

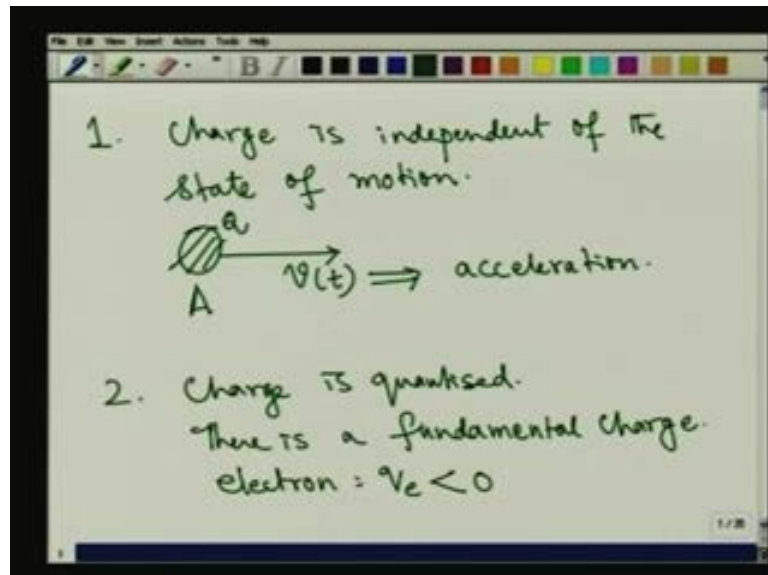
**Engineering Physics – II**  
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**Module No. # 2**

**Lecture No. # 2**

So, in the last few lectures, we have been studying the properties of charges; and there are two important properties which I emphasized.

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The first one was that charge is independent of the state of motion. So, let me make it somewhat more elaborate. Suppose, there is a body which I shall call A and it carries a certain charge  $Q$  and let us say that this body is set in to motion and therefore, it acquires a velocity  $V$ . This velocity  $V$  need not be a constant; it could become a function of time. So, in that case not only does it have a velocity, it also acquires an acceleration; we know from the relativity that when body start moving the mass is not going to be the same; it is going to be dependent on the velocity. So, by the same token we can ask the question, whether the charge carried by the body is also going to change or is it going to remain the same? The answer to that question has been furnished by any number of experiments and there is a lot of evidence for that; as I told you in the last lecture.

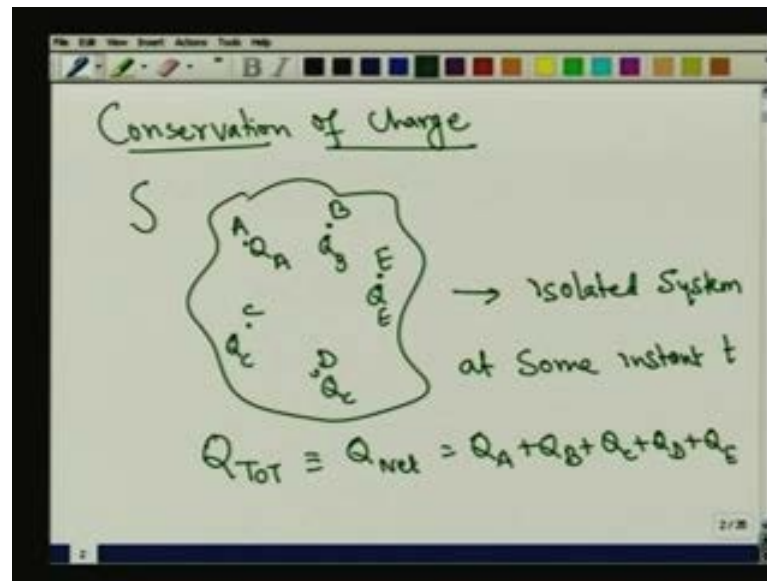
And, the answer is that charge is independent of status motion. By that we mean irrespective of whether the body is at rest or it is having a velocity, whether the velocity is uniform or it is non uniform; the charge is not going to change at all, it is completely independent of the state of motion of the body. The second important property of the charge that we studied was that charge is quantized. So, what do we mean by that? When we say that charge is quantized by that we mean that charge always comes in integral multiples of a fundamental charge; so, that means, there is a fundamental charge, **fundamental charge**. The existence of this fundamental charge is not a matter of convention; like for example, in SI units kg is chosen as a unit by a matter of convention or in CGS units gram is chosen as unit for mass as a matter of convention.

But when we speak of fundamental charge by that we mean that, if you take an electron and denote its charge by  $q_e$ , then all charges are integer multiples of  $q_e$ ; that is what we mean. Of course, by convention  $q_e$  is taken to be negative. Therefore, charges if they are positive; that means, you take  $q_e$  and multiply it by a negative integer and if you have negative charges, you take  $q_e$  and multiply it by positive a positive integer and all charges are thus integer multiples of this fundamental charge.

I have spent quite some time discussing the equality of the magnitude of the charge carried by an electron and the charge carried by a proton; I will not get into that. But at this point it is sufficient for us to mention that all matter is composed of electrons, protons and neutrons. The electron and the proton carry the same charge magnitude wise except for a sign; neutron does not carry any charge. I placed very precise upper limits on the charge carried by the neutron; therefore, it is no surprise that charge is quantized. So, in that sense the statement that charge is quantized is simply a mathematical expression of the basic fact that all matter is composed of three fundamental particles; the proton, the neutron and the electron.

Having discussed this, we now go on to discuss the yet another very, very important property of the charge and that is the conservation of charge. I am going to spend considerable time discussing the evidence for the conservation of charge. And also some amount of experimental basis; although, normally one simply mentions it and goes on to discuss the coulomb law. So, let us start with the beginning of beginnings.

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So, we have conservation of charge. (No audio from 04:44 to 04:52) What do I mean by that? Let me take a system consisting of many, many bodies; so, I have a system consisting of many, many bodies. So, I have a body A and I have a body B, C, D, E. So, let me label it A, B, C, D, E; so on and so forth. Now, let us say that the body A carries a charge  $Q_A$ , body B carries a charge  $Q_B$ , body C carries a charge  $Q_C$ , body D carries a charge  $Q_D$  and body E carries a charge  $Q_E$ ; that is what I have. The charges can be either positive or negative; we do not place any restriction either on their sign or on their magnitude.

