Engineering Physics - II

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Module No. # 04

Lecture No. # 09

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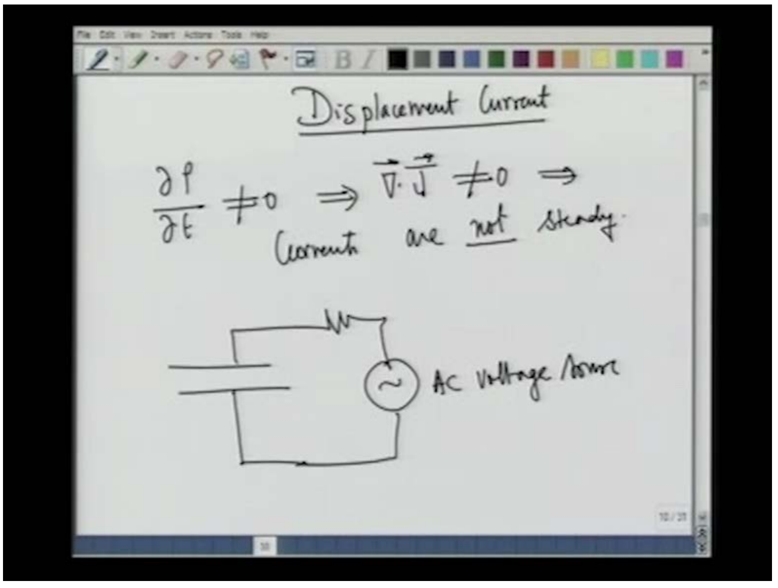

So, we promised ourselves in the last lecture that we will now look at the most general situation, where you also admit time dependent charge densities. So, when there is a time dependent charge density, delta rho by delta t is not equal to 0 and this implies divergence J not equal to 0, implies the currents are not steady.

Now, when the currents are not steady, what we have to do is to go back, look at the Maxwell's equations as we have as of now and ask, whether it is compatible with what the idea of the conservation of charge. Now, there is a standard motivation, which I would certainly like to give you. All of you are familiar with this, you have a capacitor plate and let us say, there is a resistance here and we have AC voltage source.

You people can see that you will run into trouble if you try to use Ampere's law for these expressions, because I can consider a loop like this and this is the surface that I am interested in. And what I can do is to try to evaluate integral B dot d l, which will give me the enclosed flux.

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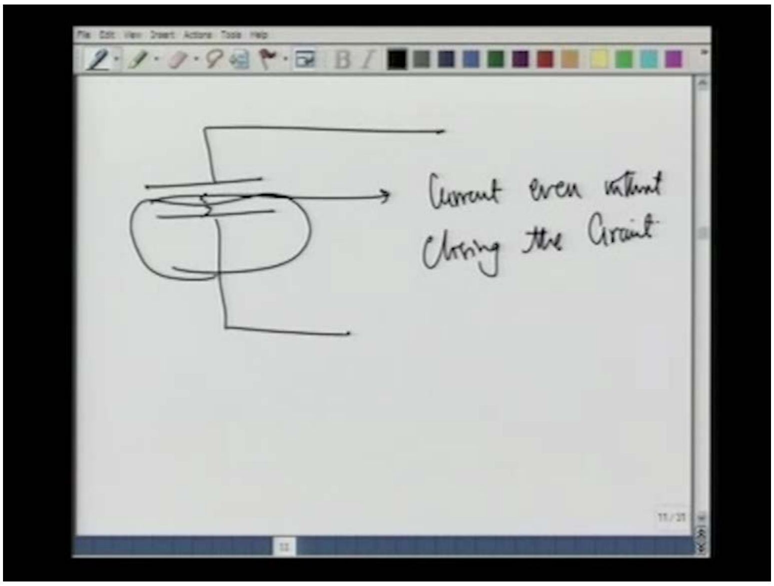

But on the other hand, I can also consider a slightly different situation, so let me write that section again. I can actually look at a surface, which is like this, which is open here. Now, I get into trouble because if I try to do curl B dot d S and integral B dot d l or whatever, the current enclosed this, the current enclosed along this particular loop is equal to 0. So, it becomes difficult to make sense out of Ampere's law because depending on the surface that you choose for that particular loop or depending on the kind of loop, that you have, you can get contradictory answers.

