Engineering Physics – II
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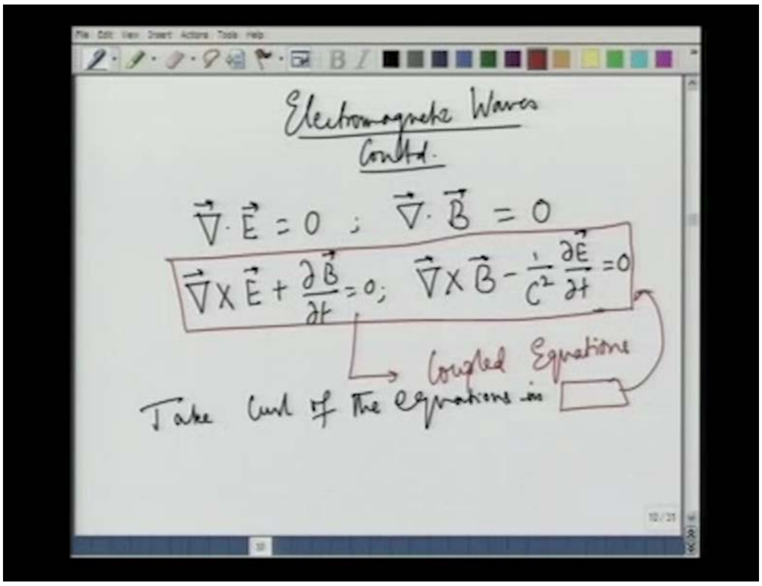
Module No. # 04

Lecture No. # 10

So, we have now been on to a very interesting topic namely the possibility of non-trivial solutions for Maxwell's equations in free space. So, this is some kind of a crucible test to find out whether the fields have significance on their own or not. As I have already told you, not only do they pass the test but they also yield us something very important and quite unexpected during the time of Maxwell, namely the identification of optical phenomena with a branch of electrodynamics.

As I kept telling you, this is one of the great unifications that was achieved by Maxwell. The earlier unification was by Faraday when he showed that the electric and the magnetic fields were related to each other, that they were not independent of each other because of the fact that time dependent magnetic field invariably produces an electric field. So, let me revise a little bit of what I had written in the previous lecture. (Refer Slide Time: 01:32)



We started with Maxwell's equations in free space. Divergence E equal to zero, divergence B equal to zero; no charge, no current, of course no magnetic charge. Then we write curl E plus delta B by delta t equals to zero and we have a generalized Ampere law: curl b minus one over c squared delta E by delta t equals to zero.

In this case the time dependent component of the magnetic field is going to produce the electric field. The time dependent electric field is going to be the magnetic field; therefore they are going to act as sources for each other. It is a kind of bootstrap. Why do I have a non-vanishing, non-trivial magnetic field? It is because I have a time dependent electric field. What is the origin of the electric field? The origin of the electric field is the time dependent magnetic field. So, the electric and the magnetic fields act as sources for each other and if these sources can propagate a certain disturbance quote unquote disturbance of wave, then we would have had what is called as a wave solution or wave equation. In order to accomplish that, what I noticed was that these two equations, that is, the bottom equations are coupled equations, linear coupled equations(( )) homogeneous end a and b. Now, I would like to decouple them. I would like to set up an equation involving entirely e and any equation involving entirely b. So, what is the trick to do that? The trick is take curl of both the equations in the box. My box is shown here; take the curl of both the equations and let us see what we get.

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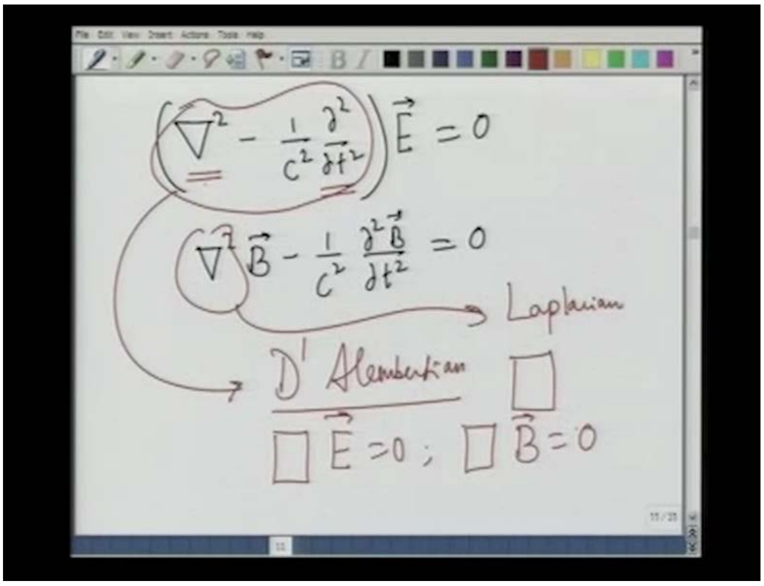

The answer is straight forward except that when we substitute, you should remember divergence E equal to zero and divergence B equal to zero. So, a simple home assignment problem for you people in vector calculus. If you did that, you get this very beautiful equation: del squared minus one by c squared, del squared by del t squared into E equal to zero. Now, you see we have made a transition from a first order differential equation to first order differential equations involving a and b; they were coupled to each other; by taking a further derivative we have been able to convert it into a second order differential equation involving only E. Well whatever equation E satisfies B also satisfies. Therefore, I will write del squared B minus one over c squared, del squared B by delta t squared equal to zero.

As I told you, these two equations are a set of six identical equations; the three components of the electric field, the three components of the magnetic field; all of them satisfy one and the same equation. Whatever I have indicated in the bracket here, this is a differential operator involving both the special derivative and the time derivative. That is the very important thing. It involves both special derivative and the time derivative and what we should remember is that, go back remember all that you studied about electrostatics; you get non trivial solutions, non-vanishing non trivial solutions if and only if both special dependence and the time dependence survives. So, this is given a name and this is called D'Alembertian. Let us not forget this by itself is called Laplacian. Laplacianrefers to the differential equation only in the special domain, but if you include the dependence both on space and time position and time then you end up with the D'Alembertian equation.

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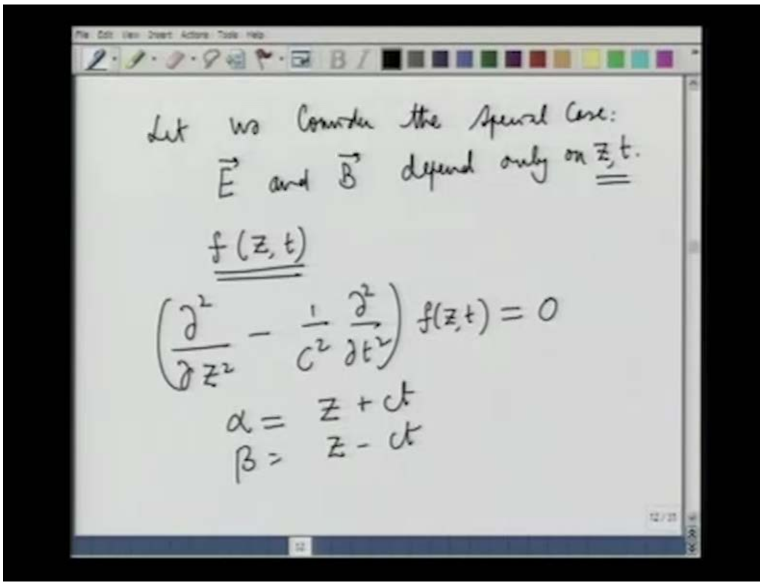

And generally this is denoted by a box. So, you can write divergence E equal to zero; for a D'Alembertian E equal to zero, D'Alembertian B equal to zero. It is written as a square box. What we want to do now is to explore these solutions. In order to simplify matters, what I will observe is that these equations are linear and therefore, the principle of superposition holds. The principle of superposition, after all holds in the original Maxwell's equations themselves. These are special cases of Maxwell's equations. Therefore, I can consider each individual case and I can write down most solutions.

