Engineering Physics – II

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Module No. # 02
Lecture No. # 01

Welcome all of you. In the last few lectures we have been trained to build the essential mathematical background that is required for us to study electric and magnetic phenomena. Today what I propose to do is to start the discussion of electrostatics beginning with the coulomb law; however, before I do that it is a nice juncture for us to actually recall and summarize all the important defects that you have obtained. So far, I should remind you, I should tell you that it is not that we have built all the required mathematical apparatus we have built the essential mathematical apparatus, and whatever is required for later use will be developed as we proceed when we discuss electric, and magnetic phenomena. I started with the notion of the coordinate system, and established the properties of three important coordinate systems.

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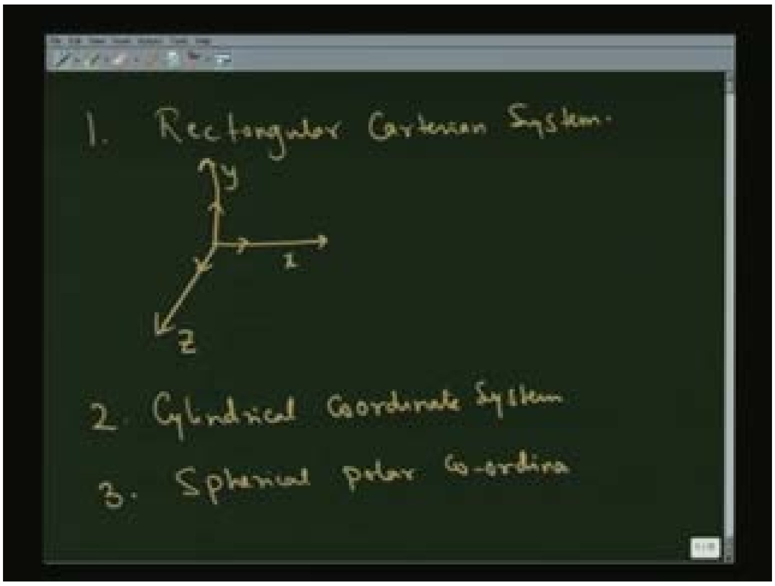

The first one is rectangular Cartesian coordinate system. Rectangular Cartesian coordinate system which is as ever describe in terms of three orthogonal coordinates bases which we denoted by the x axis the y axis, and the z axis with the unit vectors in the respective directions. This is the simplest of the coordinate systems, and then we went on to generalize this to discuss the properties of what I call as cylindrical coordinate system. Sometimes it is also called cylindrical polar coordinate system. Then we went on to discuss a slightly more complicated example, but the very very useful concept, and that is of spherical polar coordinates spherical polar coordinates. All these 3 coordinate systems are very important for us and therefore, I would urge you to solve as many problems as you can in making the transformation of the vectors, and the scalars from one coordinate system to another.

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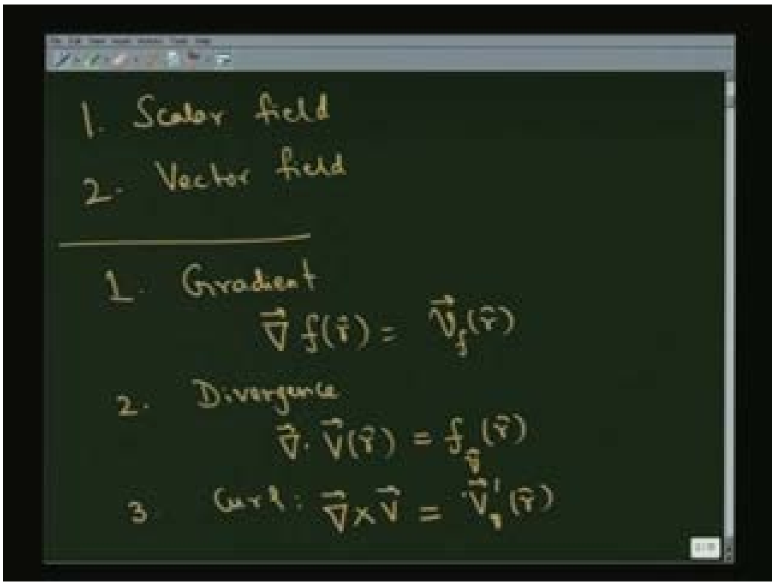

After having done that I went on to discuss 2 very important concepts namely that of a scalar field, and a vector field. As I told you scalar field occurs in the form of the scalar potential. Whose gradient is given by the electric field, and the vector fields occur, because the electrical magnetic field themselves are vector fields after having introduced these 2 fields which we will be dealing with throughout the course. I introduced three very important operations, and they are the following. The first one is the gradient which then acting on a scalar field produces a vector field. So, what do you mean by that. If you give me a scalar field f which is the function of r then I operate the gradient on it even a vector field which I shall denote by V f of r. This is a vector field generated by the scalar field by the action of the gradient.

Next I introduce the concept of a divergence. So, what does it tells you the divergence allows you to construct a scalar field given a vector field by acting as an inner product. So, what do we do now? You give me your vector field any arbitrary vector field I am going to do it in the fields of a vector inner product, and that produces a scalar field which I should denote by f of V of r. This is a scalar field generated by V.

The last important concept that I introduce was that of a curl. So, what does the curl tell you? Curl allows us you to construct one vector field given another vector field. So, how shall be denote that denoted the cross product of this gradient operator the del operator, and then with the vector field and what do we get we get yet another vector field which I shall denote as v prime of V of r.

This vector field V prime is generated by the original vector field V. All these three are very, very important, and we spend considerable time discussing the physical meaning the geometric meaning of each of these quantities, the gradient that tells you the directional magnitude of the slope at that particular point. Divergence tells you whether or not there are sources for the field at that particular point, and the curl tells you whether there is a certain rotation or a curliness associated with the vector field. At this point it is also good for us to remember that when I construct the curl of a vector field. I have 2 classes of vector field either I have the polar vector field or an axial vector field when the curl operates on a polar vector field it gives me an axial vector field, and when it acts on it axial vector field it gives me the polar vector field. That is something that we have to be remember.

At some stage we will introduce what is called as a vector potential which is a polar vector field. When the curl operates on it what do I get I get the magnetic field which is an axial vector on the other hand when I operate curl on the magnetic field itself then what do I get I get the current density, and the current density is a polar vector field. So, curl has the property of changing the nature of the vector field with respect to coordinate inversion or reflection. That is something that we have to be remember having done that we now, move on to recapitulate all the important theorems that we stated involving the divergence, involving the curl, and involving the gradient.

These are indeed very, very important, and they would develop together as electro dynamics develop in the seventeenth, and the eighteenth centuries.