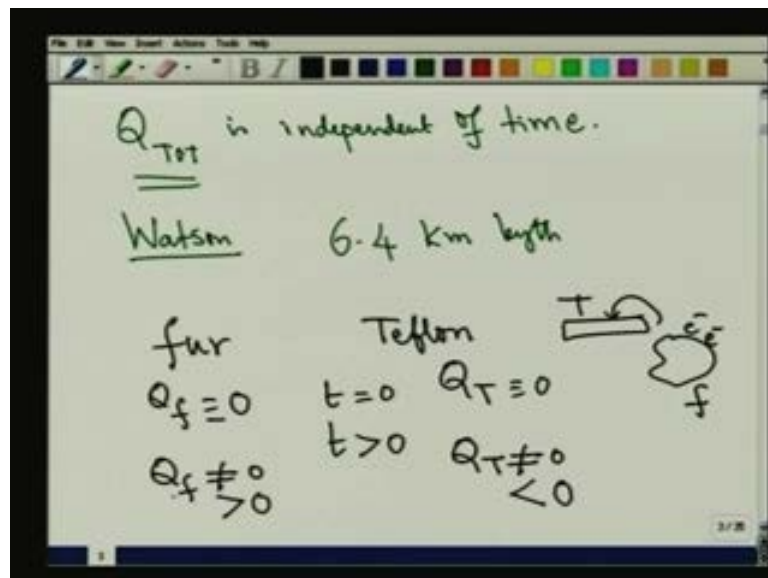
Now, the most important concept for us before we state the conservation of charge is that, I would like this S to be an isolated system; that is it is isolated from the rest of the materials in the universe and I am going to study the properties of S without it having to interact with anything else. So, let me take S to be an isolated system. Now; obviously, the total charge contained in this particular system is simply the algebraic sum of all the charges. So, let me write that down here; my  $Q_{total}$  or which is the same as  $Q_{net}$  nothing but  $Q_A$  plus  $Q_B$  plus  $Q_C$  plus  $Q_D$  plus  $Q_E$ . Obviously, when I say that this system has bodies A, B, C, D and E; I have a particular time at my mind. Therefore, let us say all these were measured at some instant t, **at some instant t**.

Now, the question that we ask is, if S has always been isolated, whether this  $Q_{total}$  remains the same for all times or it does not remain the same for all times? Let me

repeat. I said that this S is isolated, when I said this S is isolated, by that I mean that this conglomeration of bodies A, B, C, D and E is isolated from all the other objects. By that we do not mean that A, B, C, D and E are none interacting; by that we do not even mean that A, B, C, D and E should retain their identity. For example, this A could be a radioactive nucleus. In that case, initially it might carry a certain charge, after a while the nucleus will undergo radioactive decay; it might emit a beta particle, it might emit an alpha particle. In that case, what is going to happen is that the charge sitting on the nucleus is going to be different.

Not only that if A is radioactive, at a time t for example, if there are 5 bodies A, B, C, D and E; at a later time it might so happen that there are more number of bodies, because you have either emitted an alpha particle or a beta particle. It might also so happen that B is a detector or D is a detector or C can actually interact with the electron or the alpha particle which is coming. So, when we say that S is an isolated system, for us A, B, C, D and E are not sacrosanct constituents. All that we mean is that by interaction between them whatever is produced is taken into account; we do not consider the interaction of this system with the rest of the bodies.

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Now, if you grant me that, we ask what is the charge at a later time t? Now, the principle of conservation of charge asserts that this Q total which I wrote is independent of time is independent of time. So, whatever might happen, the number of particles or objects in

the system need not be a constant or the nature of the interaction between them might keep on changing; at some stage they might be far away from each other, at some stage they might be near the each other. But once I create an isolated system, the net charge contained in the system is always the same. So,  $Q_A$ ,  $Q_B$ ,  $Q_C$ ,  $Q_D$  and  $Q_E$  need not be constants of time, but once I add them all up and produce this  $Q$  total; that charge is simply a conserved quantity, it is not going to change with time whether earlier or at a later time. Having said this, it is very easy to make such a statement that something is conserved, but it is yet another matter to verify it experimentally.

So, how does one verify experimentally? Historically, this idea of conservation of charge occurred to an English physicist called Watson, who was a very famous member of the royal society of London and he made an analogy with the fluids. So, if I have an incomprehensible fluid, we know that when the fluid flows from one region to another there is no loss in the mass of the fluids; it is yet that just that it has moved from one point to another. In a similar manner Watson conjectured. It is not that he verified it experimentally; Watson conjectured that may be the amount of electricity on a body, the amount of electric charge on a body is something like a fluid. So, when electric charge moves from one body to another all that is happening is the transfer of the charge from one body to another. But the charge itself is neither created nor destroyed; this was the conjecture that he made.

Watson was a courageous person given the times that he was living in, because he knew the existence of the electric currents; in fact, he wanted to measure the speed with which the electric charges, that is the current flows from one end to the other. To that end actually what Watson did was to construct a fairly complicated contraption. He created a fiber, a conducting fiber of roughly 6.4 kilo meter length. And he wanted to charge, put some charge on the conductor and ask how it moves to the other end of the conductor.

It is a different matter altogether that Watson did not succeed in measuring the speed with which it moves; although he had an enormously large length of fiber on which the charges could pass through, because the apparatus that he had was fairly primitive, but we can see that not only could Watson conjectured the total charge is conserved. He could conjecture how the charge can be transported from one place to another; he had an idea of a continuous motion from one region in space to another region in space. And then he also set out to verify it experimentally, it is a different matter; altogether that he

could not actually verify because at that point he did not have apparatus which was sensitive enough to actually make these measurements.

So, here is the contraption that Watson actually employed to measure the speed with which the current flows. So, you can see here is the flask which is supposed to contain a lot of charge. And he wanted to conduct through this long winding wire which goes all the way C E E G F H, whatever is written here. And he wanted to measure the charge that reaches here. He wanted to connect it to another electrode, so that he can find out what is the time lag between the beginning of the flow of the current and the reaching of the charge at this particular point.

I do not want to spend much time on this, except to point out that even with whatever little they had anodes, Leyden jars and the wires that they had; zinc fiber is probably what he used. Watson did try to measure the speed with which electric current flows, he could not do that. In fact, I do not know if I had a fair idea of how much charge was there to start with. So, although Watson could neither verify the conservation of charge nor verify the idea that charge actually moves like a fluid, he did try to perform an experiment although he did not succeed.

