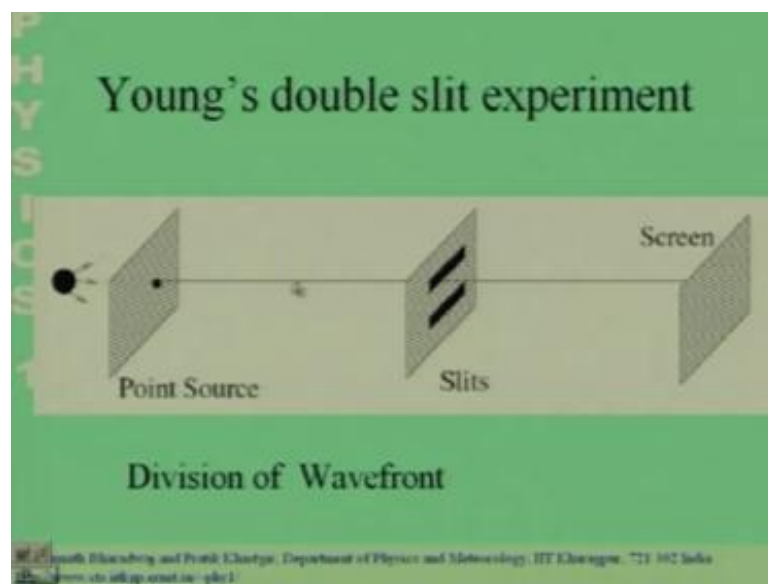


Physics I : Oscillations and Waves.
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Lecture No - 16
Interference - III

Good morning. In the last 2 lectures we have been discussing Young's double slit experiment. Young's double slit experiment works, using the principle of division of wave front.

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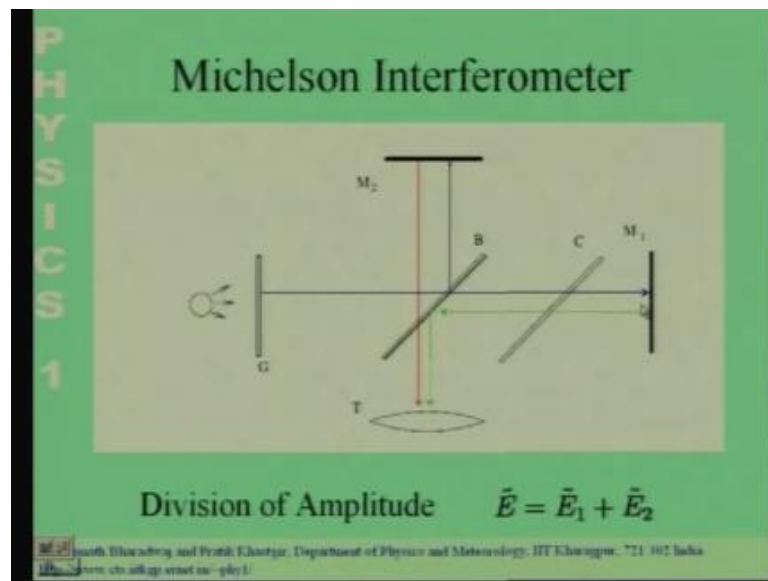
So, this is something which we have not discussed in the last 2 class. So, let me briefly tell you what this is all about the 2 slit is which are used in the young's double slit experiment divides, the wave fronts which come from this source. It divides the wave front into 2: 1 from the upper part, 1 upper part and the lower part. So, it is these so, it is these 2 wave fronts which we obtain by dividing a single wave front which are made an interfere on the screen over here.

Then, it is this which produces the interference pattern. It is necessary to adopt this kind of a technique because, usually 2 different sources if I had 2 different point sources over here, they would be incoherent that is the waves that emanated from 1 point source would not interfere with the waves that emanate from another point source. So, to

overcome this problem that is to produce 2 sources which are coherent we have to adopt different techniques.

So, the technique used here is division of wave front. The wave same wave front is divided, using the 2 slit is and then, they the 2 wave fronts are produced or the super post to produce the interference pattern.

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Now, today we shall discuss something else we shall discuss another very interesting situation. Another interesting apparatus which can be used to study interference and this is called the Michelson interferometer. The Michelson interferometer works by using a different technique, it uses division of amplitude and not division of wave front. So, let us discuss how, what the Michelson interferometer apparatus looks like and how it works.

So, this shows you schematically the Michelson interferometer you have a light source which illuminates a ground glass plate. The ground glass plate has the property, that it scatters the incident light into all directions. So, this is an extended source each point over here is different source, it is a extended source and each point over here emit is radiation in all directions.

The radiation which is scattered forward so, this is what we are going focus on. So, the radiation that is scattered forward going in this direction is incident on a beam splitter B.

So, the element B over here the optical element B is a beam splitter. The beam splitter is essentially, a slab of glass 1 of whose surfaces is semi-silvered. So, it has been given a very thin silver coating.

So, in this particular case the lower surface of this glass slab, the lower surface has been given a very thin silver coating. So, has to increase the reflectivity of the lower surface. So, the lower surface of the beam splitter of the glass slab over here has a very high reflectivity. So, the light which is incident on this and the beam splitter is kept at 45 degrees angle to the direction from which the light is incident from the ground glass plate. So, it is this angle over here is 45 degrees.

So, the beam splitter split is the incident wave front the amplitude of the it split is the incident wave front the whole entire wave front is split into 2 parts. So, the amplitude of the wave front is divided 1 propagating this wave which is the reflected part. A reflected wave propagating upwards and the transmitted wave shown in the blue reflected waves, shown in red propagating, upwards and a transmitted 1 shown in blue propagating, straight through. So, the amplitude of the incident wave is now divided into 2 parts.

So, the you have 2 parts which together whose sum. So, the sum of the electric field of this wave and this wave is equal to the incident electric field. So, the amplitude over here is divided 1 and there are 2 waves now which come out from the single wave 1 going up and 1 passing through. Now, the wave which goes which passes through, the shown in blue propagates to a mirror.

So, there mirror over here which reflects this wave back which is what you shown in green over here and then, the reflected wave again encounters the beam splitter. And we are interested in the part of the wave, that gets reflected from the bottom of the green beam splitter the bottom is semi silver it has highly reflective. So, a part of the wave transmitted through the beam splitter comes from the ground glass plate is transmitted to the beam splitter, propagates to this mirror there is another component see here which I shall come to later.

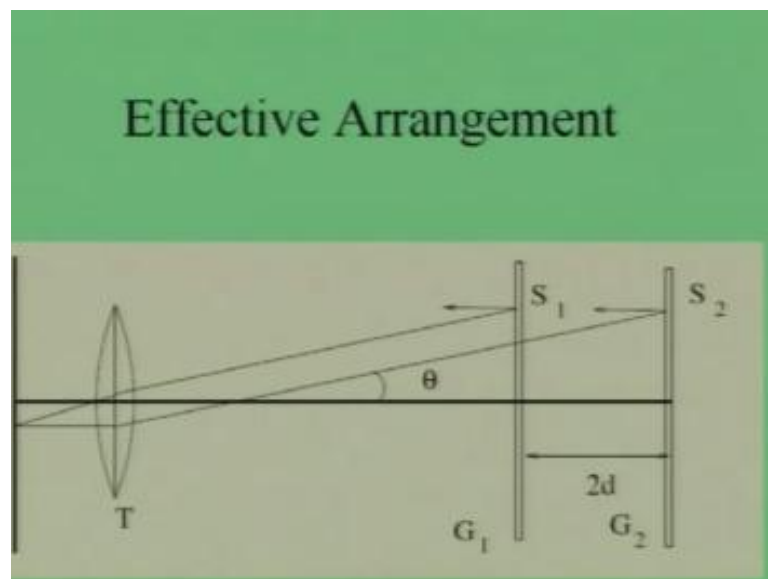
So, we are interested in the wave that is transmitted through comes to this mirror is reflected and then, comes over here and is collected this light is incident on a telescope which focuses it to the focus. Now, the other wave which is reflected, from the beam splitter. So, there were 2 waves produced at a beams splitter 1 which was transmitted and

1 which was reflected. The wave which is reflected, on the beam splitter goes up and an encounter a mirror which reflects it back which is shown over here. Then, it is a part of it is transmitted through the beam splitter and this again is collected by the telescope.

Now, the net effect of this apparatus is that you see 1 the light which goes through this and then, comes here produces an image of the ground glass plate the image will be produced somewhere over here. The light which goes through and then, comes here and is reflected here produced the another image of the ground of the ground glass plate. Which is also produce somewhere, over here the 2 images will be at different locations.

