Engineering Physics – II

Prof. V. Ravishankar

Department of Basic Courses

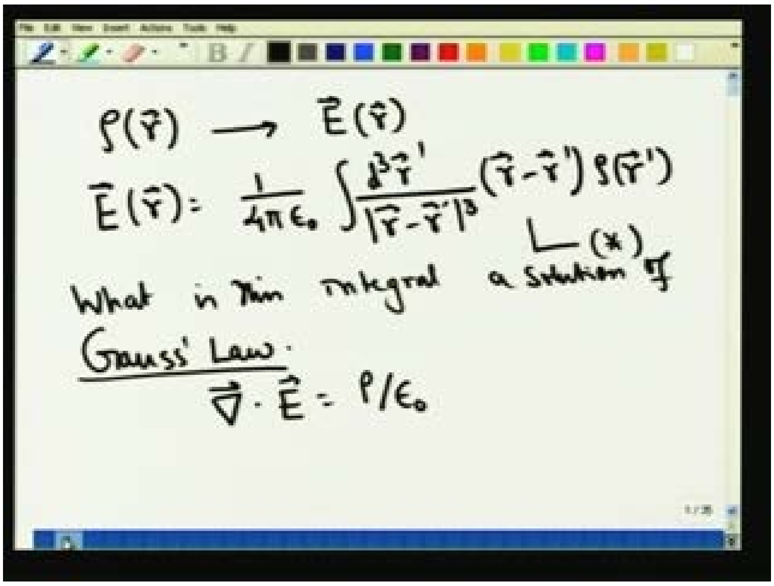
Indian Institute of Technology, Kanpur

Module No. # 02

Lecture No. # 05

In the last lecture, we spent a considerable time discussing how to get a continuous charge density given a basic underline discreet charge density, and then we also went on to write down the Expression for the Electric field given a continuous charge density distribution. So, let me recapitulate that.

(Refer Slide Time: 00:36)



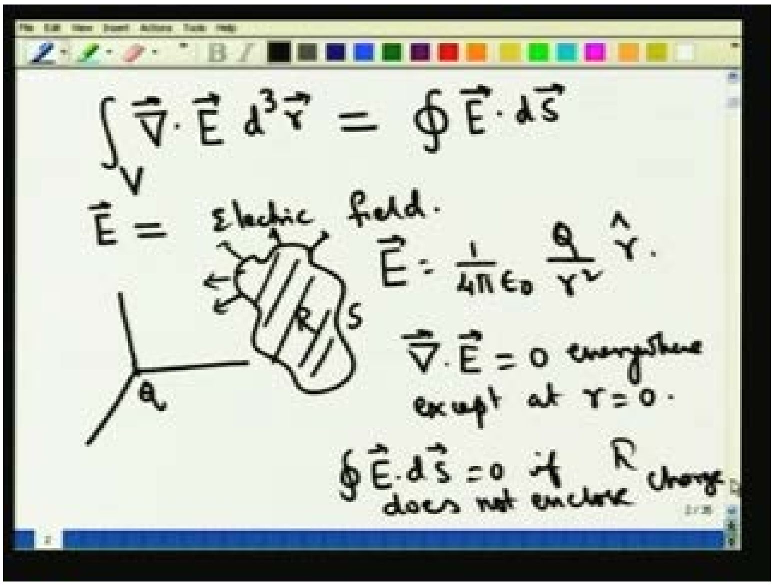
If rho is the charge density given as a function of r, we are interested only in the Electrostatic case. There is no time dependence in the time charge density, and therefore, there are no currents. Then what we find is that, given this rho of r it is going to fix the value of the Electric field everywhere and we derived an expression for this.

So, the expression for the electric field turned out to be simply one over 4 phi Epsilon naught integral d cubed r prime. Then I will write a mod r minus r prime cubed, that is what I have, and then this multiplies r minus r prime. This is indeed the generalization of the coulomb law which was initially stated for a point particle to a continuous charge distribution rho of r prime. That is what we have.

Now, after doing this, I set to myself the task of recasting this equation, E equal to d cubed r prime mod r minus r prime whole cube r minus r prime multiplied rho of r prime in terms of a differential equation. Here, given any rho of r prime, I give you the E straight away, but I want to write down a differential equation of which this equation is a solution.

So, if I were to call it as a star, I ask the question given this, what is this solution of? What is this integral a solution of? So, equivalently we ask for the differential form of this equation and that goes by the name Gauss law, and we want to demonstrate Gauss's law starting from this particular expression. So, in order to do that, I started rewriting the Gauss Divergence theorem as is appropriate to us. So, what I shall do is to briefly recapitulate whatever we did in the last lecture and then continue the team. This is the so called first Maxwell Equation that would have written once we wrote down Gauss law, and let me set Gauss's law straight away that is nothing but divergence E equal to rho by epsilon naught.

(Refer Slide Time: 03:18)



Now, let us start with gauss divergence theorem in its generality and then specialize it to the electric field as we have written so far. What does gauss divergence theorem say? It tells you that given any vector field; I shall denote it by E itself although it need not necessarily be an electric field. Calculate the divergence and integrate it over a certain volume.

Then, this value is numerically equal to what you get by integrating the same vector field, but this time, it is a surface integral over the surface that bounds this volume V, which is the reason why we put the circle here. Now, let us try to understand the import of this. The consequence of this law which we actually proved in one of the earlier lectures to our particular form namely: the Coulomb law. In other words, my electric field is E is actually the electric field and not any vector field.

In order to study the consequence to the electric field, what I will do is to repeatedly make use of the principle of superposition, that is, I will start with a point charge. Then see what the consequences are. Make use of the principle of superposition generalize it to any number of point charges, and then, since we know how to go from a discreet charge density to a continuous charge density, that is, a discreet distribution of charges to a, continue, continuous charge distribution. We will be able to write down gauss's law in the form mentioned in the previous page namely: divergence E Equal to rho by epsilon naught.

So, in order to does that, let me start with the simplest case as I told you. Take a point charge Q and let me locate it at the origin. So, this is my coordinate system and my charge Q is located here, and we all know that this charge Q produces an electric field which is given by one over four phi epsilon naught Q by r square r hat. That is what we have. R is the distance of any point p from the origin. We know that. Now, what we shall do is to make use of the equation written above and try to evaluate. Try to see as examples what gauss divergence theorem tells us.