To make my life simple, I will consider a solution which is dependent on only one coordinate, that is, what I started with. So, let us consider this special case: E and B depend only on Z and t; they do not depend on xy. What do we mean by that? As you move along the xy plane, the direction and the magnitude of E will be constant; nothing is going to change. The change will take place only when you move along the z axis. Of course, even at a given point, the field can change as time changes because it certainly depends on both of them. So, what we want to do is to put this special condition and see what happens to my equation now. I do not have to keep on writing E and B because every component satisfies the same equation.

So, let me generally consider some particular function of(Z,t). This could be any of the components of the electric field or of the magnetic field. What is the equation satisfied by f of (Z, t). The answer is very simple. The derivatives with respect to x and y drop out and what do I get? I get del squared by del Z squared minus one by C squared del squared by del t squared operating on f of (Z, t) is equal to zero. It means that I have to find solutions which satisfy this equation. Please notice that you cannot have a solution which depends only on Z and does not depend on time, nor can you have a solution which only depends on time and does not depend on Zunless you write trivial solutions like a Z plus. But these solutions are bad because they keep on going in time, they will carry an infinite energy etcetera. We are not interested in that. We want non trivial physical solutions.

There are many ways of handling these to explore that. One particular way of handling these is to introduce new variables.

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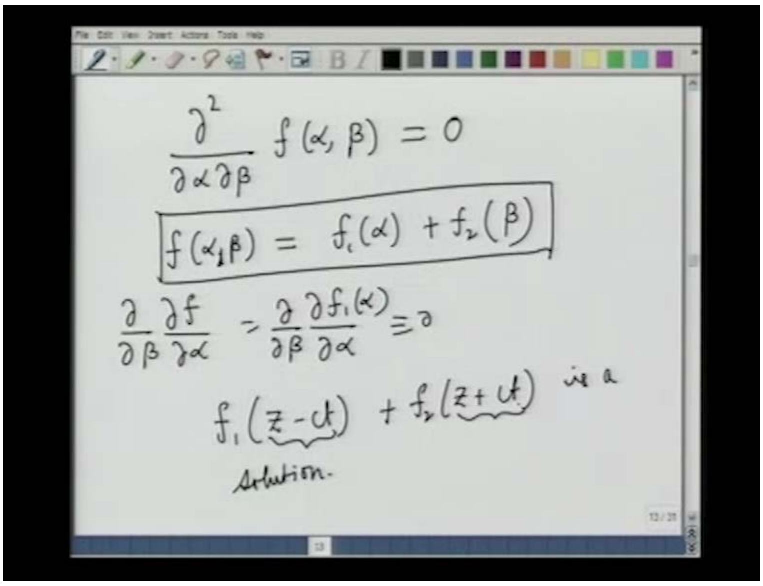

So, let me do that, let me introduce a variable alpha is equal to Z plus ct and beta is equal to Z minus ct. If I introduce these two variables, a little bit of algebra involving d by d Z and d by d alpha, then you have to consult the Jacobean of the transformation. If you did that which is a very simple exercise, what is it that is going to happen? I am going to get the equation del squared del alpha del beta. f is a function of Z and t, equivalently it is a function of alpha and beta; f of alpha beta is equal to zero and this is the equation that we are getting.

So, you operate first with respect to alpha and then with respect to beta or the other way round. I want to find out the most general solution corresponding to this equation and that is a very easy thing. I will write f of alpha beta is nothing but f one of alpha plus f two of beta. I will take a function, any arbitrary function please understand that. The solutions of this equation any arbitrary function of alpha and any arbitrary function of beta. How so? If I do del f by del alpha, I will get del f one by del alpha. The derivative with respect to beta f two will drop out because there is no alpha dependence and then if I did a del by del beta, this will be del by del beta. This is identically equal to zero because this is only a function of alpha. If you were to interchange the order by the same token, if I did a derivative with respect to beta, f two will contribute but then the derivative with respect to alpha will drop out. Therefore, this is the most general solution. So, what we have is a situation where we are not looking for one or two solutions like you did in the case of harmonic oscillator: a cos omega t plus b sin omega

t. No, we are getting an infinite number of solutions. This is perhaps the first time that you are encountering a class of solutions for these differential equations and I am telling you, give me any function of alpha or you give me any function of beta, it is identically a solution.

Now, let me substitute for alpha and beta, what is it that I am going to get? The solution that I am going to get is f one of Z minus ct plus f two of Z plus ct. This is exactly what I was telling you, that it has to be separately a function, it has to be depending on both Z and t. This analysis tells you that f depends either on the combination Z minus c t or on the combination Z plus c t.

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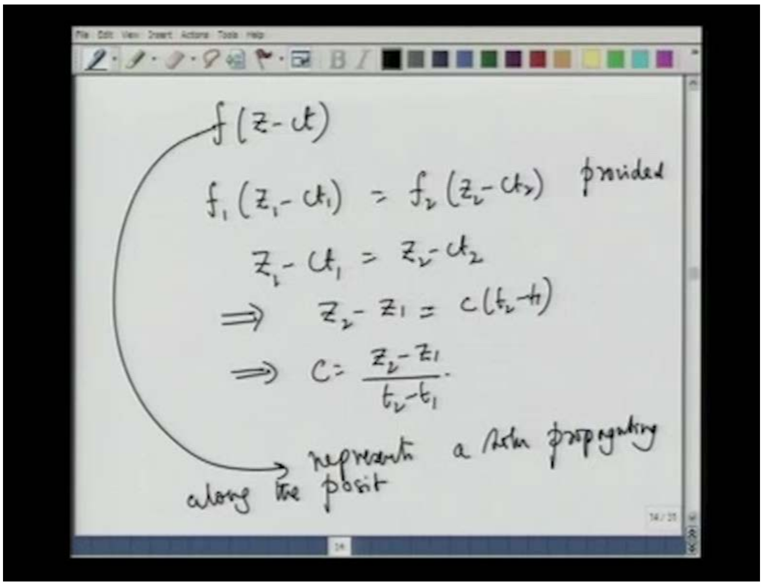

Now, let me look at the function which depends on Z minus ct. What does it tell you? It tells you that it does not depend on Z and t individually but on the combination of Z minus ct. What does it mean? It tells you that f one of Z one minus ct one is equal to f two of Z two minus ct two, provided Z one minus ct one is equal to Z two minus ct two. Because it is a function of the quantity Z minus ct, stare at it for a second and you will find that Z two minus Z one is equal to c into t two minus t one which implies c equal to Z two minus Z one divided by t two minus t one. In other words whatever was the nature of the solution at a point Z one at a time t one, will get repeated at a point Z two at a later time t two as given by the quantity c equal to Z two minus Z one t two minus t one, this is the velocity. In other words, this represents a solution propagating along the Z axis, along the positive Z axis. c is always greater than one; if t two is greater than t one, t two is greater than (( )) Z two is greater than along the positive Z axis with a speed c.

