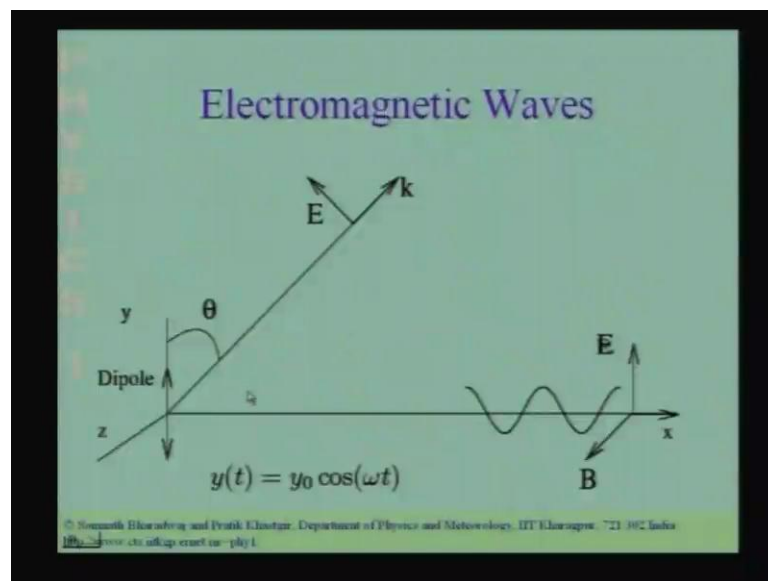


Physics - I: Oscillations and Waves
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Indian Institute of Technology, Kharagpur

Lecture - 11
The Vector Nature of Electromagnetic Waves

Good morning, in the last class we had started discussing a situation where, we have an electric dipole which is essentially a rod or a wire.

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And we have fed the situation we considered in the last class we had a dipole like this. And we had connected an oscillating voltage source to this dipole. So, there were charge particles oscillating up and down this metal wire metal rod like this. The oscillation of the charge we represented as $y(t)$ where, y is displacement of the charge along the y axis is equal to $y_0 \cos \omega t$.

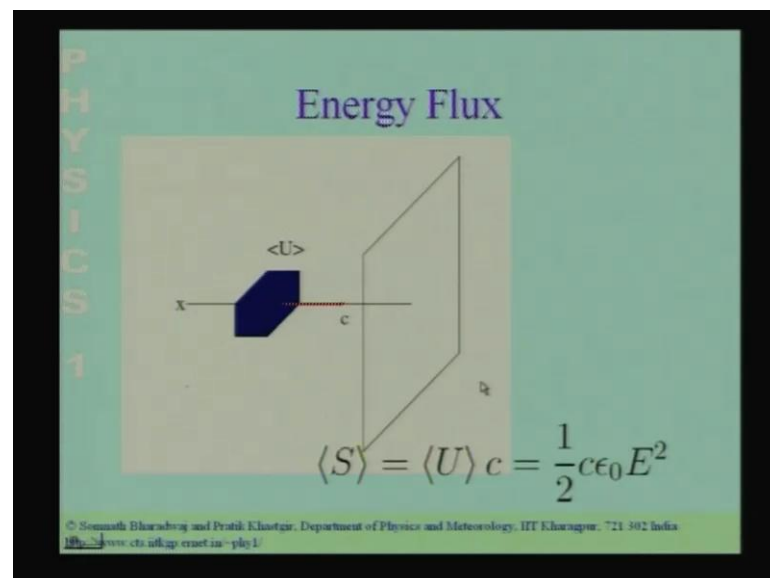
So, the charges move up and down with an angular frequency ω . And so, we consider this situation and for this situation we calculated the electric field at a large distance away from the dipole. And we saw that at a large distance the electric field along the x axis at large distance the electric field also oscillates parallel to the y axis. So, it is in the same direction as the direction in which the electron oscillates up and down which is along the y axis.

So, at a large distance x the electric field oscillates everywhere x along everywhere along the x axis the electric field oscillates up and down parallel to the dipole. And at a large distance we can treat this oscillating electric field as a sinusoidal plane wave; it behaves like a sinusoidal plane wave. So, at a large distance if you look at the electric field at the fixed instant of the time it will have this kind of a sinusoidal pattern. And with time this whole pattern moves forward which is precisely what we called a sinusoidal plane wave.

Then we also have the magnetic field which is perpendicular to the electric field and it oscillates in exactly the same phase as the electric field. In any arbitrary direction over here we have exactly the same thing we have a sinusoidal plane wave at a large distance from the dipole. The electric field is now along the so, the electric field over here is to be calculated by taking the component the projection of the dipole. Normal to the line of sight which is what I have shown here and the sinusoidal plane wave the wave propagates along this direction; the radial direction.

And we also have a magnetic field which oscillates in the same phase as the electric field, but which is perpendicular to both direction of the wave and the electric field vector.

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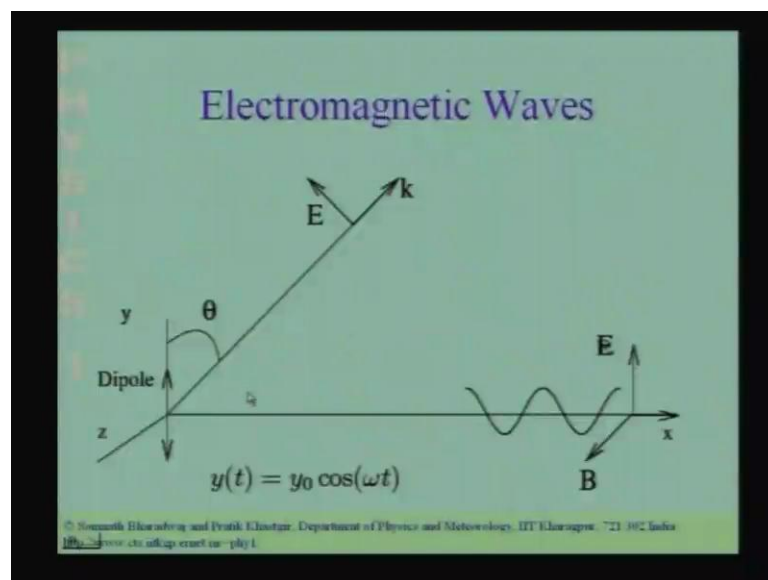


Now, in the last class having discussed the field the electric field pattern, in the last class we calculated the energy density and then we were calculating the energy flux. I told you that the average energy flux is the average energy density the time by average we mean

the time average. So, if you look at the energy density average it over a time period which is much larger than the time period of the oscillations of the dipole. The oscillation of electric field the energy flux is the average of the energy density into the speed of the light.

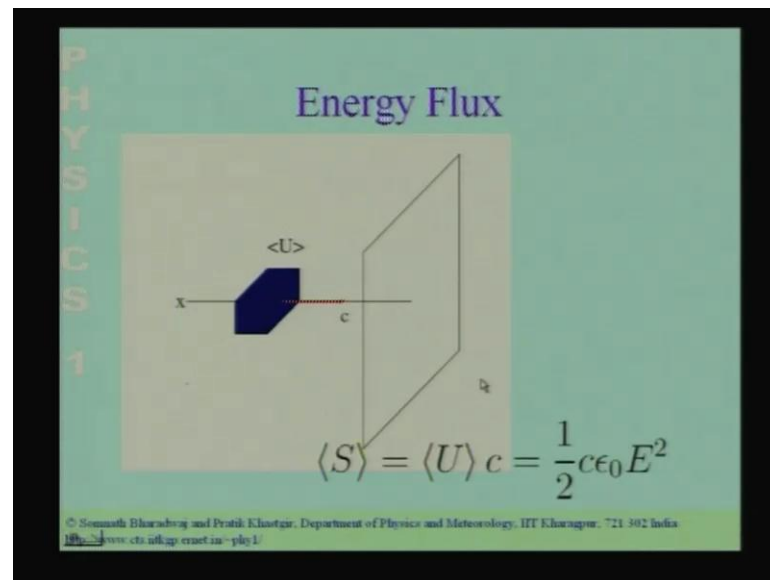
So, the whole wave propagates in a direction at the speed of light. So, the energy density also moves forward at that direction if I put a surface normal to the direction in which the wave is propagating. The energy density the energy per unit volume into c is the amount of the energy that will cross unit area of this surface in a unit time. And we had calculated this; this comes out to be half epsilon naught $c E^2$ where, E is the electric field at this point.

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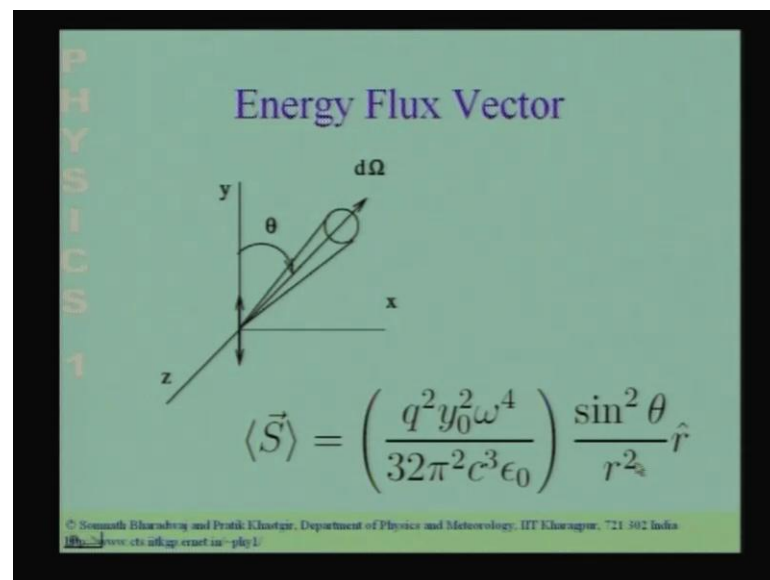
So, let me remind you from over here the energy flux is a vector along the direction of the wave which is in this direction. if I am over here, the energy flux will be a vector in the direction of the wave which is in this direction. And if I put, a surface area surface normal to this surface of unit area normal to this.

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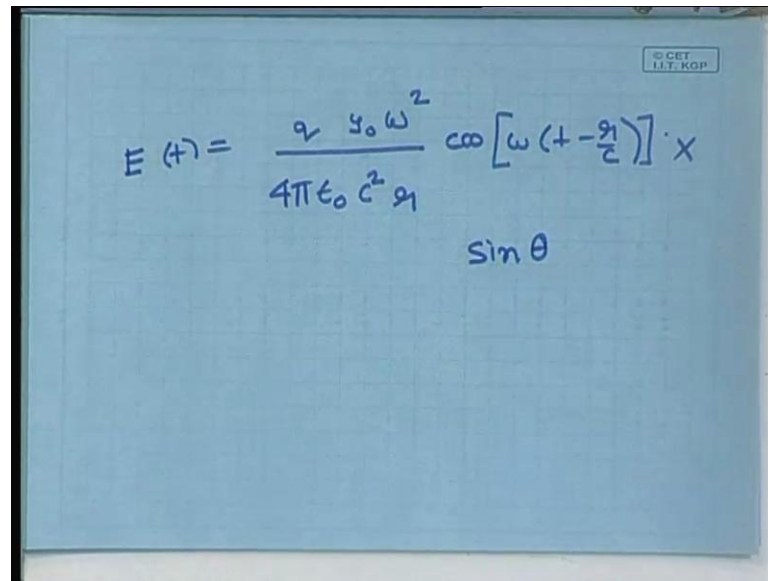
Then the energy that will cross that surface in a unit time is half c epsilon naught E square.

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So, putting in the expression for E which we had calculated we get the flux the energy flux vector. And this is given by this expression. So, we have put in the expression for the electric field at a any point over here. And we have also done the time average.

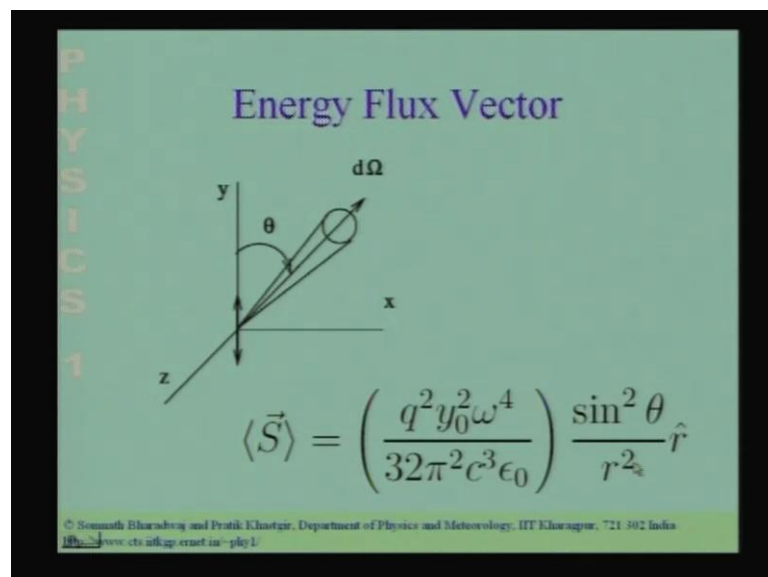
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$$E(t) = \frac{q y_0 \omega^2}{4\pi\epsilon_0 c^2 r} \cos\left[\omega\left(t - \frac{r}{c}\right)\right] \sin\theta$$

So, to just let me just remind you that the electric field at any point was q into y naught omega square where, y naught was the amplitude of the oscillations of the charge particle. This divided by four pi epsilon naught c square into the distance into cos omega t minus r by c into this whole thing into sin theta. So, we have to square this and take the time average and which gives us an extra factor of half.

