Physics – 1: Oscillation and Waves
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Lecture - 01
Simple Harmonic Oscillators

Welcome to this course Physics 1.

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Let me also wish you a Happy New Year 2007.

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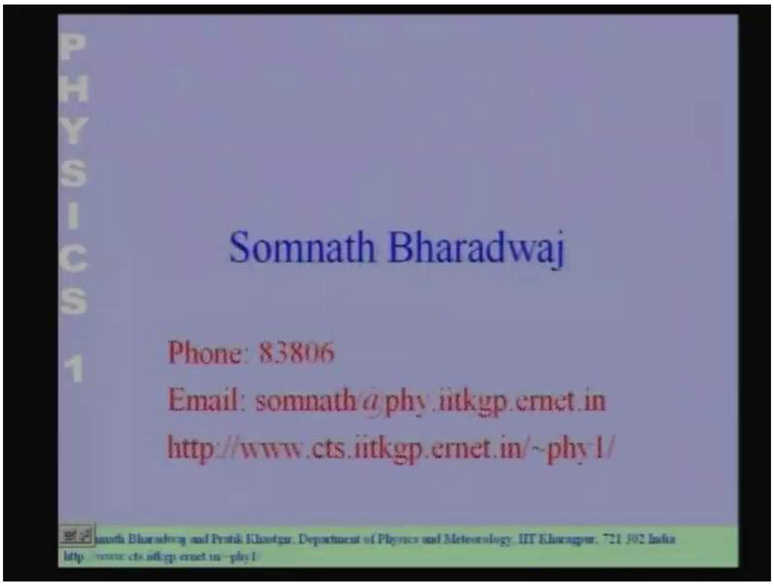

My name is Somnath Bharadwaj and in case you should need to contact me, this is my phone number and my email id. All material regarding this course will also be available at this following website displayed over here.

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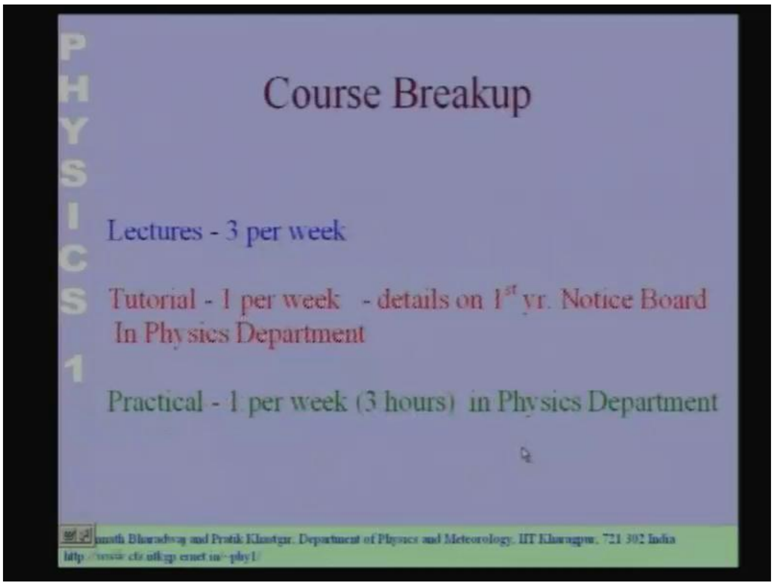

The course breakup is as follows: there will be three lectures per week in addition there will also be a tutorial. The division of the tutorial sections is displayed in the notice board in the physics department. There will also be one practical class every week this will be held in the physics laboratory in the physics department.

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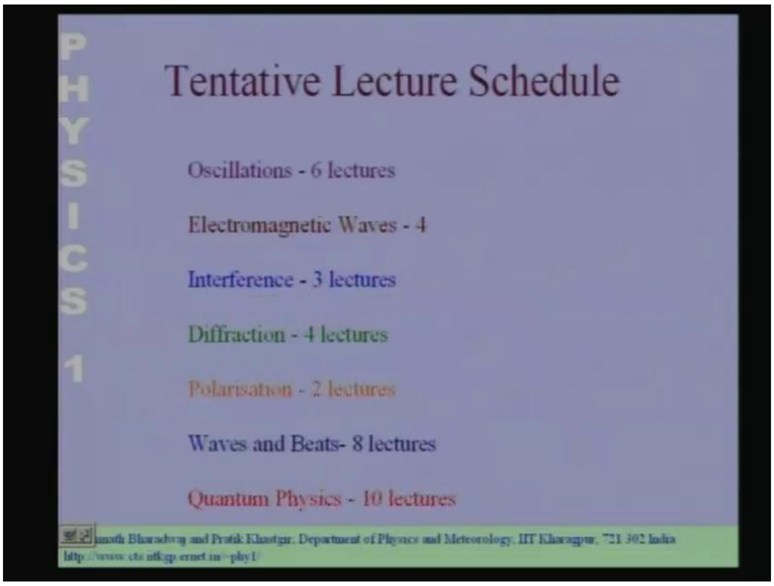

The tentative lecture schedule is as follows: we will start off with 6 lectures on oscillations and then move on to electromagnetic waves, then take up interference, diffraction, polarization, waves and beats. And finally, devote the rest of the lectures to quantum physics.

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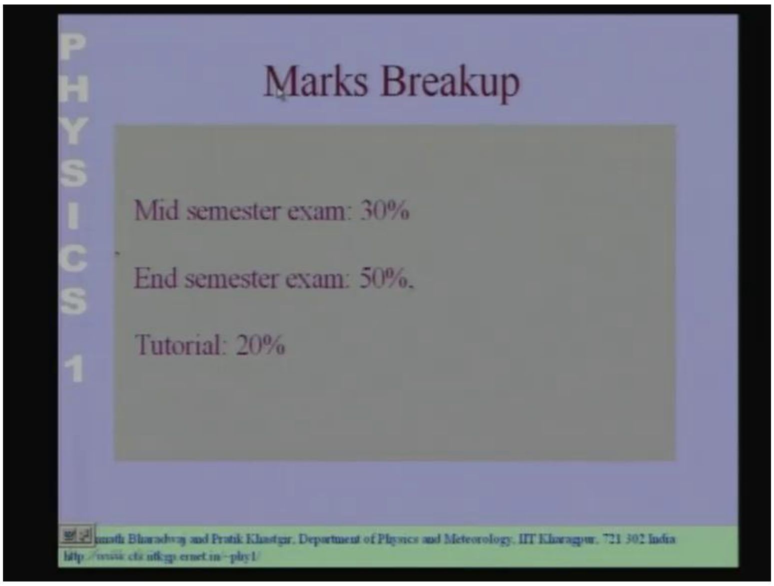

The breakup of the marks is as follows: the mid semester examination carries 30 percent weight the end semester exam 50 percent and the remaining 20 percent of the marks shall come from the tutorials.

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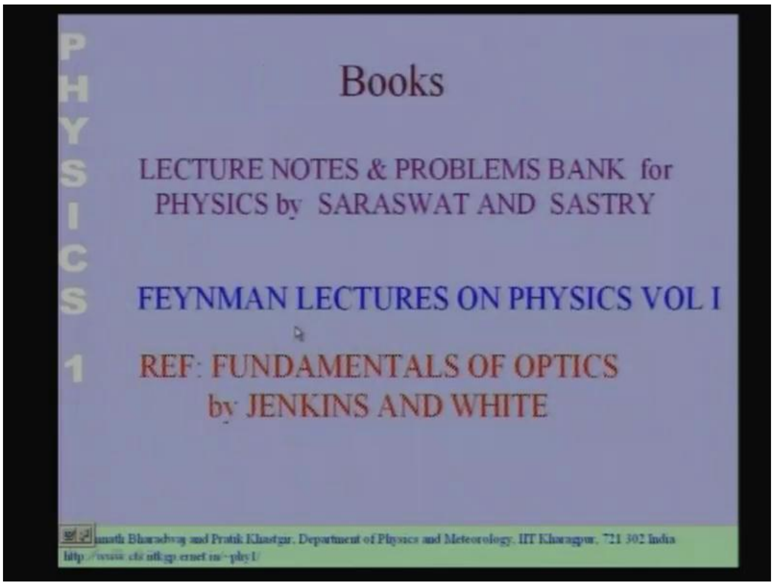

Books which you could refer to the lecture note where, professor Saraswat and Sastry this is the main reference material. You could also refer to Feynman lectures on physics volume 1 and for the part on optics you could refer to the book by Jenkins and White.