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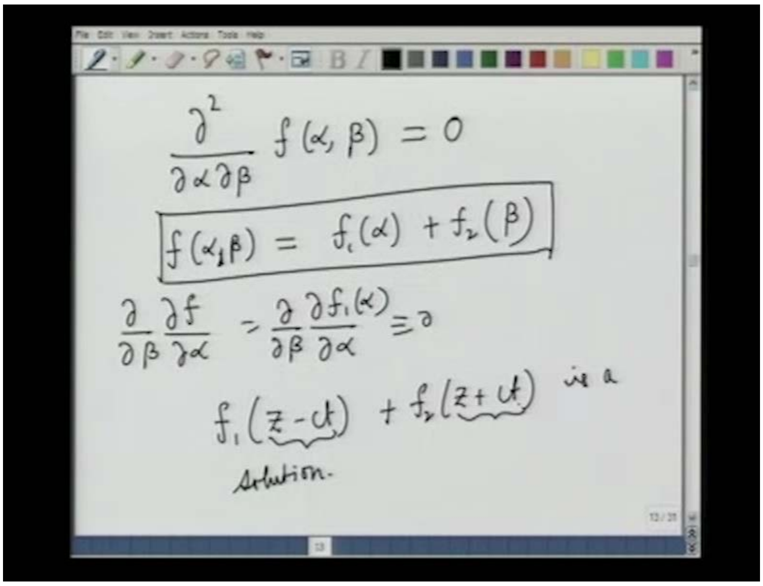

So, all solutions of the kind f of Z minus ct give you propagation with a speed c along the positive Z direction and therefore, by the same token, f of Z plus ct is a family of solutions. The most important words are propagating along the negative z direction'. Let me illustrate what I said. Suppose at Z equal to t equal to zero, my solution was like this. This is my Z axis; this is a t equal to zero. Suppose the way we are propagating, suppose I look at the solution corresponding to Z minus ct, this will be the same. So, this is my Z equal to zero. This is my Z equal to ct. This pattern will repeat at any given time t.So, in other words, at an earlier time it was somewhere here; no change in the shape, no change in nothing. At a later time it was here and at a yet earlier time it would have been here. Therefore, this solution corresponds to the propagation of the pulse along the positive z direction. In a similar manner, this corresponds to f of Z minus ct. If I were to look at f of Z plus ct, then it would have moved along the left.

Let me illustrate that here as well as I can do. So, here is a pulse. This is Z equal to zero and I have my t equal to zero. If were propagating along the Z direction, minus Z direction, this is Z equal to zero and this pulse would be here. So, if we treat them as identical, this will be Z equal to minus ct.In this case the pulse is propagating in this direction therefore; we have nontrivial (( )) solutions. So you now understand the meaning of the D’Alembertian, at least in the case when there is propagation along a particular direction, there is a dependence only on z and t but this does not exhaust everything that I have to discuss because I have to make the solutions consistent with the set of all the four Maxwell’s equations.

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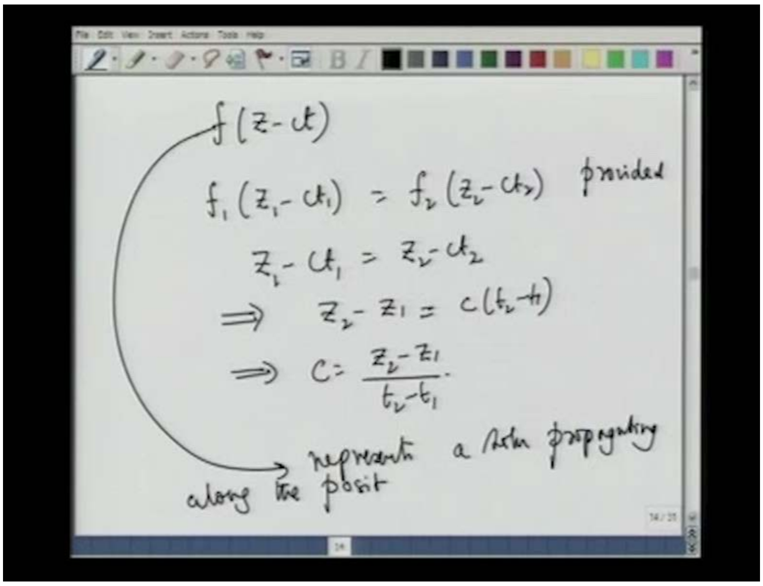

So, let me continue to look at a plane wave. So, for the first time I am introducing the concept of a plane wave. A plane wave is a solution which depends on only one coordinate. What is the coordinate that we have chosen? It is the Z coordinate. If I move along the plane perpendicular to that direction of propagation, which is what I showed just now it is a constant, nothing is changing; it is like a sheet. You will never know what is happening as you move along the x y plane that is the reason why it is called as a plane wave. What I will have to do after a short while is to introduce the concept of a monochromatic plane wave; we will come to that. That is what we are all familiar with when we do ray optics, lenses, dispersion, diffraction, reflection, and everything, almost everything is on the monochromatic plane waves. We just looked at a plane wave. Now I have to find further constraints on the plane wave by plugging into the Maxwell’s equations.

So, let me look at the electric component and please remember that the electric and the magnetic components cannot be independent of each other. So, let me write down the electric field. By this principle of superposition, it is sufficient to consider one solution;

you can keep on super posing later. So, let me take this to be some constant e naught vector into f of Z minus ct. So, I do not know the direction but I know its functional dependence Z minus c t. I ask what Maxwell's equations tell me in this particular case. May be I will do a slightly better job than that. What I will do is, I will remove this and put a unit vector n here. Please notice that for this solution which I am going to make it explicit, Z t the direction of propagation is Z and not the n. We are yet to find out the relation between the z axis and the n. So, let me explicitly mention here the direction of propagation is the k axis or the z axis. Now the first thing that I will do in order to get the solution is to remember the first of the Maxwell's equations namely divergence E equal to zero. What does it tell me? It tells me E naught is a constant and f depends only on Z therefore, delta f by delta Z; the unit vector n dot the unit vector z is equal to zero.

I am not using the notation unit vector i j k because I want to preserve k for the wave number which you are going to encounter very soon. Therefore, we find that the direction of the electric field is perpendicular to the direction of propagation. This is the propagation direction and this is the direction of the field. We have not made use of any specific consumption that it is z minus ct if you want to you could have written f of Z minus ct plus f two of Z plus ct.