As an example, let me consider a situation where I have a certain region r which is outside the charge. So, this is the region r which is bounded by a surface s and it does not enclose the charge. So, when I am speaking of this region r, I do not restrict myself to any particular shape for that particular surface. It is arbitrarily; it can be as big as or small as it is. The only condition is that it shall not enclose this particular charge Q.

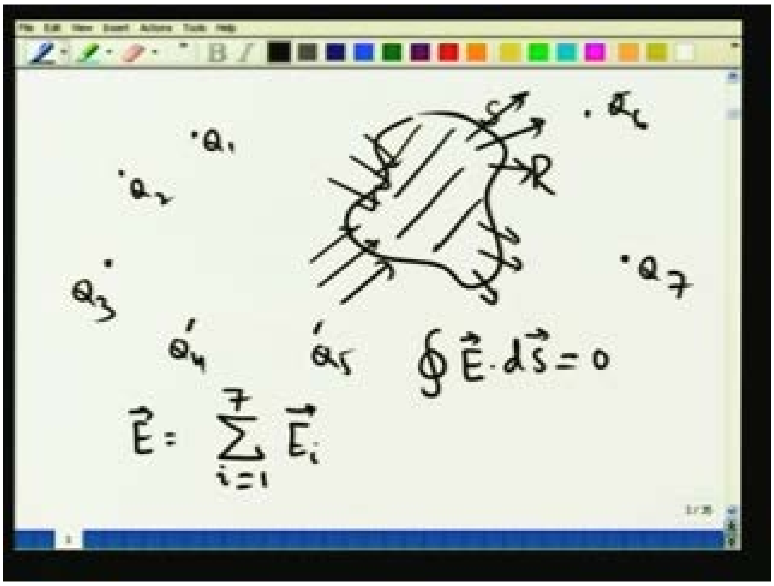
Now, if I were asked to evaluate integral E dot D s for this particular region obviously electric field is going to take complicated values. Although the form itself is simple, E dot D s is going to be complicated because the surface elements, the surface vector is going to be in different directions and have not even bothered to specify the nature of that particular surface; it might be difficult, but on the other hand, the left hand side is something easy to evaluate, because as we have seen repeatedly divergence E equal to 0 everywhere, divergence E equal to 0 everywhere except at r equal to 0, at r equal to 0.

This is a very simple mathematical exercise that we can perform. Take divergence, evaluate it in the spherical polar coordinate and you will find that this is identically equal to 0 everywhere. Since divergence E equal to 0 is true for all points in this region r by this equality. We conclude that integral E dot d s equal to 0 if r does not enclose, does not enclose the charge. This statement actually did not assume, did not make use of the fact that the charge is located at the origin.

I wrote that only in order to write an explicit equation for E, but if I had located the charge here, here or here, it really does not matter because divergence E would have been vanishing in any case because divergence E is non vanishing only at the location of the charge. Therefore, this result is independent of where the charge is located.

Now, by the same token, if I was to not taken just one charge but take several charges. So, now, let me call this as a charge Q 1. A charge Q 2 is located here; a charge Q 3 is located here; a charge Q 4 is located here let us say. Then all of them are in the region which is outside the designated region r, that is, this r does not enclose either Q 1, Q 2, Q 3, Q 4 are for that matter any number of charges.

(Refer Slide Time: 08:18)



So, let me indicate that again here. So, what I do is to take a certain region r, and here, I put a large number of charges. I could put a charge here; I could put a charge here. Let me level them Q 1, Q 2, Q 3, Q 4, Q 5 and let us say Q 6 and Q 7. If I did that, I can calculate the electric field produced by each of them in this particular region r which is bounded by the surface s.

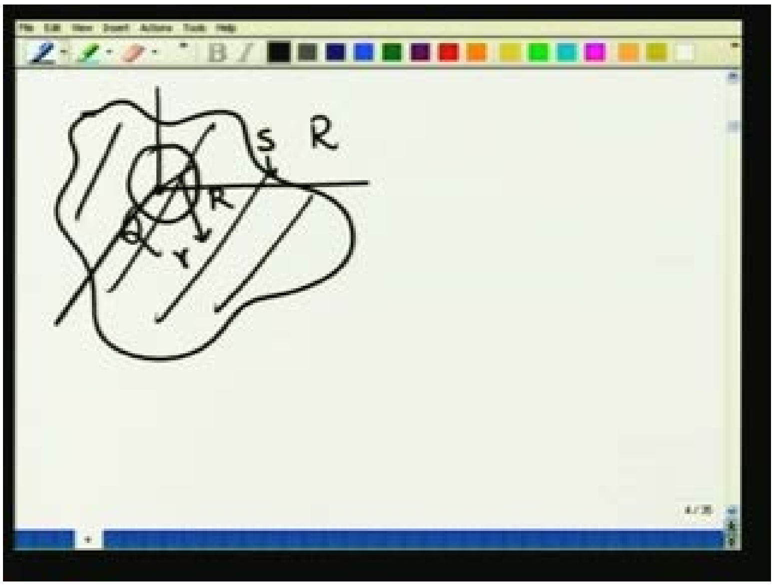
They are all going to give me the field which is simply given by a superposition of the contribution for individual terms. That is something that we have seen at great length never mind. Since for each of them divergence E equal to 0; divergence E Equal to 0 for the sum, since for each of them integral E dot d s is equal to 0, the integral E dot d s equal to 0 for the total electric field as well. Therefore, again we conclude that integral E dot d s is equal to 0 where E is actually obtained by summing over the contribution coming from all possible sources.

So, here, I have I Equal to 1 up to 1 corresponding to seven point charges. So, we have it our disposal a very nice a very beautiful theorem which tells you that, so long as a surface does not enclose any charge. If you calculate integral E dot d s, then that surface integral is going to vanish, which simply means that whatever may be the flux of the electric field that enters this particular region. For example, for Q 4, there is a certain flux that enters and there is a certain flux that is.

Similarly, from Q 2, there is a certain flux that enters and there is a certain flux that leaves. Whatever enters is exactly equal to whatever leaves out whatever exits form this particular surface; that means this region does not enclose any source or any sink. All the possible sources and sinks are located outside this region. If Q is positive by convention, we say that it is a source for the electric field. If Q is negative, it is like a sink for the vector field. That is what we have done.

