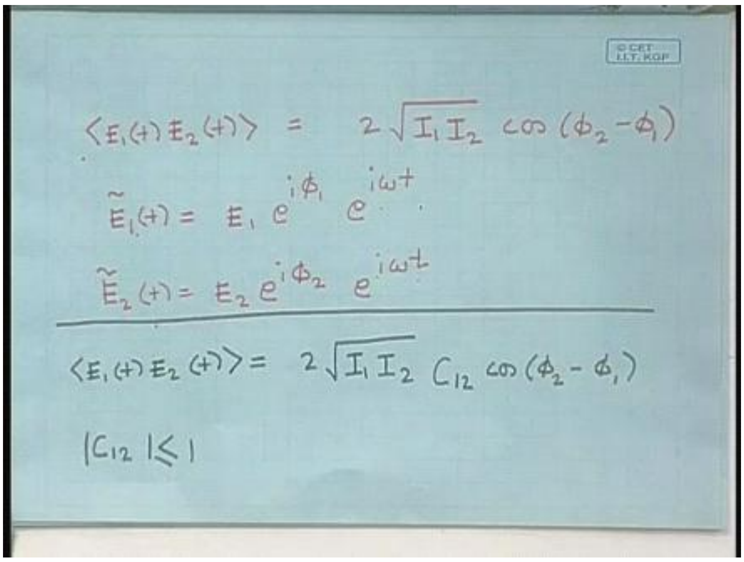

And the question was, what is the consequence of this spread in frequencies on this particular formula over here which is the main formula that we use in interference right. We have to we have to take into account the affect of the finite frequency spread of the wave. And we will we are taking into account this affect by asking the question, what is the effect? What is the consequence of this effect on this particular formula which is the central formula when we discuss interference?

So, let me tell you the answer and then we will discuss it in some more detail; the main consequence of this spread in frequencies. So, there is a spread in frequencies in in real waves in any real situation where we have waves there is always going to be a spread in frequencies. And the affect of this spread in frequencies is that the time average of E 1 into E 2 i have 2 waves. And the time average of the product of these of the displacements of the electric fields of these 2 waves is going to be less than this number. As a consequence of the fact that; we have a spread in frequencies.

So, the spread in frequencies is going to cause this time average to be less than this value which is predicted when there was a single frequency only. So, we can incorporate this by rewriting this formula as follows E 1 t E 2 t the time average of this is going to be 2 root 2 I 1 I 2 exactly the same as this when i have a single frequency. But, we now introduce an extra term over here C 1 2 and then we again write this over here cos phi 2 minus phi 1.

Where this term C 1 2 the mod of this is less than or equal to 1. And it is going to be less than 1 when we put into when we incorporate the fact that, there is a spread in frequencies in both E 1 and E 2. We have already seen that by rather by definition C 1 2 has a value 1 when E 1 and E 2 or monochromatic that is they have only a single frequency which is exactly the same for both of them. So, this numbers C 1 2 we shall call the degree of coherence are the co-efficient coherence coefficient of coherence. So, we shall refer to this as the co-efficient of coherence or the degree of coherence or just the coherence.

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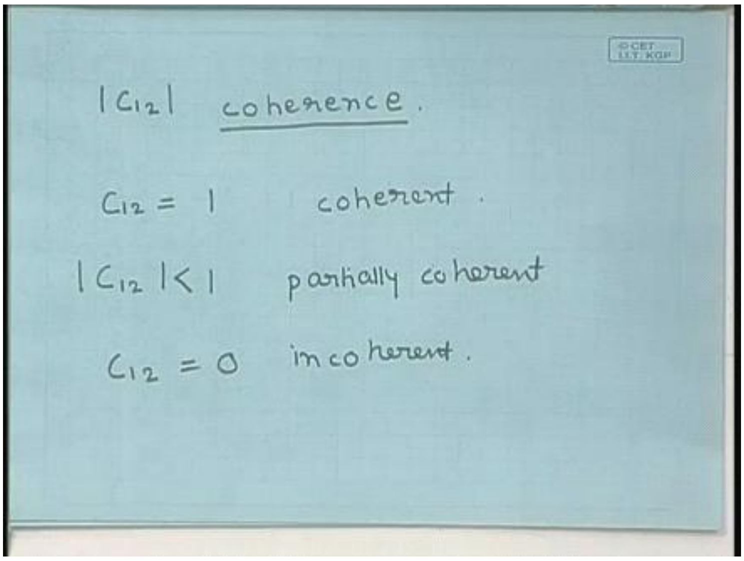

So, C 1 2 the number which we have introduce over here.

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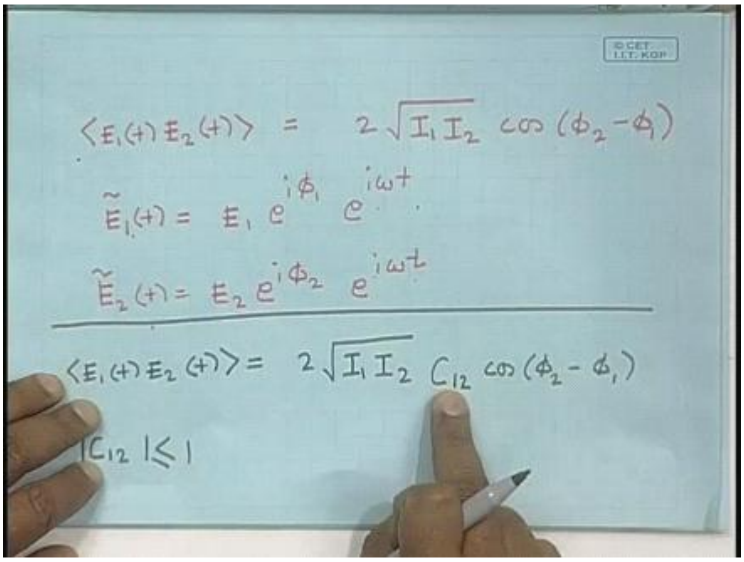

This quantifies the degree of Coherence between the 2 waves E 1 and E 2. If these 2 waves have the exactly the same frequency see the coherence C 1 2 has a value 1.