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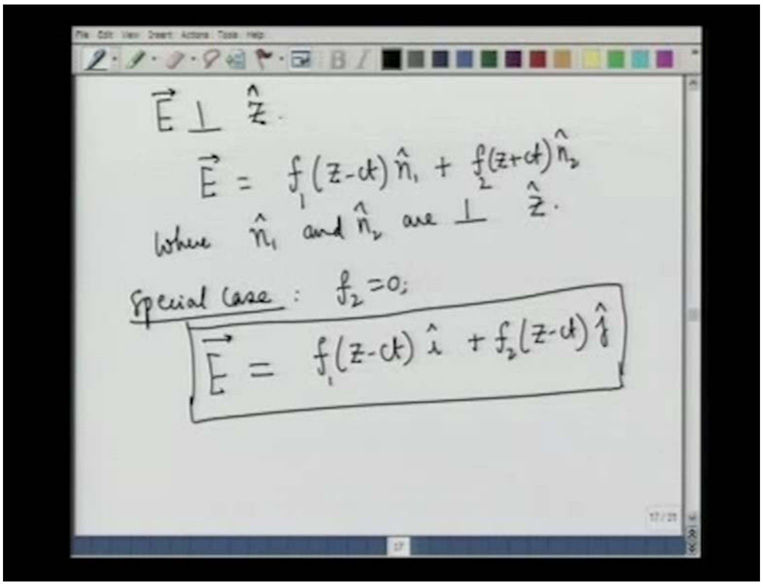

Even if you had added that term, you would have found this condition n dot Z is equal to zero for all times. Therefore, the first condition that we find is that E is perpendicular to the direction of propagation namely the Z axis. What is my most general solution for E? E is hence given by, f of Z minus ct n one plus f of Z plus ct n two. So, let me make it plus one f two n two where n one and n two are perpendicular to the unit vector Z. that Now, where I have a motion both parallel and anti-parallel to the Z axis, I will produce standing waves; I am not interested in that. Therefore, let me consider the special case, all the other cases can be obtained by the principle of superposition namely f two equal to zero. That is, if we want to have only the positive direction, then you give me this and I can write a slightly better solution. Now what will E be? My E will be f of Z minus ct i plus f two of Z minus ct j; that is what I am going to get.

There can, in principle with two different function but I am going to get i and j. Therefore, what I have is the electric field which is completely confined to the direction of propagation to a plane perpendicular to the direction of propagation. Now of all these solutions we know there are some very special solutions called the simple harmonic solutions. Now, I want to specialize myself to these simple harmonic situations because I want to be able to write down the relation between the wave number, the wave vector and the speed of the propagation. Therefore, let us look at the Z minus ct and let us write down a very specific form and then ask what the relation should be.

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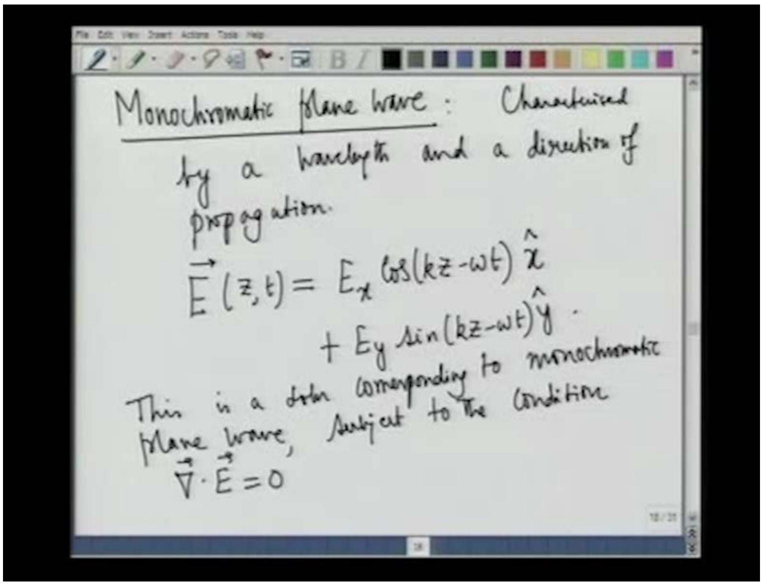

So, that brings us to what is called as a monochromatic plane wave characterized by a wavelength since we associate colors with wavelengths although it is a very tricky business it is called monochromatic under direction of propagation. If you did that, I want to write down the simple harmonic solutions; what how do they look like? My electric field will look like Z comma t; the most general solution will look like Ex cos kz minus omega t into unit vector along the Z direction plus E by sin kz minus omega t into the unit vector j or maybe I should put the unit vector iy. This is my solution. In writing these solutions, I have done a lot of simplification, face assumptions etcetera and that is something that you would have studied in great detail in your waves in your mechanics course. Therefore let us state this and let us ask ourselves what are the other conditions imposed by the Maxwell's equations. So, let me repeat whatever I have told you all this time, this is a solution corresponding to monochromatic plane wave subject to the condition divergence E equal to zero, that is the condition that we have imposed.

Now, all of you know the physical meaning of k and omega. Is that right? How do you analyze that? I am not going to work it out I will leave it as a problem for you people. You hold the time fixed and you ask how many times the wave vibrates at a given point therefore, k will essentially give you the wave number or the wavelength or equivalently you hold the position fixed. Let us say Z equal to zero and you ask how many times it vibrates in time, and then what will it give you? It will give you the frequency.

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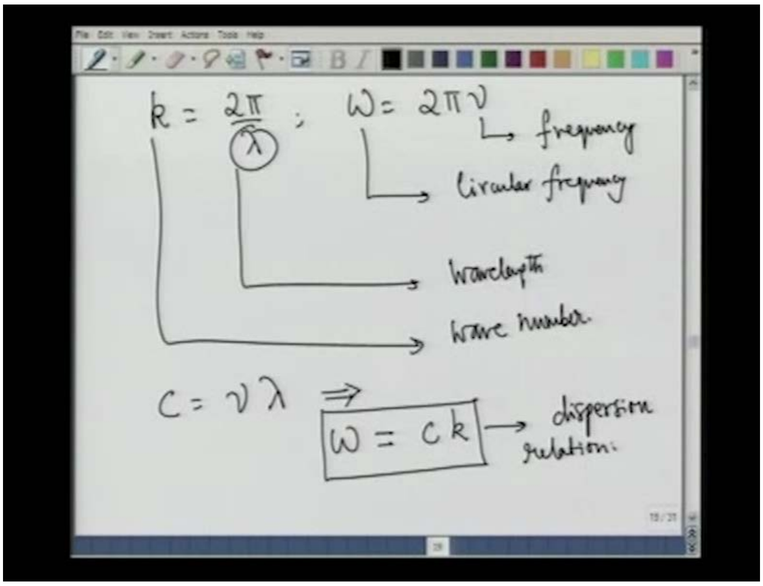

So, what is the interpretation? The interpretation is something that you people know very well from your waves whether it is transverse or longitudinal. My k is nothing but two pi by lambda and; omega is nothing but two pi nu. This is the frequency and this is what is called as a circular frequency, measured in radiance per second or whatever. nu is measured in hertz my lambda is nothing but my wavelength and what is my k. k is nothing but my wave number it has the inverse dimension of the length. There is only one thing that you should remember, it is that there is a certain ambiguity in the definition of k. Atomic physicists use remove the factor two pi and say k equal to one by lambda is my wave number, but it really does not matter.