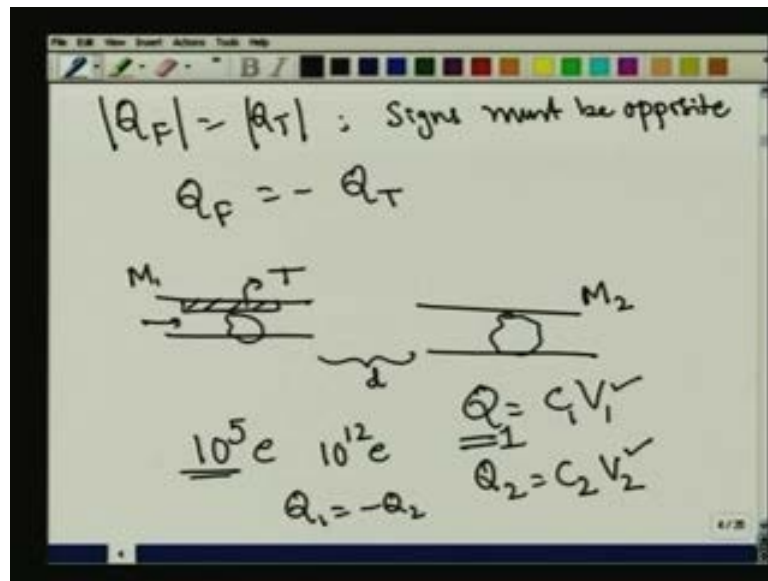
I diagnose to discuss this experiment of Watson because these two concepts, the conservation of the charge and that the charge flows one point to another through the current, all two very, very fundamental aspects which are naturally imbedded in Maxwell's equation; that is something that I would come back at a later time. So, at this point what I shall do is to continue the theme of discussing the conservation of charge and ask how is it that we are going to verify that. The really the person who actually raised the conservation of charge, the idea of conservation of charge to the level of a principle was Benjamin Franklin.

Benjamin Franklin not only raised it to the level of a principle, but he actually tried to verify that the total charge is conserved through a large number of experiments. The basic idea behind all these experiments is that you recognize that many, many objects are actually electrically neutral. Take fur, take amber or take a modern material like Teflon which is plastic all these are neutral materials. However, if you rub amber against wool or silk, we know that they will get charged and if the conservation of charge is valid then the net charge must be equal to 0.

So, what do I mean by that? So, let me look at two objects which are familiar to us in modern days. So, let us take fur. In fact, an experiment has been done in a school, where this fur is the rabbit fur; got it of rabbit and let us take Teflon. Now, what we shall do is to rub the Teflon rod against fur, fur is neutral to start with that is  $Q_{\text{fur}}$  is identically equal to 0 and the charge carried by Teflon is also identically equal to 0. Now, if I bring the Teflon rod and take this fur and rub the Teflon rod against the fur then there is going to be a charge transfer. In fact, we know today that what happens is that the electrons which are sitting on the fur migrate to the Teflon. So, what happens? Initially, Teflon was neutral, the fur is neutral, but the electrons migrate from the fur to the Teflon. Therefore, at a later time so let us say that this is the initial condition before the rubbing take place.

At a later time  $t$  greater than 0, since there is a transfer of electrons from the fur to the Teflon; so, this is the fur and this is the Teflon rod. Your  $Q_{\text{fur}}$  is not equal to 0; in fact, it is greater than 0 because it has lost a certain number of electrons. By the same token, the net charge on the Teflon rod is again not equal to 0, but it is going to be less than 0. Because there is a large number of electrons which are going to sit on the Teflon rod; therefore, at a later time we find that  $Q_{\text{fur}}$  is not equal to 0, but it is greater than 0. The charge on the Teflon is not equal to 0, but less than 0; but if indeed the conservation of charge is valid, we should have  $\text{mod } Q_{\text{F}} = \text{mod } Q_{\text{T}}$ . So, what do I mean to say? If I satisfy two conditions which I am going to write in a minute, then we can actually say the charge is conserved. What are the two conditions?

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The two conditions are simply given by  $Q_F$  must be equal to  $Q_T$  magnitude wise and the other condition is that signs must be opposite or to state it as a single condition  $Q_F$  must be equal to minus  $Q_T$ . This is the kind of experiment which Benjamin Franklin attempted in the late 17th century and it turns out that those days the results were actually temperamental. Actually, by that what I mean is that the results were temperature dependent. If Benjamin Franklin went with a sweaty forehead or wet hands, then the results were not very good because immediately whatever charge was deposited on Teflon or whatever charge was lost by fur would get depleted; because it would charge or give away the charge from the atmosphere.

On the other hand, if it was a dry good weather; then the charge could be retained. So, in any case since it was almost impossible to create ideal laboratory conditions for measuring an important, where verifying an important concept like this; Benjamin Franklin's results were at best really approximate and only pointers to the idea that charge is indeed conserved. However by performing a large number of experiments Franklin was able to convince himself that whatever result he got, the deviation from the conservation of charge could be attributed to experimental inaccuracy rather than, what? Rather than, a form of breakdown of a fundamental law of nature; therefore, he boldly enunciated the conservation of charge.



Today, it is not very difficult to verify conservation of charge with this kind of an experiment. It is not simply accidentally that I took out the example of the fur and Teflon; in fact, these experiments are performed. What you do is to have a contraption something like this. So, you have a metal pipe and they have a metal pipe here, let us say this can be looked upon as some kind of a cylindrical capacitor; so, what you do is to coat on this Teflon. So, this is the Teflon coated metal wire, a metal pipe and this is completely empty.

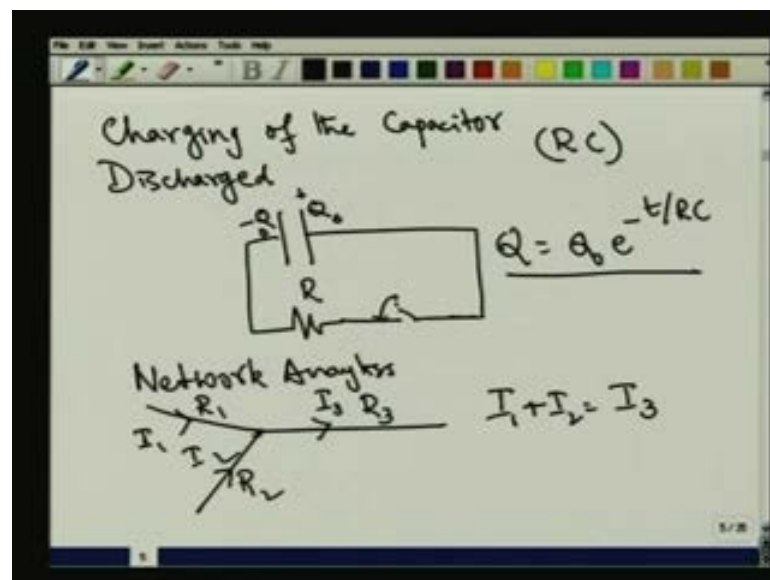
Now, what we do is to take some amount of fur and actually move the fur from one end to the other end, such that the fur rubs against the Teflon. I have given a spacing which is grossly exaggerated between the two metal pipes. So, let me call this metal pipe M 1, let me call this metal pipe M 2. The space between them is  $d$  which is very, very small compared to the size of the fur. So, the fur keeps on rubbing against the Teflon, comes here and eventually comes here and sits here.

