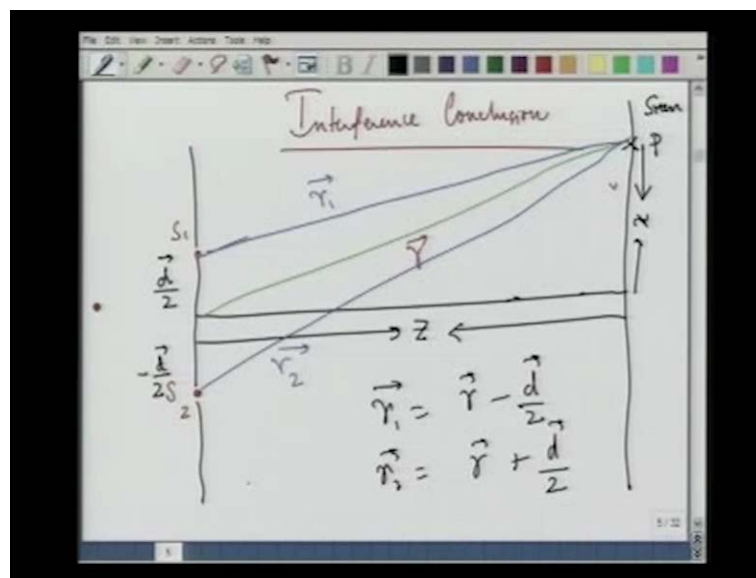


**Engineering Physics - II**  
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**Module No. 04**  
**Lecture No. 12**

So, we will conclude the discussion of interference, which we started in the previous lecture in this lecture and incidentally, we will also conclude the course with this lecture by studying or rather getting a glimpse of how interference experiments, very careful interference experiments actually lead us to the quantum theory of light, the photon concept. So, in some sense, what you are going to do is to lay down the conceptual foundations of what you call modern physics or quantum mechanics, through the so called single photon interference experiment, which cannot be understood in terms of either an ordinary **corpusus** corpuscular theory or the wave theory. And I am also going to give you experimental evidence as to how the same kind of interference pattern is seen in other systems. So, with that in mind, what I will do is to resume my discussion of the classic Young's double slit experiment.

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So, the double slit experiment's configuration is something that I wrote. So, let me very briefly summarize, what is it that I did? So, what I had was two slits. So, let me draw them here; please treat them as straight lines. So, you have the slit S 1 here, you have the slit S 2 here, grossly exaggerated. And then, this is the midpoint; the source is sitting somewhere here, and from this midpoint, you are going to draw a straight line. So, let me extend it, and somewhere, here you are going to put the screen. The dimensions are suggestive, that is the distance between the slits is small compared to the distance between the slits and the screen; so, let me indicate that here. I said that this distance is  $z$ . I am going to take a point P on the screen and let me say that this distance is  $x$ .

You have the source, a coherent source and there are two slits S 1 and S 2. They are along the  $x$  axis. This is my  $z$  axis; this is the  $x$  axis. So, let me locate the position of S 1 by the vector  $d/2$ , which is actually along the  $x$  axis and this will be minus  $d/2$ . This is my vector. And from the midpoint, I can actually draw a line, which I shall do now **and what is that?** Let me connect it to P. Let me connect it to P; let me erase this bad part of the line. Ok? And let me call this distance  $r$ . This is from the origin. The two slits are moved with respect to the origin, along the positive  $x$  axis and along the negative  $x$  axis. So, we shall indicate them through some other colored lines so that there is no confusion. So, what is it that I do? I join this two and I joined these two lines; very good. And this is what I will call as  $r_1$  and this is what I will call as  $r_2$ .

Everyone knows how to add vectors. We know how to write  $r_1$  in terms of  $r$  **in**  $d/2$ ; we know how to write  $r_2$  in terms of  $r$  **in**  $d/2$ . That is something that I wrote in my previous lecture, transparency. So, we have  $r_1$  is equal to  $r$  minus  $d/2$ . Is that right? And  $r_2$  is equal to  $r$  plus  $d/2$ . Please let us not forget our basic premise, namely,  $z$  by  $x$  is a very small number and so is  $z$  by  $d$ . So,  $z$  by  $x$  and  $z$  by  $d$  are small numbers, **s...** large numbers rather. **So,  $x$  by  $d$  not  $d$  by  $x$  much much less than 1 and  $d$  by  $z$  much much less than one;** these are the conditions that I have.

I also introduced the proper angles so that we can write down  $r_1$ ,  $r_2$ ,  $d$  and  $r$  in the... in terms of the sin and cos. So, what I do in order to do that is to draw two lines parallel. Call this angle beta and call this angle alpha. So, now you know how to write down cos alpha, sin alpha, cos beta, sin beta by simple trigonometry. And with these things, we were able to write down the expression for the total electric field. And what is the expression for the total electric field that I wrote? I wrote,  $E_1$ ,  $E$  is equal to  $E_0 \cos$ , let

me open a big bracket,  $k_1 \cdot k_1$  is the wave vector, the direction of the propagation corresponding to the ray from the slit  $S_1$ . Let me go back and show it to you. So, this is the ray from the slit  $S_1$ ,  $k_1$  is along this particular direction, dot  $r$  minus  $d$  by  $2$ ; that is what I have here at the point  $P$  minus  $\omega t$ .

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The image shows a whiteboard with handwritten mathematical expressions. At the top, the electric field  $\vec{E}(P)$  is given as the sum of two terms:  $\vec{E}_{10} \cos\left\{\vec{k}_1 \cdot \left(\vec{r} - \frac{\vec{d}}{2}\right) - \omega t\right\}$  and  $\vec{E}_{20} \cos\left\{\vec{k}_2 \cdot \left(\vec{r} + \frac{\vec{d}}{2}\right) - \omega t\right\}$ . Below this, a note states: "In the leading order when  $\frac{x}{z} \ll 1$  and  $\frac{d}{z} \ll 1$ ", followed by the approximation  $\vec{k}_1 \approx \vec{k}_2$ . The final expression for  $\vec{E}(P)$  is then simplified to  $\vec{E}_{10} \cos\left\{\vec{k} \cdot \left(\vec{r} - \frac{\vec{d}}{2}\right) - \omega t\right\} + \vec{E}_{20} \cos\left\{\vec{k} \cdot \left(\vec{r} + \frac{\vec{d}}{2}\right) - \omega t\right\}$ .