The next thing that we have to do which is indeed important for us is to ask what happens if my region actually encloses a search. This is something that we should do with a little bit greater care because the proof that I am going to give you, the demonstration of Gauss's law that I am going to give you is slightly different from the geometric demonstrations which are given in the books, but it is completely equivalent to that. Our on the one hand, it might not bring force the geometric nature completely, but on the other hand, algebraically it is simple. Therefore, it is something that compliments whatever demonstration you see the text books. So, let us do it carefully.

(Refer Slide Time: 10:59)



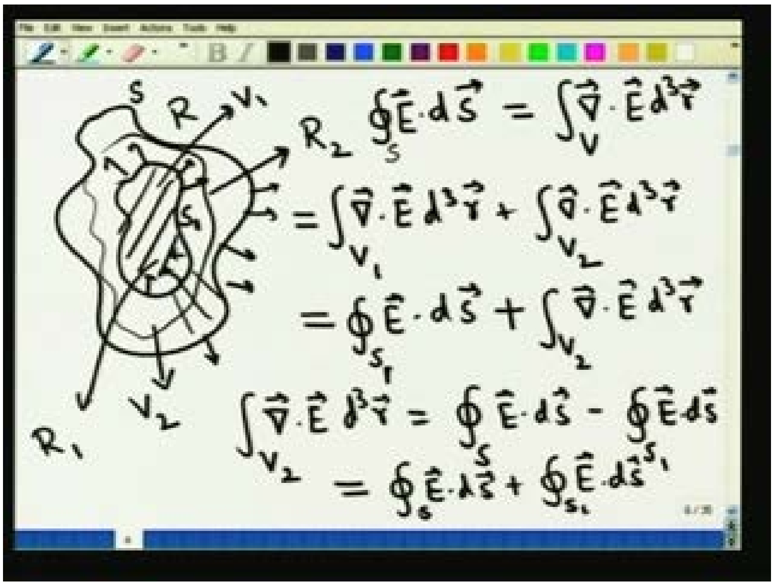
So, what is it that I want to do? I am going to take a certain charge Q. We will generalize it arbitrary number of charges later and I want to enclose it in a certain region R. That is what I want to do. Now, I am interested in calculating the flux of the electric field through the surface S that is bounding this R. So, this is the surface s that is bounding this r. Let me erase this prime. That is what I have.

So, this region is R and what I have is this surface s. It is easy to evaluate the surface integral as we have seen earlier if I locate Q at the origin and I look at a spherical surface. This surface is of course is an arbitrarily surface, but I want to exploit the fact that I know how to do the integral over a spherical surface. Therefore, in order to exploit the fact, what I shall do is again erect a coordinate system. My charge Q is located here and I will draw a sphere of radius R, capital R.

I have already employed the notation small R. I have already employed the notation capital for the region. Therefore, let me call it as capital R. Except that in this case, R is a not a variable it is a fixed quantity. That is what I had. May be it is better to elaborate not as r, but a sphere of radius a. That is to be making over life simpler.

Let us call it as sphere of radius a. So, what shall we do now? Before I jump on to evaluate the surface integral, I do which you already great experts. I want to state rather restate Gauss's theorem in a slightly different form, and then, I will come back to this particular problem. So, let us pause for a minute and let us try to write down Gauss's form in a slightly generalized form. This is a dig ration from our main theme of writing down Gauss's law, but this is something very useful.

(Refer Slide Time: 13:01)



In order to do that, let me consider a volume in a certain region r, and now, I will split this volume into two regions. This region r will be split into two regions. So, there will be an internal region and then there is an outer region. So, the internal region let me call as R 1 and the outer region which is surrounding it will be called as R 2. So, you have the total volume V which is enclosed by the surface s corresponding to the region r. Then I have the surface s; then you have the internal region R 1 corresponding to volume V 1 that is enclosed by a surface s 1. That is what I have.

R 2 is of course enclosed by s. R 2 encloses s one and it is enclosed by s. Now, I will rewrite gauss divergence theorem for this volumes and corresponding surfaces and we will find an interesting result. What is that? So, let us write down integral E dot d s. This is over the close surface and the way I have shown this is the outer surface. This is nothing but integral divergence E d cubed r over the whole volume which I shall denote by V 1 v. So, in order to be very clear, in order to avoid any confusion, I will denote the interior volume by V 1 and the surrounding volume the so called Exterior volume by V 2. That is what I will call. R 2, the region R 2 has a volume V 2. The region R 1 encloses a volume V 1, contains a volume V 1.

Now, by the additively of volume integral, my right hand side can be written in the following form. This is nothing but divergence E d cubed r over V 1 plus divergence E d cubed r V 2. We first calculate at the volume integral in the interior region, then we calculate it in the outer region. But on the other hand, in the form that I stated Gauss divergence theorem for you, the first term above that is divergence E d cubed r V 1 can be rewritten as a surface integral which involves the surface s 1. So, what is this equal to? This is nothing but integral E dot D S 1 S 1.

What about the next volume integral? We did not prove Gauss's theorem or we did not write Gauss's theorem to a case corresponding over the volume was bounding a surface and was bounded by a surface like in this case. So, in order to just evaluate that, I will rewrite it as integral V 2 divergence E d cubed r. Now, we can actually see that divergence E d cubed r over V 2 can in fact again be rewritten as a thumb of two surface integrals, not one surface integral because I started with the expression integral E dot d s that is nothing but integral E dot d s over s 1 plus V 2 divergence E d cubed r.

Now, let me add s here so that there is no confusion. So, what have you proved? What we have done is to generalize the Gauss's theorem for surfaces which bound and unbounded by a certain volume like this region R 2 and that is nothing but integral divergence E d cubed r V 2 is equal to integral surface integral E s E dot d s minus surface integral E dot d s over s 1.

