Physics I: Oscillations and Waves
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Lecture - 10
Electromagnetic Waves – II

Good morning. Let us, start off by recapitulating the things that we have learned in the last class.

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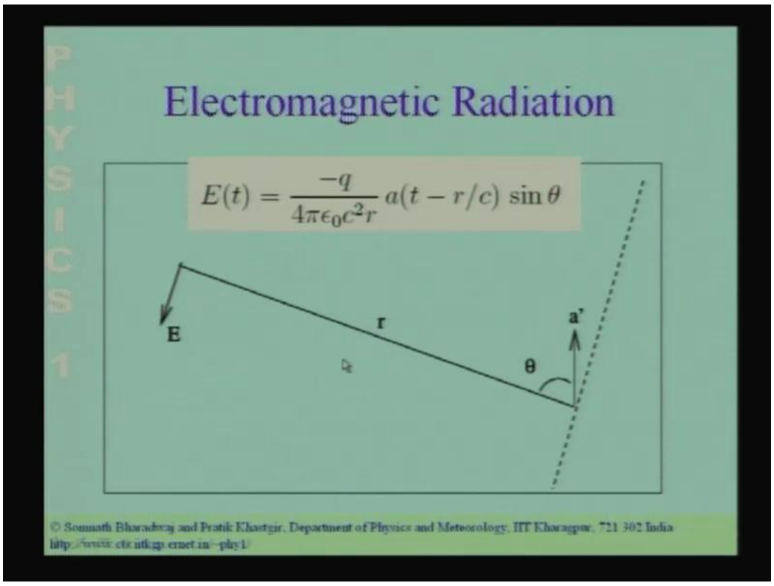

So, if you have an electron accelerating in the direction a direction of this vector shown over here. Then, in the last class we learned that in addition to the electric field which falls as 1 by r square this electron this charged particle will also produce an electric field component which falls as 1 by r. And I had told you and that it is this part of the electric field that falls as 1 by r which is responsible for electromagnetic waves, electromagnetic radiation.

So, if you ask the question what is this radiation electric field, what is the electric field. Due to this electron at the point over here the electric field at this point will be. So, the way to calculate the electric field at this point is that you should take the component of the acceleration perpendicular to the line of site from the point where, you wish to calculate the electric field to the charge. So, this is the line of site from the point where we wish to calculate the electric field to the charge. We have to take the component of the electric field perpendicular to this. Which is what the dashed line over here shows.

So, the dashed line is the direction perpendicular to the line of sight the component to the electric acceleration in this direction, is what causes the electric field over here. And the electric field is given by the expression over here. So, it is minus q where q is the charge the value of the charge divided by 4 pi epsilon naught c square r r is the distance from the point where we wish to calculate the electric field to the charge.

This into the acceleration of the charge at the retarded time, the retarded time is in the past. So, if I wish to calculate the electric field at a time t at this point. I have to look at the acceleration at a time t minus r by c. The term minus r by c takes into account to fact that, the signal takes a finite time to propagate from here to here. The signal travels at the speed c the speed of light and it takes the time r by c to propagate from here to here.

T minus r by c is called the retarded time, A is retarded acceleration. So, we have to take, we have to use the acceleration of the charged particle not at the time t, but at a rated time t minus r by c. We have to take the component of the acceleration, perpendicular to the line of site which is which is what give rise to this factor of sin theta. So, the particle the charged particle over here if it accelerates produce the electric field given by this expression.

The key point is that the electric field falls as 1 by r and it is proportional if the electric field is in the direction perpendicular to the line of site and it is proportional to the component of the acceleration. In the direction perpendicular to the line of site. So, this was the first thing that I told you.

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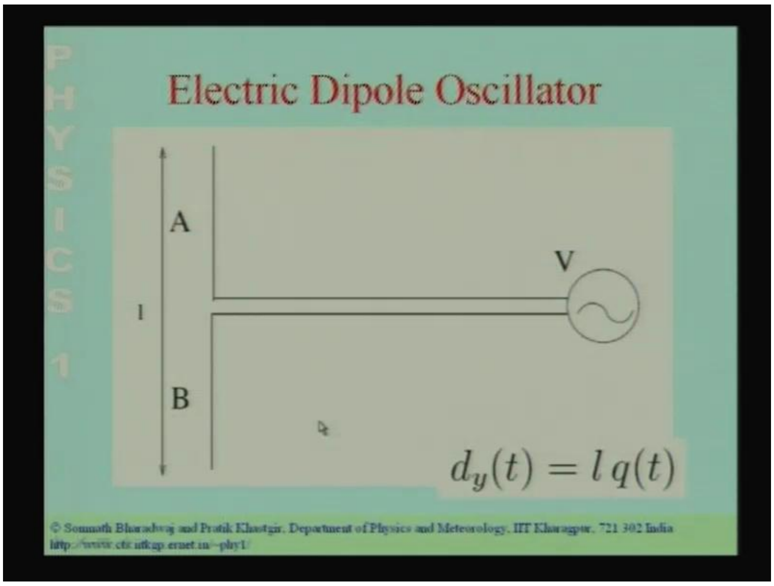

The second thing then, we moved on to a particular situation where we can apply this. And we were discussing the electric dipole oscillator. This is the device of considerable technological importance. So, we have 2 metal wires: A and B which are aligned like this and these 2 metal wires are connected to a voltage generator, which produces as an oscillating voltage at these 2 ends of these wires.

Now, if have a positive voltage here and a negative voltage here. So, the bottom of A is given a positive voltage, the bottom of B is given a negative voltage. Then, there will be an excess of positive charge at this tip of A and there will be an excess of negative charge in the tip of B. These charges the negative charge here and the positive charge here will reverse. So, there will be a positive charge here and negative charge here when, the voltage is reversed.

So, if i have a positive voltage here, negative voltage here and a positive voltage here these charges will get reversed. So, as the voltage oscillates the charges rush back from A to B. So, we have a electrons rushing up and down, from A to B and back and forth. So, you can think of it as electrons moving up and down this a single wire electrons accelerating up and down. These electrons that accelerate up and down, will produce electric fields which fall of as 1 by r.

If I am sufficiently far away, you will see an electric field component which falls of as 1 by r this is dominate thing which, you will see if you are far away. And so, we can apply the expression for the acceleration, which I just showed to you if you are sufficiently far away. And if the time the it takes for the electrons to go from here to here and then, come back. If this time period is considerably larger than the time period that light takes to cross this you can think of it, you can think of this whole device as an oscillating electric dipole. So, this is called the electric dipole oscillator.

So, when you are quite far away at a distance which is much larger than the length of this. So, when the oscillations here are quite slow you can think of this as an oscillating electric dipole. And I shall go into a little more detail of this shortly later. Now, this kind of an oscillating electric dipole has considerable technological applications. So, much of the radio receivers, radio transmitters, the antennas used over there are electric dipole oscillators.

For example: your TV you might have seen the TV antennas. They are a collection of electric dipole oscillators. They look like electric dipolar oscillator which is essentially, are long metal rod which is cut in the middle and has 2 leads connected to a voltage source or possibly to a detector.

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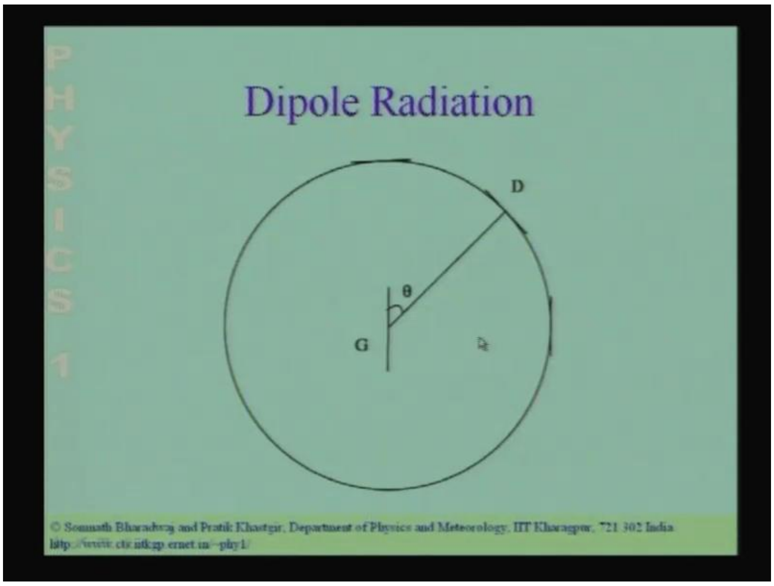

So, let me now consider the nature of this dipole radiation. So, we have a dipole over here we have this metal rod AB which I just showed you over here. And we have a signal voltage source connected to it. So, this is the generator of the electromagnetic radiation, the electrons accelerate up and down and these accelerating electrons produce electric fields elsewhere.

So, this is a common situation where I wish to transmit some kind of an electromagnetic radiation. I have a dipole oscillator and I am, I have connected a voltage source; the voltage source is the signal which I wish to transmit. So, the voltage source is driving my electrons up and down this metal, these 2 metal rods. And this gives rise electric fields elsewhere.