So, I am evaluating the net electric field at the point  $P$ . We should not forget that. So, one thing I can do is actually move it slightly to the left and right,  $E$  at the point  $P$  along the  $x$  axis; that is what I have here. This is from the slit one and from the slit two, I have  $E_{20} \cos k_2 \cdot r$  plus  $d$  by  $2$  minus  $\omega t$ . I am not going to write the expression for  $k_1$  and  $k_2$  in terms of  $\cos \alpha$ ,  $\sin \alpha$ ,  $\cos \beta$ ,  $\sin \beta$ , which I have derived here, defined here. There is an  $\alpha$  and there is the  $\beta$ ; there's a small difference.

But in the leading order... What is my leading order? In the leading order, when  $x$  by  $z$  much, much less than one,  $d$  by  $z$  much, much less than one. We have the screen far off. We can forget all difference between  $k_1$  and  $k_2$ . It is as if they are coming parallel to each other. We need not forget, we need not bother about the small difference coming, because of the change in the angle. Therefore, what we do is to say  $k_1$  approximately equal to  $k_2$ . Of course, the magnitude is the same. It was the direction alone that differed for the two cases, but in this limit we are going to make this assumption.

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$$I_{\text{Tot}} = \vec{E}_1 \cdot \vec{E}_2 \cos(\vec{k} \cdot \vec{d}) + I_1 + I_2$$

$$\cos(\vec{k} \cdot \vec{d}) \text{ is a maximum when}$$

$$\vec{k} \cdot \vec{d} = 2n\pi \quad n=0,1,2, \dots$$

$$\cos(\vec{k} \cdot \vec{d}) \text{ is a minimum when}$$

$$\vec{k} \cdot \vec{d} = (2n+1)\pi; \quad n=0,1,2, \dots$$

And if we did that, now what follows is a very **very** simple calculation that we have to do and, that is, let me write it down first. E of P is nothing, but E 1 0 cos, now I am going to use the same k, k dot r minus d by 2 minus omega t. This is at any given time at the point P, plus E 2 0 cos k dot r plus d by 2 minus omega. So, the change in the phase of the field coming from the first split and the second split is entirely due to the path difference. You need not worry about the change in the direction; that is what we said.

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Maxwell's eqns  $\Rightarrow$  additivity of fields:  
 The intensity is quadratic in the field strengths:  

$$I_{\text{Tot}}(P) = I_1(P) + I_2(P) + I_{12}(P)$$

$$I_{12}(P) = 2 \vec{E}_1 \cdot \vec{E}_2 \cos \left\{ \vec{k} \cdot \left( \vec{r} - \frac{\vec{d}}{2} \right) - \omega t \right\} \times \cos \left\{ \vec{k} \cdot \left( \vec{r} + \frac{\vec{d}}{2} \right) - \omega t \right\}$$

Now, given  $E$ , the point that I was emphasizing for you people, that is very, very important, is that Maxwell's equations are additive in the fields. So, Maxwell's equations imply additivity of fields; Fields add up. That is the reason why I was able to write  $E$  is equal to  $E_1$  of this plus  $E_2$  of this. By the way, I am making a distinction between  $E_1$  and  $E_2$  because suppose there is this source which is emitting the radiation. So, let me indicate it here. So, there is a ray which is coming here and there is a ray which is coming here. In principle, what you can do is to put a polarizer here. So, this is my polarizer. I can put a polarizer and can keep on changing the direction of the electric field which is the reason why we write  $E_1$  and  $E_2$  separately in this. There is no harm if they are both parallel to each other, if you did nothing to them, but in general they can also be different and such experiments are done.

Now, let me return to whatever I was telling you, Maxwell's equations imply additivity of fields. That is,  $E$  equal to  $E$  from the slit  $S_1$  and the  $E$  from the slit  $S_2$ . What is the relation between the intensity and the field? The intensity, let me go back to my original color. The intensity is quadratic in the field strength.  $\propto E^2$ ; more intense the beam, greater the energy carried by the beam. Therefore, the energy is also quadratic in amplitude. Is that ok? It is not linear in the amplitude.

So, suppose there are two rays of light as shown in this particular picture. So, this is carrying an intensity  $I_1$ . So, let me indicate that here, and this is carrying intensity  $I_2$ . If this were a stream of particles and if this were another stream of particles, the intensities would have added up, the intensities would have added up. But now, what adds up is not the intensity, but the amplitudes which are something like the square root of the intensity. More correctly, intensity is actually the magnitude squared of the amplitude. We have to first add the amplitudes and then only calculate the intensity and let us not forget that the addition of the amplitudes is vectorial. When it is vectorial, there is no reason that when I add these two together, the magnitude should actually increase. The magnitude can increase, the magnitude can decrease and that is what gives rise to the phenomenon of interference.

So, let me write down the intensity now. If I were to write the total intensity,  $I_{\text{total}}$  at the point  $P$ . The first term comes from the intensity due to the beam one, which is my beam one?  $I_1$ . The second comes from  $I_2$  and the third term comes from the super position of the two terms. So, let me group them here in this particular expression; this is going to

contribute to  $I_1$ , this is going to contribute to  $I_2$  whereas, the cross term between the two is going to contribute to what I will call as  $I_1 I_2$ .

What we are obviously interested is what this  $I_1 I_2$  is going to do because that is the new term which is there, which is the dot product of the inner product of these two terms. If the intensities had added, I would have got  $I$  equal to  $I_1$  plus  $I_2$ . So, very well, let us do that.  $I$  total of  $P$  is therefore, is given by  $I_1$  of  $P$  coming from this slit one plus  $I_2$  of  $P$  and the one will be  $I_1 I_2$  of  $P$ ; this is my pictorial way of writing.  $I_1$  of  $P$  is a constant, it is not going to matter to me.  $I_2$  of  $P$  is a constant, it is not going to matter to me. So, therefore, let me write down the cross term  $I_1 I_2$  of  $P$ . Apart from multiplicative factors epsilon naught etcetera, etcetera, let us not worry about that. You people can see that this is nothing but  $2 E_1 \cdot E_2 \cos(k \cdot r - \omega t)$ , multiplying  $\cos(k \cdot r + \omega t)$ .