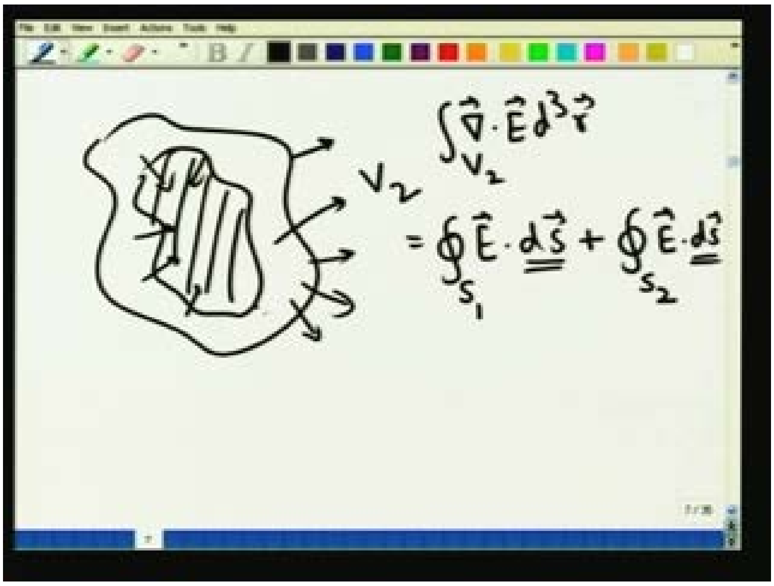
In writing these integrals, we should remember both s 1 and s have to be taken to be outward normal. What was my s 1? When I wrote down the vector corresponding to the surface element s 1, my normal's is going to be outward because I was all the time considering the interior volume R 1.

Of course for the surface element s 2, the area element is again outward normal, but when I am rewriting this expression divergence E d cubed r V 2, I am interested only in the region R 2 which is actually enclosing bounding the surface s 1. Therefore, I should characterize the surface integral not in terms of the volume that is enclosed, but in terms of the volume that is excluded.

What do I mean by that? Let me repeat. Consider the outer surface. For the outer surface, I wrote down the outward normal because I wanted the normal's to come out of the volume element. In a similar manner if I am interested in them so called inner surface, I am interested in the integration over this region and this is a volume which access a surface which accesses a boundary for that volume. Therefore, what I should do is to take actually the surface integrals actually coming outside the volume not entering the volume. That is the convention that we have always had.

If we did that, this can actually be written as surface integral s E dot d s plus surface integral E dot d s s 1, but s 1 has to be redefined; s 1 is now redefined with respect to V 2 and then we have the usual notion of a sum of two surface integrals. The only point that we have to remember which I have indicated in this figure is that unlike in the earlier cases, s and s 1 are not in the same direction, that is, if I imagine the following situation.

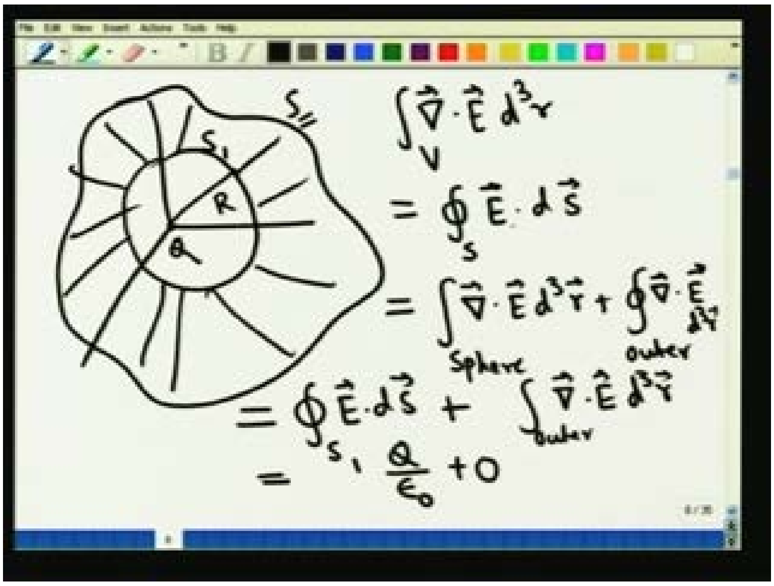
(Refer Slide Time: 19:16)



So, let me say that I have a some kind of an annular volume. There is some kind of an annular volume. We are not interested in this volume. We are interested in this volume V 2. Then I have a fluid. Let us say which is enclosed in this. The fluid can flow out of the volume either to the outer space or it can flow out of the volume into the inner space. We want to accommodate both of them. After all that is how we motivated the whole of gauss divergence theorem.

Therefore, in this case, the surface element goes from the surface element to the inner space. Here, the surface element goes from the volume to the outer space that is the reason why we write divergence V E d cubed r over the volume element V 2 is equal to integral E dot d s; obviously close surface s 1. Now, I will use the notation s 1 plus integral E dot d s 2. S 1 refers to let us say the outer surface; s 2 refers to the outer surface, and we know exactly how to determine the area vectors d s in both the cases. In both the cases, there shall get out of the considered volume. That is what we have done.

(Refer Slide Time: 20:49)



Once we state Gauss's divergence theorem in this particular form, it is now not a difficult thing to rewrite Coulomb law in its differential version. Therefore, let us return to our original problem. What is our original problem? Now, what I am going to do is to take a certain coordinate system. I am going to replace my charge at the origin of this coordinate system and let me draw a sphere. This is a sphere of radius r and then I am going to consider an arbitrary surface s.

My interest is obviously in evaluating the surface integral E dot d s corresponding to this arbitrary surface, whereas this interior region corresponding to this sphere is something that I can easily do. Therefore, let me call this as s 1. I want to make use of the Gauss divergence theorem in the form that I demonstrated just now in order to evaluate integral E dot d s whatever may be the shape of the surface. I cannot do that explicitly although I know the form for the Electric field, but Gauss divergence theorem in the form that I wrote in the previous page is going to help me and that is what I want to do.

So, we shall follow step by step the argument that we gave just now to this particular example. So, what I am going to do is to write divergence E d cubed r over the whole volume. So, I have the whole volume, full volume this by additively is nothing but integral E dot d s corresponding to s is equal to what do we have the inner volume.

So that I shall call as its sphere integral divergence E d cubed r plus integral divergence E d cubed r outer. Outer is this region. The inner is taken to be as sphere for the sake of computation, but otherwise there is no restriction on. Yes, there is no restriction on the radius of the sphere as well.

The only condition is that this sphere, and therefore, the outer surface shall also enclose this charge. Now, how shall we evaluate this integral? I cannot evaluate my left hand side obviously, but I do know how to evaluate divergence E d cubed r corresponding to this sphere because what is this relation? This is nothing but integral E dot d s corresponding to the surface s 1 plus integral divergence E d cubed r corresponding to the outer surface.