Now, the question is then, I now wish to study the electric field pattern So, let us consider a situation where I have a detector located elsewhere. So, this is what this picture shows us. I have a source the generator over here and I have a detector located a large distance away from this. So, where is the detector located over here? The detector also is a dipole. So, we have a dipole here and a dipole here.

The difference between the dipole here which is the detector and the dipole which is the generator is that the dipole over here. And which, is the detector is connected to a voltage detector could be an oscilloscope. So, if I the electric field produced by the generator when, it falls on a dipole located over here D when, the electric field from the generator falls on this.

If the electric field is aligned with the dipole, it will produce a voltage across the dipole and this will cause the current to flow in this dipole. So, if you connect a voltage detector then, it will measure a voltage across the dipole. So, you could connect an oscilloscope and you could measure the voltage across the dipole. So, you can think of this dipole as you can think of our radio antennas or TV antennas as being some kind of typical dipole.

So, I can use the dipole to measure the electric field produced by this generator which is also a dipole. So, let us consider the situation where I have a dipole located. Let us, say to start with over here. The dipole is aligned in the same direction as the dipole which is generating the signal. So, on the detector dipole is aligned in the same direction as the dipole which is generating the signal.

Now, the electron which is generating the signal is running rushing up and down this dipole the generator. So, let us ask the question what kind of an electric field will be produced over here? So, let us go back to our expression for the electric field. The electric field I told you at this point if the electron is accelerating over here, the electric field here will be parallel to the component of the acceleration perpendicular to the line of site.

So, this is the point where I wish to calculate the electric field over here, this is the point where I wish to calculate the electric field. The line of site from here to here is in this direction. So, the component of the acceleration perpendicular to the line of site is this direction. So, the electric field here is basically parallel to this. So, it is going to oscillate up and down, the electric field is going to oscillate up and down over here.

Then, it is going to produce a voltage in this dipole which you can measure. Now, let us consider another situation where I have a moved the dipole on a circle. So, the think of the dipole as being attached to this generator through a rod or something like that. I move it around, maintaining the same distance and move it to a different position shown over here. Now, if I keep my dipole over here and I keep it aligned like this then, the electric field then let us, ask the question. What is the direction what is the electric field here like?

So, you have to take the line of site from this point to the generator, this line over here shows us the line of sight from the point where I wish to calculate the electric field to the electron that is oscillating up and down. You have to take the component of the acceleration perpendicular to the line of site. So, you have to take the component perpendicular to this. So, the component perpendicular to this line of site is going to go down by a factor of sin theta.

So, if the electric field here is going to be parallel to the component of the acceleration which is projected perpendicular to the line of site. So, it is going to be along the tangent to the line of site which is the way the dipole is oriented. Now, let us again consider as another position for the detector. So, if i move the detector all the way over here, where it is directly overhead, directly overhead to the dipole.

Let us, repeat the same exercise. The line of site from the position where I wish to calculate the electric field to the dipole is this the particle the electron is accelerating this way it has no component, the acceleration has no component, perpendicular to the line of site. So, the electric field produced over here is 0. So, the electric field is maximum.

When, the at this point it falls as 1 sin theta as I moved further, up as I move as theta is varied is as theta. So, theta when theta is 90 sin theta has the maximum value as theta is decreased. The value of electric field falls and it is 0 over here. The direction of the electric field at this point is in this direction it is the tangent to the radius. So, it is the tangent to the line of site which is in this case the radius. So, the electric field is perpendicular to the radius which is the tangent.

So, at this point the electric field is in this direction at this point, the electric field will be in this direction at this point the electric field is 0. But this would be the direction it could be normal to the line of site, but it is has a magnitude 0. If I move a slight distance away, it will be the tangent. So, the electric field that is produced is perpendicular to the line of site this is the point which is important.

Another point, which is important is if I put the dipole instead of putting the dipole in this direction. If I were to put the dipole in the direction which is perpendicular to the electric field.

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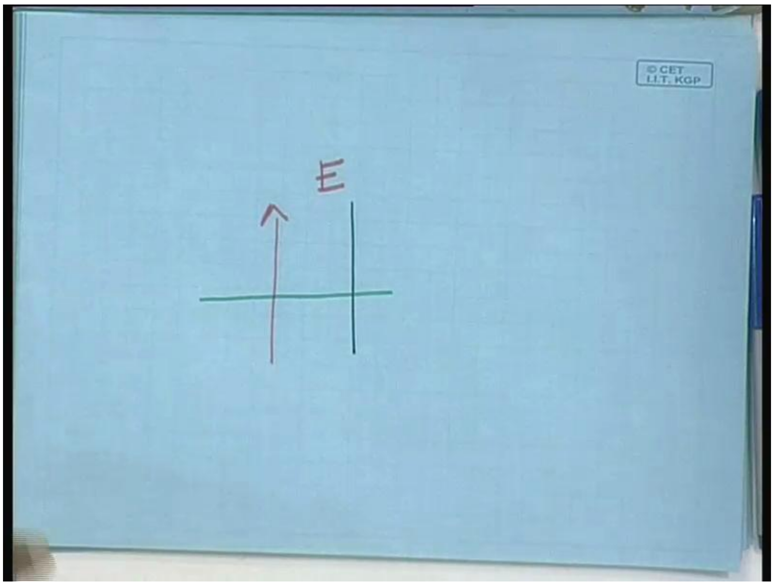

So, if I put the dipole like this and the electric field oscillates this way. So, I have a dipole in this direction and the electric field oscillates like this. Then, this electric field will not produce any voltage difference across this dipole and this dipole this detector will not be able to detect this electric field. So, this is the point which we should bear in mind.

If the detector dipole is perpendicular. So, if the metal wire AB which is which I am using to detect the electric field is perpendicular to the direction of the electric field it will not the experience any voltage difference and there will be no signal picked up here. Signal will be picked up only if the dipole is aligned. So, if the dipole which I am using to detect the electric field is aligned like this. In the same direction it is only then, that it will then there will be a voltage difference produced across this and there will be a signal picked up.

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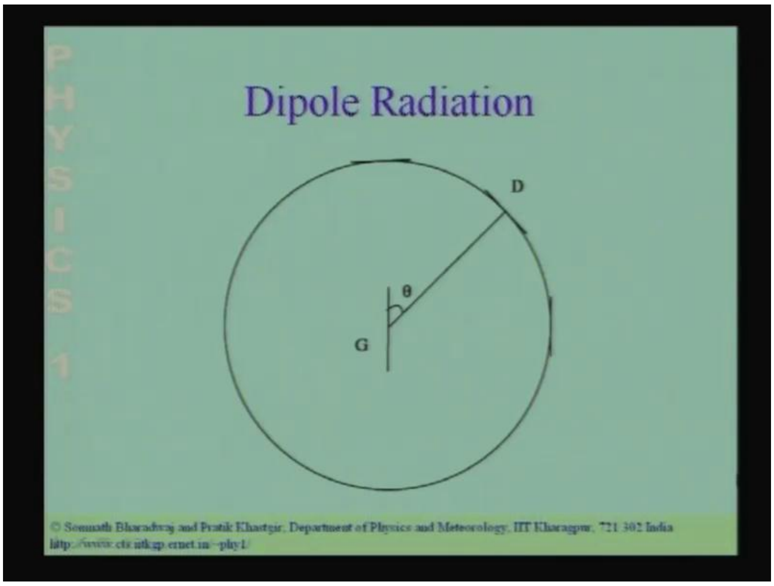

So, coming back to this picture over here if I have got to put a dipole in the perpendicular direction here I would get nothing. Right the dipole here is aligned with the electric field. So, I get a signal. So, this essentially summarizes the direction dependence of the signal of the electric field produced by the dipole oscillator.

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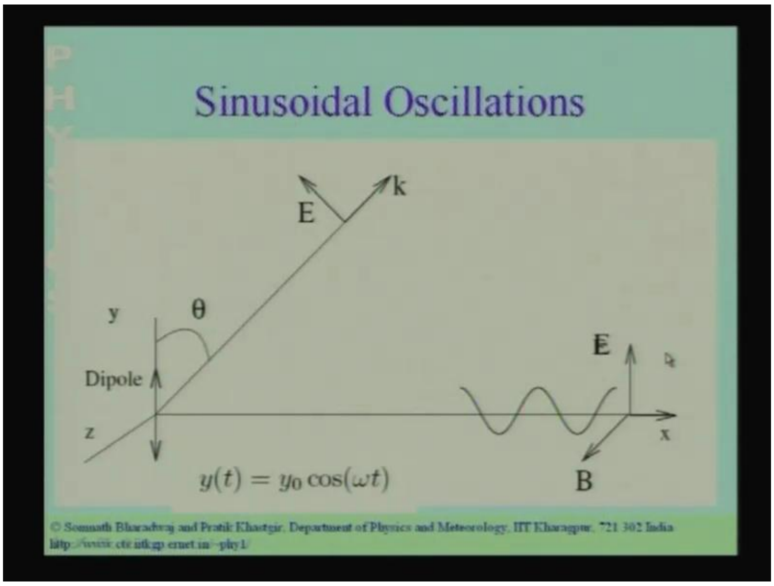

Now, we going to concentrate on a situation where the signal that I am feeding in.