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Energy Flux Vector

$$\langle \vec{S} \rangle = \left(\frac{q^2 y_0^2 \omega^4}{32\pi^2 c^3 \epsilon_0} \right) \frac{\sin^2 \theta}{r^2} \hat{r}$$

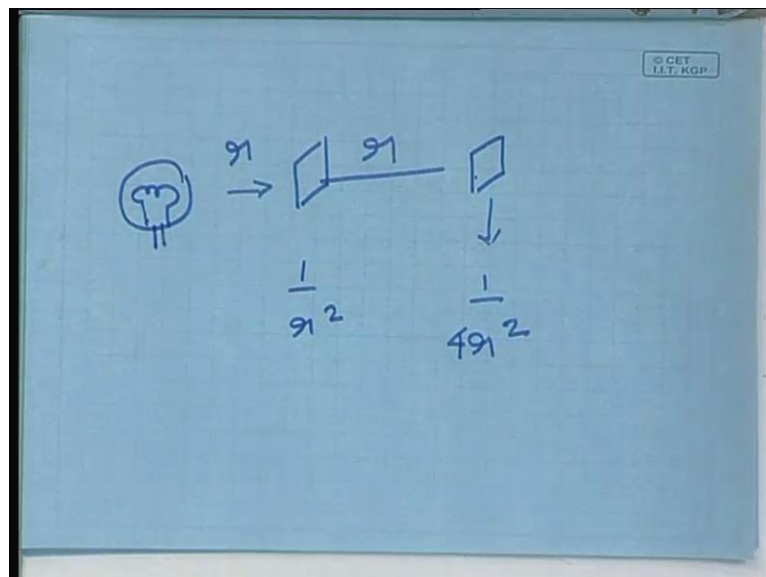
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So, when I square this I get all these factor over here q square y naught square omega 2 the power of 4 by 32 pi square epsilon naught square into c square epsilon naught c

square into sin square theta by r square into the unit vector \hat{r} . So, this is the expression for the energy flux. The point to note is that the energy flux vector points in the radial direction. So, if I am over here it will be pointing radially outwards along. So, the radial direction is defined by the position of the dipole and my positions.

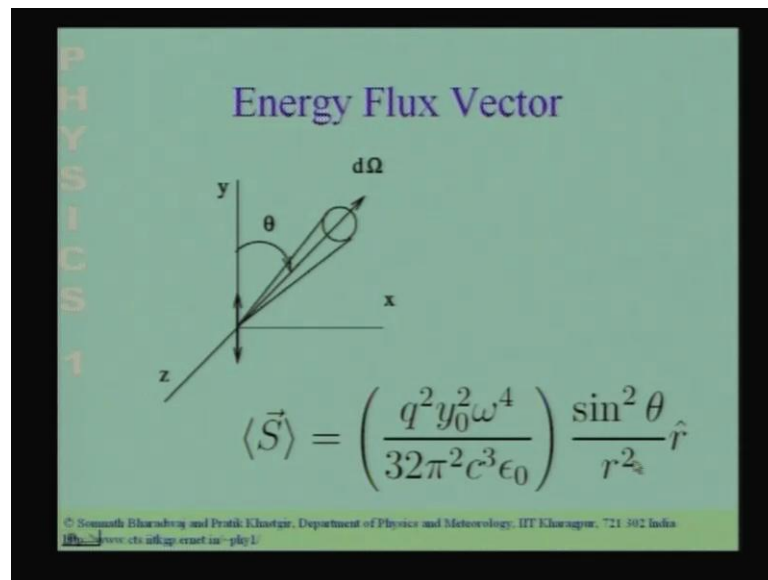
If I am looking here I have to draw the radial line from the position of the dipole to this point here. And the energy flux vector points in this direction and this gives you the magnitude the magnitude of the energy flux vector falls as $1/r^2$. So, the flux falls as $1/r^2$ this is the feature which we see quite commonly.

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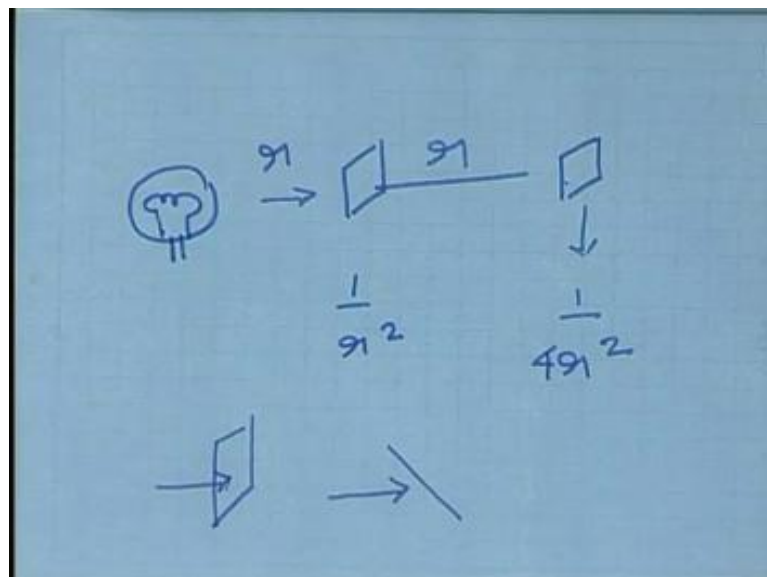
So, if I have any source of radiation could be a bulb or anything of that sort. The flux or the radiation flux falls is known to fall as $1/r^2$ which is what we see over here. So, if I put a unit area at a distance r and if I put a unit area at a distance $2r$, then the flux over here is going to be $1/4$ of $1/r^2$. The energy crossing this area per unit time falls as the inverse square of the distance which is what we have just which we can see here in the expression that we have just derived.

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And the energy flux also depends on the angle. So, if the dipole is oscillating like this the energy flux is proportional to sin square theta. And it is maximum in this direction and as I move up theta will decrease theta is 90 here as I move up theta decreases. The amount of the energy flux goes down and if I am located just in the same direction along the dipole I will get no energy coming over here.

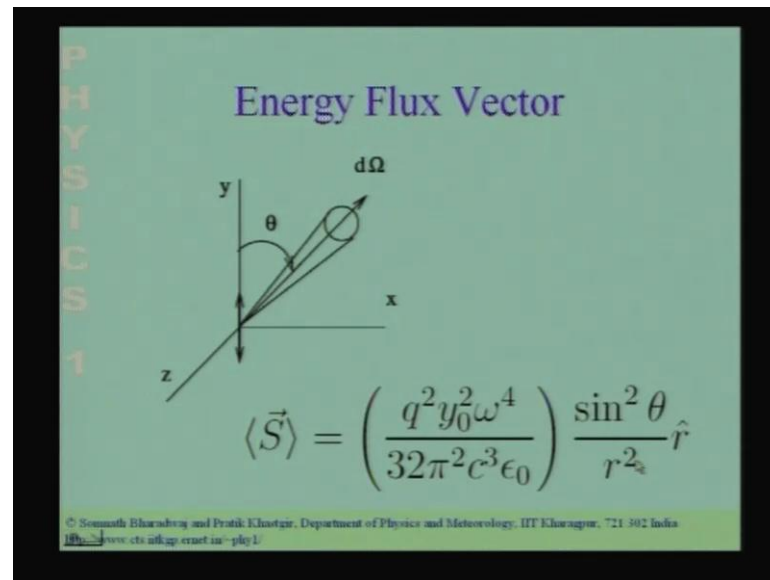
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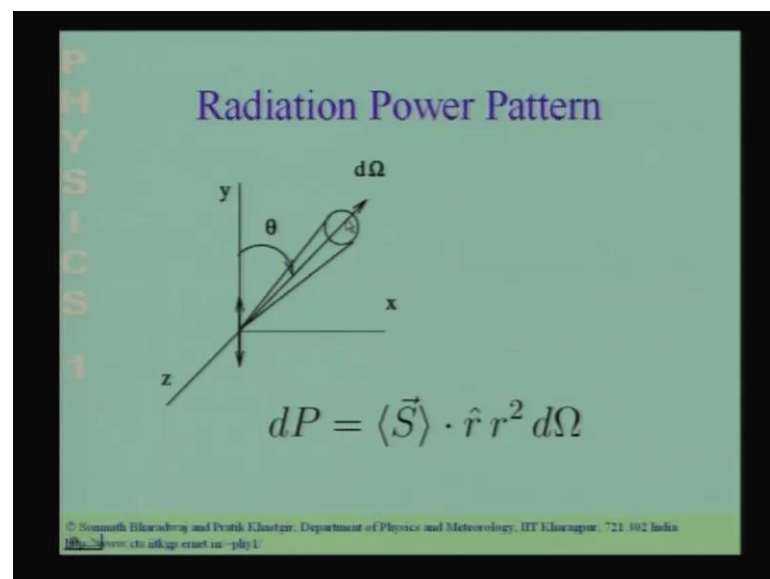
The other feature which you should note in this expression is as follows: if this is the energy flux vector. And if I put a surface unit if I put a surface area like this, then the

amount of energy that crosses this is given by the S over there. If I put the surface certain angle then you have to put in an extra factor of $\cos \theta$ which is another point which you have to keep in mind when, interpreting when applying this expression.

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Now let us move ahead. So, we have calculated the energy flux the energy flux tells us that if I am located at a distance r away from the dipole. So, my dipole is over here and I am located over here it tells with the amount of energy that crosses across a surface of

unit area per unit time over here. Now, let us now go on to a slightly different question the question which we shall take up next is I have this or same oscillating dipole.

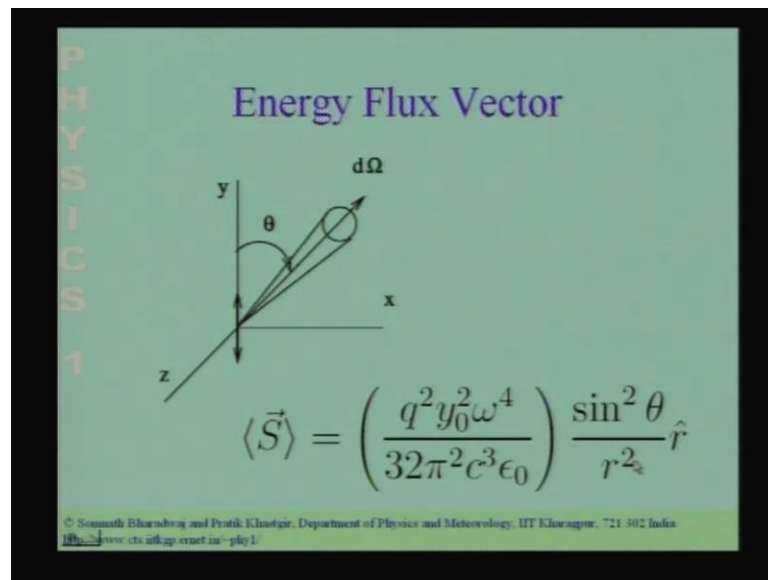
What is the radiation pattern of the dipole? So, when we talk about the radiation pattern we change our point of reference, we shift the point of reference to the origin of the dipole. So, let us go to this picture. We have this dipole oscillating up and down. And we have changed our reference point to the origin of the dipole and we ask the question. How much energy does the dipole send out per solid angle?

So, in a solid angle $d\Omega$ what is the radiation sent out by the dipole when the solid angle is in a particular direction. So, in a given direction in a fixed direction I put a solid angle $d\Omega$ and ask the question how much energy does the dipole send out in this direction; in this solid angle. So, we can calculate this as follows corresponding to this solid angle over here there will be an area.

So, the area corresponding to this solid angle the solid angle is $d\Omega$ the area corresponding to this solid angle the area subtended by this solid angle with reference to the dipole is $r^2 d\Omega$ where, the r^2 is the distance to this point. So, this is the area corresponding to this solid angle. And if I write a vector normal to; so, this area I can represent as a vector where, the direction of the vector will be along the normal to this surface.

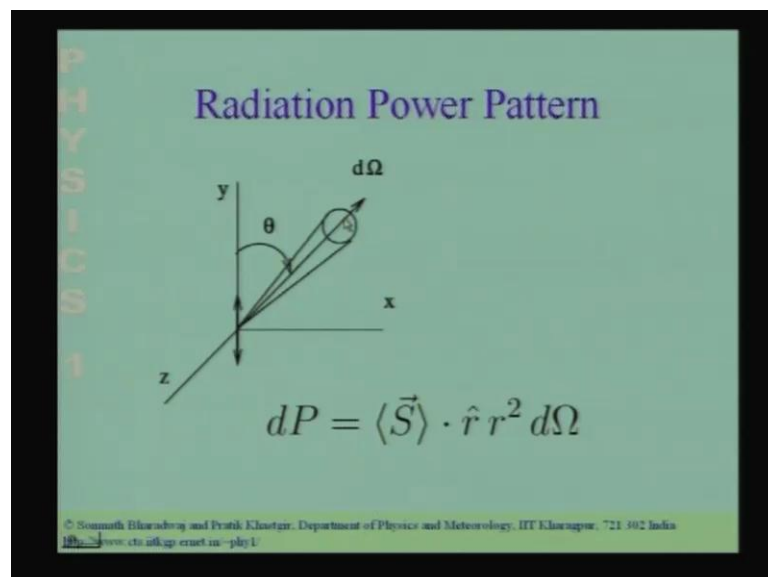
So, the normal to this surface of the radial vector \hat{r} . So, the area corresponding to this solid angle is the normal is the vector \hat{r} into $r^2 d\Omega$. And the power that goes into this solid angle is the energy flux into the area corresponding to this solid angle which is what we are over here.