So, you have the frequency and you have a wave number now the big question is what the relation between frequency and wave number is? We all know from this that c is equal to nu lambda. The frequency multiplied by the wavelength is the speed and the speed is given by c; I already argued for that. So, if I substitute for both nu and lambda, what is it that you are going to get? You are going to get the equivalent relation nu equal to omega equal to c k.

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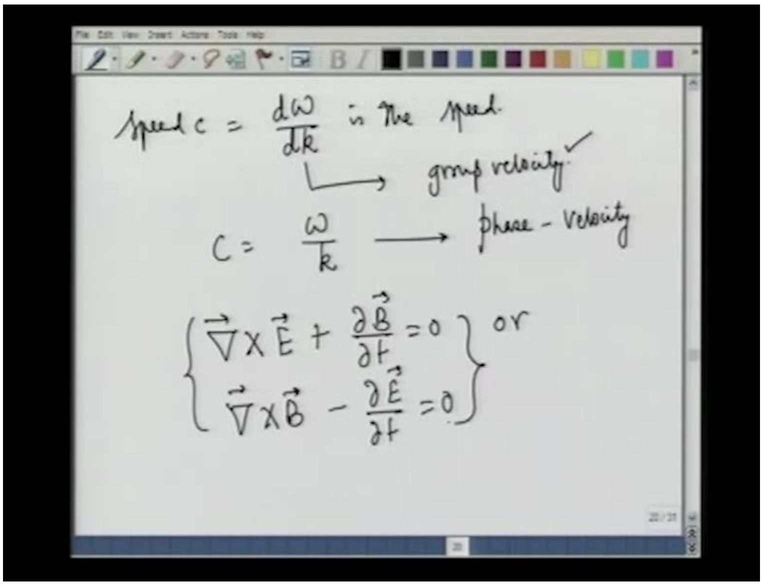

The wave number k is related to the circular frequency omega by this factor and this is called dispersion relation. What is this dispersion relation characterized by? My speed C is nothing but d omega by d k is the speed. We do not have time to get into that but in a more general context; this is called the group velocity. Let us not worry too much about it at this particular point; this is called the group velocity. Of course you could also have written C is equal to omega by k.

In the case of the simple plane wave characterized by a single frequency, a single wave number and a single wavelength, they would both be the same. This is what is called as the phase velocity. For a monochromatic dispersion free nice wave like this, phase velocity is the same as group velocity; otherwise in general the two are different. Normally almost everywhere if you want to find a speed of a wave which is propagating, we should always employ the notation definition d omega by d k and not omega by k. Those of you want to know more about it should study more about it. let us not get into this.

So, now we have established a relation between omega and k but still this is not the end of the story because I have fixed my direction of propagation and I have determine my electric field. What about the direction of my magnetic field? In order to determine the direction of the magnetic field there are two options open for you; either you use curl E plus delta B by delta t equal to zero or you use curl B minus delta E by delta t equal to zero. They will give the same information and I will leave that as an exercise for you people. How does one go about analyzing? Let me explain it to you so that you can finish this one start with the magnetic field.

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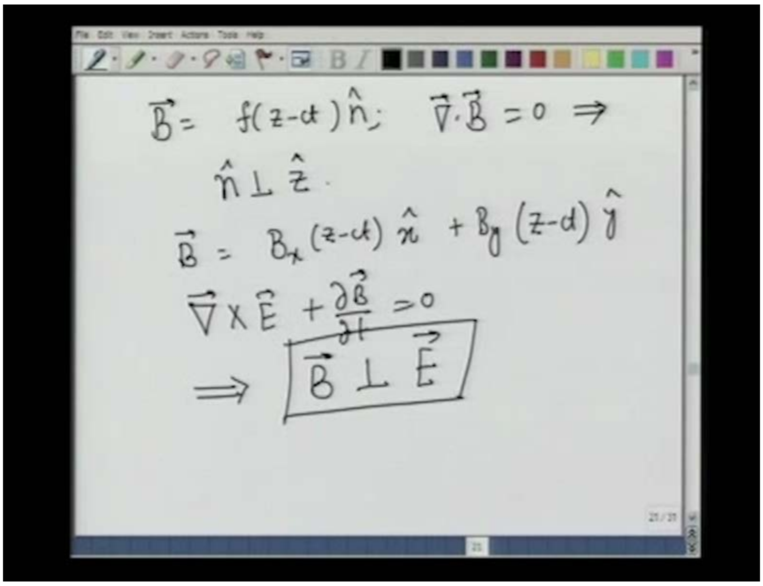

Now, the magnetic field will also be written as same thing f of Z minus ct into n; then what do you do? You will impose the condition divergence B equal to zero and you will conclude n is perpendicular to the direction of propagation. What you then do is to write down the solution for B as a linear combination of what the i and the j exactly what I did. So, my B will be Bx z minus c t into x plus By z minus c t into y; the Z component is not vanishing. What does it tell you? It is telling you that both the electric and the magnetic fields lie in a plane perpendicular to the direction of propagation, that is what it tells you and that is what we are saying. You take this and couple it with the equation: curl E plus delta B by delta t equals to zero. If you couple it with the equation curl E plus delta B by delta t equal to zero, you already know the form for E; i wrote a cos omega t sin omega t with kz throw in. When you do the curl you know how to do the calculation and compare the coefficients, what do you find? We find that b is perpendicular to e.

So, if you have a monochromatic plane wave propagating along a given direction, let us say the z direction, how do my solutions look like? My electric field will be in a direction perpendicular to the direction of propagation and at that instant, B will be perpendicular to both E and the direction of propagation.

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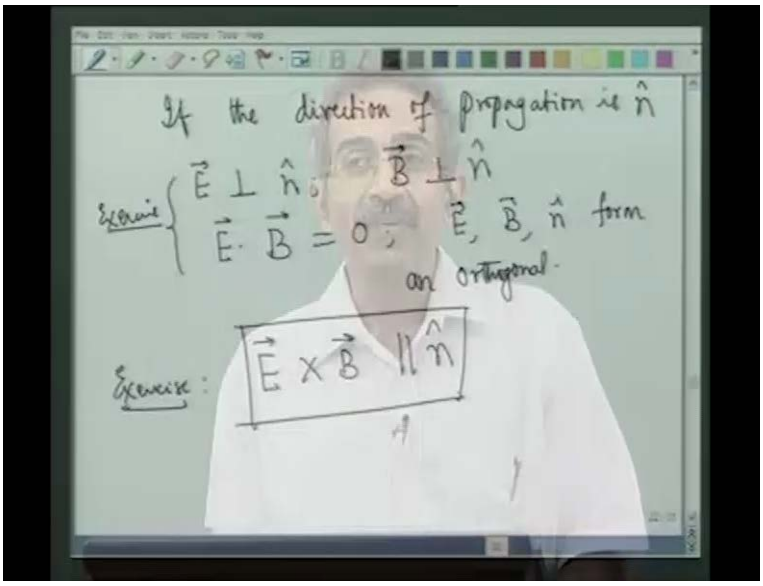

So, if the direction of propagation is n, whatever may be the direction of propagation; what are we saying? E is perpendicular to n, B is perpendicular to n. In our case the direction of propagation was Z axis. So, both E and wave are in the x y plane. The further condition is that E dot B equal to zero. In other words the electric field, the magnetic field and the direction of propagation, they form an orthogonal basis. Is that right? So, E, B and n form an orthogonal basis. What is an orthogonal basis? All of them are perpendicular to each other by ((k)) x y and z axis.