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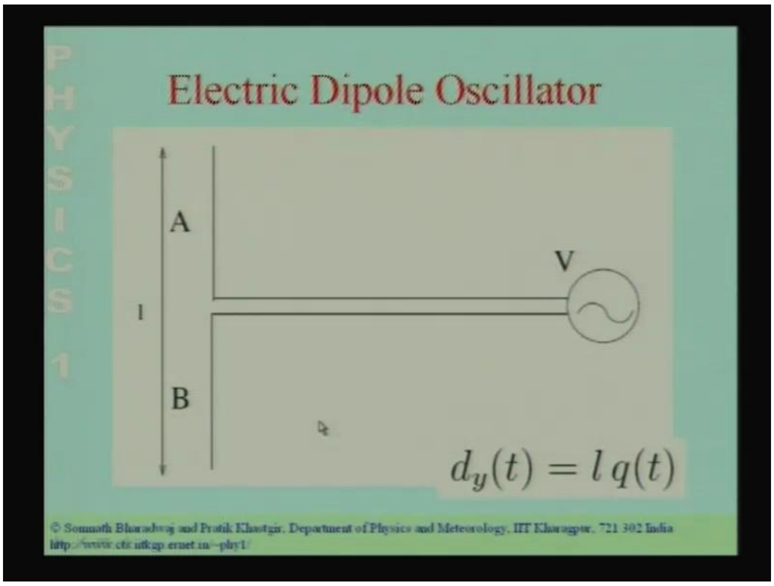

So, the signal that i am feeding in to the dipole oscillator we are going to assume that the signal is cosine is doing. So, the signal which I am feeding into the dipole oscillator the voltage generator is producing a sinusoidal voltage pattern. So, if it if the voltage generator produces a sinusoidal voltage pattern, the electrons are also going to rush and rush back and forth.

Then, the motion of the electron between those 2 wires: A and B across the 2 wires moves up and down the 2 wires is going to be sinusoidal. So, we are going to focus for the rest of this we are going to focus on this of today's lecture. We are going to focus on this particular situation. And in this situation, where the voltage oscillates as a sinusoidal. We can write down the displacement of the electron up and down this dipole oscillator.

So, Y the dipole oscillator here is aligned with the y axis and the motion of the electrons as they move up and down this we can write as y the displacement in this direction charged particle moving up and down. The displacement in this direction is yt equal to y naught which is the amplitude of the displacement into cos omega t. So, let me just remind you of the situation again.

We have this kind of an electric dipole oscillator the 2 metal rods: A and B or aligned with the x with the y axis. And we had applied, we have applied an oscillating voltage source which is oscillating in a sinusoidal fashion cos omega t. So, the electrons are also going to go up and down in a sinusoidal fashion. And we express the motion of the electrons up and down the dipole oscillator as y naught cos omega t. The electrons move back and forth along the y axis. Now, we want to calculate the electric field pattern at different points.

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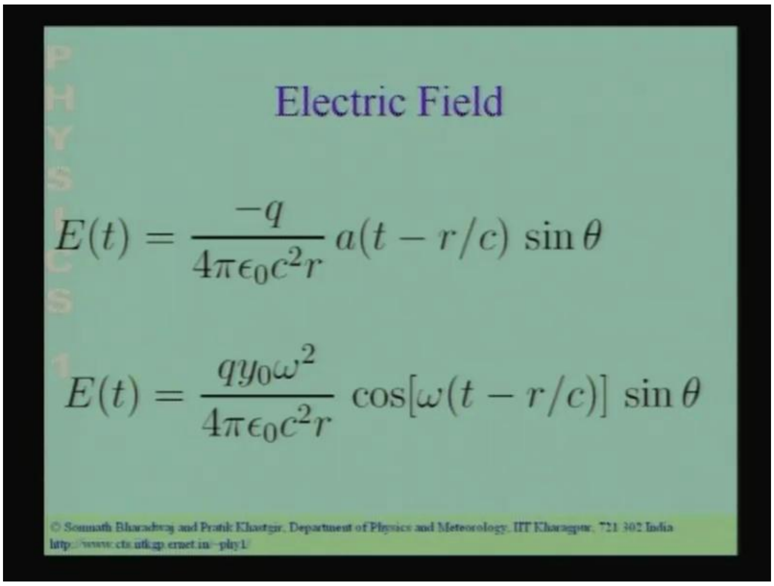

So, the electric field pattern at any arbitrary position distance r away from the dipole can be calculated in using this expression. So, the electric field at a time t a distance r away is minus q by 4 pi epsilon naught c square r. Into the acceleration at a retarded time t minus r by c into sin theta, where sin theta is the angle between the point where I wish to calculate the electric field and then, direction of the dipole.

So, in this case it is the angle with respect to the y axis. So, this is the expression for the electric field in terms of the acceleration. And in this case the particle, the displacement of the particle is sinusoidal. So, it is y naught cos omega t this is the displacement I have to differentiate this twice to calculate the acceleration. If I differentiate this twice, I pickup up a minus sign and a factor of omega square outside.

So, putting this into the expression for the acceleration, we get this expression for the electric field here. It is q into omega square into y naught by 4 pi epsilon naught c square r into cos omega t minus rc. So, is this is the cosine at a retarded time t minus rc into sin theta. So, this is the electric field this gets us the electric field at the point where we wish to calculate it and this can be applied to any at a large distance r in any arbitrary direction theta.

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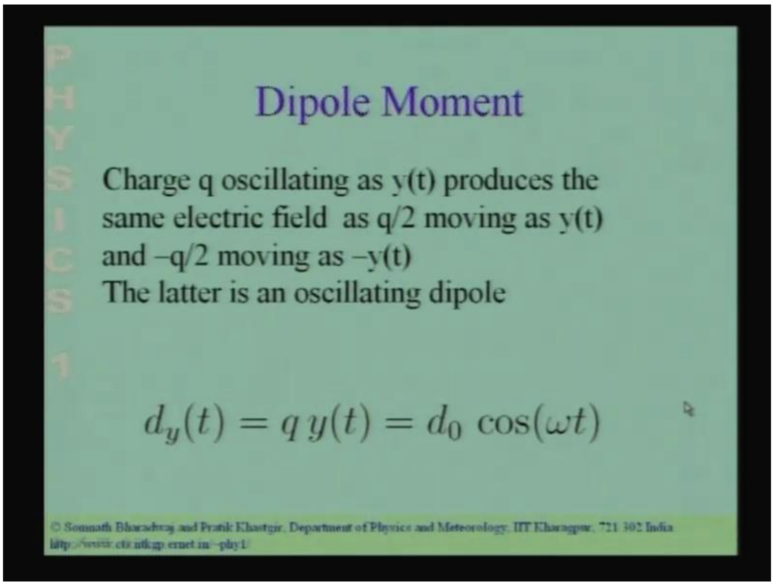

Now, it is convenient to think of this whole thing. So, we have the whole thing is that we have an electron, we have this these 2 things. So, you can affectively think of is at 1 we have single we have some charge q which is rushing back and forth up and down this dipole. Now, it is convenient for certain purposes to think of it as of 2 charges: 1 q by 2 and another minus q by 2.

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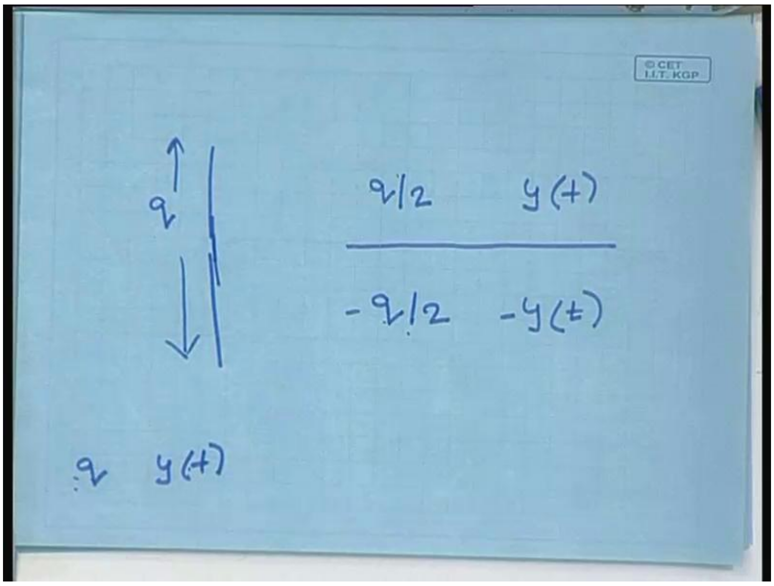

And the charge q by 2 is doing this oscillation yt which I just wrote down. And the charge minus q by 2 is doing exactly the opposite oscillation minus yt. So, a single charge moving up and down. I can think of as 2 charges: 1 of magnitude q by 2, the other also of magnitude the q by 2 with the opposite sign doing exactly opposite oscillations. Now, the point is that if I replace the a charge q with minus q and replace the acceleration a with minus a such a change does not change the electric field.

