Engineering Physics – II

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Module No. # 04

Lecture No. # 11

In the last lecture, we looked at Maxwell's equations in free space, and we studied the solution in great detail. What we did in order to simplify our life was to study a class of solutions, which I will call the Monochromatic plane wave. The solutions comprised of waves of a given frequency which are travelling along the given direction. So, what we did was to characterize the wave in terms of a frequency omega and a wave number k, and we derive the relation omega equal to kc.

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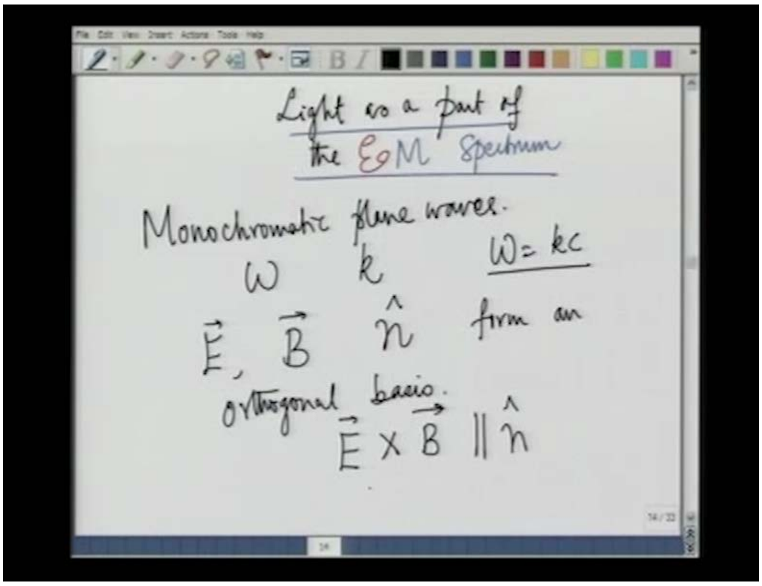

Even more importantly, we found that the electric field, the magnetic field and the direction of propagation, which I shall denote by n, from an orthogonal basis. Of course, I have to replace the electric and the magnetic fields by their unit vectors. Anyway we not only understand that they form an orthogonal basis but we also found that if you fix the direction of the propagation then E cross B, the cross product of the electric field vector with the magnetic field vector, will be parallel to n.

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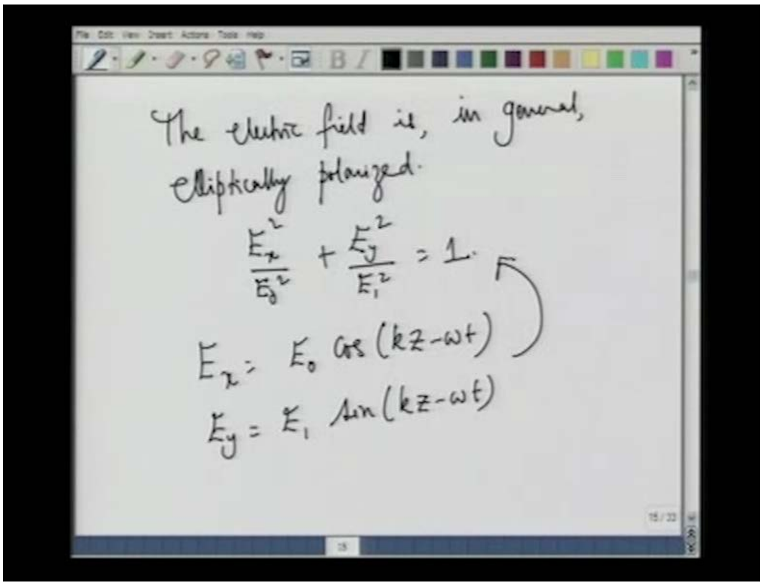

We found out that this cross product will give me the direction of propagation. What we then did was to proceed to find out the most general solution for E, because if you know the solution for E,you also know the solution for B. We also found that theelectric field is in general, elliptically polarized. Then we have Ex squared over E naught squared plus Ey squared over E one squared is equal to one for all times.

What do we get if you trace the locus of these points? We get the elliptic polarization. Of course if either E naught or E one is equal to zero, you get the linear polarization and if E naught equal to plus minus E one then you get the left circular polarization or the right circular polarization. All these are very familiar to us from optics. May be I should also supply one more missing step. What I did was to write E axis equal to E naught cos kz minus omega t and write Ey to be E one sin kz minus omega t. In writing this expression we have made intelligent use of the orientation of the xy axis and also the origin of time and the Z. That is what we have done and it is a very trivial thing to evaluate this fellow.

So, if you look at these two expressions, we can say E one equal to zero. This implies linear polarization parallel to x axis and E naught equal to zero implies linear polarization parallel to y axis. E naught equal to E one gives you right circular polarization and E naught equal to minus E one gives you the left circular polarization. So, all the polarizations are contained in this and this is precisely what the experimentalist starting from Young, Fresnel and Farrago all these people and a large number of experimentalists had seen.

There was no clue to the understanding of the origin of this polarization. Maxwell's equations tell us that if it is indeed true that what we call as optical phenomena are actually electromagnetic phenomena, then polarization is nothing but the direction of the electric field. Of course when I tell you that the polarization is nothing but the direction of the electric field, there is a certain convention because it is entirely up to us whether we want to choose the electric filed or the magnetic field because they are not independent of each other; giving me the electric field is equivalent to giving me the magnetic field and giving me the magnetic field is equivalent to giving the electric field. So, they are perfectly equivalent; they are on the same footing and there is nothing to prefer one over the other. As I told you, by convention we choose the direction of the electric field to characterize the polarization of electromagnetic waves.

What we have found so far? So far, we have found a certain plausibility argument in the sense, speeds match polarizations match; as I will show you in a short while, may be at the far end of this lecture. There was another fundamental observation which the experimentalist had made, that if you have two waves with opposite polarizations, there is no interference. That is something that cannot be understood from the corpuscular theory of light but we will understand it very easily by superposition principle. If I write E equal to E one plus E two, then what is my interference term? My interference term is nothing but E one dot E two and this will be zero if E one is perpendicular to E two

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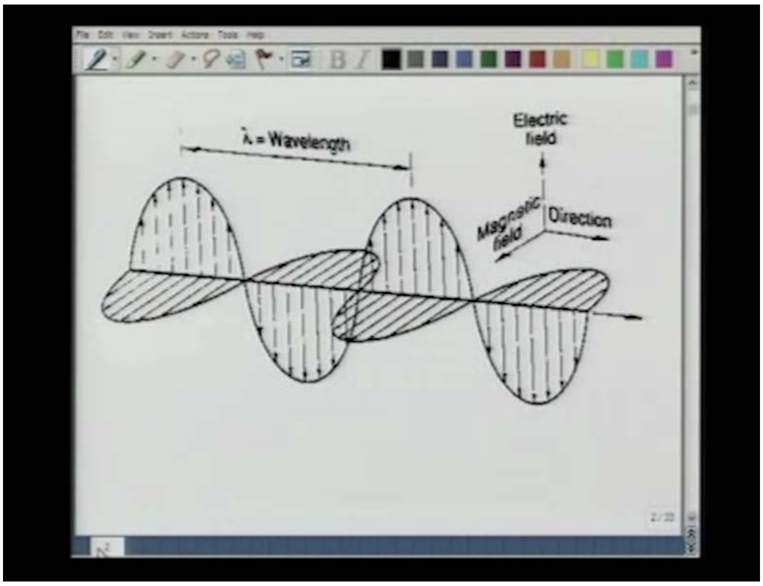

This was another fundamental observation which people had, the experimentalists had and there was not a clear understanding of how this comes about but this is a very natural consequence of Maxwell’s equation. So, you see there is an over whelming evidence in favor of identifying light with a part of the electromagnetic spectrum. But then, if that is indeed true, then the property of light should be a universal property of electromagnetic spectrum. That is, I should be able to generate and detect waves which are beyond the visible spectrum. You should go to the infrared, you should go to the ultraviolet and you should be able to generate wavelengths, you should be able to measure the frequencies and you should be able to verify that lambda mu equal to c, it is only then that we can claim that we have clearly identified and provided complete evidence for the fact that what we call as light is nothing but a part of the electromagnetic spectrum.

