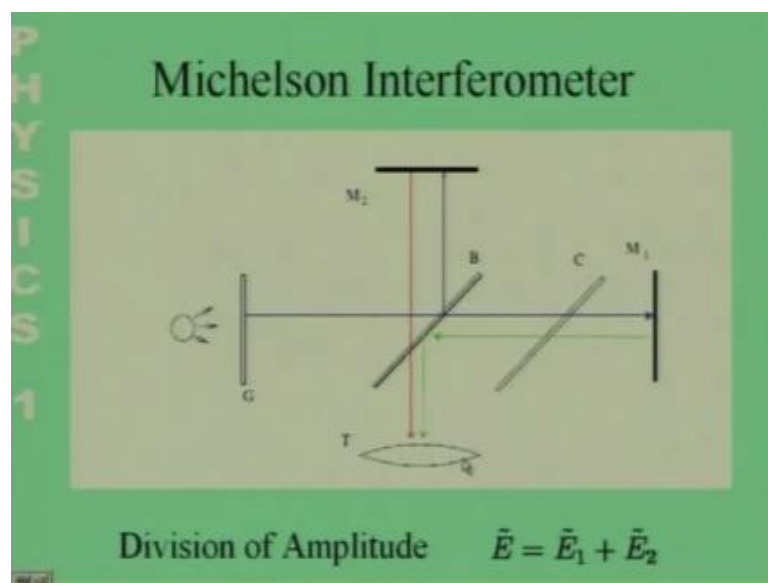


**Physics I : Oscillations and Waves**  
**Professor. S. Bharadwaj**  
**Department of Physics and Meteorology**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 17**  
**Interference - IV**

In the last class, we were discussing the Michelson interferometer. The Michelson interferometer as I told you works by the division of amplitude.

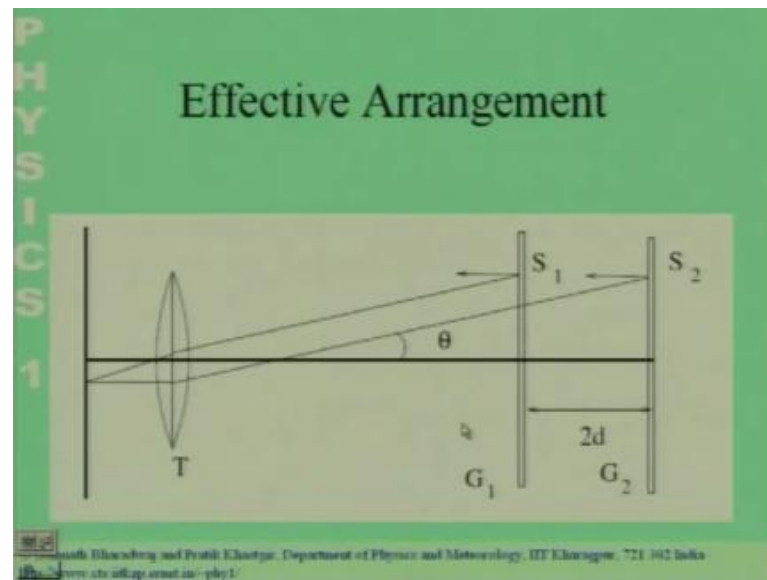
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So, we have an extended source from which the incident light falls on a beam splitter and the beam splitter divides a single wavefront into 2 wavefronts 1 which is reflected and another which is transmitted. So, the amplitude of the incident wavefront is divided to produce 2 wavefronts these 2 wavefronts traverse 2 different paths along the 2 different arms of the interferometer.

So, this is 1 arm of the interferometer 1 the transmitted wave travels along this arms comes back and then is reflected into the telescope. The reflected wave propagates along this arm it goes up and then it is reflected back passes through and again is collected in the telescope.

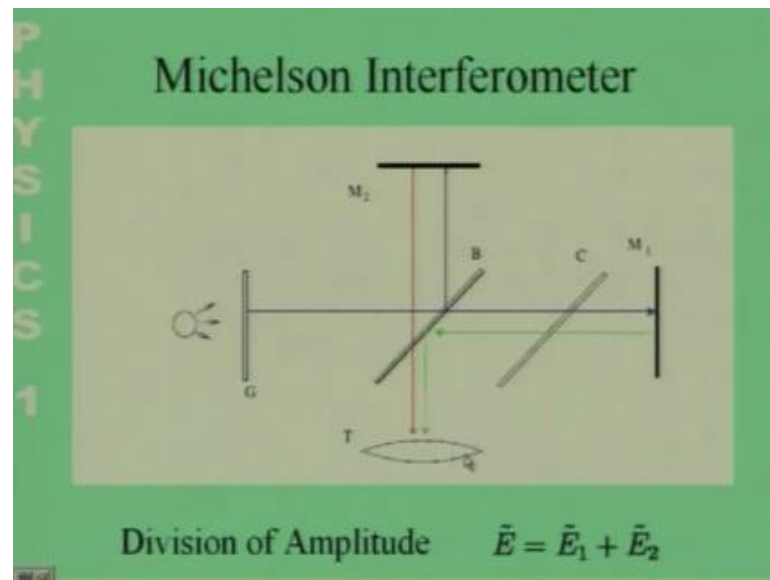
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So, we have 2 images of the source of the ground glass plate which are produced and the waves from these 2 images interfere with each other. So, we considered a particular point on the ground glass plate  $S_1$  is the image of the of this point the in the first,  $S_1$  is the point corresponding to this point in the first images,  $S_2$  is the point in the second image the waves which will be emitted from these 2 images will be coherent because they are images of the point on the source.

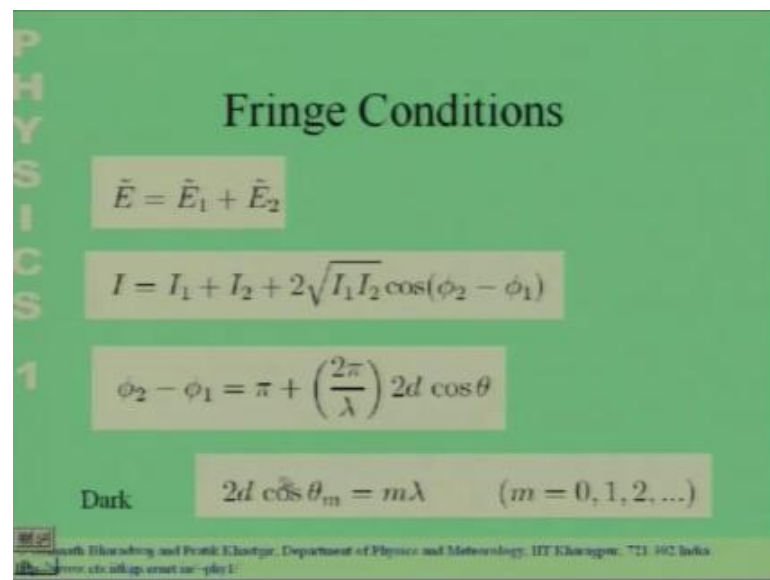
And these 2 waves will be focused to the same point by the telescope. And you will get a bright or dark intensity dark point over here. So, the intensity here will be bright or dark depending on the phase difference between these 2 waves the phase difference between these waves is  $2d \cos \theta$ . The phase difference between these 2 waves arises because of the path difference and this is  $2d \cos \theta$ . So, you have to multiplied by  $2\pi$  divided by  $\lambda$  to get the phase difference.

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There is also an extra phase difference of the pi because 1 of the waves under goes internal deflection while the other 1 under goes external reflection.

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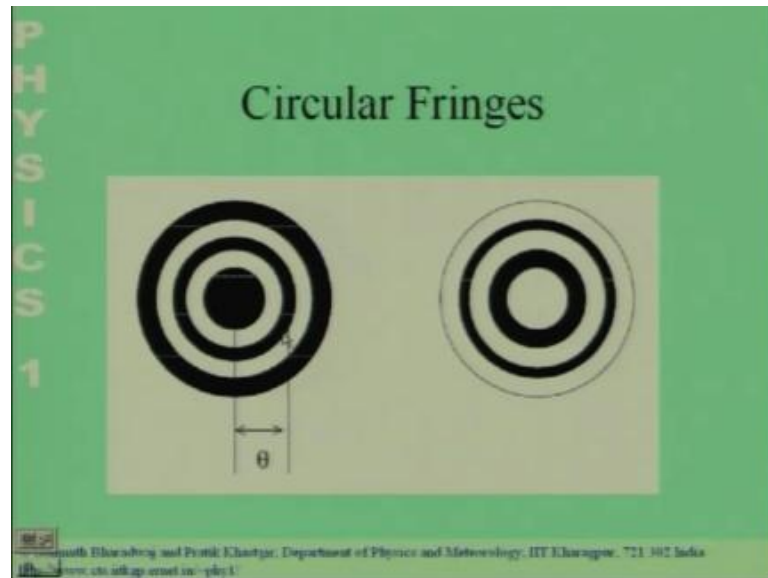


So, finally we had this expression for the phase difference between the 2 waves and you will get a dark fringe if this condition is satisfied. So, you will get a dark fringe, a dark the intensity will be dark will be very small if this condition is satisfied. Where d is the difference in the arm lengths theta is the angle at which you are looking and m is an

integer. So, if  $2d \cos \theta$  the path difference is equal to  $m \lambda$  you get a dark fringe if it is  $m + \frac{1}{2} \lambda$  you get a bright fringe.

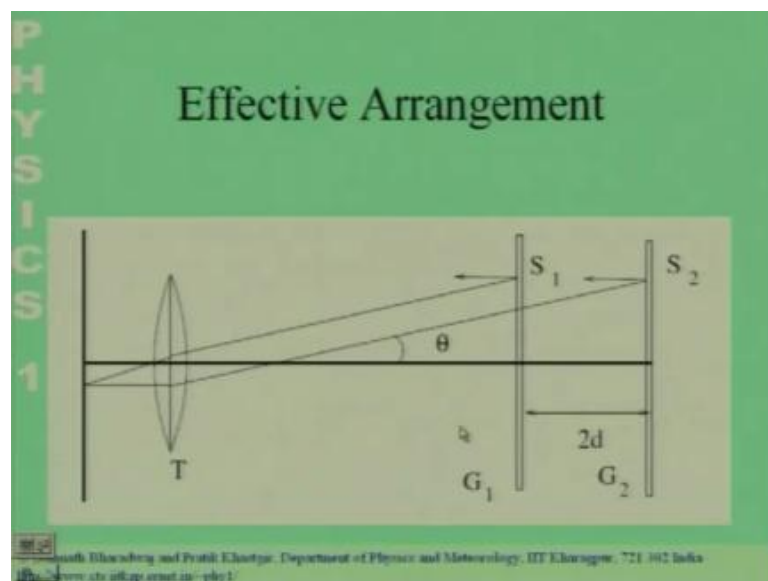
So, this was the fringe condition which we had discussed in the last class.

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I had also told you that the fringe pattern is going to be circular

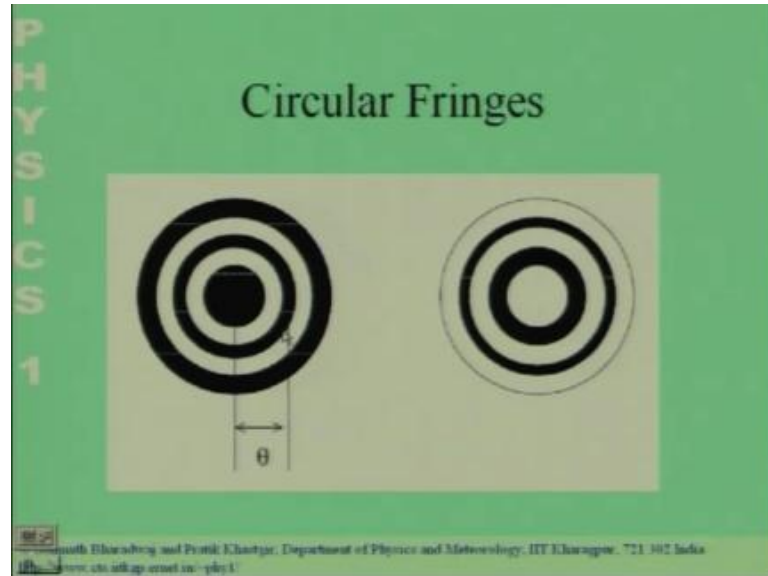
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And this is because the whole thing is symmetric around this axis. So, a point over here at the same angle  $\theta$  is going to have the same intensity as the point over here and not

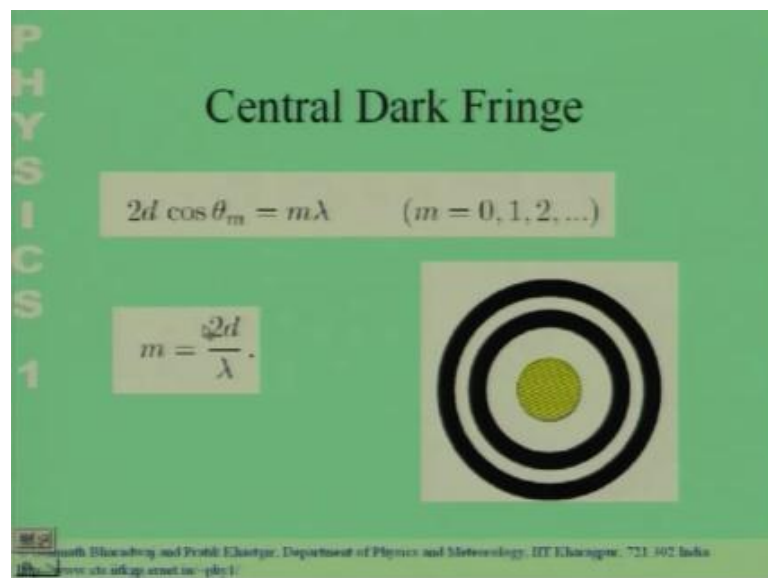
only that all points which make an angle theta with this axis are going to have the same intensity.