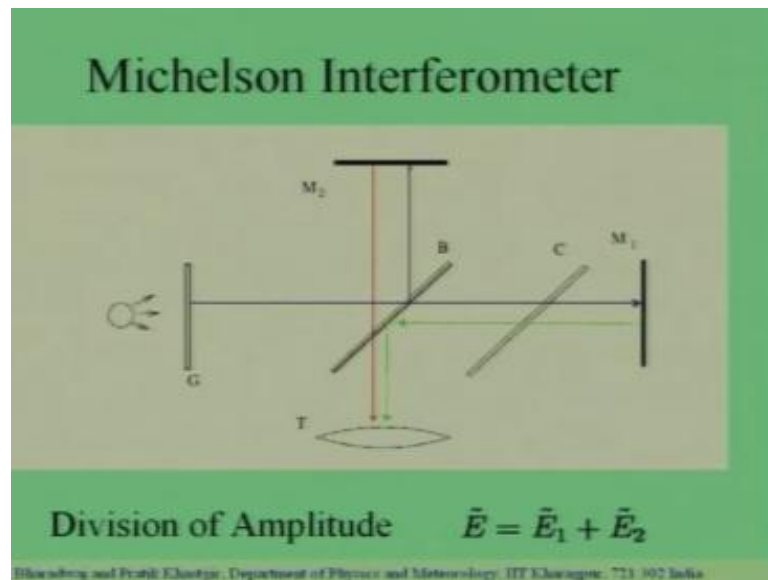
So, let me go over this point again the net effect of this Michelson interferometer is that you will get 2 images of the ground glass plate. Both of them will be located in the direction of view. So, they will be both located in this direction. They will be located somewhere above this mirror, behind this mirror M2 and the distances to these 2 images of ground glass plate will depend on the length of these arms of these 2 arms.

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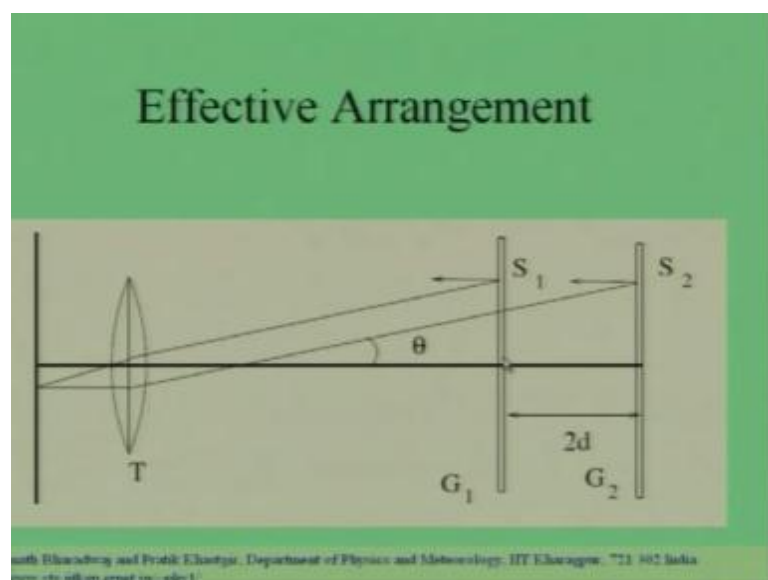
So, the effective arrangement is again shown over here the thing which I was just discussing. So, you have 2 images of the ground glass plate which are produced in the direction of the line of sight.

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Which, they are produced in this direction. So, they are there will be 2 images of ground glass plate produce in this direction. Which I put my eye here and the eye piece of the telescope I will see 2 images of the ground glass plate 1 arising from the light which is transmitted comes here and is it reflected. Another arising from the right which is reflected and then, transmitted so, 1 produce by M_1 , another produce by M_2 .

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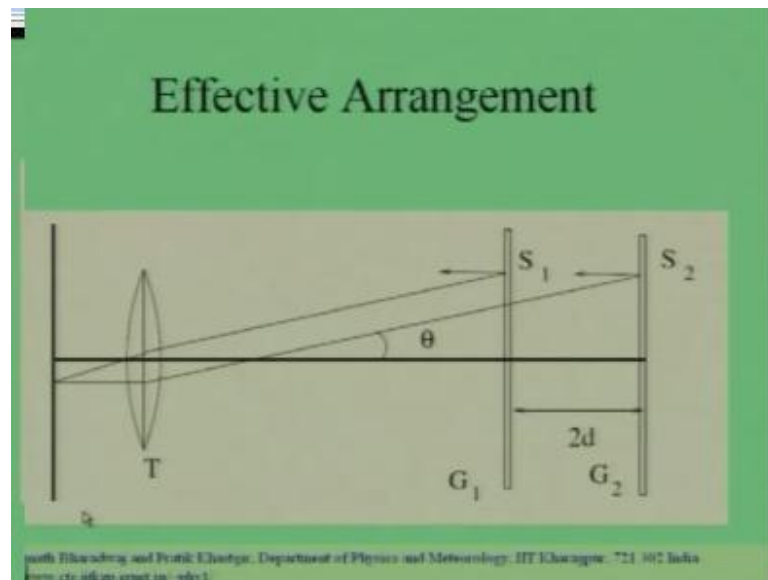
Then, these 2 images of the ground glass plate are shown over here. These 2 images will not in general co-inside and if the difference in these arm lengths. If the difference in

these arm lengths is d so, if 1 of the arm lengths is at a distance l_1 from the beams splitter. And the other arm length is at a distance l_2 from the beam splitter. Then, the difference in distance we will call d so.

So, if l_2 is larger than l_2 minus l_1 I will refer to has d . So, let me remind you again what l_2 and l_1 are: l_2 is the length of this arm over here. Let me, just change the nomenclature let me call this l_1 . Let me, call M_2 so, l_1 is the length of the arm to the mirror M_1 from the beam splitter to the mirror M_1 , l_2 is the length of the arm from the beam splitter to the mirror M_2 .

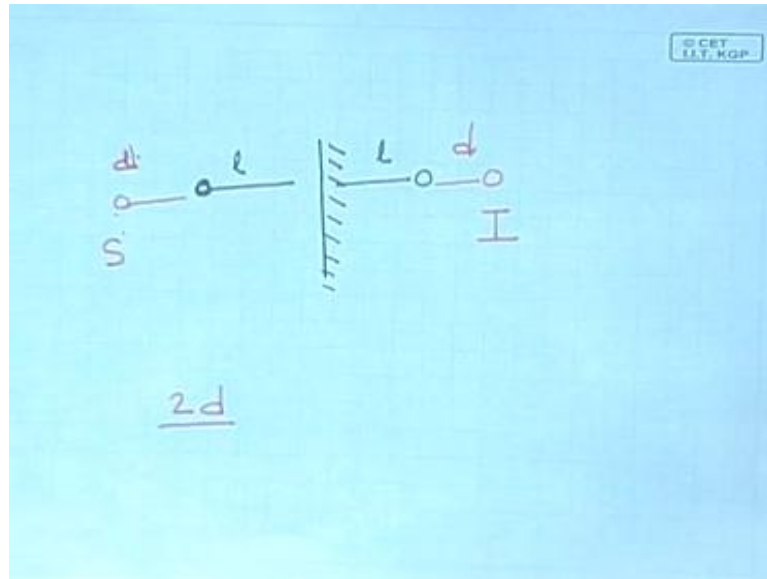
We assume that the l_2 is large for it does not make a difference which 1 you assume to the larger. If you assume that l_2 is larger then, l_2 minus l_1 we will call d ; d is the difference in the arm lengths. The point to remember is d is the difference in the arm lengths. If the 2 arms have exactly the same length, the difference d will be 0. If 1 of them is longer then, d is the difference in the lengths arm lengths. So, the 2 images which are produce by this apparatus.

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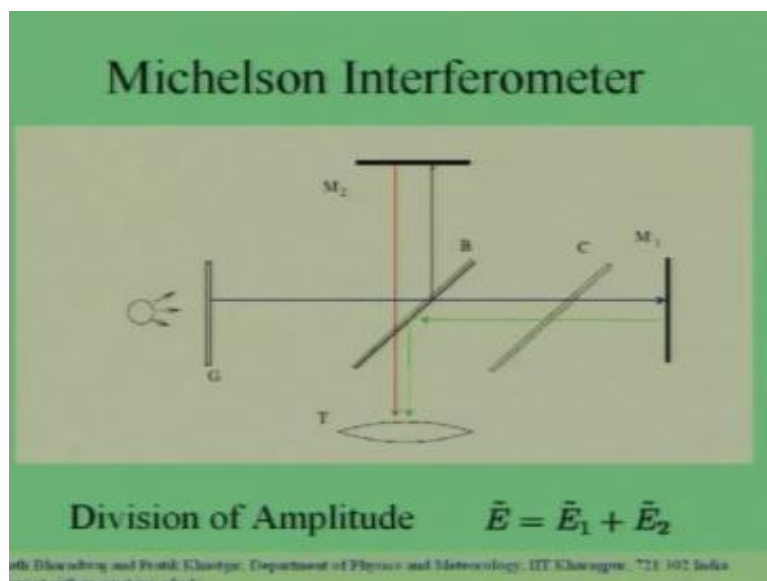
So, will be at a separation $2d$ if d is the distance the difference in the arm lengths then, the 2 images produced are going to be of the ground glass splitter are going to be at a separation of $2d$. Why it is $2d$ is should be clear from the following argument. Let me, explain to you briefly why it is $2d$ If I have.