Who are the main experimentalists responsible for demonstrating this very crucial piece of information? As I told you, the first name is that of Hertz and the second name is J.C.Bose; both of them are very great experimentalists in their own right. Hertz did a large number of experiments and all of you know that the unit of frequency is in his honors. J.C.Bose himself did enormous number of experiments in electricity, magnetism and radio antenna; he anticipated Marconi. In fact, he preceded Marconi and probably he should have got the credit. That is a realization which has come in the last ten years; people have resurrected him and have restored to him the honors and the credit that was due him. J.C.Bose further went on to even study the response of plants to various electromagnetic waves. So he was a pioneer in many areas and he was far ahead of his time even if we now take a skeptical view point about his work on the demonstrations, so called demonstration of life in plants, there are other experiments which people have glossed over, but today they have been, as I told you duly recognized for whatever his work is.

So, what I will do is to spend a little bit of time to try to explain to you how J.C.Bose was actually able to demonstrate that what we call as light is nothing but a part of the electromagnetic spectrum. Before I do that, here is a very good nice picture which probably tells you all that I have told you in words; I was drawing pictures you know but here is a nice thing. So, you have the concept of the wavelength, the electric field is along this particular direction. So, let me make a few things; if I call this the z direction. I should use some color thing; if I call this z, this is will be the x direction and this will be the y direction. You people can see in this picture that the propagation direction is z; the electromagnetic wave is propagating along this particular direction, the polarization is along the x direction. You see that? The polarization is along the x direction; it is oscillating. This is an instantaneous snap shot at a given time.

So, as you keep on moving along z at a given time, this is how it looks like. The electric field shows a sinusoidal behavior in the zx plane whereas, if you look at the magnetic field direction which is along the y direction, you see a sinusoidal behavior. Are you seeing that? In the plane perpendicular to the yz plane and that is what you are finding and that is propagation. So, here you have a complete picture of whatever I was trying to tell you. This is exactly what we want to demonstrate in other regions beyond the visible spectrum.

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So, here you have a picture of Jagadish Chandra Bose with his apparatus with he was doing his experiments in the Presidency college. So, we will learn to try to know a little bit more about the kind of experiments that he did and how he managed to actually produce and detect the waves. These are really remarkable experiments

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The first one shows a transmitter of the waves. So, how do you transmit waves? First of all, you have to produce the electromagnetic waves. How do you produce light? For example, you have a spark plug; as you keep on bringing them together, it reaches a point of dielectric breakdown and at that dielectric break down, immediately there is a charge which jumps from one electrode to another and there is a radiation that is produced. As you know, that is the principle behind ignition. We did discuss that when we were looking at electrostatics and dielectric media.

So, the next picture actually shows this spark graph in the transmitter. He was able to arrange the gaps such that enough energy will be produced and supplied so that you can produce radiation and not just in the visible spectrum but also in the micrometer spectrum; you see the gap here.

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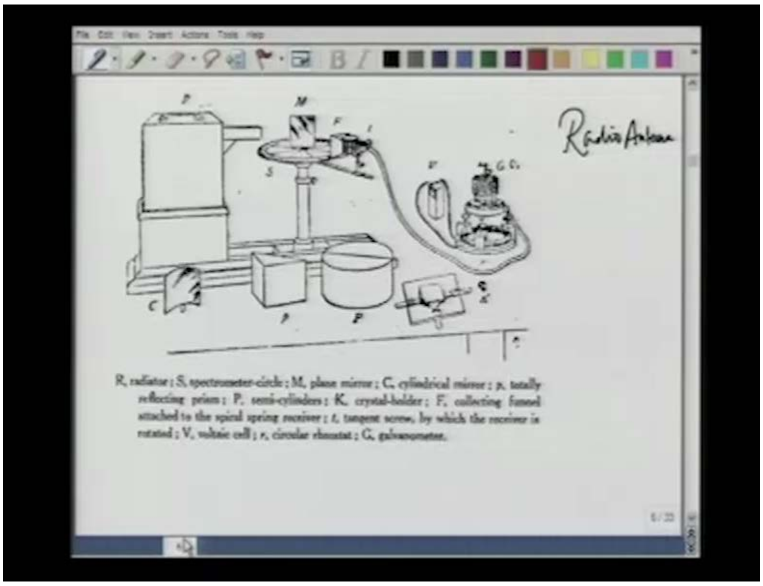

Now, this is a very remarkable set apparatus that you had. In fact, this is the one that was there in his portrait. Here you can see that he has the complete setup; you have the radiator that emits the radiation, there is a spectrometer, there is a mirror; he had totally reflecting prisms. Jagdish Chandra Bose actually performed very pioneering experiments on the evanescent waves. Those of you who will study a little bit of more of modern physics in semiconductor physics will hear a word called tunnelling; that is something that he demonstrated for the first time and then of course, you have the receiver,diastase,galvanometer, everything that is sitting here is that. So, it is such a nice apparatus, a very simple apparatus. What is it that he was able to detect?

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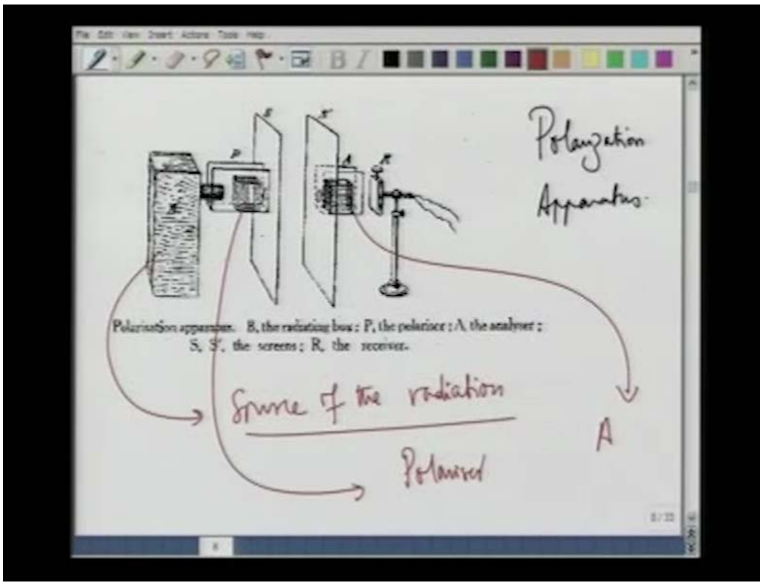

Jagdish Chandra Bose fabricated his own receiver. This is more of a large number of spiral springs which will actually respond to the electromagnetic wave in that particular wave length, the micrometer and here is a very beautiful polarization apparatus. So, this is the radiating box, source of the radiation and this is in the microwave region by the way. So, it is not the visible region and what you have is a polarizer and an analyzer. So, this is a polarizer and this is an analyzer. Of course you need a receiving antenna; I will show you receiving antenna in a while. So, you should be able to actually perform experiments you have a polarizer and an analyzer. If the polarizer and the analyzer are crossed with respect to each other, what should you receive? You should receive no radiation at all; that is the principle that you have always followed in optics. If the polarizer and the analyzer are parallel to each other, then you should receive half the intensity of the radiation that is coming, assuming that the radiations are un-polarized

So, what Jagdish Chandra Bose did was to build a very simple apparatus analog of the optical apparatus on which you have lenses and things like that, except that you cannot use a glass lens; you have to need appropriate metallic receivers or the detectors. Now let us get a glimpse of what he did; this is the kind of grating that he used in order to create polarized light. Remember? I gave you a very interesting problem where there is a frame and you put a large number of wires which act as a polarizer.