Please remember that when I am writing  $I_1$  of  $P$  and  $I_2$  of  $P$ , these quantities are time averaged, these quantities are time averaged. So, what I have to do is perform a time averaging of these also. Why should I perform a time average? Let us not forget some basic facts. If I am performing an experiment in the visible spectrum, then the frequencies of the order of ten to the power of fifteen hertz, ten to the power of fourteen hertz, what do you mean by that? In one second, the electromagnetic wave oscillates ten to the power of fourteen times. Each oscillation takes ten to the power of minus fourteen seconds or ten to the power of minus fifteen seconds and my detector cannot detect that, in particular my eyes cannot detect that. The maximum efficiency of my detector is of the order of a mini second. Even if it were to be micro second, there are already ten to the power of six, ten to the power of fourteen, ten to the power of eight times the electromagnetic wave had oscillated at that point. Therefore, it is prudent to take the time average. So, we should calculate the time average; that I will indicate by these **line** angular brackets.

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$$\langle I_{12} \rangle = \vec{E}_1 \cdot \vec{E}_2 = 2 \cos \left\{ \vec{k} \cdot \left( \vec{r} - \frac{\vec{d}}{2} \right) - \omega t \right\} \times \cos \left\{ \vec{k} \cdot \left( \vec{r} + \frac{\vec{d}}{2} \right) - \omega t \right\}$$

$$\langle I_{12} \rangle = \cos(\vec{k} \cdot \vec{r} - \omega t) + \cos(\vec{k} \cdot \vec{d})$$

And let us see how to do that. So, the time averaging is what is to be performed and what is this quantity? Let me repeat it.  $2 \cos k \cdot r \text{ minus } d \text{ by } 2 \text{ minus } \omega t$  into  $\cos k \cdot r \text{ plus } d \text{ by } 2 \text{ minus } \omega t$ . How do we do the average? When we do the average, we should only keep those terms, which do not vanish, because  $\cos$  is an oscillatory function, but there will be some combination of these two functions which will not actually vanish. How do we do that? First of all let us make use of the famous formula  $2 \cos a \cos b$  is what?  $\cos$  of  $a + b$  by  $2 \cos$  of  $a - b$  by  $2$ .

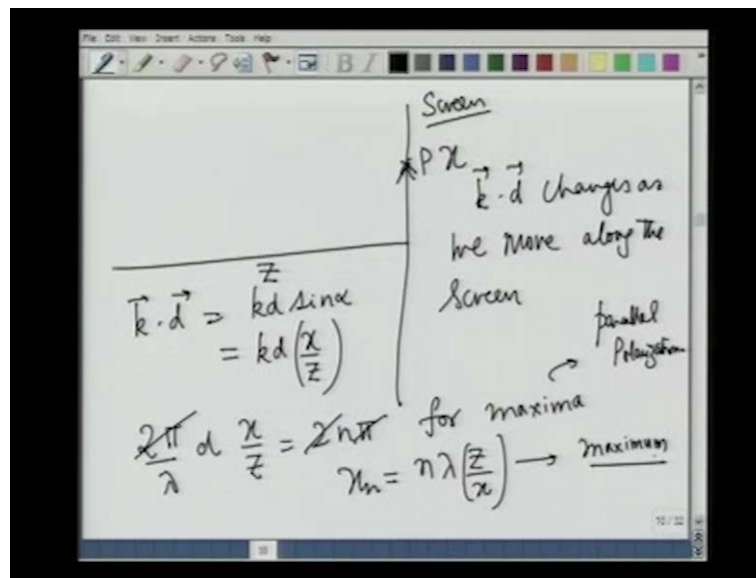
So, apart from all the other factors which I will indicate like this, what is this expression? This is nothing but  $k \cos$ ,  $\cos$  of  $k \cdot r \text{ minus } \omega t$ ; that is term number one plus. So, we are going to write product of two  $\cos$  functions as a sum of two  $\cos$  functions. The next term is the most interesting expression for us. This is nothing but  $k \cdot d$ ; this is what you are going to get. Is it ok? Now you can clearly see that my  $I_{12}$  is what I have. There is an  $E_1 \cdot E_2$ , which I will insert at this particular point.

When I perform the time average, this term gives me 0. In fact, over any period, my  $\cos$  function becomes 0. This is the only term that survives. Therefore, now we know how to write down the interference term. So, the interference term looks like  $E_1 \cdot E_2 \cos k \cdot d$ . This is my interference term and to this is added two terms which are constant; they are not going to change with  $d$  or  $k$ .

Once we have this expression at our disposal, it is a very very simple calculation for us to **(( ))** this expression and ask when this will be a minimum and when this will be a maximum. Is that ok? So, clearly one thing you can see is that  $\cos \mathbf{k} \cdot \mathbf{d}$  is a maximum when  $\mathbf{k} \cdot \mathbf{d}$  is equal to  $2n\pi$ . What are the values intakes? Zero, one, two etcetera. Why is it so?  $\cos 0$  equal to  $\cos 2\pi$  equal to  $\cos 4\pi$  is equal to 1;  $\cos$  functions starts with 1. But on the other hand,  $\cos \mathbf{k} \cdot \mathbf{d}$  is a minimum, when  $\mathbf{k} \cdot \mathbf{d}$  is equal to  $2n\pi + \pi$ ,  $n$  equal to zero, one, two etcetera.

