Physics I : Oscillations and Waves

Prof. S. Bharadwaj
Department of Physics and Meteorology
Indian Institute of Technology, Kharagpur

Lecture - 14
Interference - I

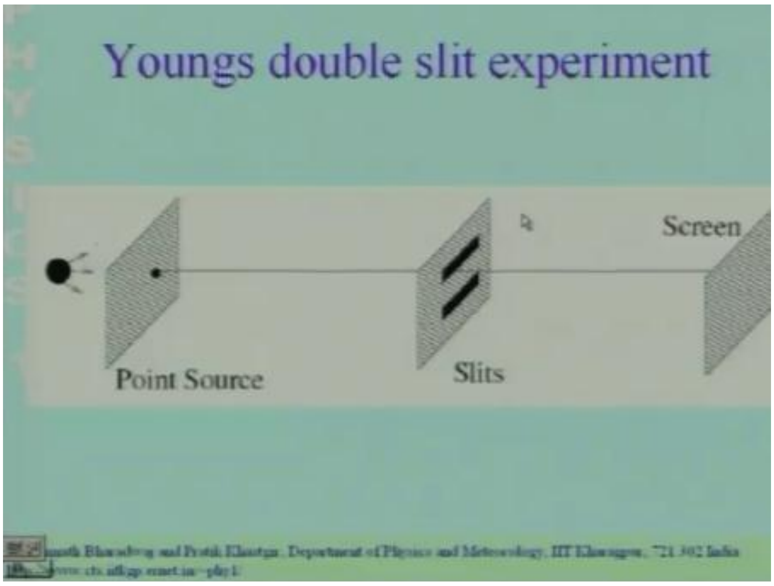
Good morning, in today's lecture we shall discuss, what happens when 2 waves are superposed. Suppose, we have a room which has 2 light bulbs a and b and we switch on only a or we switch on only b or if we switch on both of them. Now, we expect that when we switch on both the light bulbs the room will be twice as bright as the situation when we have switched on either a or b only.

This is because we expect the intensities of the waves from a and b to add up when we have both these waves incident on the same place, but, this is not always true. The reason why this is not always true is because waves can have both positive and negative value. For example when we have a sound wave the density fluctuation can be positive or negative the density may go up or it may go below the average value.

Now, if i have 2 waves being superposed and it. So, happens that 1 wave has a positive value while the other has a negative value at the same point. Then if I add up these 2 then there will be some cancellation and the resultant intensity will could actually be less than the individual intensities. So, the resultant intensity when I have superposed 2 waves could be actually less than the situation, when I have only the first wave or when I have only the second wave. This happens because the wave could have a positive or negative sign unlike the intensities.

So, this phenomena is what is known as interference and we are going to discuss the phenomena of interference, which is a very interesting phenomena and it has lot of applications it occurs in nature and it has a lot of applications. So, in the next few lectures, we are going to discuss this, phenomena of interference and today we will take up a particular situation Which is known as the Young's double slit experiment.

(Refer Slide Time: 03:31)

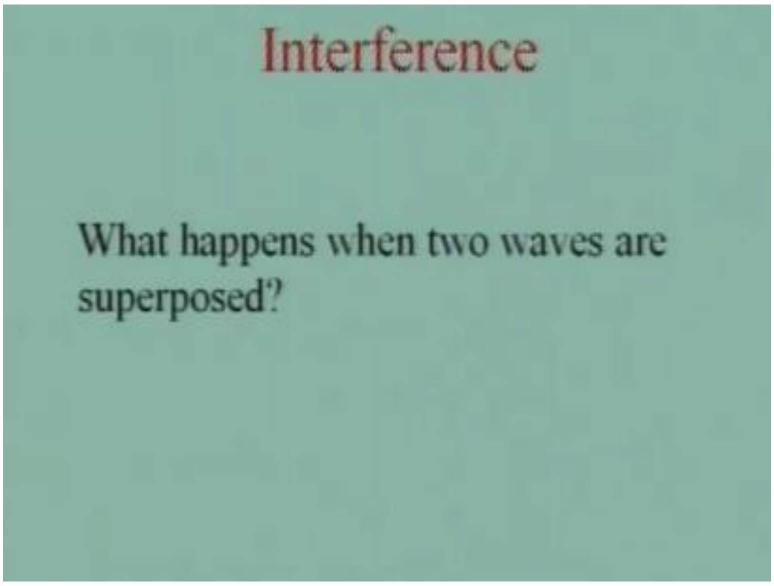


So, in this experiment we have a light source and the light from this source falls on a screen over here. The screen has a small aperture which and through which the light comes out. So, effectively this acts like a point source the aperture here is. So, small that; you can think of it as a point source far away from this point source there is another screen and the screen over here has 2 slits as you can see over here.

So, there are 2 slits in the screen over here which is at quite a distance from this point source and finally, we have a screen over here, where we would like to study the intensity pattern as a consequence of the light which comes out through these 2 slits. So, you may say that we would like to study the image of these 2 slits on the screen over here what does it look like and the screen again is at a large distance from the 2 slits.

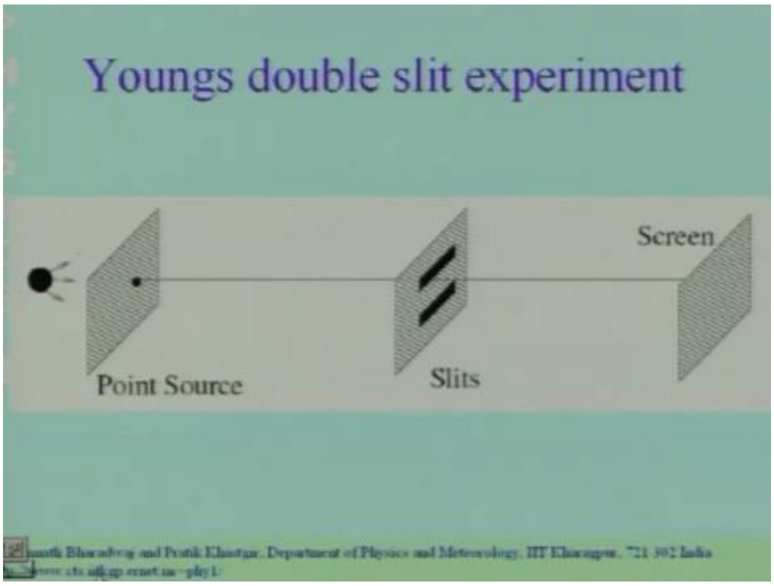
Now, the point source is at a large distance from this slit. Now, the radiation the wave that comes out from the point source is initially going to be spherical. So, when the wave comes out of the point source.

(Refer Slide Time: 05:20)



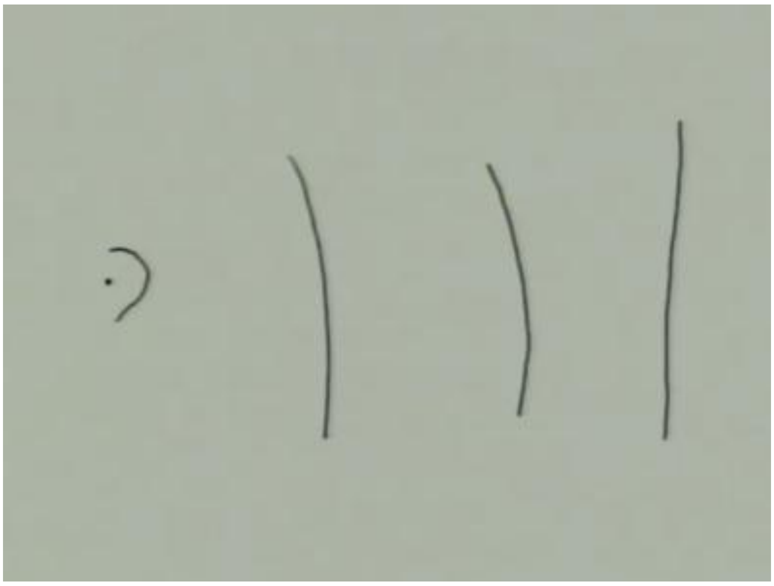
It is initially going to be spherical.

(Refer Slide Time: 05:43)



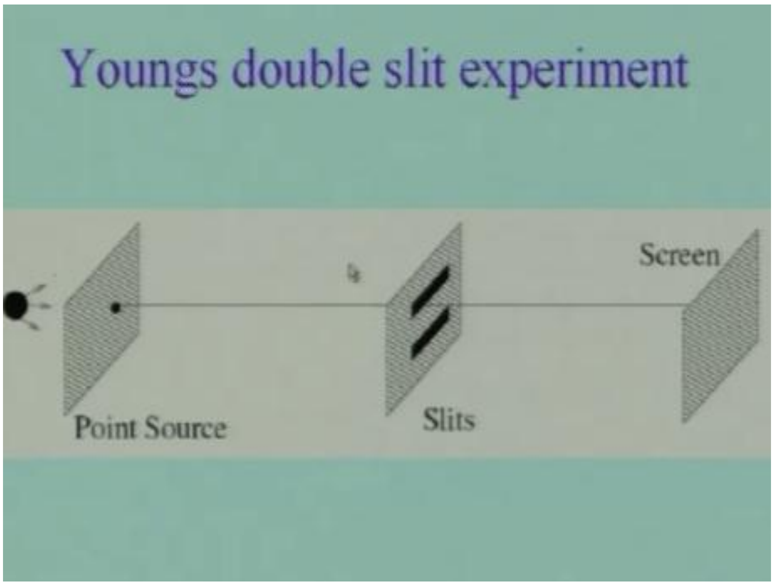
But by the time it reaches the slit, the slits which are the quite far away you would expect the 2 waves. So, let me draw a picture and explain this point to you. So, you have the point source over here.

(Refer Slide Time: 06:09)



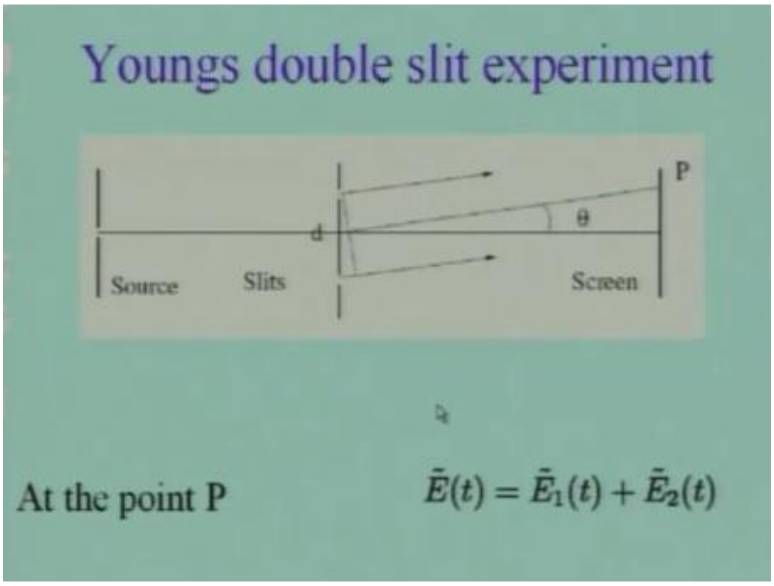
And when the wave comes out it will be spherical, but, by the time you are quite far away the radius of this sphere has become very large and for our purposes it is sufficiently close 2 a plane wave.