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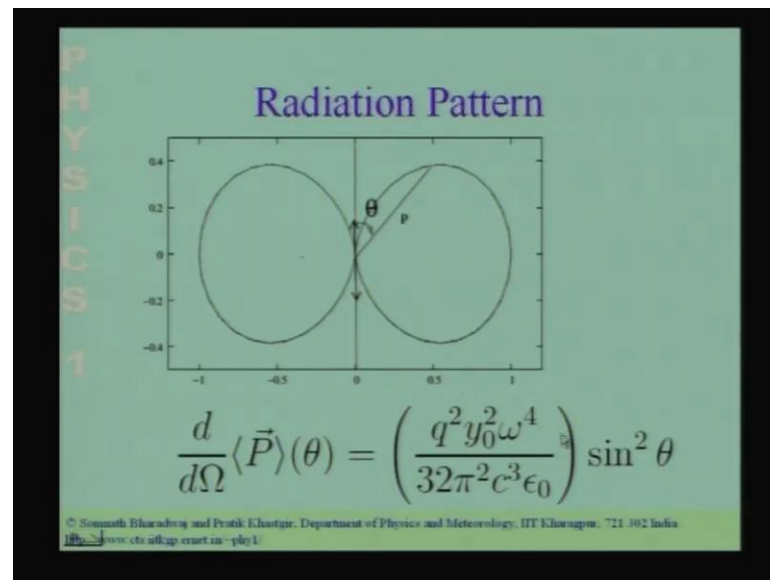
Now, putting in the expression for the energy flux vector that we just calculated; putting in this.

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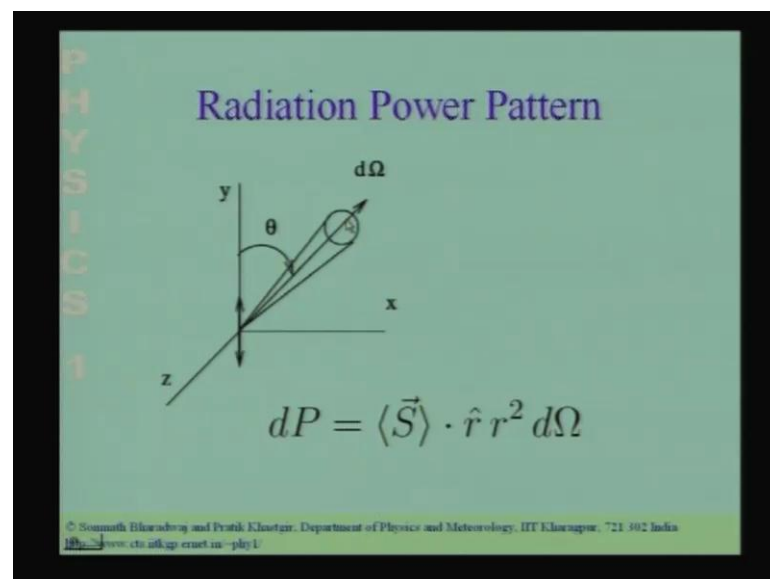
Putting in the expression for the area which, we have here the area corresponding to the solid angle

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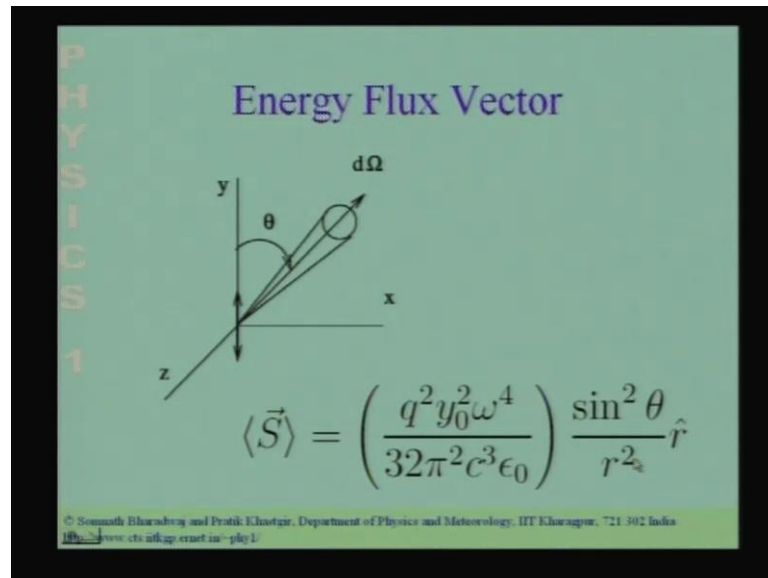
We can calculate the radiation pattern. So, the radiation pattern is the amount of the power that is that is emitted in the solid angle $d\Omega$ where, the solid angle is located at an angle θ this.

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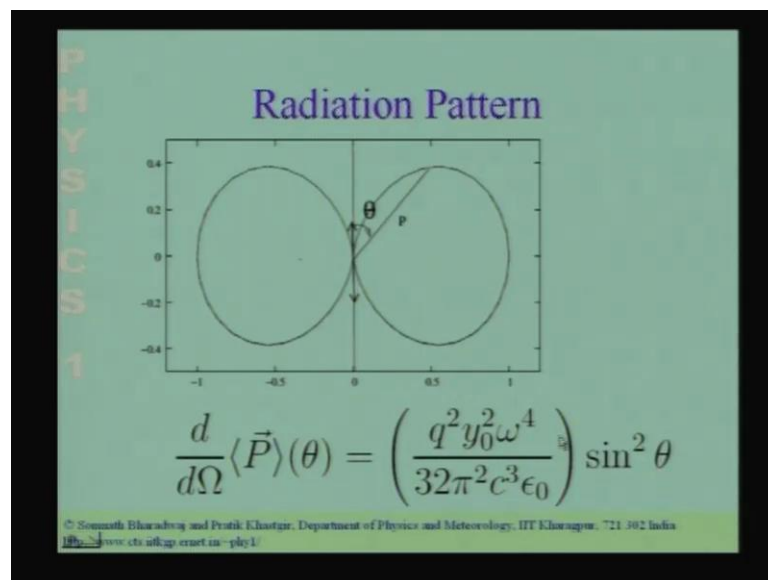
So, notice that the solid angle the area corresponding to the solid angle is proportional to r square.

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The flux the flux vector S is also proportional to $1/r^2$. So, these factors of r^2 cancel out.

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And we get the power per solid angle the power per unit solid angle to be an expression which is independent of r and this is what is called the radiation pattern of a dipole. So, the radiation the point the important point over here is the radiation pattern of the dipole the amount of energy the amount of power the dipole sends into a solid angle $d\Omega$ per solid angle $d\Omega$. Depends only of the direction at which we place the solid angle

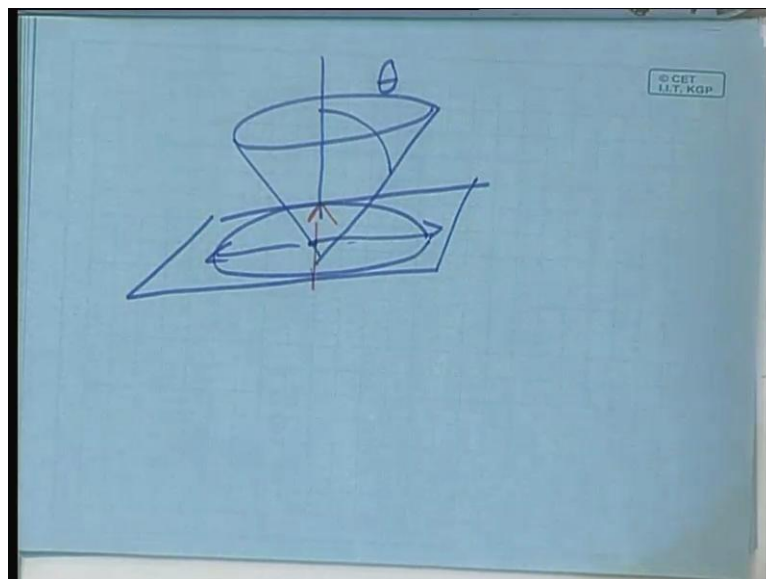
it does not depend on how far away the solid angle is located. This is an important feature of the radiation pattern.

So, the radiated power per solid angle depends only on the direction and it with respect to the dipole and it goes as $\sin^2 \theta$. So, this picture over here shows us the radiation pattern the length of the distance from the origin. So, the dipole is located here and the length of this line from the dipole to this point on the curve. So, for a particular θ I will have a different length for θ equal to 90 degrees the length is going to be maximum for θ equal to 0 or θ equal to π the length is going to be minimum.

This length tells me the magnitude of the power that is emitted in this direction power per solid angle that is emitted as a function of θ . So, the maximum power per solid angle is emitted towards θ is $\pi/2$ perpendicular to the direction in which the dipole oscillates. And as you move in this direction or in this direction the power per solid angle that is emitted false as $\sin^2 \theta$.

So, there is no power emitted in the direction in which the dipole actually oscillates. The maximum power is emitted in the direction perpendicular to the direction in which the dipole oscillates.

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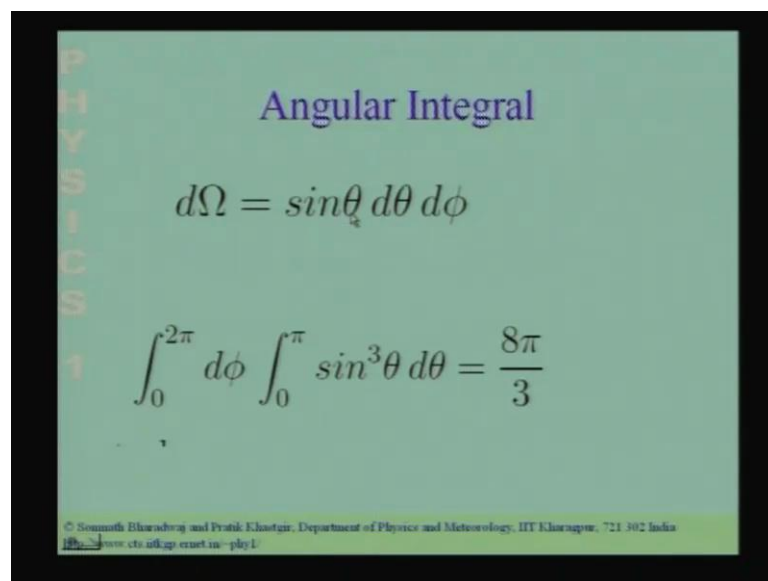


You should also remember that, the whole pattern is symmetric around the dipole. So, if the dipole oscillates up and down like this, the whole pattern is symmetric around the

dipole. So, it is if this is the plane perpendicular to the dipole the dipole is over here this is the plane. So, the whole radiation pattern is symmetric around this. So, it is maximum over here in all directions in this plane it is maximum and then if I consider a circle over here, which makes an angle. The amount of energy that is sent out per solid angle in this direction will fall by a factor $\sin^2 \theta$. We could now, move a little further and calculate the total power which is radiated by the dipole.

So, to calculate the total power we have to integrate over all solid angles this expression tells as the power that is emitted per unit solid angle per solid angle $d\Omega$. So, to calculate the total power that is radiated you have to integrate this expression over $d\Omega$. Now, if you look at this expression for the power emitted per unit solid angle it has $\sin^2 \theta$. This is the only term that comes in when you integrate this over solid angle.

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Angular Integral

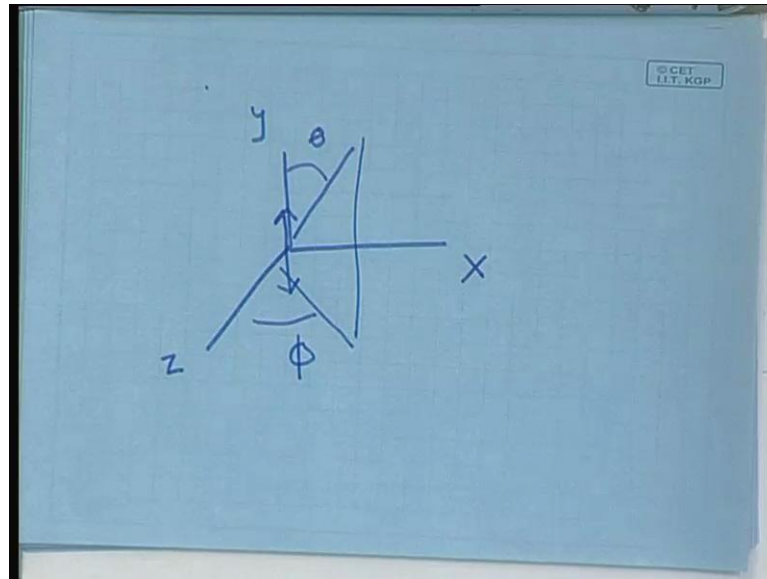
$$d\Omega = \sin\theta \, d\theta \, d\phi$$

$$\int_0^{2\pi} d\phi \int_0^\pi \sin^3\theta \, d\theta = \frac{8\pi}{3}$$

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In spherical polar coordinates this solid angle $d\Omega$ is $\sin \theta \, d\theta \, d\phi$. So, let me explain this a little bit here.