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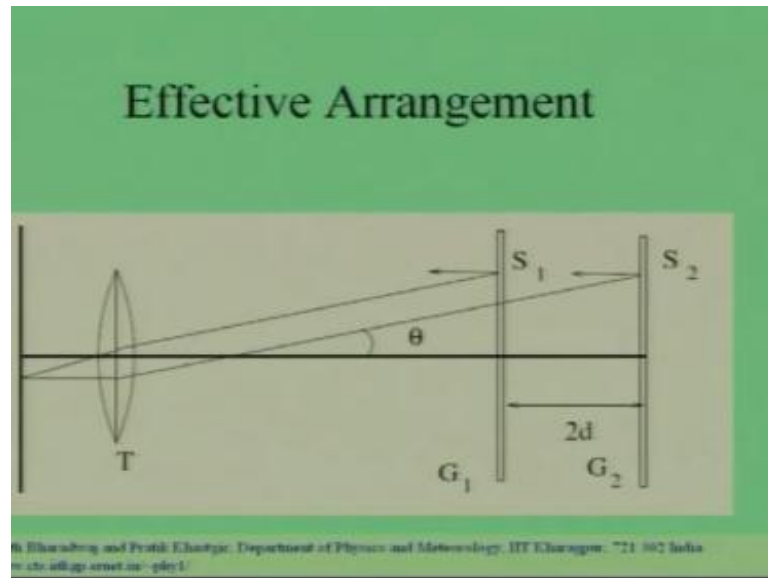


A mirror over here and if I have source over here at a distance l then, the image is the same distance behind the source. Now, if I move the source a little backwards to increase the distance from the mirror. So, if I move the source to a distance l plus d . So, this distance is d then, the images also going to move back by exactly the same amount d . And the total distance now, between the source and the image has increased by a factor of by an amount $2d$. So, if I move the source by a distance d then, separation between the source and the image increases by $2d$.

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It is exactly for the same reason, that if that separation if the difference in the arm length is d then, the difference between 2 images of the ground glass plate is $2d$, they will be produce at a distance which is $2d$ apart. Now, there is a point which I should discuss here which I have not mention till now. The point is as follows the light which propagating to M2, the light which is reflected at the beam splitter goes up and then, gets reflected down this light traverses the thickness of the glass slab 3 times.

So, 3 times because the reflection occurs at the bottom of the glass slab. So, when it comes here for the first time goes through it once. And then, it comes out twice it goes all the wave here and when, it comes back once more so, that makes it 3 times. Now, let us look at the transmitted wave: the transmitted wave traverses the glass slab only once goes through comes here gets reflected from the bottom surface does not enter. And then, comes over here to the telescope.

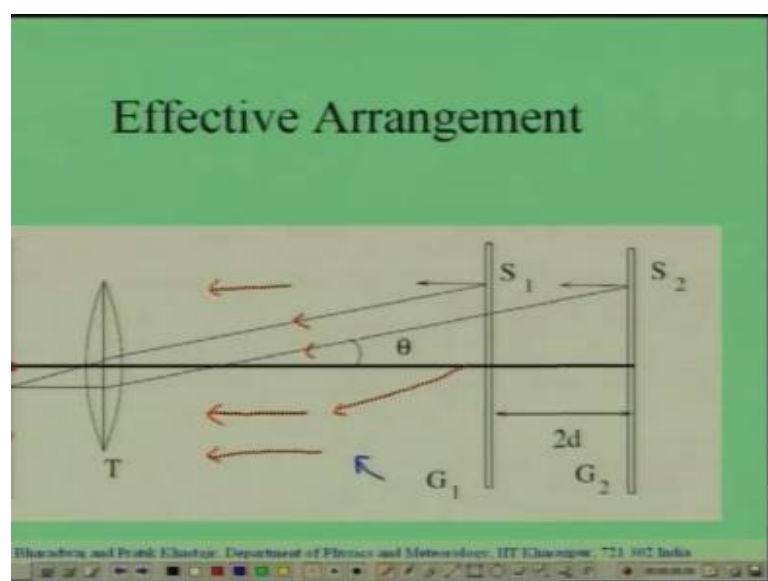
So, the reflected the 1 the wave which goes to M2 traverses the glass slab 3 times whereas, the wave that goes to M1 and then, comes here 1 traverses the glass slab only once. Now, this introduces an extra optical path difference because, the reflective index of glass is different than that of air. So, even if these 2 arm lengths are exactly the same, the optical path traverses by this wave and this wave are going to differ. And the difference is going to be exactly 2 times the path length inside the glass slab.

So, this is going to traverse a slightly larger optical path length than this. Even if the arm lengths are exactly equal. So, it has to compensate for this and it could think that you could compensate for it by moving the mirror M1 a little behind. So, as to introduce the extra path length, but it so happens that the refractive index of glass is wavelength dependent. So, you could compensate for the extra path inside glass at a particular wavelength by moving the mirror back. But then, the compensation would not be exactly correct at other wavelengths.

So, in order to account for this extra optical path length it is most convenient to introduce a glass slab which is exactly identical to the beam splitter, but which does not have the silver coating for this glass slab is called the compensator. So, it is most convenient to compensate for this extra path which this particular wave has to traverse by introducing a compensator over here. So, the compensator is a beam splitter glass slab exactly identical to the beam splitter, but it has no silver coating.

So, now the wave which is transmitted goes to M1 and comes back also, passes through the glass slab 3 times once over here. And twice over here so, the optical path through the glass is exactly identical for the wave that goes up that is reflected up and the wave that is transmitted. So, this is achieved through this optical element called the compensator.

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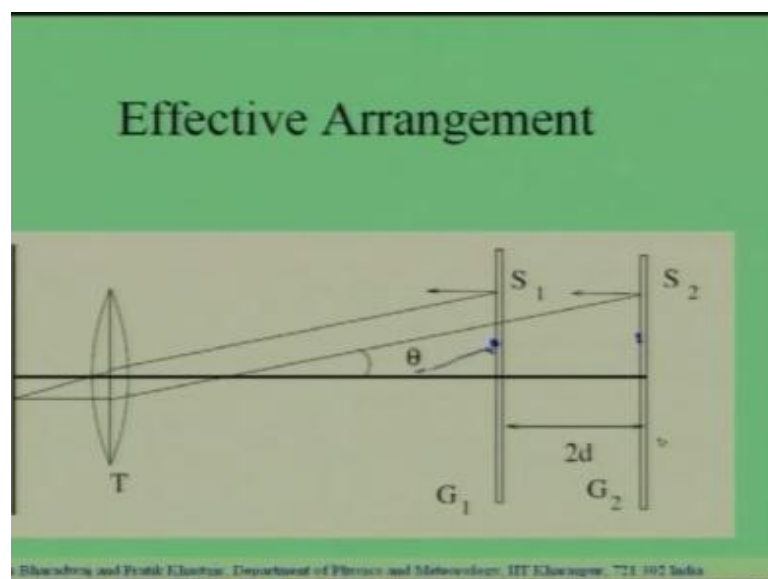


So, now any difference in the position of the images is only due to differences in the lengths of the 2 arms. And the separation between the 2 images of the ground glass plate is exactly twice the difference in lengths of the 2 arms. So, this is the telescope now let us turn a attention to the image this is the telescope. So, we can for our purposes replays the telescope by a lens and put look at, the image of the produce by this lens at the focal plane.

So, what is the role of such a lens? The role of such a lens is that, it will take all the light coming in a particular direction and focus it to a single point that is the role of a lens. So, let me draw a picture and explain this point to you. The role of the lengths is as follows, is if there was light coming in this direction a wave coming in this direction on the focal plane the entire wave so, this this this is focused to a single point over here.

So, if the wave is coming at an angle as it is shown over here. If the wave comes at an angle θ then, all of these all the waves that arrive at this angle are focus to a single point. And if the angle θ is like this upwards then, the point will be below this point over here. If I change θ the light is going to focus to a different point, if I increase the θ light is going to come over the different point here. If the light propagates this way, if the light propagates in this direction it is then, going to be focus to some point over here.

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So, this is the role of the lens. Now, let us see what lens does in this particular situation. In this particular situation, we have 2 images of the exactly the same ground glass plate. Let us, look at the image G1 produced by the mirror M1, every point on this ground glass plate acts like a source this is an extended source. So, every point over here acts like a source and every point on the ground glass plate sends out light in all directions.

So, right that is a property of the ground glass plate it scatter the incident light the light that come out from the ground glass plate is scattered in all direction, it comes out in all directions. So, let us look at the us 1 point on the ground glass plate the point S1. So, this is a particular source which on the ground a particular point in the ground glass plate which act like a source S1.

So, in S1 is the point on the image G1 corresponding to this source S1 there will be an image on the ground glass plate, G2 is the this point on the ground glass plate over here. On this image will have a corresponding, image in these point in the second image. So, S2 is the source is the image of this point over here on the second image. Now, what the lens over here does is it, focuses it brings together the radiation that is emitted from this point at an angle theta and this point at an angle theta, both of them are combined to a single point.

Not only this also, radiation emitted from this point at an angle theta. So, radiation emitted radiation emitted from this point at angle theta is also focused on to the same point over here. Similarly, radiation from this point at an angle theta is also going to be focused to the same point over here by the lengths. So, radiation from all the points on this image, all the points on this image which are emitted at an angle theta.

So, this angle as long as this angle it makes with the normally, theta all these waves are going to be focused to a single point over here. And at this point we going to have the super position of all of these waves. Now, the question is which of this radiation is going to interfere with 1 another. Here you should remember, that different points on the ground glass plate are incoherent sources.