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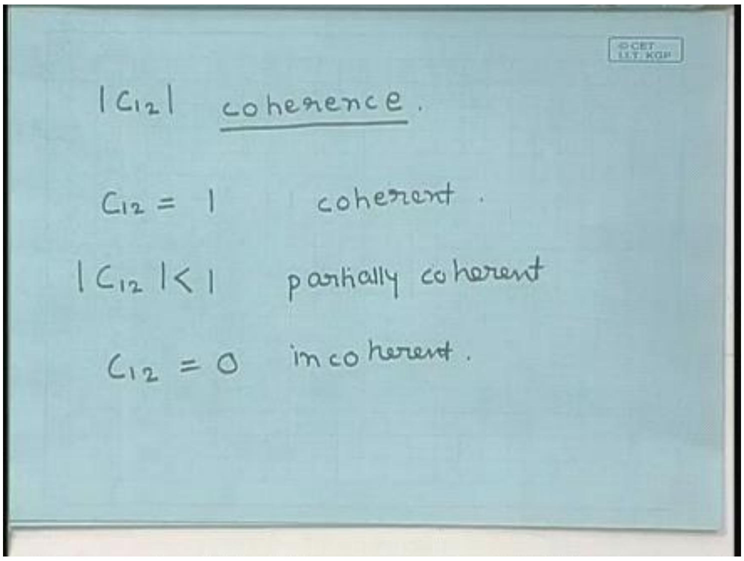

So, if this has a value 1 the 2 waves are set to be exactly coherent perfectly coherent.

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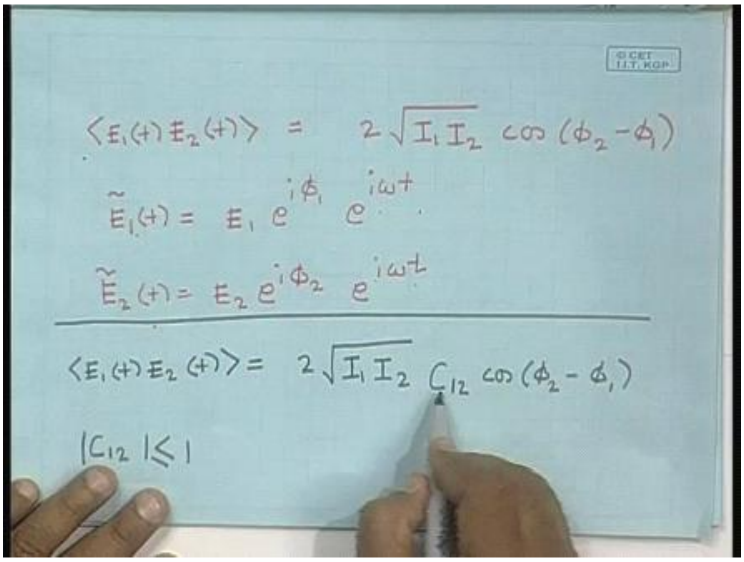

Now, i have already told you that we are going to see an example shortly. If i incorporate the fact that there is finite spread in frequencies for this and for this then the value of C 1 2 the modules of C 1 2 is going to be less than 1.

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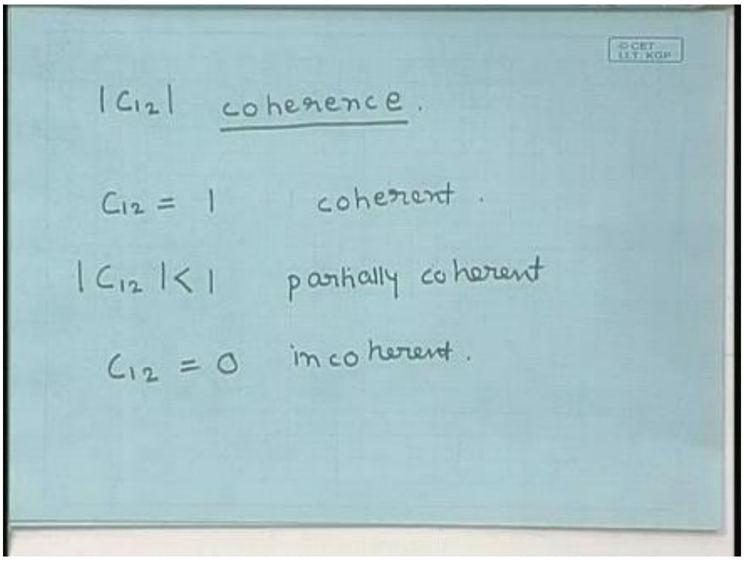

This refers to a situation which the 2 waves are set to be partially coherent. And we could also have a situation where this coefficient is 0 and the 2 waves are then said to be incoherence. Let us now discuss examples of of 2 waves which are coherent perfectly coherent incoherent and partially coherent. I will just give you simple examples to just show that this whole thing make sense. The examples at we will consider are not realistic just there are they have been constructed just to show you.

That this whole thing make sense and give you an idea of how you can go about calculating it. So, perfectly coherent we already know that if i had only a single frequency if I have a wave with only a single frequency. So, I have 2 waves E 1 E 2 both of which have only a single frequency omega then the 2 waves are perfectly coherent. Now, absolutely incoherent let me give you a situation where I have a 2 wave which are absolutely incoherent if this such a situation is very simple to construct.