The point is that in this particular set up, there is a current even without closing the circuit. My discussion is a little bit fast-track in this particular instance and that is because as I told you, this is no example of mine, this is an example concocted by Maxwell himself and that is something that your instructors will teach you at great length.

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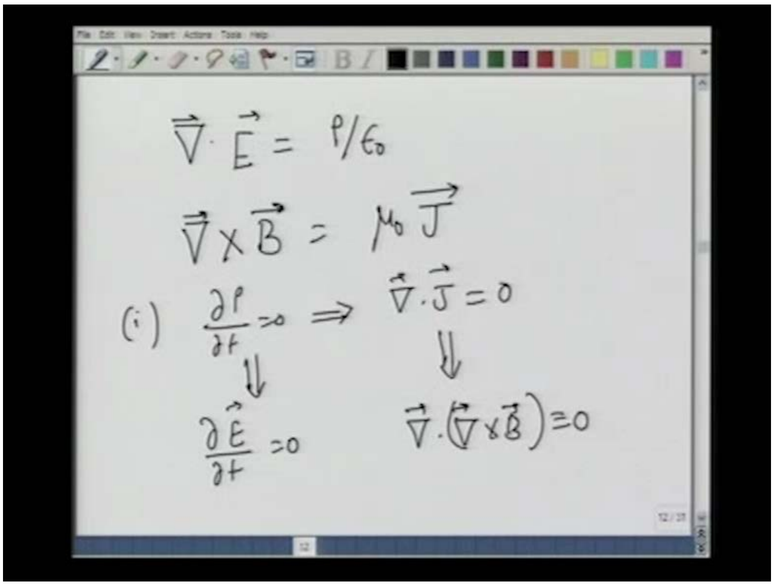

But on a more formal footing, what is it that I want to do? Let me list my equations. I have my divergence rho is equal to, sorry, divergence E is equal to rho by epsilon naught and I have curl of B is equal to mu naught J. I do not have to worry about the other 2 equations because as I told you, they are constraints, these are the source equations.

Now, suppose, my delta rho by delta t are equal to 0, case 1. This implies divergence J equal to 0 and that is consistent because this implies delta E by delta t equal to 0 and this implies divergence of curl B identically equal to 0. So, we are hope, there is no problem at all.

The real problem will start when my rho is a function of time. There is a time dependent charge density and there is an associated time dependent current density. So, let me rewrite the Maxwell’s equation, the relevant Maxwell’s equation again.

I have divergence E equal to rho by epsilon naught and I have curl B equal to mu naught J. Let us not forget the way we codify the way we express conservation of charge, that is a sacred equation, divergence J plus delta rho by delta T equal to 0, this is sacred like fundamental rights are sacred in a democratic society; this is a sacred equation, we do not want to violate that.

Now, if I look at these two equations and I start studying the properties, I have rho equal to epsilon naught delta E, therefore I have epsilon naught del dot delta E by delta T plus this is my delta rho by delta theta. I have to write my divergence J, but J is 1 over mu naught curl of B. Therefore, 1 over mu naught divergence curl of B equal to 0.

So, Gauss's law plus Ampere's law plus continuity equation implies this equation and you see trouble brewing in this particular expression because as I told you, this expression is identically equal to 0. Divergence of a curl is equal to 0; curl of a gradient is equal to 0; this is what we have been saying, therefore this implies, delta E by delta T equal to 0.

So, we are putting an unnecessary unphysical restriction, which is not allowed. We are asserting, that you cannot have a time dependent charge density and a time dependent current, which is silly because if I take an electron and if it make it, make it move, if I take charge particle and if we make it move, I know, that it will produce a time dependent electric field, I know that it will produce a time dependent charge density, there will be a time dependent magnetic field. Therefore, clearly, there is something seriously wrong with the Maxwell's equations. Seriously, something is missing and that is something that we have to explore at this particular point. A little bit of thought will tell you, that what I am telling you here is closely related to the previous picture, that I wrote. What is the previous picture that I wrote?