(Refer Slide Time: 06:32)



So, by the time, the wave from this point source arrives at the silt as the silt is quite far away from the point source the wave which arrives over here is a plane wave is well approximated by a plane wave. The point source over here is aligned with the center of these 2 slits. So, the plane wave let arrives over here on the slit is parallel to the 2 slits and this plane wave falls illuminates these 2 slits. So, let we discuss this situation.

(Refer Slide Time: 07:10)



In a picture, which looks like this is a section through the picture which I had shown you just little while ago. It is the same situation just that we have taken a section through it. So, we have the point source over here. And the wave from the point source by the time it reaches these 2 slits is to a great to a great accuracy of plane wave. This plane wave illuminates these 2 slits and we are interested in the intensity of the light on the screen over here, which is at a far large distance from the 2 slits.

So, let us focus on a point P on the screen where we would like to calculate the intensity. The electric field of the wave the electric field of the wave at the point P will have 2 contributions, 1 contribution will be the part of the wave that arrives at P through slit 1 and another contribution will be the part of the wave that arrives at the point P through slit 2. I have not put names 1 and 2 to the any of the to the individual slits, you can refer to any 1 of them as 1 and the other 1 as 2 it is your choice.

So, if you wish to calculate the electric field of the wave at this point remember it is an electromagnetic wave. So, the quantity of interest is the electric field if you wish to calculate the electric field at the point P over here. It will be it will have 2 contributions 1 that comes from the first slit E1 and another that comes from the second slit E2 and the resultant electric field over here at the point P is a superposition of these 2 contributions.

And you should remember that the electric field both these electric fields E1 and E2 and the resultant electric field t all of them are oscillating with time because we have a wave another point which I should clarify at this stage is that the electric field is a vector. And if the wave is propagating in a particular direction the vector will have 2 independent components in the plane perpendicular to the direction which the wave is propagating. So, we should really be talking of the electric field vector because it has two possible independent directions in which it could be oscillating.

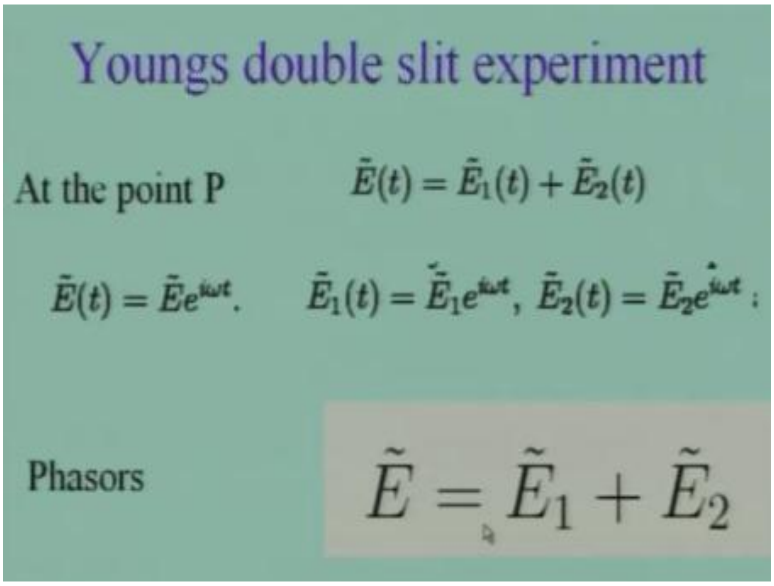
The 2 polarization states, which we have discussed in the in a the few classes ago. Now, we are in this lecture in these lectures right now, we are going to restrict our attention to only a particular direction of the electric field. So, we are going to restrict our self a to the electric field in only a single direction. Which simplifies matters because we can now think of the electric field as having only 1 component we are going to follow only 1 component of the electric field because we are going look at the electric field only in 1 direction.

So, if we restrict our attention to only 1 component of the electric field 1 of the 2 possible components then we can think of electric field as a scalar. And in the rest of this discussion, we are going to do a scalar treatment. So, bear in mind that it the scalar treatment refers to only a single component of the electric field, there is another component which will be here in exactly an identical the identical fashion.

So, the electric field at the point P is a sum of the electric field contributions from the 2 slits now, the 2 slits are illuminated by the same source. It is a same point source which illuminates the 2 slits further the wave front is parallel to the slits. So, the electric field over here at the slit 1 and the electric field at slit 2 have the same phase because this is the definition of a wavefront.

All points on the a same wave front have exactly the same phase. So, the electric field at all points, on the wave front have the same phase also the same amplitude. So, at the slit at the location of the slit. So, let us the look at 2 slits at the location of the slits the electric field is exactly identical. So, the electric field here and here are exactly identical they are doing exactly identical oscillations with the same phase and they have the same amplitude and both of these contribute to the electric field at the point P and it is these contributions that we call E1 and E2 and we have to add up these contributions to get the total at the point P.

(Refer Slide Time: 12:32)



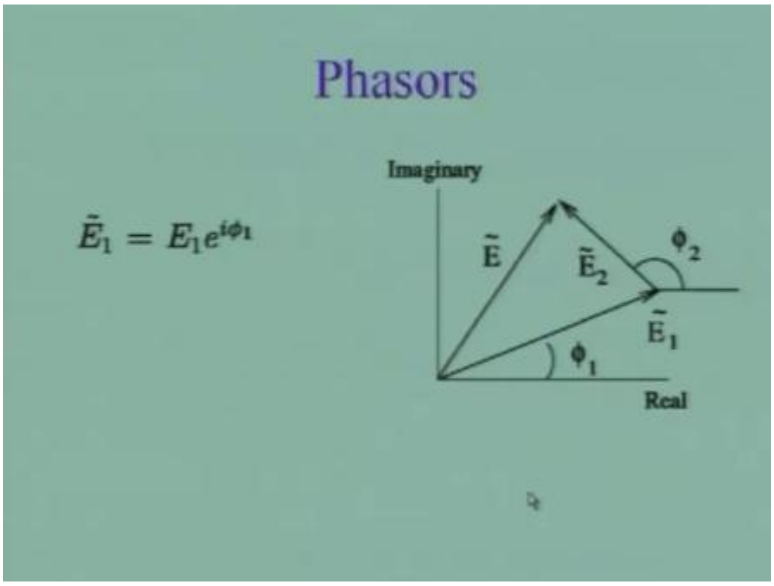
Now, so let me this shows you the same thing again at the point P the resultant electric field is the sum of E1 and E2. And we have use the complex notation which is why we have these tildes on top. Now, the contribution from the slit 1 which is E1 tilde t, this contribution we can write as E1 tilde a constant amplitude the constant complex amplitude into e to the power i omega t.

Here omega is the angular frequency of the wave and both the slits are illuminated by the same from the same source. So, the electric field at both slits are going to have the same frequency or the same angular frequency and the resultant, which is a sum of both the contributions from both the slits is also going to have the same angular frequency or the same frequency. So, the point to note is that we have a factor of E2 the power i omega t for the contribution from the slit 1, we also have the same factor E2 the power i omega t for the contribution from slit 2 and the resultant also has the same factor of e to the power i omega t.

So, we can cancel out the factors of factors of e to the power i omega t from all 3 of these. And this expression now reduces to an relation to a relation between these constant complex amplitudes E1 tilde, E2 tilde and E tilde; E tilde is the complex amplitude of the resultant E tilde is the sum of E1 tilde which is the complex amplitude of the contribution from the first slit E2 tilde is the complex amplitude of the contribution from the second slit. And we have a complex number because we have to use to complex numbers because the waves could have different phases.

So, the contributions from the different slits could have different phases. So, we have to use a complex notation the amplitude is a complex number it has both the modulus tells you the real amplitude and the phase tells you the phase with respect some arbitrary oscillation. Now, these complex amplitudes are also called phasors and this phasors. So, for the next few minutes, we shall refer to these complex amplitude as phasors. Phasors have a very nice graphical representation.

(Refer Slide Time: 15:25)

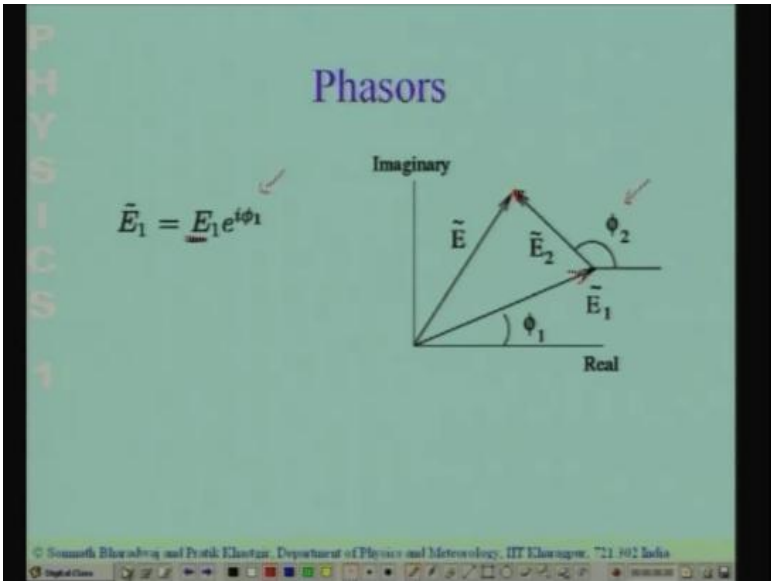


So, let me spend little time discussing the graphical representation of phasors. You might have already encountered phasors in courses on electrical technology and such courses, but, I think it will be a good idea to just revise it briefly over here. So, we have a complex number E1 tilde. So, we have the complex number E1 tilde and E1 tilde has a has an amplitude. So, we have the complex amplitude and it has a real amplitude which is the magnitude of the complex number E1. And we have a phase.

So, we can write this as E1 e to the power i phi 1. Now, we represent this complex number in a 2 dimensional plane. So, this shows you the 2 dimensional plane where we are going to represent this complex number called the phasor along the x axis. So, along the x axis, this is the x axis I am going to plot the real part of the phasor. So, which is why I have written real over here and along the y axis I am going to plot the imaginary part of the phasor.

So, along the real axis I am going to plot the real part of this place phasor and along the imaginary axis I am going to plot the imaginary part of this of this phasor. So, we are looking at the phasor E1 and in this 2 dimensional plane. So, E1 is going to be represented by a vector.

(Refer Slide Time: 17:27)

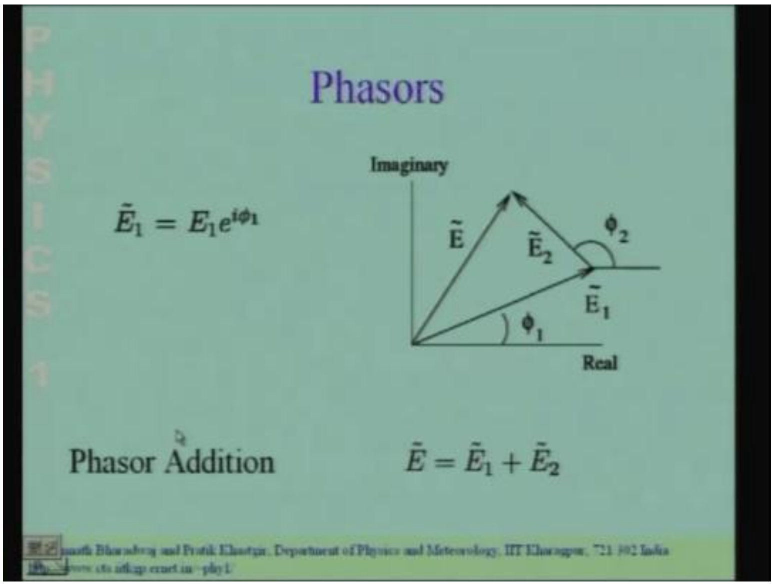


The length of this vector is the length of this vector E1 is the magnitude of the phasor. So, the length is the magnitude of the phasor whereas, the angle the vector makes with the x axis phi is phi 1 is the phase of the phasor. So, let me recapitulate again these complex amplitudes, we refer to them as phasors these phasors can be represented in they are complex numbers. So, we can represent them in a 2 dimensional plane. The x along the x axis we will plot the real part along the y axis we will plot the imaginary part.