In fact, people invented these theorems in order to understand the electro dynamic phenomenon. Therefore, it is worthwhile spending from time although

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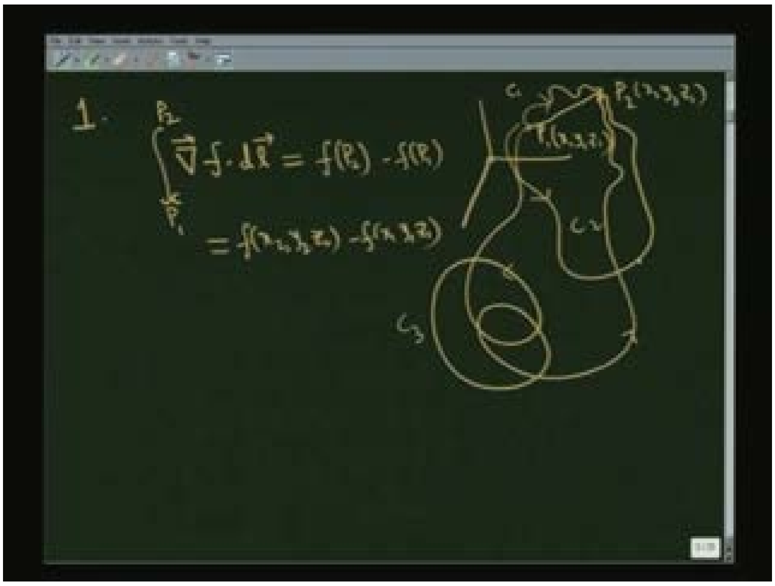

We have spent several lectures discussing that. The first one in was the gradient suppose I have a scalar field f and I construct a vector field by operating gradient on this. Now, let me take a coordinate system here, and let me say that I have a point P 1, and the point P 2. The point P 1 is denoted by x 1 ,y 1, z 1 these are the coordinate for the point p 1. x 2, y 2, z 2 are the coordinate of the point p 2. I know which to move from the point P 1toP 2 in any manner what. So, ever remember I have chosen these 2 points arbitral, and all over this place I have defined the scalar field f of course, once I operate the gradient on it I have a vector field.

Now, suppose I take a path like this which I shall denote by c 1. I will take it to another path which goes like this which I have denote by c 2, and moving from P 1 to P 2. I can take a turn away path which actually curls like this and goes comes back, and terminated tube by P 2. I can what I can do is evaluate this gradient f along this particular path as I move from the point p 1 to the point P 2. How do I do that I take the gradient, and dot it with the d l which is defined along this particular curve. So, what do I do I do it and integrate from the point P 1, and P 2.

I as it stands it appears that this equation is meaningless, because I have given. So, many return. So, may curls here I have a c 1 here, I have a c 2 here, I have a c 3 here. I can make as complicated a curve as I want therefore, I should also specify the curve along which I am going to integrate.

So, in order to do that I shall put a symbol c here, so I am integrating the gradient f dot d l along the particular curve c from a point P 1to P 2 c could be c 1 or c 2 or c 3 or any other curve that we want. Now, the remarkable result is that this object is simply given by the difference of the scalar field f between P 1, and P 2. So, how do I do that this is a scalar object notice I will simply write it as f of P 2 minus f of P 1.

Of course if you want to express this in terms of the coordinate, because I erected the coordinate system here I will write it as f of x 2, y 2, z 2 minus f of x 1, y 1, z 1; in other words what we have is the path independence of this particular scalar field. So, for an example, if I were to start with this point P 1, and P 2 the simplest way to integrate this might be actually, draw a straight line and integrate along with this and not make any complications of integrating whether very, very involved and convoluted curves that is the statements. This is not an unfamiliarly result for you, because even a mechanics you studied this when you introduce the concept of your potential and you know in a conservative force field the work done is independent of the past that is taken that is what the summarizes.

And eventually, we are going to write the electric field in terms of the gradient of a scalar potential as I told you, but is possible when the charges are static when there is no current which is varying which time or there is no magnetic field which varying in time. In that case it collapses to the examples that you have worked out in your mechanics, and this is a obvious physical meaning. It does not matter how we are moved from a point P 1 to P 2 the work then is they say is simply the difference between the energies associated with the point P 2, and associated with the point P 1. So, that is what we have in this particular result I have not proved this result. You must have proved that in your mechanics course, but anyway please go back, and recapitulate this, because that is something that we are going to use again, and again.

We shall now, move on to state 2 more theorems involving the divergence, and the curl that is sometimes the you are not familiar with your mechanics course, because there they do not deal with fluids.

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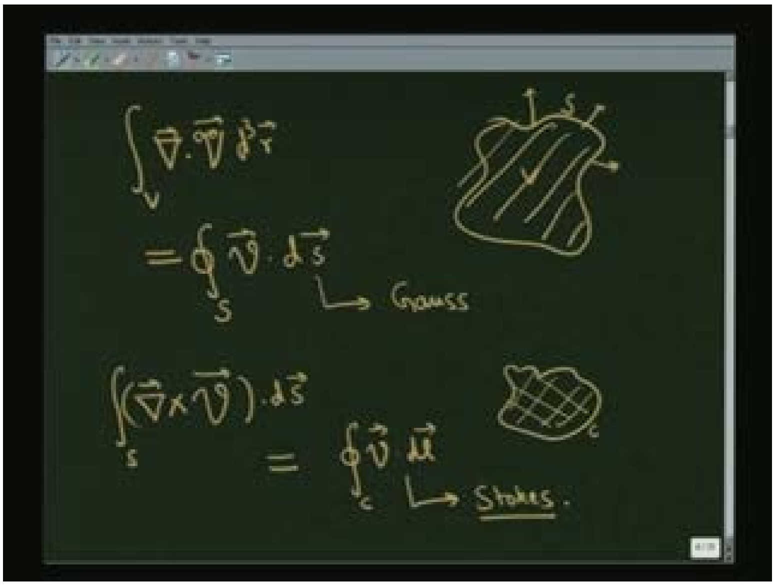

So, I spend quite some time discussing them, now let me summarize them by stating the results. What I do now, do is to consider a vector field v, and now, I consider not the integral of this few with respect to d l, but I take it divergence. Remember the divergence of the vector field is the scalar field. What I do I do with this divergence I shall now, integrated over certain volume. So, I will write it as a certain volume d cube bar which will be indicated by the symbol V here.

So, in order to avoid I will make it to curly V here, and this is volume element. Now, the result that there is a certain volume element, so this is a certain region in space this is my volume which I am going to integrate. I notice this volume is; obviously, bounded by a surface s. So, here I have a volume V which is the interior of the region which is bounded by the surface s. Now obviously, what I am going to state is the celebrated goes divergence theorem. So, what does it say? This tells you that this volume integral which is the three-dimensional integral where I am going to some more of divergence over all infinitesimal volumes, and add them up a simply given by a surface integral. So, I write the surface here, and write V dot d s.

That is what I do? Now, I have employed this rotation circle around this integral to remind you to emphasize that this surface is a bonded surface it is not an open surface. Any volume in any given region is bounded by a surface, and that surface is; obviously, not an open surface it is a cross surface.

Imagine a balloon for an example as you keep on blowing it at some points you may even like to press it there is always a certain volume which is bounded by the surface namely the balloon itself. So, what we are saying is that give me a vector field v what you can do is to integrate over the whole volume are equivalently divergence; obviously, you can integrate the whole volume or equivalently, simply take the vector field do not bother to evaluate it divergence dotted with the surface element, and integrate over the surface, and that is what you are going to get. Please remember when I write this d s I always mean the normal at a given point that is a convention, and we should not violet that. This is the famous gauss divergence theorem which we should use to fullest extend in study electrostatics.

