Engineering Physics II

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Module No. # 02

Lecture No. # 03

Before we begin today's lecture, let me introduce the gentleman here, whom you be see on the screen, he is Coulomb, whose law we have been discussing in the last lecture and which I also mentioned in the lecture previous to that. So, this is a picture which I have downloaded from internet from one of the galleries. It is also good to remember at this point that when we start discussing Coulomb's law, Ampere's law and Maxwell's equation that what we are trying to do is to provide a prospective, and these lectures are not a substitute for your classroom lectures.

So, I am not going to solve all kinds of problems or discuss all the aspects. But what I will do is to pick up a few central features, the most important features and highlight them and also indicate to you how to go about understanding the subject of electricity and magnetism. Now, let us start with Coulomb law again and as I said in the last lecture, we are still left with one more job to do before we go on to discuss things like Gauss's law or whatever and that if we state the principle of superposition.

We have so far stated the principle of invariance of charge that is charge does not depend on the state of motion of the body. We have stated the principle of quantization of charge that is charge always comes as an integral multiple of electronic charge or for that matter, the proton charge. Secondly, charge is a conserved quantity, if you give me net charge in a closed system that shall always remain the same; it is not going to change as the time evolves. Today, what we shall do is to go back to Coulomb law, and ask what would happen if you have not two charges, but more than two charges. So, let us start with three, and then go on to include more and more charges. So, we are now going to extend

Coulomb law to more than two charges.

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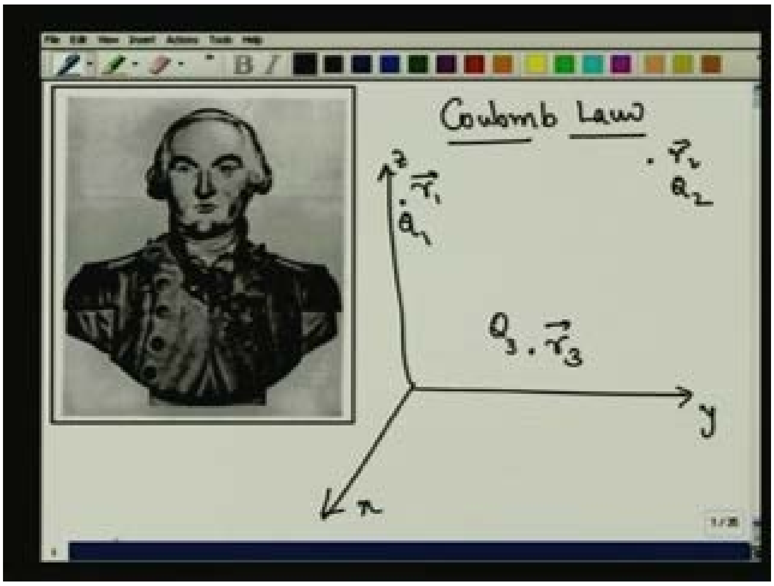

So, let us say that there is a charge Q 1 here, there is a charge Q 2 here, there is a charge Q 3 here. So, let me erect co-ordinate system so, I have my x axis, I have my y axis and I have my z axis and these charges are having locations r 1 r 2 r 3, these are the position vector at which the three charges are located. We are now interested in the force of interaction between these three charges. Remember, whatever we have done all this time in the previous lectures is to consider only two charges. So, at r 1 of course, there is a charge Q 1 which could be positive or negative, at r 2, there is a charge Q 2 which is positive or negative, at r 3 there is a third charge Q 3 which is again positive or negative.

We are interested in the force that acts on the charge Q 1 due to Q 2 and Q 3, the force that acts on Q 2 due to Q 1 and Q 3 and the force that acts on Q 3 due to Q 1 and Q 2, that is the question that we have to answer. If Q 2 were not absent; we know, what is the force that Q 3 exerts on Q 1, and what is the force that Q 1 exerts on Q 3? We know the answer. If Q 3 were absent we know what is the force exerted by Q 1 and Q 2 and vice versa. Now, the extended question that we are asking is if there are three of them how is it that the force or one let us say is to be given the locations of Q 2 and Q 3. The answer to that is given by the principle of superposition, which I shall write now, and this indeed is a fundamental principle.

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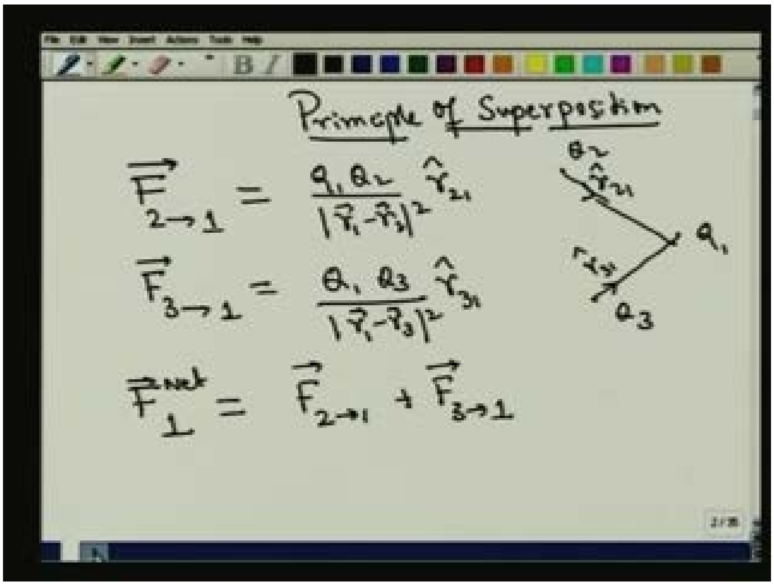

In fact, our life is somewhat simple because of principle of superposition as you will see now if the principle of superposition were not valid then we would have got into trouble. What is the principle of superposition says? Suppose, I am interested in the force on 1 due to 2 and 3, I have already indicated the positions of 1 2 and 3 in my previous figure. So, as usual I shall ask what is F 2 1, what is the force acted upon by 2 on 1. You know the answer to this, which we write as Q 1 Q 2 divided by mod r 1 minus r 2 whole square and it is in the direction from r 2 to r 1, which I shall indicate by the unit vector r 2 1, r 2 1 is in the direction of r 2 to r 1. So, if this is my charge Q 2, if this is my charge Q 1 I draw a straight line connecting them and this is the unit vector r 2 1 that is what I mean.

Now, this is entirely in the absence of the third charge, and we ask how the force is going to get modified when I introduce a third charge. So, imagine that I introduce a third charge Q 3 here, again I know that if the charge Q 2 were absent, the force on by the third charge on the first charge is would again be given by Q 1 Q 3 divided by mod r 1 minus r 3 whole square r 3 1. Needless to their r 3 1 is the unit vector in the direction of above the from in the direction that connects Q 3 to Q 1, that is what we have this is r 3 1.

Now, what we are asking is what is the net force acting on Q 1, this is something which has been assumed throughout, but it is not obvious at all. What the principle of superposition states is, if net force so let me write the net force acting on one due to charge two and three is simply given by F due to 2 plus the force due to 3. In other words, the force exerted by the third charge does not take into cognitions, the existence of the charge Q. The force exerted by the second charge does not take into cognitions, the existence of Q 3. The forces independently add up to give you the net force acting on Q 1. Now, of course, all the charges are democratic we have to treat them symmetrically. Therefore, we need not necessarily consider the first charge, you can take charge Q 2; you can take charge Q 3 on any charge in the given system. The net force acting on that is simply a sum of the forces that come from all the other charges. That is the statement of principle of superposition.

