Engineering Physics-II

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Module No. # 02
Lecture No. # 04

In the last lecture, we discussed Coulomb's law. We stated it for a single charge particle.

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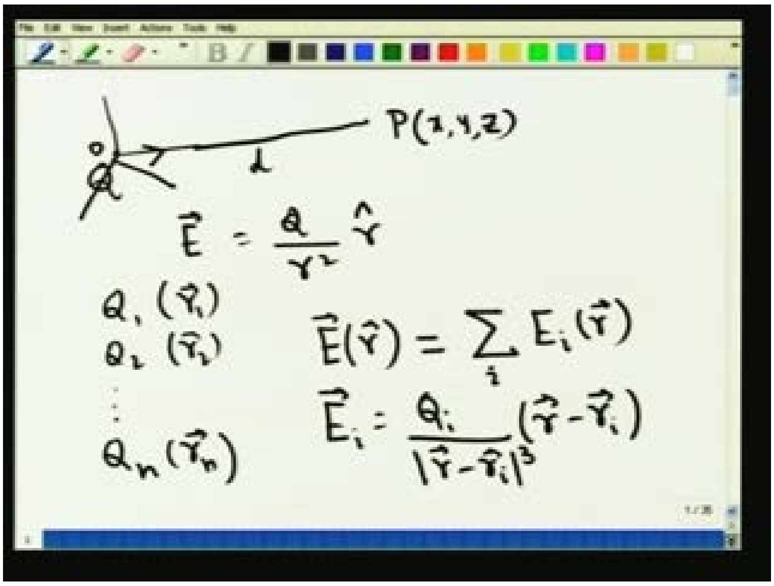

And we said that if you have a particle located at a point, and let us say that it carries a charge Q, and if I look at any point p at a certain distance d from the charge. So, let me say that this charge is located at the origin of the coordinate system, say for it is at the origin and this P has coordinates x y z, then the electric field at the point p by this charge Q is simply given by e is equal to Q by r square r hat, where r hat is the unit vector in the direction of the the direction of the point p.

What we then did was to generalize the result to a system of charge particles by stating the principle of superposition. So, what is it that we did? We said that if I have a system of charge particles; let us say Q 1, Q 2, etcetera Q n and they are located at the points r 1 r 2 etc up to r n. All these are vectors and we are interested in the field at a given point r.

If I am interested in the field at a given point r; this would be simply given by the summation of the fields produced by each of these individual charges. So, I will write that this is equal to e i of r. This is the statement of the principle of superposition. The field produced by the individual charges add up to find the total field produced by the collection of the charges Q 1 Q 2 to Q n and what is E i given by? E I is simply given by Q I divided by mod r minus r I whole cubed multiplied by r minus r i.

This is indeed the statement of the coulomb law together with the superposition principle. Now we already know from a detailed discussion in the earlier lectures that all charges quantized; it always comes in discrete units, integral multiples of either the electron charge or the proton charge and therefore, Coulomb law together with the principle of superposition essentially completes the subject matter of electro statics. Because you give me any charge distribution all that I need to know is to find out the locations of the individual charges Q 1 is located at r 1 Q 2 is located at r 2 so on, and so forth. Q n is located at r n.

Find the field produced at any point from each of them they are simply given by the usual inverse square law, add them up and then you find the total charge. But we know this is not the end of the electro statics as I said in the last lecture it is. In fact, the beginning of the subject matter of electro statics, because we have continuous charge distributions. Not only do we have continuous charge distributions; materials built out of these continuous charge distributions behave very different from each other.

You have conductors where the charges are free to move. You have insulators where the charges are not entirely free to move, but they can only adjust their positions against an external field. So, in order to discuss this very rich physics, this very rich world of electrostatics you have conductors, you have dielectrics, you have liquid crystals, you have ferroelectrics so on and so forth. We need to go beyond these basic principles and develop special techniques. In order to develop these special techniques, what I did last time was to consider a toy example of a certain charge, which was distributed over a certain line.

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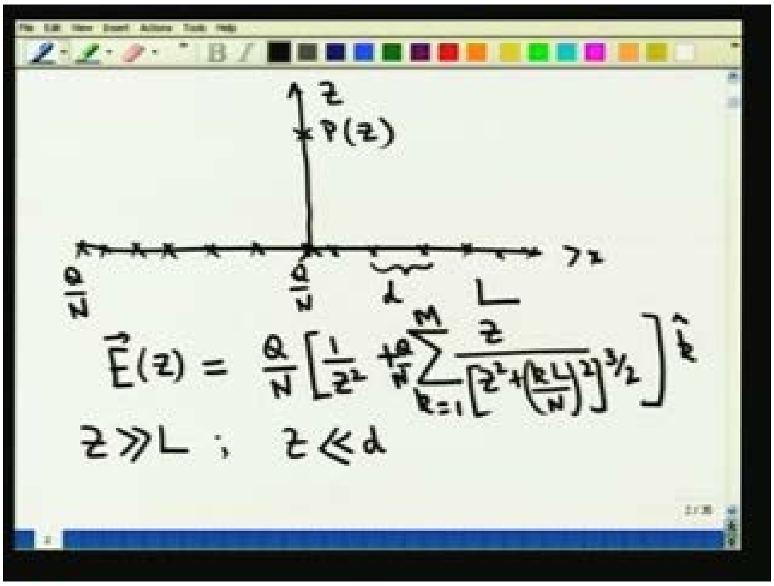

So, let me very briefly recapitulate that and see what is it that we have to do now. So, let us take a line, which is let us say along the x axis, and then what I did was to take a charge Q, and I distributed it uniformly at discrete points. So, what do I do? First of all, I mark the centre of the line, I put a certain charge. So, let me call it as Q by n, then I choose so many equal equidistant points here, so many equidistant points to the left of it. The total length is simply given by l. What I do is to distribute Q by n on each of these points.

So, if there are n equidistant points; n is taken to be odd for the sake of convenience for purposes of illustration here. If there are n equidistant points I put Q by n charge on each of them. So, that the total charge adds to Q. That is what we did. So, when we are looking at n equidistant points; obviously, I have the distance K, which is d which is simply given by l over n minus 1. I am not going to write that and then I ask what is the field produced by this line charge distribution, which is not continuous, but which is discrete along the line which is perpendicular to this midpoint of the line charge.

So, if this is the x axis. Let me call it as the z axis. I take any point p which is having a coordinate z; x is equal to 0 y equal to 0, because I will put the origin at the midpoint of this line and I ask, what is the field produced at this point? Now, thanks to the principle of superposition and of course, you people have worked out many such examples in your class twelve. It is not at all difficult to write down the field at this particular point p. All of you know that the direction of the field is simply along the z direction. What do I mean by that? By that I mean the x component and the y components of the field identically vanish, only the z component survives.

What is the expression for the field? If you people remember this is what I wrote e is along the z direction this is simply given by Q by n and what I did was to pick out the first point. So, that is simply given by 1 over z square; this is the field produced by the charge locate at this particular point. Then I was interested in the fields produced by the charges which are to the right and to the left of the origin which I wrote as summation K equal to 1 up to sum m which is simply given by n minus 1 by 2. Then I had Q by n of course, comes here because each of them carries a charge Q. I have my z here divided by z square plus k l by n whole squared to the power of Q 3 by 2.

And the electric field is along the z direction and this is the expression that I am going to get. So, you can easily see this. So, once we have done this we have a complete expression. But that is not my purpose. My purpose was actually to explore the behavior of this field in the limiting case in various cases for example, when z is far, far away from the line charge distribution and z is very, very close to the line charge distribution. If you people remember again I discussed the first two cases with some generality. What is it that I did? I looked at the case when z is very, very large. I looked at the case when z is very, very small. I should quantify the statement z is very, very large, z is very, very small. The way we do that is not tough at all.

What we say is that if z is much much greater than the length of the line charge distribution; we say that z is large. On the other hand if z is much much less than the inter spacing the spacing between the two consecutive charges here then we say z is small. The distance of your reference point is smaller than inter charge separation. These are the two extreme limits that I discussed. I am not going to work out those steps again, but you people can easily see that in both the limiting cases we are going to get the Coulomb law. So, let me write it down because it is important for us.