So, what do I do now? If I take both the polarizations to be parallel, then this will be a positive quantity and if  $\mathbf{k} \cdot \mathbf{d}$  equal to  $2n\pi$ ; that means, you have enhanced your intensity. Ordinarily if there were an incoherent superposition, your total intensity would have been what?  $I_{total}$ , it would have been just  $I_1 + I_2$ . Now it moves up by factor  $E_1^2 + E_2^2 \cos \mathbf{k} \cdot \mathbf{d}$ . But on the other hand, when the two polarizations are parallel to each other, that is, you do not tamper with whatever original polarization was, what is going to happen? If  $\mathbf{k} \cdot \mathbf{d}$  equal to  $2n\pi + \pi$ , that is,  $\pi, 3\pi, 5\pi, 7\pi$  so on and so forth, odd multiples of  $\pi$ , then this is going to be negative and the amplitude is going to be suppressed.

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So, the point that we are making is, as we move along the screen, as we move along the screen along the x axis, my  $\mathbf{k} \cdot \mathbf{d}$  is going to change.  $\mathbf{k} \cdot \mathbf{d}$  changes as we move along the screen and therefore, if I were to place my detector all along the screen, is that right?

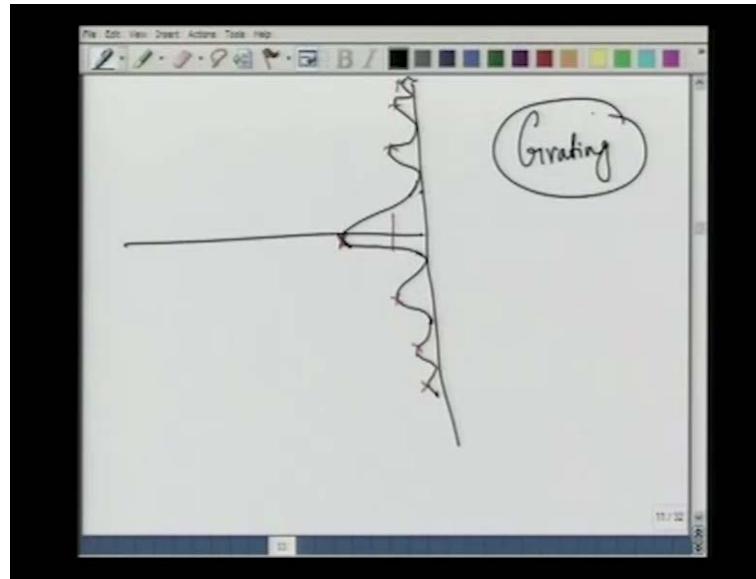


Or if I were to move my detector along the screen, you will find a series of maxima and a series of minima. It is a trivial trigonometric exercise to write down what that thing is.

Let us not forget what my  $k \cdot d$  is.  $k \cdot d$ , my  $d$  is along the  $x$  axis is nothing, but  $k \cdot d \sin \alpha$ ; that is what I have. So, I think it was in page six, the previous page where I defined my  $\alpha$ . So, your  $\sin \alpha$  is nothing but  $x$  by  $z$ . So, if I were to expand it, this is nothing but  $x$  by  $z$ ; that is what I have here. So, let me go to the next page, this implies this is equal to  $k \cdot d \cdot x$  by  $z$ .  $x$  is the distance at which my detector is and  $z$  is the distance at which my screen is from the two slits.

Let us not forget the definition of  $k = 2\pi / \lambda$ . Therefore, I have  $2\pi / \lambda$ . I am looking at a quasi-monochromatic plane wave, monochromatic plane as far as we are concerned.  $d$  is the distance between the two slits;  $x$  by  $z$  is equal to  $n\pi$  for maxima. Of course, if the polarization were anti-parallel, suppose the first beam was polarized along the positive  $x$  direction and the second beam, the beam from the second slit polarized along the negative  $z$ , then the roles should have been interchanged, you should always remember that. This is for parallel polarization and that is what we encounter most of the time unless I put a polarizer. So, let me cancel my  $2\pi$  on both the sides, I will get  $x \cdot n$  is simply given by  $n \cdot \lambda \cdot z$  by  $x$ . So, all these points are at a maximum. In fact, the intensity will be, I think twice the intensity  $I_1$  plus  $I_2$ . It will get added up, the interference. What about the minima? I will leave that as an exercise for you people to work out.

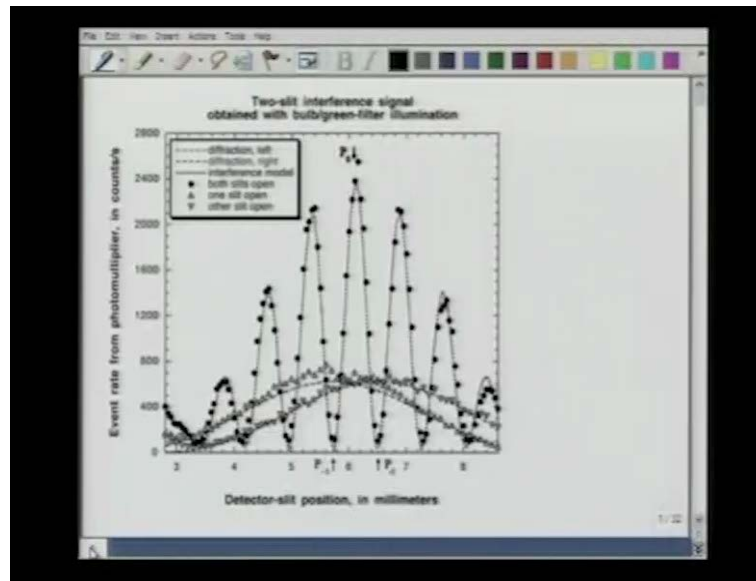
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So, speaking pictorially, if this is my screen, really speaking, I will have quite a pronounced peak and then it goes like this. The periodicity will be there, but the peak value keeps on decreasing, although this formula does not give because of losses and things like that.