Now, how would be that, how is it that the principle of superposition is valid? That has something to do with the linearity of the force on the charges Q 1 and Q 2; I will not get into that. But I will get into a slightly more detailed discussion, when we write Coulomb’s law in the differential form divergence equal to rho; at this point, let us simply take this. Now, once this principle of superposition is given, which is indeed, in fact verified experimentally, in fact it is used in almost every experiment, if it were not true due to not be possible to proceed to do any calculation at all. We shall now launch on to introduce yet another very important concept and this concept is due to Faraday and that is the concept of a field. In order to write down the concept of a field, what I will do is to write F 2 acting on 1, F 3 acting on 1 explicitly and factor out Q 1 and then we see there is something interesting.

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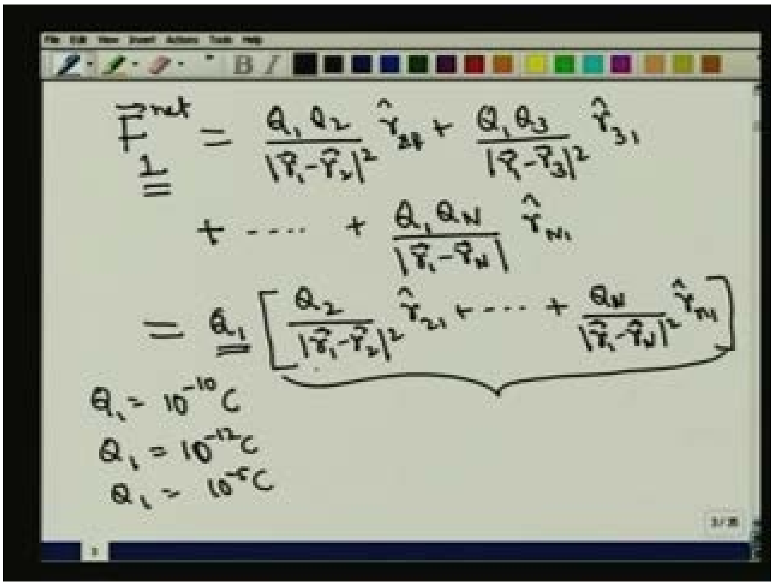

Now, suppose, I ask what is the force acting on 1, the net force? When I am writing this, I need not restrict myself only to a system of three charge particles, I can take 3 4 5 in general n number of charges, how would that be given by? That the principle of superposition that would be given by Q 1 Q 2 divided by mod r 1 minus r 2 whole square r 1 r 2 1 plus Q 1 Q 3 divided by mod r 1 minus r 3 whole square r 3 1. Please remember r 3 1 is the unit vector from charge Q 3 to charge Q 1 that is something, we should not forget plus etcetera. The last charge Q 1 will exert a force which is given by the expression Q 1 Q n divided by mod r 1 minus r N into r N 1 that is what we have.

So, what I have is to take a charge, which I call the first charge and I have put it in a environment of N minus 1 other charges and I am asking what is the net force acting on the charge Q 1? Now, you can see that each of these expressions is bilinear in the charges and since I am interested in the charge Q 1. I am asking what is the force on the charge Q 1? Q 1 is something that occurs in every single term. So what shall we do? We shall factor out Q 1 and we shall write this is equal to Q 1 into Q 2 divided by mod r 1 minus r 2 whole square r 2 1 etcetera, you will have N minus 1 such terms, then in the last term we have Q N divided by mod r 1 minus r N whole square r N 1, that is what we have.

All that I have done is to simply factor out Q 1. Now, stare at this expression ,what has been factored out Q 1 is a measure of the strength of the test charge, if you feel like because introduce a charge Q 1 at a point r 1 that is what I am doing. If you now concentrate on the term in the parenthesis in these brackets, you see that there is no reference to the charge of the particle; it refers only to the position. In other words, whatever may be the strength of your charge; your Q 1 can become Q 1 prime, Q 1 double prime, Q 1 triple prime so let us say initially Q 1 was something like 10 to the power of minus 10 Coulombs. You remove that charge, now you put a new charge which corresponds to Q 1 equal to 10 to the power of minus 12 Coulombs. Or you can build a large charge in a very small volume and then say Q 1 is equal to 10 to the power of minus 5 Coulombs. Do whatever it is; what is this expression in the bracket is not going to change.

In other words, whatever is in this parenthesis, if a property of the rest of the charges which is going to influence any test charge, whatever its magnitude may be when it is located at a point r 1. This is something which Faraday's and that is what motivated him to introduce the concept of a field. So, what is it that I am trying to say, what I am trying to tell you is the following? So, imagine that you have a large number of charge, let me again show that here so we have a large number of charges sitting here.

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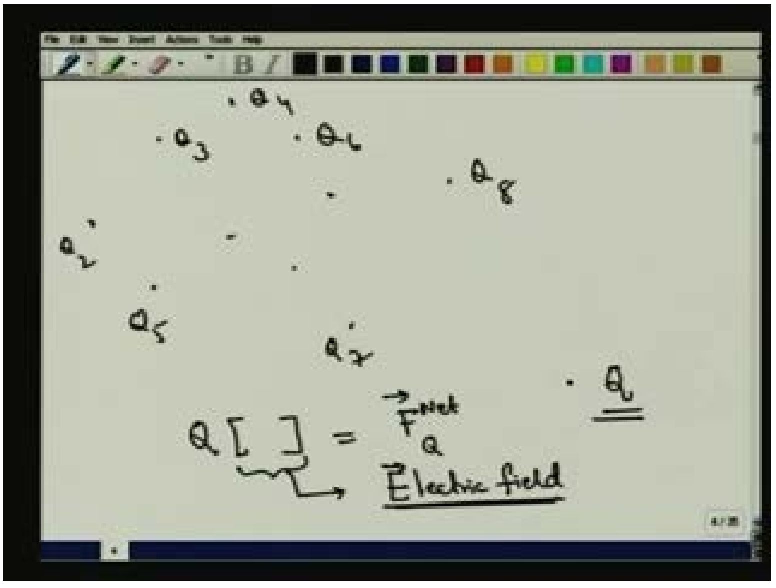

So, I can start labeling them Q 2 Q 3 Q 4 Q 5 Q 6 Q 7 Q 8 so on and so forth. So, let me throw a little bit more here. These are of course, not distributed in the plane; I am showing it on the screen here. They would be distributed over a volume in a certain region and now I introduce a test charge Q here. Now, what we are trying to say is that the combined effect of all these charges at this particular point can be qualified in the term in the parenthesis that I wrote in the previous term. Now, bring any Q that you want, you can bring a charge of strength 10 to the power of minus 10, 10 to the power of minus 5, 10 to the power of minus 8 whatever, what so ever. If I multiply this charge Q by whatever term was there in the parenthesis then I always get the force, the net force acting on the charge Q that is what we are trying to say.

In other words, when I bring the test charge Q here, it does not know what are all the individual charges that are actually contributing; there might be a dipole there, might be a nucleus sitting here, there might be a quadrupole in fact, there might be moving charges, whatever they may be of course, at this point, we are only looking at static case. What that is going to sense a field, which is produced at this particular point, which has a contribution coming from all these n sources that is the statement that we are making.