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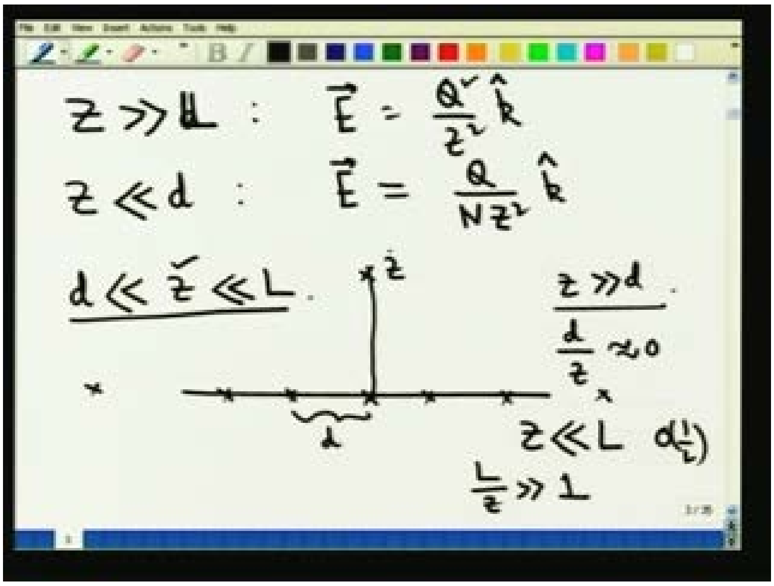

So, I say if z is very, very much greater than l; that is what I am interested in. If z is very much greater than l then I am not able to resolve the size of the system. I would not know that actually the charge is distributed over a line of length l. I would imagine that all of it is probably concentrated at the center. Therefore, the leading order contribution for the field will be simply given by Q by z square k. It is as if all the charge is concentrated at the origin. Of course, what you can now do is to imagine that I am going to dissolve the distance or resolve the size of the system. Then I will get higher order corrections which will go like 1 over z Q 1 over z to the power of four etcetera etcetera. We are not interested in that.

We are interested only in the leading order and this is the electric field that I am going to get. Now on the other hand if you go to the other limit z is much, much less than d. If z is much, much less than d of course, it is even lesser than l because I have put a large number of divisions on this line of length l. If I look at this again I get the Coulomb law because what my test charges is; is essentially the field due to only one particular charge. So, that is again given by Q by n z square k hat. So, in one case you see the field produced by all the charges distributed over the n points which is the reason why I get the total charge Q in this particular example. Whereas, here I see the field produced by only one test charge Q by n whatever is closest on the z axis and the all the others will give a (( )).

If I remember right I even wrote down the correction terms which go like proportional to z etc etc. I am not going to do it now. Now we ask a far more interesting question. What happens in the intermediate case? So, the intermediate case correspond through d less than or equal to z less than or equal to l before I proceed to discuss this example. Let us pause for a minute to examine what the situation means. So, I am sitting at such a distance. So, let me write down the figure again I am not going to write all the points. Let us say I am going to write a few points here.

So, this is the inter charge separation d and I have put my z here. So, I put my z somewhere here and I am saying z is much, much greater than d. What is the meaning of this statement that z is much, much greater than d? By that I mean that I am sitting at such a distance that to the lowest order I can ignore the separation between the two charges. So, in this situation one says that it is not going to be great error on our part if we assume the charge is going to be continuously distributed. I am not going to resolve in the 0th order this operation between the two charges. So, in other words I am going to say that d by z is very close to 0. That is the statement that we are making because z is much much greater than d.

On the other hand I also have this scale z is much much less than l. So, if z is much much less than l it is as if you will never see the end points of the line charge. So, the n points are sitting somewhere here. So, z by l is a very, very small quantity or equivalently l by z is much much greater than 1. So, in the 0th order, I can ignore the finiteness of the line charge distribution. That is in the 0th order the line charge can be taken to be of infinite length. Is that part ok? I am not saying that the line charges are of an infinite length, but for all practical purposes it is of a very, very large length which can be approximated to infinity and if I am going to work out higher order corrections they will be obviously, of the order 1 over l which we will not bother about at this particular point.

So, we are interested in finding the electric field produced by this kind of a line charge at a location z which is larger than the inter charge separation, but which is smaller than the line charge separation. So, what are we going to do with it? Well, what we do is to go back to the previous expression. I wrote the summation over the contribution coming from each of these chargers, sum them up, but that is not what we need to do. We want to motivate how we are going to get into the notion of a continuous charge density. And so, in order to do that what I shall do is to show you the graphical representation as to how the field varies in this particular regime and that is really instructive.

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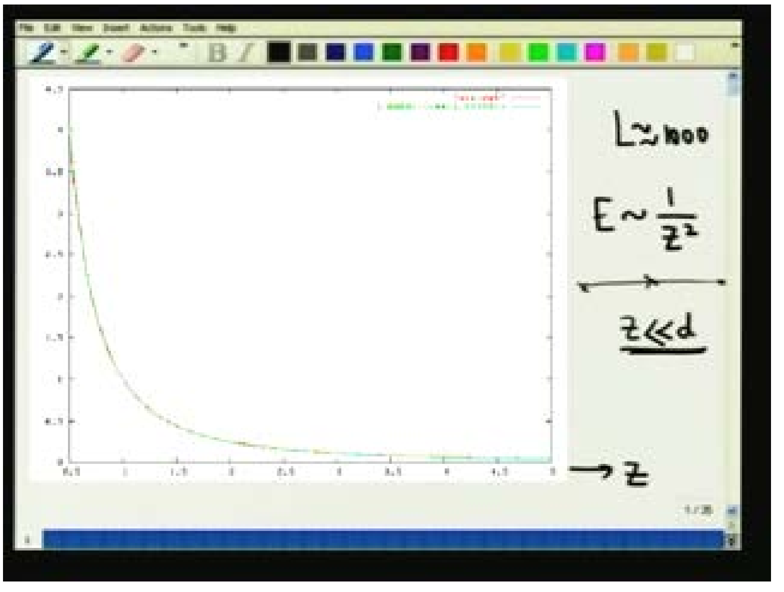

So, since I am showing to going to show a few pictures for you, graph for you; I might as well start with the simplest of the situation and let us look at the first graph here. Now, I have to explain a few notations here. If you look at the x axis the variation is from 0.5 to 5 which is the distance z. So, this is actually the distance z. That is what we have here. Now, I am drawing the field along the y axis. I am interested in the E. However, what I have d 1 is to choose a line of roughly 1000; l is of the order of 1000 and therefore, when my z is varying from point five to five you can easily see that I am sitting very very closed to the line charge distribution.

Now, the next question that we have to ask is what are the charges that we have played on this line of length l? What I have done is to put only three charges; one at the center and the other at the two ends and I am sitting very, very close to the line charge. So, what do I mean by that/ So, there is one center here and there are two end points here and I am sitting somewhere very, very, very close so obviously, what I expect is that this figure should confirm to the limit that z is much much less than d because d is actually of the order of the length of the line charge itself. It is simply given by d by 2 l by 2 I am sorry.

Now, let us look at this figure there are two curves actually which have been superposed on each other because you can see some slight variation in the color between green and the red. Actually one of them corresponds to the exact value of the field that is the red 1 you see a few red points here. Whereas the other corresponds to a curve fitting and. So, look at the curve fitting you'll be able to see that it goes like 1 point naught naught 1 x to the power of 1.99999 which is an excellent approximation to two. Therefore, we can simply say that e is indeed going like 1 over z square that is what you have seen. So, this simply verifies whatever we have actually analytically worked out that namely if you are test charge or if your point is very close to the system then all that you will see is the field produced by one individual charge.

Now, let us proceed a little bit further let us again keep only three charges and let us ask what happens if I go a little bit further. So, that is something interesting and this is what we are going to get.