So, if I change its sign of the charge and the sign of acceleration both, the electric field is not changed. So, we are essentially using this property. So, I am replacing 1 single charge q moving up and down with y of t by 2 charges 1 q by 2 moving with yt, another minus q by 2 moving exactly opposite minus yt. So, the both of them produce exactly the same electric field and sum of these 2 electric fields is exactly the same as the electric field the radiation pattern of the part of the electric field produced by this.

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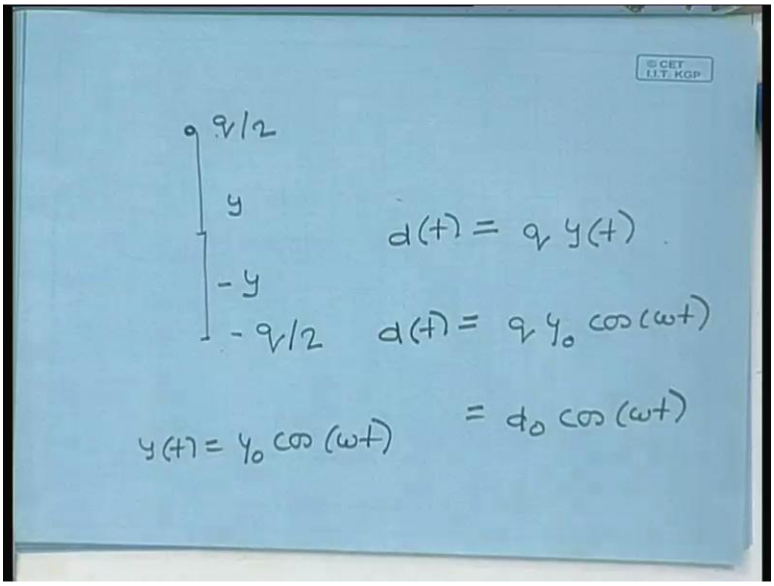

Now, if I have a charge q by 2. So, I have a charge q by 2 and it is at a displacement y. And I have another charge minus q by 2 at a displacement minus y this is a dipole with a dipole moment d this d is the charge into the distance. So, the charge is q by 2 the distance is 2y. So, this dipole moment is q into y as a function of t. So, this signal charge. So, as far as the radiation pattern is concerned the 1 by r component of the electric field is concerned a single charging move up and down and the displacement being y of t.

I can think of is 2 opposite charges of half the magnitude moving up and down. And the 1 of them moving up and the other 1 moving down by the same amount. And this can be thought of. So, the signal charge moving up and down can also be thought of as a dipole. Which is doing the exactly the same kind of a oscillation. So, the signal charge is moving as yt equal to y0 cos omega t and we can think of this as also being equivalent to a dipole dt which is q y naught cos omega t.

Which I can write as d0 cos omega t. So, which is what is given. So, what I have shown you is that, a single charge moving up and down this I can think of as a dipole which is oscillating, as an oscillating dipole. So, the dipole moment the magnitude of the dipole moment is d naught which is equal to q into y naught into cos the and the whole thing oscillates. So, it is d naught into cos omega t.

And we can now, write the expression for the electric field over here. Which we had the expression for the electric field, it can also be written in terms of the second derivative of the dipole moment. So, when we have an oscillating dipole it produces a radiation electric field pattern which falls as 1 by r. And the radiation electric field pattern, is given by this particular expression over here.

So, the accelerating charge moving up and down you can also think of as a as an oscillating dipole, as a as an oscillating electric dipole. And the expression for the electric field you can interpret in terms of an oscillating dipole. So, what we see here is that if I have an a dipole, an electric dipole positive and negative charge which is oscillating. It could be set into oscillations in in a variety of ways.

So, I could have real physical oscillations of a dipole or I could have a positive and negative charge moving up and down. I could have a dipole which is rotating, all of these situations give you an oscillating dipole and there are a large variety of other situations also. All of these can be thought of as an oscillating dipole and the electric field pattern produced by this is given by the expression over here.

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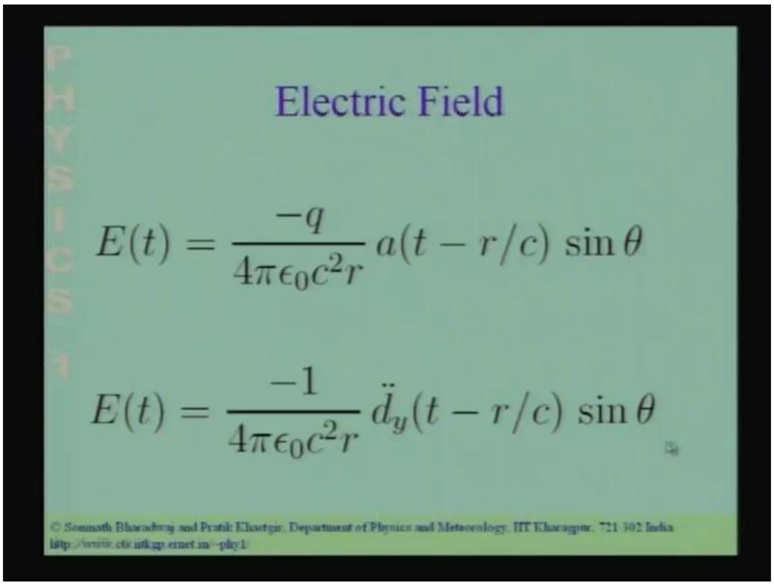

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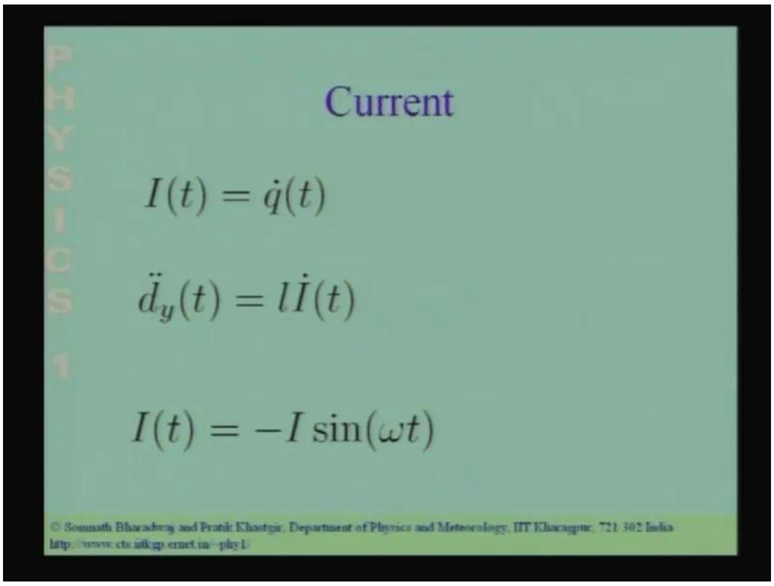

Another convenient way and another convenient way of expressing the electric field of the same, electric field is in terms of the current. So, this oscillating dipole let me is in terms of the current. So, if when I have when, I apply a positive voltage to 1 of these bits: A and the negative voltage to B and then, if the voltage is reversed there will be charges rushing from A to B. And then, when the voltage is reversed again the charges will rush back and charges keep on moving back and forth.

Now, when charge is moved you have a current. So, the current is the rate of change of the charge, the rate the current is the rate at which charge flows that is the rate of change of the charge at the 2 tips. And the dipole the rate of change of the dipole the second derivative of the dipole over here in this expression you have the second derivative of the dipole.

So, the second derivative of the dipole moment is the first derivative. So, you have the dipole is the distance the separation into the rate at which the charge changes is the displacement of the charge. So, the second derivative of the dipole moment you can write in terms of this. So, the l into i dot the rate of change of the charge, the rate of the change of the current.

And if I have current which is minus I sine omega t then, the rate of the change the second derivative of the dipole moment is going to be minus l into i into omega cos omega t.

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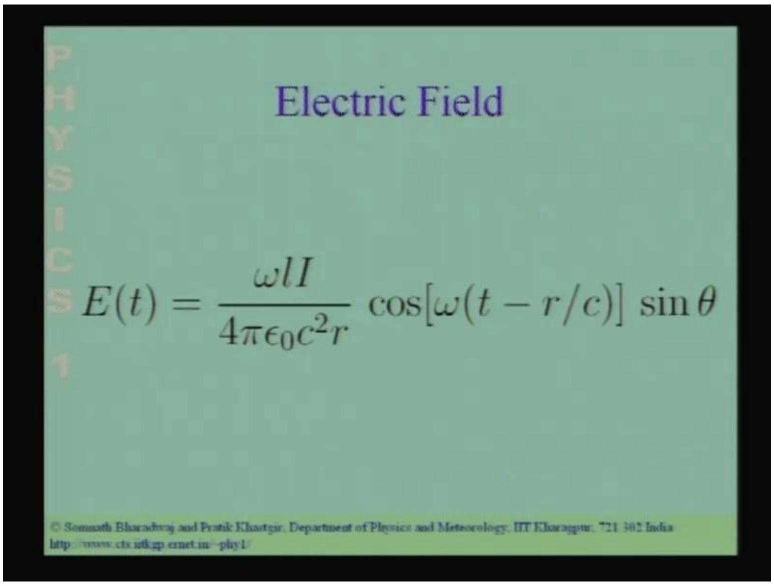

So, we can also write the electric field in terms of the current. The I here is the amplitude of the current and this is the expression for the electric field in terms of the current. So, what we have done is we have written the expression for the electric field in this situation in terms of the dipole moment and also in terms of the current. So, all of these expressions have their utility depending on the situation that you are analyzing.