Now, I know that whenever they are perpendicular to each other, the cross product of any out of them will give the other one up except for a sign. So, this is an exercise that you people should do, a trivial exercise. Let me give the next exercise so that you do not get bored by my lectures; you should have something to do you can verify that direction of E cross B is parallel to n. It is not anti-parallel but it is parallel to n. If I take the cross product of the electric field and the magnetic field, then what do I get? I simply get the direction of propagation. So, one way of saying is that E, B and n form a right handed coordinate system. Is that right? The electric field, the magnetic field and the direction of propagation, they are related to each other by these things. So, please go home and work this out. These are very simple consequences but very profound consequences fantastic.

So, now we understand what it that is happening is. Remember the great controversy that existed ever since the time of Newton and Huygens. Newton believed that light was corpuscular in nature, that it was a stream of very tiny particles which are coming. Huygens believed that what we call as light is a wave and those days it was very difficult to distinguish between the two. Snell’s laws of reflection and laws of refraction were in favors of Huygens and not in favor of Newton. If you believe in Newton’s argument, the speed of light in a medium should be greater than the speed of light in vacuum but very careful measurement showed that speed of light in a medium is less than the speed of light in vacuum. In fact, we know something more today; the speed of light in vacuum is the maximum speed attainable by anybody, any material or article. Nobody can attain that. Actually that is the upper limit which can never be attained by material objects.

And another phenomenon that people discussed discovered was the polarization as if light had sides and for some reason Newton actually used the notion of polarization in order to argue that in nature because those days’ people did not have such a sophisticated notion of a wave, whether it was a transverse wave or a longitudinal wave. There was only one degree of freedom but here, there were two degrees of freedom and Newton thought that there should be attributes of particles and not of waves.

Newton was so influential. Newton had such a great stature. So, great was his eminence that people preferred to believe Newton rather than Huygens in spite of the evidence that

Newton was perhaps wrong. It goes to show the sociology of even doing science where all of us tend to follow the bandwagon. That is what happens but luckily things did not remain as they were. You know from your twelfth standard that there is the famous Young double slits experiment that conclusively proves that light has a wave like nature. In fact, Young did something more; he looked at two different transfers polar, two perpendicular polarizations. He superposed them and showed that there can be no interference at all. If two different beams have two different orthogonal polarizations and all that falls into place in our analysis. Is that right?

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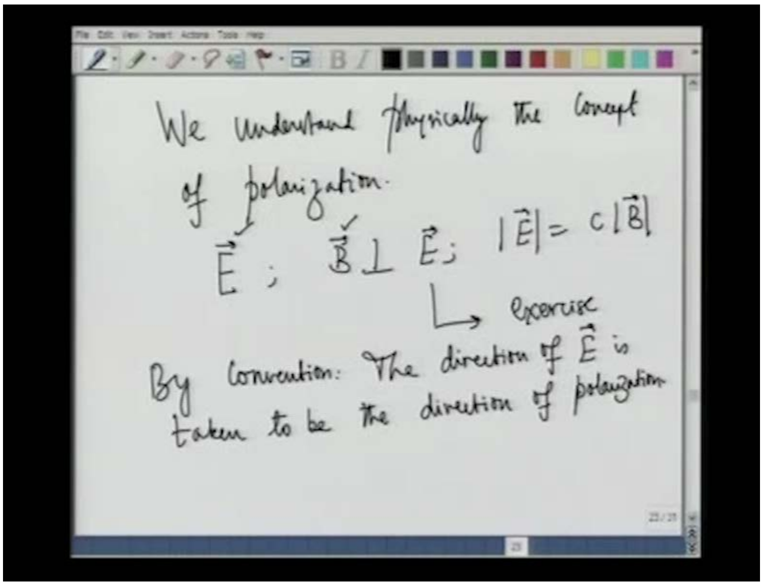

So, we see when I say that I have an electromagnetic wave and I want to identify that with light, it is not based on a numerical analogy, a numerical coincidence that the value of speed is given by three hundred thousand kilometers per second there and three hundred thousand kilometers per second here. No, there is much more than that. We understand polarization; we physically understand the concept of polarization. What is the direction of polarization? It is simply the direction of either the electric field or the magnetic field. If you give me the electric field I will give you the magnetic field, if you give me the magnetic field I will give you electric field. No big deal about that because if you give me the electric field, my B will be perpendicular to E. What about the magnitudes? My magnitude of E will be nothing but c times magnitude of B.

So, your next exercise is that simply follows from that equation curl E plus delta B by delta t equal to zero. There this factor of c which comes therefore, we understand everything about polarization, either you give me E or you give me B. By convention, that is the most important thing, by convention. The direction of electric field is taken to be the direction of polarization. So, please understand that can have only transverse polarization because the electric field can only be perpendicular to the direction of propagation. That is what we have found with this little bit of algebra, that the direction of E is taken to be the direction of polarization. So, how many kinds of polarization are there? There are as many polarizations as the most general solution for E which we can write in a second.

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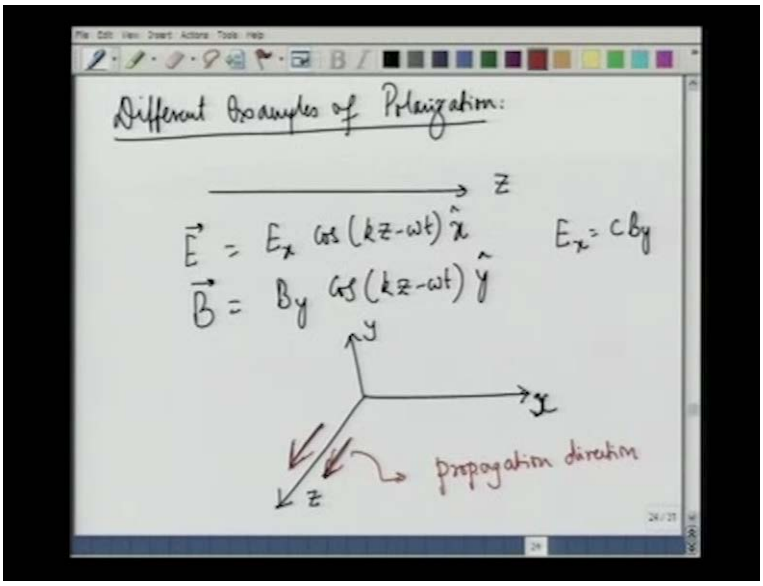

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Let us consider different examples of polarization, may be at the end of the lectures I will show you a few pictures which will illustrate all this; let us not distract ourselves at this particular point. I wrote down the most general solution. Let us take the direction of propagation to be z and suppose I take E equal to Ex cos kz minus omega t into the x axis. Now you see I do not want to use unit vector k along the z direction because I have introduced the wave number k. You will immediately find that B is nothing but By cos k z minus omega t into y direction. What is the relation between Ex and By? Ex is equal to cBy. What is the property of this? Let us do that by looking at the direction of propagation. So, let me take this to be the z axis, this is x and this is y, the wave is propagating along this direction. I should use this color. This is the propagation direction. We are looking at it at a given point as a function of time. So, if you want, you can imagine that I am sitting at z equal to zero and I am looking it as a function of time.