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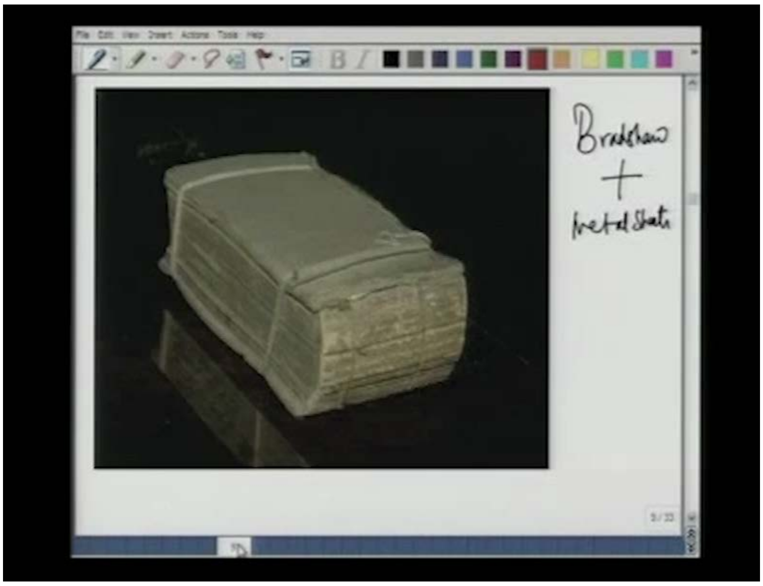

Now, what is it that Jagdish Chandra Bose did? You should remember that the resources were extraordinarily limited during those days because those were the days of the colonial rule and one had to be very innovative whether it was Bose or Raman. What they did was to make up for the lack of resources by their sheer talent, innovation and by their creativity. So, what you would see here is a bundle of Indian Bradshaw. Indian Bradshaw is a huge timetable consisting of all possible railway timings, northern zone, eastern zone, north eastern zone etc. It is easily one of the thickest books that were published by probably the government of India. What J.C.Bose did was to take sheets of Indian brads haw and place metallic sheets in between them. Is that part clear? So, the paper sheets will be the dielectrics and the metallic sheets will be the analogs of the metallic wires. This is a very beautiful grating and I will leave it as a problem for you people because you people have studied gratings quite a lot. You find out what should be the spacing between successive metallic plates if you want it to act in the microwave region.

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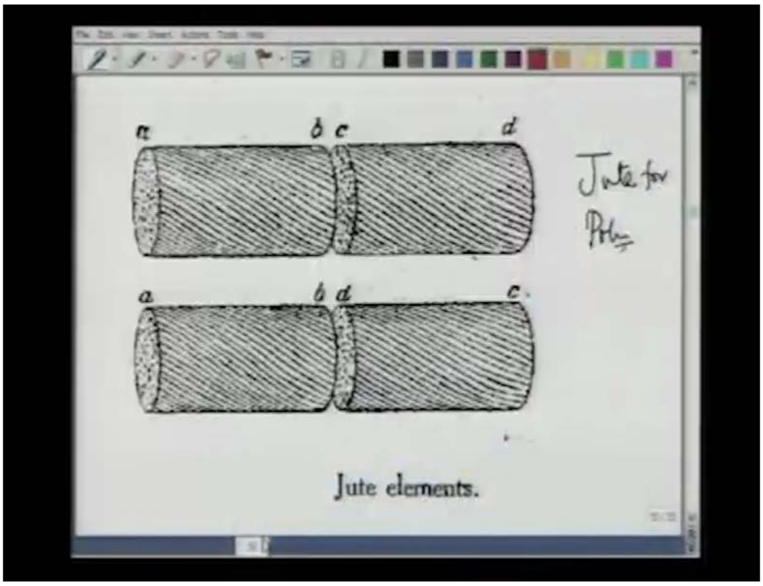

Not only that, actually he went even further to use twisted jute coirs. These are all twisted in order to actually polarize microwave radiation. Jute of course is very bad if try to polarize the radiation in the visible spectrum but J.C.Bose was able to do that. We should remember that all these materials act as excellent antenna, they act as polarizers they act as receivers. And those of you who stay in some remote corners of the country, you may remember that if you do not get an antenna and if you have a TV set, you can simply take your cable and connect to a banana plant. That will start receiving radiation or whatever the signal that is required for your TV receiver. So, it is the same principle that was used by J.C.Bose when he started looking at the receivers.

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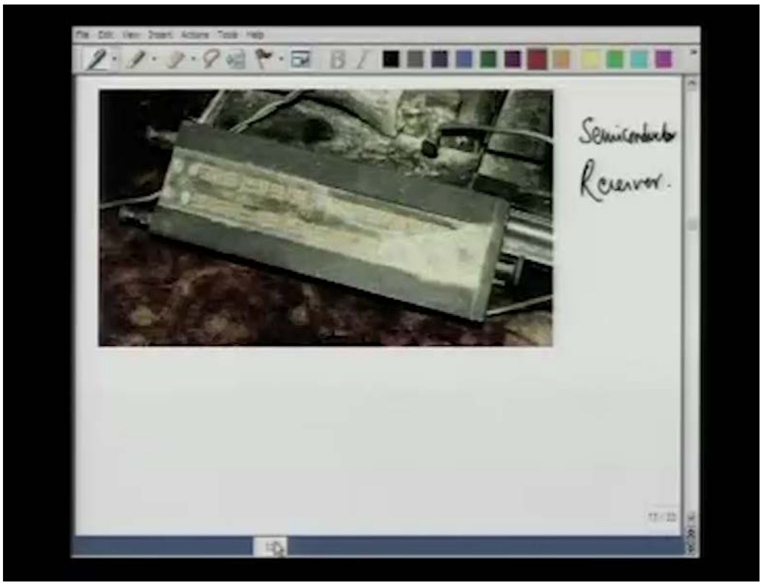

If you think that J.C.Bose was using some primitive detectors and receivers all the time, then you are wrong because for the first time J.C.Bose also used semiconductor receivers. You people will study a whole lot of semiconductor physics because all of electronics is based on them. One of the things that you know in semiconductor physics is that the so called VI characteristics, that is, voltage current characteristics. Let me spend a few minutes on that.

When we looked at conduction in an ordinary wire, what is it that we wrote? We wrote V equal to IR for a metal, for a conductor; that means the voltage current characteristic is a linear relation whereas, the voltage current characteristic in a semiconductor is not linear at all. In fact, it starts like this and it starts saturating. You see that in electronics in transistors in triodes in semiconductors because all of them are built out of semiconductors. J.C.Bose was the first person to study the current voltage characteristics. See the non-linear relation between the current and the voltage.

So, what is happening here? The resistance itself is dependent on the current that is flowing; the resistance is not independent of the current that is flowing. It is a non-linear response; it is not linear, it is linear only in this narrow region and J.C.Bose was able to study that. Mord, a great physicist makes the observation that Bose was at least thirty years ahead of his contemporaries because it was only thirty years later when transistors proper were fabricated and discovered that all these studies came to therefore.

Now thanks to Hertz and thanks to J.C.Bose, we have seen that there is an experimental demonstration where we are able to verify all the properties of light even in the invisible region. So, what do we have? We have the complete electromagnetic spectrum which this picture shows. So, where is the visible spectrum? The visible spectrum is barely visible; you can see that light bulb is what indicates it. So, let me circle it with some appropriate color.