So, this the radiation from this point is not going to interfere with a radiation from this point on the same ground, same image of the ground glass plate at the first point. Second point is: that the radiation from here is going to interfere from the radiation with the radiation from it is image on the second. So, the radiation from this point and the

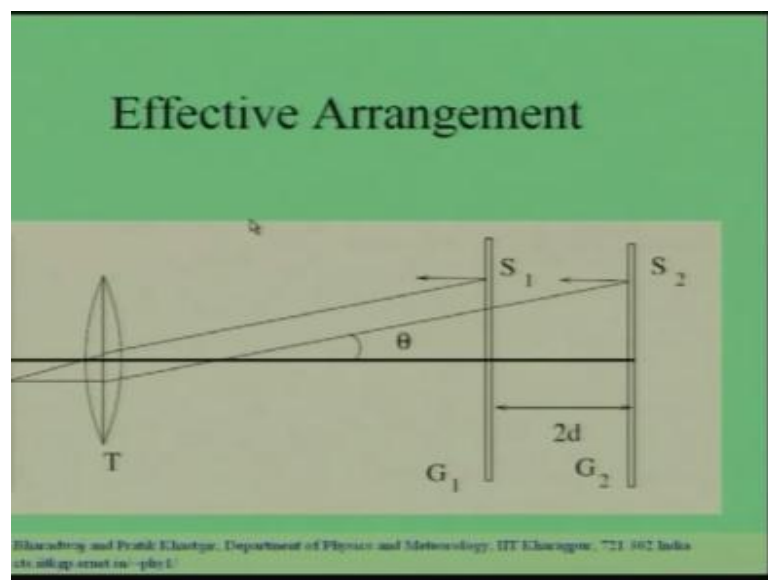
radiation from this point are going to interfere with each other. Because, they are the images of the same point on the ground glass plate it is basically the same source.

We are seen 2 images of the same source and since, these are 2 different images of the same source. The radiation from here and the radiation from here are going to be coherent. The radiation from here is not going to interfere with the radiation from any other point on the image neither, is it going to interfere with the radiations from any other point on this image. So, it will only interfere with radiation from its own image.

So, S_1 is going to interfere with radiation from S_2 , where S_1 S_2 are the images of the same point on the ground glass plate. Similarly, the radiation from this point is going to interfere with a radiation from its corresponding image on the other image from the corresponding point on the other image. So, we are going to have interference between pairs of waves which originate from 2 different images of the same point on the ground glass plate.

So, what you will get over here is a superposition of 2 waves: 1 over here 1 from this image and 1 from this image. And we have to add up the intensity from waves coming from all of such all such points.

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Fringe Conditions

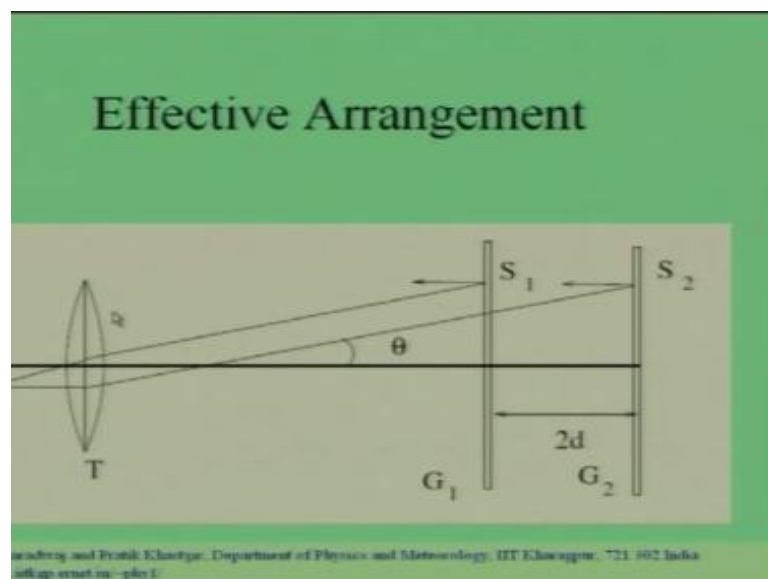
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_2 - \phi_1)$$
$$\phi_2 - \phi_1 = \pi + \left(\frac{2\pi}{\lambda}\right) 2d \cos \theta$$

Dark $2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$

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So, let us ask the question what is the intensity pattern on the screen over here going to look like right. So, the intensity pattern on the screen over there is going to be the superposition of the of 2 waves.

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Remember, again is going to be a superposition of 1 wave, which originated from this image of the ground glass plate, another wave, which originated from this image of the ground glass plate. We will focus on only 1 point on this image and 1 point on this image. The waves which are emitted at an angle theta from here and here both of them

are going to be superposed here. So, the resultant electric field at this point over here is the electric field of the wave emitted from here and the wave emitted from here superposed. Which is what I show over here.

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Fringe Conditions

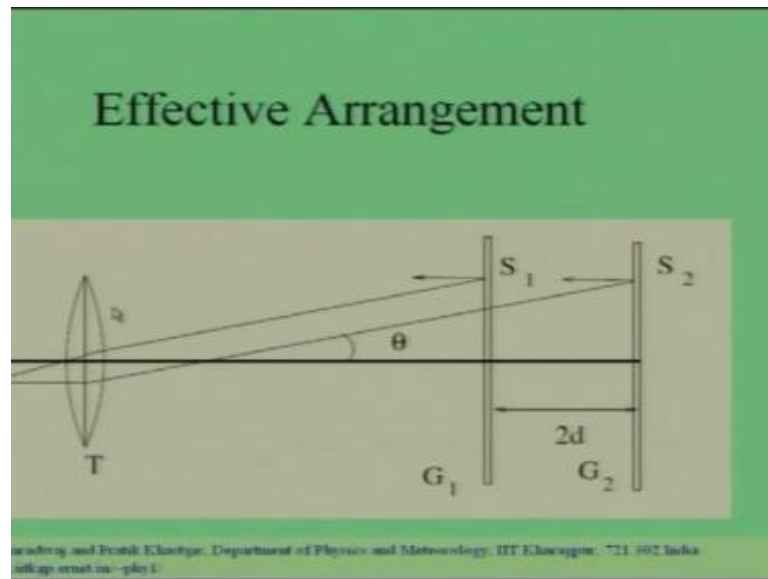
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_2 - \phi_1)$$
$$\phi_2 - \phi_1 = \pi + \left(\frac{2\pi}{\lambda}\right) 2d \cos \theta$$

mark $2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$

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So, this is the wave from the first image this is the wave from the second image both of them will get superposed on that point in the screen. So, I wish to calculate the intensity that point in the screen. So, I wish to calculate the intensity at this point arising from this image and this image together. The intensity at that point is $I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi_2 - \phi_1$ We have discussed we derive this formula 2 class back and we have discussed it.

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So, I_1 is the intensity of light coming from here, I_2 is the intensity of the light from here $\phi_2 - \phi_1$ is the phase difference between these 2. And the phase difference between these 2 waves arises because, they have to traverse different paths. These 2 images of ground glass plate are effectively sources which are separated by distance $2d$. And if we look at a wave which is emitted at an angle θ , then the path difference between these 2 is $2d \cos \theta$.

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Fringe Conditions

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_2 - \phi_1)$$

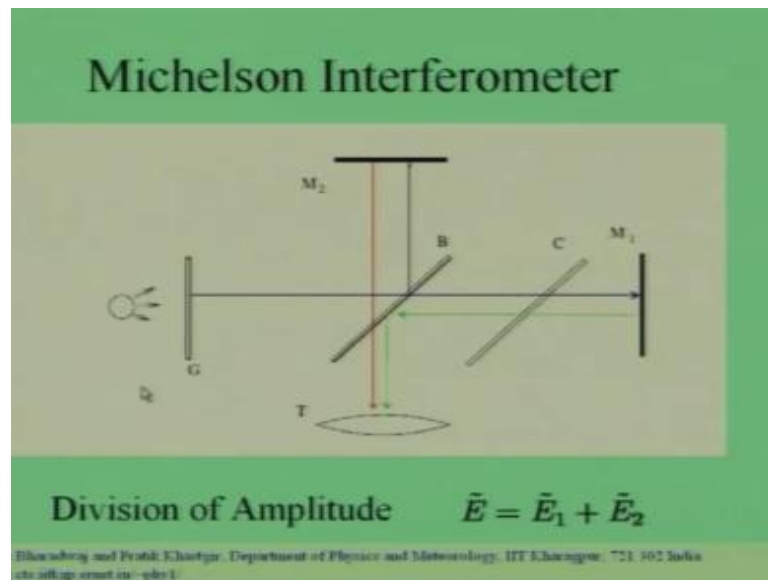
$$\phi_2 - \phi_1 = \pi + \left(\frac{2\pi}{\lambda}\right) 2d \cos \theta$$

Mark $2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$

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So, $\phi_2 - \phi_1$ will arise because of this path difference $2d \cos \theta$. You have to multiply by $2\pi/\lambda$ to get the phase difference between the 2 waves. Now, there is another effect which contributes to the phase difference. This effect is as follows.