The last result which I would like to recapitulate is what is called a stokes theorem. So, what does stokes theorem tells you? Stokes theorem always says to write a integral as a line integral that is the beauty of that, and obviously, it will involve the curl. So, again I will consider a vector field V denoted in this particular fashion. Now, what we shall do is to construct another vector field by operating curl on this what shall I do now? I shall low integrated not over a line not over a volume, but over open surface.

Let me make it precise. So, I am going to integrate it as an open surface and I will write it as a d s. So, what do you mean by an open surface a very simple way of imagining is to imagine this give you a certain loop of the boundary of an membrane. This is a line a curve c this curve c if you are enclosing this particular membrane which I am shading is a 2 dimensional surface from by the boundary which is this curve what I can now, do is to take this membrane, and stretch it out or stretch it in any way I want whatever I might do I am generating an open surface. It is open, because it is bounded by this curve c.

Now, I can make this curve c smaller, and smaller, and smaller, and then the limit that the curve c reduces to a point. It has no extension at all then it becomes a close surface in that in that case it will be enclosing the volume, but that is not what I am been doing.

I will always keep the curve c, and it shall be finite; however, small it may be, and we are going to evaluate the curl V at every point of the surface, and you do this surface integral. Stokes theorem tells you that this operation is equivalent to it can be replace by a line integral which can be written as integral V dot d l.

Obviously what bounds an open surface is a closed curve it can on be a open curve therefore, in order to emphasize that what I will do is I will put a circle here and I will put a c here. These indeed are very, very important theorem for us which is the reason why this result goes by the name of gauss. Who is son of the greatest mathematician he is called the prince of the mathematician, and this results goes by the name of stokes. The same stokes who studied the this at great extent. These are the results that we need.

Please remember that these are the very, very important theorems, and you should work out as many examples as you can haven't done this it is no time for us to really study the coulomb law. What I will first do is to state the law as you are all familiar with then I will look at each of these terms try to understand their physical significance. What their meaning is what their physical import is, and what the current understandings of these objects are coulomb introduced it in some time in 16 or the 17 century.

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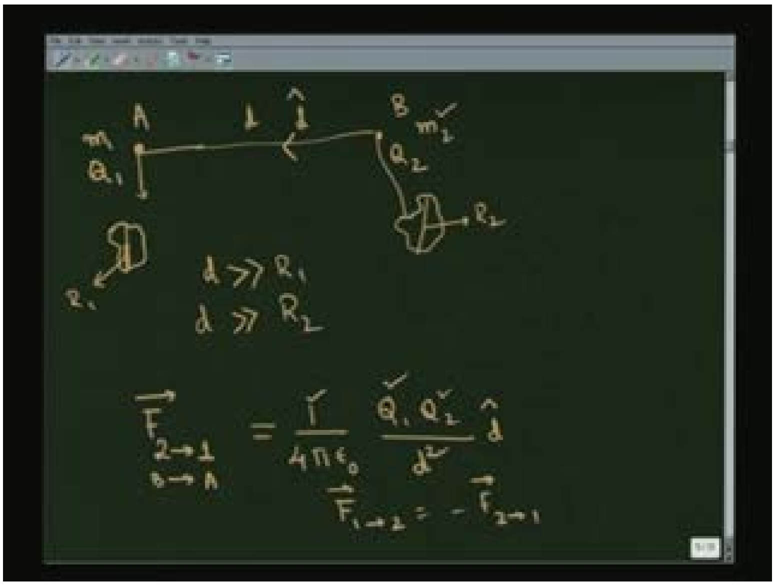

Today we are in a twenty first century; therefore, it is good to ask what is it that we move about the charges starting from the time of coulomb. So, let us start with that all of your familiar that suppose I am giving 2 bodies. So, let me call this as body A, and let me call this as a body B. Now, let us say that this body has a mass m 1, and this body has a mass m 2. Now, what I was to say is that? Apart from processing a mass it can also process another attribute, another quality. Another quantity called the charge which I shall denote by Q 1here, and Q 2 here. These chargers are produce for example, by.

These are the first experiment, and on a drive winter evening or an winter day if you are wearing synthetic clothes, and if you walk on a carpet you know that you pick up an enormous amount of static charge. We are all familiar with this if you are wearing a woolen sweater, and if you are walking on a carpet to pick up. So, much charge that if you try to remove a sweater you will see the crackling sound that comes sometimes you may even see sparks. So, so much of charge is there that it can even discharge. This is precisely the origin of the given electricity.

So, intuitively by experience although they do not know how to describe it quantitatively at this point, we know that there is some object called charge, and I am going to denote it the that quantity of charge by Q 1 for the object A, and Q 2 from the object B, and this point you know that object A will have a certain size, object B will have a certain size, that object A will have a certain shape, object B will have a certain shape, we shall ignore them. What we shall do is to assume that these are far apart. So, what do I mean by that? If I want to draw these lines, and denote this distance d. Now, let me blow this up.

So, suppose this object has a size like this this object has a size like this let us say. Now, let me say that the maximum point to point distance. The maximum distance between 1 point on A, 1 point on B is denoted by some R 1, and similarly it is denoted by R 2. We are making a statement that d is much much greater than R 1 d is much much greater than R 2. So, if I say that d is much much greater than R 1 this distance between A, and B is much much greater than R 2. We can ignore to the first all that the fact that A has a size it has a shape B has shape it has a size in that case the only relevant parameter. The only relevant distance K is given by the quantity d which measures the average distance between the body A, and the body B. That is something that we have to remember, otherwise coulomb's law is not easy to state.

Later actually, we are going to spend time discussing what happens if the finite extension of A is taken it or gone, the precise shape of A is taken it or gone, the finite extension of B is taken it or gone, the precise shape of B is is taken it or gone. That will give rise to what you call as a dipole field, quadruple field, so on and so forth. So, in order to be precise in order to know exactly what we mean when we say something. We shall say that we are ignoring the size, and the shape of the bodies A, and B.

In fact, that is precisely what coulomb did when it did the experiments. If we did that the force of interaction between A, and B is given by the following result. Now, in order to be even more careful let me ask what is the force exited by the body B on the body A. Therefore, since I am into it in employing the rotation into m 1, and 2 what I shall do is to write F 2 acting on one. If you want to write you can even say B acting on A force is a vector quantity. So, let me put a vector sign over that. So, what am I saying I am saying that I am interested in finding out the force exerted by the body B on the body A.

So, if I want to do that? This object is given by the coulomb law, and this tells you that f is nothing but one over four point epsilon naught. This tells because of a very peculiar choice of units about future you will have to say something at a latest stage, but right now, let us accept this one I have the charge Q, 1 then I have the charge Q 2, then I have the distance d square, and the next direction is that it is in the direction from B to A. So, what shall I do about that? This is what the direction. So, at this particular point since I have not erected the coordinate system if I denote this by the unit vector hat d had I will write it as a d hat, and measuring the direction from B to A of course, there is a reciprocity, and what does the reciprocity tell you.

If I am interested in the source acting on the charge two by charge 1 over 4 phi epsilon naught remains, Q 1 remains, Q 2 remains, d square remains, d will be replaced by minus d. In that case the magnitude of the force is different, but we shall now, add from the direction of A to B that is what coulomb law says. Imagine this I should hear little bit of this particular point although it is not often emphasize. When I say that? I want to measure the force acted upon by B on A actually, I should imagine that B is a very, very massive object. Speaking of at a two charges, and put them together A is going to act on B, B is going to act on A. As I told you F 1 acting on 2 is minus F 2 acting on 1.