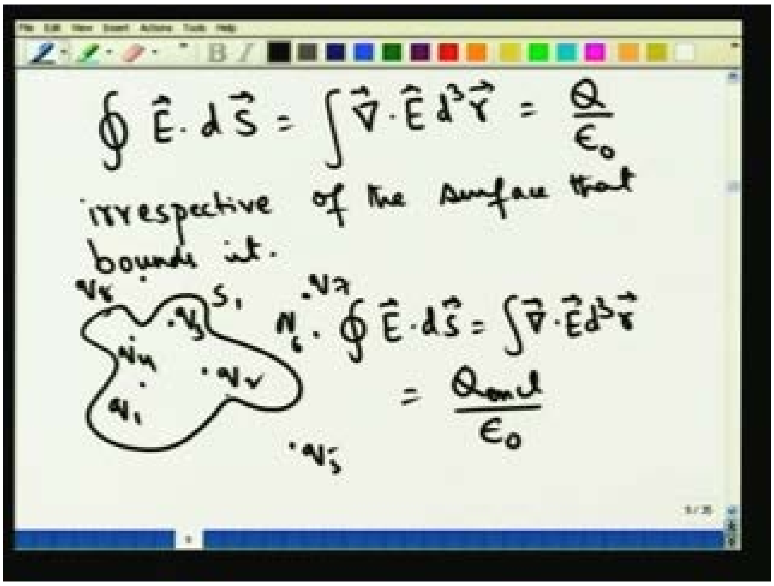
So, we are almost mimicking; reemitting the argument for this particular example, and what is integral E dot d s for the inner surface s 1? Well, remember E is nothing but Q over four phi epsilon naught by r square. Therefore, this is going to be Q over epsilon naught. That is what we have. The first integral corresponding to the spherical surface is going to give me Q over epsilon naught, but then, since I do not know divergence E d cubed r for the outer region because I do not know how to evaluate this particular surface integral. I do not know the shape of the surface. It might appear that we are in a fix not entirely.

What is it that we have to do now? Note is thus divergence E d cubed r is equal to 0 everywhere in the outer volume. In fact, divergence E d cubed r is identically equal to 0 everywhere except where the charge is located. Therefore, this is nothing but Q by epsilon naught plus 0. That is what we have here.

What is it that we have done now? This divergence E d cubed r for all of outer surface s is Equal to 0, and if you want to make your argument rigorous, I will rewrite this divergence E d cubed r as a sum of two surface integrals s 1 and s 2. Therefore, that object will also be equal to 0. The inner fellow will be equal to 0 rather the inner fellow will exactly contribute the cancel the outer fellow. Therefore, what we have proved is that the surface integral E dot d s is nothing but Q by epsilon naught because we know that divergence E d cubed r equal to 0.

What is it that we have done? What we have done is to employ the fact that divergence E equal to 0 everywhere in the outer volume. Therefore, the corresponding some of the surface integrals should also vanish, but on the other hand for the inner volume, I know that it Encloses the charge; I know how to I know how to evaluate integral E dot d s explicitly. Therefore I get the result that whatever may be the surface s as long as it encloses a charge Q, I am going to get integral E dot d s is equal to Q by epsilon naught. Now, I am going to state that result explicitly.

(Refer Slide Time: 25:49)



So, what am I going to say? We are going to say that integral E dot d s is equal to integral, this would be the volume integral divergence E d cubed r is equal to Q by epsilon naught irrespective of surface that bounds it. It can have any shape. Please remember, this result followed because divergence E was equal to 0 everywhere except at the location of the charge. Now, suppose you would had not one charge but several charges. So, again I am going to write down a surface here.

So, this is my surface enclosing a certain volume. I have a charge Q 1; I have a charge Q 2; I have a charge Q 3, Q 4 so on and so forth Q 1, Q 2, Q 3, Q 4 by the additively of the surface integrals at the volume integrals. Again we can write integral E dot d s which indeed equals the divergence E d cubed r that is a volume integral is nothing but Q enclosed by epsilon naught - where Q is the total charge.

This is indeed Gauss's law as it is required for us to state the Coulomb law the differential form. Now, there is a customary caution that I should exercise and I should sort of advertise before I proceed and that is the following. In consulting this surface I am considering the charges Q 1 Q 2 Q 3 Q 4, I should not assume that all the electric fields are coming only from these charges. I could have put a charge here; I could have put a charge here; I could have put a charge here; I could have put a charge here. Let me call them Q 5, Q 6, Q 7, Q 8. These are also going to produce non vanishing electric field in this particular region, but what is the statement that we are making.

Consider the charge Q phi. Q phi is not bounded by the surface s. So, whatever may be the electric flux that enter the surface, that flux is going to exit that surface; it is going to leave that surface. Similarly, whatever may be the electric field, that eminent from Q 6 or Q 7 or Q 8 whatever enters the region also leaves that, in other words, the source for the electric field in this particular region s 1 lies outside so long as Q 5, Q 6, Q 7, Q 8 are concerned.

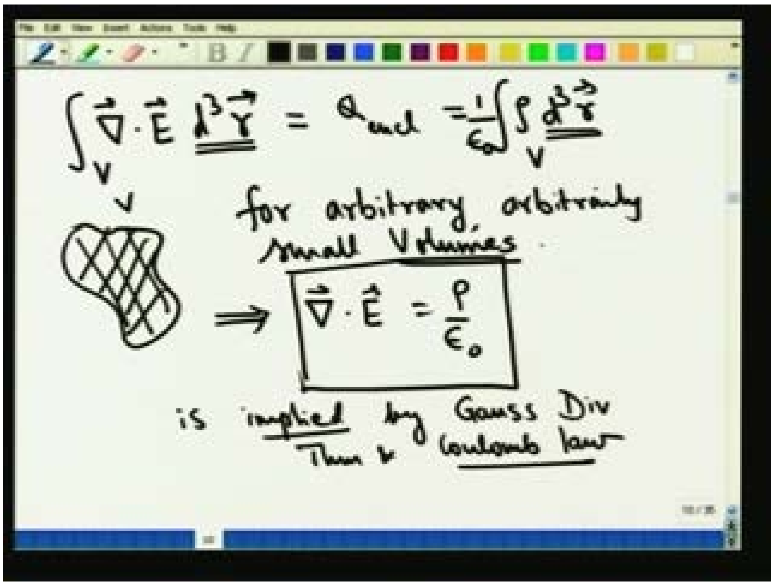
The physical surface for the electric field are actually only given by Q 1, Q 2, Q 3, Q 4. In other words, although the electric field itself it get contribution from charge distributions all over the space wherever they may be located, this surface integral, integral, dot d s, and therefore, this volume integral divergence E d cubed r is going to get contributions only from those charges which are enclose by this particular surface. That is something that we have to remember.