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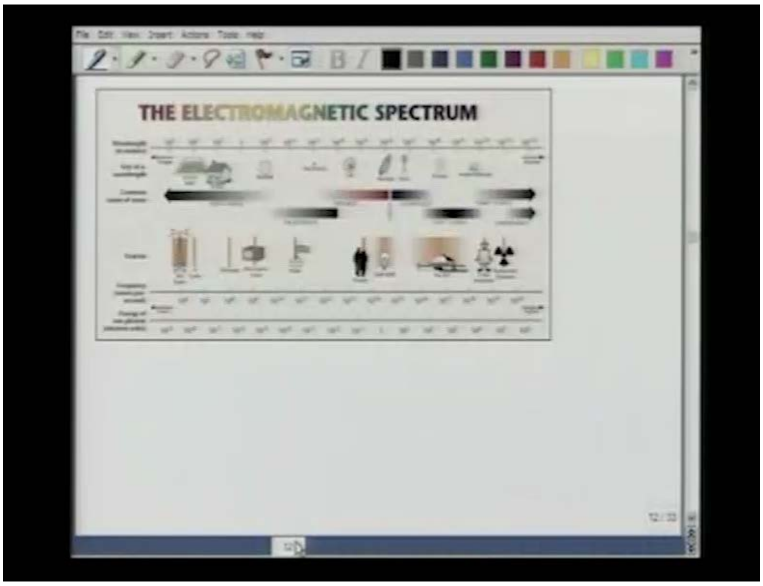

So, this is where you have the visible spectrum that has been bloated up. Whereas, if you look at the all the wavelengths and all the frequencies that has been seen so far, you start with something like ten to the power of six hertz all the way up to ten to the power of twenty hertz. That is what is shown in this particular figure. Whereas the visible spectrum you know is in something like ten to the power of fourteen to ten to the power of fifteen, that is one order of magnitude right between ten to the power of fourteen and ten to the power of fifteen cycles per second hertz. Whereas, here you are going from six to twenty, fourteen decades is that is what people have seen and invariably from the very large wavelength that is, very small frequencies to very large frequencies: ten to the power of twenty hertz.

What we find is that electromagnetic waves transverse. E and B are perpendicular to each other; the direction of polarization is given by E. You can see polarization, you can see interference and you can see diffraction in all these regions. The challenge is not so much in physics as in the technology, you should have the talent to actually fabricate the right sources, the right polarizers and the right detectors and it is a great triumph for modern technology that these things can be seen.

So, there are a whole lot of things. This is the radio wave and this is my radio wave region. So, and here you have the gamma rays at the other end which come from the emission of what the nuclei (( )). When a nucleus gets de-excited from a higher energy level to lower energy level, it will emit radiation in the million electron voltage; so that will be your gamma rays. You can also have hard x rays and soft x rays coming from atoms and molecules. You can actually prepare beams of light which travel at very small frequencies, at very large frequencies and at very small wavelengths. You can polarize them, you can scatter them against the electrons, and you can scatter them against the atom and study the final state polarization. So, whatever you do in optics can be mimicked all across the wavelength.

If that is indeed the case, then from the viewpoint of physics it is entirely a matter of mild interest that there is something called the visible spectrum. By virtue of evolution our eyes and our brain have adapted themselves to a very narrow range.

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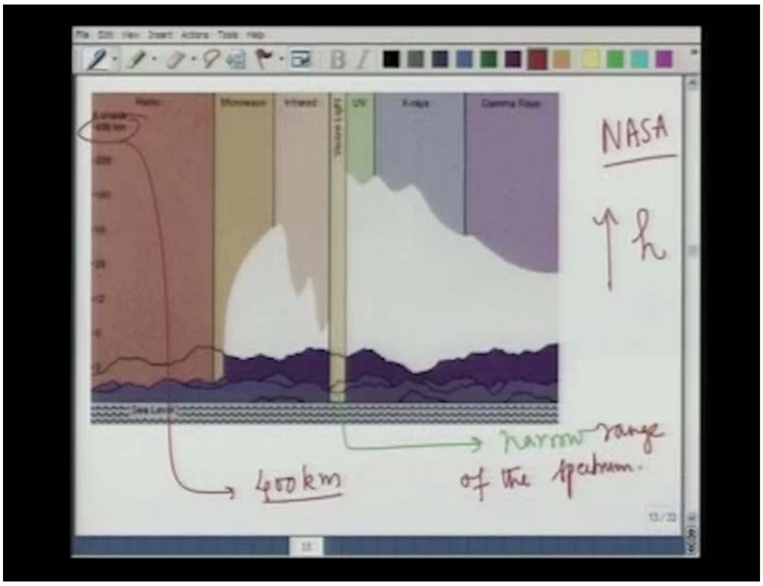

So, this is a narrow range of the spectrum. Since we have to live and survive, we have to be sensitive to the dominant wavelengths that enter the earth's atmosphere and please remember not all the wavelengths emitted by sun can actually enter the earth's atmosphere. Because of the ionosphere, there is a natural shielding because of which the ultraviolet rays cannot easily penetrate. All of you know the problem of ozone hole because which ultraviolet rays are not shielded anymore because of which people who live very near the poles are susceptible to skin cancer. It is a very big environmental problem because of refrigerators etcetera all over. So, this is the narrow region.

But if you want to explore electromagnetic waves through the complete spectrum, then you should be able to go much beyond the optical region. Here is a very beautiful picture made by the NASA people who actually explore the space. What they do is to ask, suppose I want to see more and more regions of the spectrum, how far should I go about my ground level? Of course eventually if you can go to the outer space where there is no atmosphere, then you should be able to see all possible spectra. So, this is the height that we are interested in. You are shown here three, six, twelve, twenty five etcetera units of kilometers and go all the way up to four hundred kilometers. Let us not forget that our atmosphere is about three hundred((to four hundred meters)). If you go to four hundred kilometers, you will be able to access radio, microwave, infrared, ultraviolet, x rays, gamma rays etcetera.

Today we know that our telescopes, earlier Newton built the great reflecting telescope and Hershel made its measurement with observations with telescope. Today's telescopes are not restricted to visible light; Today's telescopes are there in the infrared, there is the great radio astronomy center which is in India, there is the radio astronomy center, radio waves and then we have it in the microwave region and cosmic microwave background; all of you have heard of it which is the remnant of the big bang explosion, and there are also telescopes in the region of ultraviolet, x rays and gamma rays, you have to build them. There is what is called as the giant gamma ray bursts.

The sources are all quite mysterious and they have to build a receiving antenna in order to see that. So, in order to do that perhaps it is always better to place your telescopes somewhere up in the space. In fact, we have dedicated satellites. The Hubble telescope is very well known. There is another satellite named after the great Indian astrophysicist Chandrasekhar; that also is carrying a telescope. So, Maxwell has then immense service after he wrote the equations. Not only was he able to unify optics and electromagnetic is that optic is but a branch of electrodynamics. It has proved to be the most important tool in order to explore the universe and that is something that this picture gives us.

Now we have a complete picture of electromagnetic waves. We know that what we call as light is nothing but a part of the electromagnetic wave. What I will now do before I proceed, is to actually look at the phenomenon of interference more for the sake of completeness. We have almost reached the far end of the course and we do not have the luxury of time to actually spend a lot of time discussing interference, diffraction, gratings etcetera.

What I want to emphasize is something slightly different, namely the compatibility and the consistency of electromagnetic waves both with relativity and quantum mechanics. Now I am not able to actually derive the expressions for you because deriving an expression for the energy density is a little bit of work. You people can look up any nice book on electrodynamics; Griffiths for example, will give you a very good derivation. So, what I will do is just quote for you the result and make you some interesting observations. What we shall do is to start with an expression for the energy stored in an electromagnetic wave the energy stored in an electromagnetic wave.