That is what I say d will be replaced by minus d. Therefore, there is nobody to stop from 1 acting on 2 as much as 2 is acting on 1; however, if I just want to measure the force acted upon by 2 as an action on 1 I should ensure that B shall not mu that is very important. The forces are the same it is not under our control because the charge Q 1, and Q 2 have been given the only way we can be control is to ensure that these masses are different.

So, what do I mean by saying that? Let me make it more precise. If I am interested in measuring F 2 by into one I will make m 2 much, much greater than m 1. So, that in the first approximation in the 0 farad order I can take m two to be infinitely heavy. In that case F 1 2 is of course, equals to minus F 2 1, but the acceleration suffered by the 2 the body name 2 is. So, small that we can ignore its motion, and therefore, only the body A sees the force acted on it by B. So, if m two is much, much greater than m 1 I can write this objective relation to be m 1 a into Newtonian approximation I can calculate the acceleration a, and verify the coulomb result.

In a similar manner if I want to find out what is the force on B, because of A. So, I am interested in this on B because of A. what do I do? I go to the opposite result given m 1 much, much greater than m 2 then what happens if F 2 1 is what I shall write F 1 going to 2 will now, be given by m 1 a prime let us say I can the acceleration a prime, and I can verify coulomb law. There is 1 more point of care that we have to take it is not simply a statement as a formula when it come to coulomb law because it is a fundamental of electro dynamics, and that is the following. Suppose I consider the action of B on A, and A starts moving in a some particular fashion.

Now, if I really want to measure coulomb law I should ensure that the velocity suffered by a change into velocity of a should be very very small. So, what do I mean by that this acceleration should be small that is very very important. If we forget that then we will not be able to verify coulomb law. In fact, the best way to verify coulomb law is through the so called null experiments.

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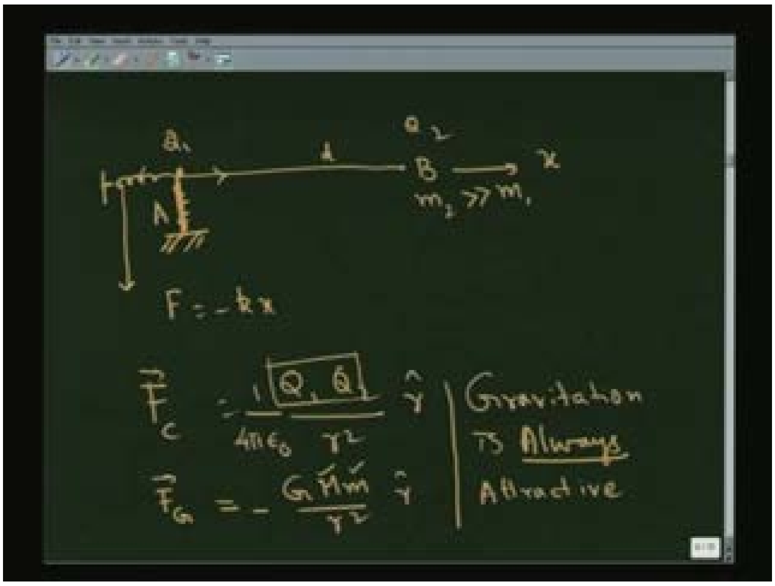

So, let us say that this is attached to a spring. So, let me denote it in the next page let us say I have the body A, I have the body B. So, these are very very massive object. So, I say m 2 is much much greater than m 1, and then there is this a this carries a charge Q 1 , this carries a charge Q 2 the distance between them is d. I want to find the force acting on A. what shall I do? What I shall do is to connect it to the spring, and leave it here instead of letting it move.

Now, when I am speaking the spring should be in this direction. So, let me draw it here because it is along the line joining the 2 charges. So, let me say that it is rigidly held at this particular point. So, that it cannot move vertically. Now, because of the force what happens Q 1 moves either in this directions or in this direction. In which direction it moves depends on the relative sign of Q 1, and Q 2 to which I will come in a minute what happens there is been to be either a in a minute what happens. There is going to be either a compression or an elongation of the spring, and we know all about the loss about the spring. F is simply given by minus k x if I take this to be the x axis. So, the force of compression or elongation depending on whether the forces is attracted or if I say the exactly minus the Coulomb force then we have null experiment which causes a displacement, but no acceleration in the final stage. So, we get an operational definition of measuring the force between 2 abstract quantities called charges which we have not yet quantified.

In this lecture I am not going to work out any problem or state any results most of the time we will spend mainly on analyzing the meaning of each of these terms. So, therefore, let me return to the coulomb law again let me write it down. F coulomb is given by Q 1, Q 2 by r square r hat that is what I am writing the meaning of this object is clear for sake of units I should introduce this quantity 1 by 4 phi epsilon naught. What I shall now, do is to steer it this expression, and ask if their anything familiar that looks like this, and the answer is obviously, yes, because this expression gives an uncanny resemblance to yet another very, very well known law which you have studied in great detail in a dynamic course in your mechanics course.

And what is that gravitational force which tells you that this is nothing but G M m by r square r hat. I should be careful I put a minus sign here to emphasize that gravitational force is always attractive. Appearances are defective by looking at this we might see that coulomb law is no different no more complicated or no less complicated than the gravitational law, and universal of gravitational is enormously successful.

We can understand the tides we can understand the falling apple, we can understand the motion of the move around the earth, we can understand the motion of the sun around earth around the sun, and so on and so forth. So, may be all that we have to do is go back revise whatever we learnt in mechanic come back, and repeat those exercises using the coulomb law, because all the mathematical techniques should be the same, because the loss are different all that I have done is to replace g by 1 by 4 phi epsilon naught capital m by let us say Q 1.

Small m by let us say Q 2, and r square let us say r hat is there, but as I told you the appearances are defective because there are also lots, and lots of dissimilarities which makes electromagnetism very, very different from gravitation. Whereas, gravitation binds the planetary system it holds a galaxy together, it hold clusters of galaxies together electro dynamic cannot do that. We cannot imagine electric electrostatic force holding the earth, and the moon. But on the other hand electromagnetism holds a molecule together, it holds an atom together, it holds clusters of molecules together, which cannot be accomplished by what it cannot be accomplished by gravitational force, and therefore, we should ask what it that makes electrodynamics is.

So, very different from gravitational phenomena although they loss appear to be the same, and these are indeed deep issues therefore, let us spend some time trying to appreciate what they are really mean. The first point of departure is in the very nature of Q. I put a sign minus here what does it mean I said that it is always an attractive force which is the reason why we always say masses are always positive. So, for the time being let me say that I will call these masses as gravitational charges in analogy with what in analogy with the electro static charges. What is the property of the gravitational charge? Gravitational charges are always attractive.

So, they all belong to 1 single species that is what it says. So, let me write it in big words gravitational force is always attractive. So, let me write them in big words gravitation is always to be underlined attractive, and here the electrostatic charges in contrast to the gravitational charges behave differently.