So, corresponding to every phasor we will have a vector in this 2 dimensional plane. The length of the vector is the magnitude of the phasor the angle the vector makes with respect to the x axis is the phase of the phasor. So, this vector over here the vector over here shows the phasor E1 similarly, a phasor E2 can also be plotted. So, we have plotted the phasor E2. So, this is the phasor E2. It makes an angle phi 2 with the x axis. So, you can see here it makes the angle phi 2 with the x axis and it has a length which is the amplitude of the complex number E2.

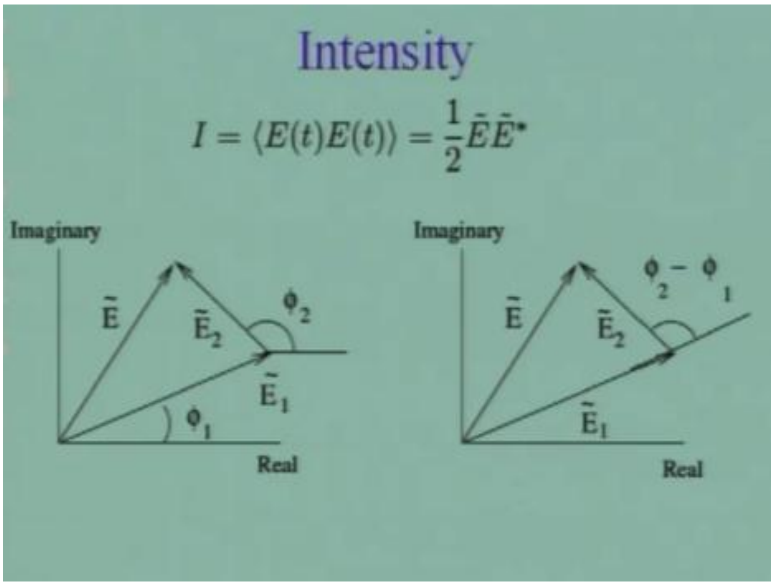
Now, in this situation, in the situation in this particular situation

(Refer Slide Time: 19:26)



We would like to add the 2 phasors. So, the sum of the 2 phasors is essentially the vector sum of these 2 vectors. So, the phasor E the phasor E is the sum. The phasor E is the sum of these 2 phasors E1 and E2. So, and I can obtain this phasor E graphically by doing the vector sum of the vectors which represent the phasors which represent E1 and E2. So, I can do a sum of these 2 phasors and this is the resultant phasor E. Now, so, this E tells us the amplitude of the electric field at the point P.

(Refer Slide Time: 20:16)



Now, if you want to the quantity of interest here. Now, if you want to the quantity of interest here, is not the amplitude of the oscillations of the electric field at the point P, but, the intensity of the wave at the point P. Let me, spend a little time discussing this point. For example, sound waves we know cover the frequency range 20 hertz to 20 kilohertz. So, there are these oscillations in the density of the air at frequencies which span this whole range 20 hertz to 20 kilohertz. How eardrum inside our ear, we perceive these oscillations, but, we do not what we record what are mind records is not these oscillations, but, the average the time average intensity corresponding to these oscillations.

Our mind we our mind really does not work at that the ear the our mind does not work. So, the human being does not work at those high speeds a require to record the oscillations. What we record is the time averaged intensity of the oscillations I do not think we have we have any. So, I mean we have any sensation of fractions of a second which are. So, small write 20 kilohertz means few thousands of a say a 10,000 20,000 of a second right.

That is the, that is how rapid the oscillations are taking place and I do not think any of us have any real feeling for a time that small the human being for example, works on the order of times scale of seconds and what we record is the average the intensity of these oscillations. So, the mean square displacement of these oscillations the some think like that averaged over something like a second. So, we in for sound we record the time averaged intensity for light.

Let us talk about light the visible light the visible range radiation. We have discussed in the last class, it oscillates at the frequency 10 to the power 14 hertz. So, it does 1 oscillation in 10 to the power minus 14 seconds and I do not I mean I do not think any of us can believe can speak about an experience on that time scale. So, we really cannot record with our eyes, we can we do not record the oscillations of the electric field, what we record is the intensity of the electric field averaged over some finite time period which is of the order of in the range of seconds could be fraction of the second.

But, not at the scale of 10 to the power of minus 14. So, what we record is the time average of the intensity in most situations. So, it is this quantity which is of interest. So, let us now take up the question of how to calculate the intensity of the light. The intensity is the time average of the electric field. So, if I want calculate the intensity at the point P I have to take the electric field at the point P.

This will be something which varies with time I have to square it take the time average there will be constant factors outside, which I have, I am not really concerned with those. So, we have dropped those constant factors up to constant factors the intensity is the time average of E square and we can write this in complex notation. So, we have discussed these right in the first class.

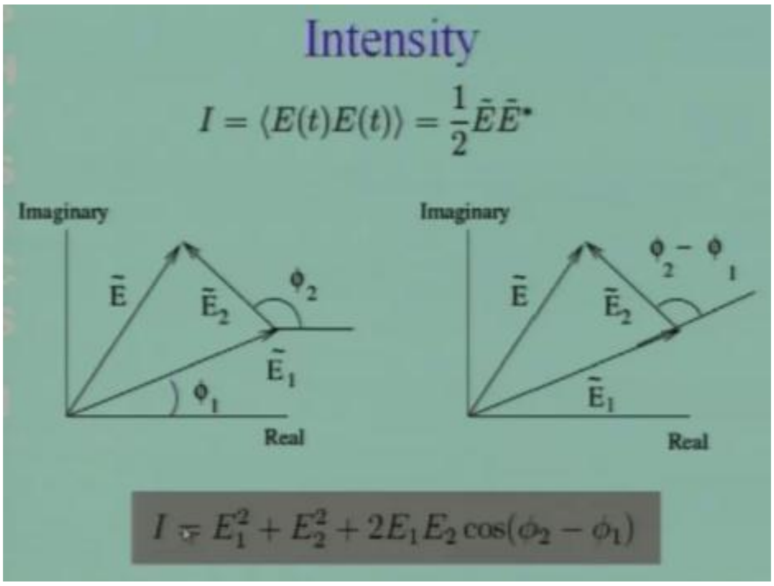
So, for an oscillating quantity we can represent Et in complex notation and we can write the time average of the energy of the intensity time average of E square as half E tilde into E tilde star. So, E tilde is the amplitude of the electric field in the complex notation. And the time averaged intensity can be written as half E into E tilde into E tilde star complex conjugate of E tilde. So, when you wish to calculate the intensity at the point P we have to take this phasor E and multiplied with its complex conjugate and put a factor of half.

So, it is quite clear that, the square of the length of this phasor the square of the length of this phasor is going to give us the intensity of the radiation right. E into E star tells us the square of the amplitude of this phasor. So, the square of the length of this phasor is going to tell us the intensity of the radiation at the point P. Now, when you want to calculate. So, let us go back, let us now look, at this picture where we have added the phasors E1 and E2 to obtain the phasor E.

So, this is the vector sum of the 2 vectors in 2 dimension a 2 vectors E1 and E2. So, the vector E is the sum of this vector E1 plus E2 to do a square to find the intensity I have to square this. So, the square of this is going to be the square of the length of this vector plus the square of the length of this vector plus twice E1 dot E2, which is going to be E1 the magnitude of this into the magnitude of this E2 into cos phi cos of the angle between these 2 vectors and the angle between these 2 vectors you can very easily convince yourself is the difference between these 2 angles phi 1 minus phi 2 phi 2 minus phi 1.

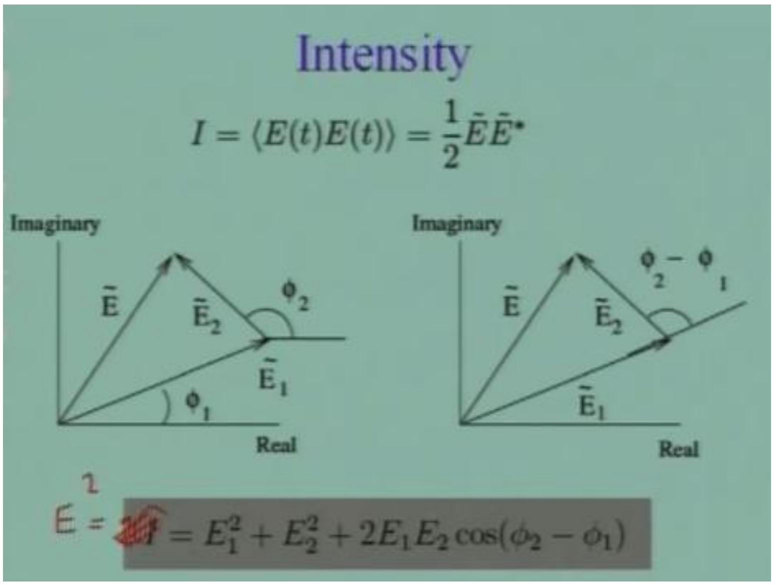
So, the length of this vector squared is going to be the length of this vector squared the length of this vector squared into twice into the length of this into the length of this into the cosine of this difference in the angle between these 2 vectors.

(Refer Slide Time: 26:45)



So, this is what is the expression for the intensity. So, this tells us the intensity corresponding to a situation, where we have superposed 2 waves. So, the discussion until now has been quite general, we have not referred to the Young's double slit experiment explicitly. We have considered, a situation where we have superposed 2 waves and we calculated the resultant intensity. And what we see is that the resultant intensity when I superpose 2 waves the resultant intensity is the square of the amplitude of the first wave which is the intensity. There should be a factor of half throughout which I have missed.

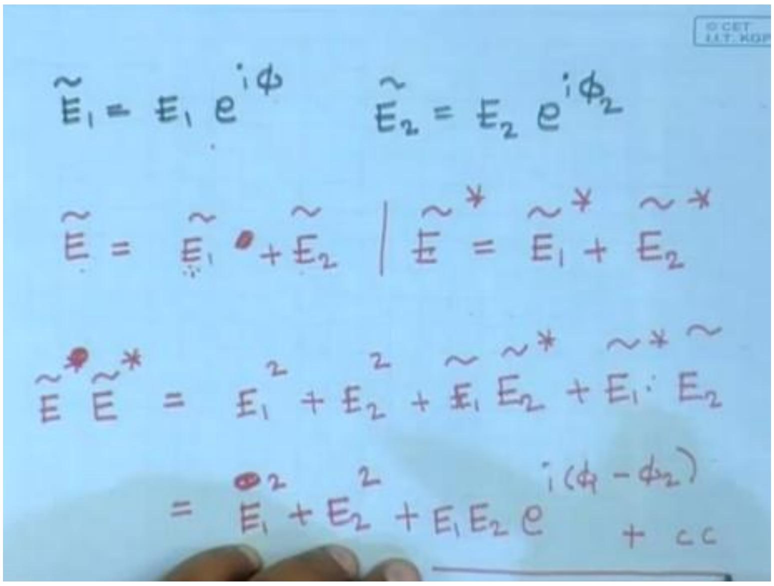
(Refer Slide Time: 27:47)



So, you should. So, this is actually twice the intensity because we have a factor of a half over there. So, the twice the intensity; so, the intensity to calculate the intensity. Or what we could is we could say that this is E square is equal to this. So, the length of this vector this phasor the resultant phasor is the sum of the individual lengths of the 2 phasors plus sum of the square of the individual length. So, we are what we want to calculate is a square of this phase length of this phasor.