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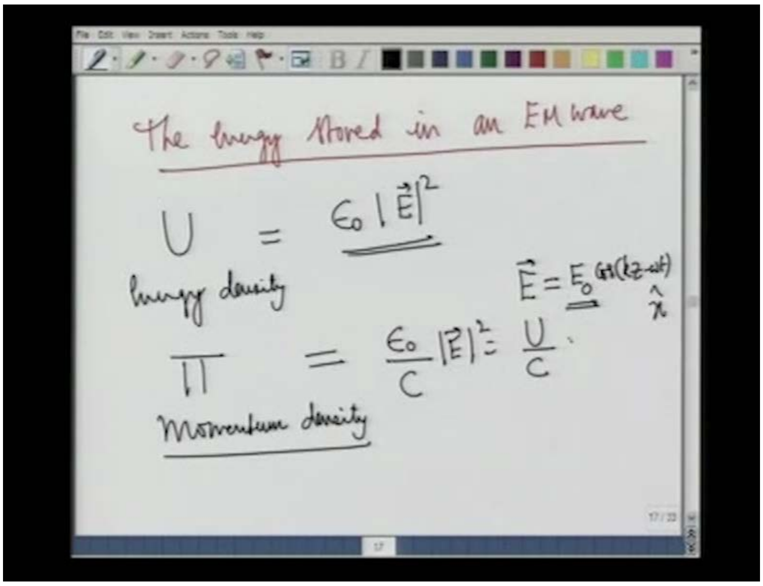

I am not interested in writing down the most general solution but I am interested in looking at a particular case, namely the monochromatic plane wave. What would be the solution? The answer is that we are not interested in the total energy but the energy density. So, this is my energy density. This is nothing but epsilon naught mod e squared. When I am writing this, I am not interested in the temporal part. What I do is to average over one particular period. So, this E is actually the amplitude. What do I mean by that? For example, if I wrote E is equal to E naught cos kZ minus omega t into unit vector x, I am actually looking at this e naught. I have averaged, I do the modulus squared of that and average over a period and what I get is epsilon naught mod E squared. This is the energy density.

Of course radiation carries energy density and in some sense sun is the source of all possible energy that we find on this earth. Photosynthesis, fossil fuel, whatever we may look at, all of them owe their origin to whatever is coming from the sun. Even our waterfalls which run on hydroelectric power are because of sun's rays because water evaporates, becomes clouds and it rains and then it becomes rivers and then it flows down a gradient etcetera. So, sun is indeed the source of all power, all energy that we have on the surface of the earth. How is the sun the source? Sun is the source because of the radiation that is emitted and this is energy density that is carried.

Now, not only does the light carry energy, it also carries momentum. Now let me write down the expression for the momentum and let me call it as pi. This is the momentum density, the momentum carried per unit volume. This is also averaged and this turns out to be epsilon naught by c into mod E square which is nothing but U by c. This is quite a remarkable expression, so, what I will do is to rewrite that expression.

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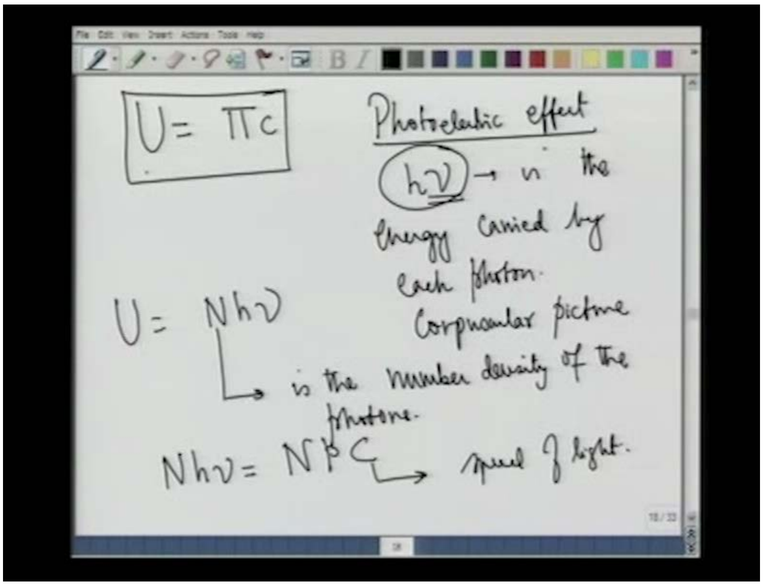

So, we have U is equal to pi c. Let us remember that in order to explain, in order to account for photoelectric effect; it is always good to go back to these fundamental experiments.

Einstein broke away from this beautiful edifice that we have constructed. It is an unbelievably beautiful formalism. Maxwell's equations, electromagnetic waves, and then light as an electromagnetic wave, wave nature etcetera and that with remarkable boldness Einstein was able to break away from that. Einstein argued that electromagnetic waves actually come in quantized units of energy. What is it that he wrote? He said that if you give me a frequency nu, a quant of energy carries an energy h nu. So, each photon is the energy carried by each photon .Let me go back to the previous line. Here I wrote E equal to E naught cos kz minus omega t into x. When I looked at the total energy carried, it was dependent on only E naught, it was independent of the frequency because in all waves, the energy is dependent on the amplitude and it is not dependent on either the wavelength or the frequency.

But here you have a peculiar situation where Planck used it in order to explain blackbody radiation and Einstein used it very boldly and asserted that photons are real. Planck actually had somewhere in his mind that photons are some kind of effective descriptions; he did not take them too seriously. In fact, Planck introduced the idea of a photon very reluctantly but Einstein embraced it wholeheartedly and he asserted that h nu will use the energy carried by each photon. Now, that does not mean you are going to give up all the expressions that we have derived in Maxwell's equations, we are going to reinterpret them. How we are going to reinterpret the conclusions and the expressions that we derive from Maxwell's equation is the great transition from the classical physics to quantum physics.

Now, of course, we are not going to get into that discussion but let us ask ourselves, suppose I admitted this kind of a corpuscular picture. This corpuscular picture should not be confused with the picture that Newton had. They are entirely different from each other. If we look at that, what my energy density would be will be nothing but the number density multiplied by the frequency. It is because I have looked I am looking at an electromagnetic wave which is monochromatic; all of them have the same frequency. Therefore, I will write u is equal to Nh nu, where N is the number density of the photons.

Actually I should be careful here, I should say (( )) number density but anyway let us not bother too much about that.

Now, if this is the number density, what about the momentum? Each photon carries a certain momentum; p equal to h by lambda for example. That is the De Broglie wave. Therefore, if I look at it now, what is it that I am going to get? I am going to get N h nu is equal to N into momentum carried by each photon into c. That is what we have. So, N h nu is simply given by N into p into c. Let us not forget the meaning of c, which is the speed of light.

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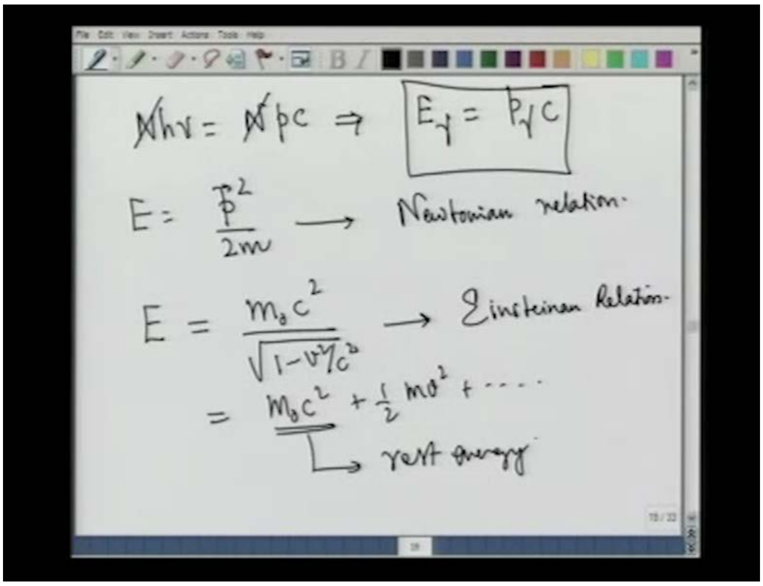

So, what I will do is to look at this equation again Nh nu is equal to Np c. N cancels, now this gives me the energy carried by each photon and also the momentum carried by each photon; so this implies the energy carried by each photon. E gamma is nothing but p gamma into c.