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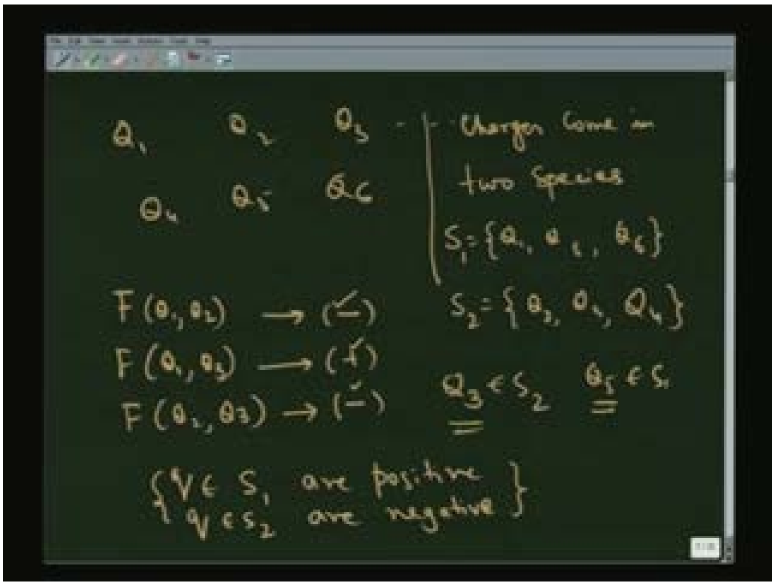

How do they behave differently? Let us ask that in order to look at that what I shall do is to collect a large number of charges. So, I have a body 1, 2, 3, 4 etcetera. Caring a charge Q 1, Q 2, Q 3 etcetera I have a large number of charges. So, I have a bag of charges if you feel like. Now, what I shall do is to take those pair wise, and start measuring the force by making use of coulomb's law.

So, what shall I do let me take Q 1, and Q 2 first, and the measure the force between Q 1, and Q 2. I am not interested in whether Q 1is act in on Q 2 or Q 2 is act in on Q 1 is ask whether these forces attractive or repulsive. That is whether the 2 charges tend to move towards each other or tend to move away from each other. I shall employ a notation I will say that if Q 1, and Q 2 is tend to move towards each other I will employ this in minus, because the distance between them starts decreasing if the charges tend to move away from each other I shall denote it by the sign plus I can make a table.

Now, let me look at for example, f of Q 1, and Q 3 it might. So, happen that they tend to move towards each other or they tend to move away from each other that is either it can be repulsive or attractive. So, let us say Q 1,and Q 3 move away from each other, and I shall denote it by plus. Now, a good question to ask is what would happen? If I consider the charges Q 2, and Q 3. What is it? That I can say about that mind you I am telling you that Q 1, and Q 2 are attracting to each other Q 1, and Q 3 are repelling each other.

Now, it turns out that if I look at the interaction between Q 2, and Q 3 it is always attractive. This is a universal law, and that is contained in the coulomb law. So, what are you saying? We are saying that there are two species of charges. So, there is some 1 species of charge if I put them together they all repel among themselves they repel each other, then there is another species of charges also which also repel each other, but to pick up one species, 1 element from a species A. Another charge from a species B they will attract each other.

So, the best way to summarize them by employing this particular notation is to say that some charges are positive, some charges are negative. So, what do I mean by that? I say that charges come in 2 species. So, I will denote them by 2 sets. So, there will be let us say Q 1, Q 5, Q 6 in this particular example, and S 2 this given by Q 2, Q 3, Q 4 let us look at that. What do I mean by saying that the charges come in 2 species. Every element in S 1 is going to buy get repelled by the other fellow. Q 1is repelled by Q 5, Q 1 is repelled by Q 6, Q 5 is repelled by Q 6.

Every element in S 2 is repelled by another charge in its own same set Q 2, and Q 3 repel each other, Q 3, and Q 4 repel each other, Q 2, and Q 4 repel each other, but pick up any charge from the set S 2 pick up any charge from the set S 1. So, let us say I pick up charge Q 3 from S 2, I pick up the charge Q 5 from S 1, Q 3, and Q 5 are going to attract each other.

That is the statement. When we say that charges come into different species. So, we need a convention in order to distinguish these 2 species; therefore, by this time you should be familiar you should be sort of striking all of you that this is nothing but the combination the multiplication of numbers plus 1, and minus 1 plus 1 into plus 1 is plus one minus 1 into minus 1 is plus minus 1, and plus into minus is minus.

Therefore, arbitrarily by choosing some convention you shall say that one of the set has positive charges the other set has negative charges. So, let us say all charges q belong to S 1 are positive, and all charges belong to set S 2 are negative. This is a convention that we need to employ, and what we now do is to say that the charge carried by a fundamental particle the electron is negative whereas, the charge carried by yet another fundamental particle the proton is positive that is the convention that we are going to employ.

So, charges come in two different species unlike masses which are always attractive there is no concept of a repulsion between 2 different masses, and therefore, science of the masses. This is not the end of the story there is yet another very, very deep difference between the charges at the mass for which we have to remember a little bit of relativity the special theory of relativity.

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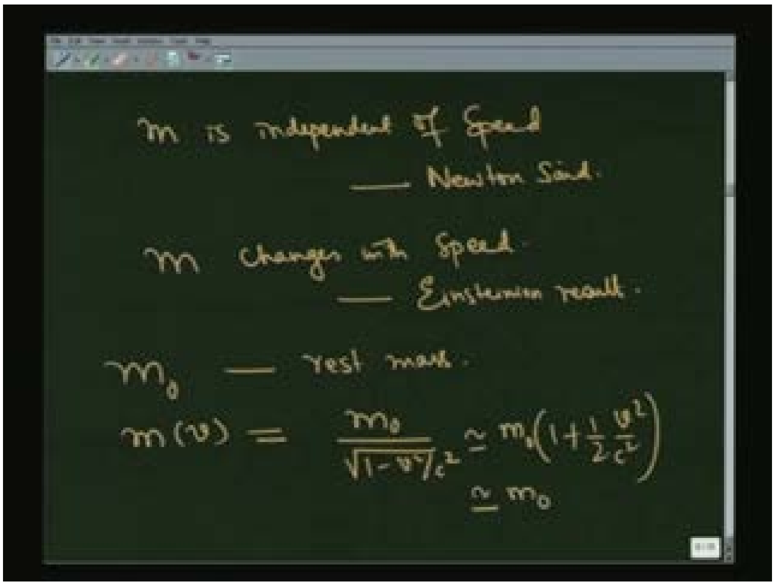

In Newtonian mechanics we know that mass is independent of speed of speed. That is one mister Newton said, nothing wrong in this because this is an excellent approximation when these speeds of the objects are very, very small. However even if you people have not studied relativity. All of you are familiar with the result with the fact that mass changes its speed. This is the famous einsteinium result. In fact, suppose an object is at rest, and this is a mass m naught. So, I will denote it by rest mass suppose it starts moving with a certain speed v.

So, now, I ask what is the mass when it starts moving with a speed v what is the formula that we have that is simply given by m naught divided by root of 1 minus v square by c square where c is the speed of light. So, you can easily see that m v equal to 0 it is nothing but the rest mass, but as v approaches c the denominator goes to 0, and the mass goes to infinity the inertia become very, very large.