In other words, what we can do is not to keep on writing forces F 1 2 F 3 3 F 3 4 etcetera, there will be n into n minus 1 by 2 terms. Instead if we are interested introducing a test charge at various points in space, we can codify the information coming from the rest of the charges by independently computing the term in the bracket. So, this is the term which is computed in the bracket and if you people have to remember that, you can simply again write it as Q 2 divided by mod r 1 minus r 2 whole squared r 2 1 etcetera, which you are all thoroughly familiar from your twelfth standard.

Now, what Faraday did was to give a name and he called it as the electric field, this is the electric field. And there was indeed a great intuition a great motivation introducing it whereas; at this point it appears as if you are doing the book keeping. It is only a matter of mathematical convenience, but Faraday had much more insight. Then what this kind of a factorization reveals? And that is something that we appreciate as we go to explore more and more phenomena in electricity and magnetism.

Now, what is this concept of the electric field and how is it that we are going to exploit that? In fact, it is using this electric field that we are able to write Coulombs law in the differential form, divergence is equal to rho by epsilon naught. We are able to make use of Gauss's law we are going to make statement that curl of u equal to 0, which is defining property of the static fields. Therefore, let us spend a little bit more time. So, I am sure that all of you people would have recognized this to be the portrait of Faraday and let us continue to understand to try to understand and appreciate what faraday meant when he introduced the concept of an electric field.

For him electric field was something that was something very real it was like a fluid. In fact, that is the beginning of the whole subject of vector analysis, vector calculus what so ever. And we should remember that Faraday was not a person who was tutored formally, he was not educated in the modern sense. But still being an assistant in a laboratory, he was able to introduce many important concepts. In fact, he contributed a fundamental law that is Faraday's law of induction therefore, it is good to spend some time trying to appreciate and understand the meaning of electric field.

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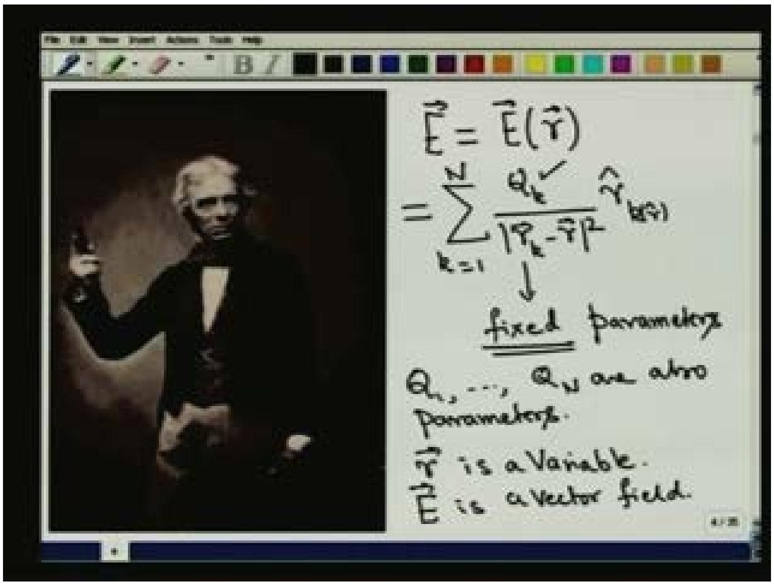

So, let us again begin with the expression for the electric field that we have written. So that is the same as the term in the parenthesis that I had written. So, I am going to imagine that there is a distribution of N charged particles; I am not going to show them again to you. And I am going to choose a point r, it could anywhere in the space. And I ask, what is the electric field at this particular point? Therefore, in order to emphasize that what I am going to do is to look at the electric field at a given point r due to charges Q 1 to Q N now, I have made a change. Earlier, I looked at a system of N charges and picked out one arbitrarily and I called it Q 1. Now, I am giving a system of N charges and I am introducing a test charge Q which I am not going to bother to write because I am interested in the electric field and then I ask what is the field due to all the N charges?

So, how will it look like? So, let me employ a compact notation instead of writing all the N terms one by one like I did in the previous screen. So that is nothing but, k equal to 1 up to N because I have N charges which are produced in the field. Each charge comes with what? Q 1 Q 2 Q 3 etcetera Q N. So, let me write a Q k here and then they are located at positions r 1 r 2 r 3 etcetera r N. Therefore, I am going to write N r k minus r square that is what I have. What else is remaining? Not only do I have to write r k minus r whole square I have to specify the direction and we know what the direction is I will write it as r k in this into r. This is a bad notation, but it will serve us at this particular point it tells you the unit vector coming from each of these fellows. So, I could have probably employed the notation r k I, but we will let me limit it at this particular point.

What is that strikes at you? What strikes at you is that the points r 1 r 2 r N r N these are fixed. In other words, these are not variables, but they are parameters. So, are the charges Q 1 Q 2 Q k so let me write that Q 1 etcetera Q N are also parameters. (No audio from 17:30 to 17:36) So, given N parameters namely Q 1, Q 2, Q N and N vector parameters, because they are also fixed namely r 1 r 2 r N. We are going to evaluate a vector function all over the space. So, what is my variable? r is a variable, please appreciate this. So, this is something which is very clear from the physical view point. So, there are N charges which are pegged at various positions. What I am going to do is to take a test charge move all around the space and I am going to measure what the force acting on it is.

And at each point once I find the force I divided by the charge carried by the test charge and that is going to give me the electric field at this particular point. In the three or four lectures, we spent a fair amount of time discussing, scalar fields, vector fields, curl divergence etcetera. And you can easily see that this quantity that I have constructed eminently qualifies to be designated a vector field. How so? Charge is something which is a scalar, it is independent of the motion of the particle, it is independent of the coordinate system. Therefore Q k is a scalar, mod r k minus r whole square is a distance between the two points it is a distance square between two points. And the distance is again invariant under all kinds of coordinate transformation that we can think of for our purposes you can rotate, you can translate for that matter, you can even perform a Galilean transformation this is not going to change.

But on the other hand the unit vector is indeed a genuine vector. For example, if I had located on of the charges at the origin, this r would have been exactly like the unit vector r, the basis vector r that you got in the spherical polar coordinate system. Therefore, this electric field that I have written is nothing but a product of a scalar field multiplied with a vector. In fact, it is a unit vector and we are summing over them. Therefore, what we get is a vector field; we conclude that E is a vector field. (No audio from 19:44 to 19:49) We are interested in the values of the fields at various points and when I mention that I am interested in the value of the field at various points of course, I am not going to go and sit on one of the charges.

Because we know that as we approach any of the charges, any of the source charges, the field is going to blow up, it is going to become infinity. But so long as we stay away from any of the charges, we can get to as close to that charges we feel like so long as we do that, we know that my electric field is a smooth function, it is a continuous function. At every point it has a definite magnitude, at every point it has a definite direction, which is exactly what we define to be a vector field. In our earlier lectures on mathematical preliminaries therefore, when we say E is a vector field, we know exactly how to deal with it and all the mathematical apparatus is already ready for us.

Now, given this it is a good question to ask whether we can pictorial represents these electric fields, because as I keep on introducing more and more charges, the electric field is going to become more and more complicated and this is something again that Faraday worried a lot about. Luckily, for us we are already geared up to face this situation, because when I discuss vector fields, the divergence property or these curl property etcetera. I also gave a way of representing the vector field at various points in space.