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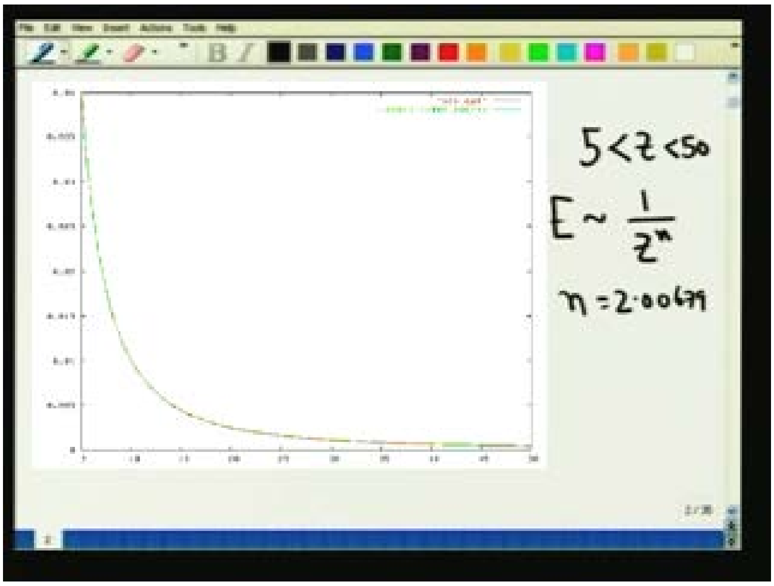
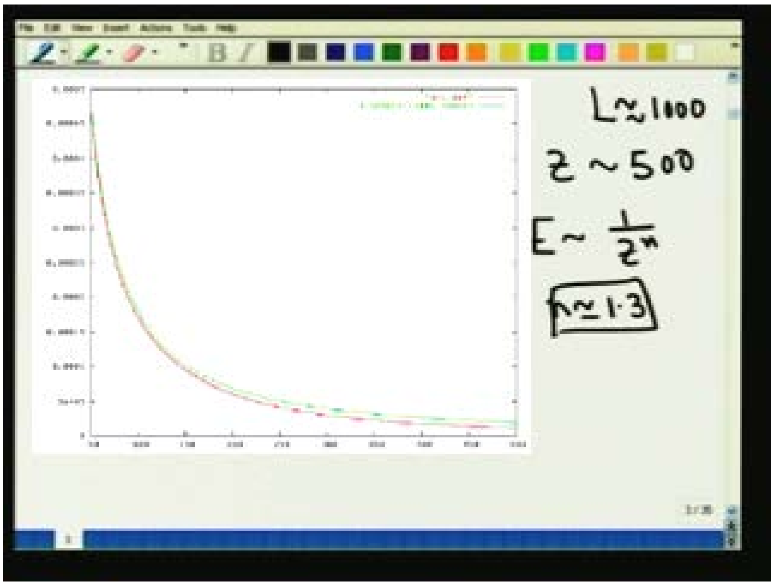


Now, what I have d 1 is again I am keeping the same number of charges, but you can see my z is varying from 5 to 50. So, earlier z was varying from .5 to 5. Now, I have 5 less than z less than 50 which is still very small compare to the length something of the order of thousand, but still a careful curve fitting shows you that there are slight deviations. In the first case we had got n is equal to 2. Therefore, if I wrote e is the of the order of 1 over z to the power of n; in the previous case we got n is equal to 2. Here we are getting n equal to 2.0678 which is a slight deviation from the value n is equal to 2.

So, although the distance is small it is not as small as the earlier case and therefore, the deviation from the 1 over z squared behavior is already something that you can see. (Refer Slide Time: 16:39)



Now, the next thing to do is to of course, to the next figure here what is happened is that I have g 1 to a scale which is comparable to the length it is still not large comparable to the length because please remember n is of the order 1000. That is what I have and here my z is all going all the way up to 500. If you look at the final value now you see that there is a major deviation. How does the exponent behave? Again if I write E is going like 1 over Z to the power of N is of the order 1.3 which is quite far away from 2.

The interesting thing is that no where you find a behavior something like 1 over z unless I constructed consider a very, very narrow regime which I am not doing. Therefore, in order to complete the tale for this particular example; let me go the next case where I make z even larger and ask how the field behaves. This is the last of the thing.

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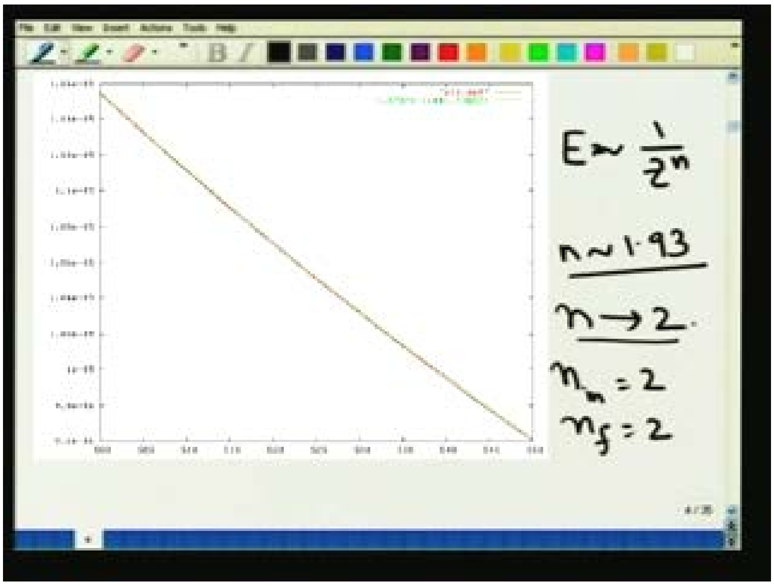

Here I am now going to a length which is only between 500 and 550. It is still not very large compared to the size of the system. It is only comparable to the of the system, but already E if I write again goes like 1 over Z to the power of Z is of the order 1.93. So, in other words, if I had continued this experiment continued this computation and g 1 to l values between let us say 1000 and 2000 or 10000 and 15000 let us say which are really large compared to the size of the system itself; then we could have said that n would have approached the value 2.

So, what is this statement that you are making? For this simple example what we find is that n extrapolates between 2 and 2. So, n initial was equal to 2 when z was very very small. n finite is also equal to 2 when z is very very small and depending on the number of charges that you put, there will be a small window in which actually n will pass through 1. That is the field behaves like 1 over z.

Now, it is deliberately that I did not show the regime where field behaves like 1 over Z here because I have put only three charges. Therefore, it is a very very narrow window in z. Is that okay between z 1 and z 2 which is a very small number, which is very difficult to show and therefore, what I will do is that in order to illustrate that I will go to the next figure.

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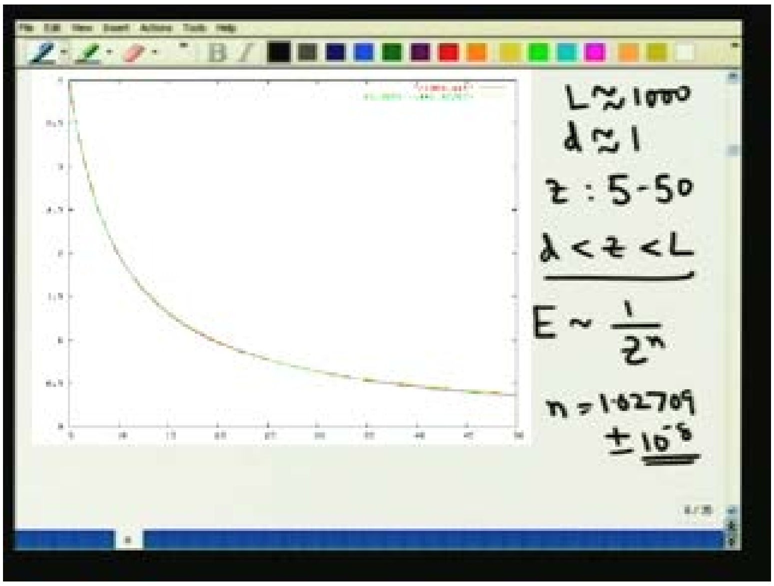

In this next figure, what I have d 1 is to put ten charges. When you put ten charges I am not interested in the extreme cases, when z is very very small or the z is very very large I am interested in the intermediate case where z goes from 0 to 50 as you people can easily see. Now, what is happening? I am going to concentrate entirely on the value of the exponent and as you people can read; you see n is of the order 1.12 which is precariously close to n equal to 1.