In other words the mass of an object is not independent of the state of its motion. That is very, very important for us to know, and it actually, goes to infinity of course, if you have a very, very small velocity, who can make a binomial expansion, and we can write it as m naught into 1 plus half v square by c square. c is an enormous number three into ten to the power of eight meters per second even the earth which is going around the sun with an enormous speed 30 kilometers per second is nothing compared to the speed of light is that therefore, then the 0 approximation is nothing but m.

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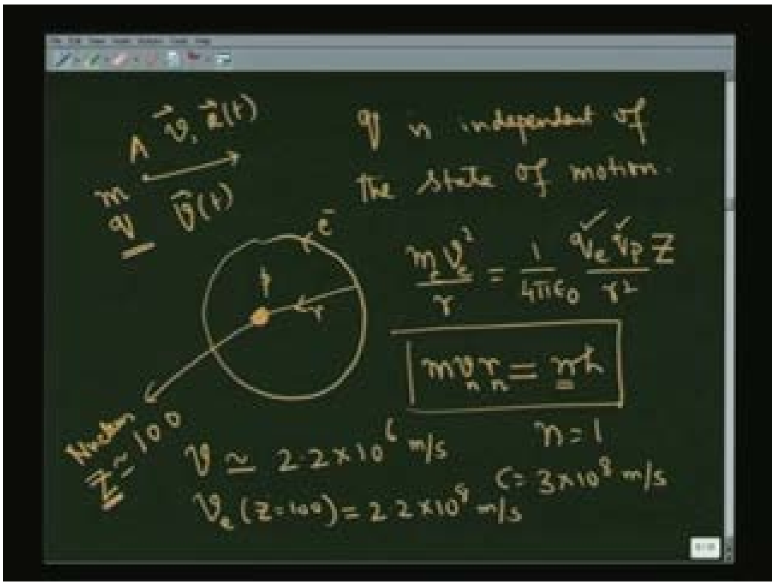

But if you got large large speed very very large velocities then we certainly going to change. What happen when it comes to charge. So, let us imagine that there is a certain body A having a certain speed, v moving in some given direction. It might be in a gravitational force or it might be attracted by a certain law which is given by hook's law. In which case not only does it have an axial velocity it also has an acceleration a. If the force is or it keeps on changing from point to point in general the acceleration is a function of time, and. So, is v a function of time. Now, this body has a mass m by that I mean there is mass m, and it has a charge q by that I mean the rest charge q, because I have to be very careful now.

We now, ask the question what happens to this charge q when this object suffers a change in its state of motion. Its starts moving with a certain speed which keeps on charging the time. The remarkable fact about the charge is that what I am going to write now, the charge is independent of this state of motion. The object may be at rest it might be hurling very fast towards something it might be going round, and round the way the electron goes around the hydrogen in the proton in the hydrogen atom. Whatever it might be its charge is something that is going to remain absolute in that sense charges are really like numbers which add up. You do not have to worry about anything else.

This is something which gloss over, but this is the most important fact that distinguishes, the electrodynamics from the gravitational law from the ordinary mass therefore, it is worthwhile asking what is the experimental evidence for this statement that charges independent of the state of motion. Today we have very, very great experimental evidence coming from accelerator physics. To which I will come in a short while, but even with your knowledge of boar model it is actually, possible to get a heat this is not a rigorous kind of argument that I am going to give, but a very good hint, a very good intuitive feeling as to why charge must be independent of the state of motion.

In order to do that let us recall boar model. So, what does boar model tell you? Boar model tells you that there is a very massive object called the proton which is sitting at the center, and then there is an electron which is going around in a circular orbit. Of course this is what is going to happen even classically even if it did not have boar model. Except that the orbit need not the circular it can also be elliptical, but boar considers circular orbits that is what you have said let us stick to that.

The difference is that boar is going to impose a condition on the angular momentum carried by the electron. Since it is a circular orbit let us say that the radius is r. What do I do? What I do is to first of all identify, the force of electro static attraction with an equivalent centrifugal force. So, what should I do I will write m v square by r is simply nothing but 1 over four phi epsilon naught q electron, q proton divided by r square. This is an attractive centripetal force. So, what are they saying? They are saying that the electro static force of attraction which is on the right hand side is equivalent to a centripetal force.

Many times you hear, you read that this is a centrifugal force which is balanced in the electro static force, please do not commit such a mistake. All that we are doing is to identify this physical force 1 over four phi epsilon naught q e, q p by r square with a equivalent centripetal force which is also attractive that is what we shall write. This is common to both Newtonian approach of the classical approach, in the boar model approach the real boar model departure is in quantized with the angular momentum.

So, what does it say next the angular momentum is simply given by m v r the product of velocity into radius into mass since it is in a circular orbit I do not have to worry about writing cross products, and all that all of you know the meaning of this this is simply given by n h bar where h bar is the famous plan constant. Now, these forces only certain values of the radius for the electron. In an ordinary classical situation depending on the initial energy, and the initial angular momentum the electron can have any radius that it wants, but this condition which is called as the quantization condition forces only certain values of the radius. So, we shall put v n, r n to emphasis that.

So, it is a matter of very, very simple substitution now, what do we do? Take this where m v square by r eliminate v or v n into r n in this particular expression, and try to get an expression for the radius, try to get an expression for the velocity; obviously, the minimum energy will be there when n is equal to 1. So, put n equal to 1, and find the expression for the velocity. If you did that I am not going to work that out I will leave it as an assignment for you you should certainly go and work it out it turns out that the speed which reach the electron go surround the proton is of the order of 2 point 2 into 10 to the power of 6 meters per second.

This is indeed an enormous speed remember I told you that the earth goes around the sun with a speed of 30 kilometers per second here we are speaking of 2 point 2 into 10 to the power of 6 kilometers per second which is two orders of magnitude 100 times larger than whatever the speed of the earth is.

Whatever you see terrestrially is of course, very very small compare to the speed of earth around the sun therefore, you can almost forget about all these objects projectiles missiles rockets whatever it might be. So, this is what we have, but this speed is still small compare to the speed of light because c is given by 3 into 10 to the power of 8 meters per second that is what we have. Now, what I shall do is to consider north the electron going around the proton, but imagine applying boar model for a very heavy nucleus. So, let us say z is equal to 100. So, when I wrote this expression I wrote the charge of the electron charge of the proton, but that means, there is only one charge sitting that is the hydrogen atom that I had.

But suppose, I replace this proton by a nucleus all of you know from again from your twelve standard. The nuclear comes with as much charge as their protons is that, and that atomic number is denoted by the symbol z. In that case the charge carried by the nucleus is given by z q p therefore, I put a z here. So, all that I need to is to take over the expression for the energy for the forces that wrote for the hydrogen atom are multiplied by z the electron is still moving in a circular orbit. In fact, if I want to do very careful I can put an e here, and a v here to emphasis that this is for the electron now, I ask what the change in the speed of the electron is when I say that I look at a z which is much much larger than the proton.

So, what do I mean let us say that I take z is roughly 100. That is what I am going to look at if you did that you can easily verify, that this expression also get the multiplicative factor z. So, what happens? So, now, I can write v e for z equal to 100 is simply multiplying the original fellow by hundred which is equal to 2 point 2 into 10 to the power of 8 meters per second which is sort of precariously close which is dangerously close to the number 3 into 10 to the power of 8 meters per second. I have to be very careful when I am making this statement. It if you gets too close to the speed of light you cannot use non relativistic mechanics.