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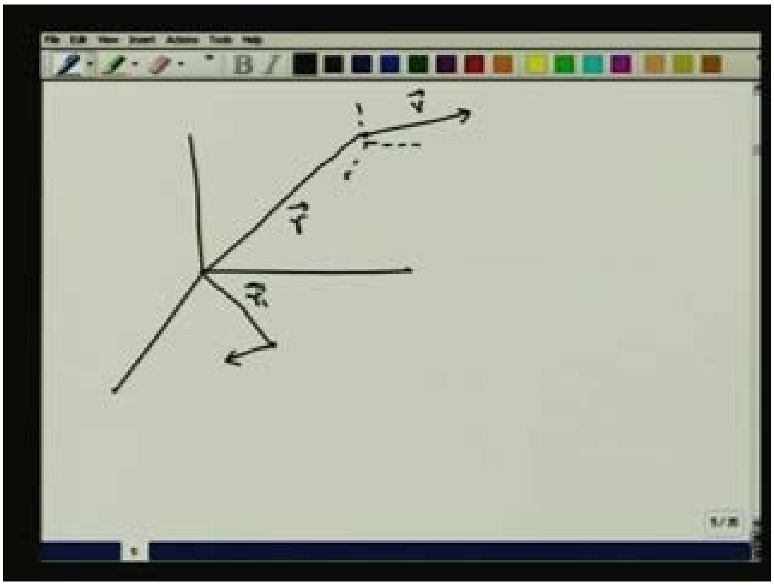

Let me recapitulate that for you. How do we do that? The way to do that as I told you is that, if you have coordinate system and have a vector field, what is it that I do? I am interested in the value of the vector field at this particular point therefore, this is my radius vector r so let me indicate it by r. At this point let me say the vector field is in direction. So, if you feel like I will introduce a coordinate system which is paralleled displaced with respect to the original fellow. That is only to book here and here with respect to this origin, this is making some angle with respect to z axis, x axis and y axis, this is my vector field. I also told you that you should remember that when I am giving the direction I am also going to give a specific length to this particular arrow which is a measure of its magnitude.

Instead of choosing this particular point if I were to choose somewhere here let us say. This will be some other r prime so this is r prime and what is that that I am going to do? If I am interested in the vector field, the vector field now could be in this direction. In showing this we have actually shorten the length of the vector that means, it conveys to us that the magnitude at this particular point is smaller than the magnitude at this particular point. Of course, I cannot be arbitrarily elongating and sort of dilating the vectors. I should have a certain scale in mind that is something that is understood.

When I say that this has this particular length and this has this particular length, I must have a scale may be every unit corresponds to one-fifth of the true length of the vector. Or if the vector field is very large, every unit length corresponds to twenty times the real length of the vector whatever the scale is, once we have specified that we know how to represent that. This is again a concept that we owe to none other than Faraday. Given this preparation which we already have, let us ask if we can depict the vector field that is the electric field for a few simple configurations. I am not going to look at very complicated fields, they are really not necessary for us. Of course, if you go and look up a book, they will give you very nice pictures involving fields produced by quadrupoles, dipoles etcetera or higher order multiples.

Today, what I will do is to just show two of the simplest situations which you are already familiar with. I will highlight a few of the aspects that are automatically displayed there and then go on to discuss the other properties of the electric field. Later, when I discuss multiple expansion, we will look at more complicated fields, we might as well go as far as octopole fields. But at this point let me look at the simplest of the situation. This depicts the so called lines of force of an electric field due to two point charges, which are of equal magnitude. What I would (( )) you people is to go and repeat this exercise, when the two charges are unequal.

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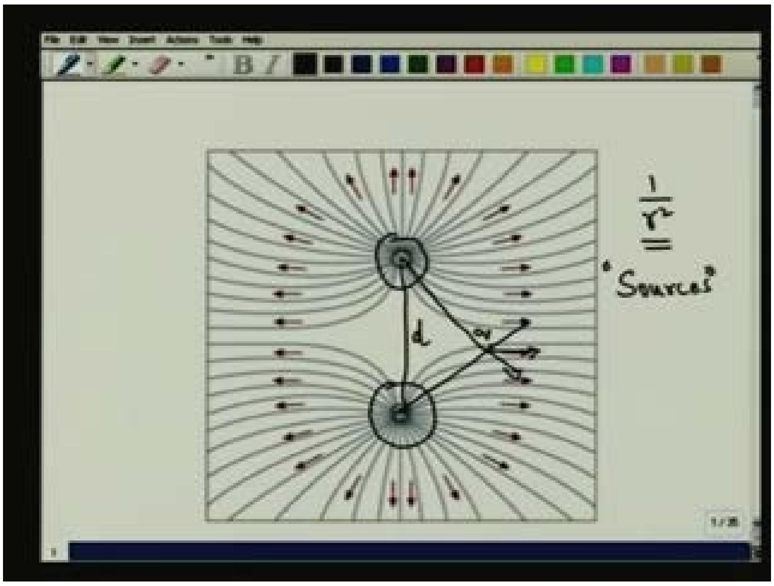

Of course, I am not going to bore you by showing the field produced by a single point particle because that is something that you are already thoroughly familiar with, we have used it many number of times. So, what is it that is happen here? So, you have a points charge plus Q sitting here, you have another point charge plus q sitting here, they are of equal magnitude. And they are at a certain distance d from each other. We have placed them at a certain distance d from each other. This picture contains a wealth of information. The first thing that you notice is that very close to the charge either of them the lines are very dense which tells you that all the lines are actually emanating from this point charge.

As you go further away the lines are becoming denser and denser. In fact, if you were to look at it and work it out with a scale, you will see that the intensity is indeed falling of like 1 over r squared no surprise about that close to this object. Now, if you look at the other charge the same thing is true. So, if I were to draw a circle here and if I were to draw a circle here, the field is going to depend simply like 1 over r square and what is my r here? r is the distance with respect to either this charge or this charge. What is the statement? The statement is simply that, if we are very close to charge one you will not see the electric field due to the other fellow. If you are very close to charge two you will not see electric field due to the first charge. That is the statement.

The next thing that you can notice is that there are these arrows that are written. All these arrows are showing you directions which are away from the charge. Why is that so? Please remember we have made a convention that there are charges which are designated as positive charges and there are other charges which are designated negative. If you have a positive charge and if you bring in yet another positive charge, they are going to repel each other. Test charges are usually taken to be positive charges therefore, if I were to bring a test charge here q and place it here. I have whether this test charge moves towards the charge here or moves away from the charge. Obviously, the answer is that it tries to move away from the source. It tries to move away from either of these sources therefore, these arrows correctly indicate that the test charge moves in that particular direction.

Of course, the test charge could have any magnitude except that it is sign is fixed therefore, when I am depicting this as an electric field, I say that my electric field is flowing in this direction. All of them are flowing outwards. If you could only recall what we said in the second or the third lecture, we said that you have electric fields and these electric fields can be produced either by sources or sinks. If there is a sink for example, like it is real sink where water flows in, then all the velocity flow is towards the sink it is flowing inside. On the other hand if you have a tap the water would be flowing outside. In a similar manner you have now a continuous nice vector field which we have called as the electric field and what is the nature of this electric field? This electric field is produced by two charges both of which act as sources.