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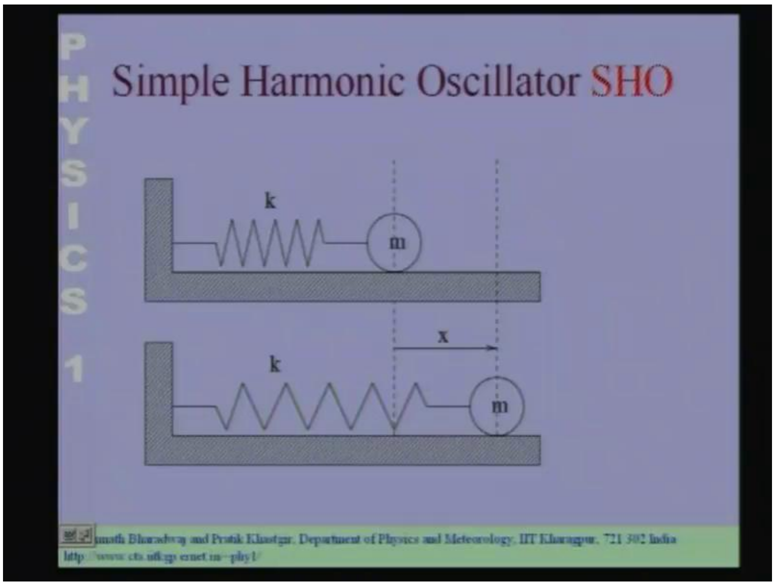

So, let us start off on the course. We start with the discussion of simple harmonic oscillators SHO. Oscillations play a very important role in all aspects of our life. There is hardly anything that does not oscillate. Fox example, your mood if I to disturb you your mood would oscillate for some time and then come back to normal. The prices of shares, this building everything would oscillate if we disturbed it.

We begin with the study of simple harmonic oscillations which are the simplest possible situation. And by the study of this simplest possible situation we hope to get some insight which will provide to be useful in the study of more complicated oscillations as we which of which, we shall take examples as we go along. We consider the spring mass system as an example of this simple harmonic oscillator.

There is the mass m attached to a rigid support with the spring. The spring has a spring constant k. The position shown over here is such that the system is in equilibrium the spring is not stretched and if the mass would remain where it is if it were left at this position. If you disturb the mass by pulling it a distance x away from the equilibrium position what happens, is that there is the force which tries to pull back the mass to the equilibrium position.

This force is proportional to the displacement through which you have pulled the mass and the constant of proportionality is the spring constant k. And the force opposes the displacement that is the force is in the direction opposite to x. Tries to pull back the mass to the equilibrium position. This force law for the spring was first proposed by Hook and it is called Hook’s law; I am sure this is familiar to all of us.

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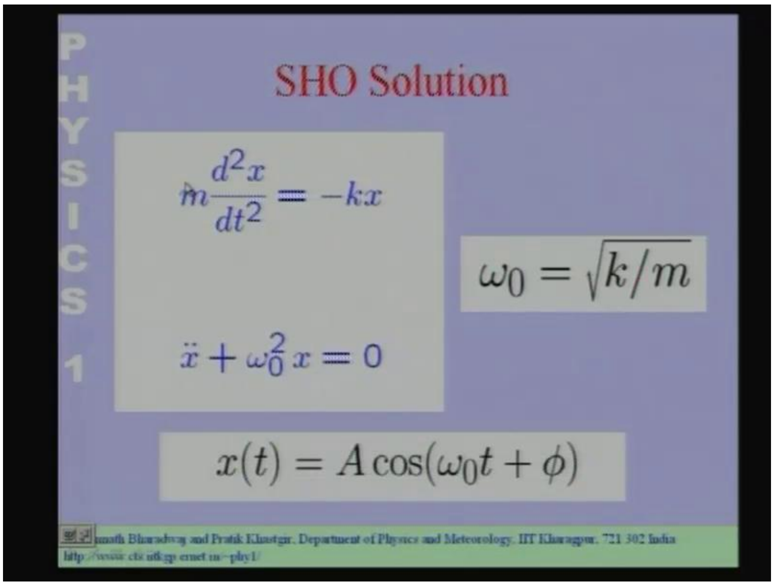

So, if you write down the equation of motion for this mass which is what we have here. The mass into the acceleration d square x d t square is equal to the force which is minus k x. So, this is the equation of motion which governs the motion of this spring if we disturbed it and leave it. This equation of motion can be cast into this form where, the dots on top denote derivative with respect to time. So, 2 dots and top indicate 2 time derivatives. So, this is the same equation as this 1 where we have defined omega naught a constant which is the square root of the spring constant divided by the mass.

So, this equation is now x double dot plus omega naught square x is equal to 0. I am sure this equation also is familiar to all of us and the solution of this equation is A cos omega t plus phi. I am sure that this 2 is familiar to all of us it is very straight forward to substitute the solution in this equation and check that it indeed satisfies this equation. Let us take a closure look at the solution the solution has three constants which appear in it. We shall discuss these constants 1 by 1.

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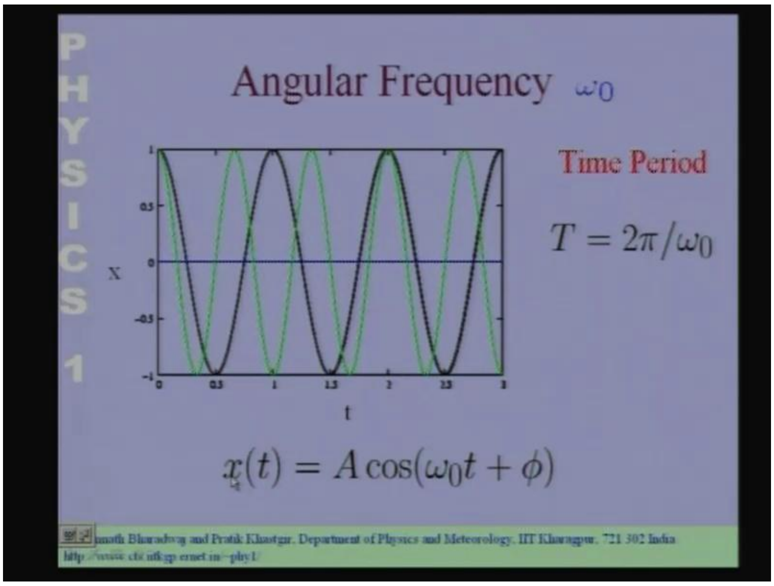

Let us take up the role of the angular frequency omega naught first. Omega naught defines the time period of the oscillations cosine the function cosine repeats whenever, this argument assumes a value 2 pi. So, every 2 pi this function repeats itself, it is a periodic function as you can see over here. And the period is 2 pi whenever the argument goes through a value from 0 to 2 pi the function repeats. So, you see that the time period of this oscillation of x t is essentially 2 pi by omega naught.

So, omega naught defines the time period of the oscillation through this relation shown over here. Now, let us see what is omega naught for the oscillation shown over here. If you look at this oscillation you see that it repeats after a time period of 1. At 0 the oscillation has a value 1 it again assumes the value 1 after a time, when 1 when t is equal to 1 it comes back to the same value. So, if the time period t is 1 omega naught has to be 2 pi.

So, the oscillation shown over here as omega naught equal to 2 pi; omega naught decides the period of the oscillation. Can you tell me what the time period for the green curve is? If you look at the green curve you will see that it is oscillating faster than the black curve which I had shown you earlier. While the black curve repeats after 1 second the green curve completes 1 oscillation before 1 second. Let us look at the green curve a little closer, it completes 1 oscillation before 1 second, it completes 2 oscillations before 2 seconds and it completes 3 oscillations exactly at 2 seconds.

So, the green curve does 3 oscillations in 2 seconds. So, the time period is 2 by 3. If the time period t is 2 by 3 then omega naught has to be 3 pi. So, we see that there are 2 curves over here: 1 which has omega naught equal to 2 pi the black curve it has a time period of 1. We have the green curve which has omega naught 3 pi it has a time period of 2 thirds.

It is very important for both scientists and engineers to be able to see an oscillation and determine it is time period and from the time period to determine omega naught. And we have briefly discussed 2 examples of how to go about during this. Let us now, discuss the other 2 constants which appear in this solution for the simple harmonic oscillation.

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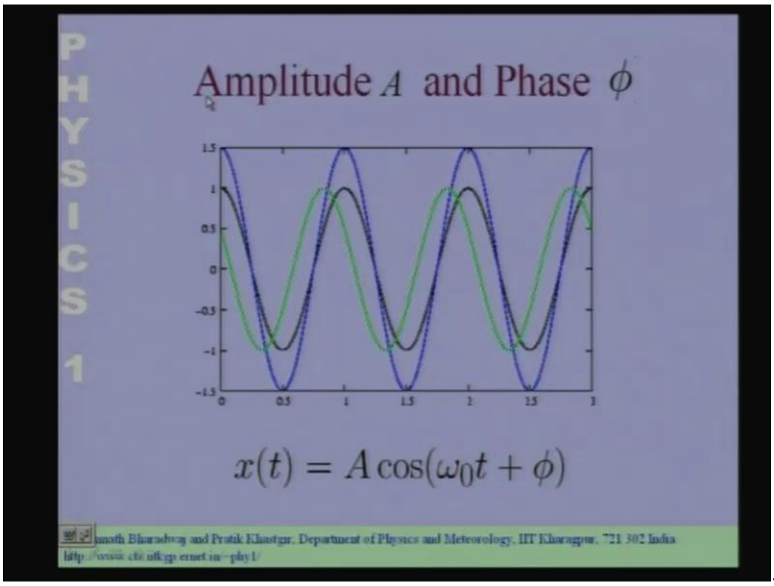

The next constant which we shall discuss is the amplitude A it occurs outside this cosine. The amplitude determines how much the particle oscillates it determines the amplitude of the oscillation. Let us first take up the black curve shown over here notice that the black curve oscillates between plus 1 and minus 1. What is the amplitude of the black curve?