Getting the expression m v n or m v square by r equal to 1 over 4 phi epsilon naught q u q p by r square that might be a wrong. It does not matter it still gives us a hint as to how the speed changes. The mass changes by an enormous quantity never mind. You look at a nucleus which has z is equal to 100 or led because z is equal to 80 which is quite close to 100, and look at the hydrogen atom. Both the atoms are what both the atoms are actually, completely neutral.

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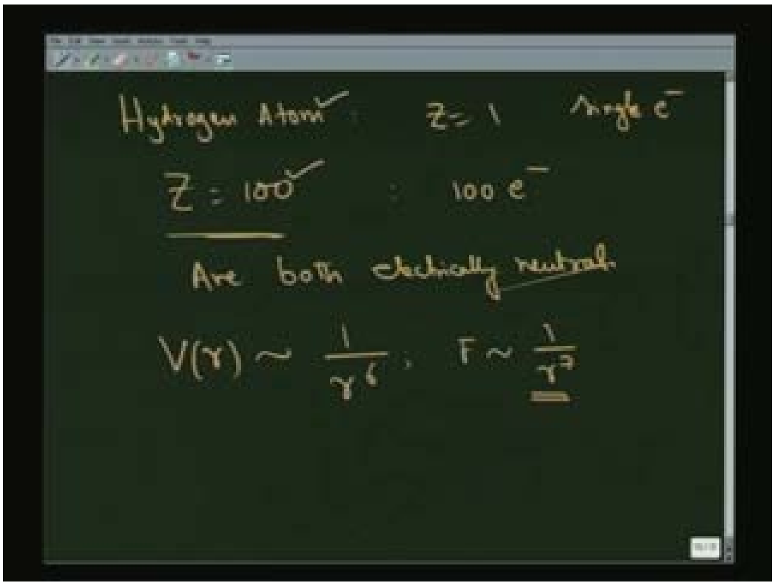

So, what do I have to say consider hydrogen atom you have z equal to 1 and a single electron electron consider a heavy nucleus for example, it could be uranium which has some to a mass number equal to 200, and 30, and atomic number very close to z.

So, consider an atom which is close to z is equal to 100. So, what is it that happens there? I have hundred electrons these 100 electrons are going to move with difference speeds according to boar. The inner most electron is going to move with a speed which is very close to speed that of light; however, it does not matter. This atom and this atom are both electrically neutral are both electrically, neutral how could it remain electrically neutral. They could remain electrically neutral, because although the charges are going to move with different speeds. Although the charges the speed of the charges depends on the orbit in which they are sitting, and also on the value of z.

The charge itself does not change, and therefore, both of them remain electrically neutral, and thus we have shown we have argued, we have hinted at the fact that this charge of an object is independent of the state of the motion. That is what we have shown. In fact, when we would go 1 step ahead, and ask what is the force of interaction between 2 atoms or 2 molecules for example, I could take 2 hydrogen atoms or it take another some other thing for example, iron or cobalt or whatever or 2 different molecules.

We know that the force always goes like 1 r to the power of 7. If I want to write down the potential energy between 2 different molecules potential goes like 1 over r to the power of 6 force goes like 1 over r to the power of 7. If there were no electrical neutrality, then there would have been a slight access of the positive charge over the negative charge or the other way round then the force would have gone like 1 over r square.

You see not only do we not see 1 over r square term. We do not see 1 over r cube, 1 over r to the power of 4, 1 over r to the power of 5 you go all the way to 1 over r to the power of 7 which again strongly hints that the net charge sitting on an atom or a molecule is 0, but it is only the electrons that are moving. The protons inside the nucleus itself is not moving therefore, you can conclude that the charges are independent to the state of motion.

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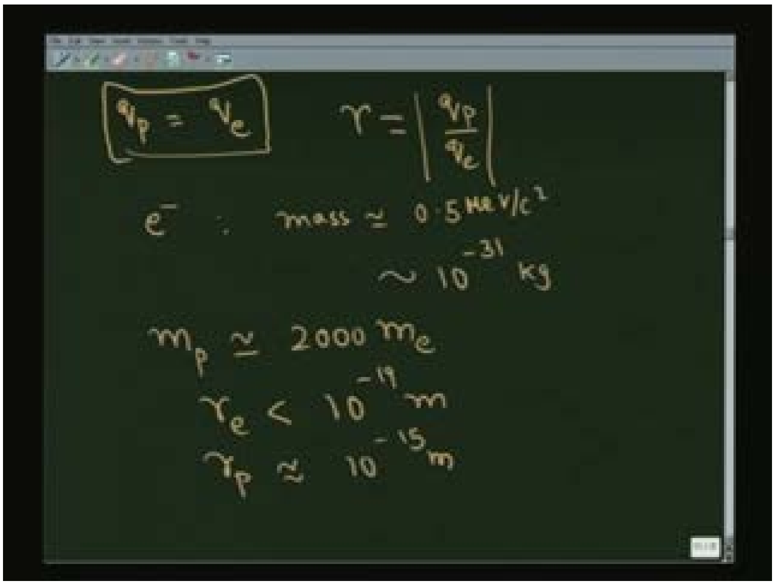

In saying this, I have actually, made use of a very very remarkable fact implicitly which I did not state explicitly. Now, it is time to state that that is one of the mysteries of nature, and we shall ask what it is, and that is when I stated this neutrality I implicitly assume that magnitude wise q p equal to q e. q p is the charge carried by the proton q e is the charge carried by the electron. Now, there is something remarkable about that what is that?

Look at a proton, and look at. An electron an electron has a mass which is given by roughly point 5 m e v by c square. If I convert it into ordinary units it is of the order of ten to the power of minus 31 kilograms that is what I have. What about a proton? Broton is roughly 2000 times heavier than the mass of the electron. So, what shall we say the mass of the proton is roughly 2000 times mass of the electron. They are two different objects. In fact, I know that electron is a point particle. It has a radius less than 10 to the power of minus 17 centimeters. So, let me write that.

So, the radius of the electron is less than 10 to the power of minus 19 meters. Whereas, the radius of the proton is of the order of 10 to the power of minus 15 meters we know that it has a finite size. Whereas, electron is a point mass. The proton is 2000 times heavier than the electron. Whereas, the electron is indeed light in spite of that there is a great great equality between q p, and q e except for the sign. If these where not. So, if the proton had carried a slightly access charge over the electron. Then my atom would not have been neutral, and I would not have been able to verify all the statements that I made in the previous page. That there is a Vander wall interaction it does not depend on the state of the motion atoms are neutral molecules are neutral, and so far so on and so forth, and this is something which has been verified to an enormous accuracy in modern day experiments in high energy physics, and let me write down the numbers for you how do the numbers look like.

So, let me construct a ratio r which is given by q p by q e. Now, q p, and q e have different signs therefore, what I shall do is to take the modulus of them, and call it as the ratio r. So, what shall we do with this now, we shall ask for the departure of this r from 1.