The relation between the energy and momentum is given by a linear relation, E gamma equal to p gamma c. At this point, it is a good thing to pause to ask ourselves why we should be looking at this kind of an equation. Let us recall what it that happens in the Newtonian case. In the Newtonian case, we wrote energy carried by a particle is given by p squared by two m. Is that right? This is the kinetic energy carried by a particle. So obviously, this relation is incompatible with this. So, this is the famous Newtonian relation which you use extensively in your dynamic course.

Now, we are going away from quantum mechanics and Planck to relativity. When Einstein developed this relativity, he gave a refinement of this formula. So, what is my relativistic formula? It is nothing but m naught c squared over root one minus v squared by c squared. This is the expression for energy and this is what is called as Einstein an relation.

If we make a tailor expansion power of v, what do we get? We get m naught c squared plus half m v squared and soon. With a great boldness, Einstein was able to identify this with the rest energy and this is the famous mass-energy transmutation; you people have studied that in your twelve standard. He will show up as mass defect production of gamma particles etcetera. So, this is the Einstein an relation. Now, what Einstein did was not only give a relativistic expression for energy, he also gave a relativistic expression for momentum.

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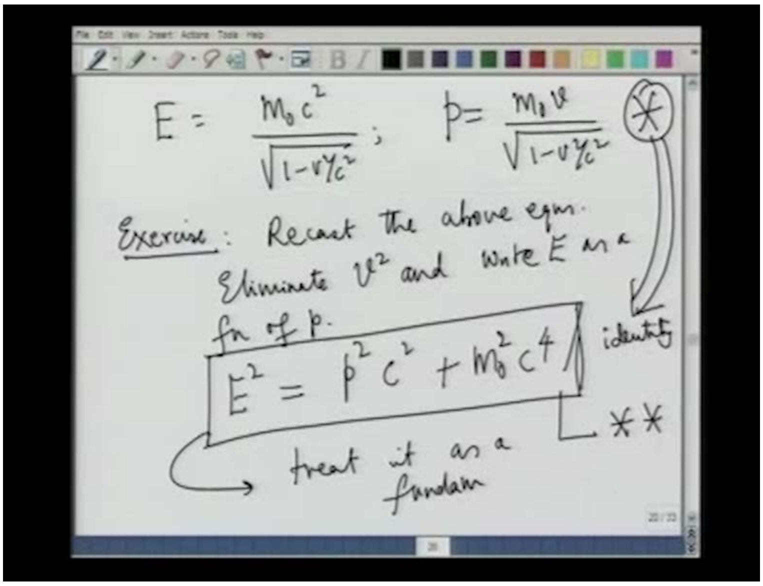

So, let me repeat, E is given by m naught c square over root one minus v squared by c squared whereas the momentum is given by m naught v over root one minus v squared by c squared. If you make a tailor expansion in power of v, a binomial expansion, the leading order time will be m naught v; m naught is the so called rest mass.

Now, I want do a little bit of algebra which I will leave it as an exercise for you people. It is pure entertainment and I would like you to see how the thought processes take place. When you derive these expressions, all of you are familiar with these expressions, you will find that if I put the rest mass is equal to zero, m naught is equal to zero. Energy is equal to zero, momentum is equal to zero; no mass no energy no momentum.

However what I can do is to recast the above equations in a slightly different form. So, exercise is for you is to recast the above equations. How do I recast the above equation? I want you to eliminate v and write E as a function of p. So, this is the exercise: eliminate v squared and write E as a function of p. After all, you wrote E equal to p squared by two m. What would that be? Let me write down the answer for you, and that is the exercise that you will work out. E squared is nothing but p squared c squared plus m naught squared c to the power of four. This is an identity and where did I get this equation from? If Iput a star here, the star implies this particular relation. Now what Einstein did was to argue that although, let me call it as a double star, I use star in order to divide a double star, speaking figuratively I will give it a five star status. E squared equal to p squared c squared plus m naught square c to the power of four; I will treat it as a fundamental expression and not as a derived expression. So, treat it as a fundamental expression.

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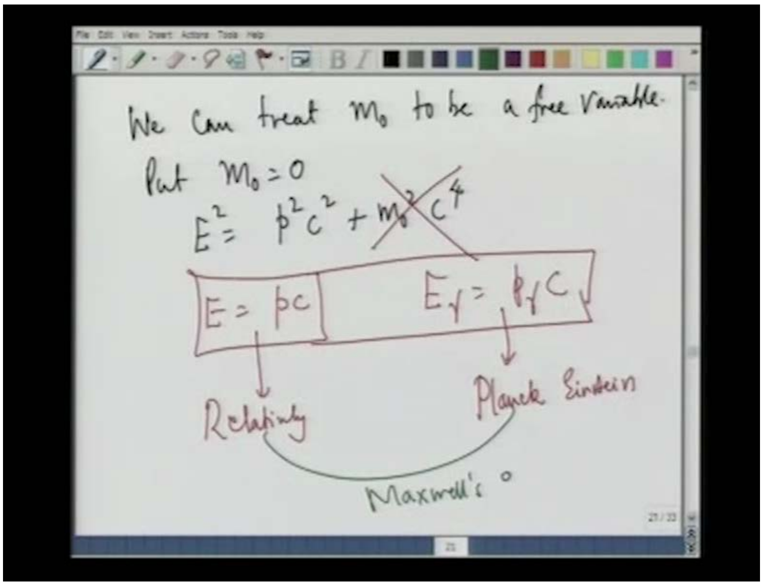

If you did that, now we are under no obligation to go back to my original equation, the star. I can choose m naught to be a free variable. So, in particular we can treat m naught to be a free variable. Put m naught equal to zero. What happens if you put m naught equal to zero? The original expression tells, if E equal to zero, p equal to zero because E is proportional to m naught and p is proportional to m naught. But in this equation, E squared is equal to p squared c squared plus m naught squared c to the power of four. If you put m naught equal to zero, this term cancels and you got E equal to p squared c. What does Planck tell us which was actually used by Einstein in his photoelectric effect? Planck exactly gives this relation: E gamma is nothing but p gamma into c. So, this is relativity; this is Planck, Einstein. Both of them are completely compatible with Maxwell's equations. Since we started with Maxwell's equations, the only thing that I did not show you was how to derive the expression for U and pi. That is a quite a standard derivation which you can find at any book; please trust me with that expression. There is nothing wrong with that.

.So, you see there is a remarkable consistency. Maxwell's equations in fact, gave rise to relativity. Maxwell's equations in fact, gave rise to quantum mechanics because of black body radiation. Radiation is nothing but electromagnetic phenomena. So, we can now reinterpret photons to be mass less particles. So, I can assert that photons are mass less and this equation tells you that all mass less particles move with the speed of light. Light is one particular example. So, perhaps I should be able to look at other examples. There are indeed such examples to a large extent neutrino is a mass less particle. You people have heard of neutrino in the beta decay problem. When it comes to a strong interaction, there are particles or gluons which are supposed to move the speed of light. They are what are called as gravitons which appear in gravitation. They also move with the speed of light. So, whatever is mass less shall move with the speed of light and whatever moves with the speed of light shall be mass less. That is, its rest mass will be equal to zero.

If its rest mass is equal to zero, you will be never be able to enter the refrain of the particle. Therefore, you'll always be moving with the speed of light and this is indeed a very remarkable conclusion that can be drawn by combining Maxwell's equations with relativity and quantum mechanics.