So, what is it that is going to happen? When it is rubbing against the Teflon rod as we saw it loses electrons; therefore, it gains the charge. And whatever charge is gained by Teflon that is transferred to the metal pipes here and in a similar manner whatever charge is contained in the fur that is also transferred to the metal pipe. So, if you could measure the voltage at which these two metal pipes are; since these are cylindrical, since we know their capacitance we can simply make use of the relation  $Q$  equal to  $C V$  measure the charge on the metal pipe 1. So,  $C_1 V_1$ , measures the charge on the metal pipe M 2 that will be  $C_2 V_2$ . Now, if  $Q_1$  equal to minus  $Q_2$  then we have verified the conservation of charge.

Now, what would typically a number in this experiments would be? If I were to write down the total charge transferred in terms of coulombs that would be a very small number,  $10$  to the power minus  $14$  coulombs or whatever. But if I were to write it down in terms of the number of charges that have been transferred, number of electrons that have been transferred; in fact, as many as  $10$  to the power of  $5$  electrons could be transferred, in fact, more. If I did a nice good experiment, we could transfer as much as  $10$  to the power of  $12$  electrons from fur to Teflon; therefore, there is enough charge that has been deposited on the two metals. If we therefore, measured carefully correctly the voltages the capacitances are known objects because of the geometry, because of the shape of this cylindrical metals; we can verify that  $Q_1$  is equal to minus  $Q_2$ .

Now, all the charge that came on metal 1 is due to Teflon, all the charge that came on metal 2 is due to fur; therefore, in measuring the charge sitting on the metal pipe 1 and the metal pipe 2, we have actually measured the charge sitting on the fur and the Teflon. Initially, both of them were neutral, therefore, since  $Q_1$  minus equal to  $Q_2$ , if you add  $Q_1$  and  $Q_2$  the net charge is equal to 0 which is what we started with. So, this is one experiment which illustrates the idea of the conservation of charge. Now, conservation of charge is so fundamental, not only do we employ them in order to verify; **verify** in experiment such as even that I mentioned in the previous example, it is routinely used in many, many other cases.

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Consider for example, the charging of the capacitor. Later when we discuss networks, we are going to spend some time; but at this point I will simply mention that charging of the capacitor. So, when the capacitor is getting charged or discharged, what do we do? The rate at which it is getting charged or discharged is controlled by the time constant which is simply given by  $R C$ . So, what do I mean by that? I have a capacitor which is connected to a resistance and then let us say that it is already charged and I want to discharge that. So, I have my circuit here, the switch is open; so, there is a plus  $Q$  sitting here there is a minus  $Q$  sitting here.

I am interested in the discharge; in order to discharge it I will close the switch. Once, I close the switch I want to know the rate at which the current flows and it gets discharged

and we know that  $Q$  is simply given by  $Q_{\text{naught}} e^{-t/RC}$ . So, initially there was a charge  $Q_{\text{naught}}$  and  $-Q_{\text{naught}}$  sitting on the capacitor, I close the switch a current starts flowing. So, at any given time  $t$ ,  $Q$  is given by  $Q_{\text{naught}} e^{-t/RC}$ , where  $R$  is resistance. In writing this equation, if we go back carefully look at the original differential equation which we solved, I will give that as an exercise for you; you will find that you implicitly assume that the charge is conserved.

The total charge is conserved; therefore, there is a current which is flowing, the positive charge is sitting here, the negative charge is sitting here. So, as the current flows the charge in the capacitor is getting depleted that is what we are going to assume. But the net charge is conserved which is the reason why you are able to write this particular equation. Yet another example where conservation of charge is explicitly and routinely used is in network analysis. So, there we do not look at capacitors, but we look at network of various resistances. So, let us look at a very simple resistance. So, how does it look like? So imagine, there are resistance  $R_1$ , there is resistance  $R_2$  and there is a  $R_3$ . So, this is a node sitting here; there is an  $R_1$ , there is an  $R_2$ , there is an  $R_3$  sitting here, what do we say?

We say that there is no accumulation of charge at the node. This suppose there is a current  $I_1$  coming here, there is a current  $I_2$  coming here and there is a current  $I_3$  flowing here. This state that  $I_1 + I_2$  must be equal to  $I_3$ . So, in saying that we are precluding either the accumulation of charge at this node or the loss of charge at this node; therefore, this linear first law of Kirchhoff it says  $I_1 + I_2 = I_3$  is imply a restatement of conservation of charge. In other words, conservation of charge is something which is omnipresent in electro dynamics, which is used everywhere so often that many times we even forget to explicitly take into account that it is the bedrock on which all of electro dynamics rests.

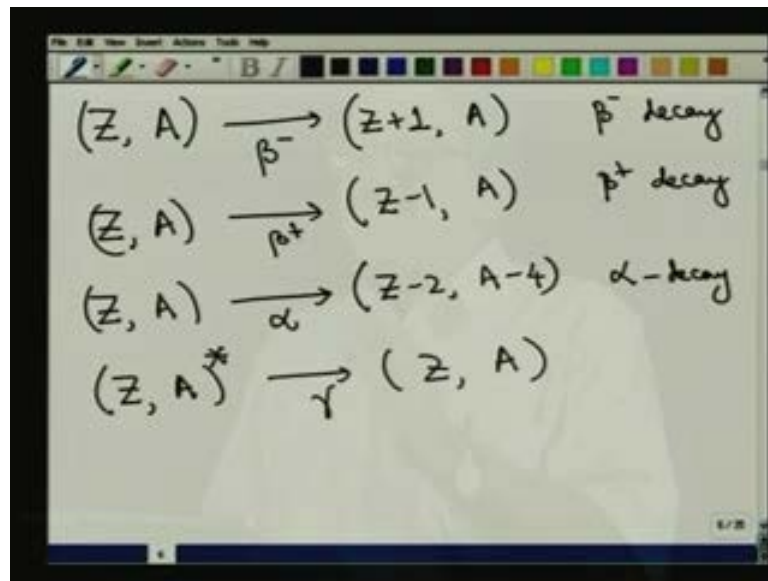
Having said that since physics is an experimental science, since we are going to study electromagnetic phenomena not in abstract, but as application to generate us power supplies, transmission lines, capacitance and what not are in fact, even the propagation of light; we should actually produce evidence which goes beyond this kind of qualitative arguments. Even the experiment which I suggested earlier cannot actually verify conservation law to a great extent, because to a great accuracy. Because if I deposit

something like 10 to the power of 8 or 10 to the power of 10 electrons on the Teflon; now this experiment certainly cannot measure this with an accuracy of let us say 1 part in 10 to the power of 10.

Suppose there were a few electrons which were lost, suppose there were a few electrons extra electrons which came from the atmosphere and got attached to Teflon, this experiment would never be able to distinguish. In other words a precision test which will actually make us believe in the conservation of charge as a fundamental law is still lacking. The precision test or a precision observation does not come from macroscopic experiments of this particular kind, but from modern physics where we look at microscopic physics. That I shall illustrate by looking at the phenomenon of radioactivity where we actually measure the total number of electrons that come from a radioactive decay, the total number of beta plus particles that come from a radioactive decay, the total number of gammas that come from radioactive decay and the total number of alpha particles that come from radioactive decay.