It is not quite 1. There is something like a ten percent error ten percent deviation, but still the minute I put 10 charges it became n equal to 1.1.2. Whereas, when I put only n equal to 3 charges that gave me exponent something like 1.4. That is something that you have to remember. By the way when I say ten actually I have put 20 charges; 21 charges to be precise. When I say n equal to 1 I have put three charges.

Now obviously, a nice question to ask is what would happen if I could define my calculation even further? Now what I shall do is to divide the line into thousand equal intervals. So, I have a length of the line of length l I shall put 2001 charges. is that I have divided into thousand divisions to the left 1000 divisions to the right therefore, the distance between them is of the order 1. If I did that again if I looked at the region between 5 and 50; now this is a very beautiful regime because d is of the order 1. l is of the order thousand and what is my z varying from? My z is varying from 5 to 50 and it eminently satisfies the condition that d is much less than z less than l.

And (( )) behold what is it that you find for the exponent again; if I parameterize e goes like 1 over z to the power of n turns out to be 1.02709. Please remember that when you are staring at this curve that are actually two curves; One of them is red another is green. The red curve is the exact number obtained by summing over all the chargers. This has been d 1 numerically the green curve is a curve fitting and it is giving a number 1.02709. Whenever I fit a curve whenever I make a statement that n is equal to 1.02709; it cannot be an approximate type. It cannot be an exact number. So, I have to place the error on it the error on it. In fact, is remarkably the error bar is remarkably stringent.

What we are saying is that; this is equal to 1.02709 plus or minus 10 to the power of minus 8. So, it is a very very accurate representation and you can, people can easily see that if I had replaced n 1000 by n equal to 10000. That is if I had put 21000 charges of that same length then what would have happened? d would have been of the order of .5 and n would have been even closer to 1.

So, what is the theme of my discussion? The theme of my discussion is that irrespective of how many charges you have and what the division is that in the asymptotic limit when the z is very, very small when the z is very, very large you have n equal to 2. But within n equal to 2 and n equal to 2; you actually the exponent actually decreases. So, let me illustrate that schematically. Let me draw a curve or is for that matter a series of curves which illustrates my point.

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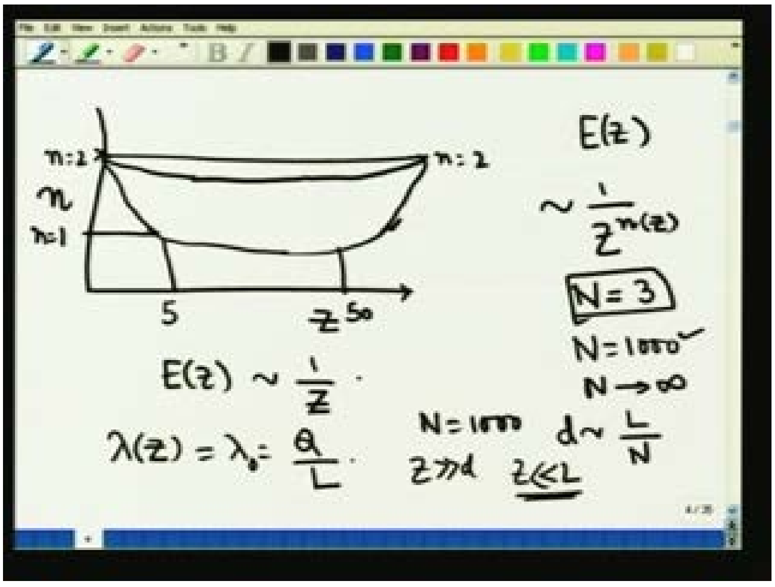

So, what I shall do is to consider the z here and then I have the exponent n is a continuous function. Now, suppose there are only three charges which will be distributed over the line n. Then I know that when z is very, very small then n is very close to 2. So, this corresponds to n equal to 2. When z is very, very large let us say. So, this is the n equal to 2 lines. So, this is going to be like this. What was the first example? We put only three charges on that line. So, of course, when we were very close n was very close to 2, when we were far away n was very close to 2.

Please mind, I am not plotting the field as a function of distance; I am plotting the local behavior of the field. So, I am going to write E of z goes like one over z to the power of n and n itself is going to change from point to point. How far you are going away? How far you are? Therefore, this object itself is a function of z that is what we are going to show. So, if n are equal to 2 or n equal to, if I had only three charges on the, this one which I will illustrate by putting a capital n. So, suppose n equal to 3; that is I have put 3 charges on the line l then, your curve would have g 1 something like this. It will be remaining very close to the value of n equal to 2 it will deep a little bit and again reaches a asymptotic value.

But on the other hand when I looked at a case like n equal to 1000; what is it that happened? The value starts with n equal to 2, comes down stays in some region for a long time and then it goes up. So, let us make it this way. And now, if you look at this region where the behavior is roughly constant that should correspond to n is equal to 1. That is the statement that we are making. This in fact, is asymptotic because as n goes to infinity; there will be a certain region in our case it corresponded to the distance from 0 equal to z equal to 0 to z equal to 5 to z equal to 50.

There my n behaved like 1; that means, my electric field is actually behave like 1 over z. This is the point that I wanted to simply make. Now, all of you are already familiar in finding out the electric field given a certain charge distribution. All of you know how to find out the field produced by an infinite line charge distribution, infinite surface charge distribution, a spherical charge distribution so on and so forth by using Gauss's law or by even by direct integration. And all that I am doing is to sort of re-derive although approximately, although numerically for you using these techniques and why is it that I am doing? And that is something that needs an explanation.

Now, I made a statement that when n becomes very, very large for example, n equal to 1000; the spacing between the charges is very, very small. Because the spacing if the of the order d equal to l by n. So, as n becomes very, very large space becomes very smaller. And now, if z is much, much greater than d of course, I want z to be much, much less than l. Let us remember that then I made a statement that I am not going to resolve the separation. If I am not going to resolve the separation; however, if I know that it is distributed over a certain length l; that means, I am indeed approximating a discrete charge distribution by a continuous charge distribution.

So, what are we saying? In this regime when z is much much greater than d and z is much much less than l if d is very small compared to l. That is if this capital n is very very large; then I can replace discrete charge density by a continuous charge density. Therefore, I automatically get a line charge density lambda z which is equal to lambda naught which is anything, but Q divided by the length of the system.

Now, the rest is all a matter of very simple integration for you people to perform. You are given a uniform line charge density. Since z is very small compared to the total length; you can take the total length to be infinite therefore, you integrate it from minus infinity to plus infinity. So, at any given distance, the field obviously, behaves like 1 over z and that is indeed the result that we have re-derived.

Why is it that I spend so much time? The reason is that we should be able to make sense physically out of apparently two contradictory statements or principles. We stated that all charges come in integral multiples of the electron charge. Therefore, charges can only be discrete. But on the other hand, in everyday example whether it is dielectrics, conductors, method of images whatever it might be we always deal with continuous charge distribution.

So, to make sense out of the statement that there is a continuous charge distribution; although the underline charges are discrete; we have to understand exactly what kind of an averaging that we are doing. In order to motivate for you the idea of an averaging over the lens case the idea of the fuzzy nature how the discreteness going to the continuous nature, I gave this particular example.