A point which I should make here is, that if you look at the expression for the electric field in terms of the dipole moment or in terms of the acceleration then, the electric field depends on omega squared. So, if you double the angular frequency the electric field will go up 4 times. And this is true if you maintain the amplitude of the displacement fixed or if you maintain the amplitude of the dipole moment fixed and double the angular frequency.

But if you maintain the amplitude of the current fixed and double than angular frequency. Then, the omega dependence note that, the omega dependence is different and the electric field will only go up twice. So, this is the point which needs to borne in mind that the omega dependence depends on the variable in terms of which you are expressing the electric field. And it will have a omega or omega square dependence depending on the variable in terms of which you are writing the electric field.

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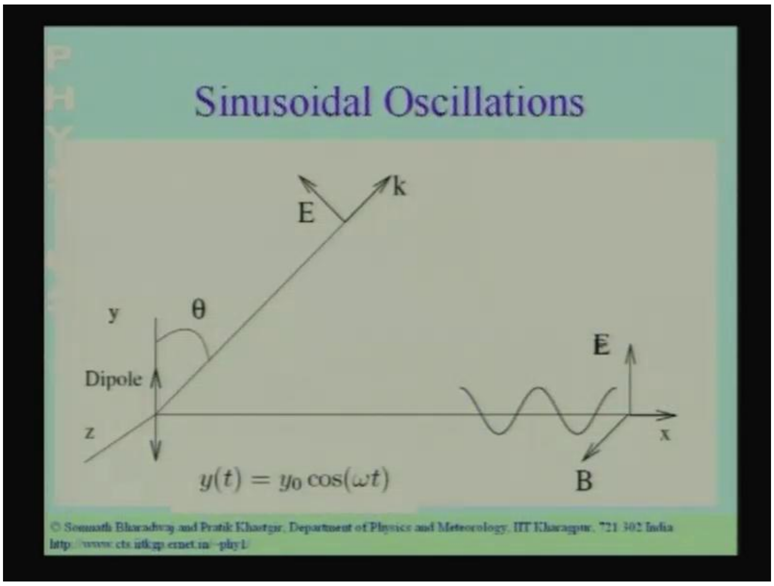

Now, let us go back to this situation which we were considering. So, we were considering a situation where I have a dipole oscillating along the y axis and we would like to calculate the electric field pattern at difference positions. Different positions means, different value of theta and different distance r. And we diagnosed a little we took a the little detour.

Where, we wrote down the electric field in terms of different variables the rate of the change of dipole moment, the rate of change of the current. But let us, now come back to a study of the electric field. So, this is the dipole which is oscillating and let us calculate the electric field at different points along the x axis for this. So, to simplify matters let us restrict our attention to points along the x axis.

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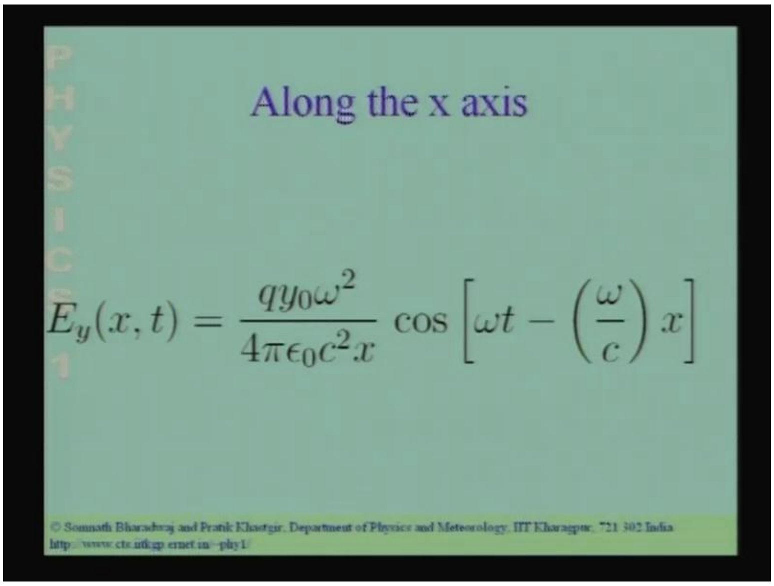

So, the expression for the electric field at different points along the X axis is this. So, we have the expression for the electric field. And we have replaced the distance r with x. And we also have this, simplification that along the when, you wish to calculate the electric field along the x axis. You have to take the component of the acceleration in the direction normal to the x axis.

In this case, the electrons, the charged particles accelerate along the y axis the dipole is aligned with the y axis. So, the charged particles accelerate along the y axis. If I take the component of this acceleration in the direction perpendicular to the x axis it is still along the y axis there is no change. The electric field will be parallel to this component perpendicular to the x axis.

So, the electric field will be along the y direction you will only have a y component of the electric field which is what I have written here. So, you are going to have an electric field along the x axis, you are going to have an electric field only along the y direction. And the magnitude of the electric field is given by this. It is has only the y component and at a distance x away you will this the magnitude of the electric field.

So, the point to note is that it will depend on the acceleration of the charges at a retarded time, the retarded time is t minus x by C. And there is a factor of omega which is there for both of these. So, this takes into account the retardation of the, retardation which has to be put in. Now, we shall consider a situation.

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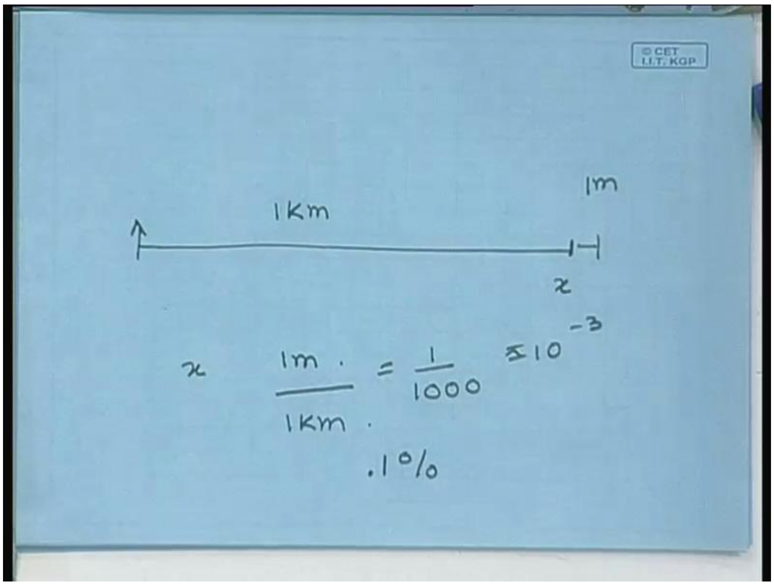

So, let me now explain to you as a the situation which we shall consider. We shall be looking at a situation where, the dipole is located at a large distance from the point where I wish to calculate the electric field. So, we could take for example: the point where I wish to calculate the electric field is located at a large distance say 1 kilometer away from the dipole.

Then, we now move from a point which is 1 kilometer away, we move to a point which is 1 meter away from this. So, this figure is not to scale you should bear in mind. So, I wish to calculate the variation in the electric field. When, I move from a point which is 1 kilometer away, to a point which is 1 kilometer and 1 meter away. So, the change in the x is 1 meter and the distance was 1 kilometer to start with.

So, you see that the fraction of the change in the distance is very small it is 1 by 1000. Which is equal to 10 to the power minus 3. So, it is a 0 1 percent change in x. So, if I move 1 meter from 1 kilometer to 1 kilometer plus 1 meter, there is a 0 1 percent change in x. Now, let us look at the expression for the electric field and ask the question how will the electric field vary? If I change x by this small amount.

So, if I change x by this small amount the term outside over here is going to change by only 0 1 percent. And I can think of this term as being roughly constant. The change in x is only 0 1 percent. So, this term here is also going to change by only 0 1 percent. But look at this term the cosine term. If x changes by point one percent it is not guaranteed that this the change in this term is going to be small there could be a very large change in this term over here. And the magnitude of this change is going to depend on the value of omega by c. Now, if omega by c is very large then, a small change in x will cause a large change in the phase. So, it will cause a large change in the argument of this cos term and this change in the argument of the cos term, may cause a significant oscillation. See cosine is oscillating

So, a small change in x may cause a well a considerable oscillation in this cosine term. If omega by c is a large number. So, let us consider the situation where we can ignore the change in x in the first term, but we have to take into account the change in x for the term inside this cosine. So, I can think of this as a constant, but I cannot think of this x as a constant.