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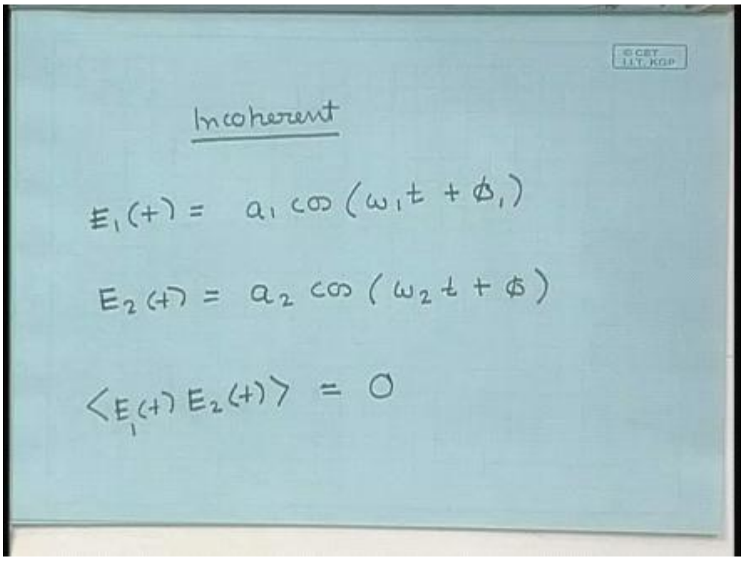

So, I will show you one situation. So, considered E 1 so we are considering a situation where I have 2 waves which have got 2 different angular frequencies omega 1 and omega 2. Let us say we are considering 2 waves which have got 2 different angular frequencies and the quantity which we would like to calculate now is the product of E 1 t and E 2 t. Now, we all know we have discuss this that if i take E 1 and E 2 to be sinusoidal waves of 2 different angular frequencies is this product is going to turn out to be 0. So, waves of 2 different angular frequencies are incoherent.

So, we do not have interfere in such in such a situation because the the term which is responsible for interference is exactly 0. The product of cos omega 1 t and cos omega 2 t if i do a time average of this product i get 0. So, we have seen 1 example we have seen an example where we have perfect coherence the 2 frequencies. The 2 waves are exactly monochromatic only 1 frequency and the 2 frequency is exactly same. Then we considered a situation which is incoherent.

So, I have 2 waves of to different frequencies which are monochromatic and 2 different frequencies. So, when I multiply them and do a time average I get 0. Now, let me give you simple examples somewhat artificial example of 2 waves which are partially coherent.

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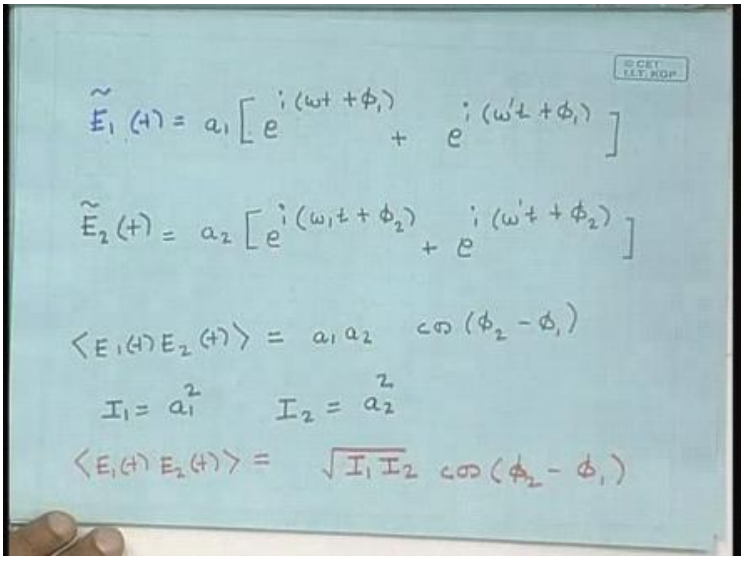

So, let us considered E 1the first wave. So, in complex notation I will write the first wave E 1 t as a 1. So, I will write the first wave as a 1 it is a superposition of 2 frequencies. So, a 1 e to the power i omega t plus phi 1 plus e to the power i omega prime t plus phi 1. So, we have a wave E 1 which is a super which is which has got 2 frequency components. So, in the complex notation I have written E 1 as a 1 e to the power i omega 1 t plus a phase phi 1 plus s to the power i omega 2 t plus a phase phi 1 the plus a same phase phi 1.

Now, let we also write another wave E 2 which is again a superposition of the same 2 different frequencies, but the phase could be different. So, we have 2 waves E 1 E 2 E 1 has 2 frequency components omega and omega prime E 2 also has 2 frequency component. So, omega and omega prime. And we would like to calculate the time average of, E 1 t into E 2 t remember that, time average that we are interested in is the time average of the real wave E 1 into the real wave E 2.

So, you could take the real part of this real part of this into the time average or it could do the whole thing in complex notation you have to be very careful there. So, let me just straight away write down the result because we know all the required in ingredients. But, before going on let me write down the result first. So, the result here is as is as follows when i multiply this with this. So, when i multiply E 1 with E 2 the real part of E 1 with E 2 I will have 1 term which will come from the product of this with this.

These are oscillations of the same frequency, but with different phase and the time average of the product of these 2 oscillation of the same frequency. But, different phases we know is going to be a 1 a 2. The time average of oscillations of the same this is going to the give me a cosine term this will also give me a cosine term. So, I will have time average of cos square and that is going to give me half into cos phi 2 minus phi 1 where phi 2 is the phase of this phi 1 is the phase of this and I will get a factor of half.

Similarly, when i multiply this term with this term sees have to multiply the this and this. So, i will have a term from here and here. This is going to give me exactly the same thing because this is a 1 into a oscillation of frequency omega prime this is a 2 into an oscillation of frequency omega prime. So, same frequencies when I multiply them and take the time average I will get a cos square term which is going to give me half. So, half plus half half from here half from here is going to give me 1. So, this is what I get.

Now, the cross term between this and this it is a product of 2 different frequencies when i take the time average I get 0 this we have just seen. Similarly, when I multiply these 2 contributions again I will get 0. So, the time average of the product of E 1 into E 2 is going to give me a 1 into a 2 into cos phi 2 minus phi 1. Now, we have to write this in terms of intensity of this wave and the intensity of this wave. So, let us ask the question what is the intensity of this wave to find the intensity of this wave I have to square this and take the time average.