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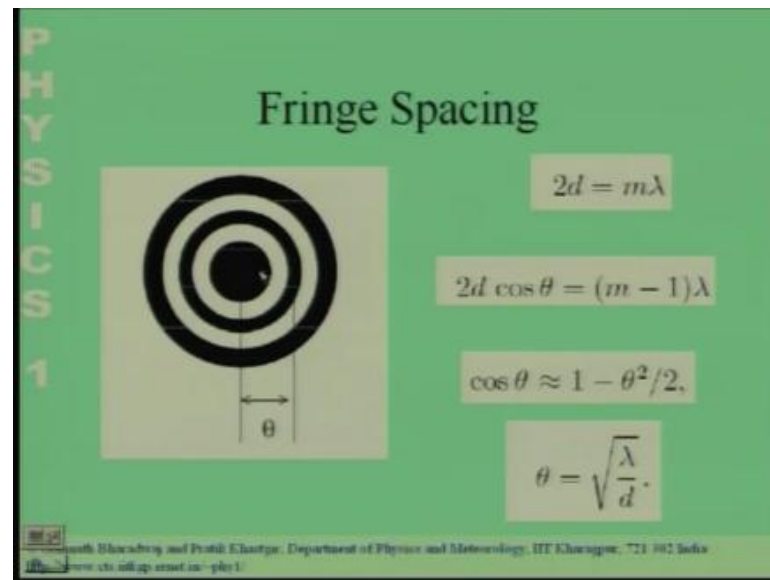
So, the fringes are going to be circular and you will have dark and bright a dark and pattern of dark and bright fringes which are circular which looks like this.

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We then had calculated the order of the central fringe when the central fringe is dark the order of the central fringe is  $2d/\lambda$ , where  $d$  is the separation between the 2 arm lengths the difference between the 2 arm lengths.

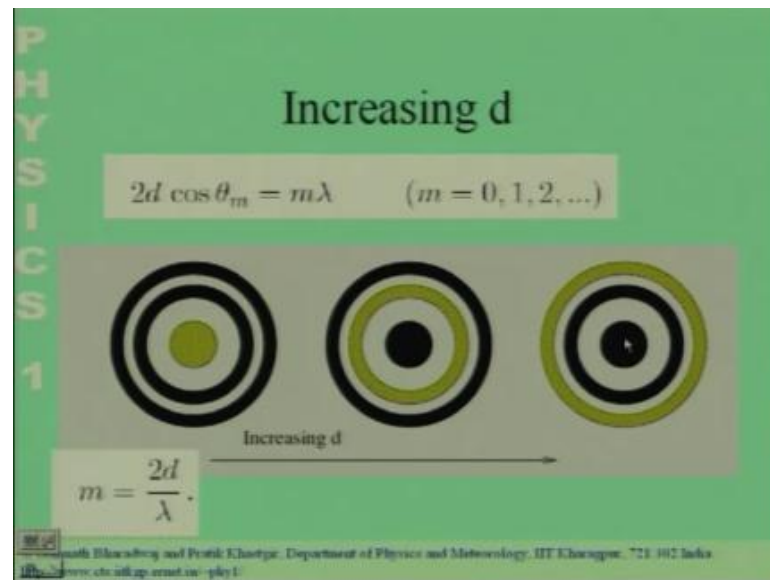
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We then had calculated the fringe spacing, which is the angle between the central dark spots and the first dark ring calling this angle theta we had found that theta is lambda by d. So, we found that the fringe spacing gets smaller as the separation between the 2 arms is increased and if this 2 arms are at exactly the same separation. The central dark spot grows till it fills the entire field of view. So, the whole field of view is dark when the 2 fringes are 2 arms are exactly of the same length.

As you increase the length the fringe spacing gets smaller and smaller and for very large links the fringes are going to be very close together.

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So, for very large difference in the length, the next thing that we had considered in the last class was what happens to the fringe of a particular order  $m$  as you increase the difference in the arm lengths. So, you had followed the evolution of the fringe of a particular order a dark fringe of a particular order. So, you have a dark fringe of a particular order at the centre. If the difference in the arm lengths  $d$  satisfies this relation that  $2d$  by  $\lambda$  is an integer  $m$ .

So, if  $2d$  by  $\lambda$  is an integer  $m$  you will get a dark fringe of order  $m$  at the centre now as you move 1 of the mirrors increasing  $d$  the this particular fringe. So, the  $m$ 'th order fringe will move outwards as you can see here. So, it will move out and you will get a new fringe. So, this fringe is now of a order which is higher than this because  $d$  has increased. So,  $m$  also as to increase. So, the central fringe now if when you increase  $d$  the central fringe is now of 1 order higher and the fringe which was at the centre earlier has now moved to a larger angle.

If you increase the distance even further the fringe will move out to even larger angle and finally, it will move out of the field of view and new fringes have appeared in the centre, fringes of higher order have appeared at this centre.

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**PHYSICS**

## Measuring Wavelength

$$2d = m\lambda$$
$$2(d + \Delta d) = (m + 1000)\lambda$$
$$\Delta d = 500\lambda$$

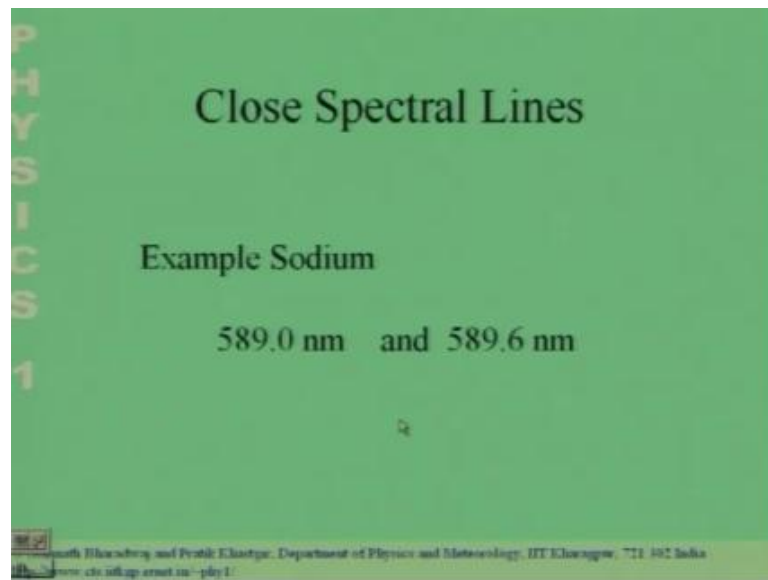
Dr. Anshu Bhattacharya and Prof. Dr. Pratik Choudhary, Department of Physics and Meteorology, IIT Kharagpur, 721 302 India  
www.che.iiitg.ac.in/~phys/

So, you can use this to measure the wavelength of light, we had discussed this in some detail in the last class, what you have to do is you have to move 1 of the mirrors. So, what you do is you move 1 of the mirrors. So, that you get let us say 1000 new fringes at the centre and you measure the distance at you have to move 1 of the mirrors. So, that you get thousand new fringes in the centre this distance we call delta d and we had seen that delta d for 1000 new fringes delta d is going to be 500 into the wavelength.

So, even if the wavelength is small delta d is something that you could measure. And from this from measuring delta d the distance that you have to move 1 of the mirrors you can determine the value of the wavelength.



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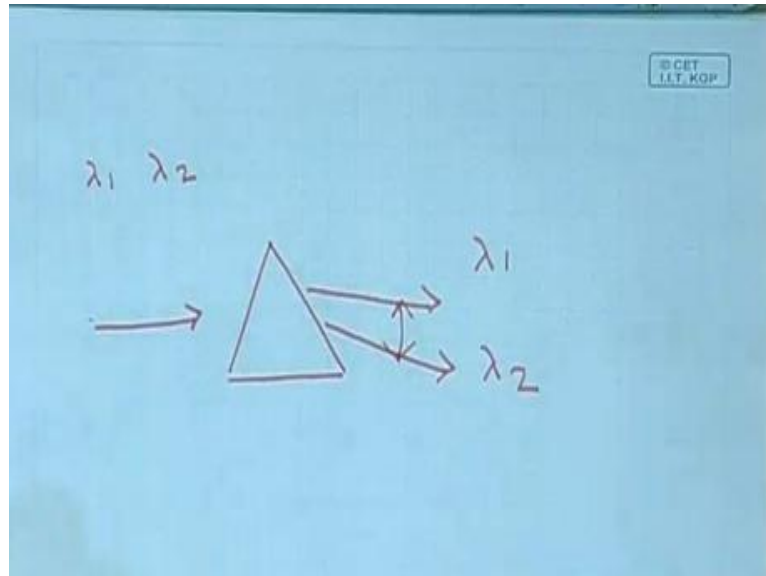
Now, today we will start off discussing another very interesting application of the Michelson interferometer. So, today we are going to consider a situation, where we have we are using some light to move the Michelson interferometer up to in the Michelson interferometer. So, we have the fringes produced by a certain light of a certain wavelength, but suppose that the light which is incident is such that it has got it is composed of 2 spectral lines, such a which are very close together such a situation is not very uncommon for example, the sodium vapour lamp.

The sodium vapour lamp which we use quite commonly in the lab also used as to illuminate the streets the light from the sodium vapour lamp has got 2 spectral lines. So, sodium has got 2 lines which are very close together, which are produced in the sodium vapour lamp these 2 lines are at 589.6 nanometers and 589.6 nanometers. So, the light which you get from the sodium lamp sodium vapour lamp has 2 spectral lines which are very close together.

The question is suppose, I have some radiations some light which is coming from a source and I would like to find out first are there is it 1 wavelength is it 2 wavelengths or multiple. And if there are 2 wavelengths then what is the difference in the wavelength between these 2 wavelengths. Now if the difference is quite large you could send the light though a prism the refractive index of the glass in the prism depends on the

wavelength. So, the prism will disperse the light and the 2 different wavelengths would have come out at 2 different angles.

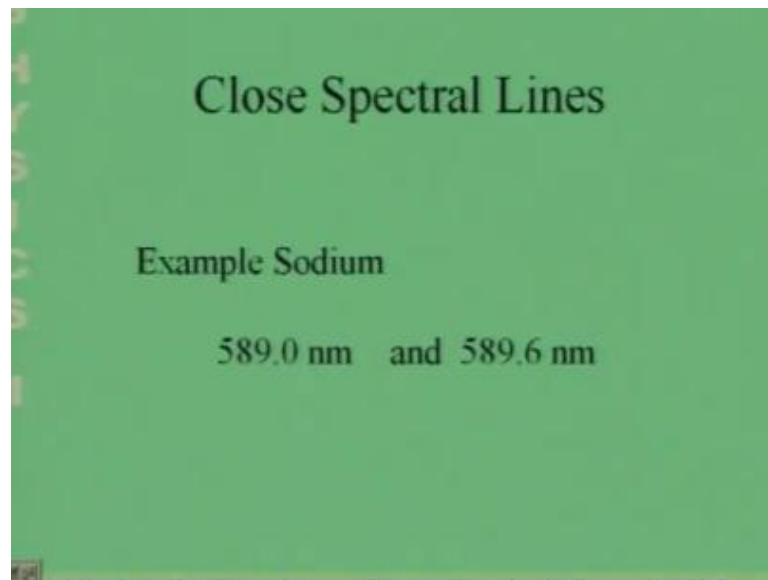
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So, the situation would be as follows, we know that the glass has a different refractive index for different wavelength. So, if you send in light which has got 2 wavelengths in it  $\lambda_1$  and  $\lambda_2$  the 2 wavelengths would get would come out at different angles even though there are incident at the same angle the light which comes in as these 2 wavelengths the light would get dispersed. And the 2 different wavelengths would come out differently and you could discriminate you could determine the fact that there are 2 different wavelength in here not 1.