The amplitude of the black curve is what the maximum value which cosine can take is 1 and the minimum value which cosine can take is minus 1 which basically, tells us amplitude for the black curve is minus 1. Let us now, look at the blue curve how does the blue curve differ from the black curve. The blue curve has a larger amplitude it is maximum value is 1.5 and it is minimum value is minus 1.5. So, the blue curve you see has a different amplitude from the black curve the amplitude is larger and the amplitude is 1.5.

Let us now look at the green curve. How does the green curve differ from the black curve? The green curve has exactly the same curve amplitude as the black curve. The amplitude of the green curve is also 1 it oscillates between the plus 1 and minus 1. But notice that at t equal to 0 the green curve has a different value from the black curve. So, this difference arises because of the difference at the phase. What is the phase of the green curve?

You can determine this by looking at the value at t equal to 0. At t equal to 0 the green curve has the value half for what value of the phase will cosine phi a value half. You can easily guess that this will occur for phi equal to 60 degrees or pi by 30. So, the green curve corresponds to a phase pi by 30 whereas, the black curve corresponds to a phase of 0. So, we have now discussed the 3 constants that appear in the solution for the simple harmonic oscillator. Namely, the angular frequency omega naught which determines how fast it oscillates which tells us how fast it oscillates.

The amplitude which tells us the maximum and minimum displacement that is how much it oscillates and the phase which tell tells us the relative phase of oscillation.

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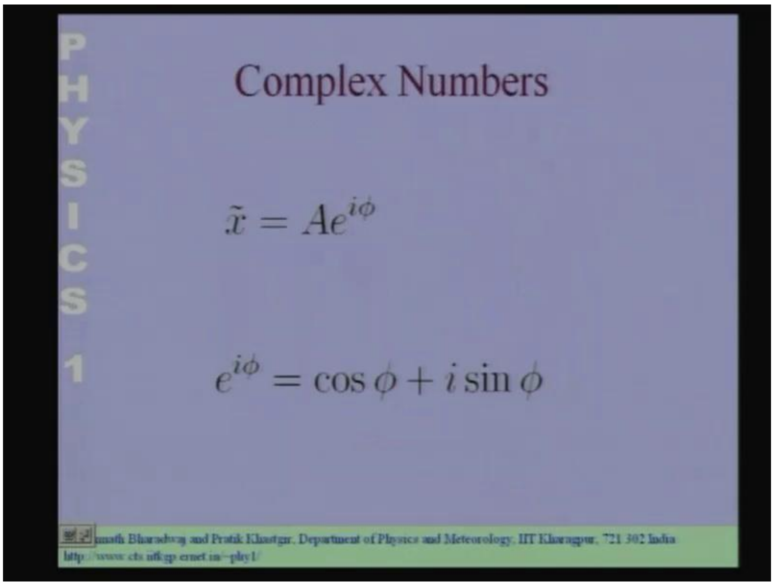

Next, we discuss how oscillating quantities can be represented using complex numbers. In this course, we shall represent complex numbers by a tilde on top of the variable. So, the variable x tilde represents a complex number any complex number can be written in terms of amplitude A and a phase phi. So, x tilde is equal to A e to the power i phi. Any complex number can be represented like this.

Now, let us ask what is the real part of this complex number I am sure all of you know that the real part of e to the power I phi is cosine phi. And the real part and the imaginary part of e to the power i phi is sin phi. So, the complex number e to the i phi essentially represents the combination of cos phi and sin phi where sin phi is the complex part is the imaginary part and cos phi is the real part.

So, if I write x tilde as A e to the power i phi the real part of x tilde is A cos phi and the imaginary part is A i phi.

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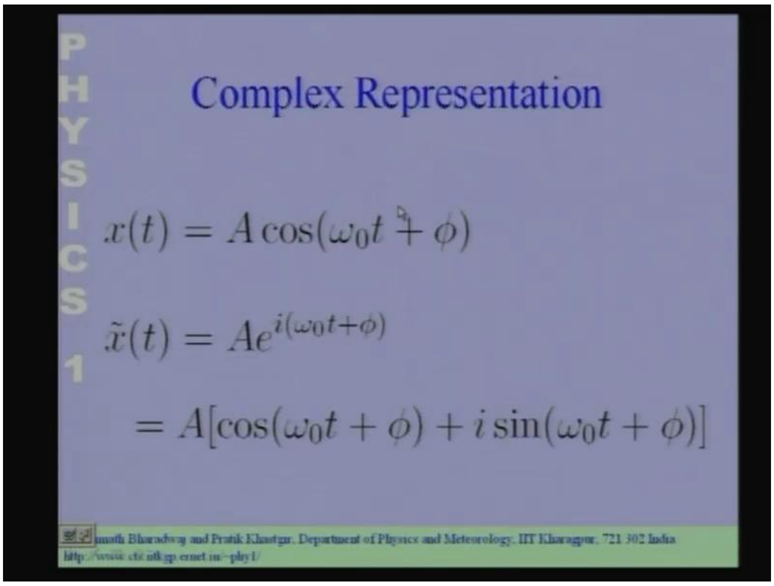

Now, let us look at the complex representation of an oscillating quantity. We have seen, that the simple harmonic oscillator is xt is equal to A cos omega t plus phi. Now, if you take a complex number x tilde which is A e to the power i omega t plus phi and look at its real imaginary parts it is real part is A cos omega t plus phi. Its imaginary part is A i sin omega t plus phi.

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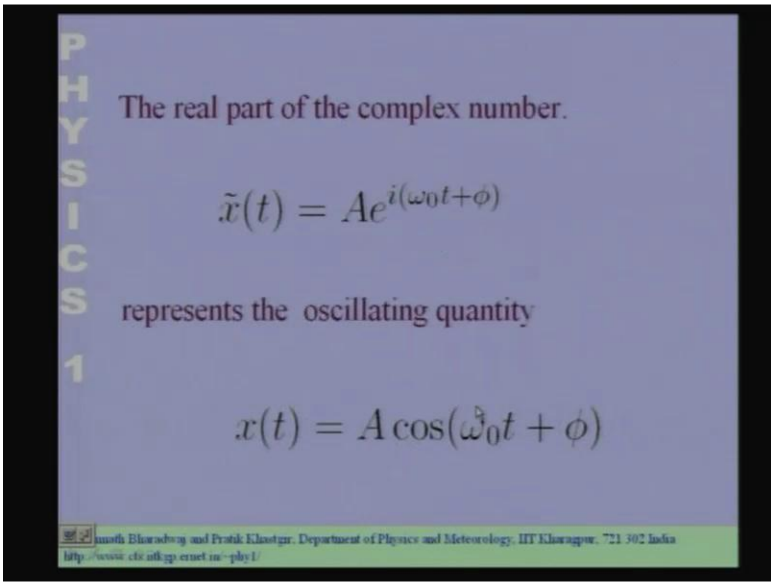

So, you see that the complex number x tilde which is A e to the power i omega t plus phi. If you take the real part of this, the real part of this gives you the oscillating quantity which you wanted to represent that is xt is equal to A cos omega t plus phi. So, we can represent this simple harmonic oscillation through this complex number you should take only the real part of the complex number. It is that which represents the oscillating quantity which we are interested in.

The real part the complex number also has an imaginary part you should ignore the imaginary part when you ask physical questions about the oscillation. The complex representation of oscillating quantities is very useful as we shall see as we go along.

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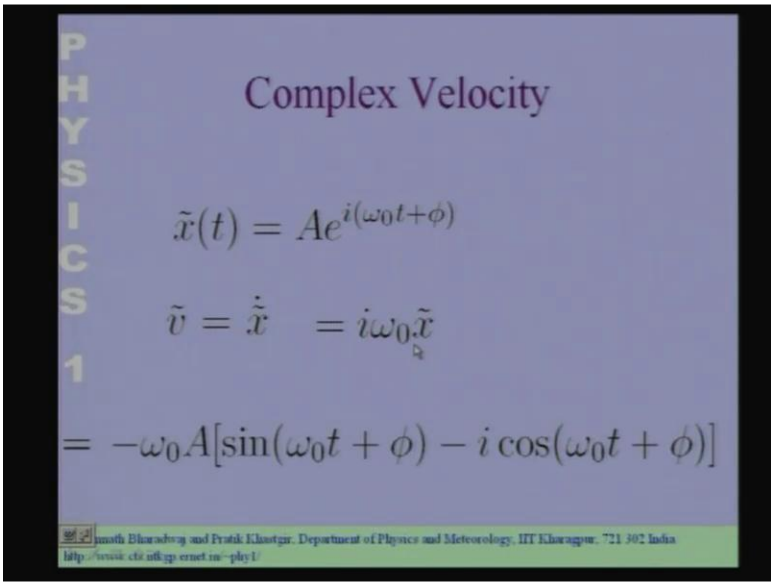

Let us calculate the velocity of the oscillator using the complex notation. How do you calculate velocity? You calculate velocity by differentiating x. And we shall do this in the complex notation. So, we have represented the oscillation of using the complex number using the complex variable x tilde which is A e to the power i omega t plus phi.

Now, if we differentiate x tilde to calculate the velocity we get v tilde which again is complex; this is the complex velocity of the oscillator. It is related to x tilde through the derivative which we represent through a dot on top. And if you differentiate this what you get is that you pick up an extra factor of i omega. So, the differentiation in this complex notation is very simple all that you have to do is that, you have to multiply the quantity which you are differentiating with the factor of i omega.