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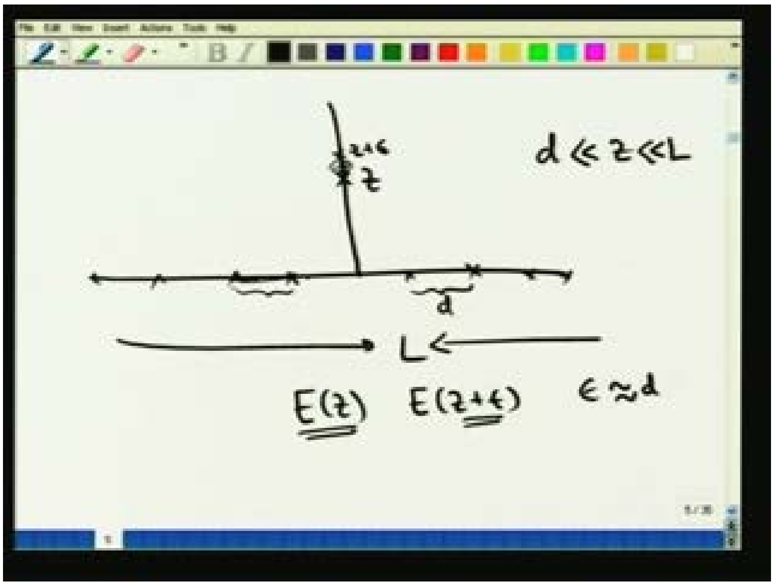

Now let me make these concepts more precise. What shall we do then? Now, again let me draw this line for you and I have the line perpendicular to that. I have various charges sitting here various charges sitting here and I sit at a distance z which obviously, satisfies my condition. So, this operation is d the total length is l. So, I have d much less than z, much less than l. When I say that my z is so large that I cannot resolve the distance between them and therefore, it appears as if it is a continuous distribution. By that I mean some more counting more. What is that?

Suppose I calculated the field at this the point z and I move a distance z plus epsilon. This is exaggerated if this epsilon is of the order d. That is I have the electric field at a point z, I have the electric field at a point z plus epsilon. Epsilon can be positive or negative and epsilon is if it is of the order d; I should not find any appreciable difference in the value of the field.

What do I mean by the appreciable difference? By that I mean if I have a measuring apparatus, some test charge that will actually sense the value of the electric field at that particular point it should not have a resolution. That actually distinguishes the value of the field at z and z plus epsilon. That is what we say. That means that my test charge does not have the sensitivity of saying that this charge is located here, this charge is located here. It is only some kind of a uniform charge distribution in between these two points or if you want to imagine you can even think that your test charge itself has a certain sized.

After all test charges are not point particles; they are also microscopic objects in everyday experiment. So, if this size of the test charges itself is comparable. So, I am going to put a test charge here. Let us say this is my test charge if that itself is comparable to d then what you are going to measure is only the mean field over the distance of the order d and then average over that. That is completely consistent with the fact that I am replacing a discrete charge density by a continuous charge density. I will get consistent results.

Otherwise of course, strictly speaking, mathematically speaking, speaking hypothetically; if you give me a test charge and an electro meter let us say of infinite accuracy then I would have been able to see the what? The discrete nature, but when we are speaking of continuous charge distributions this is what we have in mind. What we have is that the distances are sort of intermediate between the size of the system and the separation and my average in procedure take ensures that I do not have that kind of an accuracy. Even if I had that kind of an accuracy; it is not of much use to me.

This is not unfamiliar to you people because we know that deep down all atoms and molecules are discrete. Yet when we speak of this sliding of a box or some other thing over an incline plan like you have in mechanics example; you treat as if the coordinate is a continuous coordinate. So, that is the kind of situation that we are dealing with. So, what I will now do is to formalize this statement. So, that we all understand what we mean by a continuous charge distribution.

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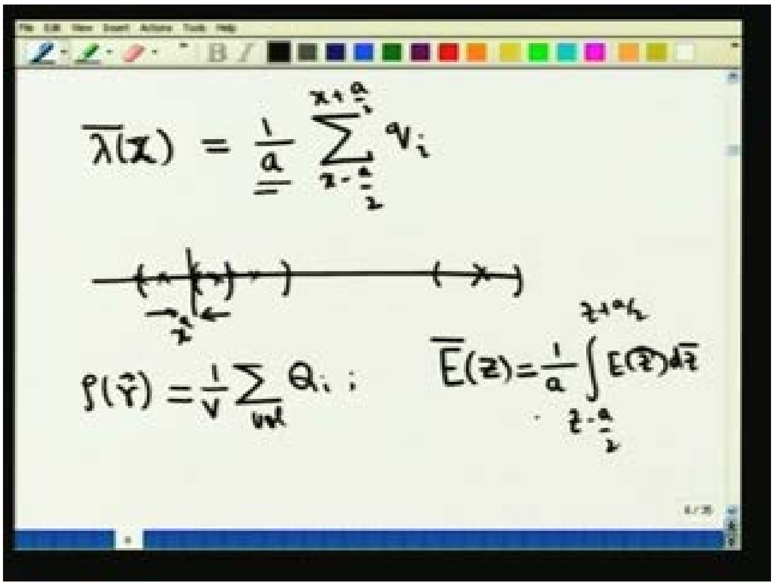

What is it that we have to do? The first step is to get into the notion of a continuous distribution from the discrete charge. So, how shall I do that? The way to do is to consider a point z or x for that matter because I am looking at the x axis and I replace the value of the charge at the point x by an average over the neighborhood of this points. Let me write down the expression for you and then I will explain that.

So, I will write it this is nothing, but 1 over a, that is what I have here. So, I will write x minus a by 2 into x plus a by 2 Q I. So, what do I mean by that? All that I mean is the following if I have my charges sitting here. What do I do? I take this point x I know that there is a charge sitting there. If I move slightly away there is no charge sitting because the charges are discrete. However, I will choose a suitable interval a.

How we are going to choose your suitable interval a depends on the reach point of your measuring apparatus depends on the desired degree of accuracy of your measurement of the fuel at the charge density. It is dictated by how macroscopic or my how microscopic your calculation is. So, what so ever it may be. Let us say that this interval is of the order a is of the magnitude a. What I do is to sum over all the charges in this interval a. In our case there were equal number of charges E not be equal number of charges. In that case we get what non uniform large density. So, sum them all over divided by the length. That will give you the average density at this point x.

Now, if I am interested in the value of x at some other point let us say here then I will construct its interval around this point. If I am interested in the value of x here, I will construct an interval here. All of length a I perform the averages and then replace them by appropriate continuous charge densities therefore, in order to emphasis that I can even put a bar. How do I generalize it to surface charge distributions and space charge distributions? Not difficult at all. What we shall do is to say that rho at any given point r, that is what I have here. What shall I do? I will take a certain volume and sum over all charges in a given volume all charges in a given volume that surrounds r.

So, imagine that r is sitting at the center of u cube of length a, then you sum over all the charges which are sitting inside the cube divided by this particular volume then you are going to get the surface, the space charge density and in a similar manner you can go on to do it for current density is later or for that matter surface densities. So, we know how to go precisely from a discrete distribution to a continuous distribution. By the same way you people should remember that an electric field can also be defined. So, if I were to calculate the mean value of the electric field in my particular example what would that be? This of course, is a one dimensional case. I will write 1 over a. It need not be the same at as this, but it is convenient to choose that. That will be z minus a by 2 z plus a by 2 E of z bar d z bar.

So, what am I doing? I calculate the electric field at every points in the interval z d equal to from z minus a by 2 to z plus a by 2 I sum over them and divided by the total length I get the mean field. So, when I say that I am approximating the discrete distribution by a continuous distribution; I should imagine that the electric fields measured are also written in this particular manner. Once this particular concept sinks in; the rest of it is quite straight forward and easy. This might appeal to be paraenetic at this particular point, but if this particular concept did not sink in; then we will end up with a variety of so called apparent paradox, contradictory statements where we do not understand the physics completely. So, that is the reason why I have spent some time. Now what I shall do is to go on to employ the concept of a continuous charge distribution, to write down Coulomb's law in a differential form to so called Gauss's law which is very, very important and very very useful.