Elementary relativity, elementary modern physics means we are not doing anything more complicated than that. Now in order to complete the cycle, what I will do is to stop this discussion for a while and go on to study yet another quintessential wave phenomena, namely interference.

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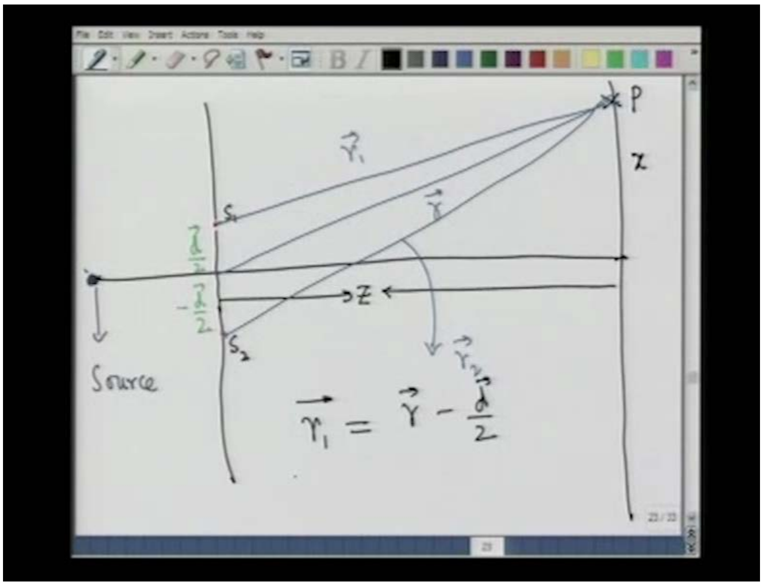

As I told you, we do not have the luxury of time to actually discuss interference in all possible aspects. What I will do is to look at the famous prototype experiment. What is that prototype experiment? The famous young double slit experiment. I will illustrate the principles for you and then what I will do is to give you examples of interferometers. Please take them as reading assignment and read them: Michelson stellar interferometer, Michelson interferometer, and then you have the famous Lloyd’s prism etcetera. You can take those examples then I will report to you I will tell you some very remarkable experiments involving interference, which when connected with quantum mechanics actually rather which when connected with Planck hypothesis will actually will give rise to a unbelievable, counterintuitive conclusions which led to the development of quantum mechanics and there we will actually conclude the course. So that when you start listening to when you start studying modern physics, you would have got some kind of a perspective as to how and why modern physics comes about.

So, let us start look at the interference phenomena. As I told you, what I am going to look at is the famous Young’s double slit experiment. In order to discuss Young’s double slit experiment, I have to make a diagram, and I have to make a nice picture. So, let us start making that, probably it is good to go to a new page and let me start writing. So, first of all I need this slit. So, this is my slit S one and this is my slit S two. You have a source of radiation somewhere here. So, you have a source of radiation somewhere here. So, this is your source and these are the two slits. I have to draw a screen now which I shall draw here. This is my screen and wherever this source is sitting, let me draw a line and this distance I will call as d rather z, I think that is a better notation. Let me call it as z; actually I am defining the coordinate system in order to discuss the experimental situation. Now I am going to choose a point p here and I am asking for the intensity. So, this point p is at a distance x from this point. So, this is zero zero, this is zero Z and this is x z. So, this is my z x plane.

Now, I have to look at the positions of the slit. So, let me do that now. Probably a good color will be this. This will be d by two and this will be minus d by two. I am putting a vector here because the position of the slit s one this is my slit s one this is my slits two is along the x axis right and the axis which connects the source to the screen is the z axis this is the z x plane that is what I have chosen.

I need a few more geometric objects. So, what I will do is I will take a line and draw this and connect it here and this is the vector r; obviously, as all of you know we are going to assume that the distance between these slits is very small compared to the distance between the slit and the screen; that is the approximation that you are going to make. So, this r is roughly the distance of the point P from the two slits but not quite so, because each of them is displaced with respect to each other. So, I need to draw two more lines. Let me draw it here. So, I have this, which I will call as r one and I have this line which I will call as r two. Please imagine that this is my straight line. Is that part clear to everyone? This is my r one and this is my r two and this is my r. So, what are the fundamental relations that I have? All of you can check that r one is r minus d by two. Please do not forget that d is along the x axis whereas, r two is simply given by r plus d by two.

Let me emphasize, my purpose here is not to discuss interference from the view point of waves as much as to show how to understand interference in terms of the superposition of electromagnetic waves, in particular the electric field vector. That is the reason why I wrote the expression for the intensity in terms of the electric field vector. What you have to now do is to write down the electric field at the point P. So, I have to write down the electric field at the point P. The electric field at the point P is by the principle of superposition coming from this and coming from this. That is, this will be E one of P plus E two of P.

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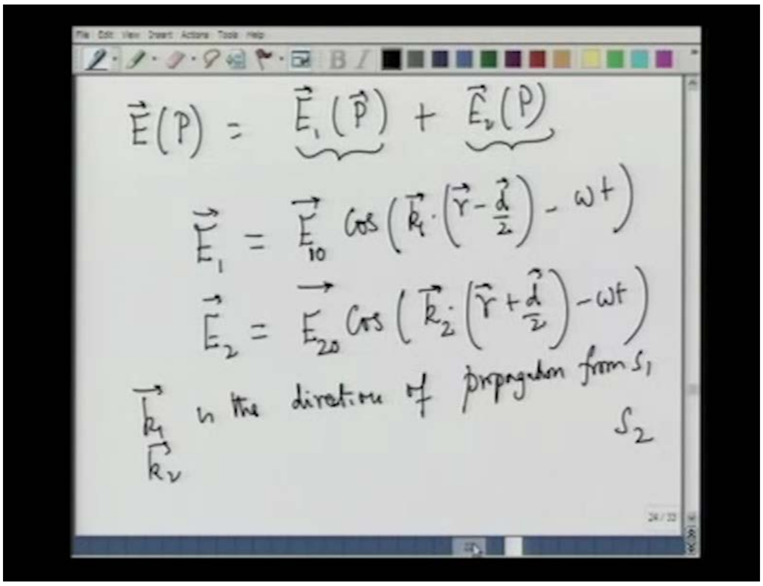

What is E one? E one is the electric field which came from the secondary source S one, that is the slit S one and E two is the electric field which owes its origin to the slit S two. So, if I know how to write down the electric field corresponding to this and the electric field corresponding to this, then we know how to add it up. So, let us start doing that. So, what is it that we have? I have E one, it is nothing but, let me write a vector E naught, then I am going to write cos of, now I should be careful with what I am going to write, k one dot r minus d by two minus omega t.

I have the direction of the electric field. And then my electric field E two will be given by, I need a better notation. So, what I shall do is to introduce E bar. So, E naught bar coming from the second slit into cos k two dot r plus d by two minus omega t. So, you have E one and you have E one naught. So, probably a better notation instead of the bar would be, let us improve upon our notation, I will call this as E one naught and I will call it as E two naught.

The naught refers to the amplitude; one and two refers to the source. What are k one and k two? K one is the direction of propagation from slit S one and k two is a direction of propagation from S two and if I were to indicate this, my k one is parallel to r one and ktwo is parallel to r two. So, that is what we have. So, these are the two directions of propagations and we have to plug these expressions. Now we need a few more geometric concepts. So, let me start introducing them. These are all quite simple. So, what I will do is, let me draw a line here and let me denote this angle by alpha. Let me draw a line here and let me denote this angle by beta.

So, when I am going to look at the components of r one and r two or correspondingly k one and k two, what I have to do is to express k one and k two in terms of the corresponding alpha and beta.