Now, this source should not be confused with the source of the electromagnetic field. It is a source in the sense of a velocity field that is the electric field is flowing outwards. And very close to either this charge or this charge you see the flow is outward. Now, if the test charge is were to be at a distance which is comparable to the separation between these two charges, then of course, it will see the combines electric field due to both of them and as this picture correctly depicts. You see at this particular point the charge q tends to move in a direction perpendicular to axis, perpendicular to the line that connects the these two charges.

Again there is no surprise about that because if I were to draw a line here and then these all my force fields, is that right? These all my forces you know that the net direction is roughly in this particular direction. And all this information is very beautifully, very simply and very elegantly codified in this particular picture. Now, in order to sort of complete the picture I will show you one more system of charges where the lines of forces are seen you could be easily guess that it is due to a dipole. Let us see how it looks.

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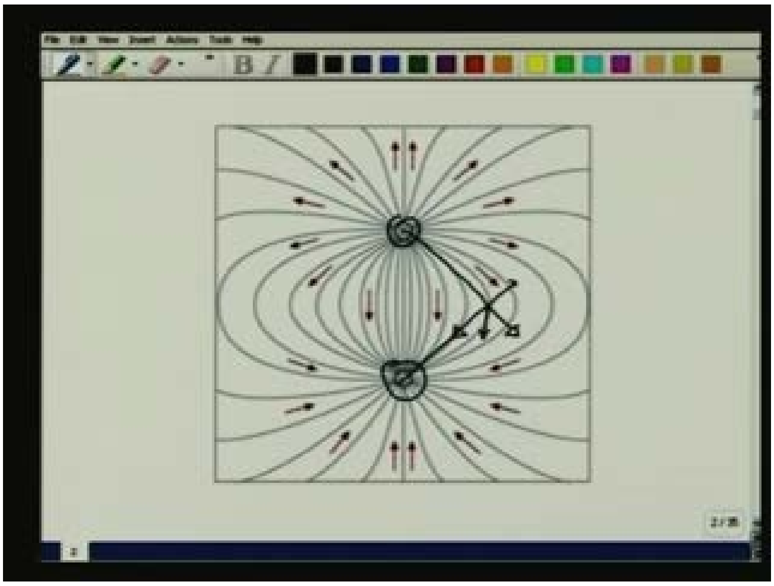

Here what I do is to take not two like charges, but two unlike charges. And you see that the lines of forces are radically different. What is it that is happening? Here you have a positive charge; here you have a negative charge. From the view point of the velocity field or any other such analogy that we had earlier the positive charge is like a source that is throwing out the fluid. The negative charge is a like a sink that is taking away all the fluid. And as you can see if you look at the negative charge the field lines are all flowing into it.

If you look at the positive charge the field lines are all flowing out. If you look at the density, the density of the line charge are lines of forces here, at the density of the lines of force here are the same. It is only the direction that is changing therefore, we say that we have a system of two equal and opposite charges. Here, again it is a very simple matter to check, what is it that happens if I were to place a test charge q here? Again you could resolve the lines of force. So, I have a force here and I have a force here. Accept that the negative force the negative charge pulls it in this direction, the positive charge is going to pull it in this direction; the net force will be somewhat here. In other words draw these lines of force draw the tangents that will give you the electric field at that particular point.

In other words what we have say all done all this time can be summarized in a single statement that the electric field is nothing but a tangent to the curves that we have drawn here. You can ask yourself what is it that happens if I go to distances far away from these two charges. In the previous system, if I go far away the distance d between the two like charges becomes a small negligible parameter; it will look like a single charge single object carrying a charge 2 q. Therefore, the field will simply fall like 1 over r square with respect to the center.

However, if you look at this dipole, I have a plus here I have a minus here, if I go far away which is unfortunately not shown in this figure, what happens? This d becomes again an irrelevant parameter that means, the two charges have collapsed to a single point. So, what is the field? The field to the leading order is 0. Of course, we know better than that if I were to make a Taylor expansion we know that the leading order contribution, the next leading order contribution gives the field which goes like 1 over r cube. So that is a property of multi-pole fields which we shall come to later.

Having done with this, I am not going to spend any time either on lines of force or the depiction of the electric fields or I would rather ask you to write down various charge configurations and try to work it out for yourselves. For example, you can take a charge plus 2 q here, a charge minus 4 q here or in the like charges case you can take a charge 2 q and a charge 8 q let us say. Work it out may be you can use a computer and convince yourself that the answers that you get can actually be pictorially represented. But at these points it is also good to caution ourselves that this should not be taken to the limit. It is not always desirable, it is not always convenient in fact it is not even expected that we should keep on drawing lines of force.

Earlier, when the methods of calculus and binomial expansion were not available, these were a means of actually studying the action of forces. Today, we have far more analytic and numerical techniques at our disposer therefore, having gained an understanding we should make use of this only whenever required and we should not spend too much time working these things out. After having done this I shall now ask what is it that we can do given a system of N charges and we are interested in the field produced by them.

In principle all problems of electrostatics have been solved once we stated the principle of superposition, why? Let us say that you give me a system of charges whenever I am depicting, I am showing you some 7 or 8 or 9 or 10 charges it need not be. So, you can give me 100 a 1000 or for that matter a million number of charges I can always find the net field simply by looking at what by simply looking at the field produced by each individual charge. So, having done this whenever we want to solve the electric field or for that matter, the motion of test particle in such an electric field all that you need is presumably a good computer. Which will keep track of all these source which are going to produce.

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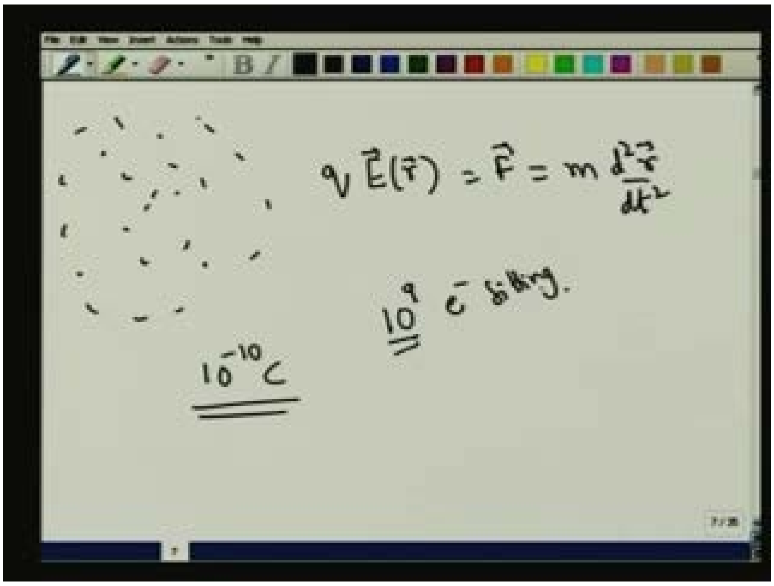

I have a charge here, I have charge here, I have charge here I have charge here in fact I can dot it like sky is dotted with many stars I will bring a test charge q wherever I want and I will try to solve. I can find the electric field all over the space multiplied by the test charge that gives me F. And if I am doing a non dilitalistic calculation, I will write it as m d square r by dt square and I will solve this and I can even find out the trajectory. So, why is it that we have to spend time discussing electrostatics anymore? What we should be probably doing is to look at magnetic, magnetism, magneto-statics or variable currents or variable charges, but that is not the end of the story.