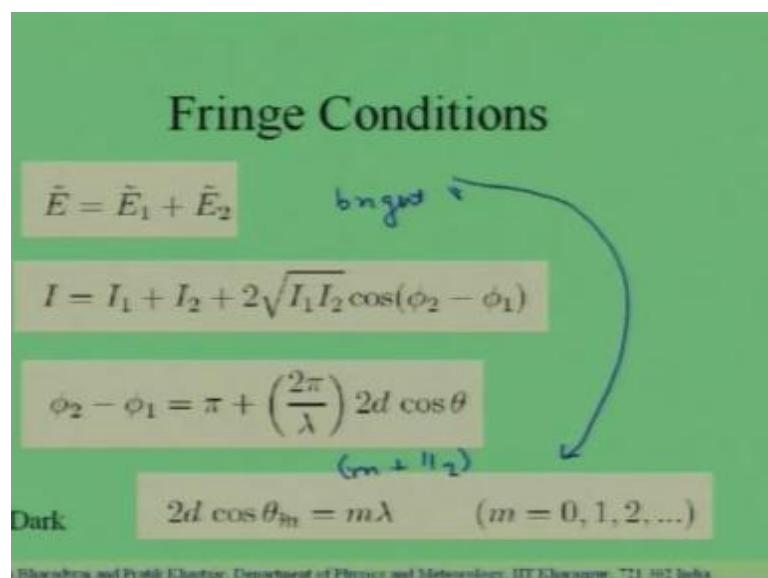
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If you follow the beam the wave which gets reflected and then, comes here notice that it undergoes an internal reflection. It undergoes an internal reflection at this over here because, it gets reflected from the interface of the glass and air. Whereas, the transmitted wave undergoes an external reflection because, it is reflected at the boundary of air and the glass.

So, there is a phase difference of pi between internal reflection and external reflection which in. So, this fact that 1 of the waves undergoes an internal reflection, another an external reflection introduce an extra phase difference of phi between the 2 waves.

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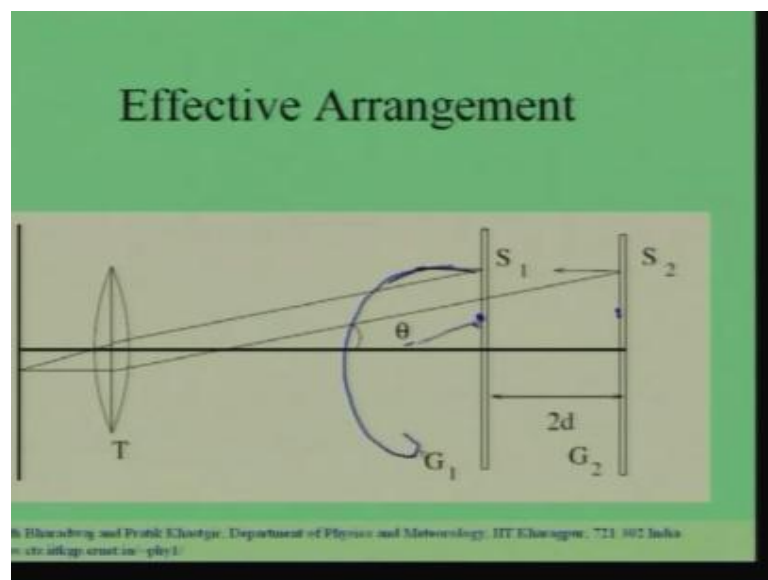


Then, this has to be included. So, the phase difference between the 2 waves is π plus the contribution due to the path difference. Because, the 2 sources do not, 2 images do not go inside. So, this is the total phase difference between the 2 waves. Now, if this is the phase difference let us, ask the question under what condition are we going to get a minima in the intensity.

The minima in the intensity is going to occur when, the phase difference is π . So, if the if you want the phase difference over here to be π . Then, $2d \cos \theta$ should be an integer multiple of the wave length. So, which is the condition over here so, this condition if this condition is satisfied, where m is an integer you will have a dark fringe. So, at that particular value of θ you will get a dark fringe. If you want if you ask the question under what condition will you get bright fringe, you will get a bright fringe if $2d$.

If $\phi_2 - \phi_1$ is an even multiple of 2π . If $\phi_2 - \phi_1$ the phase difference is $0, 2\pi, 4\pi$ then, you will get a bright fringe. If you want the phase difference to be an even multiple of 2π then, you get a condition of $2d \cos \theta = m\lambda$. $2d \cos \theta$ should be equal to $m\lambda$ plus half λ , that show the condition for a bright fringe is that $2d \cos \theta = (m + \frac{1}{2})\lambda$ for a bright fringe. The same thing just that you have to put in a factor of $m + \frac{1}{2}$ instead of m . So, this is for a bright and this for a dark fringe.

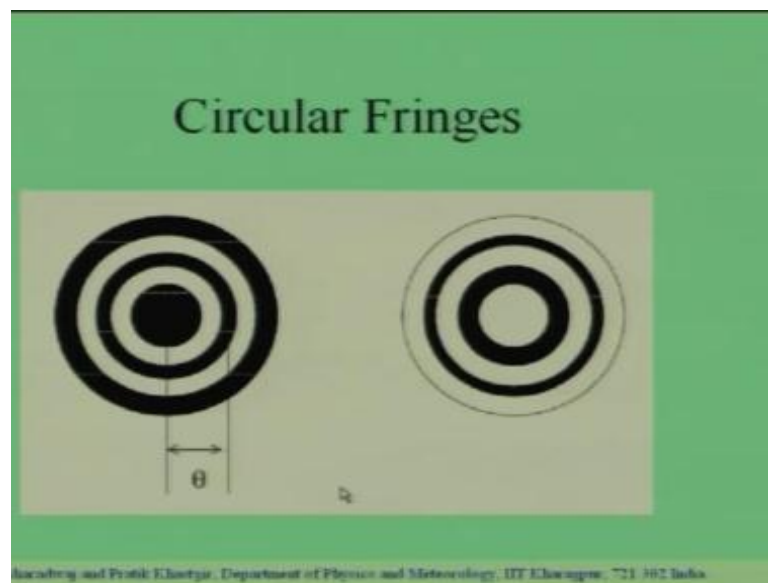
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Further, You should note that if the wave from here and the wave from here for this particular source. So, that we are focusing on 1 point. So, for this point the phase difference between it is the wave from here and it counter part of the other image. If the

phase difference between these 2 is such that it cancels out. Then, it is going to cancel out for every pair of points on these 2 screens. And you will get totally dark point over here this is the first point which you should note. Second point is that, this whole apparatus is symmetric around this central axis. So, you could rotate the whole thing around the central axis, you could rotate the whole thing around this and you will get the same condition to hold.

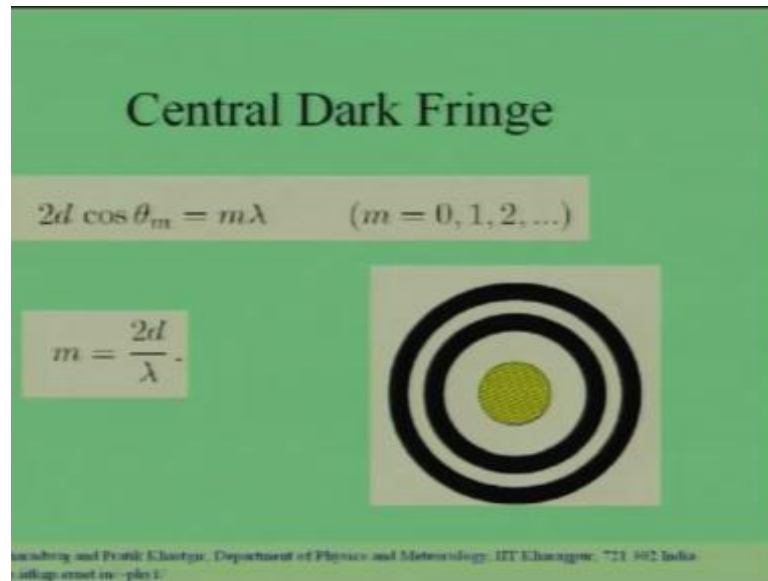
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So, what this tells us is at you will get circular fringes. So, the fringe pattern that you observe on the screen over there if you put your eyes at the focal at the eye piece of the telescope or if you put a lens and focus the light on to a screen, what you will observe over there is circular fringe patterns. You will get circular patterns of dark and bright.

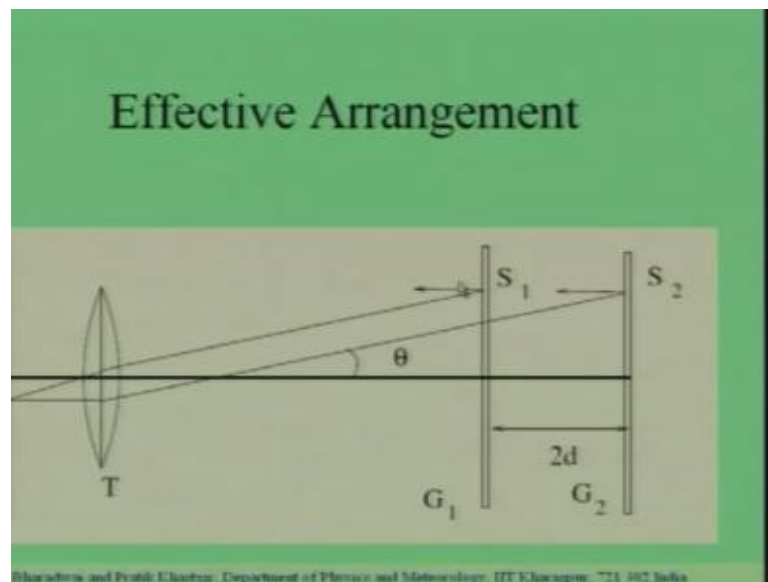
So, this is I have in the first half of this lecture I have tried to give you an idea of what the Michelson interferometer looks like and how it functions. Let us, now go into slightly more detail into the fringe pattern that you expect to see.

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So, suppose we have adjusted the whole interferometer. So, that the central fringe at the centre of the field of view you have a dark fringe. Let me, remind you once more what we mean by the centre of the field of view.