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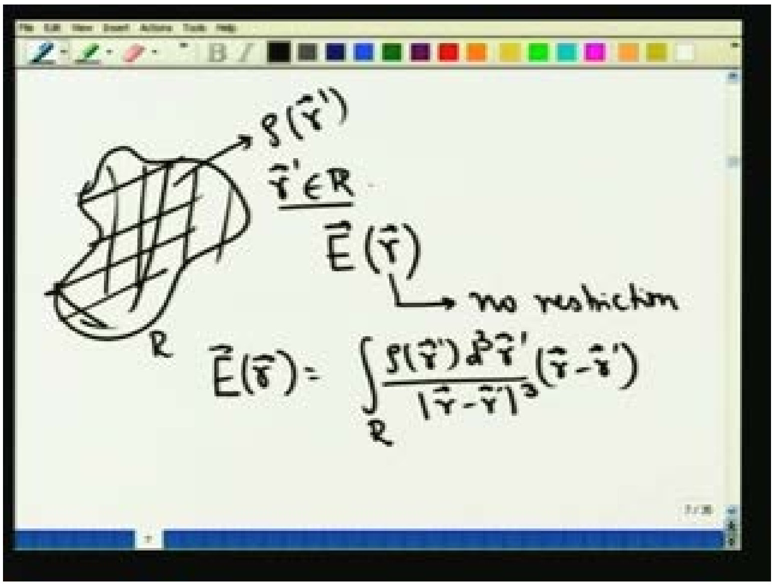

Once we understand the meaning of continuous charge density; it is not at all difficult to write down the electric field produced by a continuous charge distribution. What shall we do? let me take a region r. this is the region r and in this are charge distributions they need not be uniform. Here it is a finite charge distribution. That is something that I am emphasizing at this particular point. So, what is it that I have here, I have rho given as a function of r prime belongs to the region r. Let us remember that now I am interested r. So, what is the thing to be evaluated? I want to calculate a of r in writing a of r we are not going to place any restriction on the value of r. You can take all possible values just as in the case of point particle also, it was able to take all particular values.

However, then we worked out the field produced due to a point particle what is it that happened? What happened was that we were not allowed to sit on the point charge because at that particular point the field blew up. So, we were always to stay carefully away from the location of the charge itself. You can go arbitrarily close to the point charge and the field would diverge as 1 over r square, but here you are allowed to enter the charge distribution.

In other words, there is no restriction on the values of this r. Let us remember that and yet we are not going to get fields that are going to blow up because of our peculiar averaging procedure which is a right thing to do. So, what is this electric field given by? The electric field is simply given by a superposition principle. All of you know how to write that. So, that is nothing, but rho of r prime d cube r prime divided by naught r minus r prime cube. Let me write that and r minus r prime. This is nothing, but a continuous version of your discrete statement of the Coulomb law; rho of r prime d cubed r prime is the infinitesimal charge sitting in the volume surrounding the point r prime. So, that infinitesimal charge is producing this particular field. That is all the statement is and we are going to integrate over all r prime; obviously, in the region r. This is the electric field.

Now, given this electric field; obviously, the great advantage is that I do not have to perform a tedious summation. The whole world of analytic technique will open for us because we can use symmetries, we can perform integrations, we can study various limiting cases, we can study macroscopic electrostatic phenomena which we are really interested in and in that sense it is a major boost for us in our study of electrostatic phenomena.

But now, my right now my purpose is actually to use this electric filed produced by a continuous charge distribution to rewrite Coulomb law in a slightly different form the so called Gauss's law. That is something which is very, very important for us. In fact, this is a prototype for many, many other theories. This is something which was written even for the gravitational field by n 1 other than Laplace.

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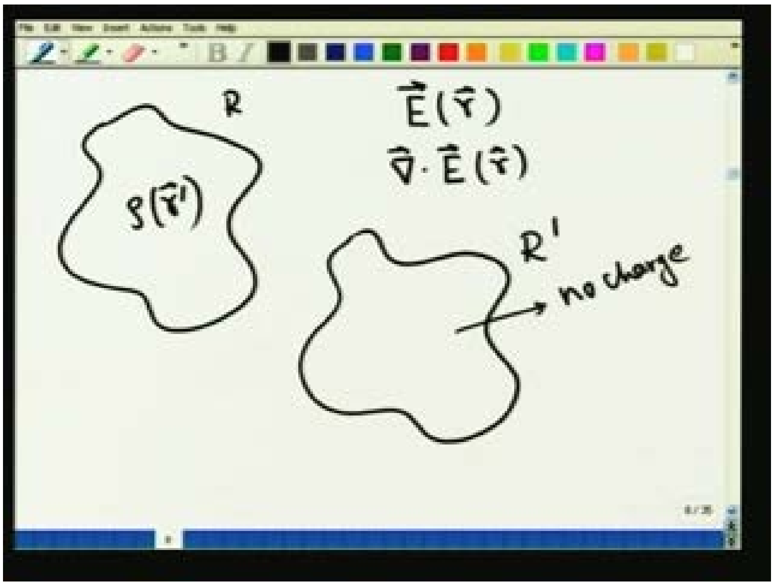

And now what we shall do is to see how to rewrite Coulomb law which is 1 over r square into a differential form for the electric field and we shall do that step by step. Let me consider a charge distribution in a region R. So, there is a certain charge distribution rho of R in a region R.

Now, I am interested in the divergence of the electric field produced by this charge distribution. So, what are the steps that I am going to do? I have my electric field which is given as a function of R everywhere. So, let me make it R prime here. Then what I will do is to look at divergence E at every point r. We have not yet done that even for a point charge. Although I have hinted at what it is formed would be in one of my earlier lectures on the divergence theorem, Gauss divergence theorem. We will revisit that. Now, in a minute then we shall ask what about the flux that goes out of any given volume. In order to simplify matters I shall consider various special cases before stating the general result.

So, if this is my region R. Let me consider another region R prime. So, this is another region R prime this region R prime is disjoined with R. What do I mean by that? There is no overlap between the regions R and the R prime which is the reason why they are separate. Therefore, if I were to take my electrometer and moving this region R prime I am not going to find any charge.

Therefore this region has no charge; no charge does not mean no field, because we know that even a point particle situated in this region would actually produce an electric field at everywhere; therefore that is going to be non vanishing the electric field in this particular region and we are; obviously, interested in what the electric field is and what the divergence of the electric field is?

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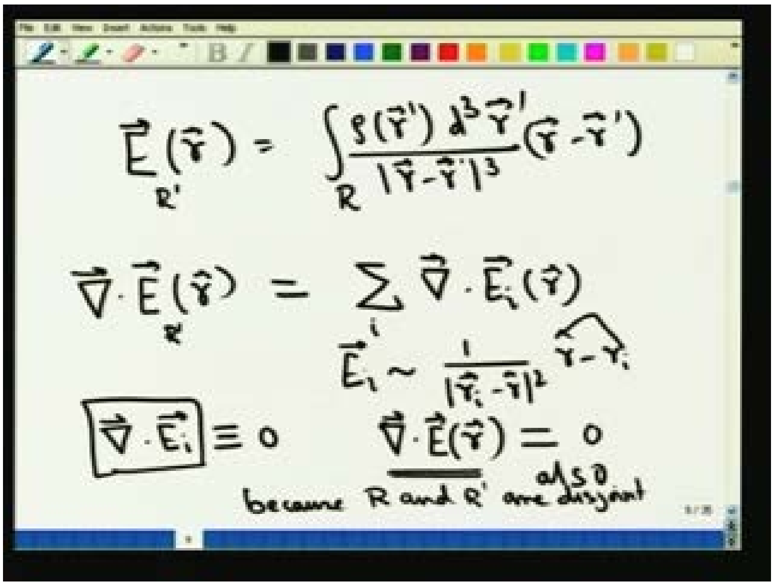

So, please remember this geometry well before I proceed to write the next step. What would be the value of the electric field my electric field? At any given point r in the region r prime. So, let me emphasis that here is simply given by rho of r prime d cubeb r prime divided by naught r minus r prime cubed into r into minus r prime. You people are quite familiar with this result. Of course, r prime belongs to region r that is something that we have to remember.

In general since you have not given me the charge distribution I do not know how to calculate the electric field. However, although I do not know what the value of the electric field is, I do know what it divergence is. That is something that we have to remember. What is the divergence of this electric field in this region? Divergence E of r in the region r prime, that is something we should not forget. Remember the continuous charge density is nothing, but an approximation to what? A large number of discrete charge densities discrete charge distribution. Each charge is going to produce an electric field which goes like 1 over r square. That is what this relation is telling me.

Therefore by principle of superposition, if I know the field produced by different point charges in the region r, find out the divergence coming from them. Add them up, I will get the total divergence therefore, if I could write it sort of symbolically. This is nothing, but summation divergence E I of r, where I is the what the contribution coming from the I th charge. Of course if you want to write it in terms of what in terms of the continuous charge density. You will replace it by an integral. Let me not do that. What matters is the concept and how does this E I of r go like E I of r goes simply like 1 over naught r I minus r whole square and of course, I have the unit vector r minus r a. So, I have the unit vector corresponding to this.