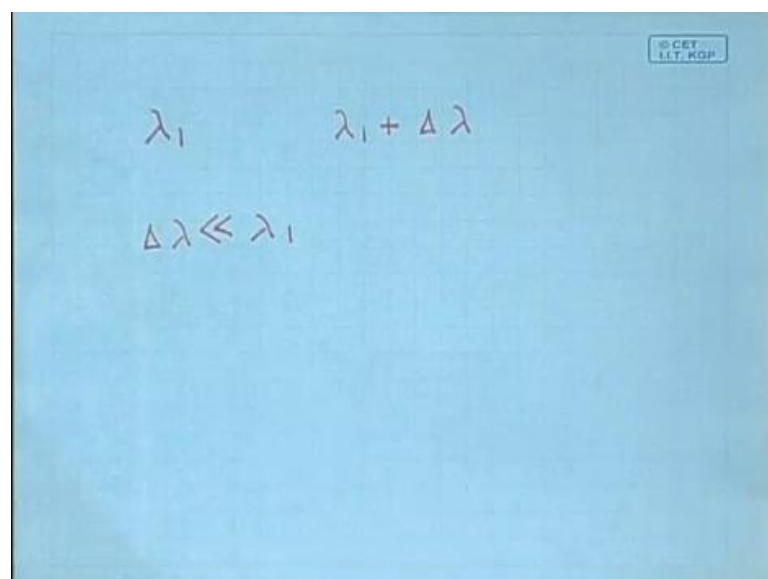
But, this will work only if the separation between these 2 wavelengths is quite large if the separation between these 2 wavelengths is not very large is very small then the difference in angle between this and this would be very small and you would not be able to measure it. So, I have given you 1 example.

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The example of sodium. Sodium has produces 2 spectral lines 1 at 589 nanometers and 1 589.6 nanometers. So, there only 0.6 nanometers apart the questions is how would you determine if I did not know the difference in wavelengths to start with how would I measure it and Michelson interferometer is a is a technique by which you could measure the difference in wavelengths. So, the situation which we are going to consider here is as follows.

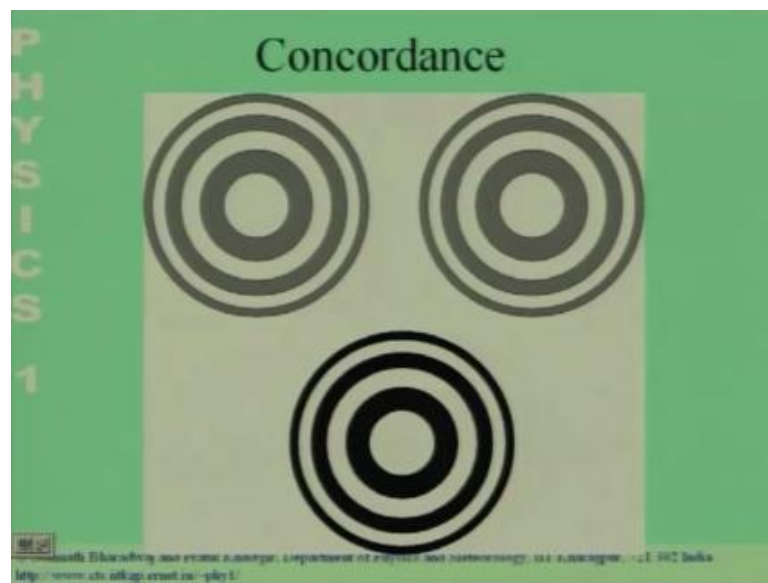
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We have light which has got 2 wavelengths 2 spectral lines in it 1 at  $\lambda_1$  and another at  $\lambda_1 + \Delta\lambda$ . So, the light which is coming into the Michelson interferometer has got 2 different wavelengths  $\lambda_1$  and  $\lambda_1 + \Delta\lambda$ . And we are going to assume that the  $\Delta\lambda$  is very small much smaller than  $\lambda_1$  and the problem is how do we measure this small difference in the wavelength the wavelength itself as we have seen is quite small it is difficult task to measure it. But you can do it using the Michelson interferometer now the question is that you have a small you have 2 light of 2 wavelengths which are coming in and the difference in these wavelengths is itself again much smaller than the wavelength. And how can you measure this using the Michelson Interferometer.

So, the way you go about doing this is as follows you first.

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So, the point first point to note is before I go into the method by which you do it the point which you should notice that if you have light of 2 different wavelengths in your coming into your Michelson interferometer, they are going to produce their individual fringe patterns. And the fringe patterns which are produce by these 2 wavelengths are going to be different because the wavelengths are different. Now suppose I adjust the difference in the arm lengths  $d$ .

So, that the fringe patterns of both the wavelengths coincide. So, let me consider, a particular situations here.

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$$\lambda_1 \quad \lambda_1 + \Delta\lambda$$
$$\Delta\lambda \ll \lambda_1$$
$$2d = m_1 \lambda_1 = m_2 (\lambda_1 + \Delta\lambda)$$
$$m_2 < m_1$$

So, we have these 2 wave lengths  $\lambda_1$  and  $\lambda_1 + \Delta\lambda$  suppose I adjust my interferometer. So, that the this wavelength  $\lambda_1$  produces a dark spot at the centre. So, my so, the condition that has to be satisfied is  $2d$  I have to adjust the difference in the 2 arm lengths. So, that  $2d$  is equal to  $m_1 \lambda_1$  where  $m_1$  is an integer if I adjust  $d$ . So, that this condition is satisfied.

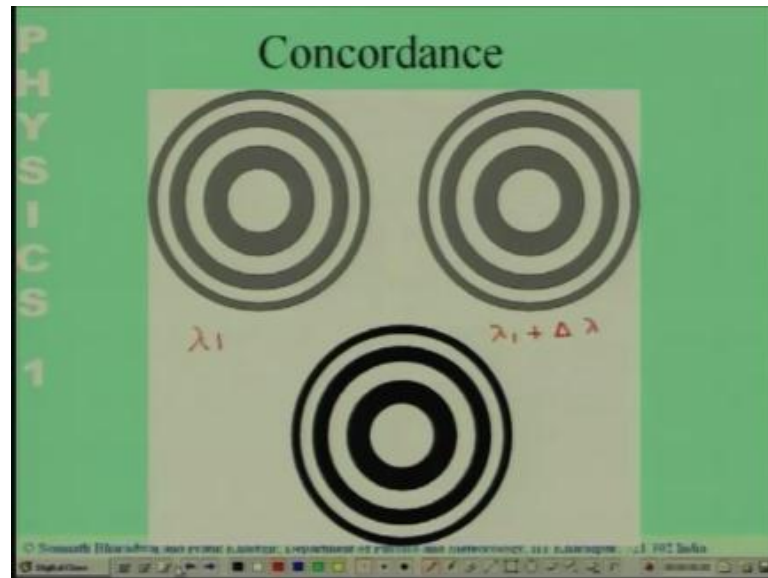
The fringe pattern produced by this wavelength  $\lambda_1$  will have a dark spot at the centre. Now I have if I choose  $d$  in such a way. So, that the wavelength  $\lambda_1 + \Delta\lambda$  also has a dark spot in the centre then they will be another integer  $m_2$  such that  $m_2$  into  $\lambda_1 + \Delta\lambda$  the other wavelength is also equal to  $2d$ . So, there will be 2 into integers  $m_1$  and  $m_2$  such that this condition both these conditions are satisfied the  $2d$  is equal to  $m_1 \lambda_1$   $2d$  is also equal to  $m_2 (\lambda_1 + \Delta\lambda)$  if I can find 2 integers.

So, I can find it  $d$  at separation in the arm lengths. So, that both these condition satisfied then both the fringe patterns will have a dark spot at the centre right and if this condition is satisfied we can also ask the question which is the fringe is the order of this fringe or this fringe which 1 is going to be larger. So, since this wavelength is larger the order of this fringe has to be smaller because the product of this and this is the same.

So, from here we can also see that if the condition is satisfied  $m_2$  should less than  $m_1$ . So, both these wavelengths have a dark spot at the centre this is of a lower order this of a

higher order. So, such a condition is referred to as. So, if the fringe is of both if the fringe is patterns of both the wavelengths coincide this situation is referred to as concordance.

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So, the picture over here shows us a situation where the 2 fringe patterns are in concordance, but this is not the situations that, we had discussed this is exactly the opposite situation where we have bright spot in the centre for both of them. But so, the bright spot and dark spot are can be handled in exactly the same way you have to just add the factor of half over here and here.

So, this shows you concordance. So, the fringe pattern corresponding to  $\lambda_1$  this is the fringe pattern corresponding to  $\lambda_1$ . The fringe pattern corresponding to  $\lambda_1$  and this is the fringe pattern corresponding to  $\lambda_1 + \Delta\lambda$  both of these coincide. So, if you can set up your interferometer. So, that you have concordance then the resulting interference pattern in the central region is going to be the some of the interference pattern.

And this interference pattern the bright spots are going to add up. So, the brightness over here it is going to increase, the darks rings are also going to add up and this dark ring is going to be darker is going to be remain dark. So, the contrast the bright the dark spot is the dark ring is going to be as dark as this 1 or this 1 but, the brighter ring the intensity is going to go get doubled.

So, the contrast between the dark fringe and bright fringe is going to become twice because you have now got 2 fringes. So, when you have concordance the fringe pattern is very sharp; very sharp in the sense that it is very distinct the fringe pattern is very distinct the contrast in the fringe pattern is very high. So, this is what happens when you have concordance when the fringe patterns of the 2 wavelengths coincide.

Now a point which you should note over here point which may arise in your mind is as follows can the 2 fringe patterns coincide everywhere in the field of view well you should if you go back to the condition for the fringe condition. Let we go back to the fringe conditions. So, what is the condition for the formation of fringe is.

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**PHYSICS 1**

## Fringe Conditions

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_2 - \phi_1)$$

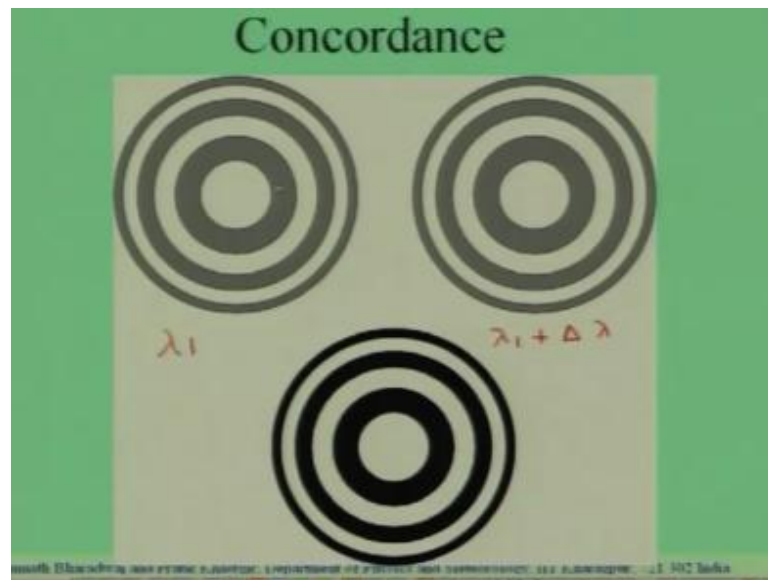
$$\phi_2 - \phi_1 = \pi + \left(\frac{2\pi}{\lambda}\right) 2d \cos \theta$$

**Dark**  $2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$

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So, the condition for the formation of a dark fringe is  $2d \cos \theta$  equal to  $m\lambda$  now you can see that if  $\lambda$  is different if I have  $\lambda_1$  and  $\lambda_1 + \Delta\lambda$ . The 2 fringe patterns cannot coincide everywhere as you increase  $\theta$  they will not coincide anymore, but you can ensure the point to note here is that you can ensure.