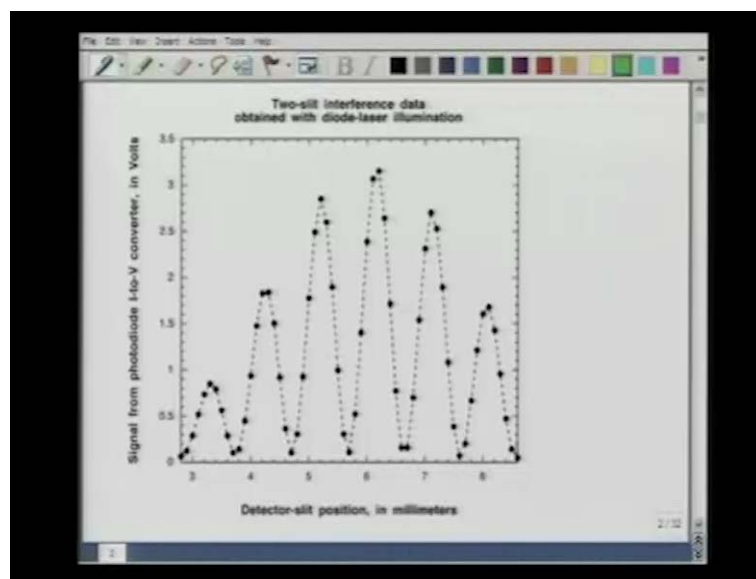
So, now you know how to look at the positions and the maxima. This is one, this is one, this is one and this is next one, so on and so forth, equally spaced. In this double slit experiment, what you should notice is that we do not get sharp minima and sharp maxima, you get fairly broad and this width, you can evaluate because I just now wrote down the cos function for you. If you want a sharp minima and sharp maxima, very very sharp lines, then you do not perform an experiment with double slit, but you perform an experiment with what is called as a grating. I am sure you people have done this experiment with gratings and as you keep on decreasing the spacing between the gratings, the maxima and the minima become sharper and sharper and it will be virtually thin lines; diffraction grating, that is what it is? **Right? Right.** But we will not get into that, but what I will request you is to please remember that this is the curve that I am going to get. Is this the curve that I am going to get? This is an experimental subject, after all, electromagnetic theory. What I have done is merely a theoretical analysis. So, let me look at an experimental result and ask if this is indeed what I get?

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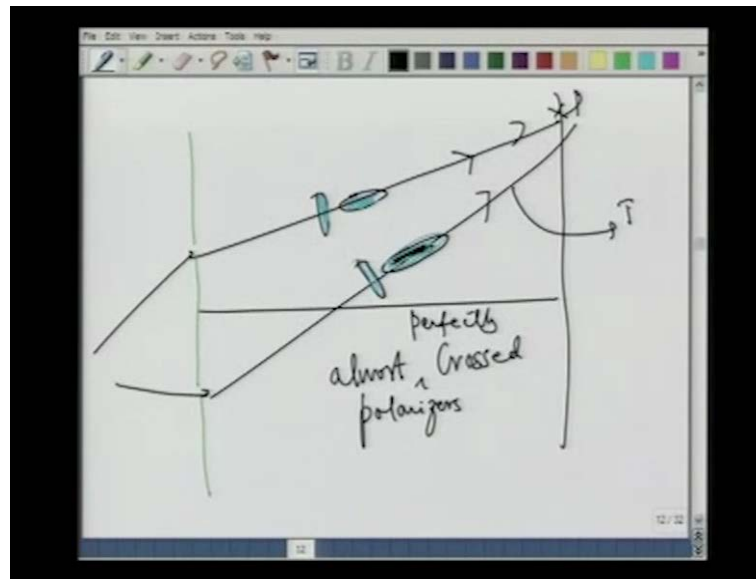


Please do not bother about the bottom curves. So, let me indicate for you what is it that you should not look at is that, right? So, let me use that. So, ignore this and what should you concentrate on? Green signal, concentrate on them. ((C)) This is an inference pattern which is taken with ordinary light. So, you can see that is what it tells you. It is obtained with dark green filter illumination. So, let me highlight that. You see that? Ordinary light. You see that this is a center maxima and it is flanked, is that right? It is flanked by secondary maxima, tertiary maxima and so on and so forth; and this shows the classic cos behavior.

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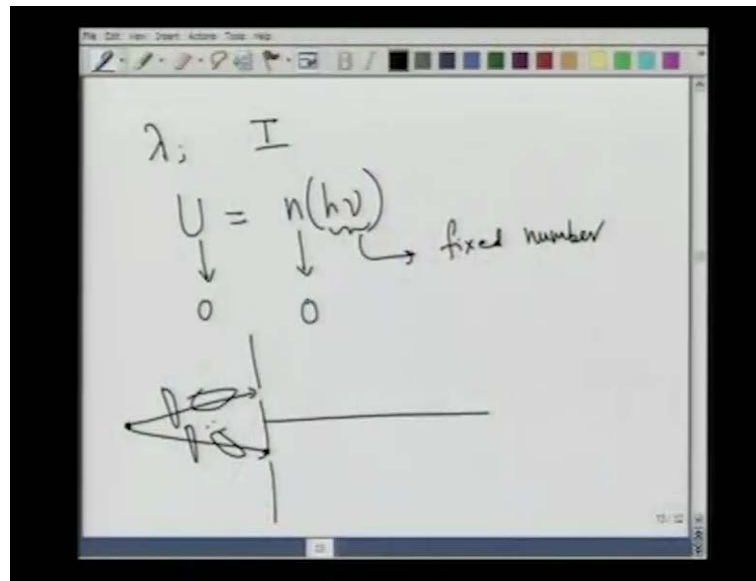


Now, what I want you people to do is to look at another figure which is there in the next slide and ask you, what is this figure representing? Look at this figure, look at this figure they look the same. You would have told me, well, you wanted us to concentrate on this particular curve. Therefore, what you did was to remove these two bottom lines and they are showing us this curve. But in reality, that is not what has happened. In reality, what has happened is something extraordinarily interesting. So, this was my grating.



Let me explain the setting for you. You still have an interference pattern. So, I have my classic double slit. Very good! And then, what I have are these two sources which are coming. Is that right? But what I do is... So, let me imagine that this was my point P. So, there is this light beam coming here, there is this light beam coming here. Here I will put two crossed polarizers, which I will indicate like this. So, let me fill it up to indicate that these are cross polarizes.