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So, we have we have the dipole oscillating like this along the y axis and you can choose a coordinate system which has got an angle theta. And an angle phi, phi is over here in this case and this is the z axis this is the y axis this is the x axis. So, when you integrate over solid angle you have to do an integral over. And the solid angle can be expressed in terms of these coordinates theta and phi which is the expression that I have over here.

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PHYSICS

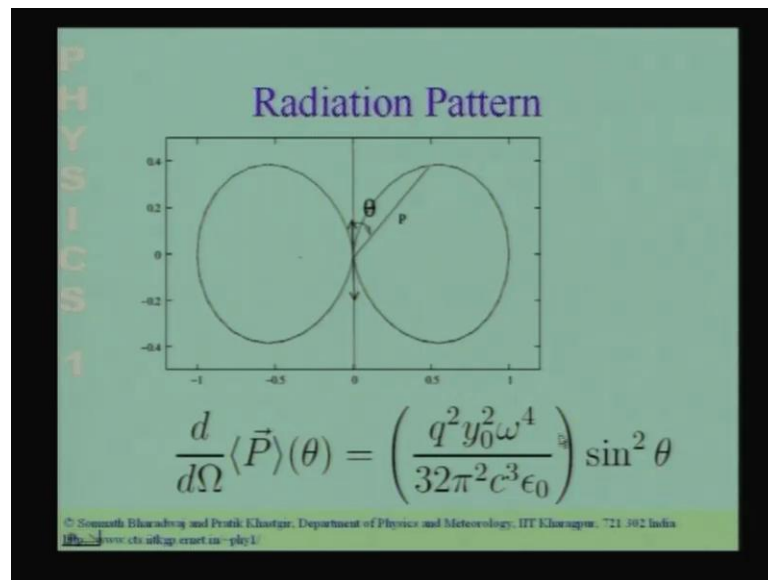
Angular Integral

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So, the d omega is sin theta d theta d phi.

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And I have to integrate the power per solid angle.

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PHYSICS 1

Angular Integral

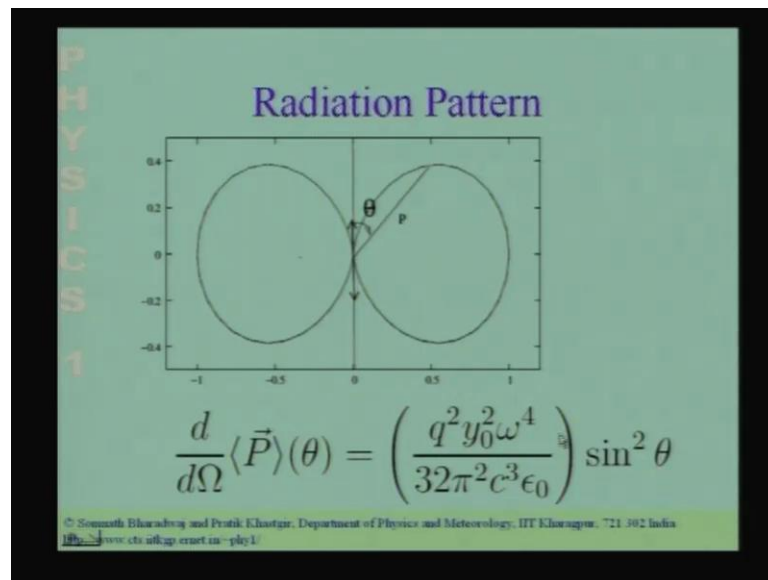
$$d\Omega = \sin\theta d\theta d\phi$$
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Over all solid angles so, I have the integral sin cube theta d theta d phi the range of theta is zero to pi the range of d phi is zero to 2 pi.

If I do this integral I will get a factor of 8 pi by 3.

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So, if I take the expression for this power emitted per solid angle. And add up the contribution overall solid angles.

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Total Power

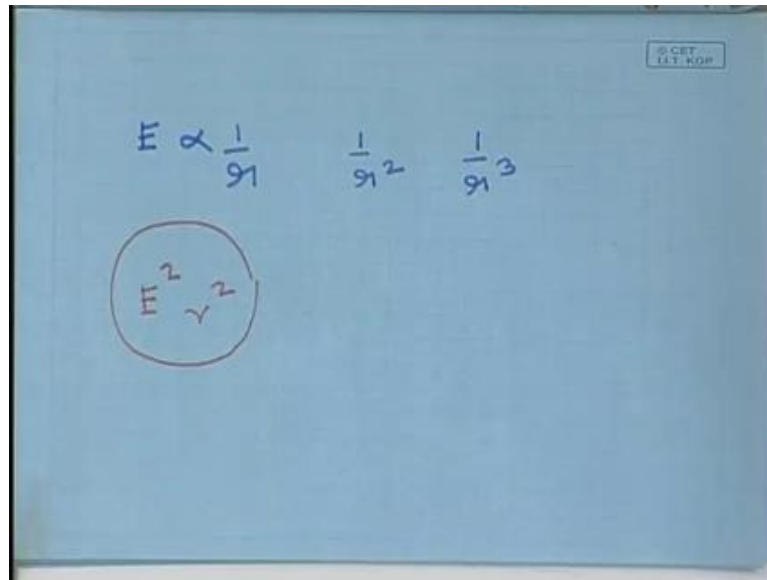
$$\langle \vec{P} \rangle = \frac{q^2 y_0^2 \omega^4}{12 \pi c^3 \epsilon_0}$$

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Then we get the total power that total power is $q^2 y_0^2 \omega^4$ divided by $12 \pi c^3 \epsilon_0$. So, this gives us the total power that is emitted by the dipole. Now, the point so this is this gives us a total power. An interesting point which you should note is that the total power does not depend on the distance from the dipole.

Let us just go back and ask the question why is why do we have this kind of a behaviour? How come the total power radiating by the dipole does not depend on how far away we are from the dipole?

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So, this you can trace back the origin this feature to the behaviour that the electric field E . The radiation part of the electric field is proportional to 1 by r . Remember, when we had the full expression for the electric field which is emitted by a charge. There were terms which had 1 by r dependence there were also terms which had 1 by r square dependence. And you so, you could have combination of such terms. And there are situations where you have terms which are 1 by r cube which have also have 1 by r cube dependence.

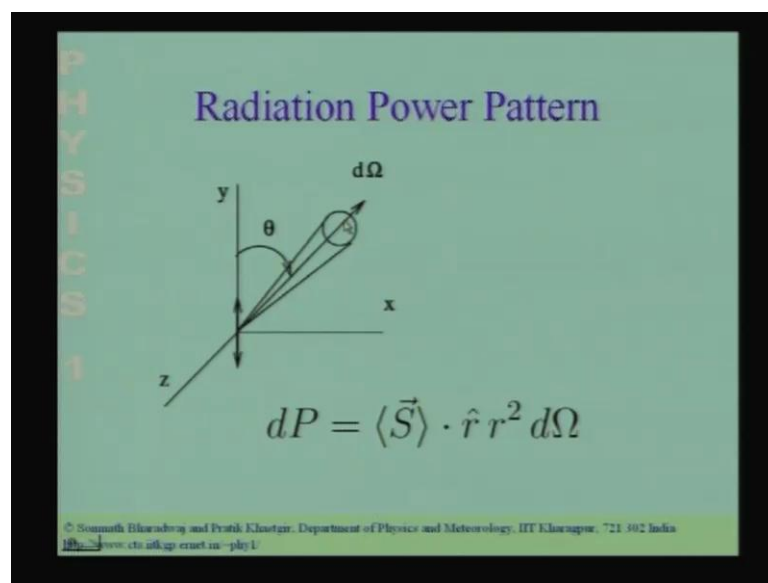
For example, if you have a static dipole remember you can recollect that the electric field falls as 1 by r cube. We have not discuss it here, but I am sure all of you would have learnt this in earlier courses that, if I have a static dipole the electric field fall as 1 by r cube. So, you could also have a situation where you have electric field a part of electric field going as 1 by r . And you could also have components in the electric field which fall as 1 by r square 1 by r cube etcetera.

Now, we had all of these terms all of these possibilities. But I focused only on the part which falls as 1 by r and I told you that this is the only thing which corresponds to radiation, the rest of them do not correspond to radiation. Let me elaborate a little on this

point again. So, when you calculate the power the total power that is emitted you have to essentially look at E^2 and then multiply it. So, E^2 gives us the energy flux the energy flux is proportional to E^2 and you have to multiply it with r^2 when you do the integral over all solid angle.

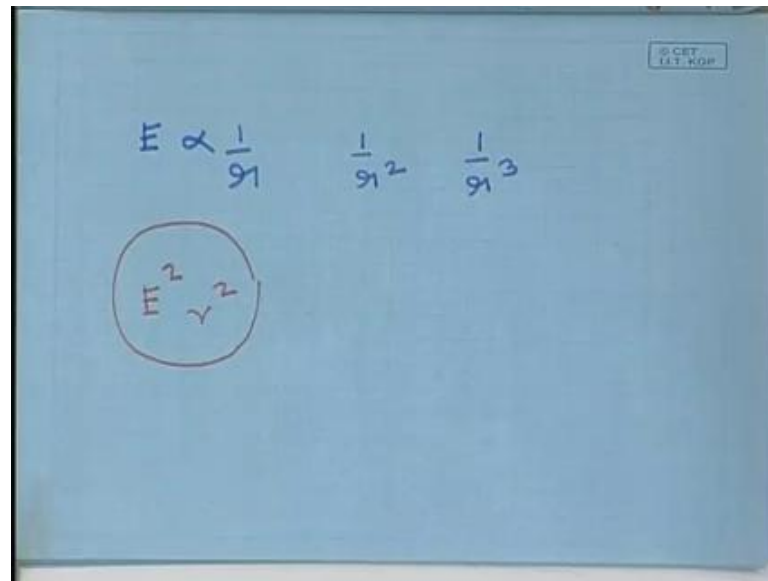
You have to multiply by r^2 and then do integrate over all solid angle over $d\theta$ and $d\phi$. So, the crucial point is that you have to look at that r dependence of this combination E^2 into r^2 .

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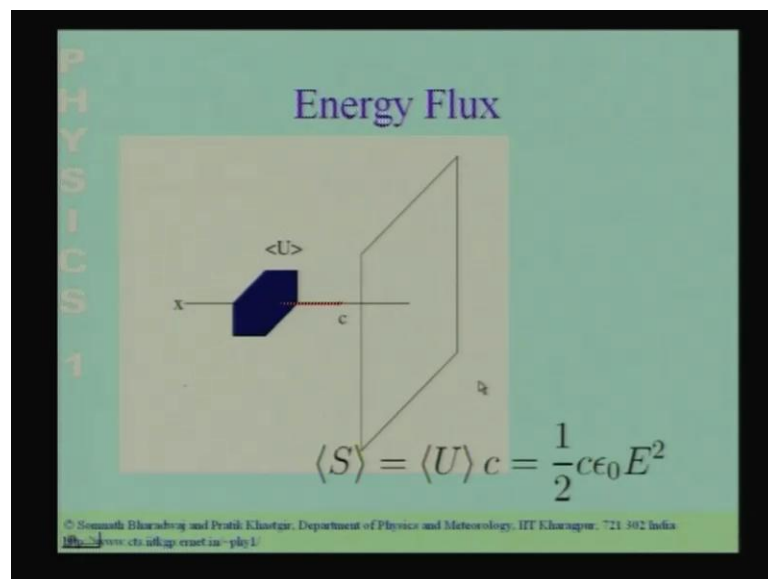
Let me just remind you again what we are talking about when I calculated it that the total radiation pattern that is when I ask the question. How much energy goes out per solid angle? We looked at the energy flux S the energy flux is proportional to E^2 right.

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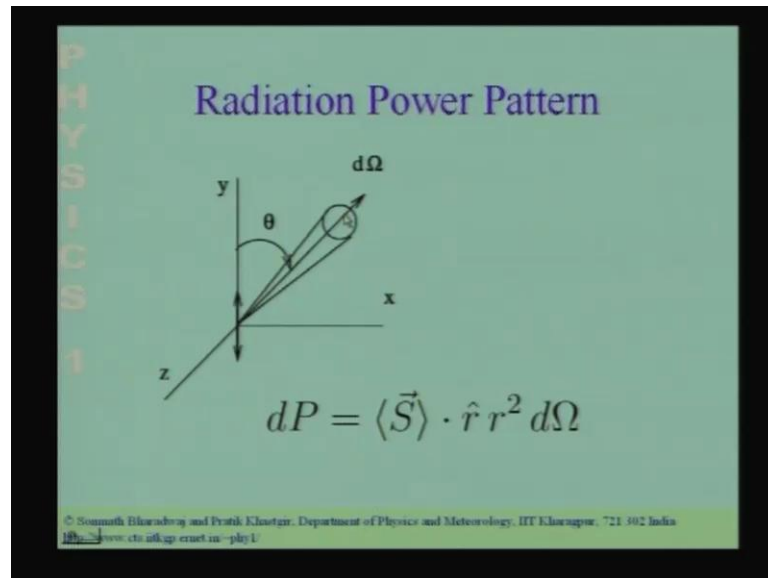
So, the energy flux as you can see here is proportional to E square.