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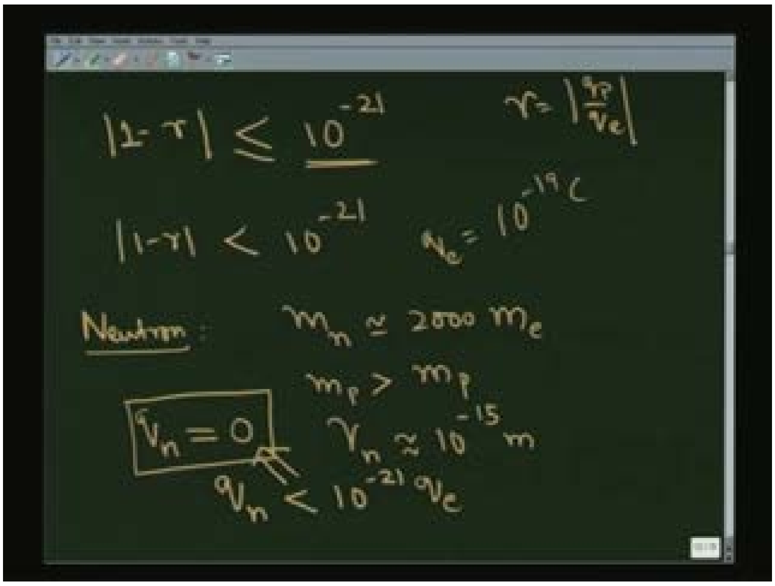

So, now, I ask what is the modulus of 1 minus r remember r is nothing but mod q p by q e. Modern day physics actually, tells you this 1 minus r is less than 10 to the power of minus 21. That is what it says? That is if you take an assembly of 10 to the power of 21 protons. If I look at an assembly of 10 to the power of 21 electrons measure the total charge there measure the total charge here you are still not able to distinguish any difference between the total charge contained in the 2 assemblies.

This is such a vanishingly, tiny number. Notice I have not even put that less equal to I have put that less than equal to. In fact, one can evenly even safely say that mod one minus r is less than 10 to the power of minus 21. May be it is less than 10 to the power of minus 25, minus 30, minus 40 future experiments field reveal that, but for our purposes we can simply say that 1 minus r is identically, equal to 0 are r is equal to 1. This In fact, is one of the surprising mysteries of nature, but it is this fact that allows you in to verify the charge neutrality of the atoms.

Now, when I am speaking of the proton, and the electron we should remember that matter is composed of yet another object namely the neutron. What about the neutron? Neutron is again a very funny object it also has a mass which is roughly 2000 times the mass of the electron. It is slightly, heavier than the proton. So, my mass of the neutron is again 2000 times the mass of the electron, and mass of the neutron is slightly greater than mass of the proton which is the reason neutron can dig into a proton you know the famous beta d k. Now, if you ask what the charge of the neutron is then it turns out that the charge of the neutron is equal to 0. That is what we find here.

I should also mention the radius of the neutron. The radius of the neutron is also again of the order of a fathometer 10 to the power of minus 15 meter therefore, it is very, very much like a proton very close to the proton in its mass very close to the proton in its radius then it when it, but when it comes to charge it is 0 it is a neutral particle which is the reason why it was called a neutron, and it was found discovered by Chadwick as you all know from your twelve standard.

What is the experimental limit on that if I express the charge of the neutron in terms of the charge of the electron it is known that q n is again less than 10 to the power of minus 21 into q e. Never mind whether you want to ask a positive charge or a negative charge it is less than 10 to the power of minus 21 times that, and therefore, we can again safely conclude that q n, q n is taken to be 0. These are indeed very, very precise limits notice these are only upper limits we are not saying that the charge of the neutron is 10 to the power of minus 21. We are saying that if at all a neutron has any charge it is less than 10 to the power of minus 21 times the charge of the electron. Therefore, we are not committing any error any approximation or any bungling if we safely said q n is equal to 0.

How do I know that q n is equal to 0. Well go back to the atoms, and look at the same atom concentrated of different isotopes. You have oxygen 16, you have oxygen 18, you have nitrogen 14, you have nitrogen 12. Now, z is the same for both of them, but the number of neutrons keeps on increasing again we know that the interaction is entirely given by the force between the proton, and the electron the neutron does not play role therefore, that gives you a hint that q n is equal to 0.

Why have I spent? So, much time discussing the proton the electron, and the neutron which occur in modern physics. The reason for that is plain, and simple, and that is the following. All matter that we know of is made of the protons, the neutrons, and the electrons. You might become a material science engineer, and electric engineer, and a electronics engineer you are going to deal with matter is that plastics, metals, whatever

whatever all of them are made up of molecules which are made of atoms, and atoms are composed of the protons, the neutrons, and the electrons.

What have I stated just now, we stated that neutron does not have any charge, and the charge of the proton, and the electron are precisely the same. With a limit given by 10 to the power of minus 21 which is vanishingly small, and whenever I say that I charged may a capacitor. There is a current flowing in a wire, there is a semiconductor which has built of a charge at the interface, because of the contact potential or whatever whenever we are saying that we are only accumulating these fundamental charges.

They might be enormous in number never mind about that there might be about suppose the total charge is 10 to the power of minus 10 coulombs. The charge of the electron is of the order of 10 to the power of minus 19 coulomb. We know that in terms of the ordinary unit. So, even if I put even if I have 10 to the power of minus 10 coulombs. I have10 to the power of 9 electrons which are sitting there .Whatever, it might be all finite chargers all finite currents are actually, built out of the basic building blocks the electrons, and what have I mean by saying that? That means, all charges are quantize in the units of the electric charge which I shall write explicitly.

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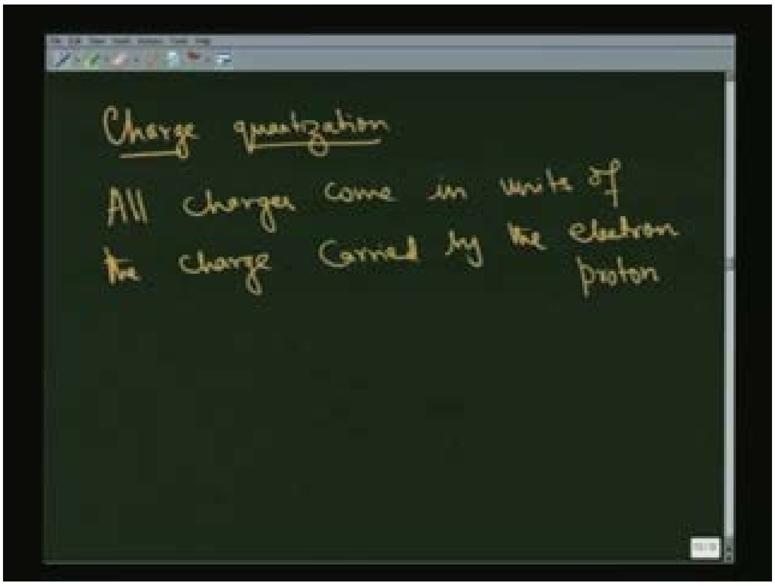

So, this is the famous charge quantization. So, what does it say all chargers come in units of of the charge carried by the electron carried by the electron. If you feel like you can also replace the electron by proton, because they are both the same except for a sign, and this remarkable thing is what is meant by charge quantization. There is yet another very important property of charge, and that is the conservation of charge that is a detailed topic which we shall take up in the next lecture.