When I square this I get a 1 square the time average of the square of this is half time average of the square of this also half. So, I get 1 the time average of the cross terms these are 2 different frequencies to the time average of the cross terms I going to be 0. So, when I square this and take the time average I am going to get a 1 square into half and another half here. So, this is going to be so the intensity I 1 is a 1 squared similarly the intensity I 2 is going to be a 2 square.

So, I can write finally, I can write the time average of E 1 t into E 2 t as the square root of I 1 into I 2 and I have this factor cos phi 2 minus phi 1. So, now let we go back to the definition let us ask the question what is the coherence of this wave over here. So, to determine the coherence we have to compare this with the definition.

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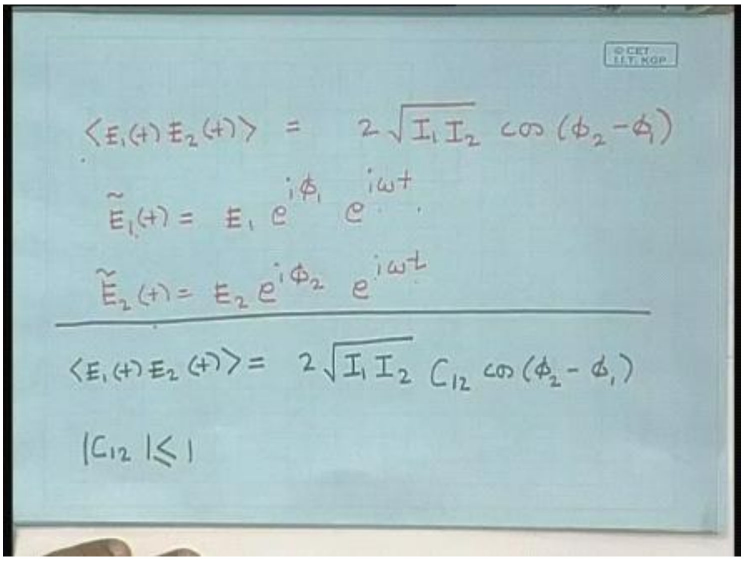

So, the definition of coherence is shown over here, coherence let we again rewind you coherence is defined as follows. If I have, 2 waves E 1 and E 2 where E 1 and E 2 both have only a single frequency which is the same. So E 1 and E 2 are both sign pure sinusoidal of a singles frequency. Then the time average of E 1 into E 2 is 2 times the square root of I 1 I 2 cos phi 2 minus phi 1. So, it is the square root of the product of the individual intensities into cosine sign of the phase difference and is a factor of 2 over here.

Now, if these are no longer monochromatic if they are no longer pure sinusoidal waves what I told you is that you have to modify this time average. So, this time average can now be written as C 1 2 into whatever we had when they was only a single frequency in both E 1 and E 2. And C 1 2 is what we use to quantify the degree of coherence the co-efficient which tells us how much coherence there is.

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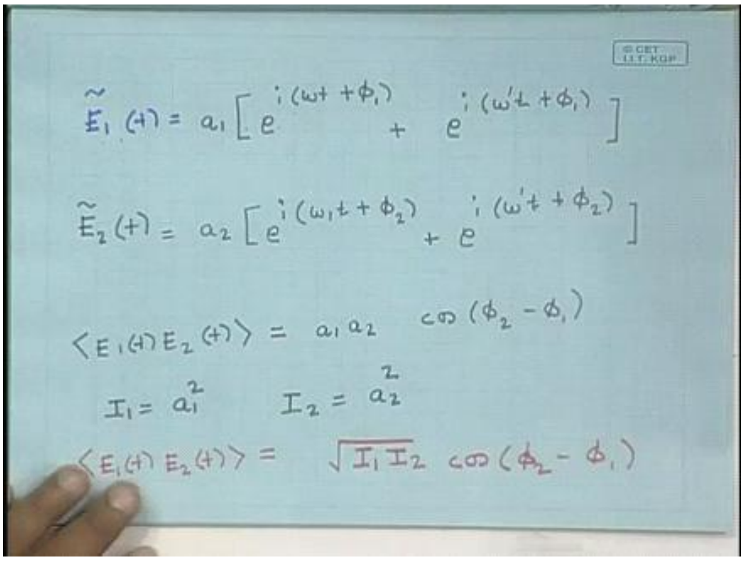

So, we have to compare this expression with what we have just calculated in the situation where the wave is not monochromatic, but has 2 frequency components. And when, the wave has 2 frequency components when both E 1 and E 2 have 2 different frequency components. We found that the time average of the product of E 1 and E 2 is square root of I 1 I 2 cos phi 2 minus phi 1.

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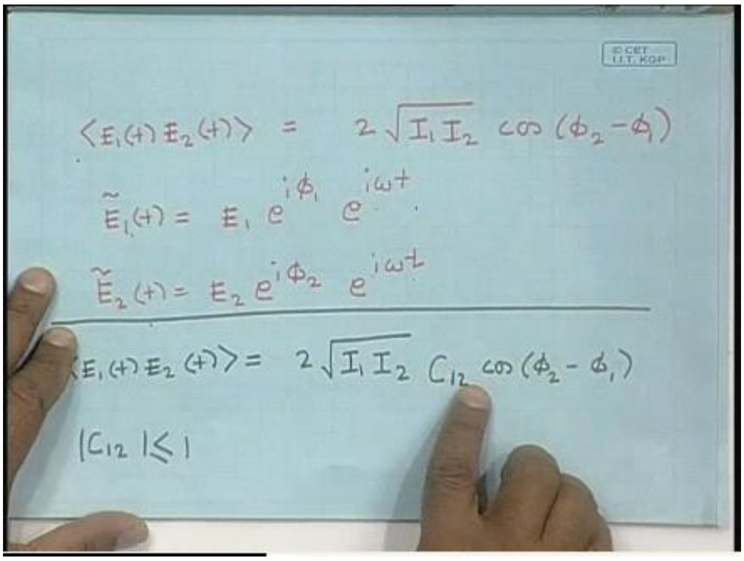

So, comparing this with the definition of this coherence the degree of coherence C 1 2.