We know the parent nucleus which has a total charge, which has a total mass; we know the daughter nuclei at every stage, because there could be a series of radioactive transitions. We can put detectors which measure the total number of alpha particles that have come, the total number of beta plus or minus particles that have come. Now, we can actually book keeping at every point in our experiment, at every point in our observation. And if the book keeping gives us an account which says that the total charge is always the same, then we have indeed verified that charge conservation is a fundamental law. So, in order to do that what I will do is to look at an example which contains the series decays involving alpha particles and beta particles. So, before I do that, let us recapitulate what happens, when there is an alpha decay or beta decay.

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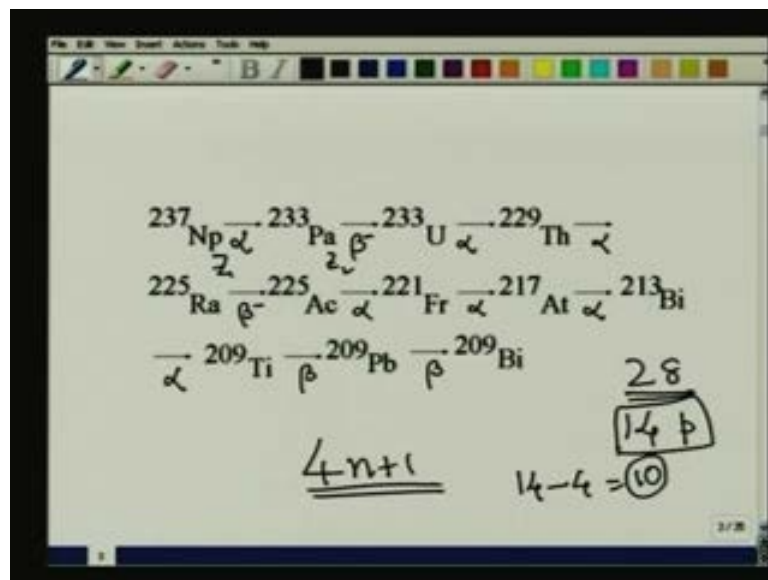


Now, let us employ the standard notation. So, you have a nucleus with a certain atomic number  $z$  and an atomic weight  $A$ .  $z$  is the total number of protons inside the nucleus,  $A$  is the total number of protons plus the total number of neutrons inside the nucleus, let us remember that. Now, if there is a beta decay, so suppose I write beta minus; we know that the atomic weight is not going to change. Because a proton is going to become a neutron, but while the neutron is going to become a proton, I am **sorry**, but then the proton neutron masses are so close to each other; we are not going to worry too much about that. However, the  $z$  value changes, under the beta decay  $z$  will go to  $z$  plus 1 and  $A$  will go to  $A$ .

Please notice, initially the total charge was  $z$  in the multiple of electronic charge; the daughter nucleus has a charge  $z$  plus 1. But then, there is an electron which is coming with beta minus, the total charge is  $z$ ; so, this corresponds to beta minus decay or electron decay. In a similar manner if I start with a  $z$ ,  $A$ , I can have a beta plus emission which is also called as a positron. In this case it this  $z$  minus 1,  $A$ . So, this is beta plus decay or the positron decay. We can have yet another situation where, I have an atomic number  $z$  an atomic weight  $A$ , I emit an alpha particle. What is an alpha particle? An alpha particle is nothing but the helium nucleus which has 2 protons and 2 neutrons. So, what happens to  $z$ ?  $z$  will go to  $z$  minus 2,  $A$  will go to  $A$  minus 4; because it has lost 4 nucleons, 2 neutrons and 2 protons, where as it has lost only 2 protons; therefore, this is  $z$  minus 2.

We have yet another example which obtains because of the de excitation of atoms or nucleate excited states; so,  $z A$  can emit a gamma particle in which case it goes to  $z A$ . How do I distinguish the parent from the daughter? Initially you have  $z$ , finally also you have  $z$ , initially you have an  $A$ , finally also you have the same way. What I shall do is to put a star here, which tells you that it is indeed in an excited state. So, in these fundamental processes, I call this as beta minus decay, I call this as beta plus decay, I call this as alpha decay. I have written down the atomic weight and the atomic mass, a number of the daughter nucleus by assuming charge conservation. If I can observationally verify that at every stage of decay, then we can really convince that charge is indeed is conserved and in order to do that let us look at the following example.

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So, this is a very famous radioactive series which you would have certainly seen in your 12th standard book and it belongs to what is called as  $4n + 1$  series, let me explain that to you. So, you start with the parent nucleus  ${}^{237}_{94}\text{Np}$ , I have not written the  $z$  values deliberately so that you can go back, write the various  $z$  values and convince yourself that everything is fine. So, now look at what is happening to  $A$ . So,  $237$ ,  $233$  etcetera, they all refer to the values of a total number of protons and neutrons, what we call as the atomic weight. Now,  $237$  is becoming  $233$ ; that means, there is a depletion of  $4$  nucleons. So, this if you balance the  $Z$  carried by the neptunium and the atomic number carried by palladium, you can convince yourself that this is actually an alpha decay.

Now, if you look at the next decay from palladium to uranium, you see that A value is not changing at all.

However, the nucleus has changed, it has gone from palladium to uranium; again if you look at this a table you will find that it is a beta decay; in fact, it is a beta minus.  $^{233}_{90}\text{Pa}$  to  $^{233}_{91}\text{U}$  is again an alpha decay, from  $^{233}_{91}\text{U}$  to  $^{229}_{91}\text{U}$  is an alpha decay,  $^{229}_{91}\text{U}$  to  $^{225}_{89}\text{Ac}$  is a beta minus decay,  $^{225}_{89}\text{Ac}$  to  $^{221}_{89}\text{Ac}$  is an alpha decay,  $^{221}_{89}\text{Ac}$  to  $^{217}_{85}\text{At}$  is an alpha decay. This is yet another alpha decay, this is yet another alpha decay and then you have your beta decay and a beta decay and that is what we have.