This in fact is only the beginning of the story because whenever we have a large number of such charges which are sitting there; it is more convenient to introduce the notion of the charge density. When I am going from the notion of these discrete charges to charge density, I am keeping in mind the fundamental fact that all charges are quantized. Even when you wrote q a q b q c etcetera in earlier examples, I had in my mind actually so many protons sitting, so many electrons sitting so on and so forth. But then in the real world, we know any reasonable charge which is you know something that we encounter in practical situation is n corresponds to enormous number of electrons.

Remember that I told you that if you rub for against Teflon, you can put as many as 10 to the power of minus 10 Coulombs. 10 to the power of minus 10 Coulombs is a small number if you look at it in terms of the Coulombs, but if you look at it in terms of electron there are already something like 10 to the power of 9 electron sitting. Or if it is the protons that are sitting it is 10 to the power of 9 protons, then we have such a large number of electron the fundamental charges which are sitting very close to each other in a very small volume. In that case it would be full hardy, it would be impractical to actually try to sum over these charges. We should replace these discrete charges by a continuous charge distribution.

So, what I we shall do is to spend the next few minutes trying to motivate as to how to go from a discrete charge distribution to a continuum charge distribution. And we will restate Coulomb law in terms of the continuum charge, let us do that. In order to motivate, what I shall do is to take a very simple example and let us analyze that somewhat slowly there is no great hurry.

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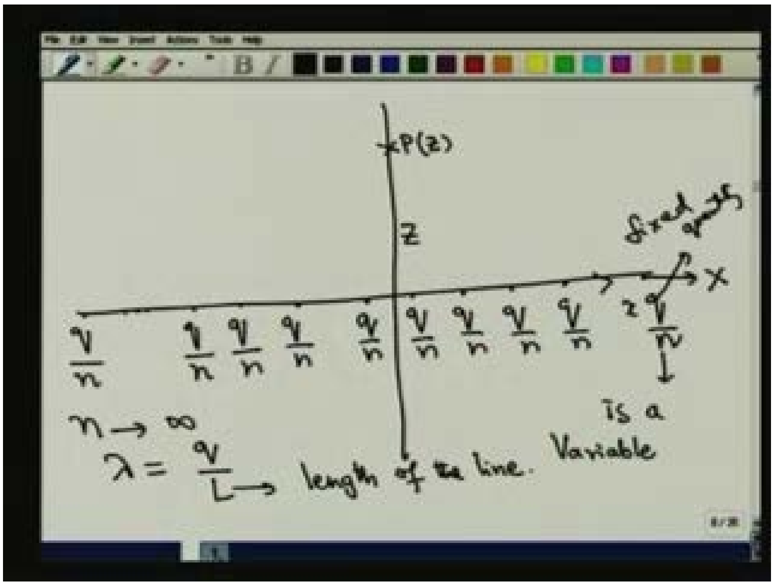

So, what I will do is I will take a line which I shall call as the X axis and then let me choose an origin here and let me put a charge q here. So, equidistant let us say at a certain distance d, I am going to put a large number of charges n number of them. This is my X axis and what I am going to do is to draw a line here, perpendicular to that I will take a point at a certain distance Z so, this is the Z X plane. This is my X axis, this is my Z axis, this is my point P located at Z and I ask what is the electric field produced by all these charges?

In order to be more careful you can easily see that if I keep on increasing the length of this line charge or if I keep on increasing the number of points, the charge also keeps on increasing, is that right? Because q q q that is what I have, but that is not I want what I want to do is to take a lump charge q and I want to distribute it over this particular line. So, what shall I do? If I take n points each of them come with a charge q by n therefore, I will write q by n q by n q by n q by n q by n etcetera. This is something of a finite length later we will worry about how to make it into an infinite length.

So, in a finite length, what is it that we are doing? We keep on increasing the number of points. In other words n is a variable, but q is a fixed quantity. No marks for guessing that in the limit, n goes to infinity, what is it that happens? There is very little charge sitting at any particular point, but if I sum over all of them I am going to get the total charge q. In other words I will go from the discrete to the continuum that is the uniform line charge density. So, in the limit n going to infinity I am going to write lambda is equal to q by L where, L is the length of the line.

I know that all of you are familiar in deriving the electric field due to a uniform line charge, electric field due to an infinite sight of charge, carrying uniform surface charge density or uniform spherical distribution of charge. My purpose is not to repeat those my purpose is somewhat different because we have to understand precisely as to what we mean by a continuous charge distribution, what we mean by an infinite length charge distribution, what we mean by an infinite sheet charge distribution, etcetera.

In reality, nobody sees an infinite line charge distribution. In reality there is nothing like an infinite sheet of charge. Yet when you dealt with capacitors, we pretended we believed that we could treat that these two small plates can be treated as infinite charges, although they might be only a few centimeters in length. In fact, it could be a fraction of centimeter in length. So, in what sense are we going to make use of terminologies like; infinite length, infinite sheet and also continuous distribution that is something that we have to worry about. So, let me analyze that somewhat in great detail and let us what we get.

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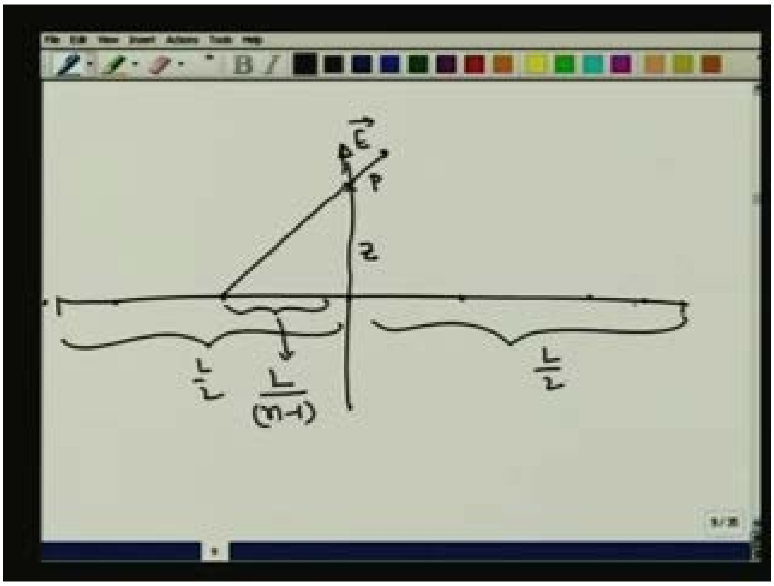

Let me take the point again, now I am not going to draw that many points I am going to draw only a few representative points. I will so I have 1 2 3 4 5 6 so let me make it 7, this is the midpoint. I draw my Z axis here and this is at a certain distance P and I am interested in the field produced by this line so, I am asking what is the force at this particular point? The way I have chosen this Z, this point P it is the line that cuts the midpoint of this line length L. so, what I mean by that? This length is given by L by 2 and this length is given by L by 2, this is the midpoint. Therefore, I know that if I were to compute the field at this particular point, it is in this particular direction. All the charges can be taken to be positive.

The electric field is in this direction because this charge is going to produce a field in this direction and the horizontal components are going to cancel each other, only the vertical component survives. What is this distance? Now, if there are 1 2 3 4 5 6 7 charges, I have correspondingly 6 intervals therefore, if they have n charges I will have n minus 1 intervals therefore, this length is nothing but L over n minus 1, this distance is nothing but L over n minus 1. This distance will be 2 into L over n minus 1, this will three into L over n minus 1 so on and so forth. And we know by principle of superposition how to determine the electric field at this particular point, what is the electric field?