The square of the length of this phasor is the square of the length of this phasor plus the square of the length of this phasor plus twice the length of this phasor into the length of this phasor into the cosine of the phase difference between the 2 waves. So, this is the most a very general expression which well which we shall use a quite often in our discussion of interference. Let us now, repeat the whole exercise algebraically we did we did this exercise geometrically. Now let us, repeat the whole exercise algebraically. So, let me do it over here for you.

(Refer Slide Time: 29:22)



So, we have 2 waves. The first wave is represented by a complex amplitude E1 tilde which is E1 e to the power i phi 1 and similarly, E2 tilde is E2 with the phase phi 2. Now, we would like to calculate first we calculate the resultant of these 2 waves which is the resultant of these to complex amplitudes which gives us. So, we can write it straight away as E1 tilde plus E2 tilde let me also calculate the complex conjugate of this. Now, let us calculate the square of the length of let me of E of the resultant.

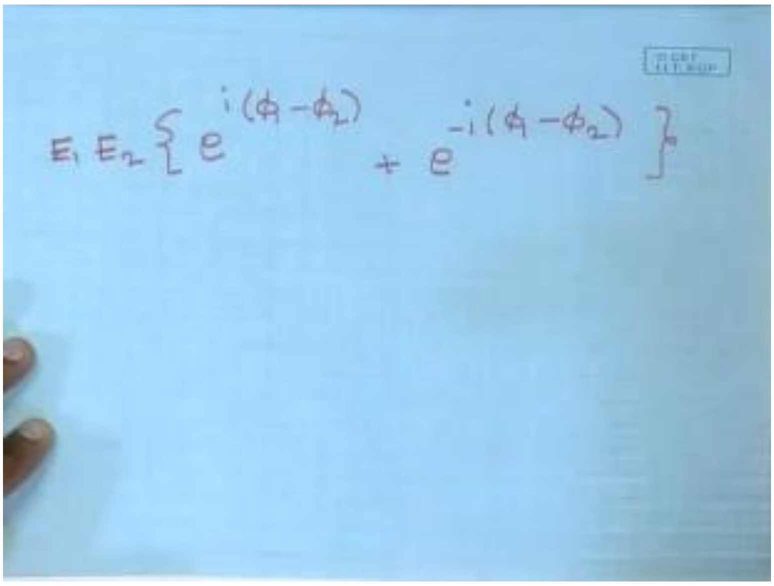
So, let us calculate this, this is going to be a some of different parts. The first part so, I am going to multiply this with this. So, the there is going to be 1 part which I get when I multiply this E1 with E1 star which is going to give me E1 square and if I multiply E2 with E2 star I am going to get E to square. Now, I am going to multiply E1 with E2 star. So, I have E1 tilde E2 star plus E1 star E2 tilde.

Let me, now simplify a little bit. This we have to retain these two terms as they are now you can write this into this using these two expression of phi E1 and E2 in terms of real part of the real magnitude and a phase

So, this whole thing will become E1 E2 into e to the power i phi 1 this will give us e to the power phi 1 this will give us e to the power minus i phi 2 because when I take a complex conjugate, I will pick up a minus sign. So, I have e to the power i phi 1 minus phi 2 plus this term notice is just the complex conjugate of this. So, I will get the complex conjugate of this.

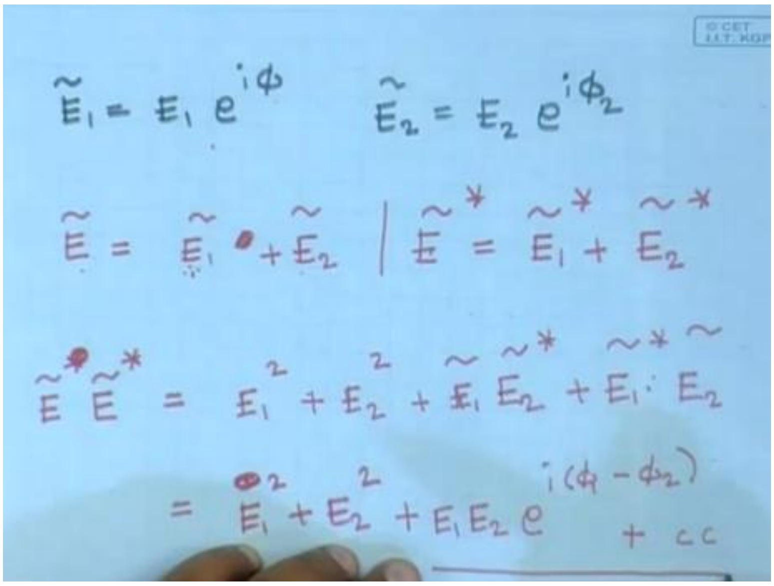
So, let me write out, write down these. So, I will have let we write down these 2 terms explicitly and then simplify them whole thing little bit.

(Refer Slide Time: 32:45)



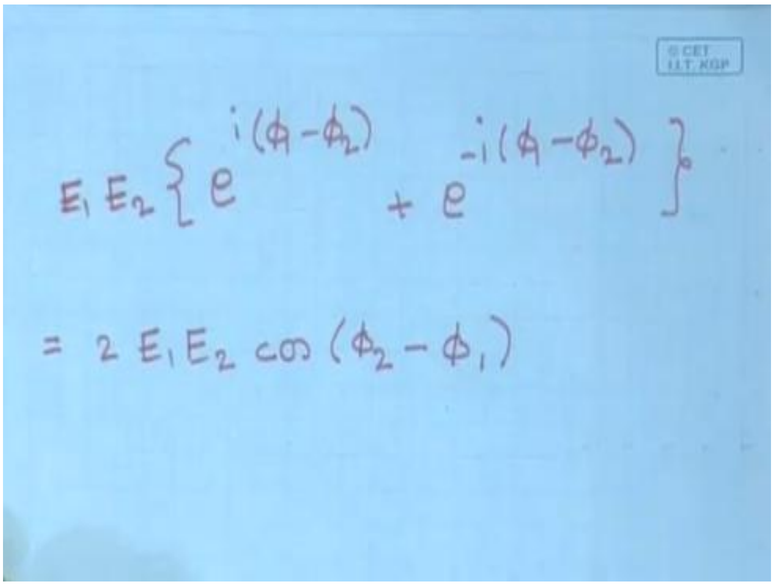
So, if I write down these two last 2 terms what I have is: E 1 E2 e to the power i phi 1 minus phi 2 plus e to the power minus i phi 1 minus phi 2. So, let me remained you again

(Refer Slide Time: 33:05)



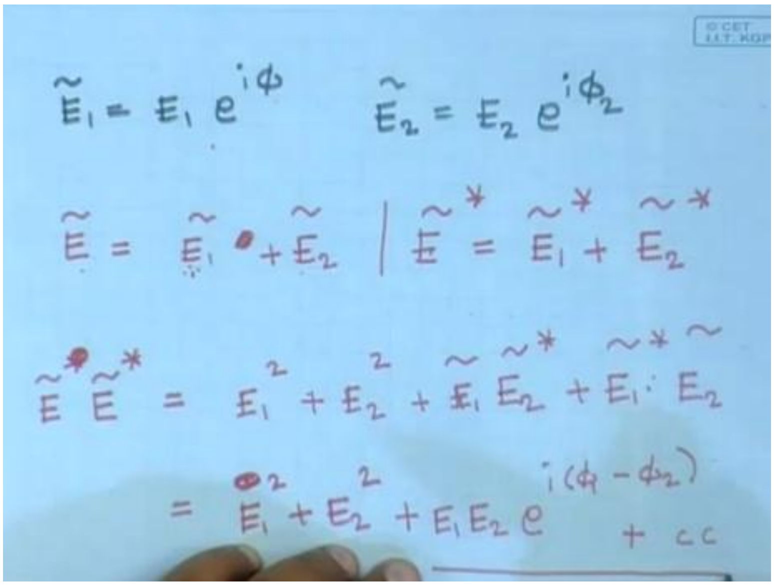
The first term comes from this the second term comes from this we are looking at only these 2 terms. So, the first term over here comes from this and the second term comes from this. So, that is what we have.

(Refer Slide Time: 33:15)



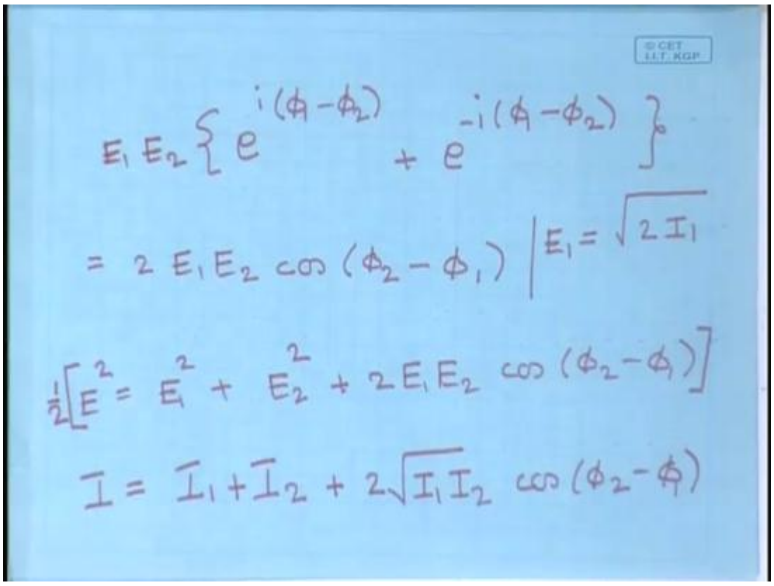
And this you can see is 2 E1 E2 cos phi 2 minus phi 1.

(Refer Slide Time: 33:35)



So, we have obtained the square of this electric field of the resultant electric field and the square of the resultant electric field E square is E1 square plus E2 square plus 2 E1 E2 cos phi 2 minus phi 1 and we could also write this in terms of the intensities.

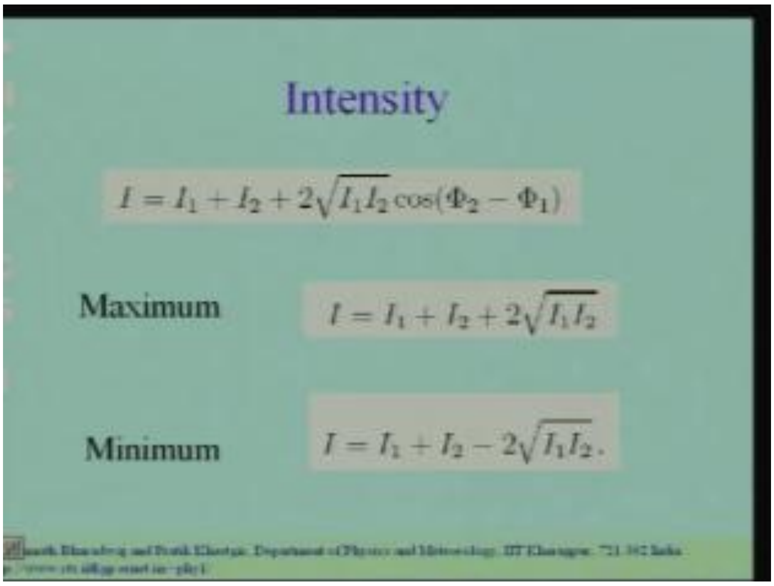
(Refer Slide Time: 33:40)



So, in terms of the intensities of the radiation we have to multiply this whole thing with a factor of half. So, half E1 square is the intensity of the first wave half E2 square is the intensity of the second wave E1, E1 is the. So, E1 square by 2 is I1. So, E1 square by 2 is I1. So, this is going to be 2 I1 the root of this. So, E1 is the root of root 2 I1.