So, what is happening to the electric field? The electric field has a fixed direction and it is moving from plus Exto minus Ex; it is oscillating along the x direction, let me show that through this blue line. My electric field is oscillating along this direction. So, this is oscillation of the E field. It is like a simple harmonic oscillator cos k z minus omega t. So, if you imagined there was an ether there are some set of particles which are moving along the x direction as the wave is propagating along the y direction, that is the z direction. It is the notion we have of a transverse wave. What about the magnetic field? The magnetic field is oscillating along this direction B field. The direction of E is fixed forever, the direction of B is fixed forever and it is propagating, so we say it is a linearly polarized light. Linearly means the electric field is moving along a line, the magnetic field is moving along a line. It is a fixed line; it is not changing its direction therefore, this is what is called as linearly polarized light.

So, how many linear polarizations are there? You have a polarization along the x direction, we have a polarization along the y direction and you can write down their superposition. That will give you the most general (( )); this is a special case.

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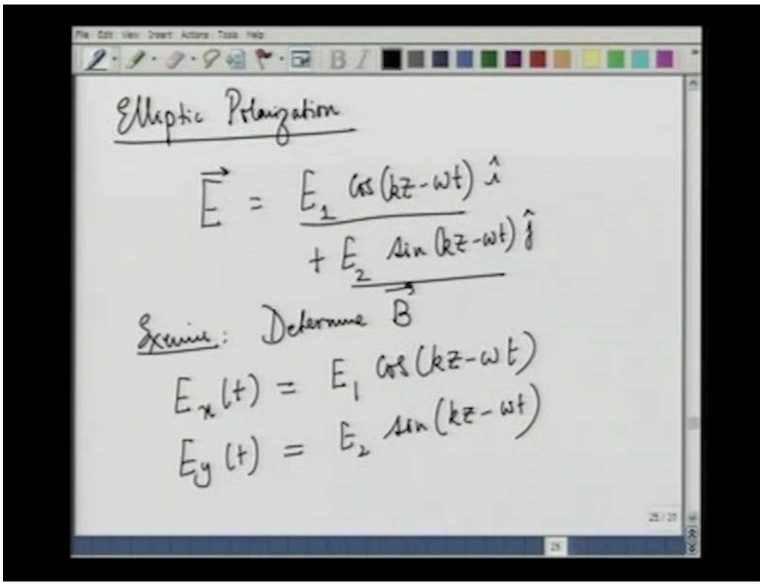

Now, without much ado let me look at the most general situation, look at what is called as an elliptic polarization. This is the most general case of the polarization, the special cases of these elliptic polarization will be the linear polarization and the circular polarization and by the way I should warn you people, in writing all these I have made very nice choices of the zero of my z axis, origin of my Z axis, zero of my clock and phase choices. Otherwise you will get an ugly equation and it is cumbersome algebra. There is no physics inside if you are going to do that.

How do I discuss the elliptic polarization? So, let me now write down the most general solution for E. I will write E equal to Ex cos k x minus omega t into i plus Ey sin k x minus omega t into j.i and j are the same as x and y. This is my solution I will leave it as an exercise for you people to write down B. B will have a very similar form except that E dot B is equal to zero; the same cos and sin will come.

So, here is an exercise determine B except that I should apologize I made a mistake. It is that I have no business to write kxi, I should write kz and same thing here, I should write kz. No problem. Determine B. Since E and the direction of propagation determine B completely, the polarization properties are by this form completely. Now what is this equation? You see, as a function of time both the magnitude and the direction of the electric field are changing. Is that right? Both of them are changing because there is a k z minus omega t cost. Suppose I specialize myself to z equal to zero, at time t equal to zero, this was along the I direction. This is because sin of zero equal to zero, cos of zero is equal to one. Then if I look, I am still sitting at z equal to zero. If I go to t equal to pi by two, then it would have rotated by an angle pi by two along the negative zy direction or whatever. So, here you have a situation where the electric field is changing both its direction and magnitude.

How do I fix that? It is very easy. What is that we do? You concentrate on this and you concentrate on this. You have Ex cos k z minus omega t and Ey sign k z omega t. So, what are we going to do with? We have to intelligently make use of the Cartesian geometry that we have learnt. We have to write down as an equation. I am writing it as E x and y. It is not a very good notation because all of x component is Ex into cos k z minus omega t. All of y component is Ey into sin k z minus omega t. This is simply the amplitude therefore, I will put a zero. E x naught E y naught to emphasize that it is a number therefore, what am I saying? I am saying the x component of the electric field is, let me call it as E one and E two to make my life simpler. Why complicate my notation E1 similarly this is E2. I have E1 cos k z minus omega t Ey of t is E two sin k z minus omega t.

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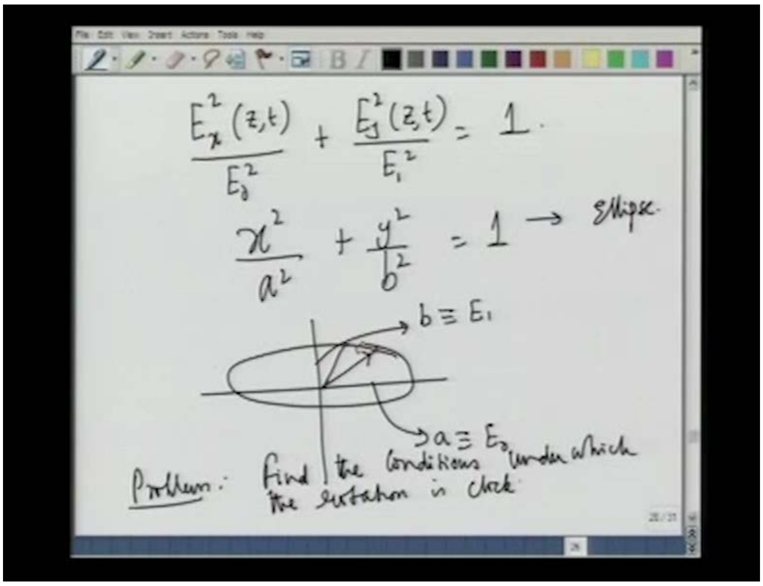

The tip of the electric field of course is the linear combination of these two. If I know Ex of t and Ey of t, then I know my electric field by the addition of this unit vectors. So, what am I going to get I am going to get Ex squared by E naught squared plus Ey squared z t by E one squared is equal to one. The right hand side is independent of time even as the electric field is moving along a particular curve. So, what does it remind you of? It reminds you of the famous equation x squared over a squared plus y squared over B squared is equal to one; ellipse.