So, in this complex notation differentiation essentially is multiplication with I omega. So, the complex velocity is equal to I omega into the complex displacement. Now, if you break this up into it is real and imaginary parts. The real part of x is cos omega t plus phi x tilde is cos omega t plus phi. When you multiply that by i it becomes the imaginary part of the velocity. So, the imaginary part of the velocity is i cos omega t plus phi and there is a factor the amplitude A outside.

The imaginary part of the displacement x tilde when you multiplied with I you get the real part of the velocity which is minus omega naught A sin omega naught t plus phi. So, you see that the imaginary part of the displacement becomes the real part of the velocity. If you now want to calculate what the velocity of the particle actually is you have to pick out the real part of the complex velocity.

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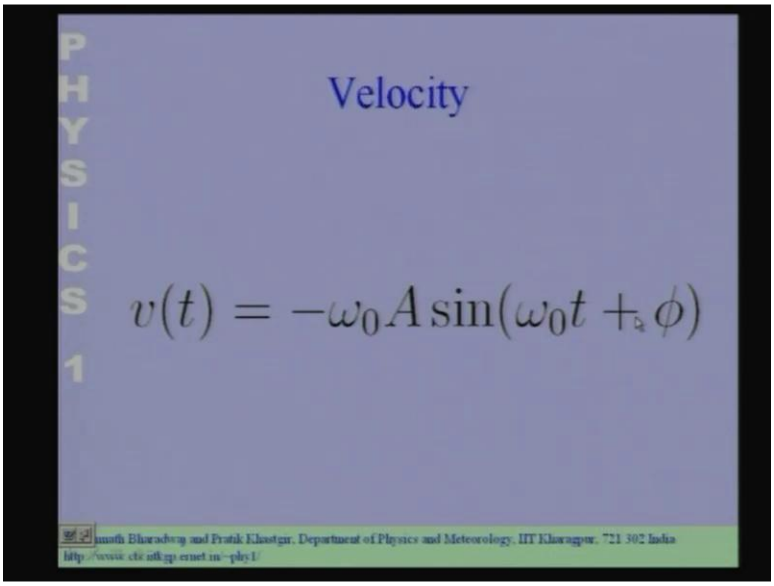

If you do that, you find that the velocity the real velocity the velocity of the particle actually is minus omega naught into the amplitude into sin omega t plus phi. So, you see the utility of this complex representation. If you see that the whole thing becomes very compact for certain purposes you do not have to really write out the full expression over here. If you represent x through a complex number x tilde you can represent the velocity by the complex number i omega x tilde; it is very compact.

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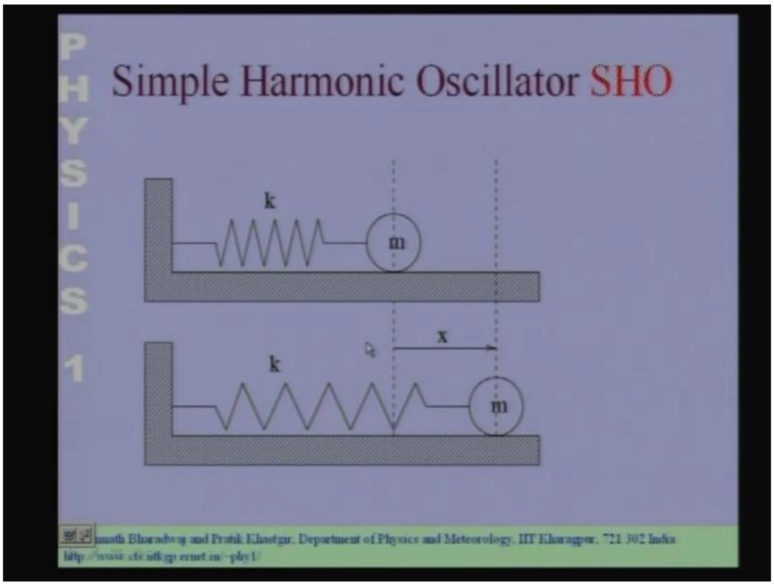

Now, let us go back to the simple harmonic oscillator which we were discussing, the spring mass system. If you displace the mass by a distance x it causes the spring to get stretched. The stretched spring stores energy this energy is the potential energy of the spring mass system. How much is the potential energy of the spring mass system?

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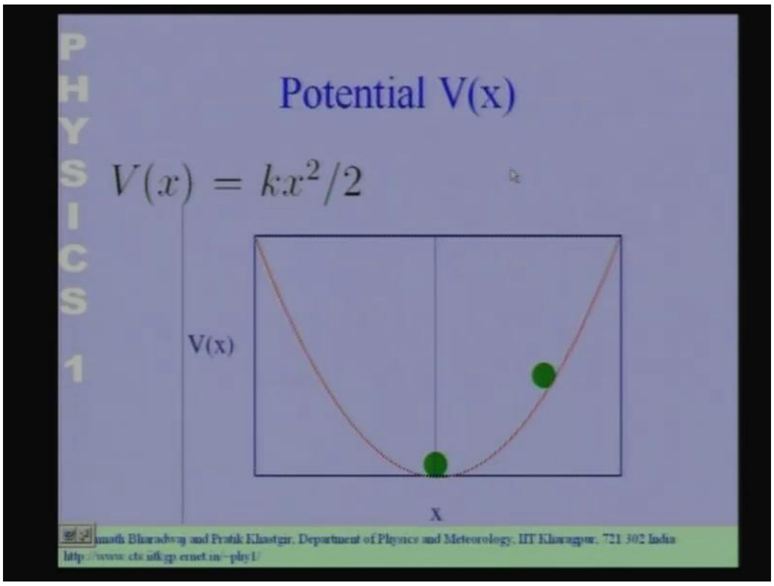

We all know, that the potential energy Vx it is half k x square this is what we have shown over here. If you keep the mass at this position it has the minimum potential energy which is 0. If you displace it at a certain distance x it gains potential energy and which we have represented by the particle moving up this parabola which is the potential energy.

So, the more you displace the particle from it is equilibrium position the more it is potential energy goes up. And it goes up as a quadratic function of the displacement half k x square.

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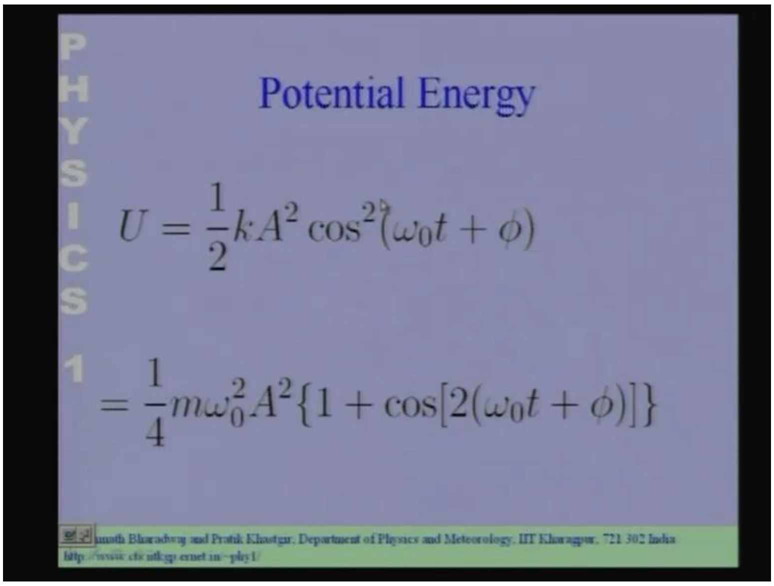

Now, let us calculate the potential energy of the simple harmonic oscillator as it oscillates. The way you calculate this is you take half k x square and you put in the expression for x which you have calculated just now. So, if you do this you get half k and we have seen that x is A cos omega t plus phi. So, the potential energy is half k A square cos square omega t plus phi. The cos square omega t, can be simplified and written as half 1 plus cos 2 omega t plus phi.

What you see is that the potential energy also oscillates just like the particle. The difference is that it oscillates at twice the frequency at which the particle is oscillating. It oscillates the potential energy oscillates at twice the frequency at which the particle is oscillating. Let us ask the question. Why does the potential energy oscillate at twice the frequency at which the particle is oscillating? Why does it not oscillate at the same frequency?

If you think about this you will see this comes about because the potential energy has a quadratic relation to the displacement of the particle. The potential energy is 0 when the particle is at the minimum. It goes up when if you displace the particle in this direction it also goes up if you displace the particle in the opposite direction. So, it oscillates at twice the frequency at which the particle oscillates. Let us now, move on to the kinetic energy of the particle.

How will you calculate the kinetic energy of the particle? We all know that the kinetic energy is half m v square. Let us put in the expression for v which we have calculated it into this expression for the kinetic energy. What does it give us?