So, using this I can write the total the intensity of the resultant wave, as the intensity of the first wave this plus the intensity of the second wave. if I express E1 as root 2 I1 and E2 as root 2 I2 then this becomes 2 root of I1 I2 cos phi 2 minus phi 1. So, what we see is, what we have just done is we have calculated the most general expression of the resultant when I superpose 2 waves and.

(Refer Slide Time: 35:36)



Let we now, look at the intensity pattern which we get when we superpose 2 waves let me just go through it again. So, when we have superposed 2 waves the resultant intensity I is the sum of the 2 intensities. So, the I1 is the intensity of the first wave I2 is the intensity of the second wave. So, this is all that I would get if there was no interference. Now, interference arises because of the wave nature. So, we have the extra term which is the cause of interference the extra term which we get is 2.

Now, I have the square root of the product of the 2 intensities into cos of the phase difference between the 2 waves phi 2 minus phi 1 is the phase difference between the 2 waves. What is the maximum value that the intensity can have. So, the maximum value is occurs when the phase difference between the 2 waves is 0 when the 2 waves are exactly in the same phase. Or if the phase difference is a multiple of 2 phi that is the phase difference is an even multiple of phi.

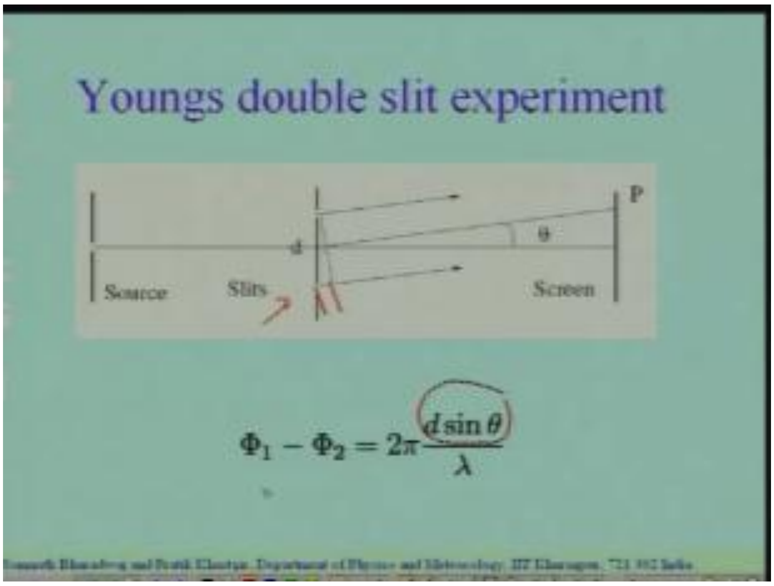
Then cosine of the phase difference becomes 1 and the this is the maximum value of the intensity that you can have it is I1 plus I2 plus 2 root I1 I2 and in the situation where the intensities of the 2 incident waves are the same you then have the resultant intensity being 4 times the intensity of the individual waves. Just remember that if I had no interference and I had superposed 2 waves I superposed 2 waves I put on 2 bulbs. Then I expect the intensity to go up twice.

Because of interference if the 2 waves are in phase instead of going up 2 times the intensity will actually go up 4 times if the 2 waves have the same amplitude. This is under the most favorable circumstances. Now, let us ask the question what is the minimum value that the intensity can have. The minimum value occurs when the 2 waves are exactly out of phase that is: the phase difference is an odd multiple of phi when the phase difference is an odd multiple of phi the cosine of the phase difference becomes minus 1.

So, the minimum intensity I is: I1 plus I2 minus 2 into the square root of I1 into I2. In the situation where the 2 waves, which are incident have the same amplitude where they have the same amplitude the minimum possible intensity is 0 these 2 terms will exactly cancel out. So, in the situation where the 2 waves have the same amplitude the maximum is 4 times the intensity of the individual wave the minimum is 0. And in situation where the 2 waves have different amplitudes you have to calculate the possible maximas and minimas and you can calculate the whole pattern over here.

So, this is a very general formula which we going to use in a large in a variety of situations, where we have 2 waves which are being superposed and the situation where we have 2 waves being superposed is quiet common. So, when we have the when we talk about interference we typically have 2 waves being superposed right. So, this expression that we have just derived is going to be applicable in all of these situations.

(Refer Slide Time: 39:12)



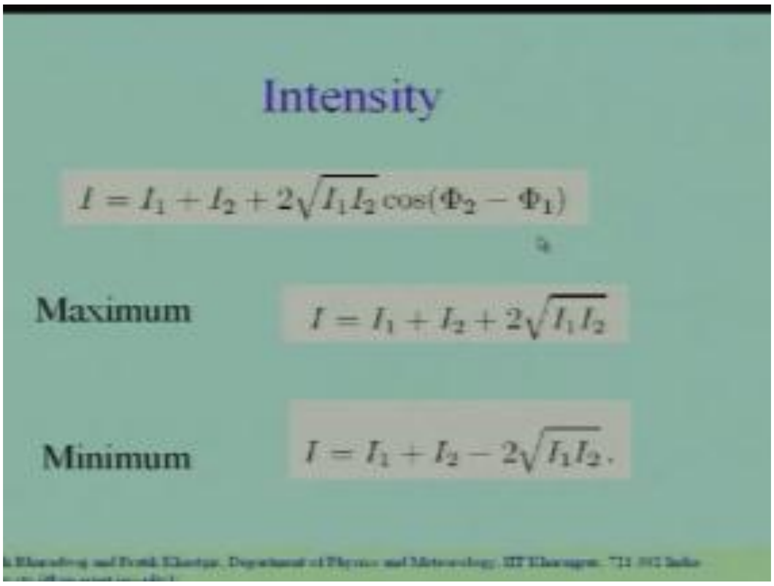
Now, let us go back to the Young's double, double silt experiment that we were discussing. In the Young's double slit experiment we have, we were looking at the intensity at point P. At the point P we have 2 waves which are incident 1 from the first slit the second from the second slit. The 2 waves from these 2 slits arrive at a with a phase difference. How much is the phase difference between these 2 waves from the 2 slits at the point P. Now, the 2 waves have a phase difference because they have to travel different paths and for the point P located over here. The wave from the, from this slit over here has to travel a larger distance and this is the extra distance which the wave has to travel.

So, let me mark this for you. So, this is the extra distance which the wave from the from the slit over here from this from this particular slit has to travel and this extra distance you can easily calculate is d sin theta. So, this d sin theta gives us the extra distance that the wave from this particular slit has to travel as compared to the wave from the slit this is the path difference. And if you want to convert this we want to convert this path difference to a phase difference.

So, to do that you have to multiplied by a factor of 2 pi by lambda a path difference of lambda corresponds to a phase difference of 2 pi. So, you divide it by lambda and multiple by 2 pi that will tell you the phase difference between the wave coming from here and the wave coming from here. So, the phase difference between these 2 waves is 2 pi d sin theta by lambda.

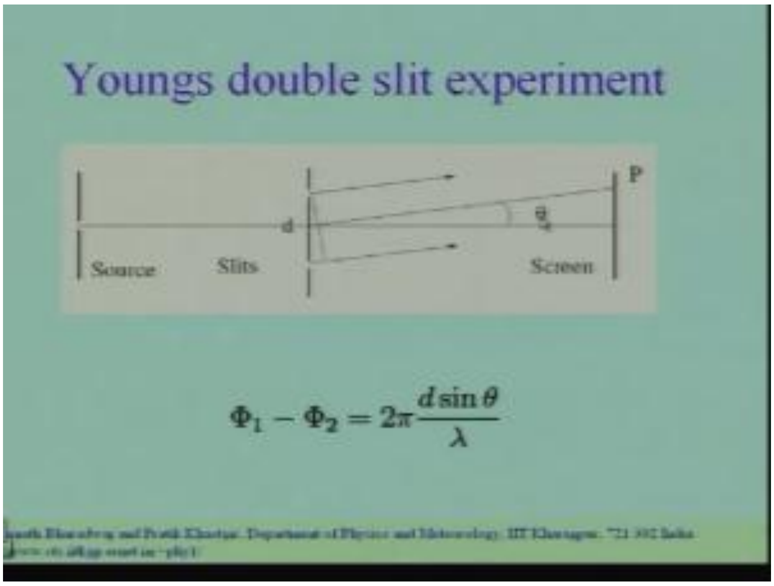
So, when you when we want to calculate the intensity pattern for the Young's double slit experiment. We have to substitute this phase difference into the expression for the intensity, which we have just calculated. So, when we wish to calculate.

(Refer Slide Time: 41:40)



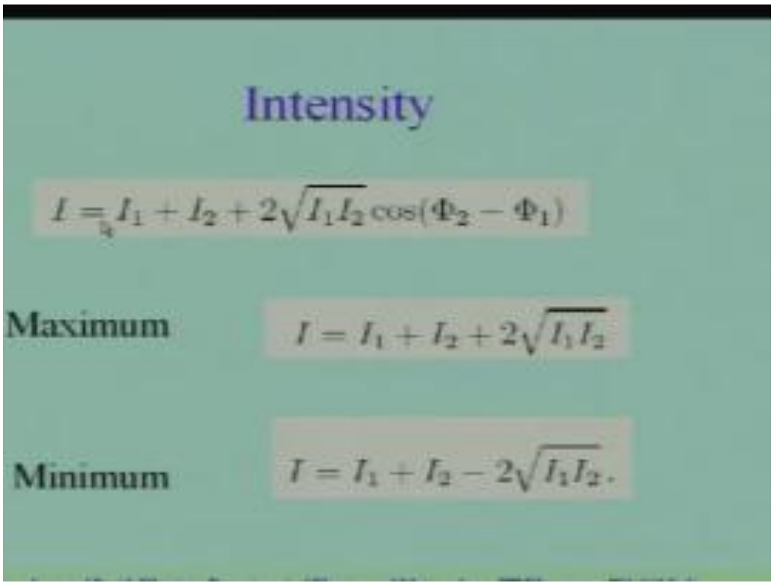
The phase the intensity pattern on the screen over there we have to plug in the phase difference which we just calculated.

(Refer Slide Time: 41:47)



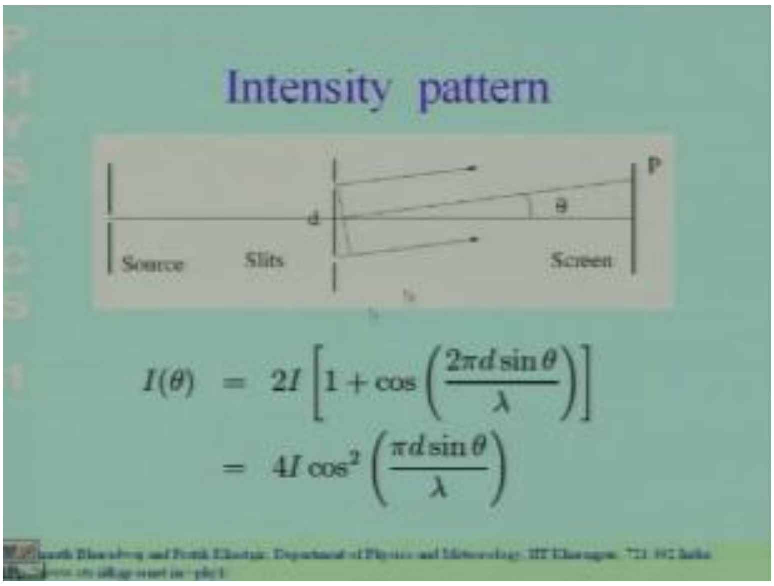
So, this is the phase difference which we just calculated we have to plug it in to the expression over here for the intensity.