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Light which is emitted parallel to the axis over here, theta equal to would be focused to the centre over here. Or if you were to place here eye at the eye piece of the telescope which you add over here. Then, the direction which you see at the centre of the field of

you at the corresponds to light which is propagated, which is embittered over here at theta equal to 0. Increasing theta means larger and larger circles over here.

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Fringe Conditions

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_2 - \phi_1)$$

$$\phi_2 - \phi_1 = \pi + \left(\frac{2\pi}{\lambda}\right) 2d \cos \theta$$

Dark $2d \cos \theta_{m\frac{\pi}{2}} = m\lambda \quad (m = 0, 1, 2, \dots)$


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So, the fringe condition is 2 for a dark fringe is $2d \cos \theta$ is equal to $m\lambda$. So, if you have a dark fringe at the centre then, as you go out you will find that the will get it'll get brighter and brighter, again is going to dark. So, you will get circular fringe like this.

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Central Dark Fringe

$$2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$$

$$m = \frac{2d}{\lambda}$$


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Now, let us ask the question what is the condition if you wish to have a dark fringe at the centre. So, this the condition for a dark fringe in the first place the centre correspond to

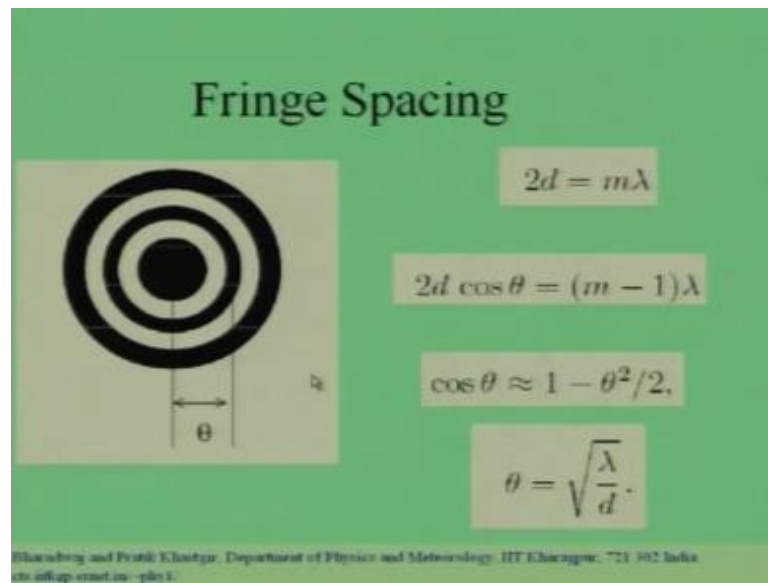
theta equal to 0. So, if you wish have a dark fringe at the centre the condition is $2d$ should be equal to $m\lambda$. So, twice the difference in the arm lengths should be an integer multiple of the wave length of the light that you are using for doing the experiment that is the first thing. So, what we see over here is that in the Michelson interferometer you will get circular a circular fringe pattern.

Then, we ask the question what is the condition that should be satisfied if you want a dark fringe at the centre of the field of view? If you want a dark fringe at the centre of the field of view, the 2 arm lengths should be adjusted in such a wave. So, that twice the different in the arm length twice d should be an integer multiple of λ . This will produce a dark fringe at the centre.

Because, the light which propagating straight down that is at no at along the axis of the interferometer. It will get a face difference, it will get a path difference which is $2d$ as it goes through. So, the phase difference going to be 2π by λ into $2d$. Which is going to be an integer multiple of 2π . So, you do not expect this to produce the dark the 2 waves to cancel out. So, what causes the 2 waves cancel out under this condition is the extra phase of π which is introduce.

Because, 1 wave under goes internal reflection, the other 1 under goes external reflection. So, this is the condition which has to be satisfied if you want to the central dark fringe and you can also calculate the order of the fringe. The order of the fringe m is $2d$ by λ . So, larger the separation of the between the 2 arm lengths, the higher the order of the central dark fringe.

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Next, let us ask the question what is the fringe spacing. So, I have already told you that in Michelson interferometer experiment, you will get circular fringes. So, suppose we adjust the interferometer in such a way. So, we adjust 2 arm lengths in such a way. So, that the difference between the 2 lengths is such that it gets a dark spot at the centre of the field of view.

As I go outwards I am going to get a bright line around it and then, I am going to get the first circular dark ring right. So, we have the dark spot at the centre.

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Fringe Spacing


$$2d = m\lambda$$
$$2d \cos \theta = (m - 1)\lambda$$
$$\cos \theta \approx 1 - \theta^2/2,$$
$$\theta = \sqrt{\frac{\lambda}{d}}.$$

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Then, we have a bright fringe around it and then, we have the first circular dark ring. And we would like to calculate the spacing between 2 dark rings. So, let us calculate these distance of the first dark ring from the centre of the central dark spot. So, the central dark fringe occurs that theta equal to 0, it satisfies the condition $2d$ is equal to m lambda where m is an integer.

Now, when we look at the first dark fringe which is the fringe over here so, we look at the fringe the first dark fringe. So, we will look at this particular fringe over here and ask the question what is the angular separation between this and this. This corresponds to an order m where m is $2d$ by lambda. Now, if I let us just go back to the fringe condition.

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Fringe Conditions

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

bringed

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_2 - \phi_1)$$

$$\phi_2 - \phi_1 = \pi + \left(\frac{2\pi}{\lambda}\right) 2d \cos \theta$$

(m + 1/2)

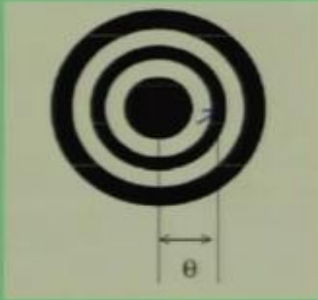
$$2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$$

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This is the fringe condition. So, if I increase theta maintaining the same distance d. If I increase theta going outwards means increase in theta then, cos theta the value of the cos theta has to will go down cos theta is maximum. When, theta is 0 at the centre now if I increase the theta the value of cos theta goes down. So, you can see the value of m will corresponding go down.

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Fringe Spacing



$$2d = m\lambda$$

$$2d \cos \theta = (m - 1)\lambda$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$2d \left(1 - \frac{\theta^2}{2}\right) = (m - 1)\lambda$$

$$\theta = \sqrt{\frac{\lambda}{d}}$$

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So, for the first the fringe condition for the first fringe tells us, that 2d cos theta is equal to m minus 1 lambda. This is the condition for the first fringe. This is the condition for

the central fringe the 0'th the central fringe at theta equal to 0. This is the condition where you will see the first dark fringe. The order of the fringe of this fringe is 1 less than, order of the central fringe. And if you assume that this angle theta is small you can then, expand cos theta as 1 approximately 1 minus theta square by 2 there will be the high order terms also which are small.

If you assume that theta is small which can be dropped and if you put this in over here. So, you will get $2d$ so, what you get when you put this over here is $2d \pm 2d \cos \theta$ 1 minus theta square by 2 is equal to $m \lambda$. Now, we subtract this from this and what you are left with is θ^2 is equal to λ / d which gives you, θ is equal to square root of λ / d .

So, the fringe spacing the angular separation between the central dark fringe and the first circular ring θ is the square root of λ / d . Just recollect, that in the young's double slit experiment the fringe spacing was λ / d . Whereas, in this situation it is square root of λ / d . So, let us go back to this expression for the fringe spacing and ask the question what happens if we increase the difference the length between the 2 arms.

So, notice that if you increase d the fringe spacing becomes smaller and smaller. So, the larger the difference in the lengths of the 2 arms the closer the fringe pattern is going the fringe are going to get. So, the fringe pattern is going to shrink, the fringes are going to be closer and closer. And if you make the 2 arms nearly equal the spacing between, the fringes is going to increase.

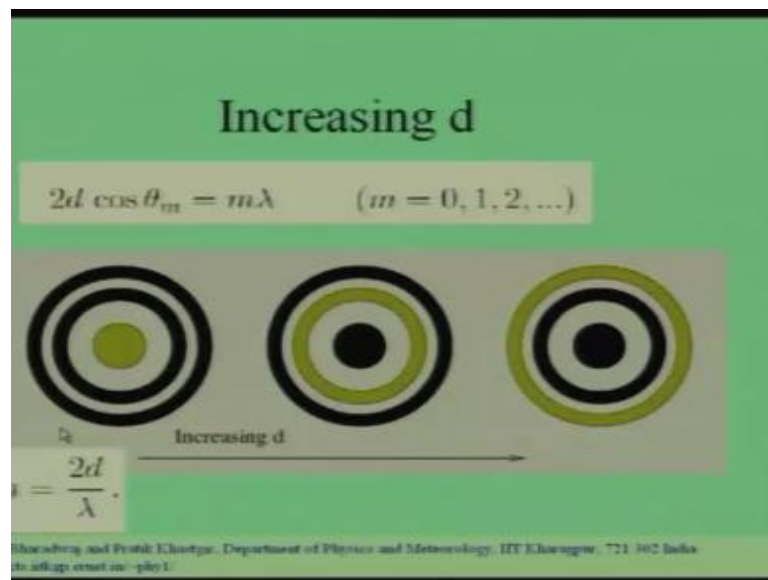
Now, you could ask the question what happens when you make both arms of equal length. Both arms of equal length corresponds to d equal to 0 right, d is the difference in the length of the 2 arms. So, if both the arms have exactly the same length, the difference d between the 2 lengths is going to be 0 what happens in this situation. So, what you see is that in this situation the entire field of view, the light from the entire field of view is going to cancel is going to be out of phase.