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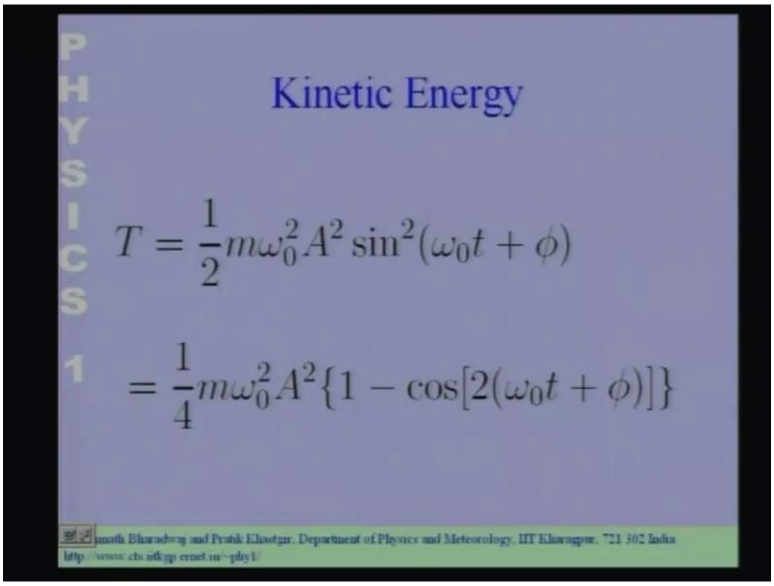

So, the kinetic energy of the particle if you put in the expression for the velocity you get the kinetic energy t which is, half m omega naught A square sin square omega t plus phi. Here, again you can simplify this expression a little bit sin square omega t can be written as half 1 minus cos 2 omega t. So, you see that the kinetic energy also oscillates at twice the frequency at which the particle oscillates.

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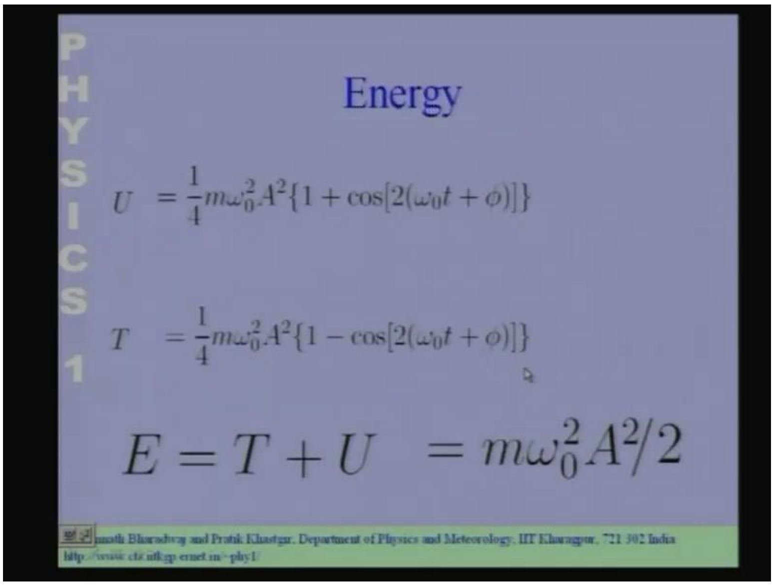

So, here we see both the expressions for the potential energy and the kinetic energy. Now, let us ask the question what is the total energy of the spring mass system. To calculate the total energy you have to add up the kinetic energy and the potential energy this gives you the total energy. Notice that, the potential energy has plus cos 2 omega t plus phi whereas, the kinetic energy has sin 2 minus cos 2 omega t plus phi.

So, both the kinetic and potential energy oscillate with the frequency 2 twice the frequency of the particle and these oscillations are pi out of phase; they are exactly out of phase. So, when you add up the potential and the kinetic energy the oscillating parts which are out of phase cancel out and you all left with the constant which is m omega naught square A square by 2. This is the conserve quantity it is the total energy of the spring mass system.

It is proportional to the square of the amplitude. So, the square of the displacement is the energy of the spring mass system is proportional to the square of the amplitude at the square of the maximum displacement.

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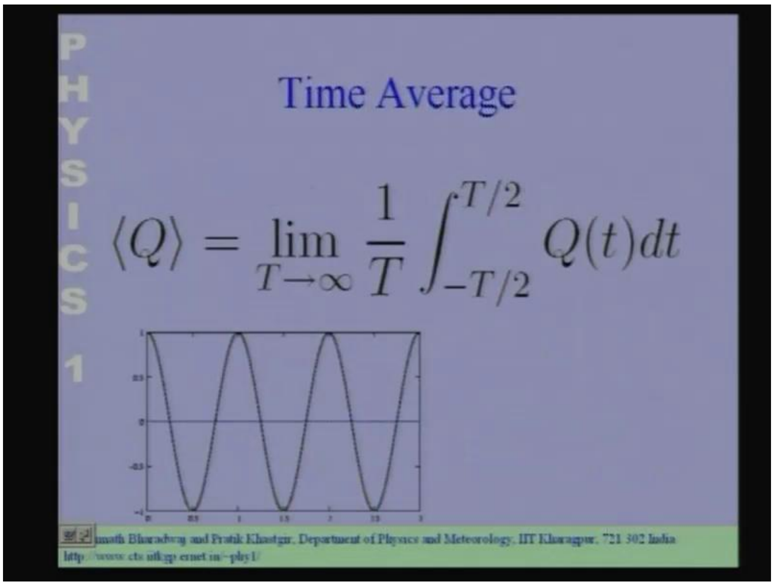

We next take up another very interesting issue that is time average. In many situations we are not interested in the details of the oscillation. So, we have an oscillating quantity, but we are not interested in the details of the oscillation. In many situations we cannot measure the details of the oscillation. For example, as I speak the vibrations in the air causes vibrations in your ear. These vibrations occur quite fast 20 hertz to 10,000 hertz. We do not we are not really interested in measuring these oscillations.

We are interested in measuring some average quantity. As I speak the sound that you hear the different words that you hear essentially corresponds to certain average quantity which is average, over these oscillations which occur at some frequency like from: 20 hertz to 10 kilo hertz. So, we are interested in some average quantity another example is the alternating current ac the voltage fluctuates at the speed of 50 hertz at 50 hertz at the frequency of 50 hertz.

But, the light that you see the light which illuminates this room does not appear to oscillate at this 50 hertz frequency. That is because, our eye is perceiving some average quantity does not perceive these oscillations; it perceives certain average quantity. So, when we are dealing with oscillating quantities quite often we are not interested in the detailed oscillations, we are interested in some average quantity.

So, the time average is very important. How is the time average of any oscillating quantity defined? So, Q of t is an oscillating quantity. Here, I show you Q of t in this case it is a simple harmonic oscillation, but the definition given over here is valid for any complicated oscillation also. So, Q of t is an oscillating quantity to calculate it is average you should add up the value of Q of t over a period of time. We have chosen the period of time over which you are averaging this we are denoting it using T.

So, we are going to average Q of t over a time T. Mathematically, you do this by integrating Q of t from minus T by 2 to plus T by 2. You could as well do it from 0 to T, but to maintain symmetry about T equal to 0 we have used minus T by 2 to plus T by 2. So, what if you want to calculate the average value of Q of t what you should do is you should integrate it from minus T by 2 to plus T by 2. That is, you should add up the values of Q by t Q t over this time period from minus T by 2 to plus T by 2.

Having added up the values you should then divide it by the time interval which is T. Now, the question is how large how should you choose the time interval over which you should take the average. The basic idea is that you should choose the time interval T over which you calculate the average, sufficiently large; so that it is much larger than the period of the oscillation.

So, far this oscillation shown over here you should average over a time T which is much larger than the period of the oscillation. So, you should add up the value of Q over this time period and then divide it by the time period this will give you the average value of Q, which we denote using the angular brackets shown over here. So, the time average of any quantity we denote using the angular brackets shown over here.

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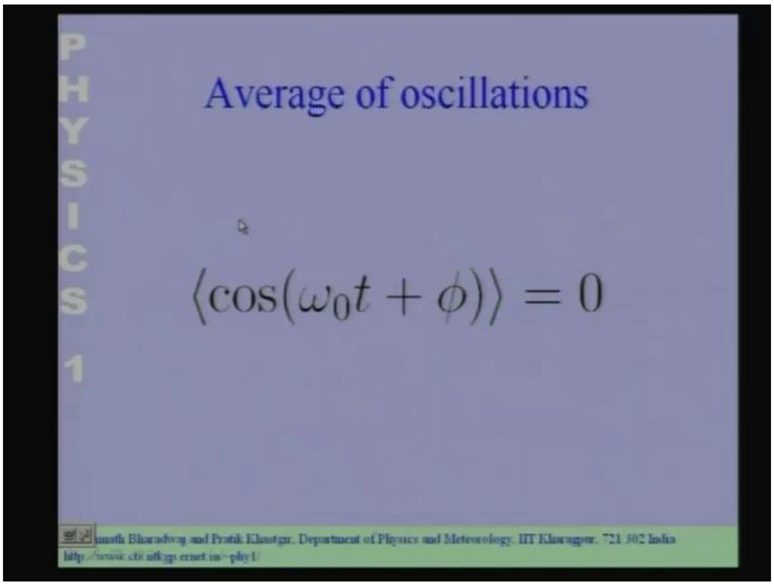

Let us take up an example. What is the time average of the simple harmonic oscillation cos omega t plus phi? How will you calculate this time average? Now, you may be tempted to say that cos omega t is a periodic function. So, we choose t to be 1 period and then it will cancel out. But, this is not the right approach what you should do is you should take cos omega t plus phi and put it into this expression.