So, if you remember that and if we can intelligently make use of this equation together with hopefully some given symmetries of the problem, then solution of many many electrostatic problems become very easy and that is where Gauss's law is indeed the most useful. However, we have not reach the end of our journey because I have still written my law in the integral form. Notice it is looking even less than a Coulomb law because it says give me Q enclose, I will give you integral E dot d s. What am I going to do with it? Suppose I take a surface and I put a charge outside. How am I going to produce the electric? Find out the electric field for reduce by that are some charge distribution for that matter.

In order to accomplish that, what we shall now do is to exploit the fact that the volume integrals contained here and the surface integrals that are evaluated here are all arbitrary. We place no restriction on them because instead of considering this surface, I could have constructed this surface. Then the relation would have been valid except that Q enclose would have been comprised of Q 2 and Q 6.

I could have constructed volume bounded by a surface which Encloses Q 3 and Q 8 so on and so forth. Therefore, we want to make use of the fact that this equation is valid for any charge distribution, for any volume element, for any surface that bounds that volume. So, if you did that, then we would have stated Gauss divergence law. How we do that?

(Refer Slide Time: 30:37)



The answer, the key is in the word arbitrary and let me exploit that. So, I am going to write divergence E d cubed r. Take a certain volume and I am going to evaluate divergence E in this region and I am going to evaluate this volume integral. Gauss divergence theorem tells me that this is nothing but Q enclosed. That is what it tells me.

I have placed no restriction on the volume. This is only for the reader purpose I wrote this, but what is Q enclosed? The total charge contained in a given volume element is nothing but the sum over all the charges, or for that matter since we are considering continuous charge distributions, this is nothing but the another volume integral rho d cubed r over the same volume. Please notice this.

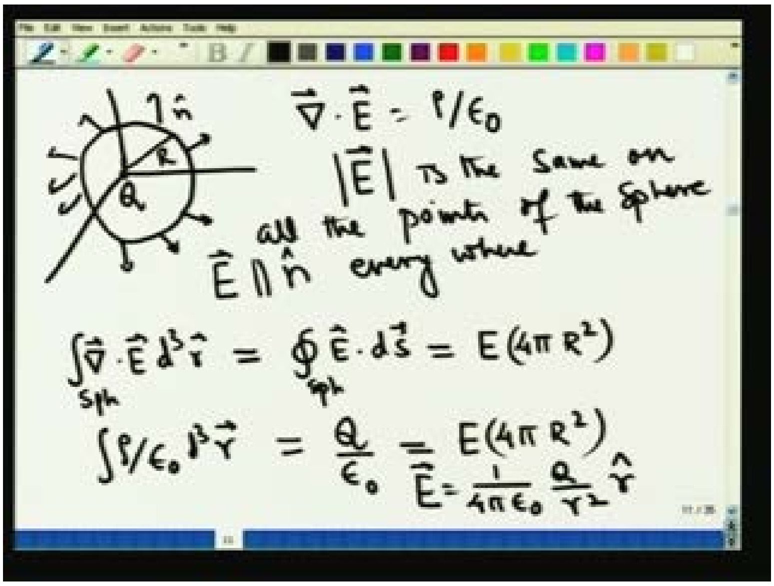
We are making an assertion that divergence E d cubed r is equal to integral rho d cubed r for arbitrary for that matter arbitrarily small volume elements. Volumes that may not call it as a volume element; that means, the left hand side which is an integral; the right hand side which is an integral has the same value, the same numerical value irrespective of the domain of integration, irrespective of over how, how, large or how small a volume you looked at.

Therefore, the only way that these two integrals can be the say is the integrants must also be the same because both of them are equal to Q enclosed. Therefore, we conclude that divergence E is nothing but rho by epsilon naught. I forgot the factor one over epsilon naught here and this is indeed the differential statement or the differential version of Coulomb law. We started with a Coulomb law generalize it to continuous distributions rewrote Gauss's theorem appropriate to this particular example of one over r square force and dot coulomb law in the found divergence E equal to rho by epsilon naught.

In other words, this equation is implied, that is something that I should emphasis, implied by gauss divergence theorem and Coulomb law. That is what the implication is, but now, I would like to assert that given Gauss's law divergence E equal to rho by epsilon naught and given some basic symmetry, we can actually derive coulomb law, that is, I would like to show that Coulomb law and Gauss's law are completely equivalent to each other.

That is the reason why most of the time divergence E Equal to rho by Epsilon naught is also called coulomb law and that is indeed the first of the Maxwell Equations. It is insulting to do it slowly because there are some very nice thing associated with it and let us see what we have to do.

(Refer Slide Time: 33:49)



We have a charge Q sitting I choose it at the origin and what I will do is construct a sphere of radius r. In fact, this sphere is what is called as a Gaussian surface, all surfaces where we use gauss divergence theorem or Gaussian surfaces, and now, I want to employ divergence E Equal to rho by epsilon naught to derive the fact that the field produced by this charge is indeed one over four phi epsilon naught Q by r squared radically outward. How should we do that?

The interesting thing here is that I have not told you how to write down the charge density for a point particle, for a point charge. We have not told you although use delta function at some point, we did not spend any time on that, but never mind we can still show the equivalence of the two forms, that is, one over 4 phi Epsilon naught Q by r squared and divergence E Equal to rho by epsilon naught for a point particle by looking at an appropriate surface integral.

However, before I look at an appropriate surface integral, we have to take some symmetry into account, symmetry considerations into account and what are those symmetry considerations. All the points on this sphere of this radius r, they are equidistant from the charge Q because the charge Q is located at the center of the coordinate system.