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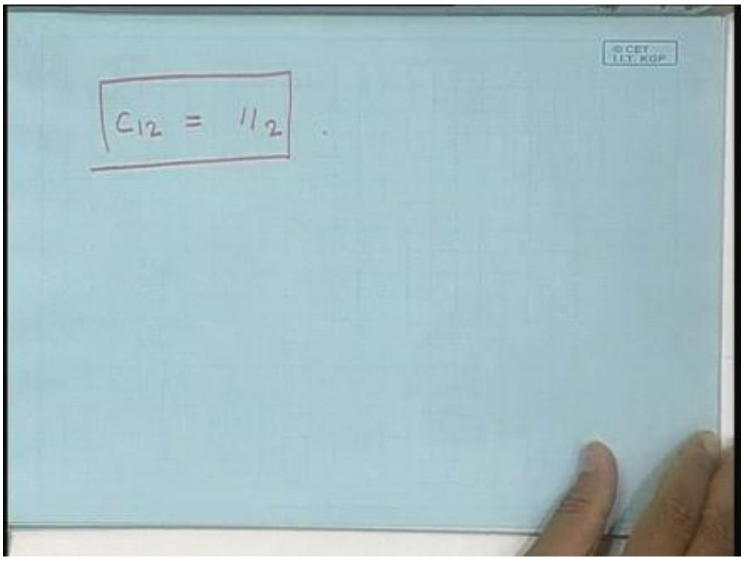

We see that in this particular situation the degree of coherence C 1 2 has of value half. So, what we see over here is that if I have a wave which is the superposition of 2 frequency components. The degree of coherence or the coherence coefficient falls from the value 1 which we have when the waves are monochromatic when there is only single frequency it is perfectly coherent. Now, when I have 2 frequency components; the coherence false to a value which is half. So, we have just considered an example where there are 2 frequency components in reality light or any other wave is going to have a spread in frequencies.

So, in reality there is going to be an average value omega bar. And there will be a spread in frequencies around this value and what I have triads to show you by doing this exercise is that, in such a situation where there is a spread in frequencies the coherence is going to fall. It is not going to be 1 it is going to be less than 1 the degree of coherence is going to be less than 1. So, that is the key point which I have been trying to convince you through this simple examples and simple and somewhat artificial possible examples and then calculations.

So, now, let us ask the question what is going to be the observable consequence of this the fact that, the coherence is has fallen what is going to be the observable consequence of this. So, let me now go back. Let we go back to the basic formula which we have for interference.

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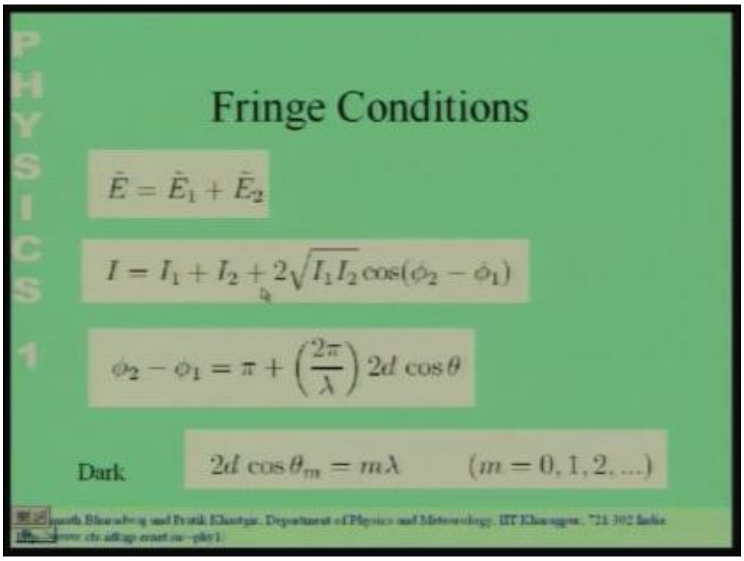

So, let me now go back to the basic formula that we have for interference. So, the basic formula governing interference is again shown over here and this formula now gets modified.

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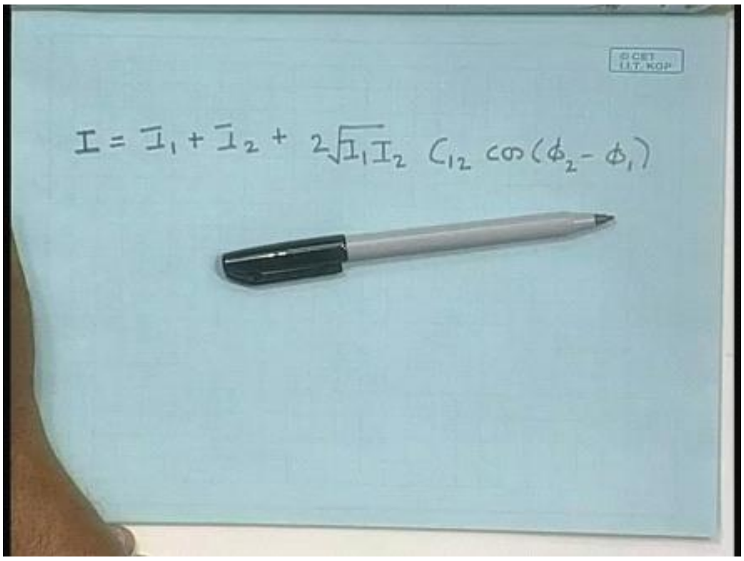

So, when you are using partially coherent light the intensity of the result tend of 2 ifs is I 1 plus I 2 plus 2 root I 1 I 2. And now, you have the degree of coherence cos C 1 2 and cos of the phase difference phi 2 minus phi 1 right. So, in this in the situation where we have perfectly coherent to perfectly coherent waves C 1 2 the coherence is 1.