So, let us integrate cos omega t plus phi from minus T by 2 to plus T by 2. If you do this integral you will get sin omega t plus phi at the 2 limits. Now, what is the maximum and what is the minimum value which sin omega t plus phi can assume. The maximum value which sin omega t plus phi can assume is 1 and the minimum value is minus 1. So, the maximum value which this integral can have when Q t is cos omega t plus phi is integral can have a maximum value of 2.

So, this expression can have a maximum value of 2 by t or if you take the limit where the time period is very large. The limit 2 by t will go to 0 which is why, the time average of cos omega t plus phi is 0.

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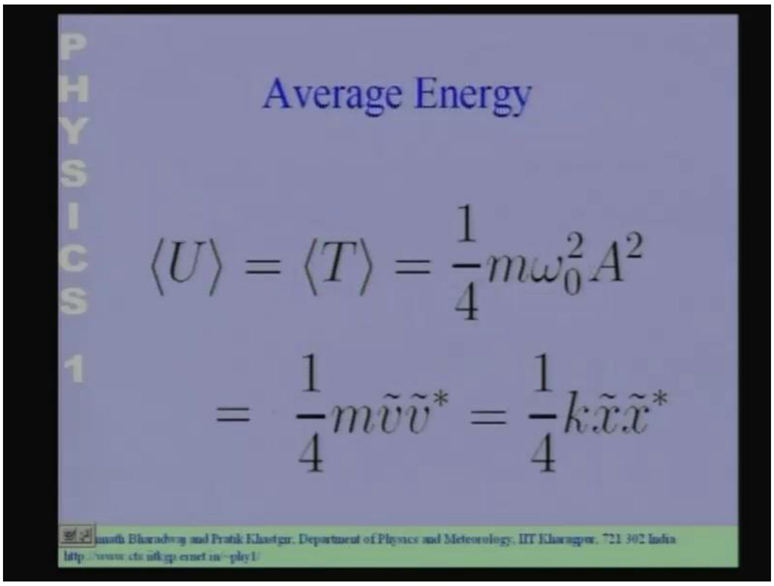

Let us use this to calculate the time average of the potential energy. So, let us look at the expression for the potential energy. We have already seen that, the potential energy can be written as 1 fourth m omega not square A square 1 plus cos 2 omega t plus phi. Now, when you calculate the time average of the potential energy you have the time average of a constant which is the constant itself and you have the time average of cosine 2 omega t plus phi.

How much is the time average of cosine omega 2 omega t plus phi? We have already calculated this we see that it is 0. So, if you calculate the time average of the potential energy U it turns out to be 1 fourth m omega naught square A square. Let us now, consider the time average of the kinetic energy. The kinetic energy has an expression which is exactly identical to the expression for the potential energy except for the minus sign over here.

Now, when you take the time average of the kinetic energy you pick up the same constant which you pick up when you calculate the time average of the potential energy and the cosine term over here cancels out. So, the time average of the kinetic energy is the same as the time average of the potential energy it too has a value 1 fourth m omega naught square A square.

Now, let us link this up to the complex notation which we have learnt. We can express the average kinetic energy and the average potential energy as 1 fourth m v tilde v tilde star where, star denotes the complex conjugate. So, you can very easily check that this m v tilde v tilde star. Remember that, v is I omega x tilde where x tilde is A e to the power I omega t plus phi. So, when you take the complex conjugate of this you pick up a minus sign in the exponent over here. And if you multiply this with it is complex conjugate to essentially to get A square.

So, v square v tilde v tilde star gives you A square omega naught square m is there and you have a 1 fourth factor. So, you can express the average potential energy or kinetic energy as 1 fourth m v v star. Similarly, you can also express the average potential and the average kinetic energy as 1 fourth k x tilde and x tilde star. There again star represents the complex conjugate of the complex number x tilde.

So, you see that the complex notation allows us to express the time average of the potential or the kinetic energy in this very convenient and very easy to remember way. So, you can remember either of this expression then it will tell you what the value is. So, the kinetic energy is half k x square and the time average kinetic energy is 1 fourth k x into x star which is essentially, mod x square.

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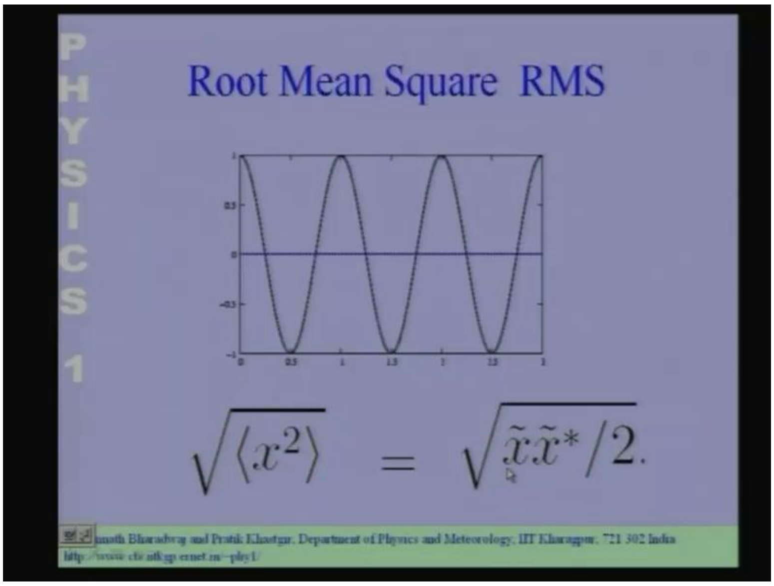

Finally, we discuss another very important concept when you want to discuss or describe oscillations. If you want to ask, if you ask the question where is an oscillating quantity? There is something which you see is oscillating could be pendulum which is oscillating back and forth, could be a voltage which is oscillating between positive and negative.

Any oscillating quantity you would like to ask the question how big is the oscillation? Well you could say let us look at the displacement and take the average displacement.

So, for a pendulum it moves back and forth you calculate the average displacement. Now, if you calculate the average displacement for any simple harmonic oscillator. So, if you look at this the simple harmonic oscillator is doing this sinusoidal oscillation if you calculate the average displacement it gives you a value which is precisely 0.

So, the oscillator oscillates it has both positive and negative values both these cancel out and the average displacement does not tell you anything because it has a value 0. It does not tell you how big the oscillations are. So, we are back to the question how will you quantify how big are the oscillations. So, you should now look at some quantity which as the oscillator oscillates assumes only positive value.

The simplest such choice is to look at x square how does x square vary as the oscillator oscillates. If you want to use just 1 number to quantify the magnitude of the oscillations you could look at the average value of x square. This is what is called the mean square. So, you take the variable x square it and then find the mean the average value of the square of x the average displacement square.

So, we know that x is A cos omega t plus phi for a simple harmonic oscillator we know this we have to square this. So, if you square it you will get A square cos square omega t plus phi. We have to calculate the time average of this quantity. So, we have to calculate the time average of x square that is what the mean square oscillation.

If you calculate the time average of x square which is essentially the time average of A square cos square omega t cos square omega t you can simplify as half 1 plus cos 2 omega t. The time average of the cos 2 omega t term is 0 as we have already seen. So, what you all left with is essentially half A square. So, the mean square displacement is essentially half A square.

Now, the quantity which we use to quantify the magnitude of the oscillation how much, how big are the oscillations? Is the square root of this, it has a same dimension as the displacement itself. This is called the root mean square RMS this is a very important number in quantifying oscillations. So, the root mean square: you take the square, find the mean and then take the square root of that which is what is shown over here.

You take the square of the variable find the average value and then take the square root this is called the RMS value. And if you calculated this if you calculate this as we have just done it turns out to be half A square. So, when you take the square root you will get A by root of 2. Now, this has a very simple representation in the complex notation. So, if you wish to calculate the root mean square of the displacement x you take the complex displacement x multiplied by x star divide by 2 and take the square root.

So, the mean square is essentially x into x star by 2 and the root mean square is square root of x into x star by 2. This is true not only for a simple harmonic oscillator, it is true for any oscillation which you represent using complex numbers. So, you could represent any arbitrary oscillation also using complex numbers and this expression that the RMS is x into x star by 2 the square root of that.

This holds even for any arbitrary oscillation you could check this by taking x to be a super position of oscillations with 2 different frequencies and calculating it is root mean square.

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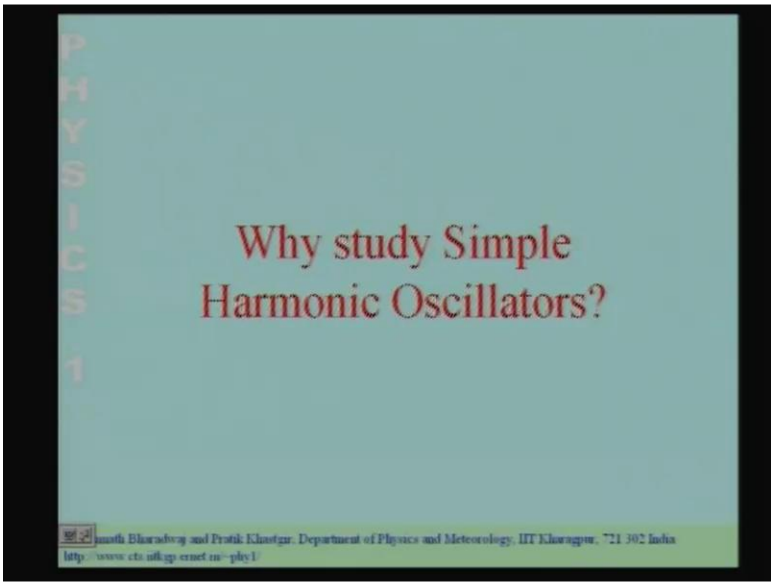

We have been discussing simple harmonic oscillations in sub detail until now. Let us now, take up the very interesting issue why study simple harmonic oscillators. What is the importance of simple harmonic oscillators?