Therefore, by symmetry, the magnitude of the electric field should be what the magnitude of the electric field should be the same all over the sphere. On the other hand, I have a point charge Q and it does not care how I erect my coordinate system. I could have taken this to be x y z or rotated it any manner. The field produced does not care for the coordinate system. In other words, the field should be completely isotropic and the way it is going to be completely isotropic is that the field should be radically outward. These are the two important things that we have to notice.

These facts, these principles were actually implicit in the Coulomb law. We are restating it explicitly now. So, what are these statements that we are making? The electric field, the modulus of it is the same on all points of the sphere, all the points of the sphere, and secondly, E is parallel to m everywhere. E is parallel to m everywhere where m is the outward normal; m is the outward normal that is nothing but the surface element.

So, if were to integrate the left hand side and the right hand side over a sphere of any radius r, however small or however large is that the electric field is always parallel to the surface element. If I had taken Q to be negative, the electric field would have been anti parallel, and then, now, it has the same magnitude everywhere. Therefore, I now make use of divergence theorem and I will write divergence E d cubed r corresponding to a sphere of radius r.

What is this? This is nothing but integral E dot d s over this m spherical surface. That is what I have, but on the surface, electric field has a same magnitude; it is always parallel. Therefore, this is nothing but E into 4 pi r square - where r is the radius of the sphere. Therefore, we evaluated the left hand side over an integration over arbitrary sphere. The only condition is that that sphere should enclose the point charge.

What about the right hand side? The right hand side is also very easy to evaluate rho by epsilon naught d cubed r is the total charge contained in that volume. Now, where is the total charge contained in that volume? You take a sphere of radius epsilon how so ever small it may be. You take a sphere which is as large as you want irrespective of the radius of the sphere. So, long as the sphere encloses the charge Q, the total charge encloses always one in the same namely the charge itself.

In other words, this density has a peculiar property that it is 0 everywhere. It is infinite at one particular point in a manner that this volume integral is independent of the volume that you consider so long as include the point where the charge density is blowing up.

That is the meaning of a delta function. Therefore, this is nothing but the charge of this sphere.

Of course, I have my epsilon naught which I should write. Therefore, to rewrite it, this is nothing but E into four pi r square. Therefore, we conclude that for any sphere, for any radius r are any distance r from the charge Q my E is nothing but one over four phi epsilon naught Q by r square. I will make it a variable now into r hat because we already made use of this.

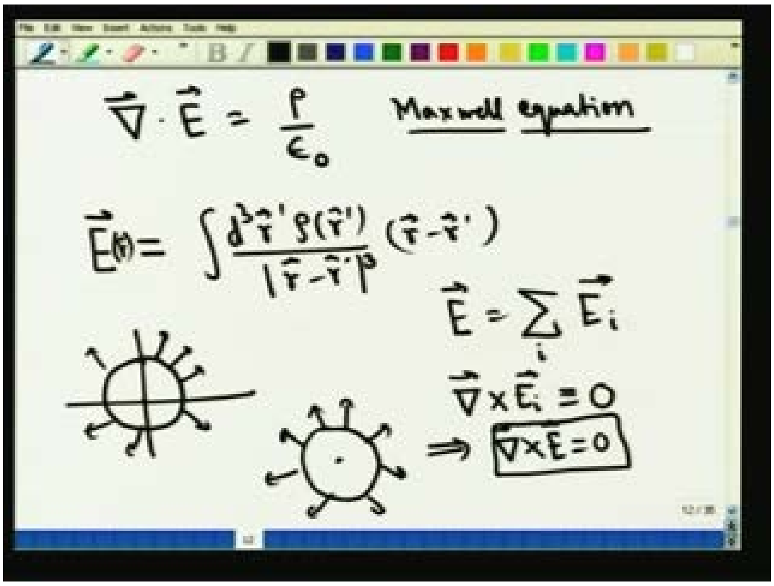
In fact, I do not have to assert whether the field is parallel or anti parallel to the surface. If Q is negative, it will be anti parallel to the surface element because the field line will be coming inside. If Q is positive, the field will be going outside. Therefore, what we have done is to quote unquote derive Coulomb law given Gauss's law.

Now, this derivation also gives you some idea as to why we include at the factor 4 pi in the definition of the coulomb law. After all as we discussed Q epsilon naught four pi, they come in a specific combination Q over 4 phi epsilon naught. The reason why one over four phi epsilon naught is absorbed into the definition of coulomb law is that, in that case, divergences E equal to rho by epsilon naught is a very simple form, that is, Gauss's law can be stated in a simple form without involving the factor 4 pi.

It's a matter of Esthetics for those people who use this equation periodically. In any case, 4 pi is a geometric factor which owes its origin to the fact that we are looking at spheres the elementary spheres of this kind in order to demonstrate the equivalences. Now, this form of Coulomb law is fundamental to electrostatics, because as I told you in my previous lecture, electrostatics does not simply deal with discreet number of charges. It deals with continuous distributions of charges. Electrostatics is fundamentally concerned especially for engineering students with macroscopic physics, dielectrics, conductors, pyroelectrics, liquid crystals so on and so forth.

And in all these macroscopic cases, we look at macroscopic charge densities. We are not interested in elementary charges corresponding to that of electrons. We are interested in large charge distributions, and whenever we have such large charge distributions, when we want to study the properties of this media, whether it is a conductor or a dielectric, it is always convenient to study electrostatics not with the original coulomb law, but with Gauss's law.

(Refer Slide Time: 41:04)



Therefore, for that reason, this equation which is indeed fundamental namely divergence E equal to rho by epsilon naught is called the first of the Maxwell Equation. So, this is a Maxwell Equation number one, and as we shall see in this lecture or at least certainly in the next lecture, we are going to use this study a variety of macroscopic properties. However, before I do that, this yet another important concept, important idea that I have to discuss and that is the idea of a potential, and in order to do that, let us go back either to the Coulomb law written in terms of rho of r prime d cubed r prime etcetera in integral over the charge density or let us look at this and see what it is.

Now, it is actually convenient to start with the expression for the electric field and what does it say. It says the expression for the Electric field due to a charge density rho is nothing but d cubed r prime rho of r prime bought r minus r prime cubed r minus r prime. That is what we have. This is the Electric field at any point r.

So, remember, I have completed my program. I ask the Question what is the differential equation corresponding to the solution. What is the differential equation? The differential Equation is divergence E equal to rho by epsilon naught. Except that, this was a complete solution, whereas they said differential equation. Therefore, I have to specify a boundary condition which I have glossed over.