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That they coincide in some central region the fringe patterns coincide in the central of the field of view. So, we are not really concerned whether they coincide or not in the outer parts of the field of view here, what we are interested in is that the fringe pattern should coincide at the centre of the field of view and this is going to happen.

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The handwritten derivation on a blue background shows the following steps:

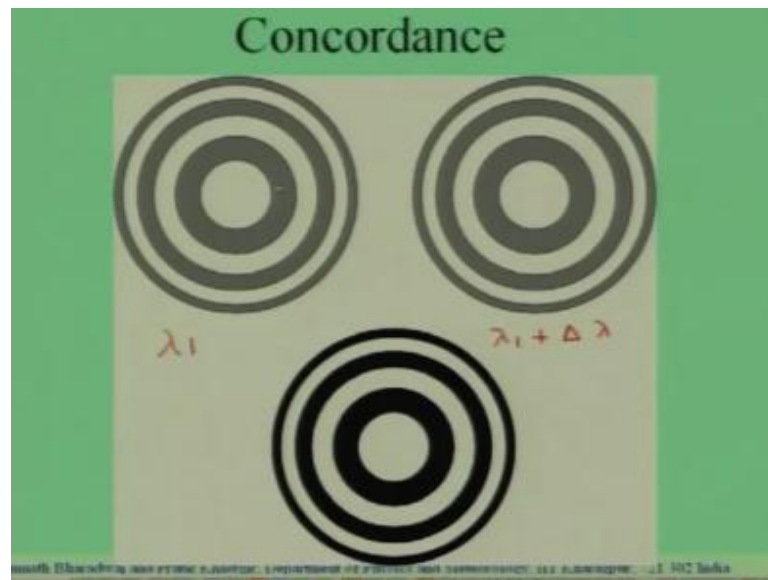
$$\frac{\lambda_1}{\Delta\lambda} \ll \frac{\lambda_1 + \Delta\lambda}{\lambda_1}$$
$$2d = m_1 \lambda_1 = m_2 (\lambda_1 + \Delta\lambda)$$
$$m_2 < m_1$$

A small logo in the top right corner reads "© CET I.I.T. KGP".

If this condition over here, is satisfied if  $m_1$  and if I can find  $m$  integers  $m_1$  and  $m_2$ . So, that this condition is satisfied.



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Then we are going to have a situation where the fringe pattern coincides at the centre of the field of view. Now, let us ask the question that I start from concordance and I slowly move 1 of the mirrors. So, as to increase the separation between the 2 arm lengths. So, the what I do is I start from concordance.

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The handwritten notes on a blue background show the following derivation:

$$d + \Delta d \quad m_1 + \Delta m_1$$

$$m_2 + \Delta m_2$$

$$\Delta m_1 = \frac{2\Delta d}{\lambda_1} \quad \Delta m_2 = \frac{2\Delta d}{\lambda_1 + \Delta\lambda}$$

$$\Delta m_2 < \Delta m_1$$

Arrows point from the inequality  $\Delta m_2 < \Delta m_1$  to the terms  $\Delta m_2$  and  $\Delta m_1$  in the equations above.

So, I have a particular value of  $d$  and I increase 1 of 1 move 1 of the mirrors. So, as to make the separation between the 2 arm lengths  $d$  plus  $\Delta d$  and look at the intensity at the centre.

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Handwritten notes on a blue background:

- Top left:  $d + \Delta d$
- Top right:  $\lambda_1$  and  $\lambda_1 + \Delta \lambda$  (both underlined)
- Middle left:  $\Delta \lambda \ll \lambda_1$
- Center:  $2d = m_1 \lambda_1 = m_2 (\lambda_1 + \Delta \lambda)$
- Bottom center:  $m_2 < m_1$

So, at the centre we look at the intensity at the centre. So, the point to be note if I increase the value of  $d$   $2d + \Delta d$  the value of  $m_1$  is also going to increase the value of  $m_2$  is also going to increase if I restrict my attention to the centre. The centre is  $\cos \theta$  equal to 0 at the centre the order of the central fringe is going to increase for both  $m_1$  and  $m_2$ .

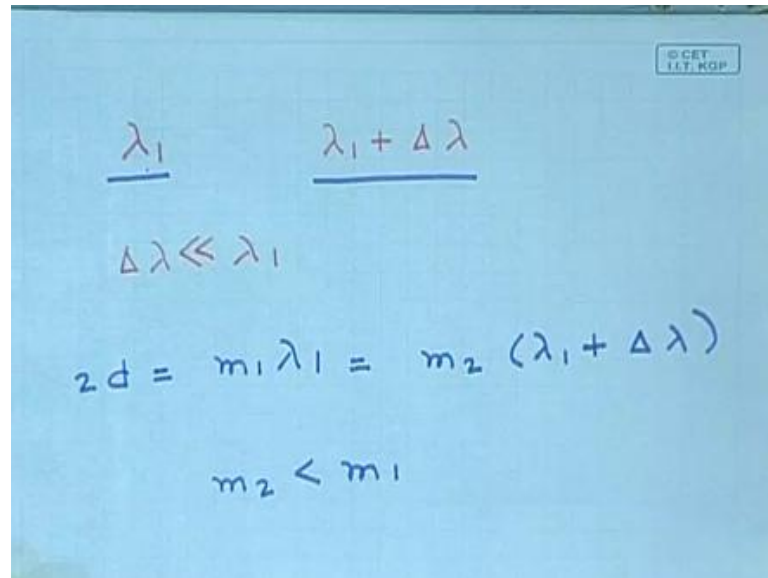
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Handwritten notes on a blue background:

- Top left:  $d + \Delta d$
- Top right:  $m_1 + \Delta m_1$  and  $m_2 + \Delta m_2$
- Center:  $\Delta m_1 = \frac{2 \Delta d}{\lambda_1} \quad \Delta m_2 = \frac{2 \Delta d}{\lambda_1 + \Delta \lambda}$
- Bottom center:  $\Delta m_2 < \Delta m_1$  with arrows pointing up to  $\Delta m_2$  and  $\Delta m_1$  respectively.

So, for the wavelength  $\lambda_1$  I will have  $m_1$  plus  $\Delta m_1$  for the wavelength  $\lambda_1 + \Delta \lambda$  I am going to have  $m_2$  plus  $\Delta m_2$ .

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Handwritten notes on a blue background:

$$\underline{\lambda_1} \quad \underline{\lambda_1 + \Delta \lambda}$$

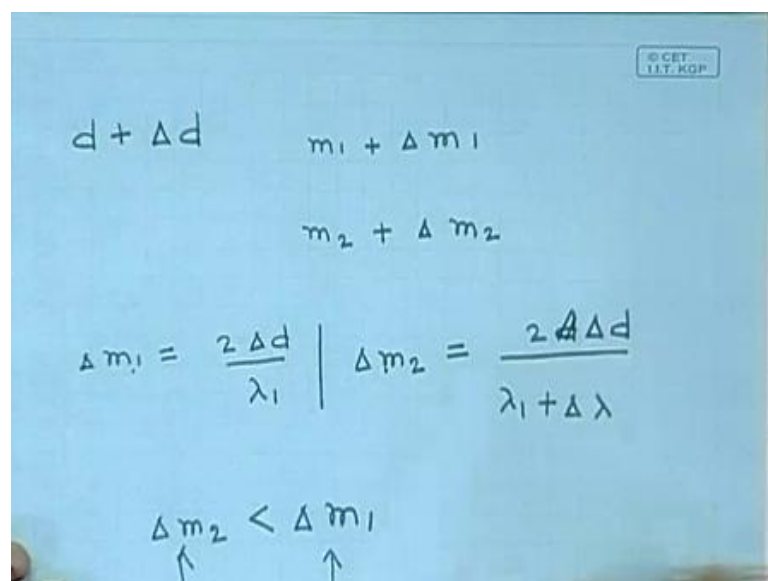
$$\Delta \lambda \ll \lambda_1$$

$$2d = m_1 \lambda_1 = m_2 (\lambda_1 + \Delta \lambda)$$

$$m_2 < m_1$$

Now, let us ask the question how much is  $\Delta m_1$  and it is quite clear from here that  $\Delta m_1$  say if I increase the separation in the 2 arm lengths  $m_1$  will increase by an amount which is twice the separation by  $\lambda_1$ .

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Handwritten notes on a blue background:

$$d + \Delta d \quad m_1 + \Delta m_1$$

$$m_2 + \Delta m_2$$

$$\Delta m_1 = \frac{2 \Delta d}{\lambda_1} \quad \Delta m_2 = \frac{2 \Delta d}{\lambda_1 + \Delta \lambda}$$

$$\Delta m_2 < \Delta m_1$$

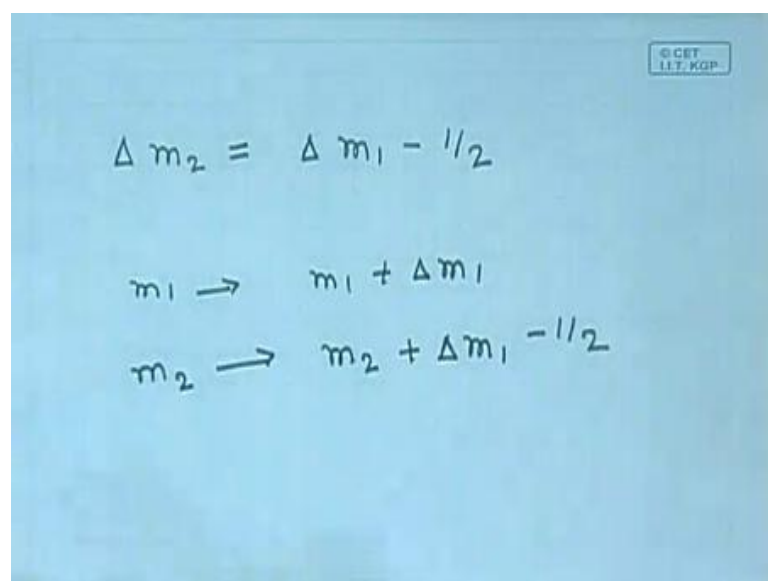
Arrows point from the  $\Delta m_2$  and  $\Delta m_1$  terms in the inequality to the corresponding terms in the equations above.

So, what we see is that  $m_1$  the order of the central fringe for  $\lambda_1$  is going to go up by  $2 \Delta d / \lambda_1$ . The order of the central fringe for the wavelength  $\lambda_1 + \Delta \lambda$  is going to go up by  $\Delta m_2$  which is  $2 \Delta d / (\lambda_1 + \Delta \lambda)$ . This will be  $2 \Delta d / \lambda_1$ . So, the point to note over here is that if I move 1 of the mirrors. So, as to increase the separation in the lengths of the 2 arms I move 1 of the mirrors by an amount  $\Delta d$  for the wavelength  $\lambda_1$  the order of the central fringe increases by a certain amount. For the wavelength  $\lambda_1 + \Delta \lambda$ , but the second wavelength the order of the central fringe increases by a different amount.

So, initially both the wave lengths had a dark fringe at the centre, but as I change the difference in the arm length, they both no longer remained dark and there is a difference in phase. So, there is a difference in the order of the fringes in between the 2 wavelengths. So, that fringe pattern is now no longer in concordance. So, this is going to be increased by a certain amount. This is going to increase by a certain amount and both the fringes at the centre do not have the same intensity because they are of different orders.

And when you can also note that  $\Delta m_2$  is less than  $\Delta m_1$  because the wavelength here is larger. So, if I move 1 of the mirrors by fixed amount  $\Delta d$  the change in this is less than the change in this. Now, suppose I keep on increasing  $\Delta d$ . So, that the difference in this and difference between these 2 is such that.