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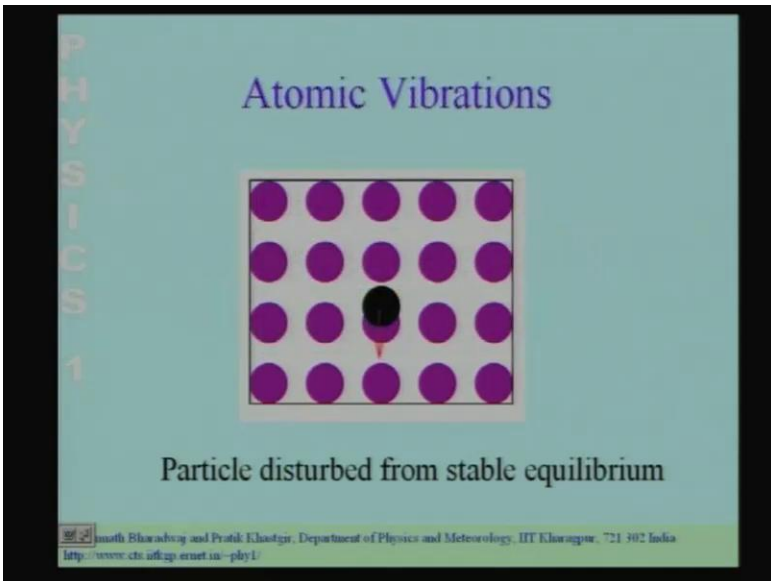

The importance of simple harmonic oscillations lies in the fact that if you disturb any system which is in equilibrium and leave it. If you disturb a system which is in equilibrium; if you disturb it slightly and then leave it, it turns out that in most situations the system will execute simple harmonic oscillation. It is a very generic phenomenon. For example let us consider the atoms in a solid the atoms in a solid we know are in equilibrium.

When the atoms in a solid we know form a lattice and in this lattice configuration the atoms are in equilibrium. For example, if you look at sodium chloride or steel or diamond in such solids the atoms have a periodic arrangement the crystal lattice. And if the atoms remain at the lattice positions the system is in stable equilibrium. Now, consider the situation where you disturb 1 of the atoms s. When the solid is at a finite temperature the atoms in the solid keep on doing small vibrations.

So, let us consider a situation where only 1 of the atoms has been disturbed and all the other atoms remain in the lattice positions. In such a situation it turns out that if you disturb 1 atom and leave it there then, the forces exerted by the other atoms or such that it will make this oscillate do simple harmonic oscillations. Now, I should point out that the situation which I have discussed here is a little over simplified in reality.

If I disturb 1 atom and leave it, it will produce disturbances in the other atoms and the whole system will oscillate as a set of coupled simple harmonic oscillators. This again is something that we shall discuss later in this course that is coupled oscillations. And you could even have waves in the solid which propagate in the solid because of these disturbances. But in principle hypothetically if you could hold all the atoms fixed and just disturb 1 atom. Then the forces exerted on it by all the other atoms would be such that it this atom would execute simple harmonic oscillations.

And this is true not only for atoms in a solid it is true for any system which is in stable equilibrium and you give a small disturbance. Let us see how this comes about.

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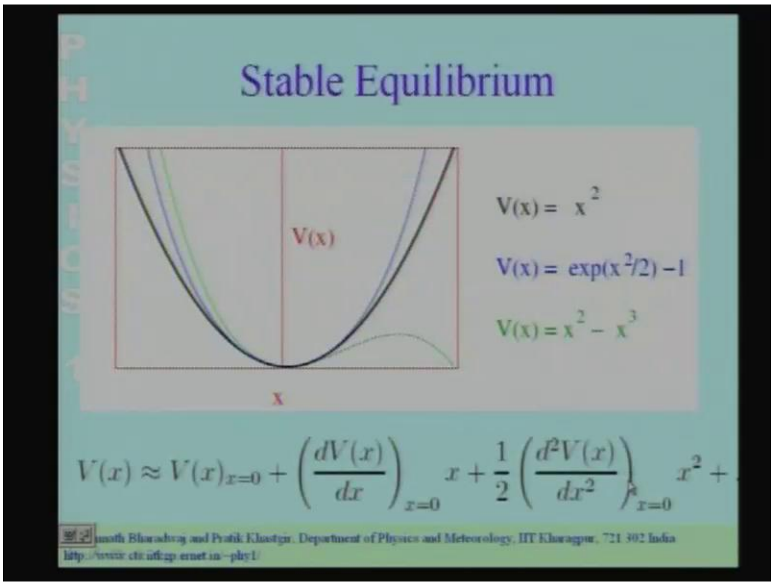

So, here I show 3 different potentials all of which have a stable equilibrium at the point x equal to 0. The 3 potentials which we consider are shown over here. The potential in black is the simple harmonic oscillator potential Vx is equal to x square. The potential in blue is an exponential the Gaussian potential and the potential in green is an asymmetric potential. On 1 side it behaves, but similar to the simple harmonic oscillator for, but on the other side you see that it goes up and then falls over here.

Its mathematical expression is given over here. For all of these potentials if you look at the behavior near x equal to 0 you see that, they are quite similar and they are very well approximated by the simple harmonic oscillator. This is a mathematical fact which I shall explain over here next. If you do a Taylor expansion of the potential around x equal to 0 then, the first term in the Taylor expansion is the value at x equal to 0.

The second term in the Taylor expansion is the derivative at x equal to 0 into x. The third term in the Taylor expansion is half x square into the second derivative of the potential near x equal to 0. So, we have done a Taylor expansion of the potential near x equal to 0 and we have retained only the first free terms. We expect these to be valid first small x.

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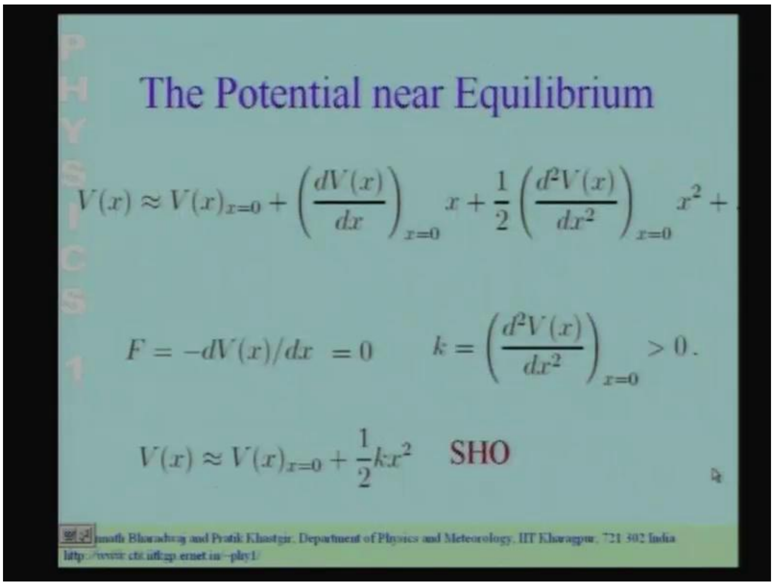

Now, if you have done the Taylor expansion near a point of equilibrium then, the first derivative the first derivative of the potential gives us the force. At a point of stable equilibrium the force acting on the particle is 0; the force acting on the system is 0. So, we expect the first derivative to vanish at a point of stable equilibrium. Further, a point of stable equilibrium is minima of the potential. The second derivative of the potential should be positive at minima.

The first derivative should vanish and the second derivative should be positive. So, the second derivative at x equal to 0 is a positive number which we denote using k. So, you see that for small x at is around the equilibrium point the potential Vx can be represented by a Taylor series. And the non 0 terms in the Taylor series are a constant. And then we have half k where k is the second derivative at x equal to 0 into x square. This is exactly the simple harmonic oscillator potential.

So, what we have seen is that around the point of stable equilibrium, if you do a Taylor expansion of a potential the Taylor expansion of a potential it essentially that lowest order non 0 term is half kx square which is the simple harmonic oscillator potential.

Which is, why? You see that all of these potentials near x equal to 0 or very similar to the simple harmonic oscillator potential; very close as you go further away they deviate.

So, you can represent all three potentials near x equal to 0 using the simple harmonic oscillator potential. So, essentially if you had a particle in this potential over here or in this potential here if you want to disturb the particle slightly from equilibrium, its behavior would be exactly the same as the behavior in this potential which is the simple harmonic oscillator. This essentially, substantiates the statement which we have made earlier. That any system if you disturb it slightly from stable equilibrium behaves like a simple harmonic oscillator.