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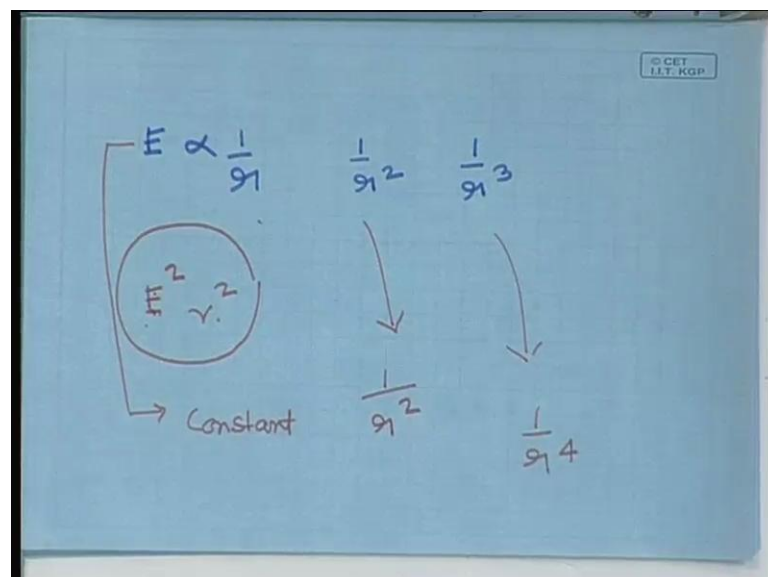
So, you have to look at E square.

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And then when you ask the question how much energy is emitted per solid angle You have to multiply it by the area corresponding to the solid angle which has a factor of r square. Now, you have when you do the solid angle integral that r does not come into the picture anymore. So, all the r factors are here you have to and the r factors have a dependence which is E square into r square.

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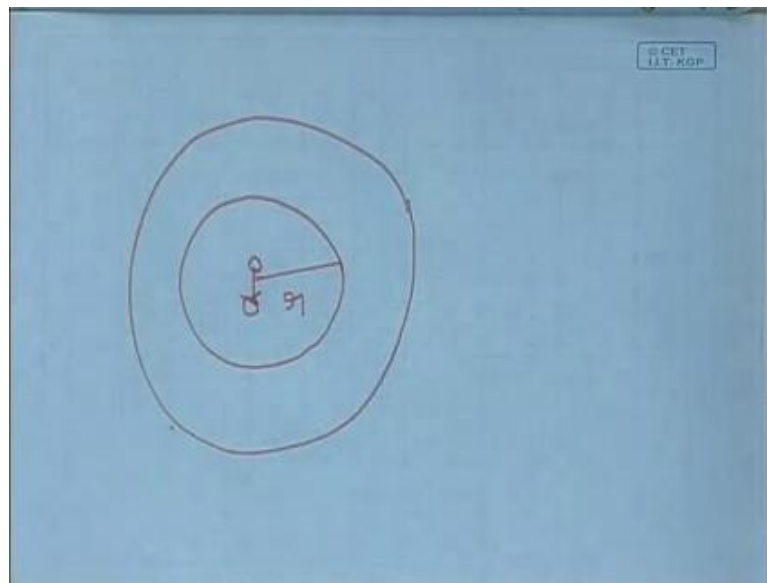


So, if you ask the question what is the power that comes out you have to look at the r dependence of E square into r square. Now, if you have a $1/r$ electric field then for the

1 by r electric field this E square into r square is a constant. So, it is for this particular electric field it is a constant it does not depend on r . whereas, for a 1 by square electric field if I look at E square into r square the whole combination.

So, when I square 1 by r square I will get 1 by r to the power four multiplied by r square. So, the whole combination will fall off as 1 by r square. And if I had 1 by r cube then the whole combination would fall off as 1 by r to the power this will fall as this and this will fall as this.

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So, what we see from this is that if I had a source some kind of a charge particle doing some kind of a motion over here. And ask the question what is the power which comes from this charge, which produce by this charge which is oscillating or doing something that crosses this surface at a distance r . Then we find that if the electric field is 1 by r the power that crosses this is also the same as the power that crosses this and it does not depend on how the large I make the sphere.

So, if I have a 1 by r electric field and I take the limit of r going to infinity I will find that, there is some power being transmitted there also. So, if I have a 1 by r electric field there is power being radiated sent out all the way to infinity. And it is this power that goes out to infinity which we refer to as radiation. So, let me repeat this if I have a situation where, charges combinations of charges produce a 1 by r electric field. The

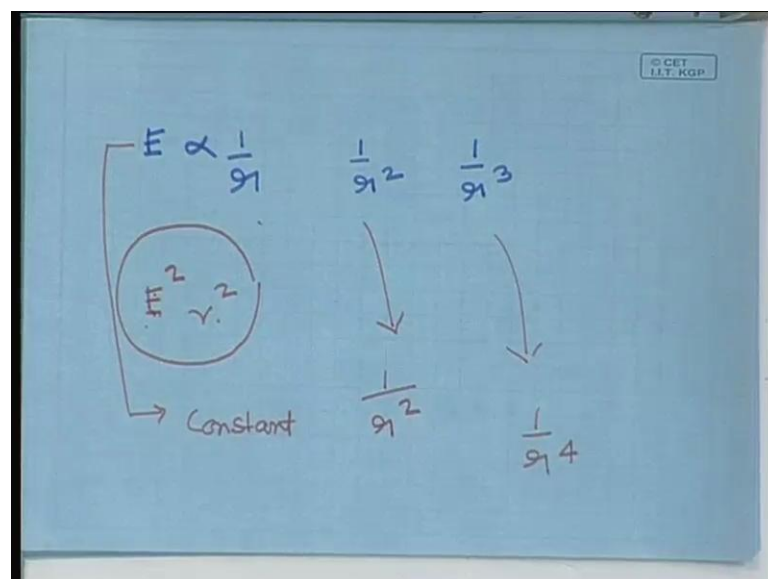
power which is being sent out does not depend on the distance from these charges it is a constant.

It does not depend on the distance and; however, large I make this sphere the amount of power that crosses it is a constant. So, if even if I make it infinitely large there will be the same amount of power crossing it. So, this set of charges which are which are producing a $1/r$ electric field or putting or sending out energy all the way to infinity. And it is this that we referred to as radiation. So, a radiating set of charges send out power.

The power which is send out the total power which is send out does not decay away as I move further and further away from this set of oscillating; the set of charges which are producing the electric fields. And it is this phenomenon that we referred to as radiation. On the contrary if I have a set of charges which produce a $1/r^2$ electric field the $1/r^2$ component of the electric field the power which is being sent out from this. The power which is being sent out or from this depends on the size of the sphere across which, I am measuring the power.

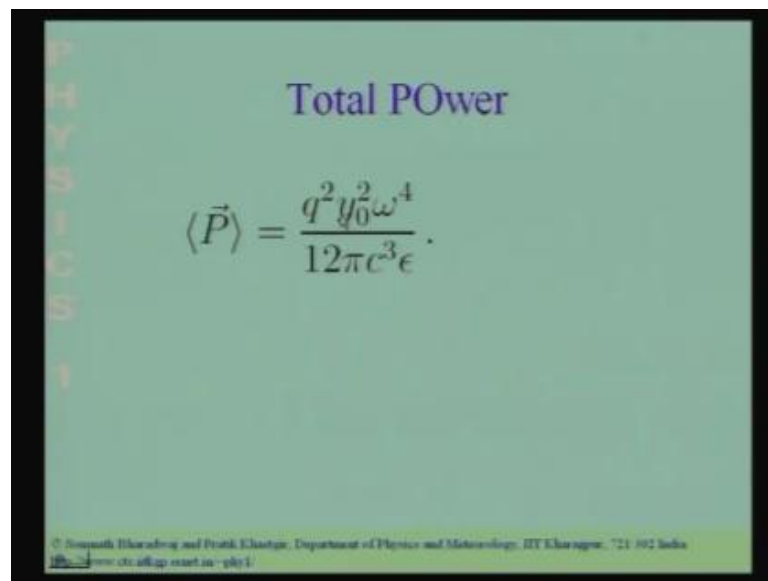
And it falls as $1/r^2$. So, if I keep on looking at larger and larger spheres after sometime I will find that very little power is coming out. So, such a set of charges do not send out power actually the power is the energy is confined to a certain region. The power which comes out from this is confined to a certain region and no power is lost basically to infinity.

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So, electric fields which fall faster than $1/r$ do not correspond to radiation, we do not refer to these as radiation they do not carry away power to infinity. It is only the $1/r$ electric field which carries away power to infinity which we referred to as radiation. So, this tells you why we ignored all the terms which fell off faster than $1/r$. Because, they do not give rise to any power being transmitted to infinity and there is no radiation; they do not correspond to radiation. So, having explained to you the reason why we took this $1/r$ term only.

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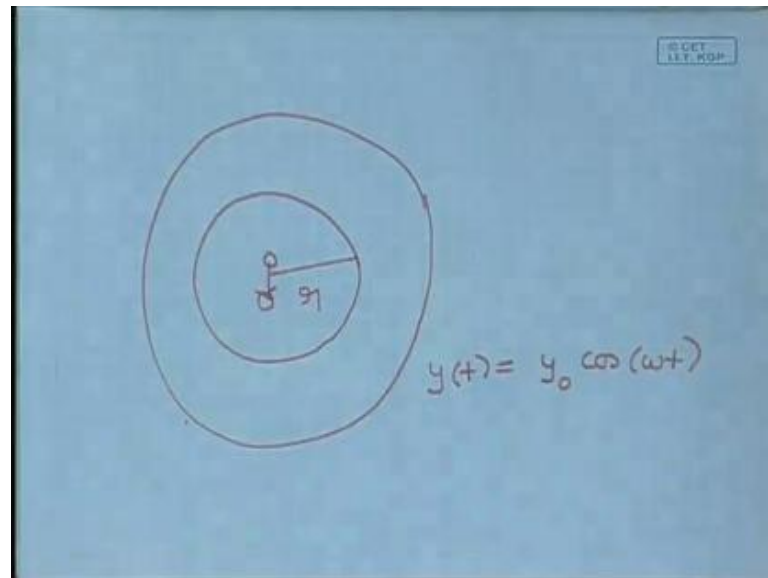
Total Power

$$\langle \vec{P} \rangle = \frac{q^2 y_0^2 \omega^4}{12\pi c^3 \epsilon}.$$

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Now, let us go back to the issue that we were looking at the total power that comes out from this oscillating dipole, the total power that comes out from oscillating dipole. If you write it in terms of the magnitude of the oscillation of the charge which moves up and down.

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So, remember that the charge we had assumed at the charge moves up and down the dipole the displacement of the charge was given by $y_0 \cos \omega t$. And the total power that is emitted depends on this amplitude y_0 squared.

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PHYSICS 1

Total Power

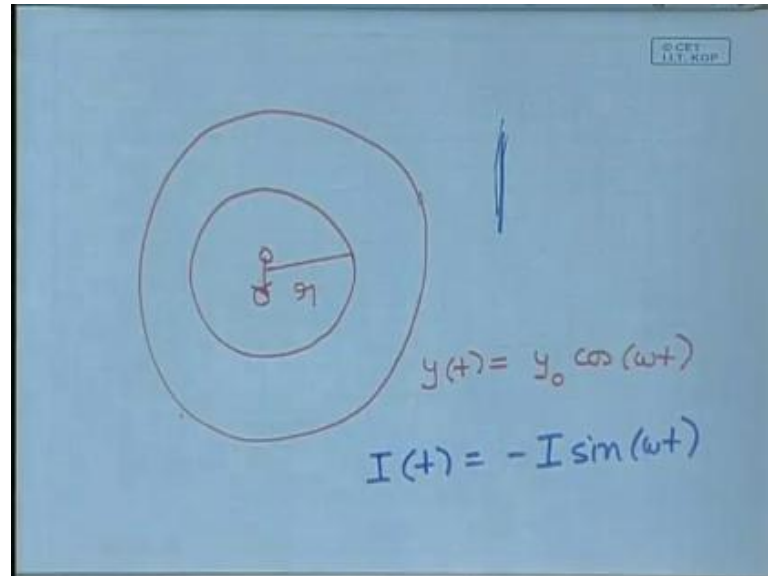
$$\langle \vec{P} \rangle = \frac{q^2 y_0^2 \omega^4}{12\pi c^3 \epsilon}$$
$$\langle \vec{P} \rangle = \frac{I^2 l^2 \omega^2}{12\pi c^3 \epsilon}$$

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So, if I double the amplitude the total power will go up 4 times it also depends on the angular frequency to the power 4. So, if I make the charge oscillate twice as faster the total power that is emitted will go up by 2 to the power 4 which is 2 4 8 16 so it will go

up 16 times. Now, we could also express the total power that is emitted in terms of the current.