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$$\Delta m_2 = \Delta m_1 - 1/2$$

$$m_1 \rightarrow m_1 + \Delta m_1$$

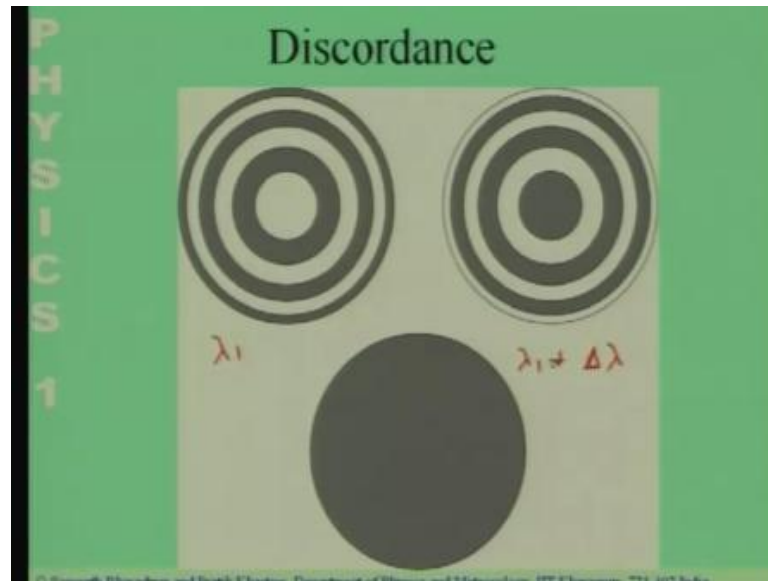
$$m_2 \rightarrow m_2 + \Delta m_1 - 1/2$$

Delta  $m_2$  is equal to delta  $m_1$  minus half. So, what have been done, we have moved 1 of the mirrors this is going to change. The order of the central fringe for the 2 wavelengths and we move the mirror. So, that the change in the order of the fringe for 1 wavelength and the change in the order of the fringe for the other wavelength, they differ by half. Now, when the difference is half remember. So,  $m_1$ ,  $m_1$  was an integer and  $m_1$  has gone to  $m_1$  plus delta  $m_1$ ,  $m_2$  was also an integer this has gone to  $m_2$  plus delta  $m_1$  minus half.

So, the point to note over here is that this in this situation what we are going to have is that the bright fringes of 1 of the wavelengths are going to coincide with the dark fringes of the other wavelength because if this is an integer and I make delta  $m_1$  an integer then I will have a dark spot here dark ring corresponding to this. And I will have a bright ring corresponding to this because this is an integer to start with the this is also an integer. And this is factor of half. So, when I have half integer multiple of lambda I get a bright ring that is what we had seen earlier.

So, if I if this is dark this is going to be bright or if this is dark this is going to be bright. So, if I move delta  $d$  if I move 1 of the arm mirrors. So, that the difference between delta  $m_1$  and delta  $m_2$  is half integer. We are going to have a situation where the bright rings of 1 wavelength coincide with the dark rings of the other wavelength such a situation is what is referred to as discordance. So, let me now show you what we mean by discordance.

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So, this is the situation referred to as discordance, notice that for 1 of the wavelengths  $\lambda_1$ . So, for 1 of the wavelengths we have a dark spot at the centre for the other wavelength we have a bright spot at the centre. Now, if you superpose. So, if is  $\lambda_1$  and this is  $\lambda_1 + \Delta\lambda$  we have a bright spot at the centre for  $\lambda_1$  we have a dark spot at the centre for  $\lambda_1 + \Delta\lambda$  if I superpose these 2.

The bright spot here is going to fall on the dark spot here, the dark spot here, the dark ring over here is going to fall on bright fringe over here. And the resultant pattern that I have which is superposition of these 2 is going to be more or less uniformly illuminated. So, the fringe pattern as a consequence is not going to be very bright this not going to be very distinguishable and the fringes have essentially washed out.

So, in this discordance the fringe pattern gets washed out. So, what we see is that I start from a situation where 2 fringe pattern coincide we have concordance the fringe pattern is very intense the contrast is very high I move 1 of the mirrors slowly the fringe pattern will get washed out. Now, if I keep on moving it even further the order of these fringes.

(Refer Slide Time: 27:47)

$d + \Delta d$        $m_1 + \Delta m_1$   
 $m_2 + \Delta m_2$   
 $\Delta m_1 = \frac{2\Delta d}{\lambda_1}$        $\Delta m_2 = \frac{2\Delta d}{\lambda_1 + \Delta \lambda}$   
 $\Delta m_2 < \Delta m_1$   
           ↑                    ↑

So, if I keep on moving the mirror even further. So, that  $\Delta d$  goes up. So,  $\Delta d$  I keep on increasing  $\Delta d$  I will first reach a situation, where these 2 numbers differ by half and then I keep on increasing  $\Delta d$  even further then I will reach a the situation where these 2 numbers differ by an integer right if I move  $\Delta d$  sufficiently if I move 1 of the mirror sufficiently far away. So, that  $\Delta d$  is sufficiently large.

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$\Delta m_2 = \Delta m_1 - 1/2$   
 $m_1 \rightarrow m_1 + \Delta m_1$   
 $m_2 \rightarrow m_2 + \Delta m_1 - 1/2$   
 $\Delta m_2 = \Delta m_1 - 1$

So, that  $\Delta m_2$  the 2 the changes in the 2 orders of the fringe of orders of the 2 fringes differ by 1 right.

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$$\begin{aligned}
 & d + \Delta d \quad m_1 + \Delta m_1 \\
 & \quad \quad m_2 + \Delta m_2 \\
 & \Delta m_1 = \frac{2 \Delta d}{\lambda_1} \quad \bigg| \quad \Delta m_2 = \frac{2 \Delta d}{\lambda_1 + \Delta \lambda} \\
 & \Delta m_2 < \Delta m_1
 \end{aligned}$$

So,  $\Delta m_2$  has to be less than  $\Delta m_1$  this we have already seen from here. And if I increase  $\Delta d$ . So, that these 2 numbers differ by 1 which is the condition this condition under this condition.

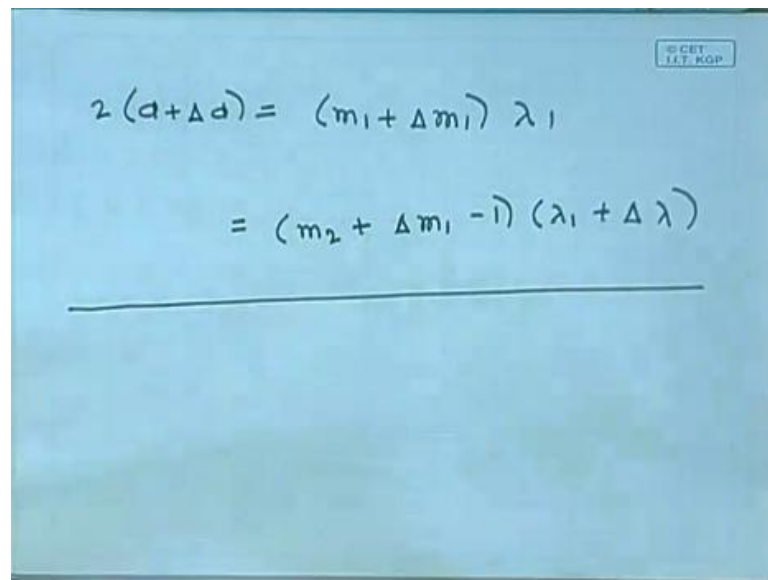
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$$\begin{aligned}
 \Delta m_2 &= \Delta m_1 - 1/2 \\
 m_1 &\rightarrow m_1 + \Delta m_1 \\
 m_2 &\rightarrow m_2 + \Delta m_1 - 1/2 \\
 \Delta m_2 &= \Delta m_1 - 1
 \end{aligned}$$

The bright fringes of 1 of the wavelength again going to coincide with the bad fringes of the other wave lengths. So, when this condition is satisfied. So, let me look at the full expression.



(Refer Slide Time: 29:12)



The image shows a blue background with handwritten mathematical equations in black ink. The equations are:

$$2(d + \Delta d) = (m_1 + \Delta m_1) \lambda_1$$
$$= (m_2 + \Delta m_1 - 1) (\lambda_1 + \Delta \lambda)$$

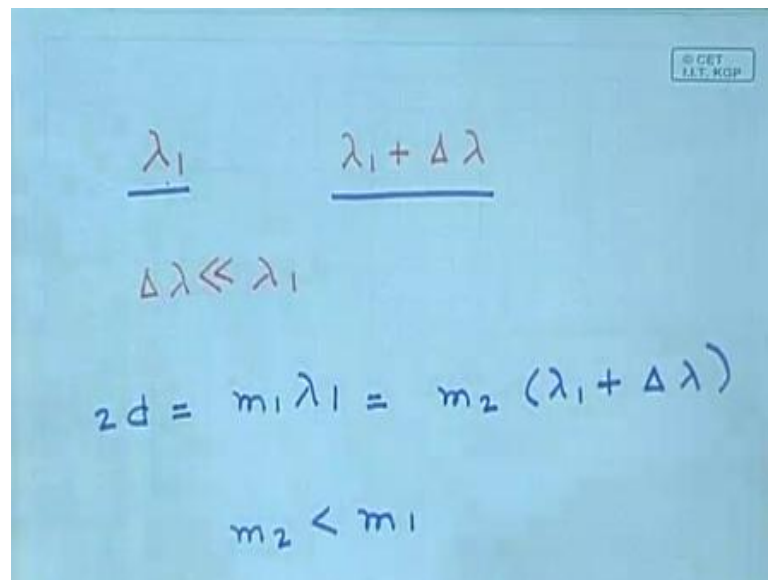
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So, this condition essentially means that  $2d + \Delta d$  is equal to  $m_1 + \Delta m_1$   $\lambda_1$  this is also equal to  $m_2 + \Delta m_1 - 1$   $(\lambda_1 + \Delta \lambda)$ . So, if I change  $\Delta d$ . So, that the order by which this fringe changes 1 more than the order by which this fringe changes we are back to a situation, where we have concordance because the order of the 2 fringe systems exactly differs by 1 and if 2 fringe.

So, in the situation where I have dark spot for  $\lambda_1$  I will also have a dark spot for  $\lambda_1 + \Delta \lambda$ . So, if this condition is satisfied the 2 fringe pattern are again in concordance. So, if we can go from one concordance to another we have gone we know that  $\Delta m_2$  will be  $\Delta m_1 - 1$ .

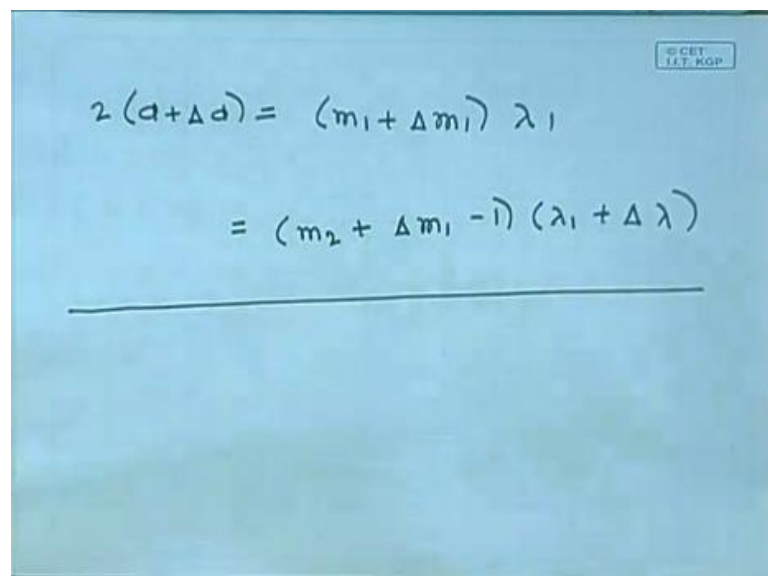
Now, we can use this condition. So, we know that at the second at the first at the second concordance this has to be satisfied at the first concordance.

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$$\lambda_1 \quad \lambda_1 + \Delta\lambda$$
$$\Delta\lambda \ll \lambda_1$$
$$2d = m_1 \lambda_1 = m_2 (\lambda_1 + \Delta\lambda)$$
$$m_2 < m_1$$

This has to be satisfied.

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$$2(a + \Delta a) = (m_1 + \Delta m_1) \lambda_1$$
$$= (m_2 + \Delta m_1 - 1) (\lambda_1 + \Delta \lambda)$$

---

We have gone from this situation to this situation. Now we could subtract this equation from this equation and see what it gives us. So, if I subtract this equation from this equation what we get let me write that down here.