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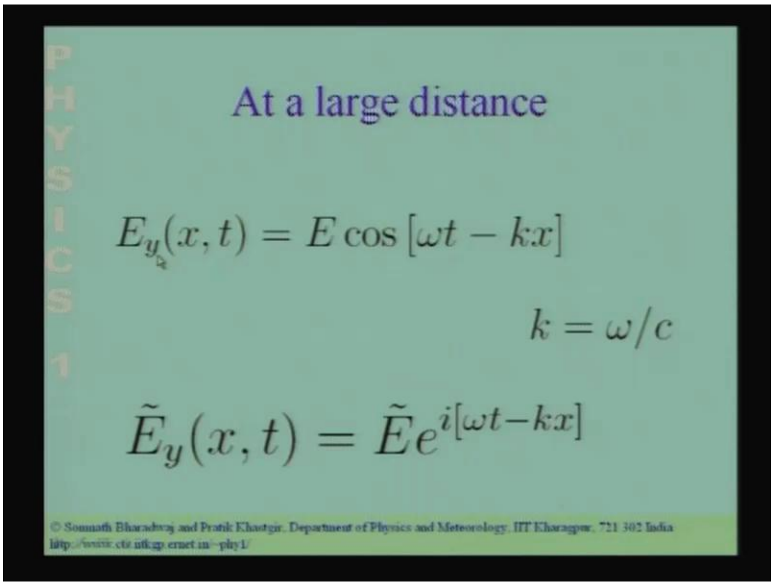

So, if I make this assumptions. So, if I work in this regime we can write down the expression for E the y component which has a y component. So, the electric field which has only a y component the expression for that, as i vary x and as a function of time is a constant into cos omega t minus kx where I have used k to denote omega by c. So, I have used k to denote this coefficient omega by c which multiplies x.

So, the point to note is that the expression for the electric field. If I vary x by a small amount at a large distance from the dipole oscillator is just the expression for the sinusoidal plane wave which we had studied earlier. So, this is the expression for the sinusoidal plane wave which we had studied earlier. So, what does it tells tell us it tells us that if we give a sinusoidal voltage to this dipole oscillator far away it produces an electric field pattern.

So, at a fixed time this electric field pattern is sinusoidal and as time varies the whole sinusoidal electric field pattern will move forward. The electric field is only along the y axis and it has a sinusoidal pattern like this as time increases, this whole thing will move forward it will behave like a sinusoidal plane wave. So, this oscillating dipole produces a sinusoidal plane wave at a large distance from the dipole.

So, what we see here is that if I have a sinusoidal, if I have a dipole oscillating like this and it is doing sinusoidal oscillations it will produce a sinusoidal plane wave at a large distance. So, if I go a large distance away from this oscillator. The electric field pattern produced by this will be a sinusoidal plane wave. The electric field pattern at a fixed time will look like the sinusoidal pattern shown over here. And this whole pattern will move forward in time as we have studied in the lecture on sinusoidal plane waves.

So, coming back to our expression for the electric field you can write it in this form omega t minus kx where we have identified the wave number k with omega by c. This is a sinusoidal plane wave which, propagates along the plus x direction. Now, let us ask the question what is the wave number? The wave number here is omega by c. Now, once you know the wave number and the angular frequency, you can calculate the phase velocity and the phase velocity is omega divided by k. So, omega divided by k gives us c.

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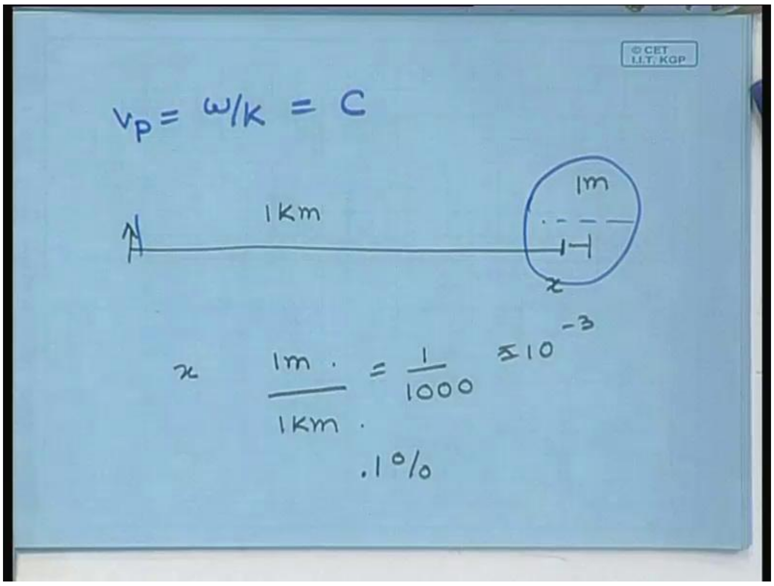

So, you find that the electromagnetic wave in this situation, the electromagnetic wave has the phase velocity which is the speed of light. So, the dipole oscillator over here if that be the sinusoidal voltage produces a sinusoidal plane wave at a large distance. So, over here this the electric field pattern produced by this is a sinusoidal plane wave which keeps on propagating outwards.

Then, you could express this in the complex notation. So, E tilde y as a function of x and t is E tilde e to the power i k omega t minus kx, this complex amplitude has both the magnitude and the phase of the electric field.

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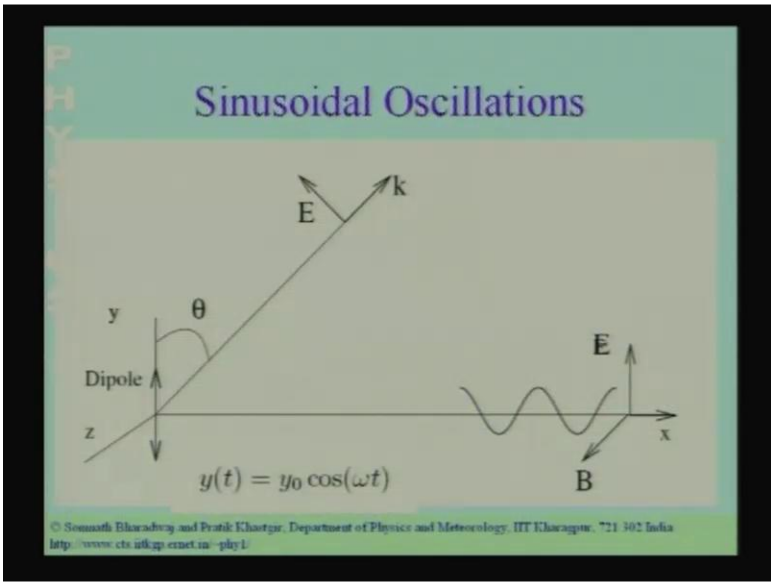

Then, Coming back to the our picture we have the dipole oscillating like this. And if you ask the question, what is the electric field at some arbitrary point here far away. Then, you have to take the component of this oscillation that it is the component of the y axis in the direction perpendicular to the line of site. So, you have to take the component perpendicular to this and this is the direction of the electric field which is shown over here.

So, the electric field is going to oscillate perpendicular to the direction of line of site. So, it is going to oscillate in this direction. The wave is going to propagate along the line of site. So, which is the direction k this is the wave vector it points from the dipole to the point where I have calculate the electric field. So, the wave is going to propagate in this direction and the electric field is going to oscillate in this direction.

So, it is magnitude is going to be smaller than the magnitude of the electric field here. And this is going to be smaller by a factor of sin theta, but theta is the angle between this direction and the dipole. And if I look in this direction theta is going to go to 0. So, sin theta becomes 0, there will be no electric field produced by the dipole oscillator in this direction.

So, for any other direction other than these this dipole oscillator far away is going to produce a sinusoidal plane wave. The magnitude of that, the magnitude of that sinusoidal plane wave is going to fall as sin theta.

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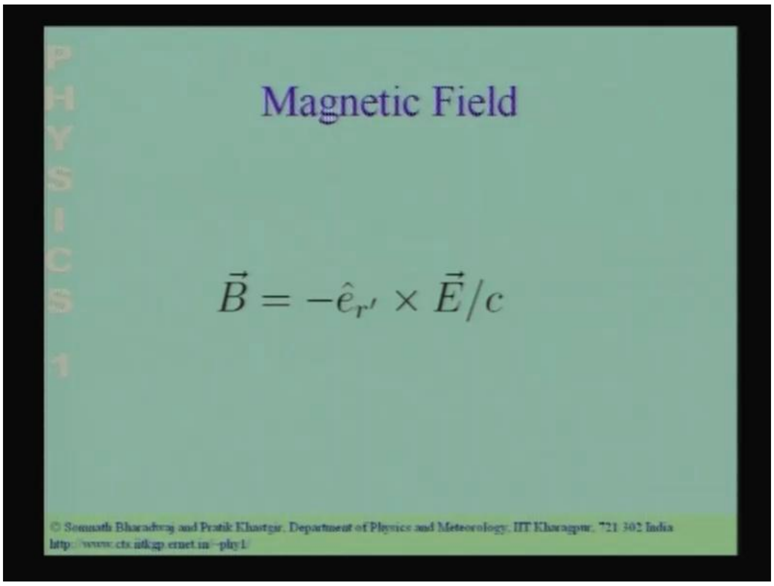

Now, next let us calculate the magnetic field. The expression for the magnetic field is given over here. You have to take the unit vector along the line of site to the dipole. And do a cross product with the electric field divide by c and there is a minus sign here. So, in the problem which we are dealing with this is the point. Let us, again go back to the x axis. The line of site is along the minus x axis. So, the E cap vector is minus i.