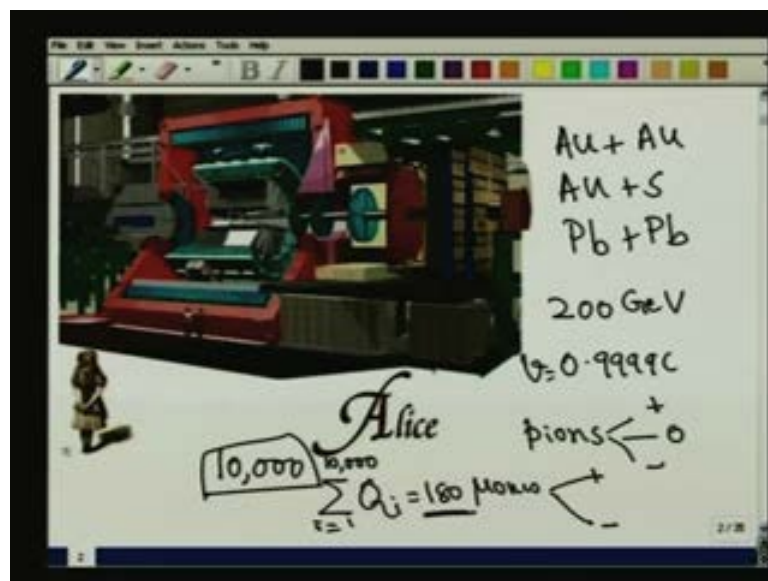
So, what is it that I am trying to do? What I am asking you is to take this series, the  $4n + 1$  series; why do we call it  $4n + 1$ ? Every time it decays either it will lose an alpha particle or it loses a beta particle. Please, match the various Z values which I have not written here; so, there is a Z 1, there is a Z 2 so on and so forth and ask what should be the final Z. And if charge is indeed correctly conserved, is conserved then we should be able to balance the total number of alpha particles that are lost, the total number of electrons that are lost; we should be able to observe them by putting detectors all around the place and convince ourselves that charge is indeed conserved.

For example if you look at how many alpha particles have been lost from the parent I have 1, 2, 3, 4, 5, 6, 7; 7 alpha particles have been lost. That means, each alpha particle carries a charge Q, there is a loss of 28; 7 into 4, 28 nucleons and 14 protons, there is a loss of 14 protons. On other hand, if I ask what is the number of electrons that have been emitted, again let me count I have 1, 2, 3, 4; 4 electrons have been lost. That means, 4 electrons have been emitted, that means that much charge has been gained. Therefore, I should take this 14 and subtract 4, I should get charge 10.

The net charge that is there in the emitted particles should be equal to 10 and indeed if you put the detectors and ask for alpha particles and ask for beta particles, believe me the detectors actually see that many alpha particles and that many beta particles; therefore, this is a very very good evidence that charge is indeed conserved. This is only one of the examples that are even more complicated decays, where the nucleus goes through many many isotopes, isobars and different nuclei. You can look up any book on modern physics; you will always find that the total charge is conserved.

In these decays the conservation of charge is not an act of faith; it is actually experimentally verified, because what experiment is do is to put alpha particle detectors, put beta particle detectors by creating the appropriate magnetic fields and they are able to verify conservation of charge. In fact, the situation in current day physics, the most up to date physics is something even more spectacular. So, I shall mention the last example, it is actually not an example, but an illustration of how charge conservation is extensively used and verified in modern accelerator physics and go on and then go on to study other phenomena involving coulomb law.

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This is an example of a very, very sophisticated and a modern detector. This detector has been called Alice; let me not get into the expansion. But in order to give an idea of how huge it is, here is a little girl and you can see that it is many many, many times bigger than us; she looks like a small doll in front of this detector. So, where is this detector used? This is a charged detector; it detects charges of all kinds. In modern day accelerated physics experiments, we have heavy nuclei coming with enormous velocities coming and hitting against each other. So, typically it could be something like gold against gold. So, gold against gold or gold against Sulphur or it could be lead against lead.

Now, what is the energy with which these nuclei come and hit against each other? For example, in the laboratory the energy carried by huge proton in lead could be something



like 200 GeV. Please remember, the rest energy of a proton is 1 GeV, close to 1 GeV; therefore, it has an energy which is 200 times its rest energy. If I were to convert it into the velocity or the speed, the speed corresponding to that is very very close to the speed of light  $0.9999c$ ; whatever, that is the speed with which it comes. And when 2 nuclei with such enormous energy come and collide against each other, they produce a large number of particles.

Please remember, what I said earlier. I said I consider a close system with body A, B, C, D, E, F; I said the identity of the bodies in the close system is not sacrosanct, that is exactly what is happening. The 2 nuclei come and hit against each other, they come and collide against each other and when they collide the nuclei breakup. When the nuclei breakup, it is not as if the final state or in the after the collision is over there are only that many particles as there were protons and neutrons in the initial nuclei. You produce a large number of electrons, a large number of the particles called Pions which come in 3 species; they can be charged, have a positive charge, they can have a negative charge or they can have a 0 charge.

You can produce a large number of other set of particles called the Muons, I am sure you people have heard of that. These also come in 3 species, positive charge, **sorry**, 2 species; positive charge and the negative charge and then you can produce anti protons, protons, neutrons, etcetera, etcetera. What this detector does is to measure each and every individual charge. At these energies it is not difficult to estimate the total number of charges; in fact, experimental itself estimated that, the theories have estimated that and it comes to close to 10,000 charges. So, there are 10,000 charged particles of various charges because either there are Pions or Muons or whatever, this detector measures them; actually adds them up and in principle and also in practice it is possible to verify that the charge conservation is indeed valid.

Now, if you look at gold against gold; for example, gold I think has a  $z$  value close to 90. Initially the total charge is given by **(( ))**. Whereas, in the final state there are close to 10,000 charged particles, but still the net charge if I add the charges of all these objects. So, if I write the charge  $Q_i$  and write  $i$  equal to 1 up to 10,000, what does it come to? It comes to whatever the initial ways it is closed to 180; I am assuming that gold has 90, it might have a different value; it is close to 90, so, whatever the total initial charges that is

what you are going to get. This indeed demonstrates for us in a very **very** spectacular manner, in a convincing manner that charge is indeed comes conserved.

So, now we know that charge is invariant; it is independent of the state of motion. Charge is conserved, charge is quantized; therefore, we have been able to appreciate the meaning of the term Q 1, Q 2 that occurs in the coulomb law. Let us proceed to study coulomb law further; we stated coulomb's law given two charges.

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The image shows handwritten notes on a whiteboard. At the top, the Coulomb force vector is given as  $\vec{F}_c = \frac{Q_1 Q_2}{r^2} \hat{r} \left( \frac{1}{4\pi\epsilon_0} \right)$ , with the term  $\left( \frac{1}{4\pi\epsilon_0} \right)$  circled and labeled "Coulomb". Below this, the charges are defined as  $q_e = -1$  and  $q_p = 1$ . The total charges are then expressed as  $Q_1 = N_1 q_e = \frac{N_1}{-1} q_p$ . To the right, the permittivity of free space is given as  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{V}\cdot\text{m}^2$  and the permeability of free space as  $\mu_0 = 4\pi \times 10^{-7}$ . A diagram shows two parallel wires with current  $I$  flowing in the same direction, separated by a distance  $d$ , with a force  $F$  between them. The formula for the force per unit length is given as  $\frac{F}{A} = \frac{\mu_0}{4\pi} \frac{I^2}{d}$ . To the right of this, the relationship  $I = \frac{dQ}{dt}$  is written.