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$$\begin{aligned} 2(d + \Delta d) &= (m_1 + \Delta m_1) \lambda_1 \\ &= (m_2 + \Delta m_1 - 1) (\lambda_1 + \Delta \lambda) \end{aligned}$$

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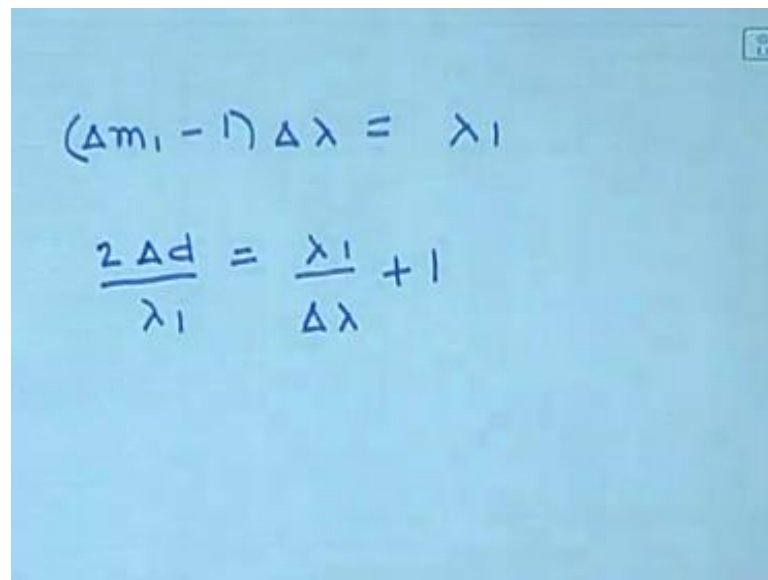
$$2 \Delta d = \Delta m_1 \lambda_1 = (\Delta m_1 - 1) (\lambda_1 + \Delta \lambda)$$

$$\Delta m_1 = \frac{2 \Delta d}{\lambda_1}$$

So, if I do the little bit of algebra what I will get is  $2 \Delta d$  is equal to  $\Delta m_1 \lambda_1$  and  $\lambda_2$  into  $\lambda_1$  plus  $\Delta \lambda$  gets cancelled out this is also equal to  $\Delta m_1 - 1$  into  $\lambda_1$  plus  $\Delta \lambda$ . So, let us now see what are the consequences of this the first consequence of this is that  $\Delta m_1$  is equal to  $2 \Delta d$  by  $\lambda_1$ , which we have already worked out earlier.

Now let us see what this tells us let us see what this part of the expression tells us. So, let me just focus on this and when I equate this with this the term  $\Delta m_1 \lambda_1$   $\Delta m_1 \lambda_1$  cancels out. So, what this gives me what this gives me is over here let me write it down.

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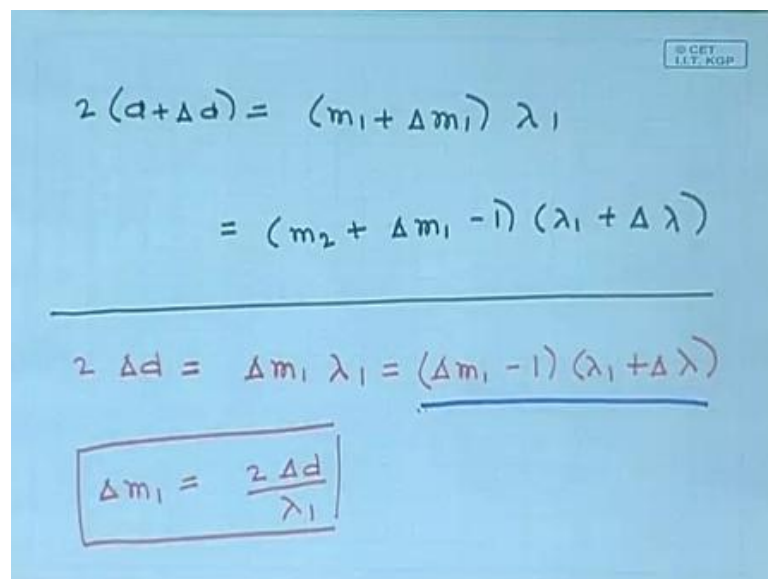


Handwritten equations on a blue background:

$$(\Delta m_1 - 1) \Delta \lambda = \lambda_1$$
$$\frac{2 \Delta d}{\lambda_1} = \frac{\lambda_1}{\Delta \lambda} + 1$$

What it gives me is delta m1 minus 1 into delta lambda is equal to lambda 1.

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Handwritten equations on a blue background:

$$2(d + \Delta d) = (m_1 + \Delta m_1) \lambda_1$$
$$= (m_2 + \Delta m_1 - 1) (\lambda_1 + \Delta \lambda)$$

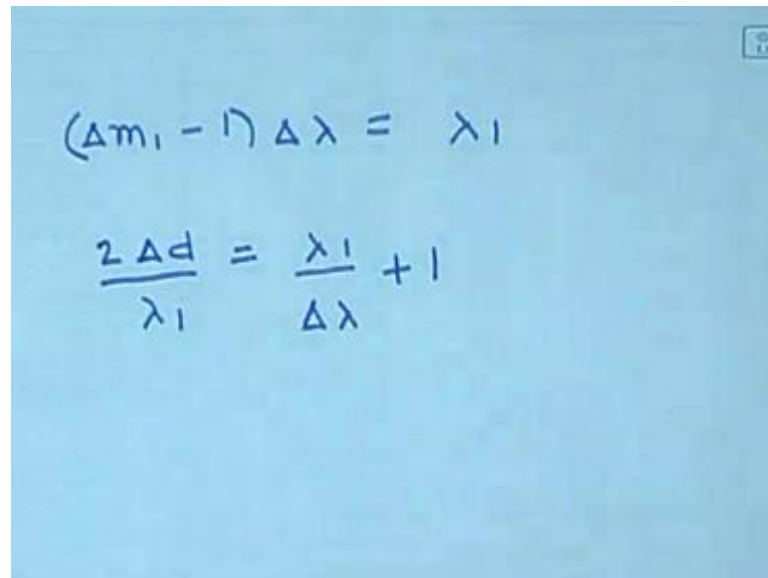
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$$2 \Delta d = \Delta m_1 \lambda_1 = \frac{(\Delta m_1 - 1) (\lambda_1 + \Delta \lambda)}{1}$$

$$\Delta m_1 = \frac{2 \Delta d}{\lambda_1}$$

So, I have just rearrange the terms over here after having cancels out cancelled out this term delta 1 into lambda 1 delta m1 to lambda 1 from both sides.

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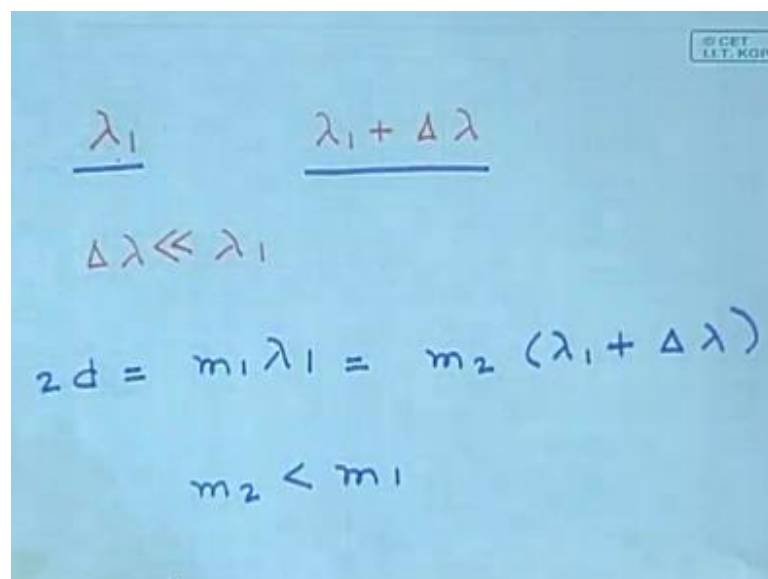


Handwritten equations on a blue background:

$$(\Delta m_1 - 1) \Delta \lambda = \lambda_1$$
$$\frac{2 \Delta d}{\lambda_1} = \frac{\lambda_1}{\Delta \lambda} + 1$$

This is what I get and this we could we could write this as a as a  $2 \Delta d$  by  $\lambda_1$  this is  $\Delta m_1$  is equal to  $\lambda_1$  by  $\Delta \lambda$  plus 1 and right at the start we had assumed.

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Handwritten equations on a blue background:

$$\frac{\lambda_1}{\Delta \lambda} \ll \frac{\lambda_1 + \Delta \lambda}{\Delta \lambda}$$
$$2d = m_1 \lambda_1 = m_2 (\lambda_1 + \Delta \lambda)$$
$$m_2 < m_1$$

That the light which comes into the interferometer components 1 with wavelength  $\lambda_1$  another 1 with wavelength  $\lambda_1 + \Delta \lambda$  where  $\Delta \lambda$  is much smaller than  $\lambda_1$ .

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$$\begin{aligned}(\Delta m_1 - 1) \Delta \lambda &= \lambda_1 \\ \frac{2 \Delta d}{\lambda_1} &= \frac{\lambda_1}{\Delta \lambda} + 1 \\ \Delta \lambda &= \frac{\lambda_1^2}{2 \Delta d}\end{aligned}$$

So, if I use this assumption then you see that this ratio  $\lambda_1$  by  $\Delta \lambda$  is much greater than 1. So, we can forget about this term 1 over here and what this tells us is that  $\Delta \lambda$  is equal to  $\lambda_1^2$  by  $2 \Delta d$ . So, this is how you can determine the difference in the wavelengths of the 2 light of the 2 lines which you have in your light in 2 spectral lines.

So, let me again go through the steps having finished this little bit of algebra the basic point is that you adjust your interferometers. So, that you have concordance the 2 fringe patterns coincide next you move 1 of the mirrors. So, that you go from concordance to disturbance a discordance if you go into discordance the fringe pattern which was very bright earlier now get washed away. It will become very hazy keep on moving the mirror until you reach another concordance. Fringe pattern again becomes bright.

So, if you move 1 of the mirrors the fringe patterns starts from a very bright situation becomes hazy and again becomes very bright again. Measure the distance that you have to move the mirror to go from 1 concordance to the next this distance we call  $\Delta d$ . So, you determine this distance experimentally you also determine the wavelength of the light experimentally by the method which we had discussed earlier.

So, since these 2 wavelengths are very close you can determine average wavelength by using the method, which we had determine which we had discussed earlier count the number fringes etcetera. So, you know the value of  $\lambda_1$  you know the value of  $\Delta d$

$d$  and  $\lambda^2$  by  $2\Delta d$  gives us the difference in the 2 wavelengths right. So, So, this tells us how we can measure the difference in the intensity in the wavelength of 2 very close spectral lines in your light another point which I should also mention that suppose, I give you certain light a light source and ask you the question does this light source have 1 wavelength is it 1 spectral line or is it 2 spectral lines. If so, then what is the difference in wavelength.

So, the first thing that you would have to do there is then you would have to adjust the interferometer. So, that you get very intense fringes

Now, if there is only 1 spectral line then if I what you would see is that as I move with 1 of the mirrors the fringe pattern would remain as, it is it would get there would be newer and newer fringes coming in, but there would be no very great difference in the intensity but, if instead, if I found that as I move 1 of the mirrors the intensity of the fringe pattern gets diminished the fringe pattern is washed away.

And then if I move it further it again comes bright this immediately tells me that there are 2 different wavelengths not 1. And then I can use the technique which I have just discussed to determine the difference in these 2 wavelengths. So, this is a very interesting application of the Michelson interferometer. So, Michelson interferometer is essentially a very useful technique for measuring the wavelength of light and determining the differences in wavelength if there are 2 spectral lines in your light.

Now, let me also next move on to other very interesting applications of the Michelson interferometer the let me, before going into that.

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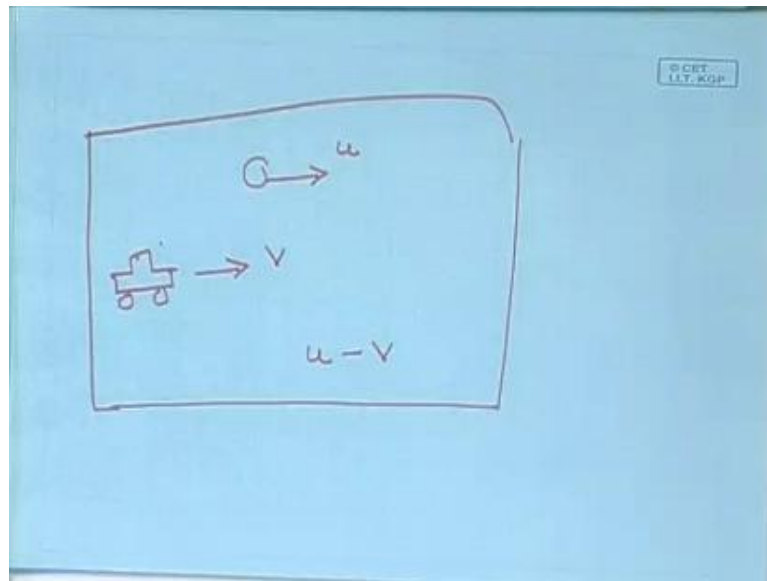
Observer	Year	Arm length (in m)
Michelson	1881	1.2
Michelson & Morley	1887	11.0
Miller	1902-1926	32.0
Michelson et al	1929	25.9
Loomis	1930	21.0

Let me show you how this how the lengths of Michelson Interferometer meters has a progressed over time. So, in 1881 Michelson did this Michelson interferometer experiment using a interferometer, where the arm lengths were 1 point 2 meters then there is this very interesting experiment by Michelson and Morley done in 1887, which used an interferometer of length 11 with arms of length 11 meter.