(Refer Slide Time: 41:51)



And in the Young’s double silt experiment the 2 waves have the same amplitude. So, I1 is equal to I2 we will express this as I.

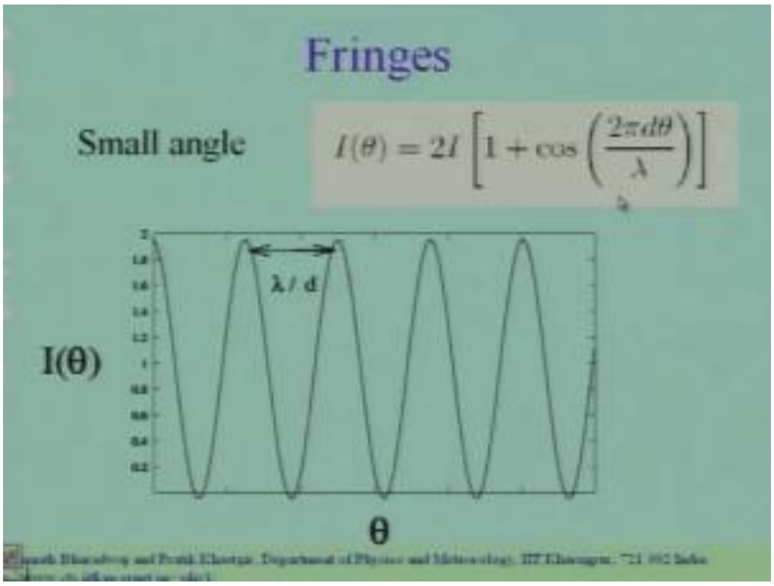
(Refer Slide Time: 42:06)



So, the intensity pattern on the screen over here, is given by the expression over here I the intensity over here is represented by I. This is a function of theta and the functional dependence on theta is shown over here. It is 2 I with this I over here is the intensity of the wave from 1 of these slits. So, the intensity on the screen over here, when both slits are open is 2 I1 plus cos 2 pi d sin theta by lambda.

This caused this term inside here is the phase difference between the 2 waves and you can simplify this and write it as the cos square of 2 pi d sin theta by lambda, with the factor of 4I outside. Now quite often it is the small angle. So, it is the angles near this line near the line to the centre of the 2 slits which are of interest. So, small angles which are of interest. And why small angles are particularly of interest shall become clear when we have studied diffraction.

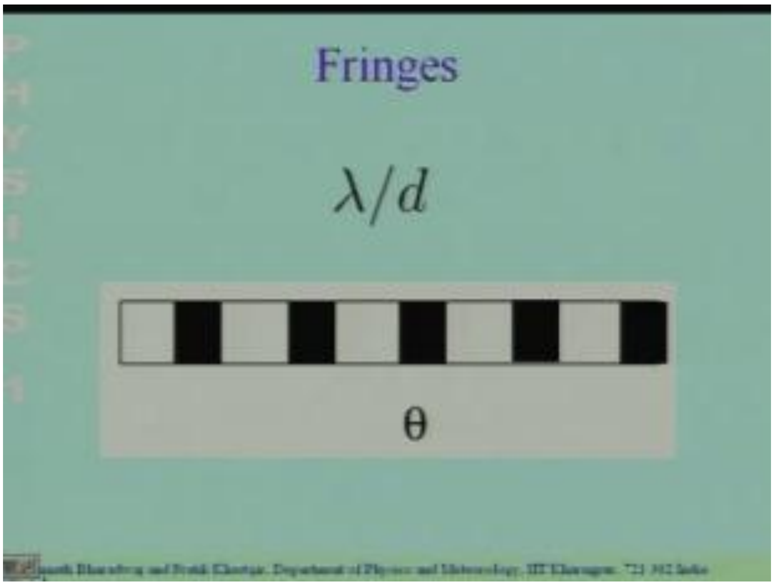
(Refer Slide Time: 43:20)



So, restricting ourself to small angles when theta is small you can replace sin theta by theta and the fringe pattern the pattern the intensity pattern is now this, given by the expression over here, you have all that you have done is you have replaced sin theta by theta. So, what is the what does it look like: so, the intensity pattern as the function of theta looks like this: you have these peaks in the intensity and you have places where the intensity become 0 the peaks in the intensity are periodic.

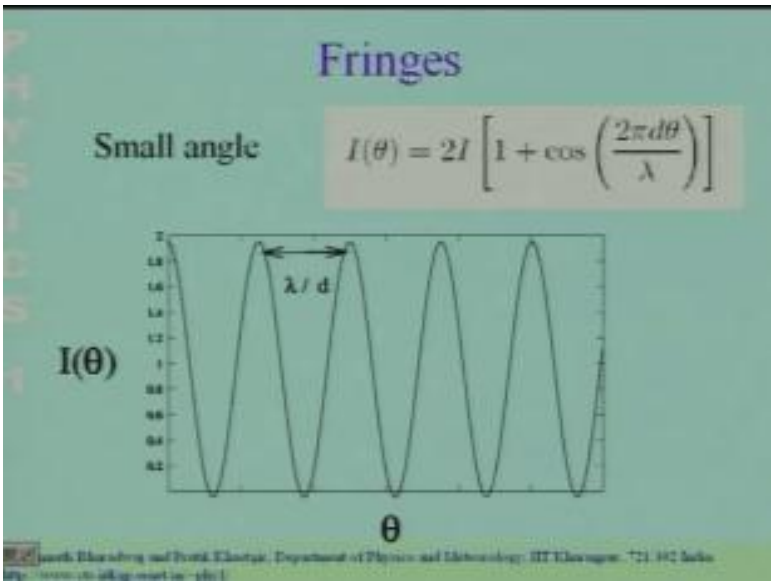
Let us ask the question what is the period of the peak in the intensity. So, the period of the peak in the intensity the value of theta after which the whole pattern repeats is lambda by d. So, if you look at the intensity as the function of theta the whole intensity pattern repeats with after a after an angle lambda by d radians.

(Refer Slide Time: 44:21)



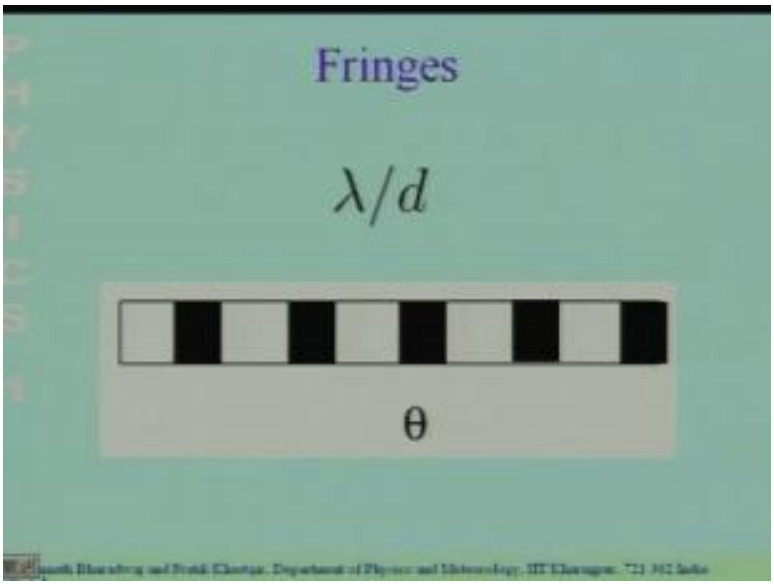
And this is what it will look like. So, you will have these bands bright bands.

(Refer Slide Time: 44:29)

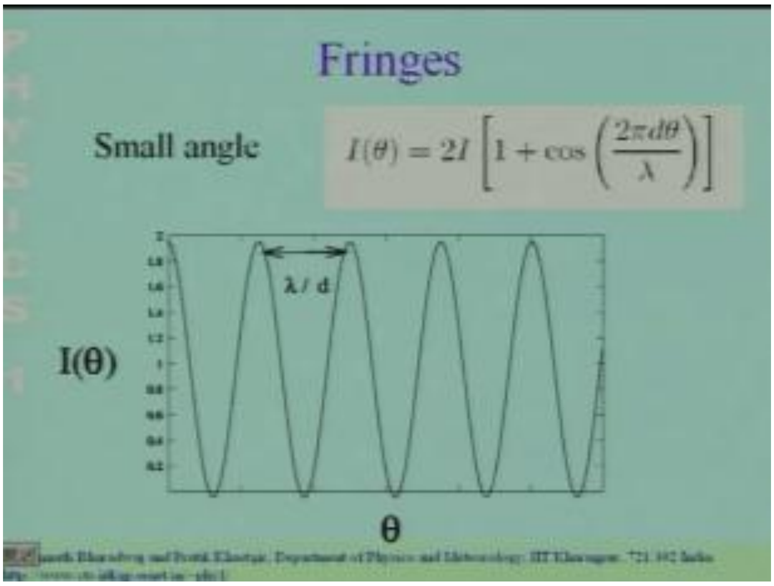


The bright bands correspond to the maxima of the intensity and then you will have these.

(Refer Slide Time: 44:33)

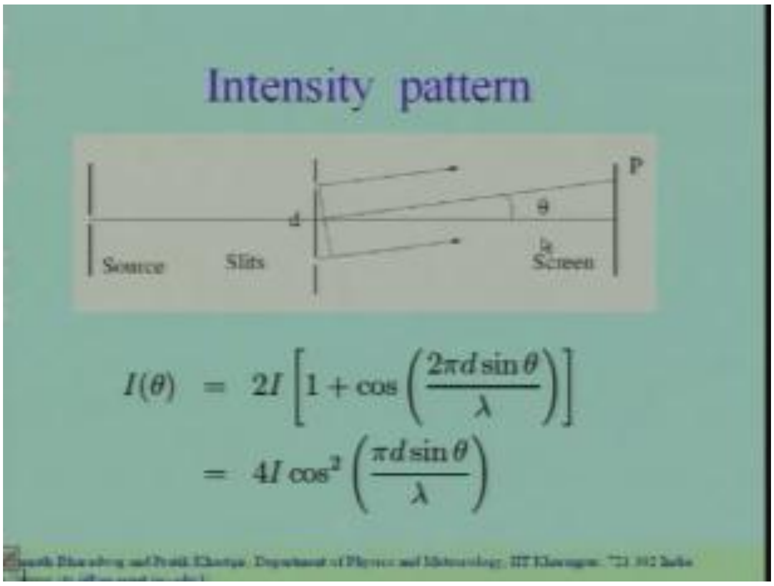


(Refer Slide Time: 44:35)



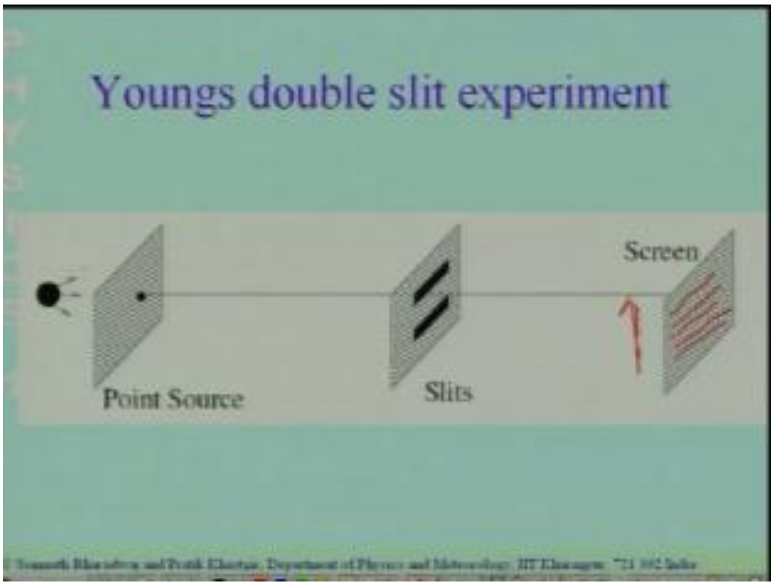
Dark bands, the dark bands correspond to the minima of the intensity where the intensity is 0 and the whole thing is aligned in the same direction as the slits.