It is a very simple exercise for you people to find out the divergence of this function E I. So, I will leave it as an exercise for you people to work it out. You can check that, divergence E I is identically equal to 0. Please remember this r is located away from the point where the charge is located. What am I saying? This r is belong to the region r. The points belong to the region r prime. r prime is disjoined from r. In other words I am not evaluating the divergence, wherever the charge is located therefore, a simple evaluation of this divergence a. All that you need to do is to substitute for the divergence in the spherical polar coordinate system. If you did that this would be identically equal to 0.

Therefore, by principle of superposition I conclude that divergence of E at any given point r to the whole charge distribution is also z. This was coming from an infinitesimal charge element. This is coming from the total charge element because so let me write it explicitly because R and R prime are disjoined.

So, although we do not know the detailed form of the electric field it can be as complicated as it can be. It might be impossible to evaluate it analytically. You might have to evaluate it too numerically. Never mind about that I know for sure that divergence E equal to 0 in the region outside.

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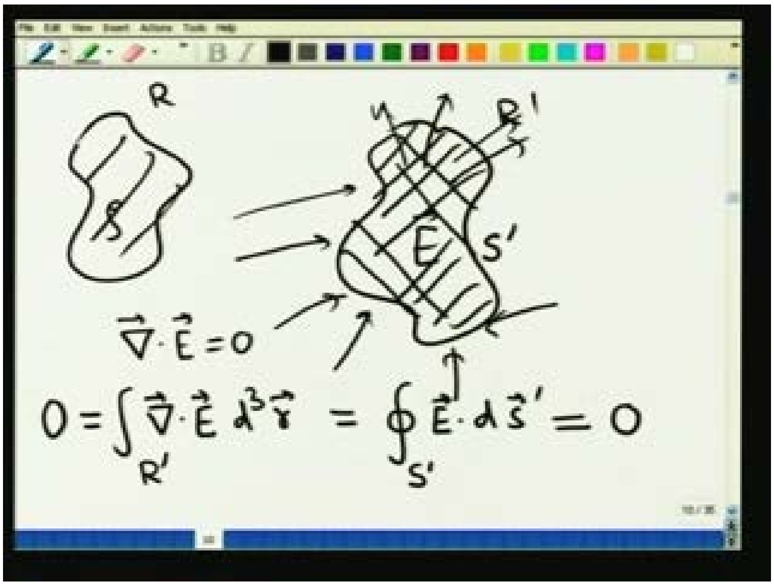

So, to summarize I had my charge distribution in this region R. I am going to look at the electric field in this region r prime. So, this is where rho is this is where my electric field is. I claim irrespective of the nature of rho how we distributed it? What you did with it? How much charge you put? I know that divergence e equal to 0 in the whole volume.

Mind you, I do not have to specify what this r prime is. The only condition is that r prime should be away from r. It should not be the same as r it can be. In fact, even as close to as you want, but there should be no overlap. Once you note that the divergence e equal to 0 throughout the volume what I can do is to actually integrate this divergence over this volume. So, I will write divergence e d cube r over this particular volume. So, that I will indicate it by the region r prime and now Gauss comes and tells us that this is nothing, but the flux of the electric field around the surface.

So, you have the region r prime which is bounded by the surface s prime. It is quite a complicated surface look at this. What is this? By gauss divergence theorem, which you have actually demonstrated it for a cubical volume and a cubical surface in our earlier lecture is simply given by the surface integral over the surface s prime integral E dot t s prime, but then the left hand side is equal to 0. Therefore, so, is the right hand side; that is what we have this is the statement of Gauss divergence theorem.

That divergence e equal to 0 is a consequence of the Coulomb law. Therefore, integral e dot d s prime over arbitrary surface equal to 0 is a consequence of Gauss divergence theorem provided this surface s prime does not enclose any charge. This is something that I have been repeating often, so that that is something that settles in you completely without any confusion.

Now, once we stated e dot d s prime equal to 0; we are making an interesting statement about the flux. We are saying that look here this charge density is going to produce an electric field and if I say that the field lines are entering this particular volume; it need not be perpendicular it can enter anyway it wants and some of the filed lines will obviously, be going out. What this statement says is that whatever flux enters this particular volume, it also flows out. Remember what we said about the flow.

If there is no source, if there is no sink the total flux tend towards this 0 because it is something like an incompressible fluid. If it is incompressible whatever gets in, also gets out. That is exactly what is happening and this statement some kind of a conservation. Whatever gets in is also get in going to get out. Nothing is going to get accumulated, nothing is going to get lost irrespective of the nature of the boundary that encloses this volume irrespective of the nature of the volume is the statement of this theorem.

Now, of course, this is not going to be of much help to me unless I look at the overlap of this region r prime with r. If we did that then; obviously, divergence e d cubed r will not be equal to 0. By the same token integral E dot d s will not be equal to 0. If we stated that result then, we would have stated what divergence e is everywhere what integral d dot d s is everywhere. However, small the volume element may be; however, large the volume element may be, whatever the shape of the surface may be and that is what did. The Gauss's law and we shall do that now.

In order to write down Gauss's law again it is convenient to make use of the principle of superposition. Without principle of superposition you would be nowhere. So, let me start with a point charge. Whenever I have a point charge, it is convenient to locate it at the origin or to put it more precisely, choose the origin of your coordinate system on the point charge itself.

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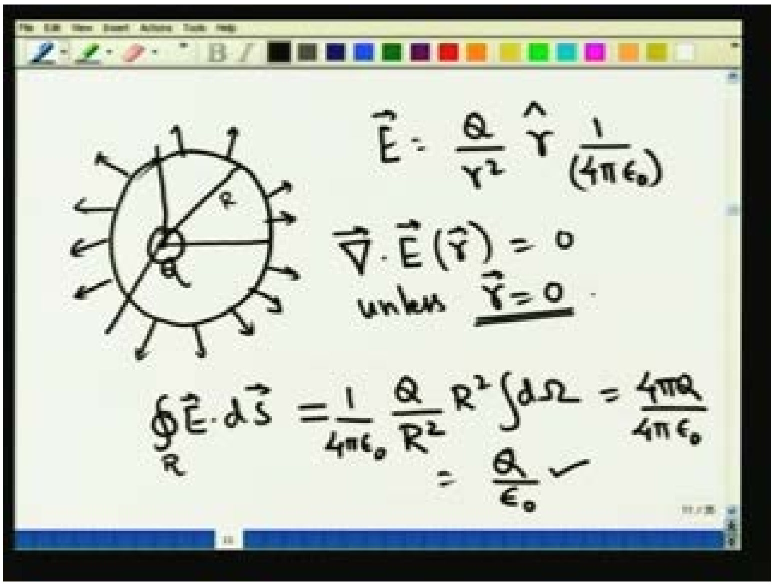

So, let me do that. So, I have the x and the y and the z axis. I do not have to write them now and here I have put a charge q. Now, what I shall do is to consider a sphere of radius r which is surrounding this charge. So, this is a sphere of radius r and I am interested in finding out the electric flux. What is the field produced by the point charge? E is simply given by Q by r square r hat because I have located it at the origin. Now again let us calculate the divergence everywhere. What am I going to get?

A simple calculation will tell show you that divergence E at any point r equal to 0 unless r is equal to r 0. At r equal to 0, your field is going to blow up like 1 over r square. So, the derivative at this particular point does not make sense at this juncture. We are going to make sense in a short while. Is that ok? You do not know how to find out the derivative because the value of the function itself is blowing up. So, in order to be careful we say divergence E r equal to 0 unless r equal to 0.

However, if I were to construct a sphere of radius r as I have done. Now, the electric field is everywhere on this sphere of radius r. So, how does my electric filed look like? It is going to be radial if I take a positive charge it is going to be radically outward. If it is go if I take a negative charge it is going to be radically inward. This is a well defined object this is a finite object. Therefore I can evaluate integral E dot d s without any problem, because I am sitting at a finite radius r where r is not equal to 0. So, let us do that.