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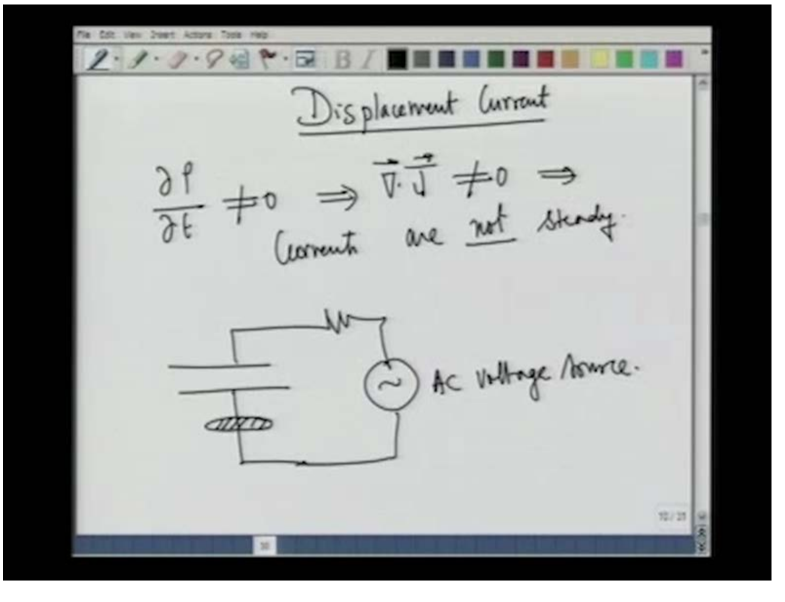

This picture, where we do not know actually how to calculate the integral I dot d l is that H dot d l, that is something you do not know how to calculate. Therefore, what I will now try to do is to ask, whether we can modify the Maxwell's equations in such a way, that it becomes consistent with the conservation of charge.

All this time we have experiments driving the theory, Coulomb, Cavendish made very, very careful measurements and they got the inverse square law. People like Olmsted made very careful measurements and Ampere and Biot-Savart variable to write it as the Ampere law. Careful experiments by Faraday, he remember the discussion that we had on the relative motion between the source and the loop, is that ok, between the source of the magnetic field and the changing flux and the loop that gave rise to Faraday's law of induction.

Here, our consideration is not driven by experiments, but driven by theory and that is how science and technology develops a new experiment, will give rise to a new theory and sometimes the internal consistency, the requirement of internal consistency, requirement, that whatever I do, be consistent with well known, well established laws like conservation of energy, conservation of angular momentum, conservation of momentum, conservation of charge, but also gives rise to what new physics, and that is exactly what Maxwell did.

In other words, now we are in a slightly different situation from what it was all this time. It is the theory that is going to drive the experiments, ok, very well. Now, if my theory is going to look at the experiments, let me go back and rewrite my Maxwell's equations again.

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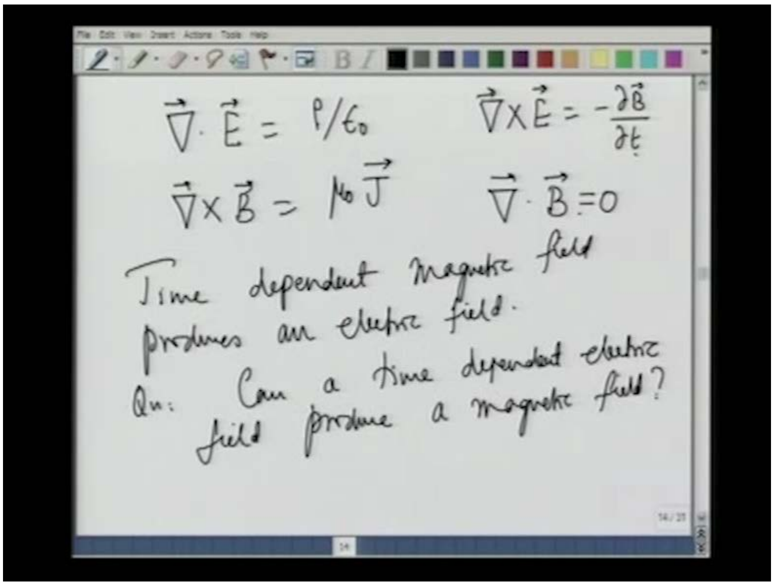