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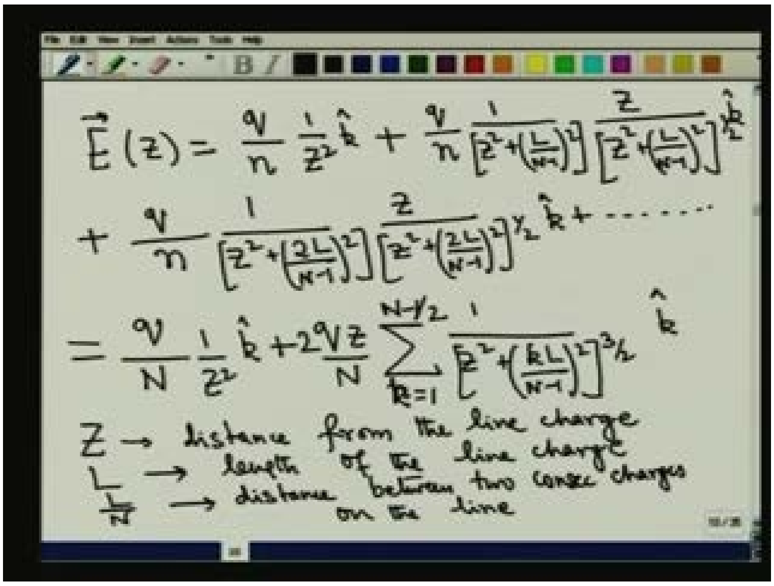

The charge which is sitting bang at the center is something that we can write straightaway so, I am going to write E of Z that is nothing but let me pull the q by n out in the first place, q by n I have 1 over Z squared k. This is the field produced by the charge right below the point Z when I draw the perpendicular, it goes going to hit the charge, what is the next term going to be? Again the next charge is going to be q by n, but that is given by 1 over Z squared plus L by N minus 1 whole squared is what I get because that is the distance, that is the length of the hypotenuse. I have to add a cos theta to evaluate the vertical component and everyone knows that is given by Z divided by Z squared plus L over N minus 1 whole squared to the power of half, that is what I get.

Now, I look at the next term, what will that be equal to? That will again be given by q over n. Now, I move a distance 2 L over N minus 1 because all the charges are equally placed so, how does it look like? That will be Z squared plus 2 L over N minus 1 whole squared Z over Z squared plus 2 L over N minus 1 whole squared to the power of half, this is the cos theta factor corresponding to that, this of course, is at the unit direction k, this in the unit direction k etcetera, this is the force. This is the exact expression for any point Z which moves along a line which bisects the line.

So, let me again employee a summation convention because I do not want to keep on deleting every term, how does it look like? I shall keep the first term apart I am actually employing both small n and capital N for denoting the number of divisions. Let me employee capital N in future so, q by N 1 over Z squared k that is what I have here. Now, I can employ as I told you the summation convention, this will be given by q Z by N and then I have a summation, where what does it take values? n equal to 1 up to capital N because there are that many charges N minus 1 because that is the number of intervals that I have. I have 1 over Z squared plus L k I should use the notation k, k L over N minus 1 whole squared to the power of 3 by 2. And this is also in the direction of k.

Please do not confuse the summation index k with the unit vector k, this unit vector k we know the significance and we know how to handle this, this is the exact expression. If I am only going to count N to the right or left, then you have to put a factor 2 here otherwise, you do not have to do. So, let us assume that we are going to put a factor of 2 because we are only going to count how many of them are to the right, let us say in which I will also out a factor of 2, no harm about that. In which case this will be given by N minus 1 by 2? We know how to again take care of these situations.

Given this exact expression, we shall ask what is the value of E as a function of Z? When I am making this Z word, we should remember that there are three length scales in the problem, what are the three length scales? One is Z which is distance from the line charge. (No audio from 44:29 to 44:36) The second is L which is the length of the line charge and then you have L by N which is the distance between two line charges, is that right? So, distance between two consecutive charges on the line. The answer that we are going to get for the electric field depends on the relation between Z L and L by N.

We are going to show that the Z is very large compared to L, what does it mean? You are so far away; you will not be able to see the line charge at all. Then all this is practically a point charge that is what you are going to get, and then what is it that you will conclude? You will say that the field is simply the Coulomb law. On the other hand we will show that if Z is very small compared to L by N not only is it small compared to L, it is small compared to L by N. If you do that, we will show that the major contribution comes form only one charge and all the other charges have no role to play. The most interesting limit is when Z lies between L and L by N. And please remember N is a large quantity because we want to eventually go to the continuum limit and if we did that, we will find it will tell you the field due to a continuous line charge distribution and that is what I want to motivate.

In order to do that let me analyze my expression, in order to analyze I have to rewrite it let me rewrite that again.

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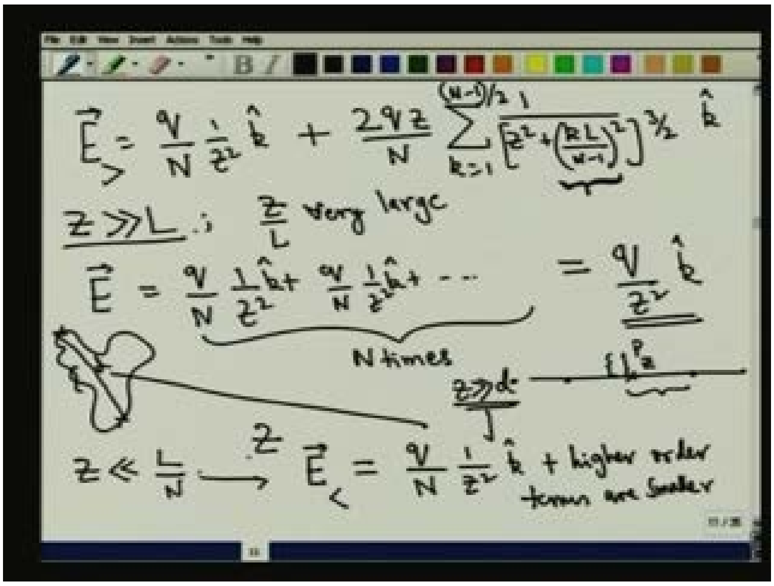

So, my electric field is simply given by q by N 1 over Z squared into k that is the unit vector hat, then I have 2 q Z over N. I had k equal to 1 up to N minus 1 1 over z squared plus k L over N minus 1 whole squared to the power of 3 by 2 into unit vector k. As I told you I want to discuss various limiting cases, the first limiting case is one Z is much greater than L. I have a line charge I have a point Z, I am looking at the limit when Z is much greater than L. so, whenever we say that something is large or something is small, it better be a dimensionless quantity.

So, we are setting precise meaning we are giving a precise meaning to statements like what is large and what is small. What we are saying is that Z by L which is a dimensionless quantity is very large, it makes sense. It does not make sense to say Z is very large or L is very large because that depends on the units that we employ as we all know. If Z by L is very large one can easily see that the maximum value that k L over N minus 1 can take is L, we know that. Because the maximum value k takes N minus 1, but L is much smaller than Z because therefore, Z gives the leading order contribution. In other words, this term which is written in brackets round brackets can be ignored when Z by L is very large.