This experiment was very interesting and you may be aware of the of the very great outcome of this experiment, this experiment Michelson and Morley experiment they use a Michelson interferometer. This experiment showed us that the speed of light does not change let me, explain to you what we mean by this if we have.



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Let us say, we have a ball going with speed  $u$  in this direction and there is a car going with speed  $v$  in the same direction then if I am sitting on the car I will see that the speed of the ball is  $u$  minus  $v$ . So, this is the law of addition of velocities and it is a this is key this is very this is a very fact that the velocities add up is goes by the name of the Galilean theory of relativity time. The time nothing happens to time, but velocity adds up the law of addition of velocities and the question is does this also hold for light.

And there is no reason why you would not expect it to hold it to also hold for light, but Michelson and Morley they showed that such law of addition of velocities does not hold for light and this fact that.

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Observer	Year	Arm length (in m)
Michelson	1881	1.2
Michelson & Morley	1887	11.0
Miller	1902-1926	32.0
Michelson et al	1929	25.9
Joos	1930	21.0

It does not hold for light required us to revise our whole idea of how to go from 1 inertial frame to a moving inertial frame. It require us to abandon this Galilean principle of relativity, Galilean ideas of relativity where you had have the laws of addition of velocity and it was finally, Einstein's theory of special relativity which could really in its I mean put all the put this thing together.

Then the fact that you have to how to go from 1 inertial frame to another maintaining the incorporating the fact that the speed of light does not change its an observe fact which was established by Michelson and Morley in their experiment that the speed of light does not change even if we move after that there were Michelson there were Michelson interferometer.

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Michelson Interferometer		
Observer	Year	Arm length (in m)
Michelson	1881	1.2
Michelson & Morley	1887	11.0
Miller	1902-1926	32.0
Michelson et al	1929	25.9
Joos	1930	21.0

Experiments with consecutively longer and longer arm lengths a let me quickly.

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From here jump right into the present times at present there is a very large very large Michelson interferometer experiment going on the arm lengths of the of this particular Michelson interferometer experiment are quite large. The arm lengths are four kilometers in length and this picture shows you the this particular Michelson interferometer. So, this Michelson interferometer is this is this is an experiment called LIGO L I G O laser interferometric gravitational wave observatory.

And this shows you a picture of LIGO it is the Michelson interferometer which has got 2 arms you can see over here as we have studied and the arm lengths are 4 kilometers much larger than the arm lengths.

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Michelson Interferometer		
Observer	Year	Arm length (in m)
Michelson	1881	1.2
Michelson & Morley	1887	11.0
Miller	1902-1926	32.0
Michelson et al	1929	25.9
Joos	1930	21.0

You see over here 20 meters 21 meters etcetera.

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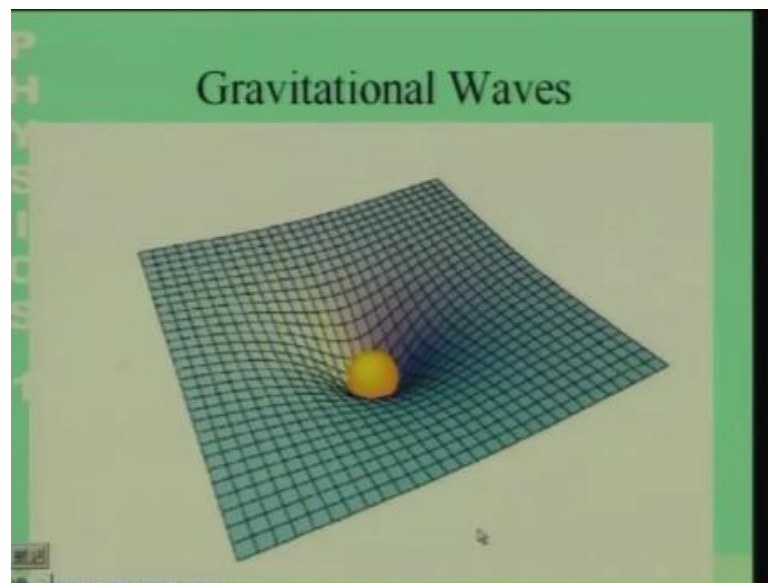


Now, what is LIGO all about. So, LIGO is a joint project mainly involving the it is joined it is a joined project mainly involving the California institute of technology. And the Massachusetts Institute of Technology Caltech and MIT and some other American

universities it is an American project by various American sources including the American government. And there are 2 such experiments being conducted in the USA 1 at a place called Hanford and another at a place called Livingston both in the united states of America.

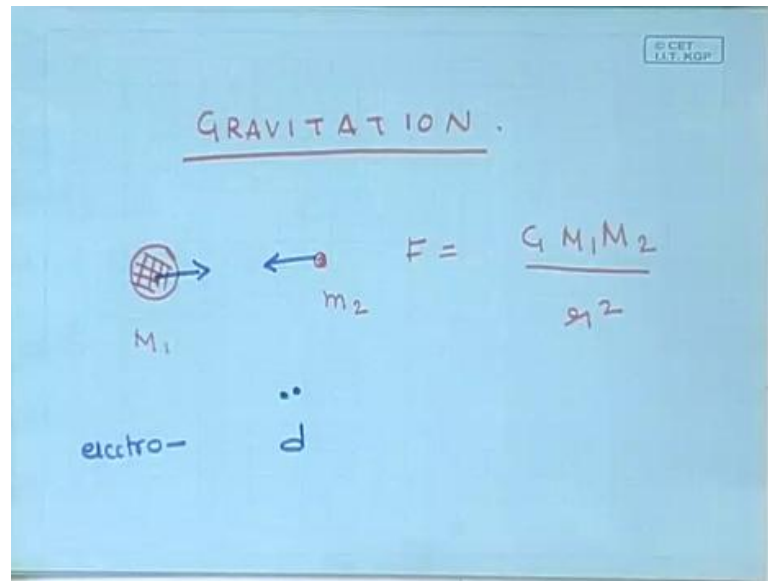
And both these experiments are Michelson interferometers with arm lengths of four meters as you can see here. The these Michelson interferometer arms are vacuum chambers. So, this enormous vacuum tubes to enormous vacuum tubes and you have.

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So, you have let me I will go into what it does now before going into what it does. So, what is the purpose of let me go little a bit tell you a little bit about the motivation for buildings such a enormous interferometer. So, to understand the motivation to get some idea of this motivation, we have to take a little if we have take a look at gravitation.

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So, we will take a little look at gravity now I am sure of all of us here know Newton's the laws of gravity as they were proposed by Newton and what Newton proposed was that there is a gravitational force exerted by massive objects. So, all massive objects exert gravitational force and the gravitational force is also experienced by all massive object.

So, all massive objects experience exert a gravitational force and also exert gravitation forces and the gravitational force between 2 massive objects 1 of mass  $m_1$  and another of mass  $m_2$  is  $f$  is equal to  $G m_1 m_2$  by  $r$  squared. This is what Newton's laws of gravity predicts and the gravitational force is attractive it will cause these 2 objects to come together.

Now, we have a few lectures ago discussed the same kind of a situation, but within the context of charges if I have 2 charges  $q_1$  and  $q_2$  Coulomb had Coulomb had the we see that these charge charges exert forces on each other. And Coulomb had proposed law governing these forces which was experimentally which has been experimentally verified and the laws exactly similar to this except that we have to use the charges instead of the masses.

Now, we had well discussing Coulomb's law we had seen that there are certain problems with Coulomb's laws and we have to replace this the Coulomb's laws are not exactly correct and there are modifications which arise and these modifications incorporate 2

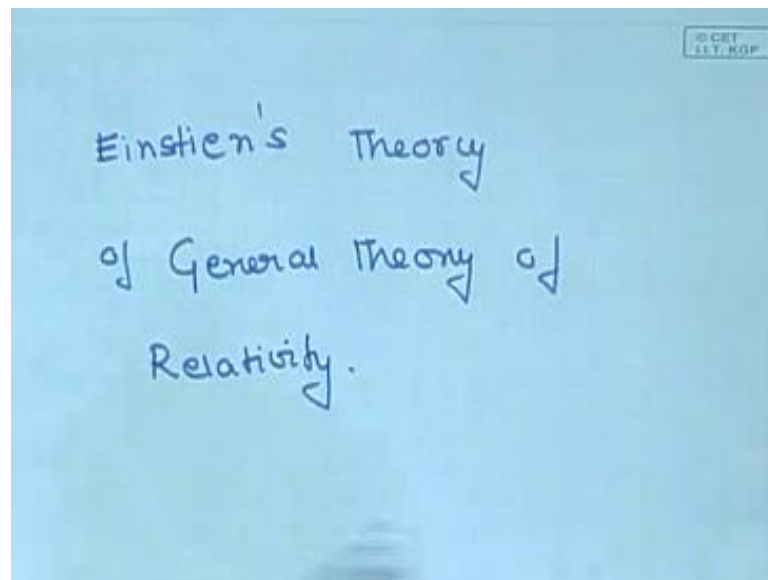
things I mean 1 of the main things which is incorporated is the fact that no signal can propagate instantaneously. And Maxwell's theory gives the correct results for the electric field produced by charges. And once you incorporate those corrections, we get a new phenomena which is electromagnetic radiation we have already discussed this.

And we saw that electromagnetic radiation arises when a charge particle accelerates and there we discussed, the situation where we have an oscillating dipole. We have a positive combination of a positive and negative charge and we found that if the dipole moment is a second derivative. So, for electromagnetism we found that if there is a dipole and if there is a second derivative of the dipole which is equivalent to saying that 1 of the charges accelerates you have a part of the electric field which falls as  $1/r$ .

Which we refer to as radiation this carries away energy from the oscillating dipole and we have an electromagnetic wave which carries away energy. The electric field so, the electromagnetic wave disturbance in the electric field which propagates and its falls off as  $1/r$  as you go away from the dipole. Now, let us look at Newton's law of gravity you see that Newton's laws of gravity is exactly similar to Coulombs' law and it has the same problem that it does not incorporate the fact that.

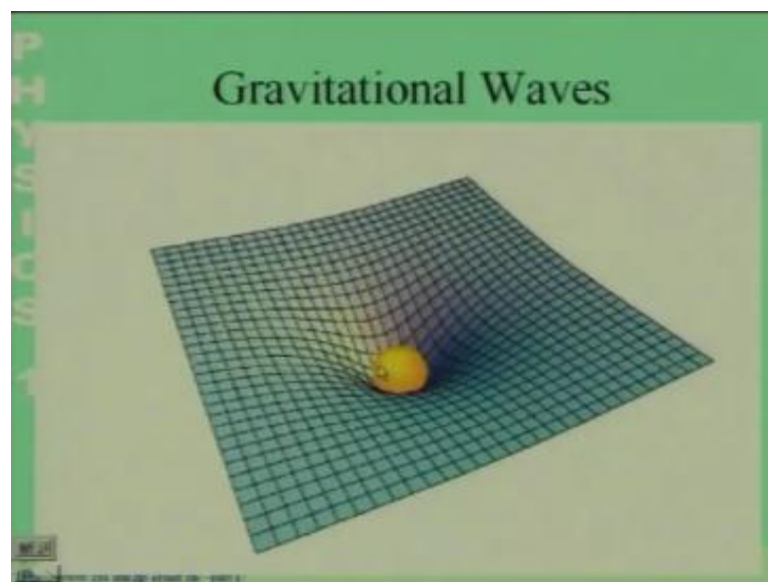
It takes a finite time for signal to propagate the gravitational force on this would change instantaneously if we were to move this mass. So, it should be quite clear that the Newton's laws of gravity that this particular law force, law of gravity is not the final story the as far the gravitation is concerned the final story as it stands.

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Now, was proposed by Einstein's. So, you have Einstein's theory of general relativity which is the correct theory which is believe to be the correct theory for gravity in Einstein's theory of general relativity.