There is very little of algebra to do in this lecture, but there is really deep profound physics, profound for us to appreciate. So, let us enjoy it, it is really worthwhile doing that. Let me write the Maxwell’s equations, I have divergence E equal to rho by epsilon naught. I have curl of B equal to mu naught J and as I emphasize for you people, to you people, that when it comes to the electric field, there are two kinds of sources for the electric field, one is the material chargers, which will produce a field, which is curl free and other is changing magnetic field, that is the famous induction law. So, let me write it here.

I have curl E equal to minus delta B by delta T. For those of you who are familiar with the Fourier transform or whatever, I will give you a jargon; others might completely ignore this. The charge density rho produces what is called as the longitudinal component of the electric field; the time dependent magnetic field produces what is called as a transverse component of the electric field; longitudinal component of the electric field, transverse component of the electric field; that is what I have.

There is no longitudinal component of the magnetic field because there are no magnetic charges, so I have this equation. So, let me contrast it with divergence B equal to 0. Divergence B equal to 0 tells you, that there is no longitudinal part. Now, the first observation, of course, is that these equations, as we have written are in contradiction with the idea of the conservation of charge.

The other thing that you notice is that you have a time dependent magnetic field producing an electric field. So, let me write it explicitly, time dependent magnetic field produces an electric field; even at locations where there is no magnetic field it produces an electric field.

The equations are all, of course, very similar to each other otherwise. So, the question, that we ask is, can a time dependent electric field produce a magnetic field; should a time dependent electric field be producing a magnetic field?

Now, we can put all the pieces of the jigsaw puzzle together and ask, whether I can admit such a situation there, a time dependent electric field can actually produce a magnetic field, such that my electrodynamics, electromagnetism as written by Maxwell's equations would be consistent with the conservation of the charge? That is the mission that we have.

The answer to that is very clear because all that I have to do is to look at my previous thing. I have divergence of delta E by delta T; I have mu naught J. So, if I add an extra term such that this term gets a cancellation coming from that extra term, then we have, say, out of conservation law. And it is clear to us while looking at this particular expression, that a time dependent electric field could also produce a magnetic field.

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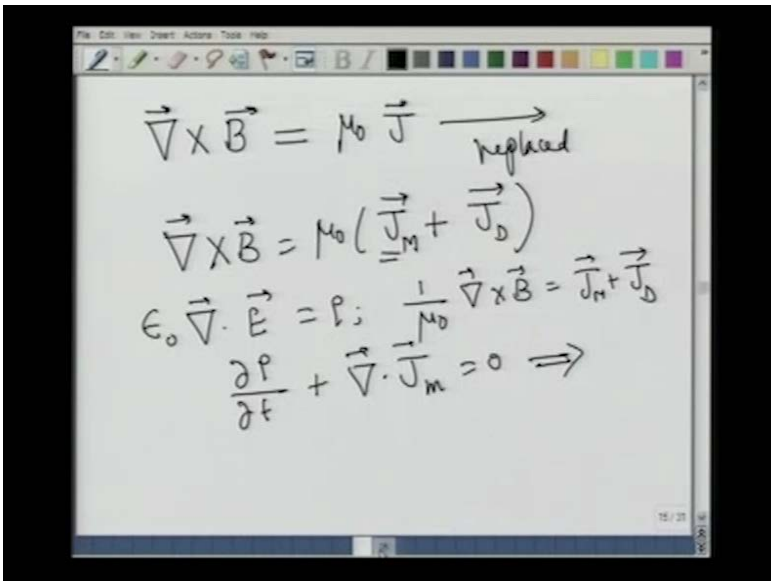

So, let me write that in words. What we shall do is to look at the Ampere's law and generalize it; curl B is equal to mu naught J will be replaced by the generalized Ampere's law replaced by curl B is equal to mu naught J plus. I will want to introduce another current, I will call it as J D and whatever contribution comes from this, J D should cancel the contribution coming from delta E by delta t.