When we say ignore by that we mean it is not going to contribute in the zeroth order, but when we start computing perturbations, it is going to contribute that is the whole subject matter of multiple analysis. But here if I look at Z very large, what am I going to get? I am going to get in this limit E equal to q by N 1 over Z squared let me now write the unit vector k. Then I have q, q by N Z each of them is going to be 1 over Z squared because this term can be ignored. Since, I am only looking at the right hand side I am going to divide it by 2, it is going to supply for me exactly N minus 1 terms of this form therefore, this will be q by N 1 over Z squared etcetera N times. Which is nothing but q by Z squared let me put the unit vector k here, unit vector k which is essentially the field produced by a point charge distribution.

What are we saying? All that I am saying is that if you give me a line charge and if you sit far away, then you are not able to resolve the length of that line charge distribution. For that matter it need not be a line charge distribution, it could have been a surface charge distribution or it could be a matter charge distribution of some funny shape. So, the charge could be distributed like this, it is very difficult to compute the field produced by them. But still if I am going to sit at a point Z which is far away, what do you mean by far away? Calculate the maximum distance on one point on the surface charge, volume charge distribution to other fellow. So, let us say this is the maximum distribution let me call it d, if Z is much greater than d, then to the leading order all this lump looks like a single point charge and it is going to give me an expression which is like a Coulomb law.

So, all that we are saying is that whenever we speak of a point charge or a distribution, the length scale is of paramount importance and that is what this example illustrates. Now, let me look at the other limit where my Z is actually is much less than L by N. By the way before I proceed, I urge all of you to complete the series, what do I mean by complete the series? When I wrote this term E is equal to q by Z squared k hat, what is it that I assume? I kept only the first term. However, if you want to make a binomial expansion which I am sure all of you are familiar with, what is it that you do? You pull the Z squared out; you will get k L over N minus 1 whole squared divided by 1 over Z squared to the power of 3 by 2.

Now, make a binomial expansion in this particular variable, it will give you systematic corrections to this point charge distribution. And I ask you to rewrite them term by term and that will give you higher order corrections to the leading q by Z squared term that is what I am asking you to do. Now, if you look at Z much less than L by N that is another very easy and a very simple limit to calculate. Because now what happens is that the term in this round bracket whatever I indicated by the flower bracket here is going to dominate over Z. L over N minus 1 for that matter N or N minus 1 really does not matter because N is large is much greater than Z 2 L over N is of course, even greater than Z, 3 L over N is greater than Z so on and so forth. Therefore, now in my expansion I am going to keep this term and I am going to drop the term Z.

However, I cannot drop the first term q by N 1 over Z squared because that is something which I have removed, please do not forget that. I will give you another alternative situation where I can think of a slightly more complicated physics. But at this point remember that my point P or the Z is so chosen that it is bang on the top of one of the charges, let me illustrate that here. So, there is a charge sitting here and this point P is sitting here, this is at a distance Z, let us not forget that. Now, there are other fellows which are sitting here.

Obviously I have exaggerated this figure to tell you that this distance is much greater than this distance. If I did that corresponding to this Z much less than L by N, what will my electric field will be? So, let me denote it by a symbol N, less than here and this will be, this will be greater than. This tells you Z is much greater than L this tells you Z is much less than L by N, this electric field has a leading order contribution which is given by q by N 1 over Z squared k hat. So, the leading order contribution is given by q by N 1 over Z squared k hat which is the first term.

Now, what about the second term? You can easily see that you have a 2 q Z by N, there is a separation factor N and Z is in the numerator instead of Z being in the denominator therefore, all the higher order terms are much smaller. (No audio from 53:09 to 53:17) In fact, again if you were to make a binomial expansion, the next order correction will give you a term of order Z by L cubed so on and so forth, remember Z by L cubed that is what we have. Therefore, all of them will be heavily suppressed, is it a surprising result or is not surprising? Let us spend one or two minutes on that before we conclude this talk and we can easily see that there is nothing very surprising about that.

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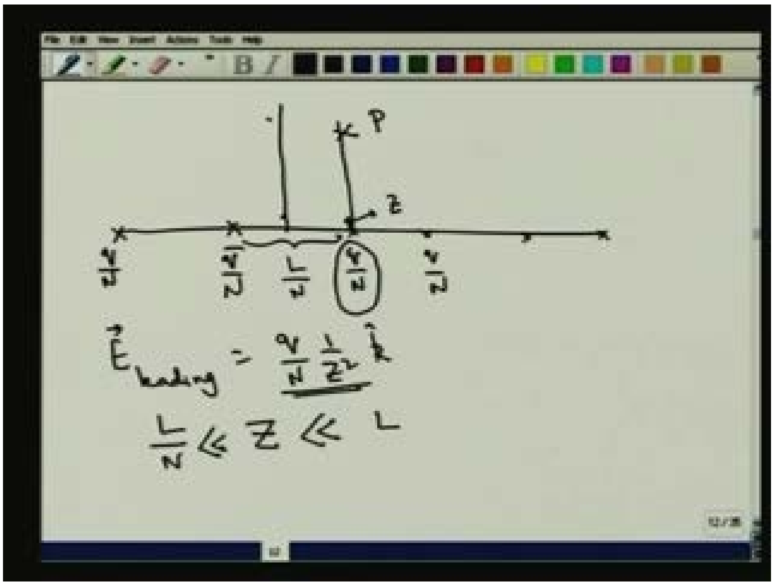

In order to see that there is nothing very surprising about that let me again show the charges here. So, I have q by N q by N the line is going to get extended q by N q by N and I am going to put a point charge somewhere here. When I am putting a point charge here, as I told you this distance which I have called Z is much smaller than this separation which is L by N. Therefore, it is close to one particular charge and it is far away from all the other charges, which is the reason why in our expansion we automatically got a leading order contribution. In other words although q by N is a small quantity the Z is so small, the distance is so small compared to L by N, what we have is that the field that this test starts is predominantly due to the single charge q by N.

In other words, E leading is simply given by q by N into 1 over Z square k hat if you feel like, that is what we get. As we try to take contributions from all these other charges, their distances are becoming larger and larger because these distances are large. Although this distance is small, the perpendicular distance is small, the separations are large. Therefore, they are all quadratically suppressed because force goes like one over squared and this is the reason why this is a leading order contribution, that is all what this thing is saying.

Now, the only cautionary statement that we have to make is that when Z is very small we are also very sensitive to how far we move this way and this way. Instead of taking the bisector or the perpendicular line along the bisector, when we are looking at these short perpendicular distances we have to be very careful about where the perpendicular is drawn. All this time I do the perpendicular around the midpoint, where there was a charge sitting. However, if you were to shift the perpendicular somewhere here and then my point was sitting here, then the leading order contribution will come from the first two nearest neighbors. And all the other contributions will be suppressed which will I which I will again leave as an exercise for you.

Now, comes the third and the most interesting case and that is the limit where Z is much less than L, but L by N is also much less than Z, that is we are going to look at a point P which is at an intermediate distance. This intermediate distance is such that this distance is larger than all the interchange separation, but it is small compared to the overall length of the line charge. This is what is going to give us a precise idea of what a continuous charge distribution is, this is a important concept which we should take some time in order to discuss and I will postpone that to the next lecture.