(Refer Slide Time: 44:39)



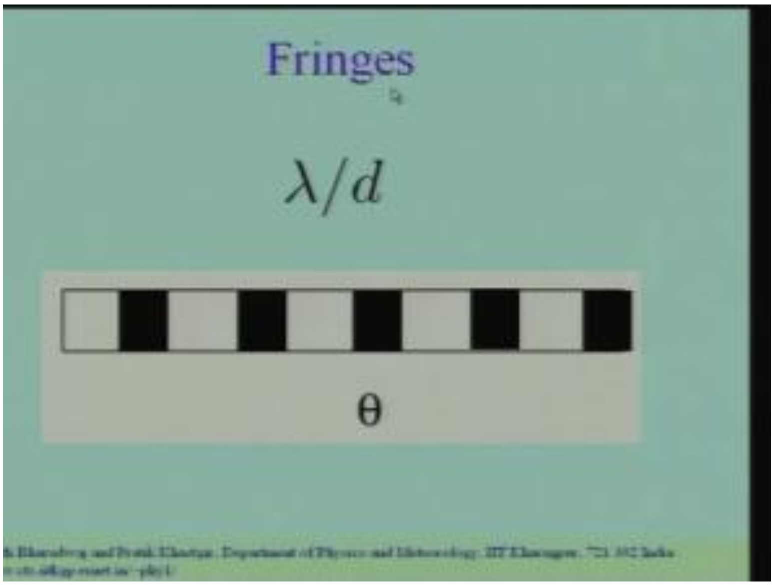
So, the 2 slits are here. So, the fringe pattern is going to be in this direction. So, the intensity is going to vary in this direction if you look.

(Refer Slide Time: 44:56)



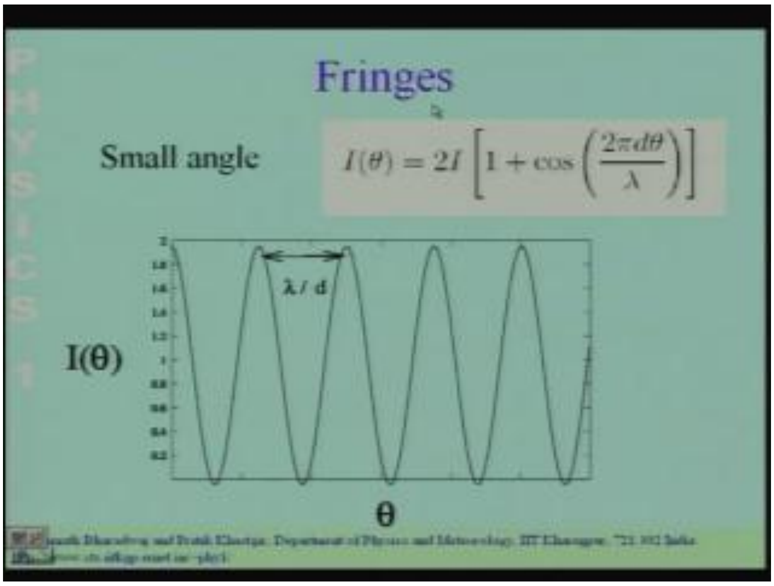
So, in this picture the intensity is going to vary upwards. So, in this picture the intensity is going to vary in this direction and the intensity is going to be constant in this direction over here. So, you will what you will get is you will get lines over here which look like this the interference pattern is going to be lines which look like this.

(Refer Slide Time: 45:35)

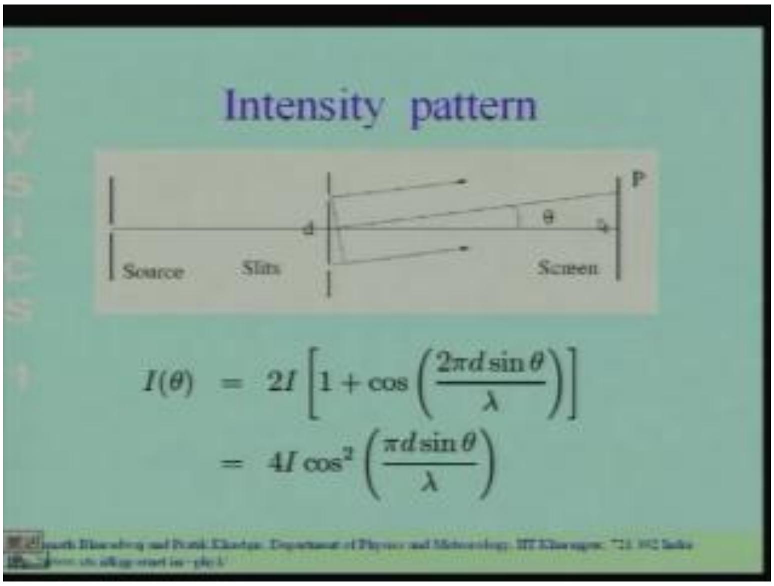


And it is these lines which are shown over here and these lines will have a spacing lambda by d and these alternating patterns of dark and bright lines is: what is referred to as fringes. So, you will observe fringes on the screen over here.

(Refer Slide Time: 45:53)

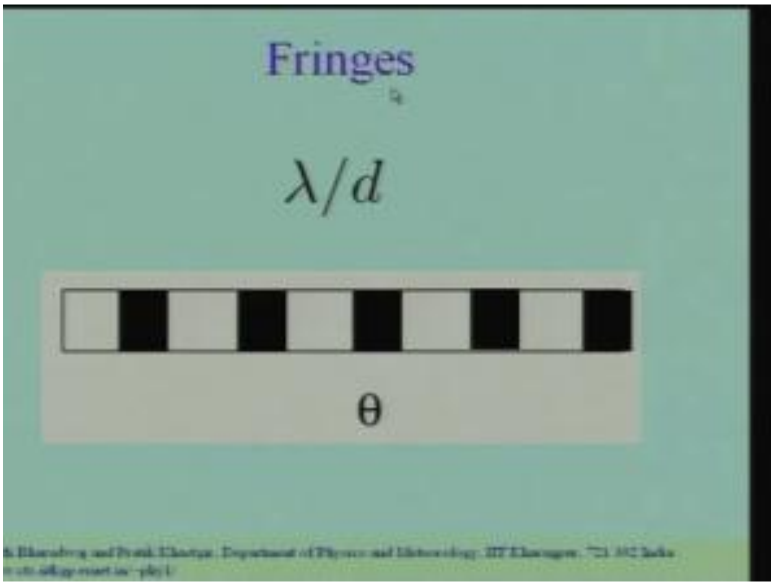


(Refer Slide Time: 45:56)



So, on the screen over here, you will get a fringe pattern the which refers to a pattern of alternative bright and dark lines.

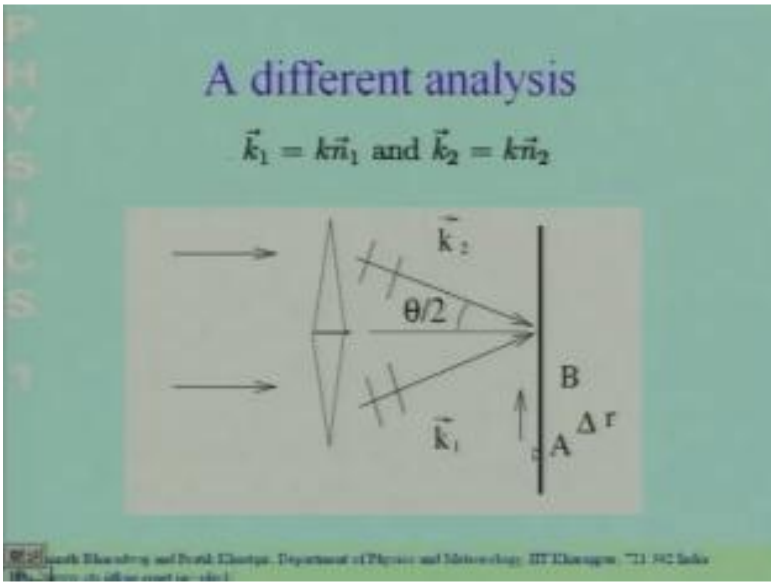
(Refer Slide Time: 46:02)



To, the spacing of lambda by d. So, we have just finished discussing the Young's double slit experiment the Young's double slit apparatus. And we found that it will produce a set up fringes the fringes are parallel to the direction of the slits. Now, let me repeat the same calculation. So, let me repeat the same calculation in a different way.

So, we will do a different analysis of the same calculation and the different analysis that we will do is quite interesting and it gives us insight and it is also useful when dealing with certain problems so, the situation. So, it is remember we are dealing with the same Young's double slit experiment, but, the situation now is we discuss the situation in terms of different things. So, the situation here is that we have a screen.

(Refer Slide Time: 47:07)

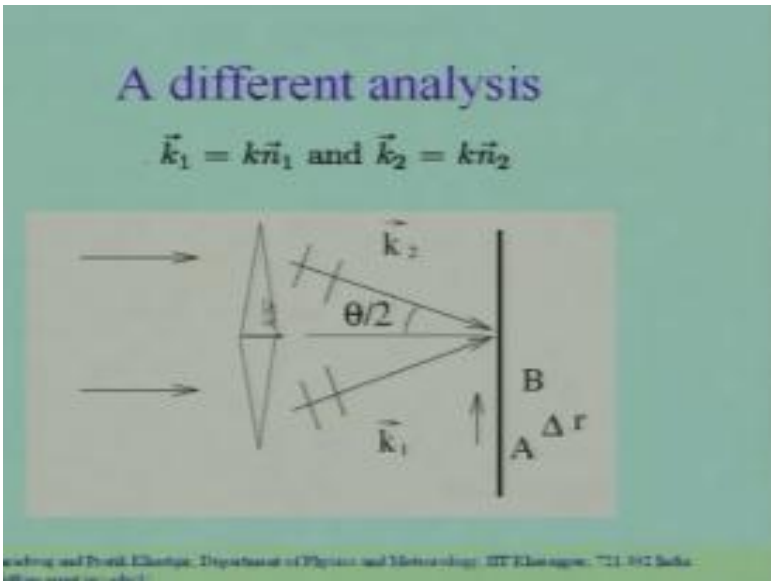


So, this picture shows you the screen and on this screen we have 2 waves which are incident on the screen both the waves originate from the same source, but, they are incident on the screen in different directions. So, we have 1 wave with wave vector k1. So, there is a wave with wave vector k1, k1 the wave vector k1 we can write as the wave number k into the direction n1. So, n1 is a unit vector in the direction in which the wave is going which is shown over here.