Obviously, this J D cannot come from any material source because whenever, there are current charges in motion, all those effects are taken into this. So, if you feel like, I will call it as a J matter, like rho matter. I have my J D, therefore now all that you have to do is to compare the left hand side and the right hand side; let me do that, there is no harm interpreting it. What are the equations that I had?

Divergence E epsilon naught, divergence E is equal to rho 1 over mu naught curl of B is equal to J m plus J D delta. Now, again, we wrote rho by delta T, etcetera, etcetera and what is my driving equation, delta rho by delta t plus divergence J m equal to 0.

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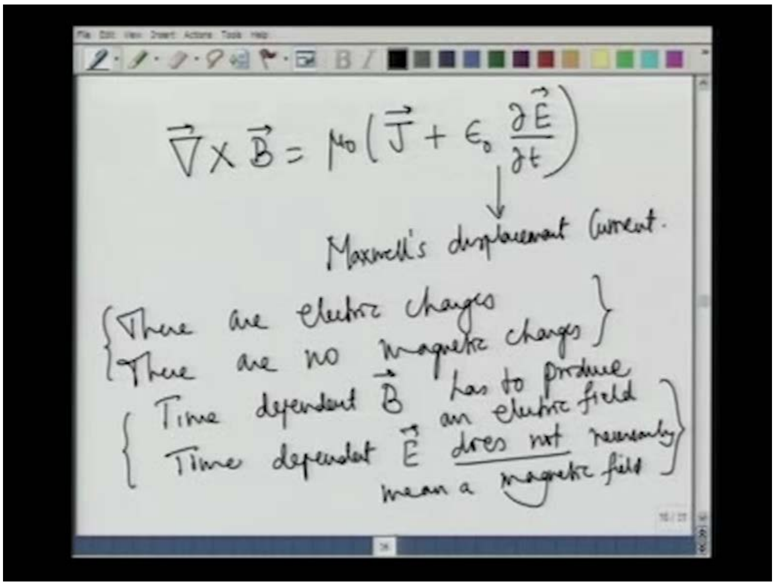

All this imply, it is a matter of straight forward substitution, that my Maxwell's equation get transformed into the following equation, curl B is equal to mu naught J plus epsilon naught delta E by delta T, and this was introduced by Maxwell and it is called Maxwell's displacement current.

This was a theoretician’s construction, you should remember, that Maxwell himself was a very, very great experimentalist, not only being a great theorist. In his Treatise on Electricity and Magnetism, which is remarkably fresh book even today, he says, that he was able to do careful experiments, where he measured the distribution of the charge density on a conductor electrostatic case. He was not happy to write some simple boundary conditions, like the way we solve our problems and propagate it, of a, as a book or as a theory. In fact, he did experiment to verify every one of the consequences coming from Coulomb’s law boundary conditions and it is only then, that he was able to write Maxwell’s equation.

So, now, what Maxwell is doing is to try to save the conservation of charge, which is very, very sacred. Therefore, we introduce the displacement current. And now, you see that even if there is no material current, even if J is equal to 0, a time dependent electric field can produce a magnetic field. So, this is the symmetry that you have established. But in doing so, we should not hasten to conclude, that there is all kind of symmetry between electric and magnetic field; that is incorrect. How? Because we have established the symmetry to the extent, that a time dependent magnetic field produces an electric field; a time dependent electric field can produce a magnetic field. But there, the analogy is even, this analogy is actually incomplete; there are electric charges, there are no magnetic charges, this is the 1st contrast.