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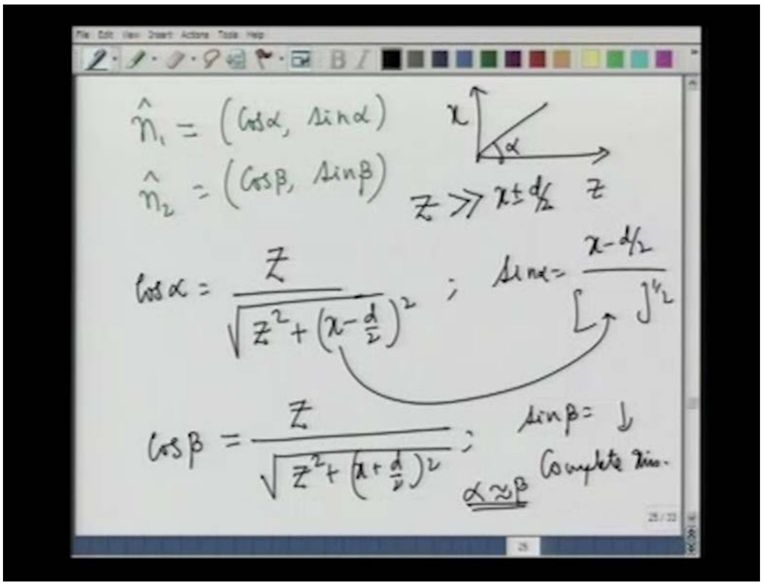

So, what will be the expression for k one and k two? That is the question that we are asking. So, let us start writing down. You people can easily see that n one is nothing but cost alpha sin alpha and n two is nothing but cost beta sin beta. So, how about writing this? I have my zx plane; this is the reason why I am writing a two dimensional vector and I am saying this is my alpha. So, nis cos alpha sin alpha similarly n two is cos beta sin beta.

What you have to now do is to actually try to see the difference between these expressions but once you give me this I know how to write down my expression for cos alpha and sin alpha because of the geometry which I introduced in the particular figure. So, you people can say what is my cos alpha that is nothing but z divided by root of r one squared and similarly for beta. So, let me go back to this page and let me write down the expression. So, what will be my cos alpha?

My cos alpha will be simply given by z over z square plus x minus d by two whole square; Pythagoras theorem sin alpha will be x minus d by two divided by half. Whatever was here will now come here now. Let me look at the corresponding expression for cos beta. What will this be? This will be the same thing: z over root z square plus x plus d by two whole squared and you can write similar expression for sin beta; complete this now. Let us not forget what our approximation was. Our approximation was that z is much much greater than x and d. Is that part, ok? z is much much greater than x plus minus d by two. If you did that, you will find that alpha is approximately equal to beta. You do not have to make any distinction between r one and r two.

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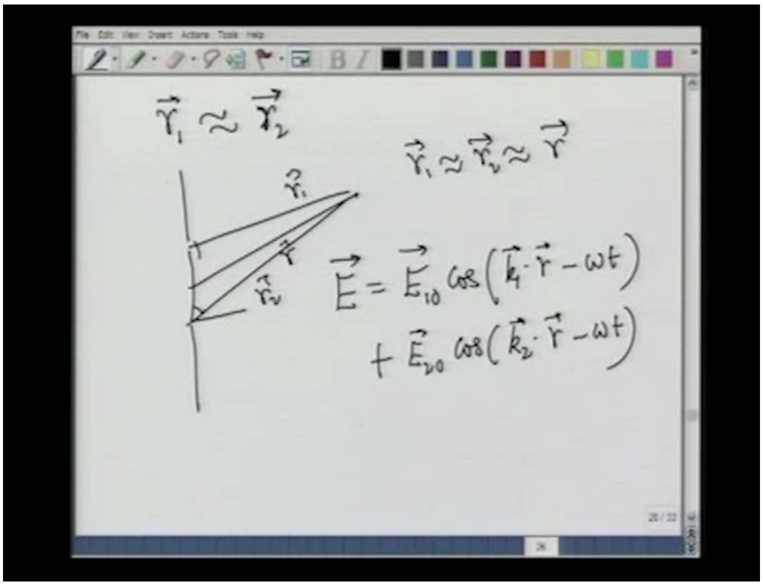

So, what we are going to now do is to replace. r one is approximately equal to r two. So, let me illustrate that in the picture again. You had your unit vector r here. This was your r one and this was your r two. We are saying that these angles are so small. So, we are saying that r one approximately equal to r two approximately equal to r;we are not going to make any distinction.

So, if you remember this long distance approximation that this screen is quite far away, then it is a very simple exercise for us to write down the total expression for the electric field at the point p. So, what is my electric field now? That is nothing but E one zero cos k one dot r minus omega t plus E two zero cos k two dot r minus omega t. We do not bother about what happens with it but we keep track of the signs of k one and k two, the pictorial sign behind k one and k two.

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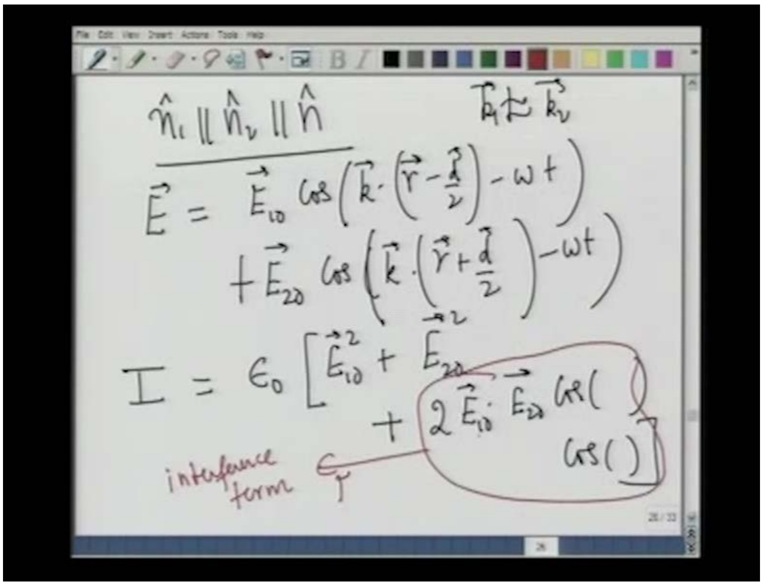

So, what do we conclude? If the angles are roughly the same, then roughly speaking, we will have n one parallel to n two parallel to n. That is, we make no great distinction between k one and k two. So, we shall now write the total field to be E one zero cos k dot r minus d by two minus omega t. That is, I have put k one approximately equal to k two plus E two zero cos k dot r plus d by two minus omega t; big bracket.

The rest of it is now standard. I am interested in the intensity coming from the electric field and this is something that you should notice. If I had a stream of particles, the energy would have been additive. The total energy would have been E one plus E two plus E three. But from the simple principle of superposition, energy is not additive but amplitude is additive, the field is additive. So, what do I get? I get the total intensity given by, apart from a multiplicative constant epsilon naught. Let me look at the energy. I will have E one zero squared plus E two zero squared. As usual, I am doing the time averages. Now, comes the most important expression: two E one zero dot E two zero into the product of the two cases which I am going to examine at some length. All of us will recognize this quantity to be the famous interference term. Now you see, whatever observation the experimentalist and the optics people had falls right into the place because if E one zero is perpendicular to E two zero then there is no interference. So, E one perpendicular to E two means no interference.

So, all that remains for me is to actually explore the consequences in these two cross products and show the bright and the dark fringes which you are familiar with, which I will do in one minute. After that what I would like to do is to describe an experiment involving interference, exactly the same setup but with some polarizers and analyzers thrown in. That will throw you a lot of surprise; that is what is called as a single photon interference experiment. I will show you some remarkable results and tell you why and how actually what we call as quantum physics or modern physics comes about and then we will conclude the course and that we will leave for the next lecture.