And, we said that is simply given by  $Q_1 Q_2$  by  $r$  square  $\hat{r}$ ; that is what we said. And we have spent a considerable amount of time discussing the properties and the important physical aspects create into the charge  $Q_1$ . I should add of course, the factor  $1$  over  $4\pi$  epsilon naught here, because we will have to make a few statements about that; because it is a matter of choice of units. Now, this is not the end of the story for us because we have to again ask as to how charge is measured?

I have said that charge is already quantized; it always comes in unit of electron charge. Therefore, I should in principle not even be asking this question because without any loss of generality, I can set charge of  $q_e$  is equal to  $-1$ . If you feel like we can take this to be equal to minus  $1$  because it is a negatively charged particle by convention, then  $q_p$  can be set equal to  $1$ ; that is what we should do because this is indeed the basic building of all the charges. If I did that the charge contend on any other body is uniquely defined because I can always write any  $Q$  to be an integer multiplied by  $q_e$ . Because there is

always an integer number of charges that are sitting; if this  $n$  is positive we know that  $Q$  is negative, if this  $n$  is negative we know that  $Q$  is positive.

So, if I write  $Q$  equal to  $N q e$ , we can simply say that the charge for this corresponding to let us say  $Q/N$  is simply given by  $N^{-1}$ . In other words, you would have agreed to measure all charges in the unit of electron charge. Anyway, since we want to measure charges in terms of a positive unit; instead of saying  $q p$ , you can write  $N^{-1} q p$ , where  $N^{-1}$  is nothing but minus  $N$  and to say the charge is nothing but  $N^{-1} q p$  that is what we can say.

If we did that, then we would have to ask the question; what the factor  $1/4\pi\epsilon_0$  is doing here? Because I know the left hand side by a balance measurement as I discussed in my previous lecture (( )) measurement, I know the right hand side I know the charge sitting on anybody because I know there is an integer multiple. So, how do I measure the force? (( )) what I would do is to take an electron; I will take another electron and place them a meter apart. Let us say that I want to employ SI units, I place them a meter apart and ask what is the force of repulsion between them? This force of repulsion is a precisely measurable object.

Now, once I measure the force of repulsion between them, what I can do is to simply write, express the electron charge in terms of the force and then we would have had a complete dictionary of writing down all possible charges and all possible forces. So, what am I saying? What I am saying is that measure all charges in units of the electron charges or the proton charge, measure the force between the 2 electrons or 2 protons when they are a meter apart, find out the force; that force is a measurable object. And fix the magnitude of the electron charge in terms of the force; that is what we are saying. If we did that, then there would be no ambiguity what so ever, there would be no confusion what so ever. And we have taken the course to a method which is completely demonstrative, which is completely based on measurements.

However, that is not the way you are defined, whatever is the charge in your SI units, what you are given is a something which is something which is something more complicated and that is a coulomb. I will not get in to the definition of coulomb at this particular point, because as you all of know all of you know coulomb itself is defined in terms of ampere; the current that flows in a wire. So, what you do is to take 2 wires

which are a meter apart and ask what is the current that should flow, such that the force between them should be some amount of whatever  $10$  to the power of minus  $7$  Newton's or whatever. And then you say  $dq$  by  $dt$  is the coulomb and you write that.

Having written that in S I units, what one does is to smuggle in this quantity called  $1$  over  $4\pi\epsilon_0$  and if you look at your books  $\epsilon_0$  is giving you a value. How much is  $\epsilon_0$ ?  $\epsilon_0$  is given to be  $8.85$  into  $10$  to the power of minus  $12$  coulomb square per Newton square. So, coulomb square plus Newton meter square because that comes from the dimensionality of this and one wonders where this number  $8.85$  into  $10$  to the power of minus  $12$  came from. If I could jump a little bit, I must you could anticipate the next question; although we are not dealing magnetism. You remember there is yet another quantity  $\mu_0$  which is the permeability of vacuum and that is given by  $4\pi$  into  $10$  to the power of minus  $7$  S I units Newton per ampere square; let us not worry about that unit at this point. So, if we are going to measure currents, if we are going to measure forces, we ask where this  $\epsilon_0$  is coming from. Where is this  $\mu_0$  coming from?

Now, let me make this question more precise. You are given this expression for the Coulomb force and the left hand side is a completely measurable object. Whereas, right hand side has two unknowns, the charges and  $\epsilon_0$  and you can see that what is measurable is always the combination  $Q_1 Q_2$  by  $\epsilon_0$ . In a similar manner, if I were to take two straight wires and pass a current  $I$ , let me take them to be the same at this particular example. And let us say that the distance between them is  $d$ , what is the force that we are going to write? We write the force is simply given by  $\mu_0$ , maybe there is a factor of  $4\pi$  and we are going to write  $I^2$  by  $d$ , this is the force per unit length.

Again, you see that the force on left hand side is a completely measurable object because I am going to use some mechanical contraptions; whereas, the current always comes in a combination  $\mu_0 I^2$ . In other words the charge cannot be measured independent of  $\epsilon_0$ ,  $\epsilon_0$  cannot be measured independent of charge, the current cannot be measured independent of  $\mu_0$ ,  $\mu_0$  measured cannot be measured independent of  $I^2$ . So, while when there are two measurements to perform  $F_c$  at the  $F$  and you have four quantities; the charge,  $\epsilon_0$ ,  $I$  and  $\mu_0$  and we ask how many of them are fundamental?

However, we know that the current is nothing but the rate at which the charge flows; therefore, if you give me the charge or if you give me the current, since time is an independently measurable quantity, I know how to fix one given the other. And in SI units by convention, we start with the definition of the ampere and go on to define the coulomb in terms of the ampere; coulomb is defined in terms of the ampere, that is something that we should not forget. So, of the four quantities there are three quantities which are independent, which I shall call as the current, the epsilon naught and mu naught. But then there are only two equations,  $F_c$  and  $F_{\text{magnetic}}$  or  $F_{\text{ampere}}$  if you feel like and we have to ask how these three quantities can be determined in terms of two absorbable or two measurable objects?

Books on electrodynamics tell us that the way out is simply to give the value of mu naught and then deduce the value of epsilon naught, in a manner which I will tell you in a minute. In fact, if you book up look up elementary books on electricity magnetism, what they give is simply mu naught value  $4\pi \times 10^{-7}$ ; simply the value of epsilon naught  $8.85 \times 10^{-12}$ . But if you look at it little bit more carefully, you can find that these two can also be fixed because I know my  $Q$  and  $(\ )$  because I know my current and the charge, by taking the ratio of  $F_{\text{coulomb}}$  with respect to  $F_{\text{magnetic}}$  which I shall do next.