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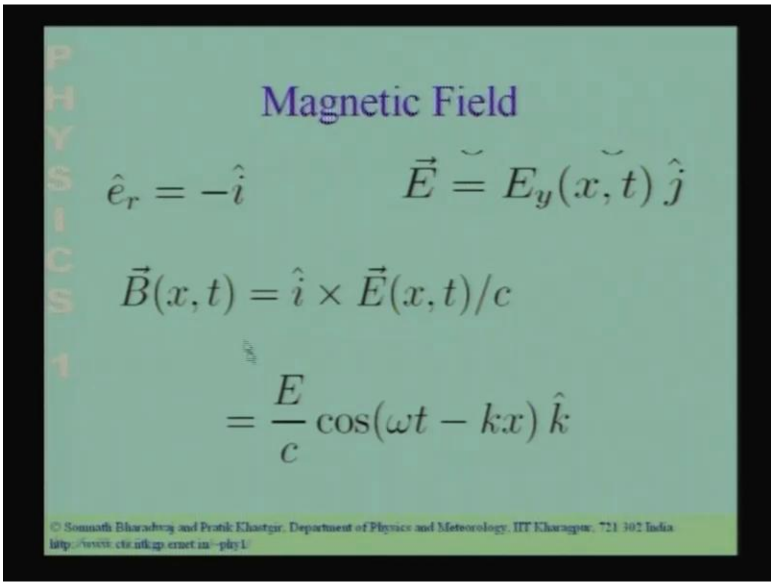

So, the E cap vector is minus i and the electric field is along the y axis. So, it is along j the magnetic field is going to be minus E cap which is i cross E by c. So, the magnetic field is E by c, the magnitude of the magnetic field is E by c. It is in the same phase so, it has cos omega t minus kx and it is in the z direction i cross j is the unit vector k. So, it is in the z direction.

So, going back to our picture. Along the x axis the electric field is going to oscillate in the same direction as a dipole which is along the y axis. And the magnetic field is going to be perpendicular to both, the direction of the propagation of the wave and the electric field the magnetic field is going to be along the z axis. So, this is a typical feature of electromagnetic waves.

In electromagnetic waves, the electric the wave propagates in a particular direction. The electric field is perpendicular to that and the magnetic field is perpendicular to both the direction of propagation and the electric field. The oscillation of the electric field and the magnetic field or both in exactly the same phase. The magnitude of the, magnetic field is a factor one by c smaller than the magnitude of the electric field. So, this is the feature which is typical of all electromagnetic waves.

So, in this part in the previous lecture and this part of the lecture we started of with a loss which govern the electric field and magnetic field produced by an by a charge. And I showed you that, there is a term which falls as 1 by r. So, this is term arises only when there is an accelerating charge and if I have a sinusoidally accelerating charge or a dipole which is oscillating it then, I showed you that such a thing produces a sinusoidal plane wave and electromagnetic wave at large distances.

Then, this electromagnetic wave has a direction of propagation the electric field is perpendicular to that the magnetic field is perpendicular to both electric field and that direction of propagation and it is in phase with the electric field. So, this is very generic feature of electromagnetic radiation. And I also showed you, how such electromagnetic radiation can be produced, how it can be generated using dipole oscillators. Let us, now move ahead.

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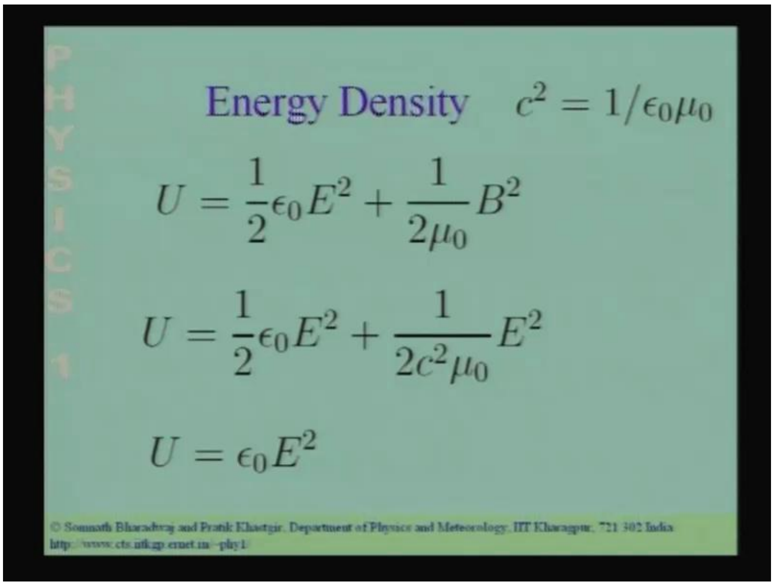

So, let us calculate the energy density in this electromagnetic radiation. Now, all of you must have already learnt that energy that there is some energy in electric and magnetic field configuration. And the energy in the electric field configuration is half epsilon naught e square. The energy in the magnetic field configuration is half b square by mew naught.

So, let us now calculate the energy density that is energy density. So, let us now calculate the energy density in this electromagnetic wave produced by the oscillating dipole. So, this is the expression for the energy density as I just told you which I am sure is familiar to all of us. And for the electromagnetic wave we saw, that the electric and magnetic field they are not independent they are both produced by the same source.

The magnetic field the magnitude of the magnetic field, is the magnitude of the electric field divided by c. So, I can replace the term arising from the magnetic field and write it in terms of the electric field as 1 by 2c square mu naught square into E square. We also know, that the constants epsilon naught and mu naught are related to c square they are not independent, they are related to c square and c square is 1 by epsilon naught into mu naught.

So, using this expression we can write the expression for the energy density in terms of the electric field as epsilon naught E square The 2 terms exactly turn out to be exactly equal. The electric field and the magnetic field it turns out contribute the same amount to the energy density and the energy density is epsilon naught E square. So, we let us just go back to the situation we have an oscillating dipole or we have a charge going up and down that an angular frequency omega.

So, this produces an electric field far away which also does oscillation that exactly is the same angular frequency. So, the electric field is also oscillating at an angular frequency omega. Now, the instantaneous energy density in the electric and magnetic field, the magnetic field is also oscillating at the same frequency. So, the instantaneous energy in the energy density in this electromagnetic field we just calculated that it is epsilon naught into E square.

So, if the electric field oscillates with the angular frequency omega, the energy density depends on E square. So, we have already seen right in the first lecture that the energy density is going to also oscillate. The instantaneous energy density is also going to oscillate and it is going to oscillate at twice the angular frequency of the electric field. So, the instantaneous energy density that is what we have calculated here is going to oscillate at twice the angular frequency at which the electric field is oscillating.

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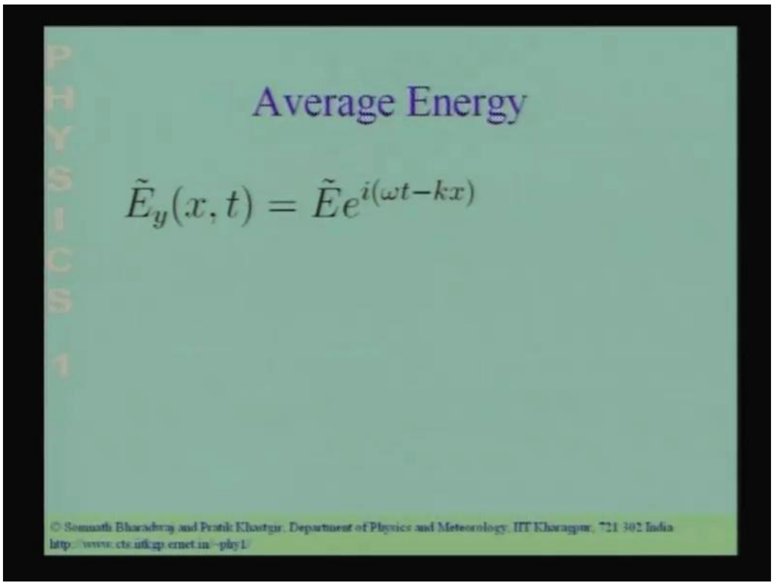

Now, the quantity of interest is not the instantaneous in most situations the quantity of interest is not the instantaneous energy density, but the time average energy density. In most situations, the electromagnetic field oscillates quite fast and the quantity that we measure is not the oscillating energy density, but the time average energy density. For example: the bulb which illuminates the room is the is emitting radiation, this radiation is oscillating at a frequency the value of that frequency we shall discuss after 1 or 2 lectures.