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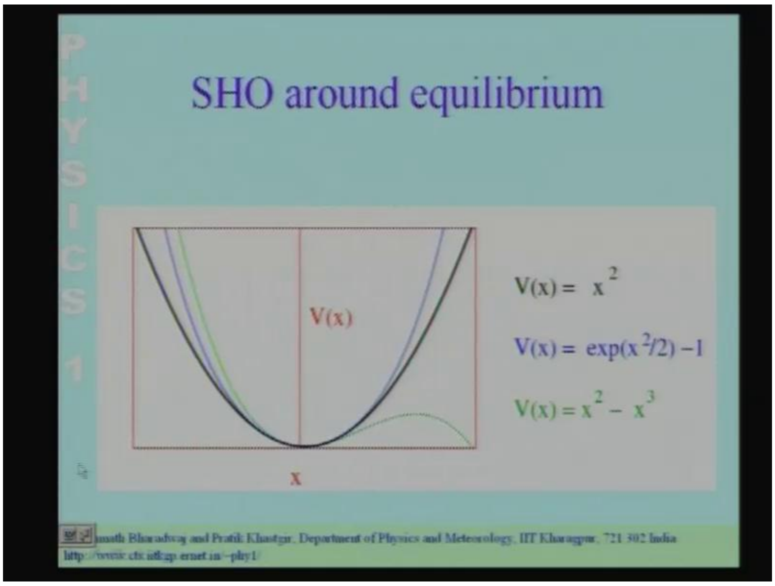

Which again you see in this picture over here.

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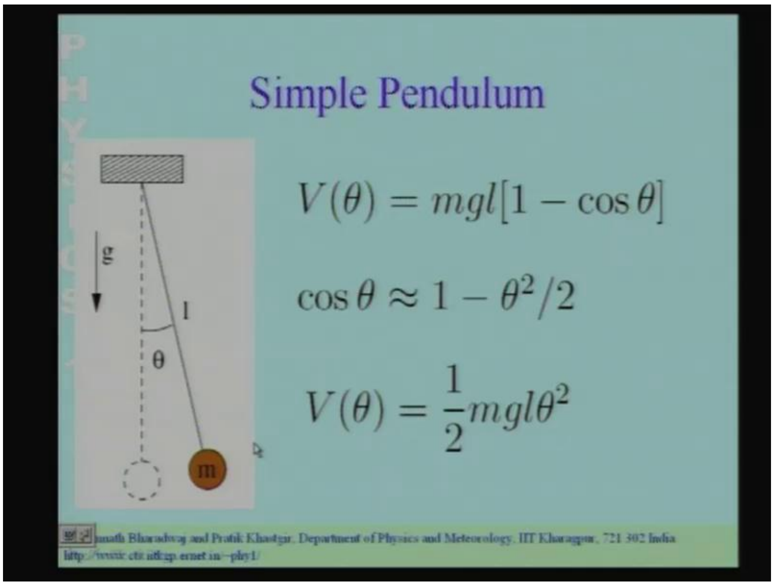

Let us now, consider 2 examples of simple harmonic oscillations. The first example that we consider is a simple pendulum. If you take a simple pendulum which is a mass suspended from a point by a rigid rod, the rod which holds the mass is assume to be mass less. The entire mass of the system is in the mass over here it is suspended from this point by a mass less rod.

Now, if you take this and displace it slightly then it gains some potential energy. The potential energy is a function of the displacement which is an angle over here theta. The potential energy is mg into the length of the rod into 1 minus cos theta. This is the amount by which the mass goes up and the particle gains potential energy gravitation of potential energy because, if it goes up a certain distance. And this is V theta mgl 1 minus cos theta.

Now, for small angles theta you could do a Taylor expansion of cos theta. How much is, what is the Taylor expansion of cos theta for small theta? For small theta the Taylor expansion of cos theta is 1 minus theta square by 2. So, for small displacements theta the potential is half mgl theta square. So, you see that the potential is a simple harmonic oscillator potential where the variable the displacement variable is theta the angular displacement. Let us now write down the equation of motion governing this mass.

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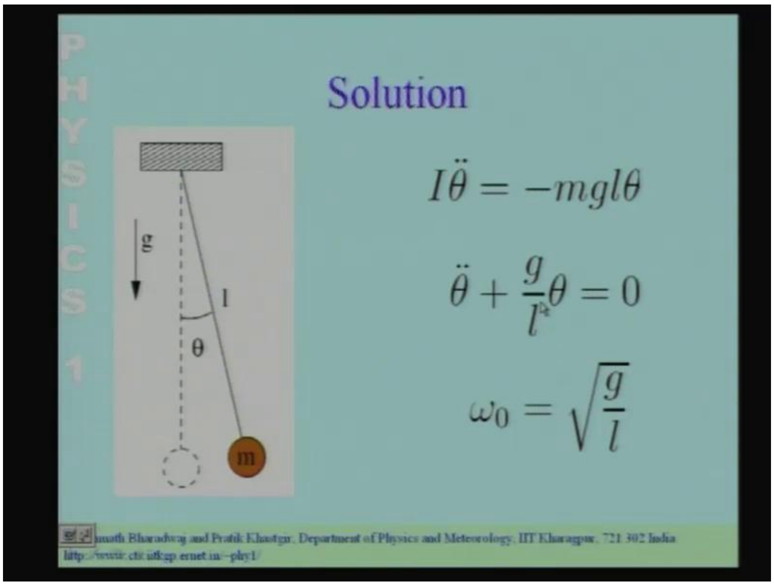

The equation of motion governing this mass is shown over here. The motion here is not linear it is an angular motion. So, you have to replace the mass with the moment of inertia and the derivative of the potential with respect to theta gives you the torque. So, this is the angular acceleration into the moment of inertia. This is equal to the torque the moment which is essentially, the moment the rate of which is equal to the torque.

So, I theta double dot is equal to minus mgl theta which is the simple harmonic oscillator equation. If you compare this, to the simple harmonic oscillator equation the spring mass system, which we had studied earlier, you find that the angular frequency is the square root of mgl by i. Now, the moment of inertia of the system the moment of inertia of the system is the mass into l square. So, if you replace I using l square m then the mass and the length 1 the length cancel out. What you all left with this omega naught is equal to square root of g by l.

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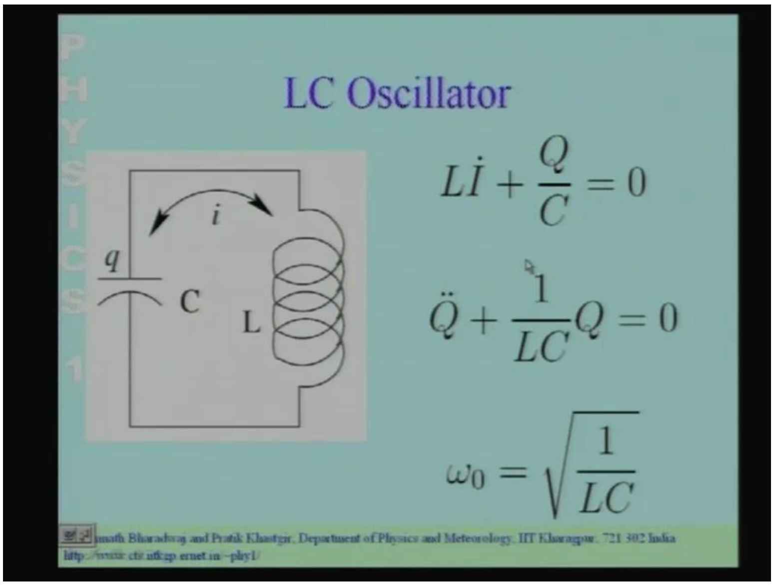

Let us take up another system which is the LC oscillator here we have an inductance and a capacitance. Consider a situation where the capacitance is charged it has a charge q and then it is left at the circuit. The equation governing this circuit is shown over here it the voltage developed across the inductance is the inductance into the derivative of the current. And the voltage developed across the capacitance is Q by the capacitance C.

So, the equation governing this circuit is given over here. If you differentiate this equation once you will get this equation can be there is no need to differentiate it once. You can write this equation in terms of the charge itself as Q double dot plus 1 by LC into the charge is equal to 0. Can you identify the angular frequency at which this the current or the charge in the circuit will oscillate.

If you compare this to the equation for the spring mass system you will see that the angular frequency omega naught is square root of this factor over here, square root of 1 by LC. So, you see that this, the charge here will do simple harmonic oscillations the capacitance will get charged. And then, it will get discharged current will flow the voltage will develop across the inductance and back and forth.

If you look at this equation you will see that 1 that l inductance plays the role of the mass and 1 by the capacitance plays the role of the spring constant. So, let me now summarize what we have learned about the simple harmonic oscillator. We started the course by considering a prototype a spring mass system and we saw that if you disturb the system from its equilibrium position it will oscillate. These oscillations are sinusoidal that is cos omega t plus phi.

We next saw how you can describe this oscillation using complex numbers then we discussed the kinetic energy, potential energy, the total energy of these oscillations. Then we discussed how to calculate time averaged quantities. Finally, I gave you the motivation for studying simple harmonic oscillators. I told you that any system in stable equilibrium if you disturb it slightly it will execute simple harmonic oscillations around the equilibrium position. And finally, I gave you 2 examples the simple pendulum and LC circuits.

There are a large variety of situations which where, simple harmonic oscillations are useful in interpreting the oscillations. And we shall not be discussing all these varieties some of these examples will be you will encounter them, if you take a problems on simple harmonic oscillators.

Thank you.