The 2nd contrast is something, which is deeper and which is not easily appreciated. Time dependent magnetic field B has to produce an electric field, produce an electric field, produces a figurative has to be accompanied by an electric field. You cannot have a time dependent magnetic field without an accompanying electric field that is Faraday’s law; curl E plus delta B by delta equal to 0. But time dependent electric field does not necessarily mean a magnetic field. Let me illustrate it with a following problem. This is actually a problem, which I have worked out in some other context.

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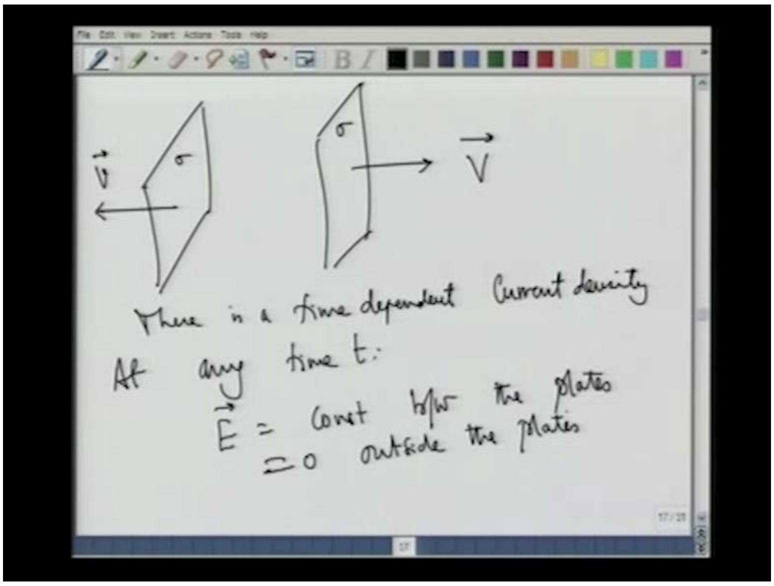

So, what we shall do is to take two plates, infinite plates, this carries a surface charge density sigma; this carries a surface charge density sigma. This is moving with a velocity v along the z direction perpendicular to the axis, along the axis perpendicular to the plane. This is moving with a velocity v in the opposite direction.

You people can easily see that there is a moving sheet of charge; therefore, it is producing a current. So, there is a time dependent current density. In fact, the current at any given time is localized only on that particular sheet. Now, I ask you, what is the electric field and what is the magnetic field? These are infinite sheets, so what do you answer?

So, at any given time, at any time t, E equal to constant, that is independent of z between the plates equal to 0 outside the plates. So, as a function of time, my electric field is a function of time at time t equal to 0. It was concentrated in this region at time t equal to 1; it is concentrated here at time t equal to 2; it is concentrated here. Therefore, if I were to write a profile, how does it look like?

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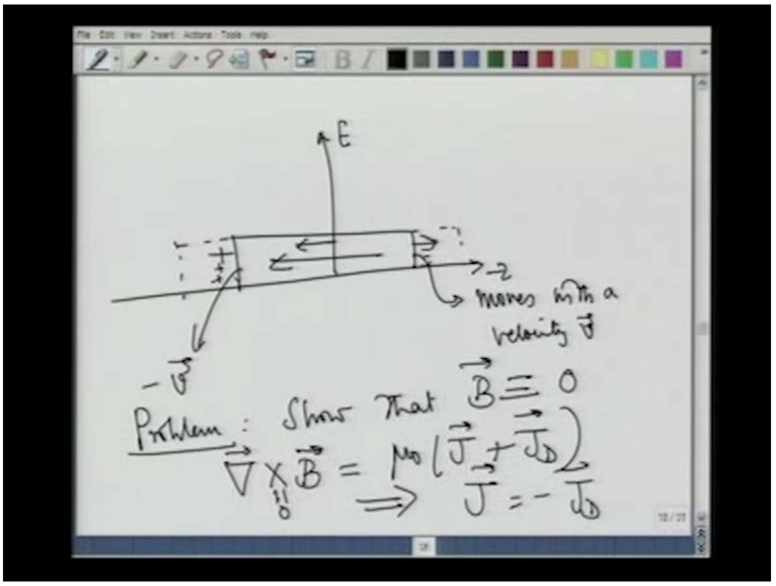