But it is an oscillating as we have seen the electromagnetic radiation is an oscillating electric field. So, the value of the electric field is oscillating. But we see a steady illumination that is because, our eye and most optical devices measure only the time average energy density. They the record the energy over a time period which is much faster than the time at the, rate at which the electric field is oscillating.

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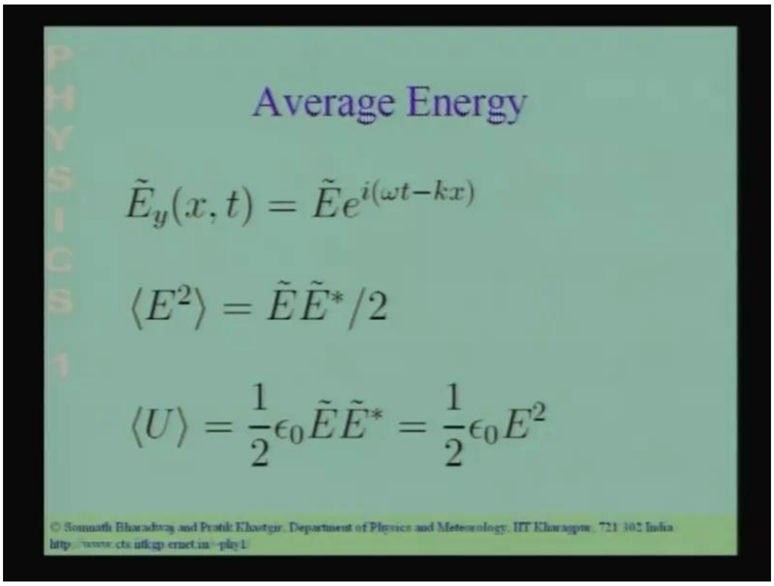

So, they only measure the time averaged energy density. So, we have we wish to calculate this quantity which is of practical interest. So, to calculate this we express we electric field in the complex notation as you can see here. And then, the time averaged energy density is the, time average of E square which in the complex notation is E into E star by 2.

So, this gives us gives us the average energy density it is half epsilon naught E square. Where E is the magnitude of the oscillating electric field. This the expression for the average energy density in an electromagnetic radiation.

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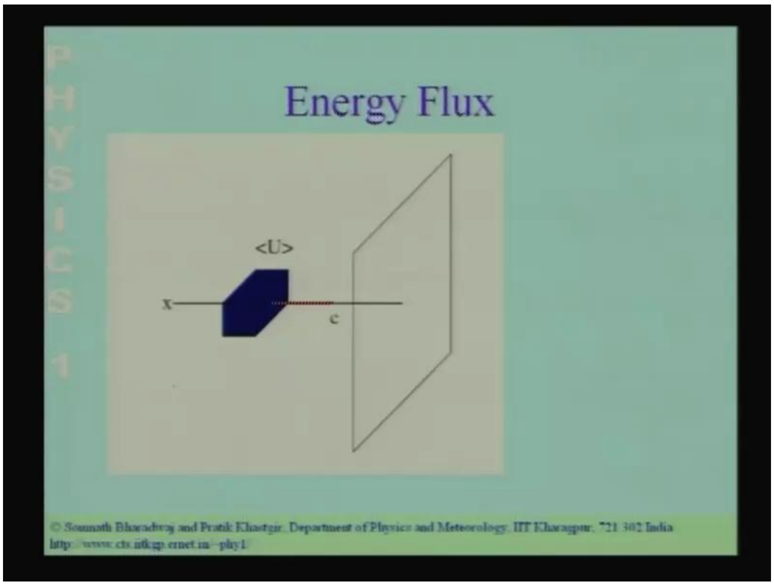

Now, remember that the we have, we have energy in the electromagnetic field, but the electromagnetic field pattern is a sinusoidal plane wave. The sinusoidal plane wave is propagating forward in this case along the x axis we were discussing waves along the X axis. So, the sinusoidal plane wave is propagating forward along the x axis and it is propagating forward at a speed c.

So, the energy density in this electromagnetic field does not remain fixed over here the whole thing propagates forward at a speed c the speed of light. So, let us now ask the question we take a surface perpendicular to the direction in which the wave is propagating. So, in this case the wave is propagating along the x axis. So, we take a surface perpendicular to the x axis. And ask the question how much energy crosses this surface, crosses a unit area of this surface per second.

So, what is the energy density? By energy density we mean, the energy the energy flux. So, this is what energy flux: the energy flux is the energy which crosses per unit area of the surface in a unit time. And this is the energy density into c because, the whole thing is moving forward at a speed c. So, if you take a unit area and ask the question how much energy will cross it in a second. The amount of energy that will cross it in a second is the energy density into the speed at which it is moving which is c. So, this gives us the energy flux.

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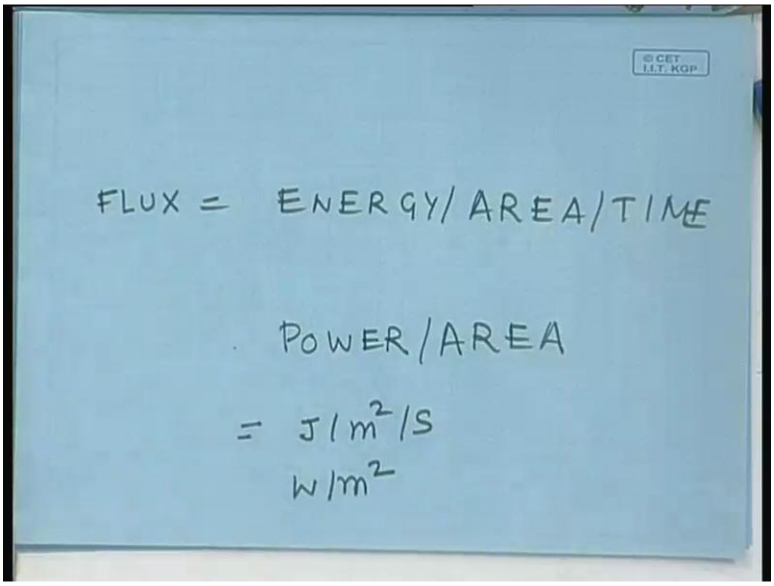

Let me, just elaborate a little of this. So, energy flux is the energy per unit area per time per unit second. And this is the quantity which is of considerable practical importance because, whenever we have a detector for example: we have a detector of some sort which is measuring radiation. The quantity that we detect is the amount of radiation, the amount of radiation energy per unit area.

So, we have the area of the detector if i the double the area of the detector, I will get twice the radiation of the energy. So, the amount of the energy per unit area of the detector per second. And this has a unit of so, energy per unit time is power this is also you can think of it as power which crosses per unit area. And this has got units of joules per meter square per second or you can also say, that it has units of watt per meter square.

The power which crosses per unit area, the power of the radiation the power from the radiation per unit area. And this is the energy density into c.

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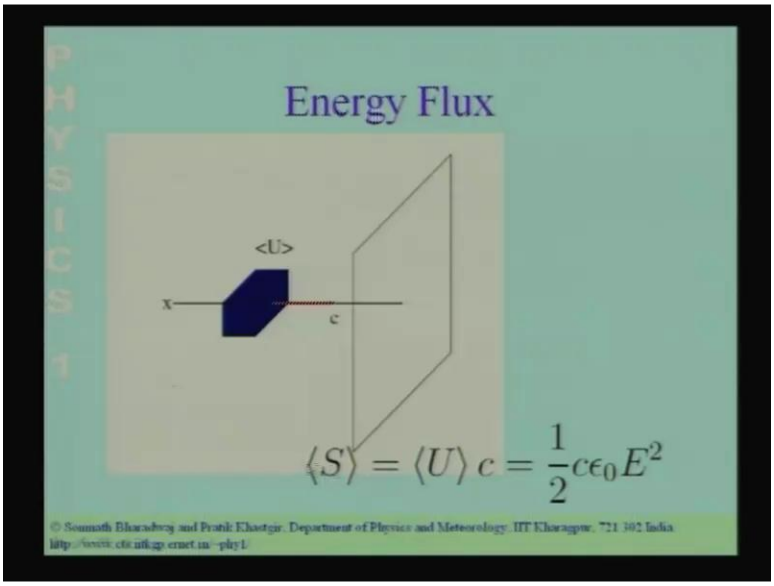

So, calculating this energy density into C we have half epsilon naught into c into E square. Which gives us the energy flux. You should also remember, that the energy flux is a vector, it is a vector in the direction. So, this energy is moving in the direction in which the wave is propagating. So, it is a vector and if I want to calculate it at some other point the energy would be moving in some other direction.

So, along the x axis the energy is flowing along, the x axis it is the energy density into c. If I want to calculate the energy flux here it could be a vector along the this direction along the direction of the wave. And it would be the energy density here into c. It should be in this direction. So, in the next let me stop here for today. In the next class, we shall calculate this expression for the flux and go ahead for the further from there.