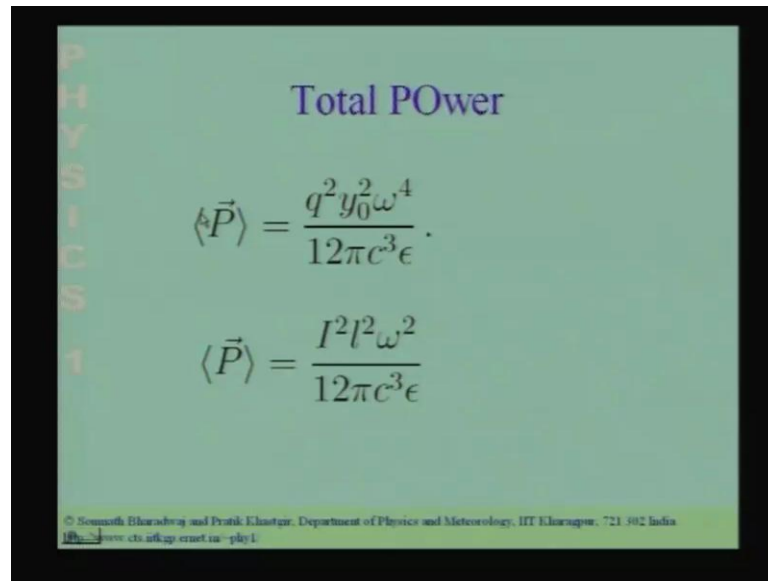
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So, if I have this charge particle going up and down this dipole the charge particle moves up and down this dipole. Then there will be a current. So, if I express the amount of total power which comes out, this is radiated out; in terms of the magnitude of the currents.

So, if I express the current like this and if I write the total power which is emitted from the dipole in terms of the magnitude of the current and the angular frequency ω . Then the expression for the power assumes this form shown over here.

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Total POver

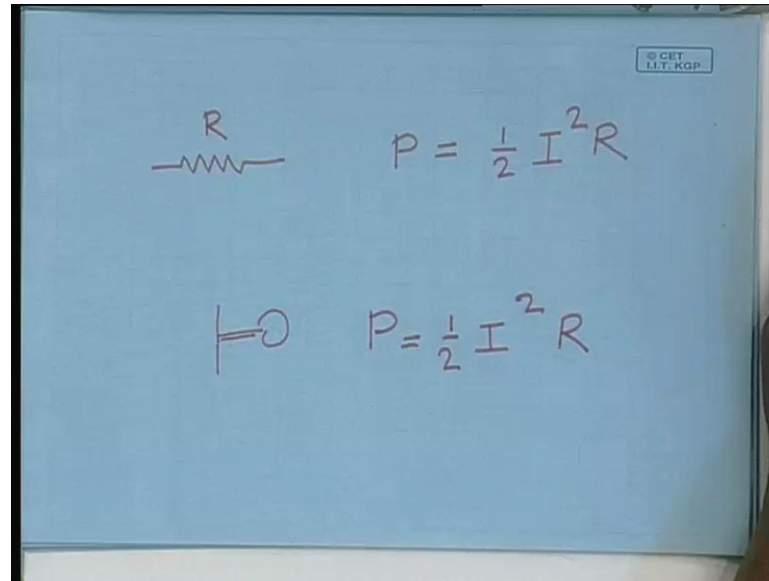
$$\langle \vec{P} \rangle = \frac{q^2 y_0^2 \omega^4}{12\pi c^3 \epsilon} .$$
$$\langle \vec{P} \rangle = \frac{I^2 l^2 \omega^2}{12\pi c^3 \epsilon}$$

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It depends on the magnitude of the current square and it depends on the square of the angular frequency. So, if I maintain the magnitude of the current fixed and increase the angular frequency twice. Then the power will go up 4 times whereas, if I maintain the displacement of the charge fixed and increase the angular frequency 2 times the power will go up 16 times.

So, this is the point to bear in mind that the angular frequency or the frequency dependence depends on the variable which in terms of which I have expressed the power. There is also another convenient expression for the power. So, note that the power that is emitted by this oscillating dipole is proportional to the current squared is proportional to the square of the current.

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So, if I have a current flowing in a resistance R the power emitted the power lost in the resistance is half I square R . In the resistance the power is dissipated as heat and the amount of power dissipated at as heat is half into the current magnitude of the current square into the resistance. Whereas, here we see that if I have a dipole and I feed a current into this.

Then again there is some power emitted and this power is proportional to the currents squared. So, I could define an equivalent effective resistance corresponding to the dipole or an impedance of the dipole equivalent resistance of the dipole in this case. So, corresponding to this dipole the dipole converts some of the energy. So, dipole actually also dissipate some power, but here the power is dissipated as radiation not as heat. So, the power that is lost as radiation I could represent using an effective resistance.

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PHYSICS 1

Total POver

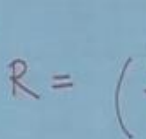
$$\langle \vec{P} \rangle = \frac{q^2 y_0^2 \omega^4}{12\pi c^3 \epsilon}.$$
$$\langle \vec{P} \rangle = \frac{I^2 l^2 \omega^2}{12\pi c^3 \epsilon}$$

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So, that the power is half $I^2 R$ and if you put in the numerical values of all of these constants. And if you express the angular frequency ω using the wavelength you get an a very convenient expression for the resistance.

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$$R = \left(\frac{l}{\lambda}\right)^2 790 \, \Omega$$


 $P = \frac{1}{2} I^2 R$

The resistance comes out to the 1 by λ^2 square 790 ohms. So, let me repeat again what we mean by this if I have a dipole of length l and I send a current through this. Then the dipole will dissipate away energy will dissipate power in the form of radiation. The power which comes out in the form of radiation we have seen is proportional to the

magnitude of the current squared. So, I could write it as an effective resistance into the current magnitude of the current squared this effective resistance can be calculated from the expression for the power.

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Total Power

$$\langle \vec{P} \rangle = \frac{q^2 y_0^2 \omega^4}{12\pi c^3 \epsilon}$$

$$\langle \vec{P} \rangle = \frac{I^2 l^2 \omega^2}{12\pi c^3 \epsilon}$$

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The expression for the power notice depends on omega square.

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$$R = \left(\frac{l}{\lambda}\right)^2 790 \Omega$$

↑
 l
 ↓

$P = \frac{1}{2} I^2 R$
 $\frac{\omega}{k} = c \mid \frac{\omega}{\frac{2\pi}{\lambda}} = c$

The dispersion relation for radiation where omega is the angular frequency of the radiation; the dispersion relation for the radiation is omega by k is equal to c. So, it basically tells us that omega by 2 pi lambda is equal to c. So, I can replace omega and

write it in terms of lambda. So, omega is inversely proportional to lambda. Which is why you see that, the resistance the value of the resistance is comes out to be inversely proportional to lambda squared.

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Total POver

$$\langle \vec{P} \rangle = \frac{q^2 y_0^2 \omega^4}{12\pi c^3 \epsilon}$$

$$\langle \vec{P} \rangle = \frac{I^2 l^2 \omega^2}{12\pi c^3 \epsilon}$$

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Because the expression of the power has a omega square over here. So, if you express this omega in terms of the wavelength of the radiation that comes out. You get you can write this expression in this form.

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$$R = \left(\frac{l}{\lambda}\right)^2 790 \Omega$$

↑
l
↓

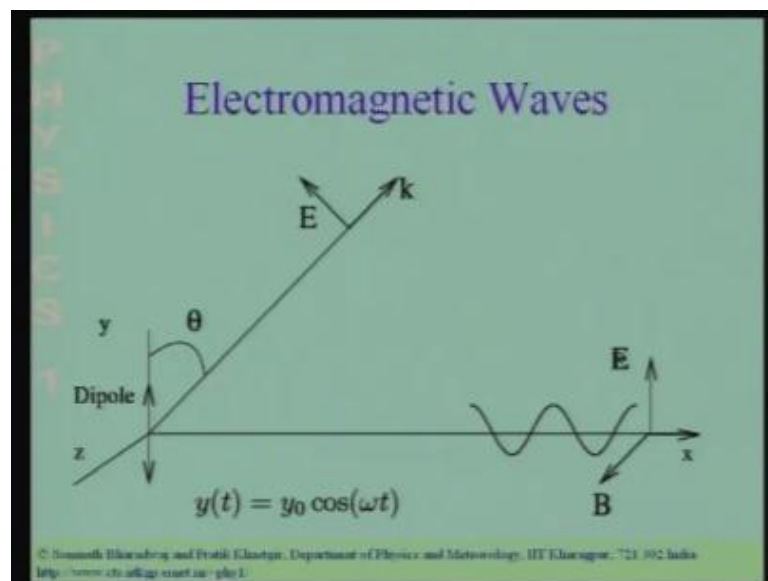
P = $\frac{1}{2} I^2 R$

$$\frac{\omega}{k} = c \mid \frac{\omega}{\frac{2\pi}{\lambda}} = c$$

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The equivalent resistance turns out to be the length of the dipole divided by λ^2 into 790 ohms. So, we shall come back to applications of this formula when we discuss problems on the radiation that comes out from a dipole. Today, let us go on to discussing another aspect of the radiation that comes out from the dipole. And the aspect that we are going to discuss let me let a before going on to this, let me again just recollect the quantity that we are the situation that we are discussing.

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So, the situation we have been discussing until now is that we have a dipole aligned along the y axis and we are feeding in a sinusoidal voltage. So that, the currents so that, the charges move up and down the y axis as $y(t)$ is equal to $y_0 \cos \omega t$. And this produces a sinusoidal plane wave far away. And along the x axis if you look at this sinusoidal plane wave along the x axis you will see the electric field the oscillating up and down along the y axis.

So, the electric field here oscillates in the same direction as the dipole. And the magnetic field also oscillates perpendicular in the same in the same phase as the electric field. But, it is perpendicular to the direction of propagation of the wave and the direction of the electric field.

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PHYSICS 1

Equivalent Resistance

$$\langle \vec{P} \rangle = \frac{1}{2} R I^2$$
$$R = \left(\frac{l}{\lambda} \right)^2 790 \, \Omega$$

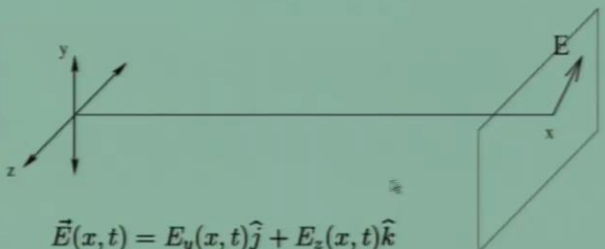
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So, here I have the expression for the equivalent resistance.

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PHYSICS 1

Crossed Dipoles


$$\vec{E}(x, t) = E_y(x, t) \hat{j} + E_z(x, t) \hat{k}$$

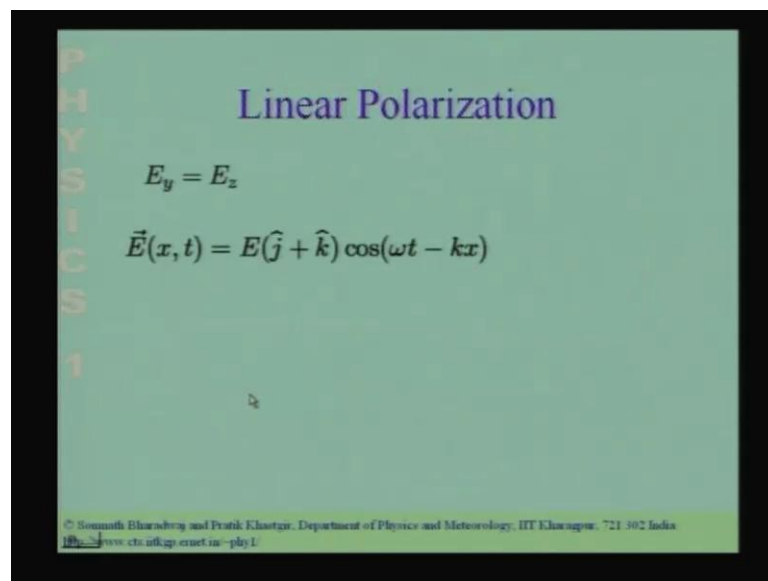
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Now, let us come to the situation that we now on to discuss the situation that we would now like to discuss we have 2 crossed dipoles. So, instead of having 1 dipole along the y axis we now have 2 dipoles 1 along the y axis and another along the z axis. So, we have these 2 dipoles and we feed exactly the same current to both of these dipoles. So, let us first consider a situation a simple situation where, we have 2 dipoles. They are perpendicular to each other that is why we referred to them as crossed dipoles.

So, we have 2 dipoles 1 along the y axis another along the z axis. We will study the electric field pattern at a point which is a large distance away along the x axis direction. So, if the point we are looking at is perpendicular to both the y the dipole along the y axis and the dipole along the z axis. So, the question is what do we see at this point over here which is far away along the x axis. Now, as we have already discussed the dipole along the y axis will produce an electric field along the y axis. And the dipole along the z axis is going to produce an electric field along the z axis.

So, the quantity which you measure over here a large distance away is going to be a superposition both of both of these. So, you will have the electric field along the y axis produced by the dipole along the y axis. And you will have the electric field along the z axis produced by the dipole along the z axis.