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So, similarly I will put two cross polarizes, right? So, maybe I should write it up and then fill it up. Ok. These are not perfectly crossed, but almost crossed polarizers. You know very well that if the polarizers are perfectly crossed, the intensity that comes out will be zero because the first polarizer cuts off half the radiation, the second polarizer cuts of the other half of the radiation. Therefore, almost perfectly crossed; it is not perfectly crossed, but almost perfectly crossed so that the intensities are very very small. Intensities are very very small. How small? That is the question.

In order to give an idea of how small these intensities are, let me go back and tell you what is it that I did. If you give me a wavelength and if you give an intensity, I can immediately associate the numbers of photons per unit volume. Why? Because I wrote the energy density is simply given by  $n$  into  $h\nu$ . As my intensity keeps on decreasing, my energy density keeps on decreasing because there is very little radiation in any given unit volume. This is a fixed number. This is a fixed number because I am not tampering with the frequency and this goes to 0. So please tell me what happens when it goes to zero. When it goes to zero, the only way the right hand side can go to zero is  $n$  goes to zero.

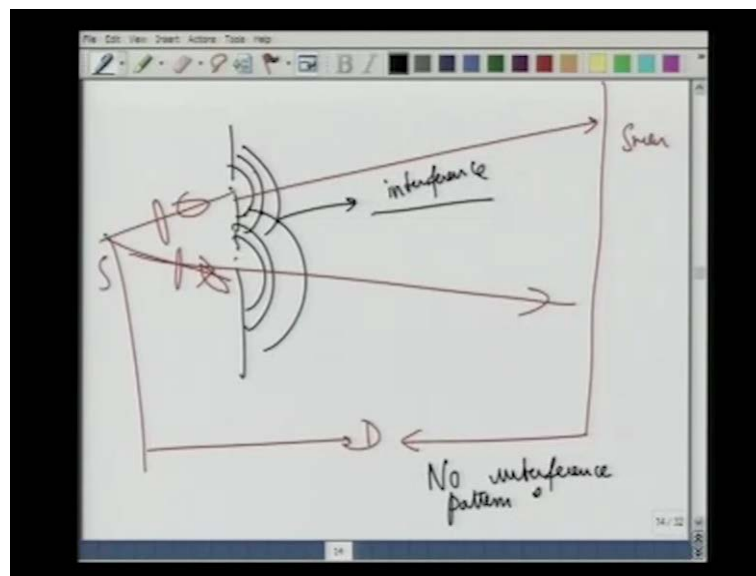
So, you can make an estimate for ordinary sun's light. What will be the number of photons? Assuming that the peak frequency is in the blue-green region that will turn out to be a large number, ten to the power of eighteen ten to the power of twenty. Now, if

you actually made the intensity so small that there is only one photon between the slit and the peak.

So, the intensity is so small, only one photon (( )) actually between the slit and the detector; that is what we do. Now when it comes to only one photon between the slit and the detector, let me go back; that means, actually what I had is the polarizer is shutting it off everywhere.

Now I would like to ask, what is it that happens with the interference experiment? You would expect that nothing much should happen because there is nothing that is reaching the screen. In order to make it a little bit closer to reality, what I will do is I will consider slightly different configuration. In fact, that is what I should have considered in the first place. So, there are these two rays coming here, I will put perfectly, almost perfectly crossed polarizers and analyzers here and then I will ask if there is an interference pattern. Is that part? So, you can imagine that the experiment is done in two parts. One thing if you put it between the place the polarizer and the analyzer in between the two slits and the detector. That is not of any great interest to us, but what is of great interest to us is this particular configuration.

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Now, normally how do I understand interference? Let us go back to the Huygens picture. In Huygens picture you assume that there is a source here, it produces secondary waves and there is a source here it produces secondary waves. It is this interference that gives

rise to interference pattern; super position. Now what I have done is to put a source here and I am sending a radiation; let me indicate in the brick red color. Now, I am putting a cross polarizer. Crossed polarizer; that is what I am doing. I cannot use the wave picture when I am using the photon picture therefore; there is no question of expanding. And this is my source and this is my screen. Now, between the source and the screen, let us say the distance is some capital D.

Suppose there is only one photon that goes at a given time, then the photon should occur either this path or this path; it cannot take both the paths. Is that part right? Because we do not know how to divide a photon; a photon is an indivisible unit. Therefore, in this configuration, we should not expect any interference pattern. So, let me write it boldly, no interference pattern expected. Now what I do is to return to my other slide. This experiment is actually taken from laser beam. The intensity became so small that there was only one photon on an average between the source and the screen. You do not expect any interference pattern. However what people did was to put the lot of diode detectors, that is, a lot of photo detectors. You wait for a long time and you find what is it that you find? Perfect interference pattern; perfect pattern of interference.

In other words, even when you go over to the photon picture of Planck and Einstein and imagine those photons to be particulate that they are going to behave like corpuscles. Even then if I did a careful experiment maintain the coherence and all that, I still find an interference pattern. That means, there is much more to Maxwell's equation and its solutions than simply look up on it as some ordinary view.

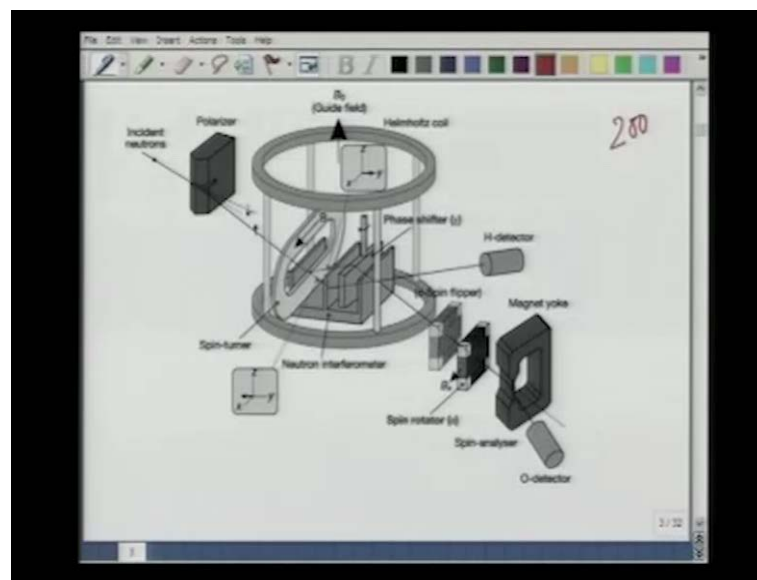
Conversely, if you take the notion of quantum very seriously, corpuscular nature there is much more to the nature of the particle aspect of light than simply imagining that it is like a small bullet which is moving, because classically, bullets shall not interfere and classically waves shall not show particulate behavior. But here they have a peculiar situation where I have only one photon at a given time therefore, it is particle like, but I still produce this interference pattern.