So, the entire field of view is going to be dark. So, as you bring the 2 arm lengths make the 2 arm lengths nearly equal the fringe spacing is going to increase the dark central dark spot is going get bigger. And the first circular fringe also going to bigger and the spacing between them in to increase. Until finally, when you make both the arm lengths

exactly equal the whole field of the central dark spot is going to fill the entire field of view which is going to be dark.

Let me, recapitulate what we learnt over here if you increase the separation between the 2 arm lengths, the fringe are going to closer and closer. If you make the arm lengths very close to each other, the lengths very close to each other. The fringe are going to be far apart and finally, if you make the arm lengths exactly equal, the whole field of view is going to be covered by the central dark fringe. So, the field of view going to be dark.

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Let us, move on to the another question now and the next question which we going to discuss is what happens if I increase d . So, we have adjusted the Michelson interferometer. So, that we have a dark spot at the centre for a certain value of d . Now, we move 1 of the mirrors slowly. So, as to increase the difference in the length of the 2 arms.

So, by moving 1 of the mirrors we slowly increase the value of d . The question what is going to happen? And we shall follow the evolution of a fringe of a particular order as we increase the separation between the 2 arm lengths. So, at the start we have a certain value of d and for this value of d , we will get we have arranged the apparatus. So, that we have a dark spot at the centre.

So, the order of the central dark spot is decided rather decides the separation in the 2 arm lengths. And to start with the central spot is of order m which is $2d$ by λ . Now, let us follow what happens to this same fringe as I increase d . So, m is fixed it refers to a particular dark fringe as I increase d . So, if I increase d you want to maintain the same value of m .

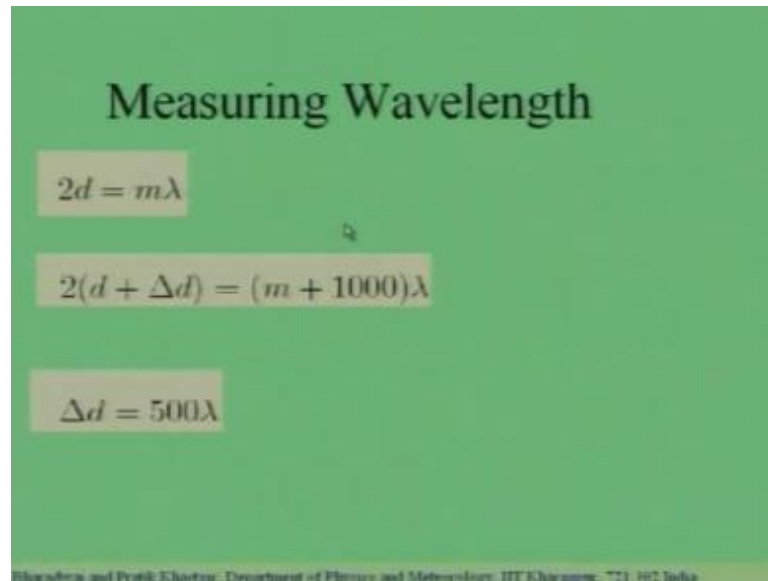
So, you are starting from θ equal to 0 for θ equal to 0 $\cos \theta$ is 1 θ m is the angle of the m 'th order fringe it is 0 to start with. Now, you increase the a little bit and ask the question where is the m 'th order fringe going to occur? If you increase d then, the value of the $\cos \theta$ has to go down to maintain the equality and $\cos \theta$ will be reduced if θ increases. So, if you increase the value of d , the value of θ corresponding to the m 'th order fringe also increases.

So, this fringe is going to go out. If you as you increase the separation between the 2 arm lengths this fringe is going to get bigger and bigger and you will get another dark fringe at the end at the centre. So, as you increase that arm length this fringe is going to go out and it will move outwards if you move without sufficiently. So, that we have to other dark fringe at the centre this fringe is going to go out.

The dark fringe in the centre has a is of a larger order right. D has increased you are looking at the centre so, $\cos \theta$ is still 1. So, d has increased and you have a dark fringe of a centre m should have g_1 up by 1. The fringe you are the focusing on of order m has now g_1 out. It has g_1 to a larger value of θ . If you continue to increase d , this particular fringe is going to move even further out in the field of view. And if you keep on increasing d this fringe going to move out of the field of view.

So, what you see is that as you increase d of fringe of a particular order appears at the centre. And there it moves outwards and finally, it moves out of the field of view. So, right as you keep on increasing d newer and newer and higher and higher order fringe is appear at the centre they move out and then finally, go out of the field of view. This is kind of the thing that you will see if you keep on increasing the separation between the 2 arms. Newer and newer fringes will appear at the centre and they will move out and slowly go out of the field of view.

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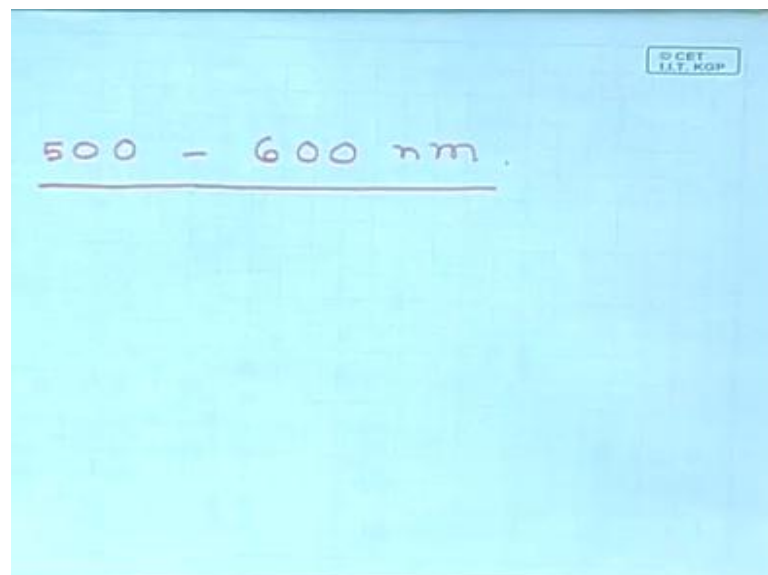
Measuring Wavelength

$$2d = m\lambda$$
$$2(d + \Delta d) = (m + 1000)\lambda$$
$$\Delta d = 500\lambda$$

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Let me, now discuss some very interesting uses of the Michelson interferometer. 1 very interesting application and quiet useful application of the Michelson interferometer is to measure the wave length of light. The visible light as we have learnt, it has a wave length somewhere around 500 to 600 nanometres.

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500 - 600 nm

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So, this is an extremely small length scale and it is not possible I mean such a small length scale is beyond direct human propection. And it is not possible to measure such small length scales using meters or a micrometers. So, it is not possible to measure such

small length scales of the order of 100s of nanometres directly, using meters which can be controlled by the by the bias straight away.

So, the problem is how do we measure the wave length of light. Which we which I have told is in the range 500 to 600 nanometres.

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Measuring Wavelength


$$2d = m\lambda$$
$$2(d + \Delta d) = (m + 1000)\lambda$$
$$\Delta d = 500\lambda$$

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So, the Michelson interferometer gives us a method by which you can measure the wave length of light. So, suppose we adjust the Michelson interferometer.

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Increasing d

$$2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$$


Increasing d

$$r = \frac{2d}{\lambda}$$

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So, that there is a dark spot at the centre. So, I have adjusted the Michelson interferometer. So, that there is a dark spot at the centre. Now, I have already told you that if I increase the difference in the arm length, if I increase the by moving 1 of the mirrors. Then, this particular dark fringe is going to go outwards and a new dark fringe will appear at centre. So, this particular dark fringe now becomes like this bigger and there is a new dark fringe at the centre.

So, I start the experiment where there is a particular order dark fringe at the centre, I do not know what the order is, but there is some order of dark fringe at the there is a dark fringe of the some order at the centre of my Michelson interferometer. Now, I move 1 of the mirrors so, as to increase d . And I count how many new fringes appear at the centre. So, from here to here 1 new fringe has appeared, here to here 2 fringe is appear.

The fringe which was there earlier at centre has now move out over here. If I increase the length even further, this is going to move even further and I will get a third. So, here there is a 1 new fringe, 2 new fringes I will get a third new fringe if I keep on increasing the d , I will get more and more new fringes. So, suppose I start with a particular d . So, that I have a certain order of fringe is which I do not know to start with.

Which I do not know, but I increase d and I count how many new fringes appear at the centre. Then, I keep on increasing the until thousand new fringes appear at the centre. So, this will require a bit of patience you have to keep on moving the mirror and counting how many and count how many new fringes appear at the centre thousand requires quite a bit of patience. Students in our laboratory typically, go for 100 fringes.

So, this experiment is there in our third year physics laboratory, not in the first year in this particular course, but advance physic students do it. There they do not go all the way to thousand they sit look through the eye piece move 1 of the mirrors. And count the number of fringes, new fringes that appear and the go till somewhere like hundred, but 1000 will give you more accurate result.

So, you move 1 of the mirrors count how many new fringes appear in the field of view keep on the moving the mirror until thousand new fringes have appeared the field of view. So, after 1000 fingers have appeared this is the condition that will be satisfied.