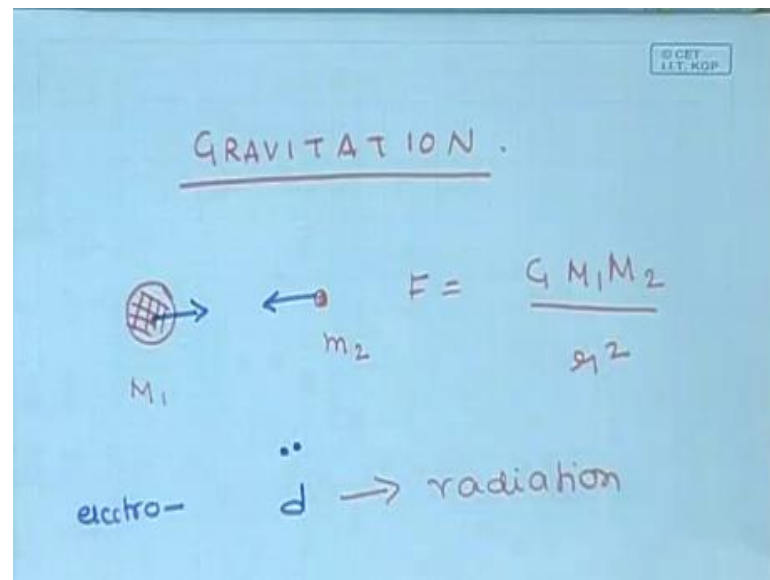
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Gravity manifests itself the gravity is manifestation of the curvature of space time. So, in Einstein's theory of general relativity massive objects produce a curvature in space time. And gravity what we call gravity is the manifestation of this curvature of space time.



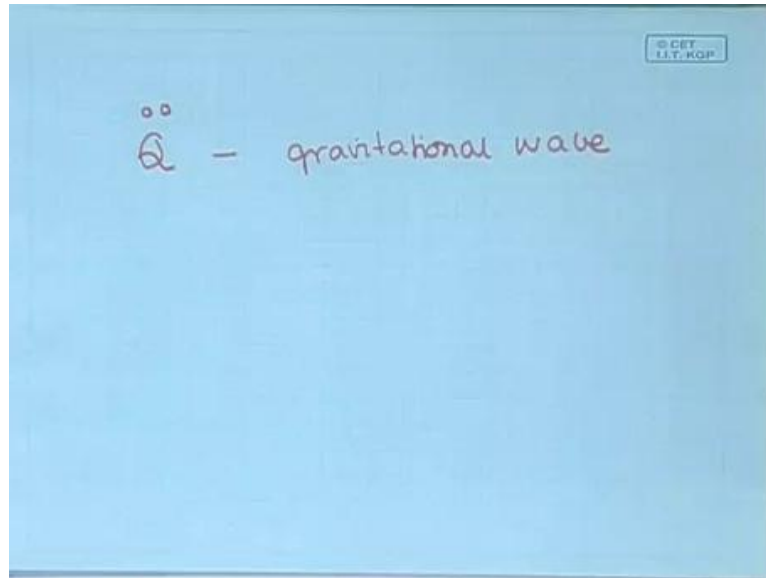
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Now what is crucial over here, for our discussion is the following: that exactly analogous to a situation where for charge particle, we had a dipole. So, for charge particle we had a dipole if a dipole oscillates if I have the second derivative of a dipole then you have radiation electromagnetic radiation which is disturbance in the electric field which propagates and falls us 1 by are exactly analogous to this now in the gravity the source of gravity is mass.

If you have a mass distribution and if you go to the centre of mass of this mass distribution then the dipole moment it is 0 that is because for charges you have 2 opposite charges, but for mass there is opposite mass all masses are positive either positive or 0. So, if you dealing with mass the question of that the dipole moment does not rise, but you could have a quadrupole moment. So, if you have a mass distribution.

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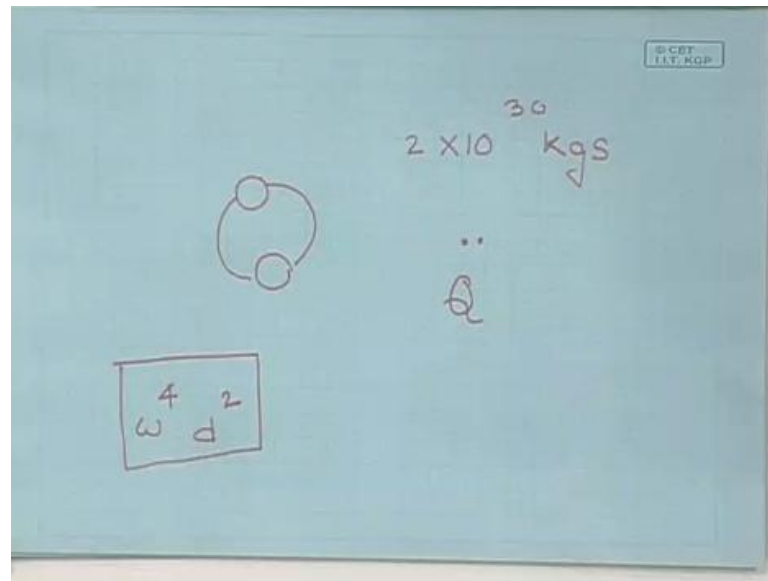
And the mass distribution has a Quadrupole moment  $Q$  and if there is a second derivative of the quadrupole moment. So, if it mass these masses are accelerating relative to 1 another. So, you have a second if you have you have quadrupole of which is has a derivatives second derivative in this situation you have a phenomena called gravitational radiation or a gravitational wave this is just analogous to an electromagnetic wave except that now the disturbance is in the gravitational field not in the electric field.

But, there is a disturbance in the gravitational field, which propagates and if falls of as  $1/r$  as you move away from the source. A disturbance in the gravitational field is also a disturbance in the curvature of the space time in Einstein theory of general relativity. So, let me again recapitulate what I have told you in the Einstein theory of gravity; gravity is a manifestation of the of curvature of space time now very similar to electromagnetic radiation in this Einstein theory of gravity if you have quadrupole.

And if the quadrupole changes that is if it has a second derivate you will produce the disturbances in the gravitational field which propagate like a wave. And it is these disturbances that we refer to as a gravitational wave it is a time depend disturbance in the gravitational field which is also propagating. And such disturbances are produced when you have a rate second derivative of the quadric pole moment.

So, you could have for example, my hands like this there will be a change in my quadrupole moment, but the gravitational waves produced by this change are going to be negligible, but there are situations as to physical situations where we have starts.

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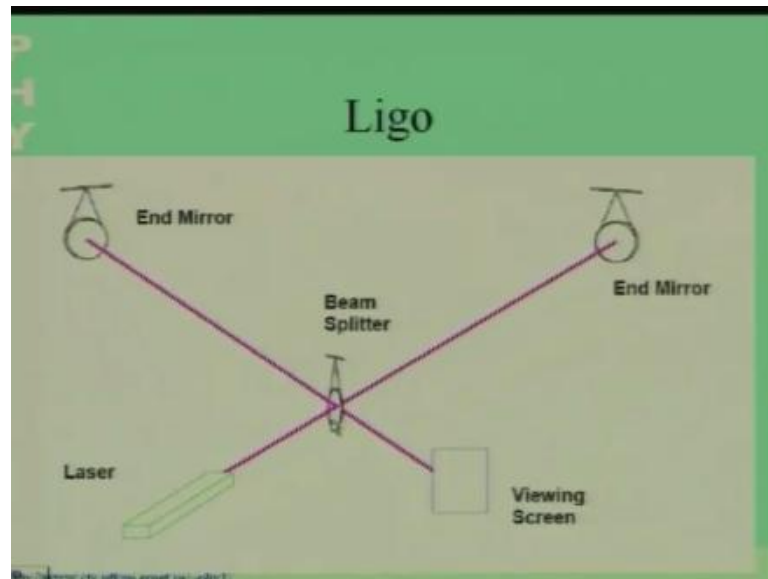
So, if you have 2 very massive stars in close orbit the mass of a star like our sun is 2 into 10 to the power 30 kg's and you could have very massive stars which are may be 10 times larger than this. So, if you have 2 such stars going around in an in an orbit then the quadrupole moment of this system will have a second derivative. So, such a system is going to emit copious amounts of gravitational waves and such a system loses energy and angular momentum through the emission of gravitational waves.

So, the orbit is going to shrink. And these 2 these 2 stars shown over here going to slowly come closer together. And if they come closer together the angular frequency of the of the of the arbitrary motion is going to increase, we had seen for the dipole radiation in an electromagnetism that the intensity of the radiation the total power emitted goes as omega to the power 4 into the dipole moment amplitude of the dipole moment square.

So, you will have something will similar, over here for gravitational radiation here again if the angular frequency goes up the total power is going to be increase and if the amplitude of the quadrupole moment goes up, again the amplitude is going increase. So, as you have if you have 2 very massive objects like: 2 very massive star and if they go around in a circular orbit they will lose energy through gravitational radiation. They will get it smaller and smaller orbits, which will be faster and you are going to get more and more gravitational waves coming out.

The gravitational wave that comes out is.

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Time dependent disturbance in the gravitational field that propagates. So, when the gravitational wave from a distance star is incident upon a Michelson interferometer like this. The gravitational acceleration on these 2 mirrors in the Michelson interferometer is going to experience the gravitational acceleration of this mirror is going to be disturbed and it going to be a time dependent disturbance in the gravitational acceleration.

So, this particular mirror is going to do vibrations like this and this mirror also going to do vibrations like this. It is. So, happens that in gravitation waves the oscillations produced in the 2 mirrors are out of phase. So, this mirror is going to come inwards this mirror is going to out and then it is going to rivers. So, this mirror is going to come in and this mirror is going out and.

So, this is the effect of gravitational waves which if they are incident on this Michelson interferometer, it is going to make the mirrors oscillates. And these oscillations of the mirror are going to produce shifts in the fringe pattern which you will see over here. So, if you can detect these shifts in the fringe pattern. The shift in the fringe pattern is going to tells us that the gravitational wave has pass through.

So, the main problem over here is that the displacements of the mirror which are produce by gravitational waves. The strains which are predicted from various sources of

gravitational waves are extremely small and in order to detect, it is a big technological challenge to detect these small strains produced by gravitational waves. And these enormous Michelson interferometer experiments have been build with the purpose of detecting these time varying strains, which will be produced if a gravitational wave passes through the interferometer, people who work on the in this area they know the kind of signal you would expect.

If a gravitational wave pass through and they could then tell you that the that a they would they have techniques by which they can differentiate between disturbances produced by the gravitational waves and other disturbances. And that is what they are trying to they trying to detect gravitational waves there has not been a positive detection has yet, but the work still goes on the main problem is that the strains are extremely small and for a fixed value of the strain.

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$$\underline{\underline{\Delta l}} = [\text{strain}] \times \underline{\underline{l}}$$

$$\left[ 2\pi \frac{\Delta l}{\lambda} \right]$$

The shift in the length, which is produce by a fixed value of the strain is the strain into the length in to the length of the arm. So, the method to increase the displacement of the mirror is to increase the length of the arm, because the strain is  $\Delta l$  by  $l$  which is why such an amount huge amount of effort has gone into to build a Michelson interferometer with arms which are 4 kilometers long.

Now, there is an effort un going effort which to have Michelson interferometer in space gravitational wave detector in space, the whole Michelson interferometer the mirrors the

source everything is going to be orbiting in space. So, you can have really long arms. So, for the same amount of strain you will have a larger displacement remember that the Michelson interferometer measures, the phase change which occurs between 2 arm lengths. And phase change is  $2\pi \Delta l / \lambda$  and the phase change manifest itself as a change in the fringe pattern. So, the larger the  $\Delta l$  the more is going to be the change in the fringe pattern and it is going to be easier to detect the gravitational waves.

So, here I have tried to give in the last part of this lecture, I have tried to give you an idea of 1 very important application of Michelson interferometers, it is in the which is in the area of detection of gravitational waves. The detection of gravitational waves is very important for 2 main reason 1 is that it would be a an independent verification of Einstein's theory of relativity a direct detection would be a very important verification of Einstein's theory of relativity.

There are other indirect detections of gravitational waves, but a direct detection would go a long way towards verifying Einstein's theory of relativity and another very interesting thing is that all astronomical observations till date have been have largely been based on the electromagnetic radiation. There are some based on neutrinos from the sun for example,, but gravitational waves would open a new window into astronomy and that is why there is large amount of interest in detecting the gravitational waves using these Michelson interferometer. So, let us stop of Michelson interferometer here and we shall continue on a different topic tomorrows lecture.