Historically this is not the way Maxwell's equations gave rise to modern physics or quantum mechanics. Historically what happened was, as we all know there was the black body radiation problem, there was the photo electric effect, then there was the problem of the Rutherford model of the atom, because accelerating charge particles have to

radiate eventually electron should collapse inside the nucleus; then there was the problem of quantum scattering. But today we have actually realized we have reached the stage where technologically we can realize experimentally and all these conceptual things and this is an experiment done very, very recently, sometime in 1998 or so. And its only about ten years ago and you find this very beautiful pattern.

So, if you think that therefore, quantum mechanics should be there only for light, we are mistaken. In fact, this phenomenon is seen in almost every other quantum object. People have performed single atom interferometry, single molecular interferometry, single electron interferometry and also single neutron interferometers.

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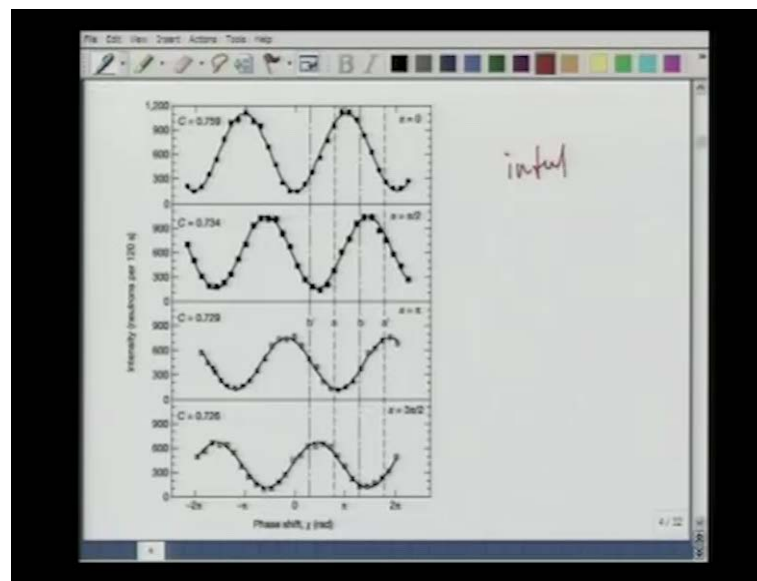


So, for the sake of completion, let me show you this very beautiful experimental setup. This is from a result which was as close to as possible. This is an experiment reported in nature in 2003 by the famous (( )) group I think. So, you have a reactor which will produce thermal neutrons and if you believe in (( )) germer experiments which you have studied, what should it do? You should show wave like behavior. This is the interferometer. One thing I would like to concentrate on this particular region. These are called silicon interferometers. This is the great advance in technology; these are perfect single crystals. Perfect means what? Not a single defect and these are of millimeter length, few millimeter length and defects may be of the order of microns so or even smaller than that. So, it is as if, you know you cannot find even a small blemish



anywhere; all were completely perfect, even the cutting is perfect, the planes are perfect; that is what they did. And what you do is to ensure by putting again lot of moderators you know which are familiar from your nuclear reactor such that there is only one neutron at any given time. Then you put your detector and ask for the intensity profile and lo and behold, what do you get? You get the beautiful **cos** pattern, which is again an interference pattern.

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So, this is the remarkable thing. Maxwell's equations naturally explain to you what the idea of interferences when it comes to electromagnetic waves. It tells you why perpendicularly polarized light beams do not interfere with each other. But then, taken to its limit, when the intensity becomes very very small and therefore, quantum mechanics has to be invoked. Again you find that there is an interference pattern and is that interference pattern restricted to Maxwell and dynamics **(())**? No, it seems to be common to all quantum substances, to all quantum systems whether it is an electron or a molecule or a proton or a neutron or for that matter even macroscopic quantities like Bose-Einstein condensates. I am sure all of you have heard of Bose-Einstein condensates because people got a noble prize just a few years ago; the famous MIT group and the French group. Is that ok?

In order to understand this, we need to reinterpret our solution for E and B. And that will take much, much beyond our course. E and B will be interpreted as probability

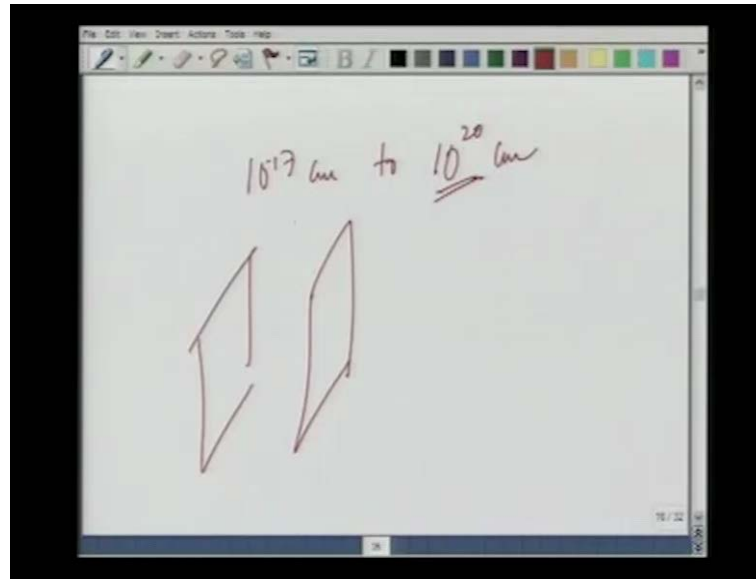
amplitudes; they will not be interpreted as functions but they will become operators that indeed is the great revolution which was initiated by Bohr, Heisenberg, Schrödinger and **dirac**. **dirac** was the first person to quantize electromagnetic field and Pauli. And today we have a theory called quantum electrodynamics, the quantum version of Maxwellian electrodynamics where we can understand all these things.