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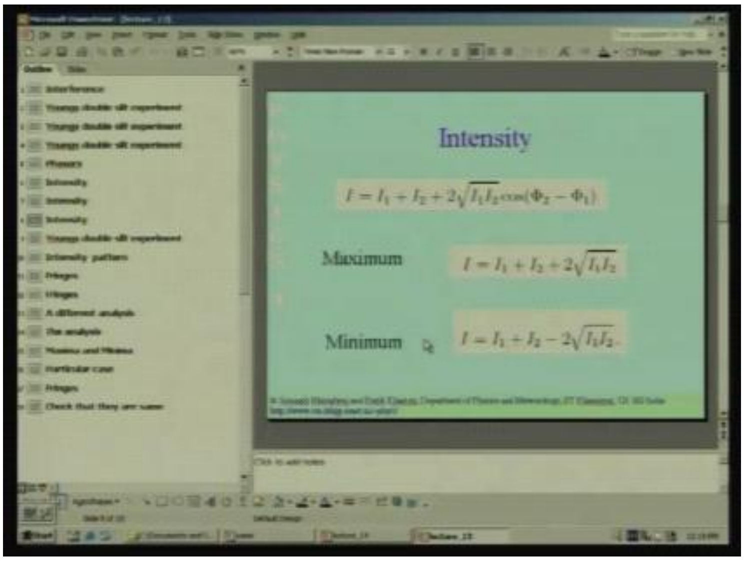

And the maximum intensity is what I have shown over here and the minimum intensity and the maximum intensity there are both shown over here the maximum and the minimum.

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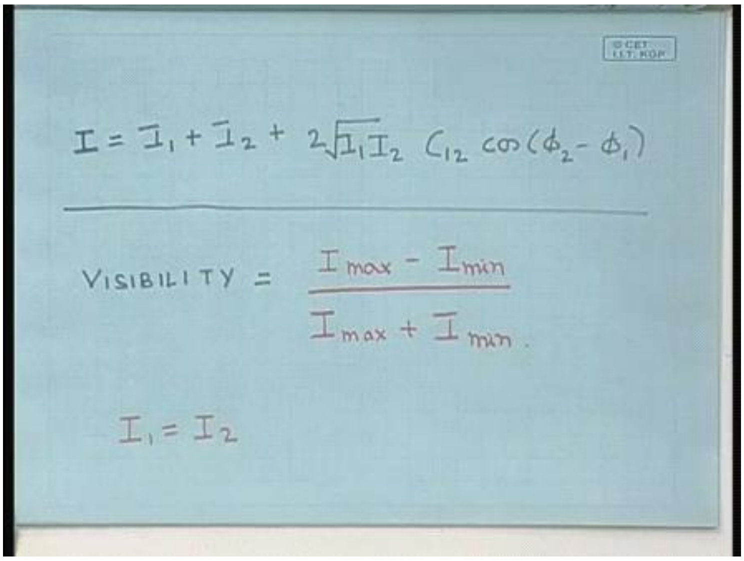

Now, the affect of this coherence of this of partial coherence so, if C 1 2 the degree of coherence is less than 1 main effect is it reduces the difference between the maximum and the minimum intensity. It reduces the contrast of the intensity pattern. So, there is the concept of something called the visibility which is defined as follows. Let we introduce this new concept new quantity the call the visibility the visibility is defined us follows I max minus I min divided by I max plus I min.

So, what does the visibility tell us? The visibility tells us how sharp how much contrast do we are not how sharp how much contrast do we have in our fringes how bright are the fringe is the fringe pattern. If the fringe if the whole field of you is more are less nearly uniformly distributed the difference between the maximum and the minimum intensity is going to be very small. So, the visibility is going to be small where as if the fringe pattern is very bright. Then the difference between the maximum intensity and the minimum intensity is going to be very large.

The visibility is going to be a large number visibility is a dimensionless number remember same as. So, is the coherence the coherence the degree of coherence is also a dimensionless number. Where, visibility is a dimensionless number which can be directly measured from the intensity from the fringe pattern. You have to measure the intensity at the place where, it is brightest in the fringe pattern measure the intensity at the place where it is the darkest in the fringe pattern and from there you can determine the visibility.

So, for the situation where I have 2 waves of exactly the same intensity I 1 and I 2 which is the situation which we that we have been considering in both the Michel's and Young's double slip experiment. We have been dealing with waves which where the 2 we have been dealing with the situation where the 2 waves have nearly equal intensity. In such a situation in the presence of this coherence the degree of coherence factor which arises when we use partially coherent light.

Let us calculate the visibility. So, when such a situation the maximum intensity that you have is 2 2 I 1 plus 2 I 1 into C 1 2.

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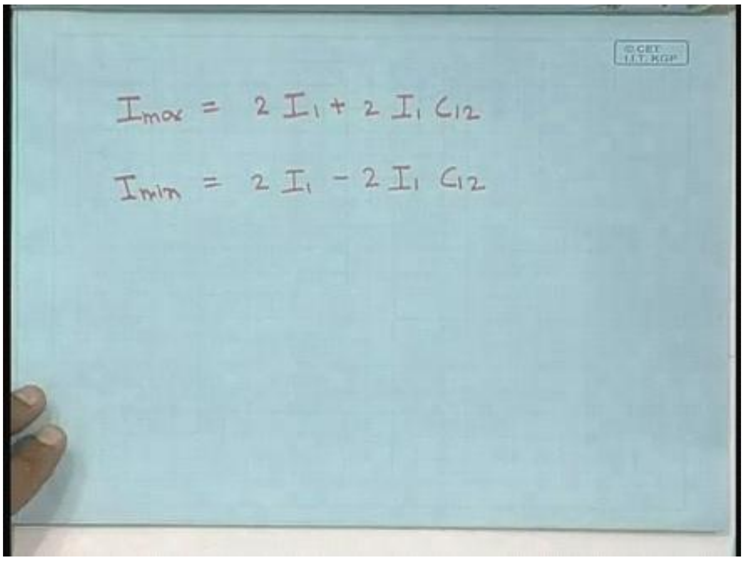

So, I max the maximum intensity the minimum intensity is when the cosine term is minus 1. So, this will be 2 I 1 minus 2 I 1 C 1 2 see we have assume that I 1 and I 2 are same.