So, if I were to write down integral E dot d s at a radius r a sphere of radius r. So, what is that? My surface element is simply given by sin square theta d theta r d phi at r square. So, I have Q by r square because I am evaluating at a fixed r. This small r attain the capital this one. Then I have my r square integral d omega which is nothing, but sin theta d theta d phi which will turn out to be four pi Q. That is what I am going to get.

Unfortunately, I have not been very careful in writing down these expressions. Now let me fill in all the details which I had sort of omitted. This is not exactly Q by r square, but it multiplies a factor 1 over four phi epsilon naught. So, that gets inherited here. So, we get a 1 over four phi epsilon naught here. Now let me put a four phi epsilon naught here this is nothing, but Q divided by epsilon naught. What are we saying? Divergence e equal to 0 at all points except the location of the charge at the location of the charge. I do not know how to evaluate the divergence E.

If I were not ambitious if I did not make to sense may I did not wish to make sense out of gauss divergence theorem. I would have said divergence E is not defined at r equal to 0; however, I do not want to stop at that; obviously, because I want to write something called the gauss law right Coulomb law in the version of Gauss's law.

So, what I do is instead evaluate the surface integral E dot d s which turns out to be nothing, but Q by epsilon naught. The remarkable fact about this expression is that this surface integral is independent of the value of r. You can take this sphere r as large as you want, as small as you want. I can shrink this sphere to an epsilon value. The only condition is that you should be non 0 finite number or I can group this sphere to a very large number. Never mind this point charge is like a luminous source which is emitting radiation. Is that ok? Therefore, if you are interested in the total intensity the, of like a bulb, total power emitted by that then it is going to be independent of the surface. It is only that the angular distribution becomes smaller and smaller. Is that ok? Because as I go farther and farther; the area of element becomes larger and larger.

Apart from that it is always given by Q by epsilon naught which is at measure of the strength of the source of the electric field. That is what we are trying to say and that is what we find. So, independent of, so, please remember independent of how small or how large the radius is I have integral e dot s which is equal to Q by epsilon naught which is not going to be equal to 0, provided Q is not equal to 0, provided this sphere is going to enclose that. Let us just remember that.

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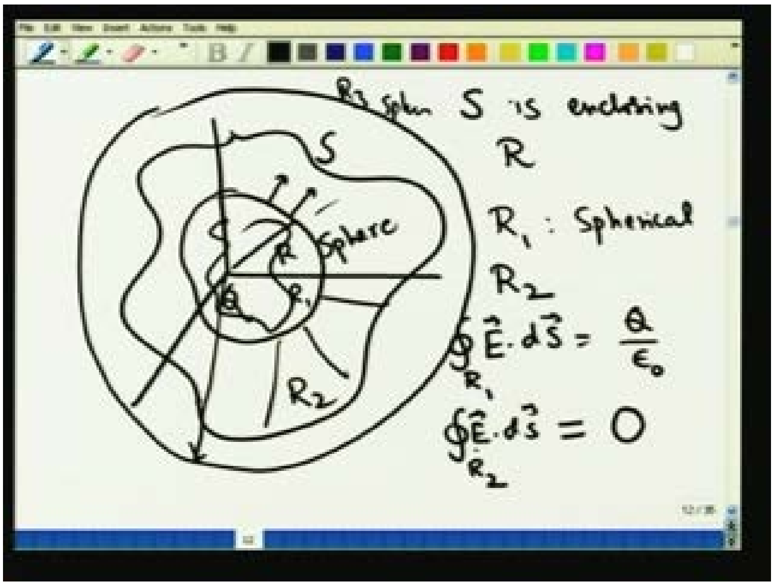

What I shall now do is to consider a slightly different situation. I shall not consider a spherical surface, but I shall consider a complicated surface. So, to repeat let me rewrite the diagram. I have my charge located at origin and I now consider a volume which is bounded by a complicated surface like this. Well, divergence E equal to 0 everywhere except at r equal to 0. That follow simply from the Coulomb law, but I cannot evaluate integral d dot d s because in the original case when I wrote down this sphere. So, let me write down the sphere here again. This is my sphere of radius r. This is a sphere. Let me make it explicit because it does not look like a sphere.

Here, my electric field was perpendicular to the surface everywhere. Therefore, I could actually evaluate the surface integral. However, for example, if you look at some particular point like this, you see if I were to draw a line from here to here you see the field line is not normal to the surface. Therefore, integral d dot d s is something that I do not know. How to evaluate? Since I have not specified the functional form of the surface it appears that I do not know how to do this surface integral.

So, I have complicated my life a bit, only a little bit. In order to illustrate something for you what shall we do? What we shall do is you look at again this whole region r. What is my whole region r? Whatever is enclosed by the surface s, the surface s is enclosing the region r s is enclosing r. Now, this region r shall be split into two parts; one is the interior region which I shall call r 1 which is spherical.

So, r 1 is spherical r 2 is the intermediate region. Therefore, this region what I have written here this is r 1 and this annulus what you see here annulus like thing is r. Now, let us look at the electric field in the region r 1 and in the region r 2. I know what integral e dot d s in the region r 1 is. Integral d dot d s in the r region r 1 is nothing, but Q by epsilon naught. What is integral e dot d s in the region r 2? Is that ok? Integral e dot d s in the region r 2.

Well you might not know how to evaluate the surface integral, but so what? In all of r 2 divergence e equal to 0 because the charge is not located in r 2 the charge is located in r 1. Therefore, since divergence e equal to 0 by Gauss divergence law this is identically equal to 0.

So, we have shown that integral d dot d s is equal to Q by epsilon naught if it is in the region r 1 integral d dot d s over the region r 2 equal to 0 and in fact, in order to make life even more clear; in order to make the concept even clearer; I can construct a bigger region r 3 which is also spherical. This is spherical again this is r 3 over region r 3. Again you will find that the surface integral is not equal to 0, but it is given by Q by epsilon naught. So, what can we conclude from this? Since the electric field is like an incompressible fluid; since the fluid is going like 1 over r square since whatever flux enters a volume it has to leave. We can sum these three examples an outer sphere and an inner sphere and an arbitrary surface, which is enclosed in between them.

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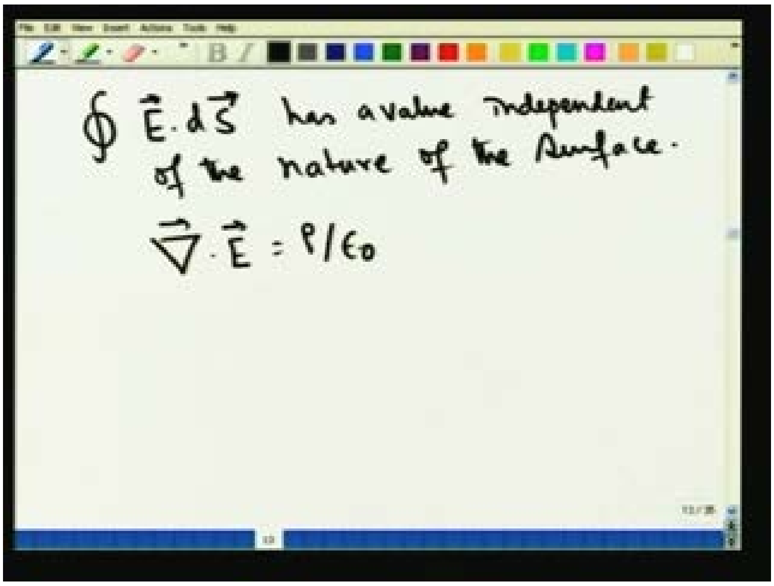

We can conclude that the value of integral e dot d s should be independent of the nature of the surface; it really does not matter whether you take the spherical surface. It really does not matter whether you take a more complicated surface. It is not a property of the surface it is simply a property of the charge that is encloses.

So, the first step that we have taken is to state a very beautiful result namely integral E dot d s has a value independent of the nature of the surface. It depends only on how much charge it encloses. What I shall do in the next lecture is to use this fact to state Coulomb’s law in a differential form namely divergence e equal to rho by epsilon naught and that is what allows us to solve many many problems.