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Handwritten notes on a whiteboard:

$$\frac{F_M}{F_C} \sim \frac{(\mu_0 \epsilon_0)}{c^2} = \frac{1}{c^2}$$

$c$  Speed of light

$$\epsilon_0 = 8.85 \times 10^{-12}$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$c^2 = 9 \times 10^{16} \text{ m}^2/\text{s}^2$

$\mu_0 = 4\pi \times 10^{-7}$  is a definition

So, what I can do is to calculate the ratio of  $F_{\text{magnetic}}$  to  $F_{\text{coulomb}}$ . Please remember the  $F_{\text{magnetic}}$  that I am writing here is nothing but the force per unit length. If I did that, it will simply turn out to be proportional to the quantity  $\mu_0 \epsilon_0$ ; that is what we are going to get. And then we are going to get some other quantities and you can work it out and check that this object indeed has the dimension of  $1/v^2$ , velocity square.

Now, if you quote go back and remember what you did in your 12th standard, when you set up the wave equations you recognize this  $\mu_0 \epsilon_0$  to be nothing but the inverse square of the speed of light, which is nothing but  $1/c^2$ ; this is indeed measurable. So, what are we saying? What we are saying is that you give me the definition of ampere, I will give you coulomb and between  $\mu_0$  and  $\epsilon_0$  both of them cannot be independently measured; both of them cannot be independently specified. However, the combination  $\mu_0 \epsilon_0$  can indeed be independently specified because that is nothing but  $1/c^2$ , where  $c$  is nothing but the speed of light.

So, go back I wrote the value of  $\epsilon_0$ .  $\epsilon_0$  was whatever I wrote  $8.85 \times 10^{-12}$  in SI units, I wrote  $\mu_0$  equals to  $4\pi \times 10^{-7}$ . If you multiplied  $\epsilon_0 \mu_0$  and if you took the reciprocal, it will simply turn out to be  $c^2$  which is nothing but  $9 \times 10^{16}$  meter per second square; because  $c$  is nothing but  $3 \times 10^8$  meter per second squared.

In other words, what we have gone the, is to get into a situation what we have done is to get into a situation which is somewhat sticky. Because we said we want to study electro dynamic phenomena move on to discuss electro dynamic phenomena in terms of absorbable. But then you have ended up in a situation where either of  $\epsilon_0$  or  $\mu_0$  can be specified, but they cannot be measured independent of each other.

Now, if you look at the book like Berkeley physics there is a very beautiful discuss on this, where Purcell very clearly mentions that  $\mu_0 = 4\pi \times 10^{-7}$  is a definition. Please notice that this is a definition, it has no sacrosanct status; heat is not an observable. Speed of light can indeed be measured; due to the speed of light I will define  $\mu_0$  to be  $4\pi \times 10^{-7}$ ,

once you give me this definition epsilon naught get automatically fixed. I already know ampere, I already know the currents in units of ampere; therefore, I can deduce what my coulomb is and therefore, I have all measurements in electrodynamics, this is the situation. Now, you might wonder why we are getting into such a complicated situation. Well, we did not take the elementary view point that all charges are built out of protons and electrons, the reasons are historical.

When electricity was first discovered or coulomb faraday and Cavendish made their famous experiments, atomic nature of matter was not known; people did not know that there are electrons and protons. On the other hand, there was a belief that there was an all pervading medium called Ether which is supportive, which is going to support the electric and the magnetic fields. And people believed that this ether medium has elastic properties just as air has bulk modulus it has density so hence and so forth.

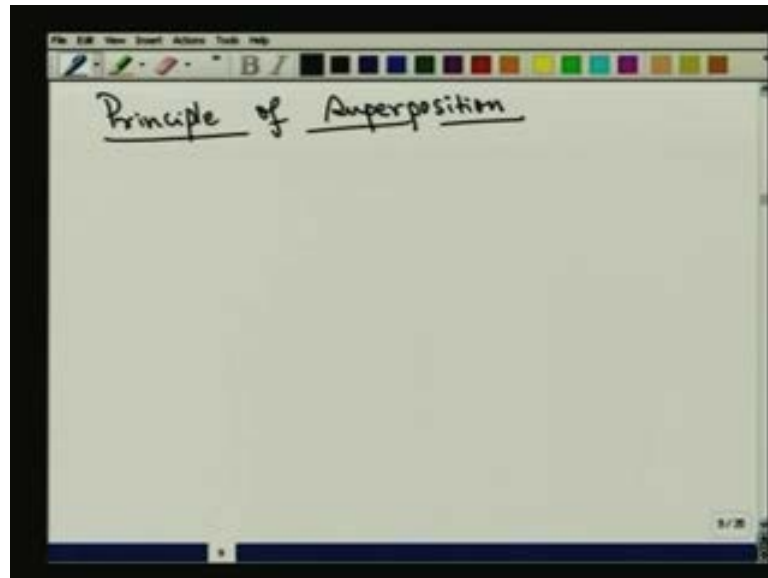
In a similar manner people believed that ether also has elastic properties and in order to describe those elastic properties, although they could not themselves measure those properties at that particular time; they introduce quantities like epsilon naught and mu naught. That is Maxwell hope that future experiment will fix the values of mu naught and epsilon naught. Today we know better, we know that there is no medium called ether; therefore, today it is largely a matter of convention that we still flick to a figure, stick to mu naught and epsilon naught.

Now, when I am saying that it is a convention, I should not (( )) this convention, because we know that S I units are eminently practical units, is that ok? Whatever we measure in S I unit is immediately related to whatever you see in your day to day life. For example, if you look at the current which is flowing through a wire in your home, you have 10 ampere plug, 20 ampere plug, 5 ampere plug so on and so forth.

And, if I were to try to measure the same thing in terms of electron units, it would have been an enormous number; I would have say I had to say something like 10 to the power of 20 electrons are flowing which is an infinity number. Therefore, although it might appear from the view point of physics that mu naught and epsilon naught are somewhat artificial, they are eminently practical; therefore, what we shall do is to continue to use them. But then we shall not forget that their practicality does not invest, does not give epsilon naught and mu naught any physical significance. And at this point let us continue

to study the other properties of the electrostatic interaction; but I will come back to this particular aspect when I discuss electromagnetic waves at a later time.

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So, you have been able to sort of get a feeling for the charges, for the epsilon naught, for pi is a number, distance is a measurable object; therefore, we can now say that we have a good appreciation, a fairly good appreciation of what coulomb force is. But then we proceed to ask the next question, what happens if I have not two charges, but more than two charges? That is going to lead us to what is called as principle of superposition. It is indeed a very, very important concept and therefore, we shall postpone a discussion of this principle to the next lecture.