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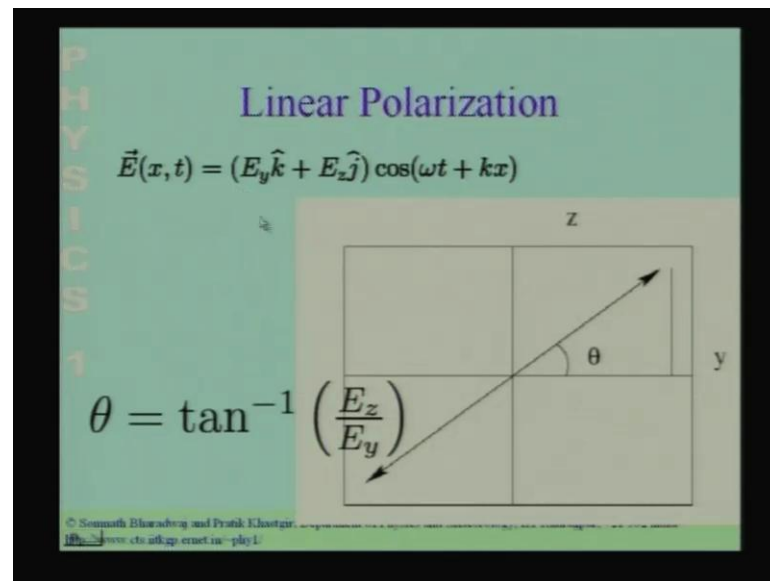
The situation which, we are considering their both being fed exactly the same current the same signal is being fed into both them. So, the magnitude of these 2 electric fields is the same and they both oscillate; the 2 electric fields are going to be oscillating in the same phase. So, we have $\cos \omega t - kx$. So, it will only change as I move along the x axis and with time both of these are going to be oscillating in the same phase.

So, I can take that factor E outside E_x is equal to E_y I can take the factor E outside and write it as the unit vector \hat{j} plus the unit vector \hat{k} into a constant E into $\cos \omega t - kx$. So, this the total electric field at the point x Now, let us fix the value of x and let us

fix it so that, kx is a multiple of 2π . So, if kx is the multiply 2π I can ignore it for the discussion

So, we will we will consider a fix the behaviour of the electric field at the fixed value of x and we will choose x . So that, k into x is a fixed number which is a multiple of 2π .

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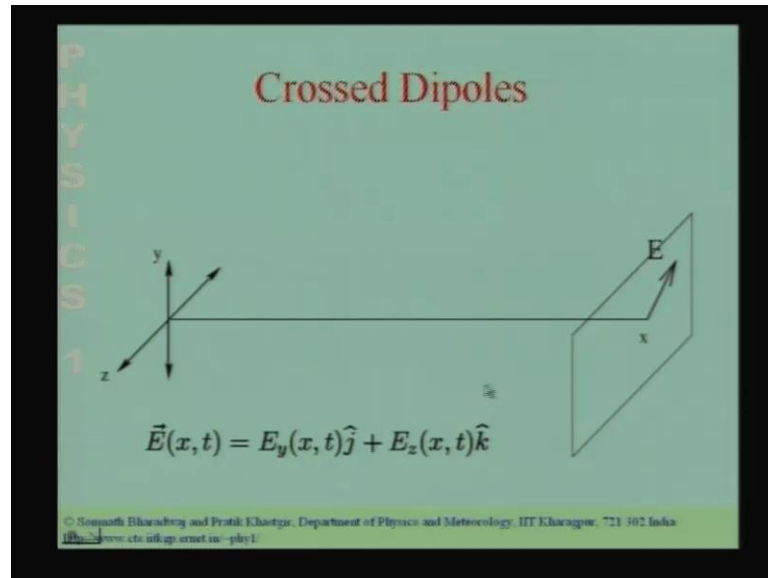
Then ask the question how does the electric field vary over there? So, let us take the time instant t equal to 0. At the time instant t equal to 0 E_x and E_y both of them have the same value and they will have a value equal to E . So, at t equal to 0 the electric field vector is going to be magnitude E into j plus k . So, it is going to point at in a direction at 45 degrees to the y axis this is the y axis, this is the z axis.

So, the electric field vector at the point x at a fix value of x is going to point at 45 degrees to the y axis when t is 0. When t is 0 this whole think is $1 \cos \omega t$ is 1. And this has the maximum value which it can assume. Now, as t increases this $\cos \omega t$ is going to come down. So, both the y component of the electric field and the z component of the electric field is going to come down by exactly the same amount. And then when ωt is $\pi/2$ it is both of them are going to be 0 and then when it crosses $\pi/2$ both of them are going to be negative.

So, the electric field is going to become like this and its going to oscillates back and forth like this. So, this kind of behaviour of light is called linear polarization. So, the light is

said to be linearly polarized in this case the light is linearly polarized at 45 degrees to the y z to the y z y axis and z axis. And it will the light the electric field vector will oscillate at 45 degrees to these axis.

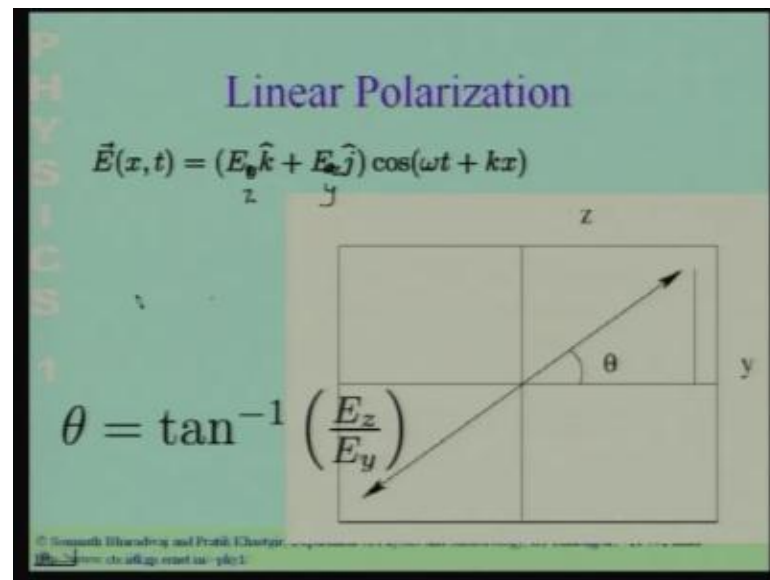
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So, let me again just remind to you of the situation that we are discussing we are discussing a situation where exactly, the same signal is fed to these 2 dipoles which are crossed. And in this situation the electric field over here is going to oscillate at 45 degrees to the y z axis.

Now, let us next consider a situation where the amplitude of the currents being fed to the y and z axis differ but, the phase of the current being fed here and here or exactly the same.

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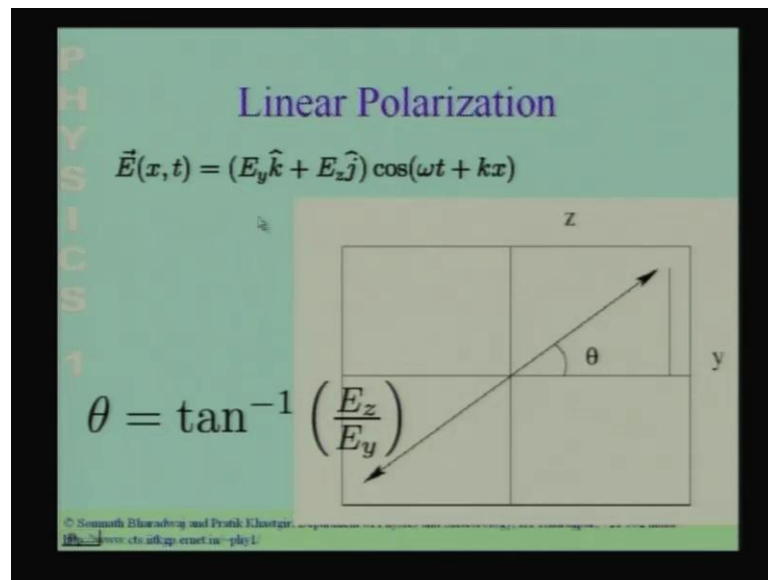


So, this is the next situation that we are going to discuss. So, in this case the electric field at a fix point x is E_z this should be E_z into \hat{k} plus E_y into \hat{j} . Let me just write it correctly over here. So, this is actually this should be E_z into \hat{k} and E_y into \hat{j} there was a small mistake over there. And the magnitude of E_z and E_y are different because that the currents being fed into 2 dipole magnitude of the current being fed into the 2 dipoles is different.

But, the phase is the same. So, I have the same factor $\cos \omega t + kx$ multiplying both the y component of the electric field and the z component of the electric field. So, the electric field at that point can be written as like this. Now, if I fix the value of x again so that, kx is a multiple of 2π . So, I can ignore this term. So, we have this factor E_z into \hat{k} plus E_y into \hat{j} multiplying $\cos \omega t$. So, when $\cos \omega t$ is 0 I have the electric field vector its y its y component is E_y its z component is E_z .

So, the electric field vector at this instant of time t equal to 0 is along the direction θ where, θ is \tan^{-1} of E_z by E_y .

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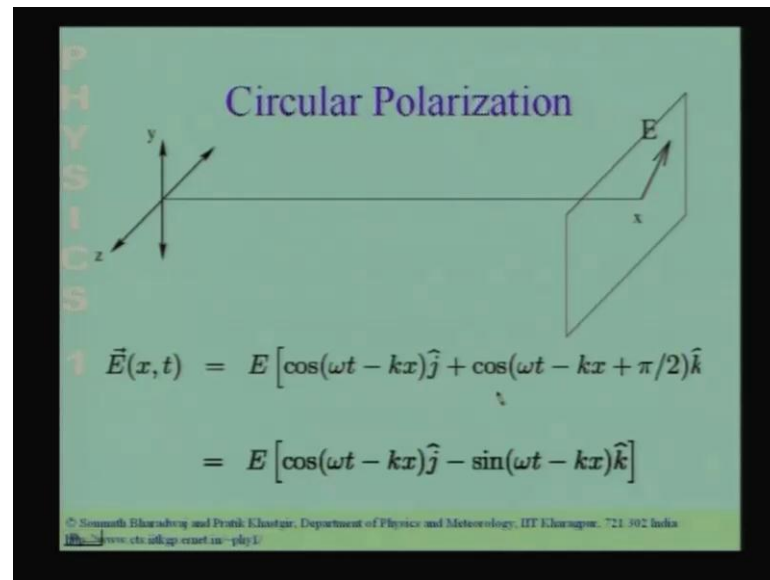
It has a magnitude theta has a magnitude which is E_z square plus E_y square the square of that. Now, as time increases $\cos \omega t$ is going to go down and both these vectors are going to go up by this go down by the same amount. When ωt is $\pi/2$ it is going to be at 0 and then again it is going to increase. So, you can easily see that the electric field vector is going to oscillate back and forth.

Then it is going to oscillate along this line and the oscillation is going to be at an angle theta to the y axis where theta is \tan^{-1} of E_z by E_y . So, such a situation the situation that we have just discussed when you feed current of the same phase, but possibly different amplitudes to the 2 crossed dipoles. The electric field at that point x which is a distance away oscillates along the line; at a fix position the electric field oscillates along the line.

Such a situation is referred to as linearly polarized electromagnetic wave the electromagnetic wave is said to be linearly polarized because; the electric field oscillates along the line. And you have this situation when both the dipoles are fed with the same current with possibility different amplitudes, but the same phase. Now, this is also referred to as plane polarized light because if you look at the electric field along the entire x axis.

Then the electric field at different x points will appear to oscillate along the plane the plane may be at an angle to the y z y z directions. So, this is called plane polarized electromagnetic radiation or plane polarized light.

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Now, let us now move on to considering a slightly a different situation. The situation which we are going to consider next is as follows we have now changed the phase of the current that is being fed in. So, the current that is being fed in to the z dipoles is given an extra phase of pi by 2 the same current the magnitude of the current being fed in to that 2 dipoles. The 1 along the y axis and the 1 along the z axis are exactly the same. So, currents of the same magnitude are fed to both the dipoles, but the dipole along the z axis has an extra phase of pi by 2.

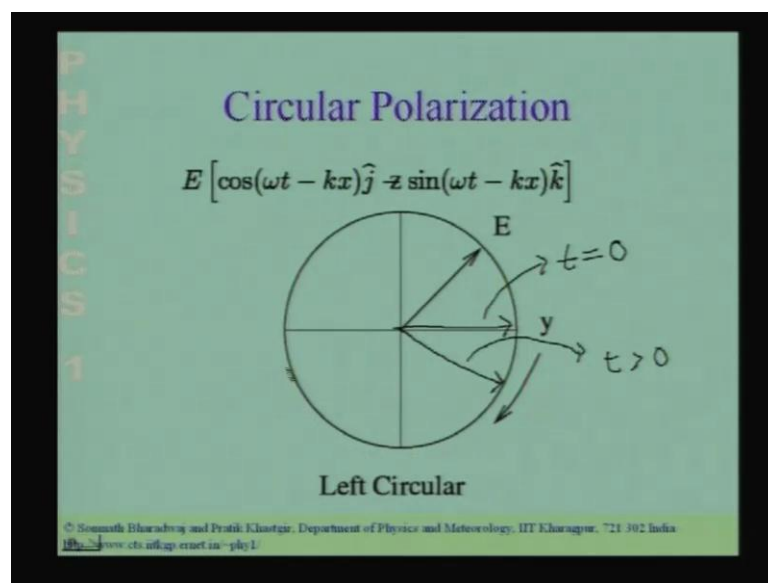
So, now let us again study the behaviour of the electric field at this point x over here. So, the electric field at this point x is a superposition of the electric fields produced by these 2 dipoles. The dipole along the y axis produces an electric field $E \cos \omega t \text{ minus } kx$ into j. The dipole along the z axis produces the same electric field along the z direction, but with an extra phase of pi by 2. So, the electric field along the z direction has an extra phase of pi by 2.