I will return to that letter, but right now let me Explore yet another property. What is that? Remember, what is we said about the field produce by a point charge. How does a field produce by a point charge look like, if I were to locate it at the origin and if I were to draw its sphere, it would be radically outward everywhere.

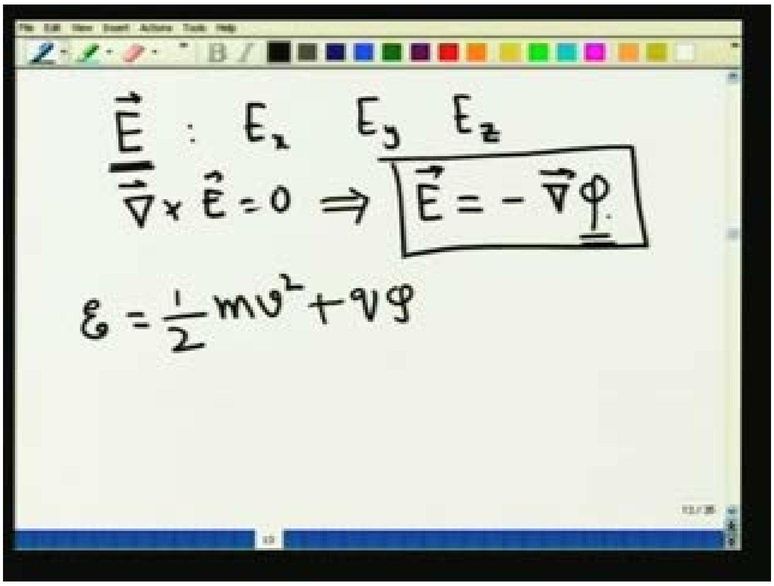
Take another charge particle let us say that I locate it at here, draw its sphere around this, it will be radically outward everywhere. In other words, although the combined electric field might have a very complicated form, if I wanted to draw the field lines, we know that the contributions of each of these individual charges is such that the, the, lines are always diverging or if you feel like converging, if you consider a negative charge from its source, that is what we have.

Consider each of them. Make use of the principle of superposition. E is Equal to summation E I - where I is the summation over each of the sources. Consider the it source coming from the it source. If I were to locate it at the origin for the sake of convenience, you can easily see that curl of E I is identically equal to 0. That is what we have here.

How do I know that? I know that E I behaves like one over r squared with respect to this particular point. It is radically outward curl of a radically outward or an invert normal vector is always identically equal to 0. There is no curliness associated with the field lines. There is only a divergence associated with this field lines. Therefore, we conclude that curl E is equal to 0.

So, strictly speaking, divergence E Equal to rho by epsilon naught does not exhaust all of electrostatics. In spite of the fact, that I claim, that I showed the equivalence between the coulomb law in this. We have to supplement it by the condition that curl E equal to 0. If curl E is taken to be equal to 0, then together we divergence E equal to rho by epsilon naught. That will give us all results which would obtain starting from an original formulation of the Coulomb law.

(Refer Slide Time; 45:11)



So, what is the meaning of the statement? That curl of E is equal to 0. Let us do some parameter counting. Parameter counting is dangerous. It is not strictly useful always, but here, it will give us an idea or a hint. When I write down my Electric field, I have the situation there are three components of the Electric field. So, what is it that I have? I have E x; I have E y; I have E z.

Now, we are asserting that curl of E is equal to 0. Now, if curl of E is equal to 0, that means E x E y and E z cannot be independent of each other. Is that right? If E x E y and E z could take arbitrary functional forms, that is, if E x could be an arbitrary function x y z, E y could be an arbitrary function x y z, E z could be an arbitrary function x y z, then curl of E equal to 0 is not necessarily guaranteed.

But I want the most general form for the Electric field consistent with the condition curl of E is equal to 0, and therefore, we ask what is the constraint? What is the condition that I can impose on E such that I will automatically get the result that curl of E equal to 0 out. In other words, I would not like to introduce an auxiliary Quantity, a fundamental Quantity. From which, the minute I derive my Electric field, this constraint curl of E equal to 0 is automatically satisfied.

Let me give an example. Suppose I am sitting on the surface of a sphere and there is a complicated force acting upon me. Now, the force might be anything, but so loss, so long as I am considering to move on the surface of a sphere, it is always useful to employ spherical polar coordinate system. Where you have only degrees of freedom corresponding to theta phi, your r is a fix parameter. In a similar manner r, I asked whether it is possible to write E x, E y and E z, that is, this vector field E as a function of some other object such that curl of E is automatically guaranteed.

Well, all of you would certainly know what the answer is because we know that its curls of E is equal to 0. Then E can be simply written as the gradient of the scalar function. So, I will write it as that E equal to minus gradient phi. What we are saying is that, you need not specify for me the three components of the Electric field. It is sufficient for you to specify for me one single scalar field, which is shall call as the scalar potential or the potential in short. At this particular point, you calculate the gradient for me it will give me the Electric field.

This is a theorem which is implicit in the result that we stated Earlier. Is that all that you have to do is to show that every time curl of E equal to 0; it can be written as gradient phi, and every time E is minus gradient phi, curl of E is identically equal to 0. I will leave that as an exercise because we have already spent a long time discussing the physical meaning of divergence curl gauss theorem and gauss theorem so on and so forth.

So, what we shall do is to write E Equal to minus grads phi - where phi is a potential. This potential is not new to you. You are already familiar with the concept of a potential because in your mechanics course actually you use the potential, which actually gave you the potential Energy, the sum of the kinetic Energy plus the potential Energy was also a constant.

Even here we can actually show that if I can write down the total Energy of a particle, what is the total Energy? Imagine there is a charge particle with a mass m. It has a kinetic Energy half m V squared, then it is moving in a potential field phi. What does it mean? If I calculate the gradient of this phi, it is moving in an Electric field which is E equal to minus gradient phi. Then if I multiply it by Q phi, one can show that this total energy is indeed a consult quantity, and for this region, this is called as the potential function or the potential field.

So, curl of E equal to 0 tells me that E can be written as minus grad phi, and in many many calculations, it is of course much more convenient to determine phi rather than E, because here, you have to determine three functions, whereas here you have to determine one function and we shall illustrate the utility and the importance of the scalar function, the so called this potential function in the next lecture.