We have another wave k2, which is also incident on the same screen, k2 is in a different direction from k1. So, the wave number corresponding to this wave is the same both the waves originate from the same source. So, they have the same wavelength and the same wave number. So, the wave numbers are the same, but, that directions are different. So, the unit vectors n1 and n2 which tells us in which direction these 2 waves are going are different.

So, in this case 1 wave is coming like this another waves is coming like this you can realize this in practice. So, you can realize this situation in practice by using 1 possible way of realizing it in practice there are many ways is by using something called a biprism. So, in a biprism you have 2 very thin prisms as shown over here.

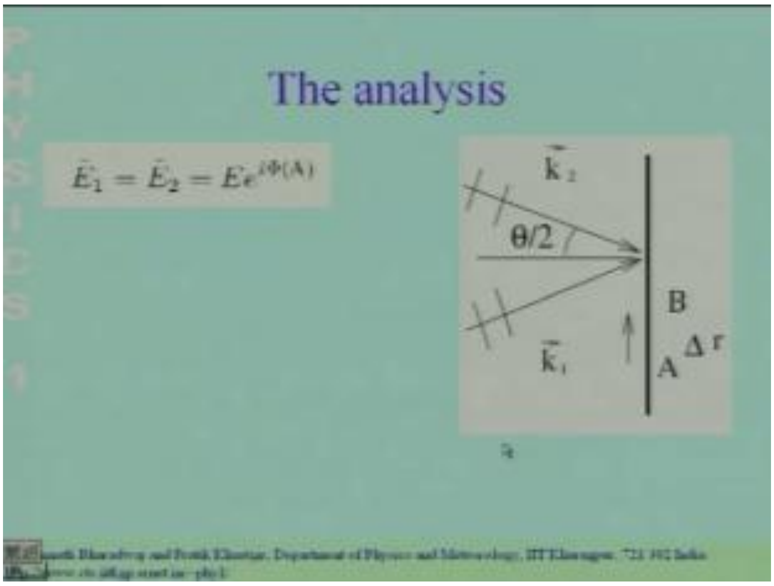
(Refer Slide Time: 48:47)



So, these 2 thin prisms which are fixed like this. This whole thing is illuminated by a plane wave and you can generate a plane wave by putting a point source very far away. So, you have a plane wave incident on these 2 prisms. The upper prism bends the light downwards. So, the light which comes out is a wave traveling in this direction the light which comes out of here is a wave traveling in this direction. Both of these waves originated from the same source.

So, there will be coherent they will have the same wavelength and you can do interference with them. So, we would like to analyze what happens on the screen over here. Now, let us look at the screen and identify a point where both the waves come with the same phase. So, the point A is the point where both the waves have exactly the same phase.

(Refer Slide Time: 50:05)

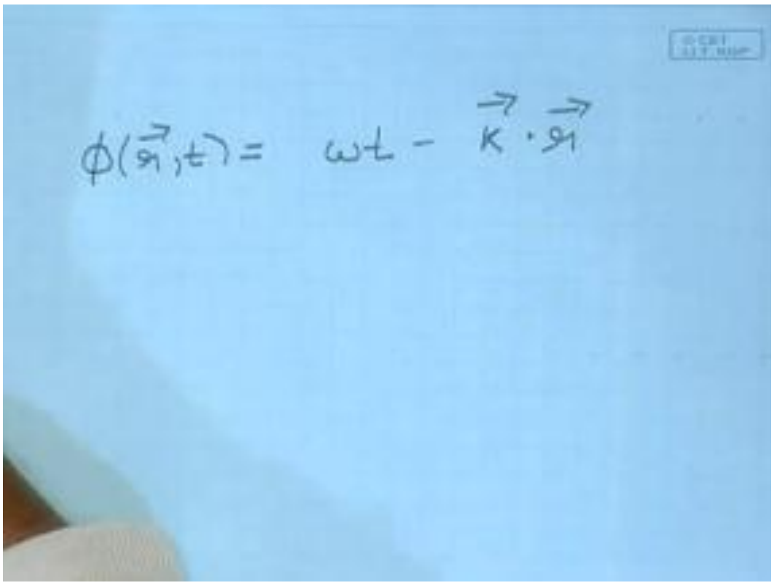


So, this shows you the same picture and we have identified a point on the screen, which we referred to as A where both the waves k1 with wave vector k1 and k2 both of them have the same phase. So, the electric field of the wave of this of the first wave E1 and the electric field of the second wave E2 will be exactly identical and they can both be written as E e to the power i phi A, where phi A is the phase of both the waves at the point A. I have already told you that we have chosen A so, that the phasors are the same.

So, they have the same value and we can write it like this. Now, if the 2 waves arrive at the same phase of the point A their oscillations are going to be exactly identical. And when I superpose these 2 waves they are going to add up and the point A is a point where I will have a maxima in the intensity. So, the intensity is going to be maximum at the point A. Now let us consider, the situation where we move away from this point where the intensity is maximum we move away a distance a displacement delta r.

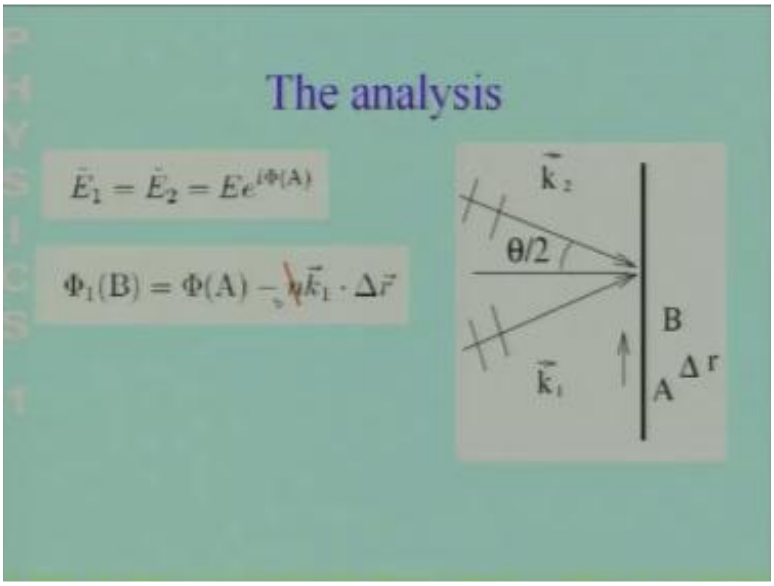
So, we move to another point B which is at a which is displaced by delta r from the point A. The question is by how much does the phase of the first wave change when I move from A to B.

(Refer Slide Time: 52:15)



Now, we have studied in the lecture, on plane waves that the phase of a wave is omega t minus k dot r where k is the wave vector corresponding to the wave. And the question which we are asking in this particular situation is that at the same time.

(Refer Slide Time: 52:27)

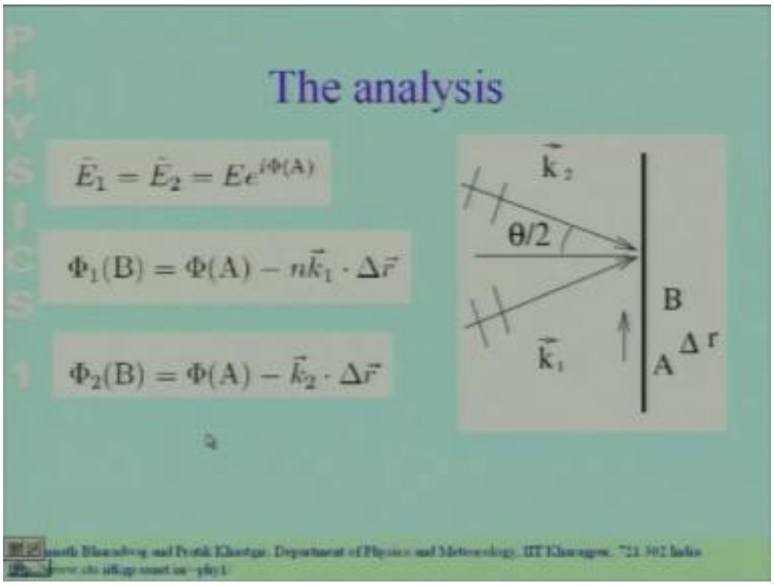


How much does the phase of a wave change when I go from the between the points A and B? So, applying the definition of the phase we see that the phase changes by an amount minus k1 dot delta r, please ignore the n over here, it is typographical error. So, it should not be there.

So, when I move from the point A to the point B the phase changes by an amount minus k1 dot delta r where delta r is the displacement between the points A and B. So, we can say that the phase at point B of the first wave phi 1 B is equal to phi A minus k1 dot delta r. Similarly, you could ask the question. How much does the wave? Does the phase change for the second wave, if I move from the point A to the point B.

So, it will be the exactly the same as this the only difference will arise because the wave vector of the second wave is not k1, it is k2 these 2 waves being in different directions. So, for the second wave.

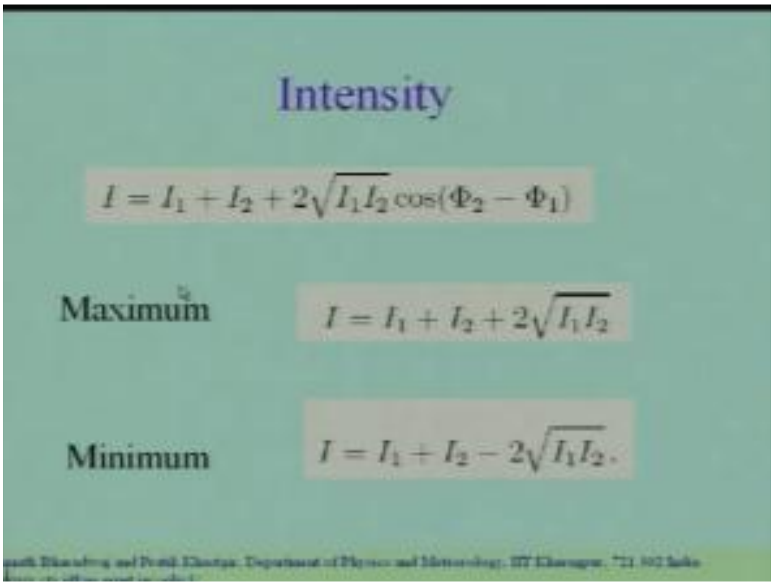
(Refer Slide Time: 54:02)



For the same point B the phase is going to change by a different amount and the phase at the point B for the second wave is going to be phi A minus k2 dot r. So, the point to note here is that; at the point A both waves oscillate with the same phase. So, we going to have a maxima in the intensity. Now, we move a displacement delta r away to the point B at the point B the phase of the first wave has changed by some amount. The phase of the second wave has changed by a different amount because the wave vectors are different and the 2 waves are no longer going to oscillate at the same phase.

So, when I move away from this point, which is the maxima the 2 waves arrive at a different phase and if the 2 waves arrive at a different phase you can see that the intensity is going to fall.

(Refer Slide Time: 55:02)



So, at the point A the 2 phases were different. Now, when I have moved to a point B the 2 waves arrives arrive with a difference in phase. So, the intensity the cos the cosine is going to be less than 1 and the intensity is going to fall. So, you are going to have an intensity pattern on the screen. In the next lecture, we will continue our discussion of what that intensity pattern is going to look like. So, let me stop here for today and continue from here in the next lecture.