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Measuring Wavelength

$$2d = m\lambda$$
$$2(d + \Delta d) = (m + 1000)\lambda$$
$$\Delta d = 500\lambda$$

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So, you would have moved the mirror 1 of the mirrors by distance delta d and 1000 new fringes have appeared. So, you are going to have m plus 1000. So, you started with this and after 1000 new fringes have appeared you are going to have this. Now, you could subtract these 2 you will get a relation that delta d is equal to 500 times lambda. Now, how much is delta d let us estimate this.

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500 - 600 nm.

$$\Delta d \sim 500 \times 600 \text{ nm.}$$
$$\sim 3 \times 10^8 \times 10^{-9} \text{ m.}$$
$$= \cancel{30 \text{ } \mu\text{m}} \quad 3 \times 10^{-4} \text{ m}$$

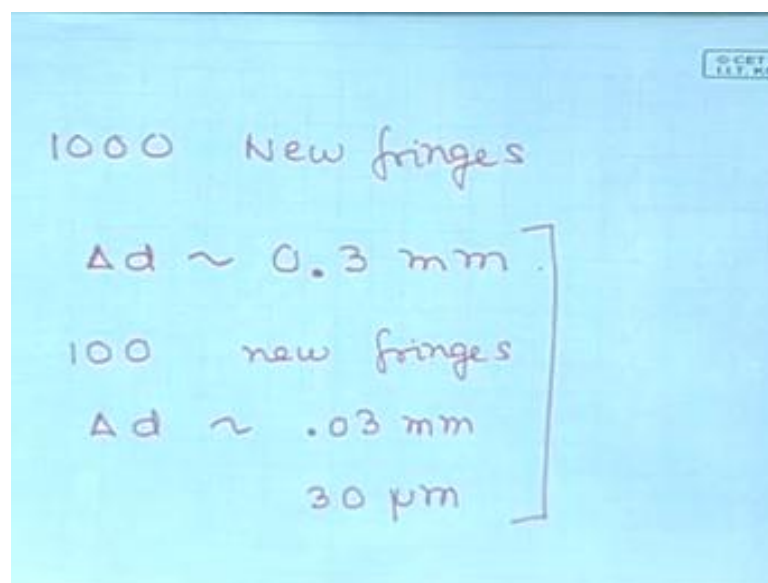
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So, delta d is the order of 500 times the wave length of light which is somewhere in the range of 600 nanometres 500 to 600 nanometres. So, this the order of the 3 into 10 to the

power 3 into 10 to the 30, 10 to the power 4 into 10 to the power minus 9 meter that is a nanometre which is equal to 30 micrometers, micrometers is 10 to the power 6.

So, 10 to the power 4 so, 10 to the power 500. So, that is 10 to the power 6 you will get 10 to the power 5 here not 4. That 1 2 1 this should be 500 1 2 3 4 0 and 30 gives you 1 more 5 0s. So, 10 to the power 5 into 10 to the power 9. So, you will be at 3 into 10 to the power minus 4 meters which is 0.3 millimeters which you can measure without great difficulty.

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So, delta d for 1000 new fringes at the centre delta d is of the order of point 3 millimeters. If you have 100 new fringes delta d is of the order of 0.3 millimeters, which is 30 micrometers both of these can be measured using a micro meter screw gage.

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Measuring Wavelength

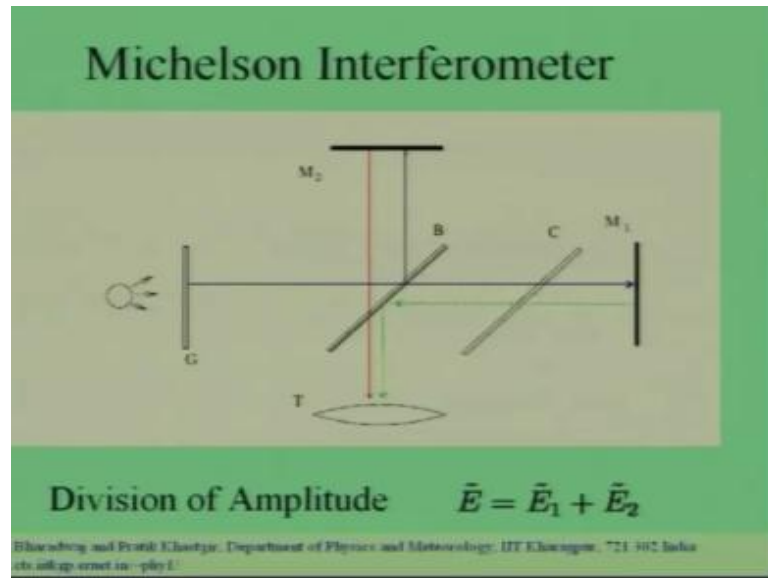
$$2d = m\lambda$$
$$2(d + \Delta d) = (m + 1000)\lambda$$
$$\Delta d = 500\lambda$$

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So, you can measure the distance you have to move 1 of the mirrors in a order to produce 1000 new fringes or you could measure the distance you have to move the mirrors 1 of the mirrors to introduce 100 new fringes. And once you can if you can measure this, you can determine the value of the wave length of light lambda. So, delta d is of the order of 500 lambda if you have 1000 new fringes.

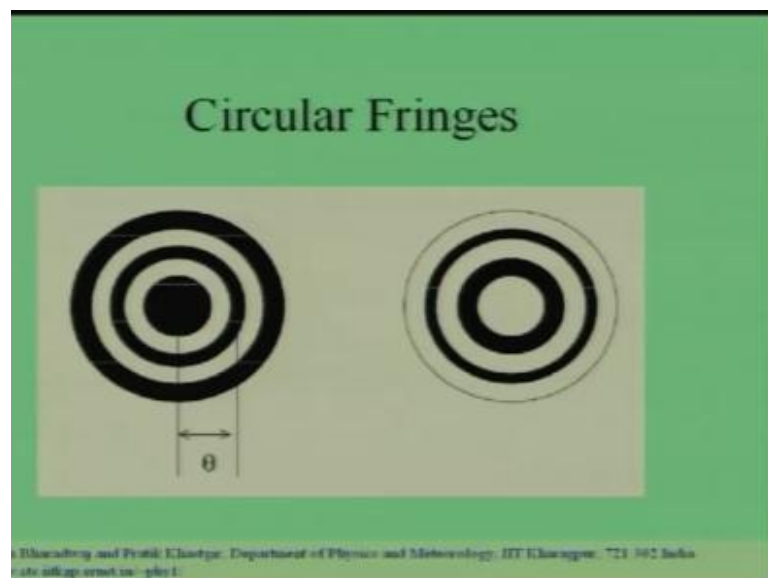
So, even though lambda is extremely small 500 lambda is not so small, you can measure it using a micrometer. And such measurements can be used to determine, the wave length of light.

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So, in today's lecture we have learnt about the Michelson interferometer. The Michelson's interferometer achieves interference through the division of amplitude. The same wave is divided into 2 waves of lesser amplitude and these are sent to through different paths and then, made to interfere.


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This interferometer Produces circular fringes and the fringe condition.

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Central Dark Fringe

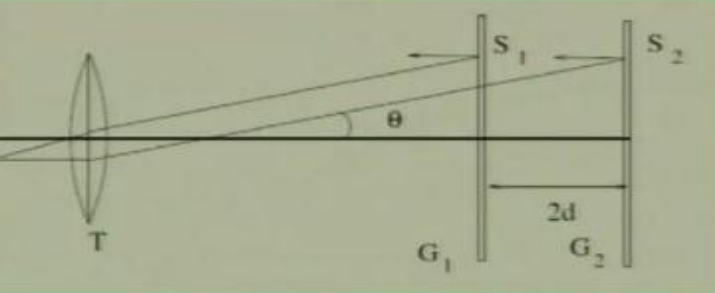
$$2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$$
$$m = \frac{2d}{\lambda}$$


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The condition for there to be fringe is $2d \cos \theta$ should be equal to $m \lambda$ θ is the angle with a line of sight to the centre.

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Effective Arrangement



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Central Dark Fringe

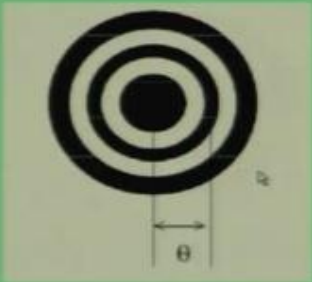
$$2d \cos \theta_m = m\lambda \quad (m = 0, 1, 2, \dots)$$
$$m = \frac{2d}{\lambda}$$


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So, this angle over here is theta. So, this is the line of sight theta is the angle over here. And the condition is for dark fringe is $2d \cos \theta$ should be equal to $m \lambda$, this will give you a dark fringe of order m at an angle θ . For a bright fringe you can have to replace this by $m + \frac{1}{2}$.

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Fringe Spacing


$$2d = m\lambda$$
$$2d \cos \theta = (m - 1)\lambda$$
$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$
$$\theta = \sqrt{\frac{\lambda}{d}}$$

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And we have studied various properties of fringe pattern. The fringe spacing, how the fringe pattern evolves if you increase the distance between the 2 mirrors. And then, we studied 1 application of the Michelson interferometer. We studied how you can use the

Michelson interferometer to measure the wave length of light. In the next lecture, we shall continue our discussion of the Michelson interferometer.