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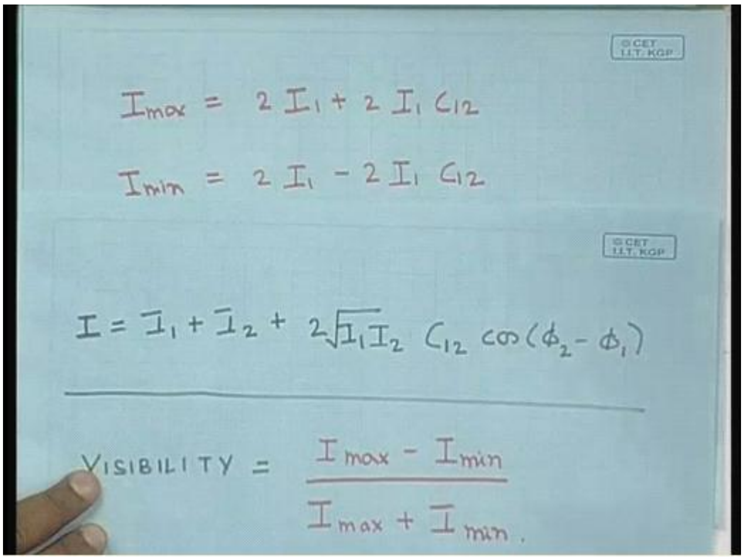

We have calculated the maximum and the minimum intensity using this over here in the expression for the visibility.

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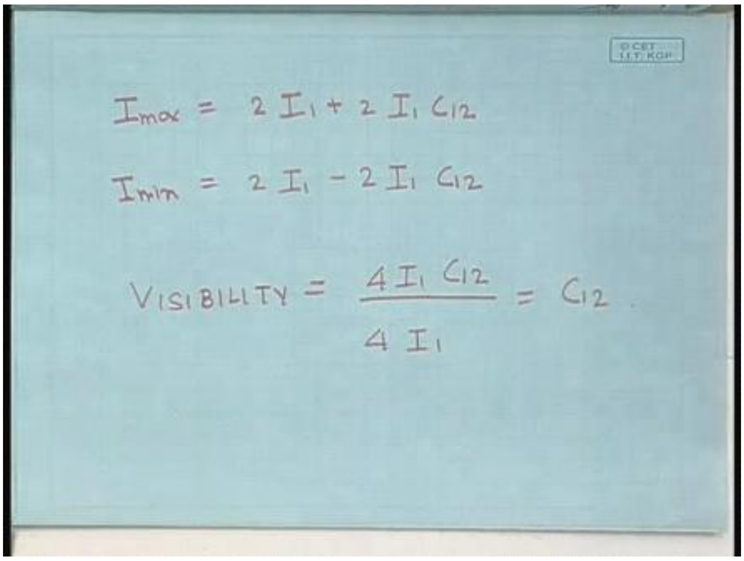

The visibility is equal to. So, when I take the difference of these 2 I will get four I 1 C 1 2 when I take the some of these 2 I will get 4 I 1 which is just the same as the degree of coherence of the coherence coefficient. So, the visibility something with at you can measure from the fringe pattern lets us measure the degree of coherence. So, this tells us how this gives us a method by which you can measure the interference interferometry gives us a method by which you can measure the degree of coherence of the light of the wave that you are dealing with.

So, coherence has a very intimate link with the phenomena of interference now, let we go back. So, in last few lectures we have been discussing a few cases of particular situations where we have interference. So, let we go back to these situations that we are have been discussing.

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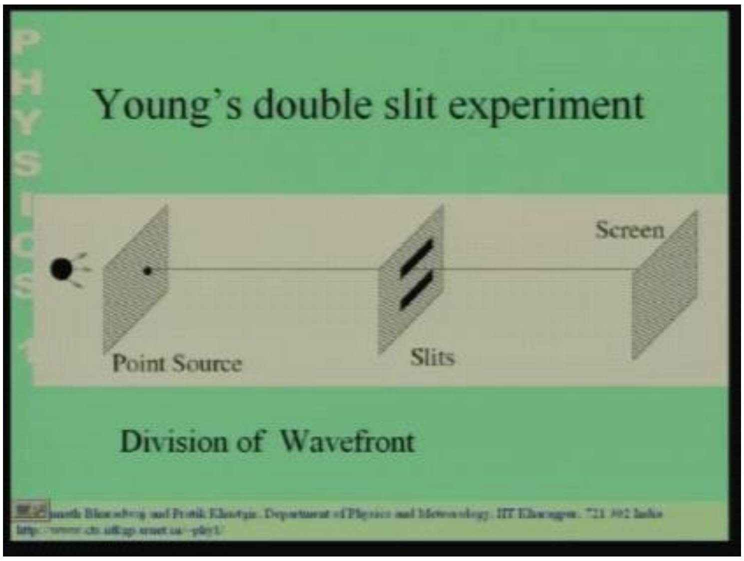

So, the first situation that we discussed was the young's double slit experiment in the young's double slit experiment. Let me just in the young's double slit experiment if you remember we had a source which emitted a wave. So, this was the source which emitted a wave. And then the wave front was incident on 2 slits. So, the wave fronts that come out from here are incident on 2 slits the role of those 2 slits is that it divides the 2 wave fronts it divides the single wave front.

So, it takes the wave from 2 different parts of the wave front and these interfere these are superposed these arrive at the screen by through separate paths. And on the screen you measure you have the superposition of 2 different parts of the wave front through different slits. So, you have produced 2 waves on the screen over here through the division of wave front.