The point that I want to emphasize at this stage, although you may not be able to follow what I told you just now is that in doing that we have not touched Maxwell's equations. Maxwell's equations are still correct only the interpretation has changed. In other words, Maxwell himself dreamt little of how important his equations will be. So, this is almost reaching the fag end of the course. The only point that I would like to make at this particular juncture is to quote from whom? From Feynman. In his lectures, Feynman says that thousand years hence when people write the history of civilization, human civilization, what is it that people would be interested in? All the great advances, what were the people, what were their intellectual pursuits and what were their ambitions; what were their aspirations?

If you ask these questions, Feynman says that the great development in electrodynamics that took place over three hundred years, **right**? Starting from the fundamental experiments of Coulomb and Cavendish, all the way up to the great equations written by Maxwell; that will take several pages. Whereas, what we consider to be very important event today, like American civil war will not even find a place in the footnote that is what Feynman says.

In other words what I am trying to tell you is that what we have try to accomplish in the last twenty eight lecture are, including this, it will be the twenty ninth lecture is to gain a glimpse, to gain an insight, to gain an appreciation of this immense intellectual endeavor where hundreds of people, hundreds of nameless people participated. People did painstaking experiments, people try to interpret them, people repeated the experiments, people modified the experiments, people corrected immense data; they were not happy to simply tabulate the data. They look for the inner truth, whatever the inner content is and ultimately what Maxwell could do was to bring all the equations together given the Maxwell's equations which seem to be established very, very firmly.

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We have found no violation all the way from something like ten to the power of minus seventeen centimeters to all the way up to something like ten to the power of may be twenty centimeters, some galactic range or whatever. Probably this is even a larger number; let us not worry about that. It spans something like forty orders of magnitude, something which no theory can boost of. So, what is it that we try to learn in electro dynamics, let us ask ourselves? It is humanly impossible to cover all the topics that are there even in your syllabus let alone the subject itself. Is that ok? In twenty lectures, for that matter even in forty lectures although we exceeded whatever our quota is by a few lectures.

But what we have try to do at every point and every lecture was to try to gain insight every discipline has many, many aspects. One is information which is very crucial for us because we use information to our advantage. Another is technique, which is very, very crucial to us because without technique information is of no use because we do not know how to manipulate. The third one is the insight. So, there is a saying we should not miss the wood for the trees, **right**? We should be able to get a perspective; we should be able to get a broad picture; that is the third one. And doing all this we should be actually be able to appreciate what kind of a human endeavor it is. There is no way that one can retain all the information, but if you revive the crucial information, crucial pieces of information in your mind because they are the landmarks in the development of the subject, then you're safe all the other bits and pieces of information can always be picked

up. In a similar manner, it is impossible to remember all the techniques you do. So, many courses: vector and analysis vector calculus, differential geometry, differential equations, calculus, **is that right?** calculus of variations. How many techniques will you remember? But if you remember the basic techniques which actually gave rise to the field then you can always relearn the techniques even if you have forgotten.

So, we have to remember some basic techniques, you have to have the basic pieces of information. More than anything else, you should have a capability of penetrating into what is the true essence of the subject and that is something that we have tried to understand. So, we started with the humblest of the beginnings, we started with the idea of a scalar field, a vector field, then we defined very, very important quantities without which electrodynamics can be cannot be understood. What were they? Gradient, curves, stokes theorem, gauss's theorem. All these theorems were not created in vacuum. They were created, because people wanted to understand physical phenomena, flow of fluids irrotational behavior, rotational behavior so on and so forth. Then we started from the most basic of the experiments like Coulomb's experiment, Cavendish experiments...

Then we introduced the medium. We studied the modification brought about by the medium to the electro dynamics in free space. We were lucky enough to codify that in terms of the permittivity. What is it then we did? We studied electrostatics in medium, then we looked at the conductors, we looked at the magnetic field, we looked at Ohm's law which is a great embarrassment and which is also a great source of all kinds of applications; that is something that I discussed. Then logically proceeding, I made a time dependent magnetic field which will give rise to an electric field; that was Faraday's law of induction and finally, anchoring ourselves very, very firmly on one fundamentally very well established fact, namely the conservation of energy. What is it that I read I modified Maxwell's equations. I followed Maxwell, the way he modified his equations; we introduced the displacement current and that gave rays to electromagnetic waves which we identify with light.

In other words over a set of about thirty lectures you have covered the great developments that have taken place over last three hundred years. So, what you should do use to listen to this lectures carefully, think over the concepts that are introduced, solve as many problems as you can, formulate problems for yourself, if possible please

go to your laboratories and design experiments for yourself and try to verify for yourself every single statement that has been made.

Infinitely long solenoid is a figment of our imagination. But take a long solenoid, try to keep away, take a magnetic meter, measure the magnetic field and see whether it agrees with your calculation. It is very easy to make a calculation of an infinite sheet, but suppose this is a finite sheet and I take this point and I will ask you what is the electric field at this particular point it is not easy at all, but make a measurement and try to evolve an approximately, mathematically to solve.

So, what we should do is not try to become bookish and try to solve problems in the book which you anticipate in your examination, rather I would exhort you, I would urge you, I would request you to formulate for yourself interesting problems. Try to solve them by any technique that you can, go to the laboratory, make a measurement and see whether it agrees or not. If you did that you would be at the same time the innovative, rigorous, careful and disciplined and that I believe is the purpose of courses like NPTEL.

So, let us conclude the course. At this particular point. The only statement that I would use to make is that all these lectures would not have been possible, but for the real initiative taken by a host of people from our institutions, the IITs and also the enlightened attitude taken by the government of India. So, probably all of us and you students especially would be grateful to the organizers to the people who actually conceived the idea called NPTEL, who have collected a band of lot of people to give lectures to you and the best way that you can actually return the benefits, that you might get from this course or any other course is to understand, imbibe and try to teach your own students at a later stage Thank you.