So, now we know what the most general motion of the electric field in a plane perpendicular to the direction of propagation is. So, I have this; I have my ellipse. So, this is my b, this is mya. In this case b is identically equal to E one a is identically equal to E naught and of course depending on whether E naught is greater than E one or E one is greater than E naught. You get the so called oblate ellipse or the prelate ellipse, it really does not matter. This is my solution and my electric field rotates. So, let me show that for you here. So, that is where it is. At a later time it will come here. So, this is where it is and it is rotating. The rotation can be clock wise, the rotation can be anti-clockwise. That depends on the relative sign of E naught and E one. So, here comes a next problem for you: find the conditions under which the rotation is clockwise or anticlockwise. Well you know normally we say clockwise is positive anticlockwise is negative.

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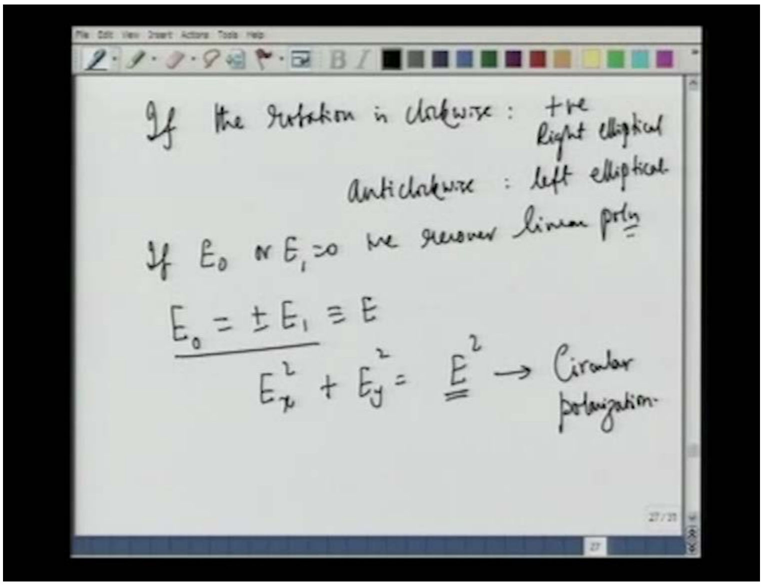

But in n this case we have to depend on the convention which was introduced by optics people long before all this was known if the rotation is clockwise, then it is positive or right sense, right elliptical. If it is anticlockwise then it is left elliptical. When does the ellipse degenerate into a straight line well when the semi minor axis goes to zero, it degenerates into a straight line. So, if E naught or even equal to zero, we recover linear polarization. This is certainly a better word then plane polarization but never mind. So, we have the linear polarization, people sometimes call it the as the plane polarization. If E naught is not equal to E one, then you have the elliptic polarization and the special case is E naught equal to plus minus E one. When E naught is equal to plus minus E one, what do we get? We get Ex squared plus Ey squared. This is equal to E squared identically equal to E.

This is the magnitude of my electric field and everyone identifies this to be what? The locus of a circle. So, this is my circular polarization. I am depending on the relative sign between E naught and E one. You have the right circular polarization or the left circular polarization. What is the most important thing that you have to notice here? The most important thing that you have to notice here is the principal of superposition therefore, I can build either the right circular polarize beam or the left circular polarized beam by superposing two linearly polarized beams but they should be off by a factor of phase of pi by two because cos of x plus pi by two sin pi sin x apart from the plus or minus sign.

So, if you can give a phase difference of pi by two with a suitable sign then I can get that conversely I can superpose two circularly polarized beams to get a plane polarization. There is nothing sacrosanct about either plane or circular or anything, any of them can be used as a basis because polarization is a vector and it needs two element basis elements. You can either chase them to be the linear polarization basis or the circular polarization basis and this is in fact the principle behind creating a circularly polarized wave. You sendone beam and then you rotate its polarization by half and then superpose the two of them together and then you are going to get a circular polarization. This is something that we understand very well here. So now you see the concept of the polarization witch appears to be somysterious before the advent of electromagnetic waves falls right in its place.

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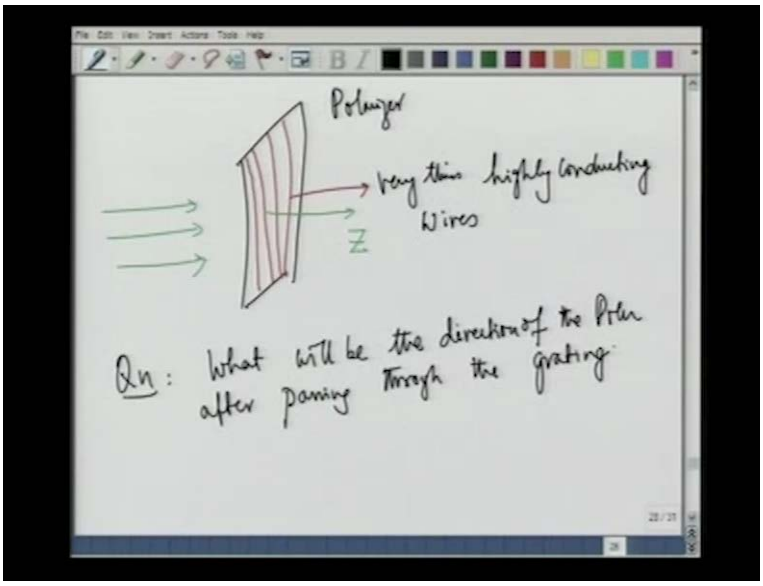

Now if it is indeed the situation, let us ask ourselves whether we understand the physics completely. One way to understand to know for ourselves whether we understand the physics completely or not is to pose a problem and let me give you a problem. So, this is a polarizer. It consists of a transparent material; I will do is I will draw these lines. Now, these lines are very dense running across the whole length of my polarizer. So, this is actually very thin, highly conducting wires. So, let me imagine the plane of this object is in the x y plane. So, this is my z axis perpendicular to the plane and here comes an un-polarized electromagnetic wave. It is an incoherent superposition of both the plane waves and what comes out is a polarized wave.

A good question to ask is what will be the direction of polarization after it passes through the polarizer. So, question: what will be the direction of the polarization after passing through the grating? There are two possibilities: the polarization can be along the axis which I will indicate, the other possibility is the polarization is perpendicular to axis but in the plane.

So, what is the question that we are asking? Is this direction question mark or what is the other color that we have used in this direction? I would not give you the answer. In order to answer this you need very little physics. In fact, whatever we have covered in last few lectures remember that these wires are conductors, remember that my electromagnetic consists of the electric and magnetic lines, remember that wires have free charges which are allowed to move etcetera, and remember that whenever there is a current set in a wire there is going to be a joule dissipation etcetera.

So, if you now intelligently use all the facts at your disposal, you will actually be able to answer the question whether this will be polarized along the x direction or the y direction. In fact, let me give it a name, let me call this x and let me call this y. So, the question is it y or is it x, I will leave that as a problem for you people.