Now, this extra phase of pi by 2 in this cosine term over here can be a used to write this term in terms of sin. So, the same expression is written over here and we have just replaced $\cos \omega t \text{ minus } kx \text{ plus } \pi \text{ by } 2$ with $\text{minus sin } \omega t \text{ minus } kx$. So, what

we see is that, if I put an extra phase of π by 2 to the current being fed into this dipole which is along the z direction. Then, the electric field along the y axis oscillates as $\cos(\omega t - kx)$ whereas, the electric field along the z axis oscillates as $\sin(\omega t - kx)$ with the minus sign over here.

So, now let us look at the behaviour of the electric field with time at a fix point x and again we will choose x so that, kx is a multiple of 2π . So, what you have is the electric field is $E \cos \omega t$ into \hat{j} minus $\sin \omega t$ into \hat{k} .

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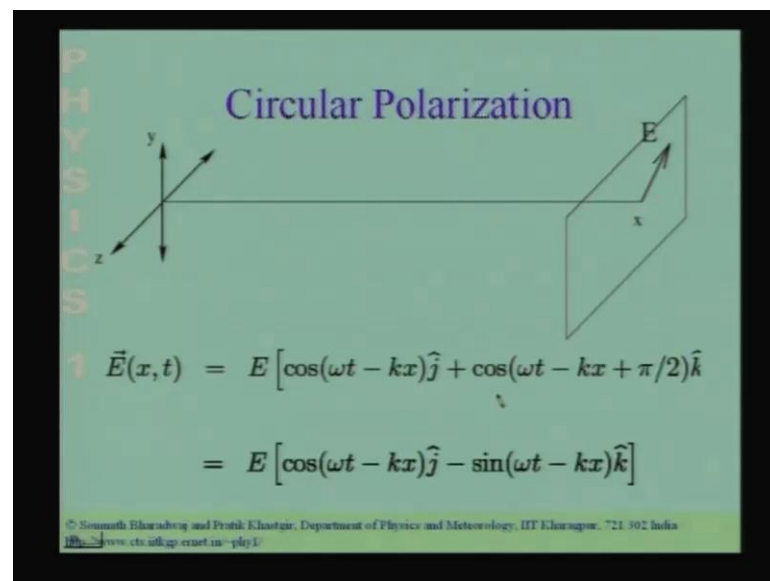


So, let us plot the behaviour of this. So, we have written here again the expression for the electric field at a fix point x. And we have chosen kx to be a multiple of 2π . So, at t equal to 0 the electric field is along the y direction. So, the electric field at t equal to 0 is aligned over here.

So, let me draw that so at t equal to 0 the electric field is aligned like this. Now, what happens as time increases. So, if you increase ωt the values of $\cos \omega t$ are going to go down and $\sin \omega t$ is going to increase from 0. But, you have a minus sign here, so, you are going to have a negative z component and the y component is going to come down. So, the y component is going to go down and you are going to get a negative z component.

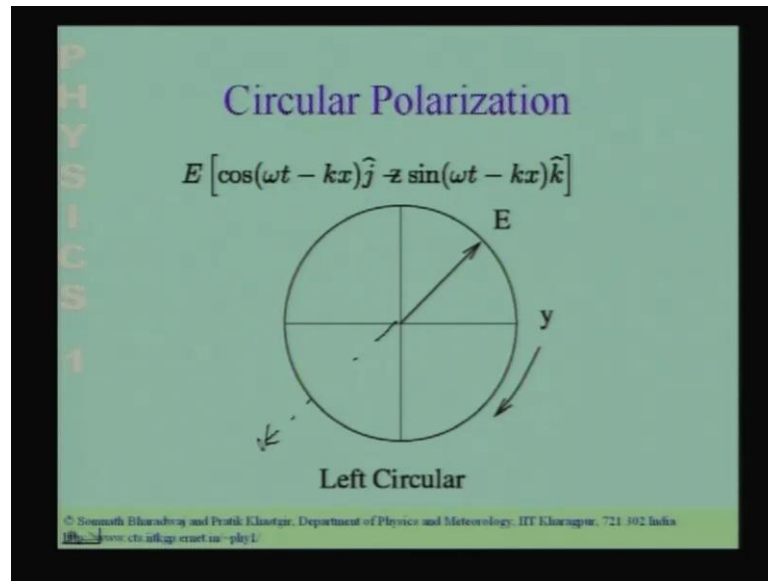
So, the electric vector is going to look something like this. So, this is t greater than 0 and as time evolves the electric field is going to go around like this. And we have the electric field going around in a circle the magnitude of the electric field is fixed at a value E . And let us now discuss briefly the direction of the circle. So, in this situation the electric field goes around like this. The arrow over here shows you the direction in which the electric field rotates.

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Now, let us ask the question in which direction is the wave propagating? Let us go back to the picture of the wave, so the wave, just remember that the dipoles are here and we are looking at the electromagnetic field along the x axis. The wave is also propagating along the x axis.

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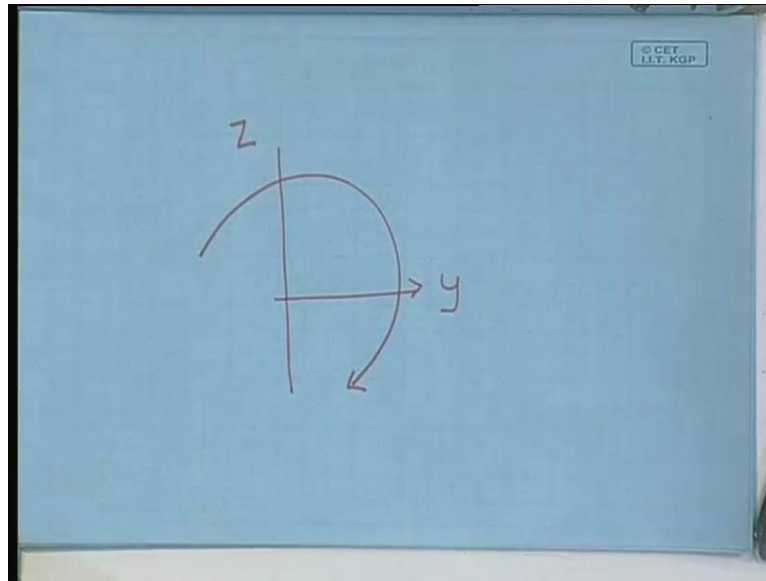


So, in this picture this is y and this is z. So, if you do y cross z you will get a vector pointing out of the screen. Now, so the wave is actually propagating outwards I could draw it over here. So, this is propagating out of the screen. So, the wave is propagating forward out of the screen. And the electric field is going around in the direction shown by this arrow.

Now, we will adapt a convention for the circularly polarized light we will adapt a convention where, if the wave propagates in the direction of the thumb. And if the electric field goes around like this. So, if the wave propagates like this and if the electric field goes around like this, so the electric field goes around like this. We will call this right circularly polarized light whereas, if the wave is going like this.

If the electric field rotates in this direction we will call this left circularly polarized light. Now let us just go back to the situation which we have over here.

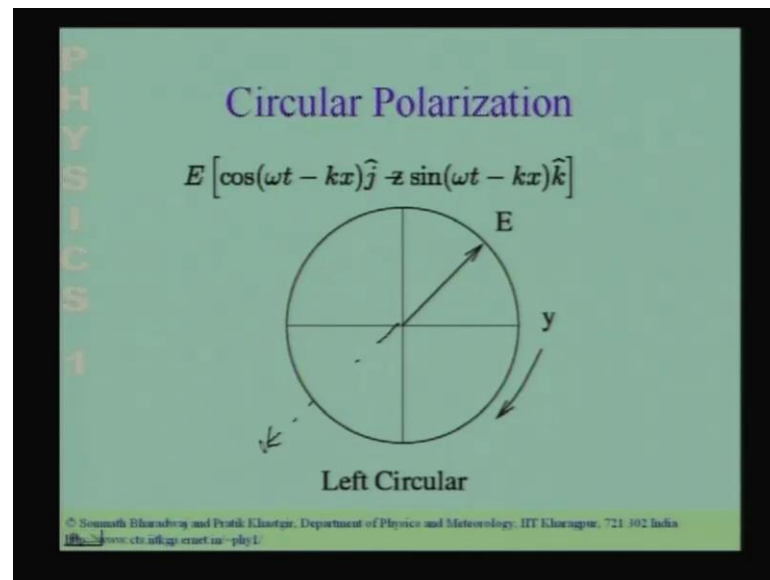
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Let me draw it for you over here it will help us understand it. This is the y axis this is the z axis and the electric field is going around in a circle like this and the waves is coming out of the screen. So, it is coming out this way which is the x direction. So, the question is, is this light is this radiation left or right circularly polarized.

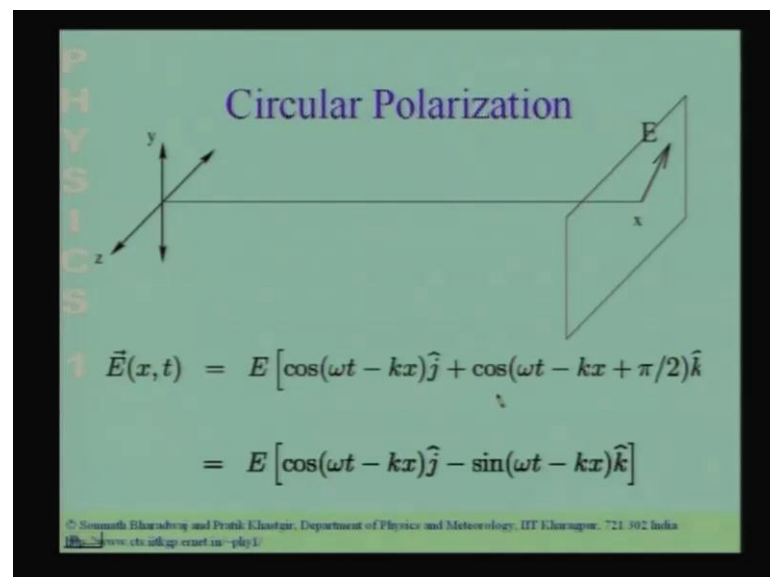
Now, if I put my right hand with the thumb pointing out towards the direction in which the wave is going you see that, this does not correspond to right circularly polarized light. Whereas, if I put my left hand here with my thumb pointing outwards towards the direction in which the wave is propagating it matches with the direction in which the electric field is rotating.

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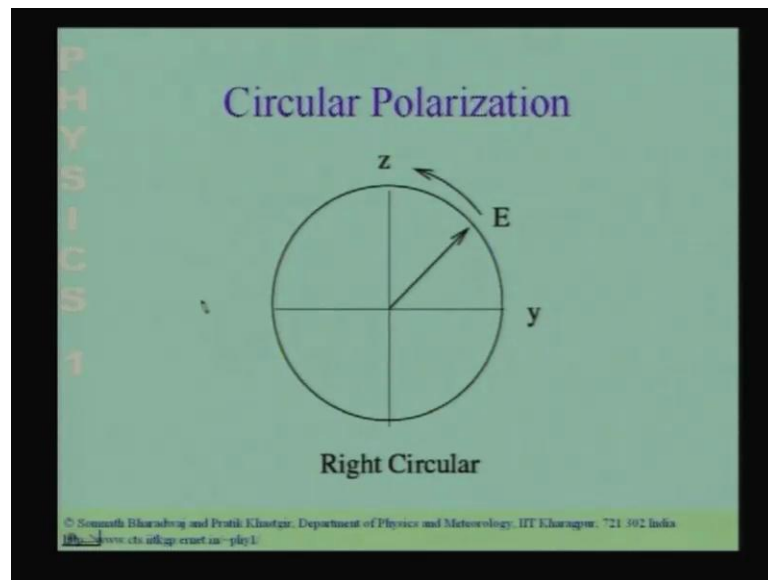
So, the situation that we have so the situation over here situation which we have studied corresponds to left circularly polarized light.

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So, the situation where we have put an extra phase of pi by 2 to the current to the oscillations along the z direction gives us left circularly polarized light or left polarized light electromagnetic radiation.

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You could also have right circularly polarized electromagnetic radiation. For in right circularly radiation electromagnetic waves the electric magnetic field will go around in exactly the opposite direction. Question is how will you produce right circularly polarized radiation? How will you produce right a circularly polarized electromagnetic wave in the situation which we have been discussing namely, 2 crossed dipoles. I am sure you can work out the answer to this question. So, let me stop here for today and resume the discussion in the next lecture.