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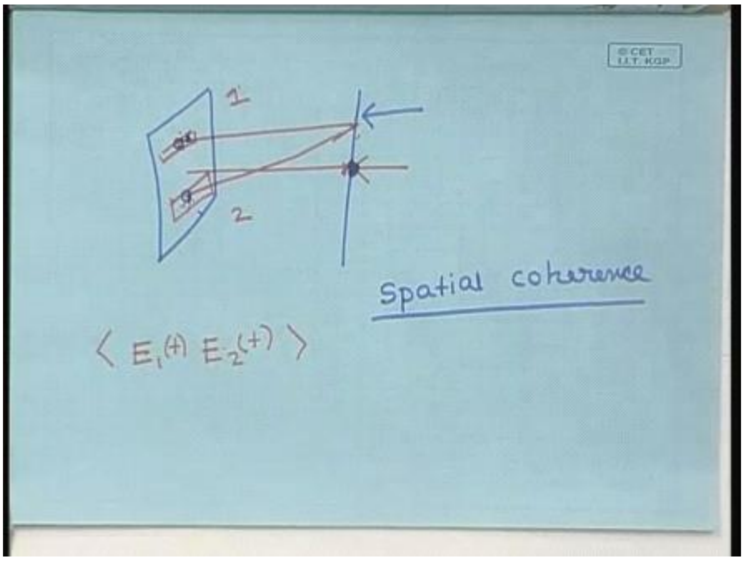

So, in the young's double slit experiment what you have done is you have this is let us say this is a these are 2, this is the way front you have take in 2 different points over here. And on the screen over here what you do is you superposed the electric field the or the wave at these 2 points these are the slits. And over here on the screen you superpose the electric field. So, we can call this is slit 1 this is slit 2 what you the what you have over here is E 1 into E 2 and you take the time average of E 1 into E 2.

Now, there is another point which comes in over here different points on the screen receive these 2 waves with different time delays. So, what you do what you achieve in the young's double slit experiment is that you are able to determine the coherence between the electric between the wave at 2 different points the 2 different points being the 2 different slits right. So, you are able to determine in the coherence between the waves at 2 different points at which are the 2 different positions which are the 2 slits.

So, on the screen over here you essentially measure the coherence of the wave at these 2 different points. So, at the central position you measure E 1 and E 2 at the same instant of time the time average of this, this is only at the centre. IfIi move to a different point on the screen if I move to this point on the screen. I will be measuring E 1 and E 2 with different time delays because the paths which they have to propagate are different. So, over here i would be measuring E 1 and E 2 with different time delays let us focus on the interference that, we get on the intensity at the screen over here.

So, at this centre of the screen at the centre of the screen we are measuring E 1 the electric field here the wave at this point E 2 the wave over here the product of this. And the time average is what is measured over here. So, this is so what we have doing is we are measuring the coherence between the wave at 2 different positions at the same time. We can refer to this as spatial coherence. So, we have could a displacement in the position and we are measuring the coherence between the wave at 2 different points at that particular displacement.

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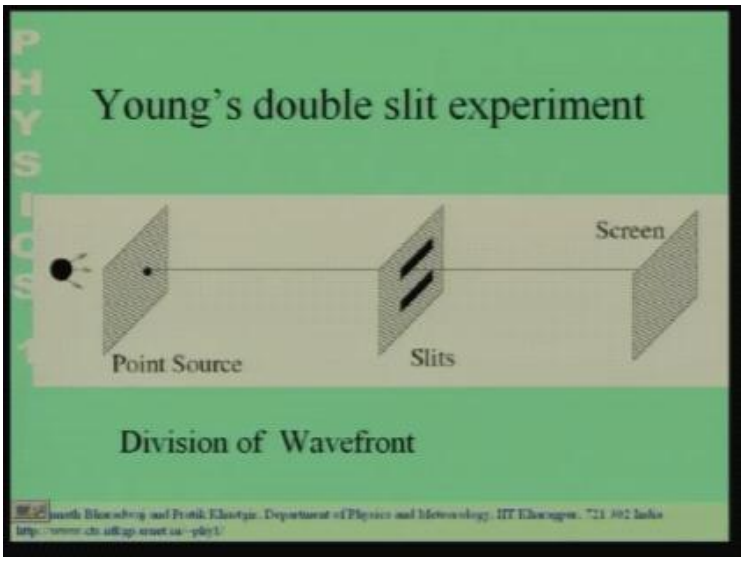

So, the point at the centre of this screen over here is measuring the spatial coherence of this wave the wave that is produced by this source from this source over here. So, the at the centre of the screen you essentially measure the spatial coherent of the wave at between these 2 points between these 2 slits. And we have already calculated this.

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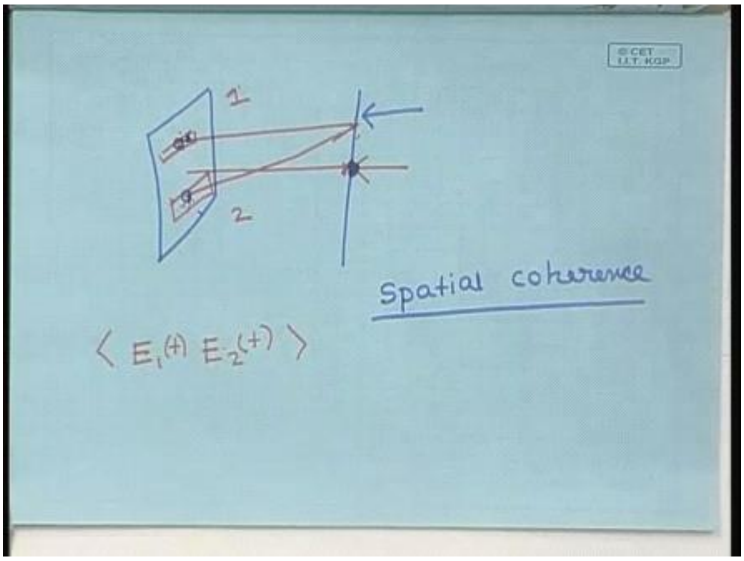

So, we have calculated this quantity at the centre of the screen. We shall take up this the interpretation of that in terms of coherence in the next lecture.