My electric field is like this, this as a t. At a later time, the field will be here. In other words, this boundary of the electric field moves with a velocity v and this boundary moves with a velocity minus v. Therefore, I have a time dependent electric field. Now, I ask what my magnetic field is? So, I have my E and then I have my, as a function of z, E is along this direction here, so the electric field. So, suppose, this is positive and this is negative, maybe I should go back and look at it. If this is plus sigma, this is minus sigma because I want the field to be only between those 2 areas, so electric field is from plus to minus. So, the field is in this direction.

What about the magnetic field? Here is a problem for you – show, that B identically equal to 0. So, what is happening in this particular scenario? I have curl of B is equal to mu naught J plus J D; J is not equal to 0, J D is not equal to 0, this is equal to 0, implying J is equal to minus J D.

This is a very cute example, very nice example. Therefore, I invite all of you people, all the students to work this out and convince yourself (( )) example and a very simple example, and that will actually illustrate the limitations to the analogy between the electric field and the magnetic field. It is certainly true as we will see in a short while, that electric and magnetic fields are sources for each other and those of you who will pursue higher studies in physics and study relativity, will also realize, that electric and magnetic field components are facets, two different facets of the same force, called the electromagnetic force. That is also true, but then, that does not mean, that I can replace electric field by magnetic field everywhere or by magnetic field by electric field everywhere, that is something that we have to do both in mind and that is the point that I have been trying to emphasize at this particular point.

We have very little time and we have the subject matter of electromagnetic waves to discuss. Therefore, I will not spend too much time on these displacement currents, except that I request you to solve all those, 3 problems, 2 problems - one was the open circuit problem and another was the problem of receding sheets.

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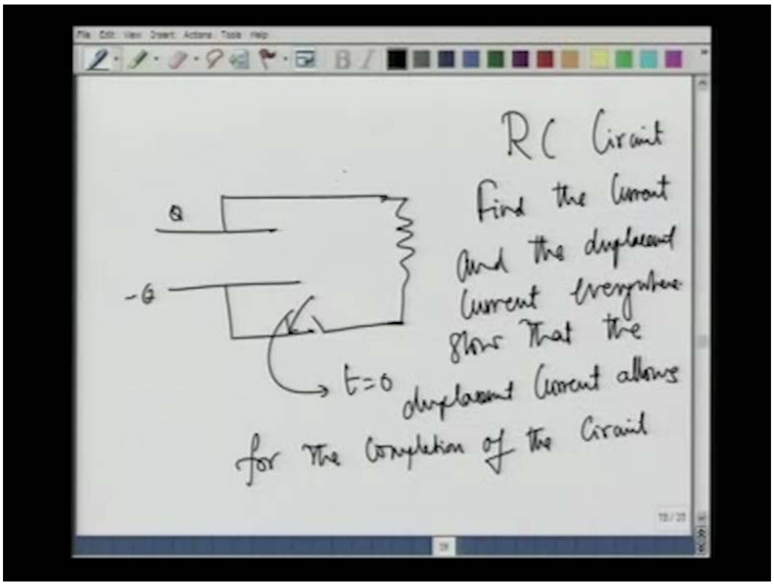

Let me make the problem of the open circuit even better. So, what I will do is I will take a capacitor plate, very well, and then, there is a resistance and let us say, it was charged to Q and minus Q initially. Now, let me imagine, that the circuit was open. So, there was a charge plus Q sitting here, there was a charge minus Q sitting here and that time t equal to 0, I place the circuit. I did not solve any problem involving networks and things like that, but all of you know how to set up the equation for this. This is a great RC circuit, the current starts flowing, the charge starts decaying, there is a decay constant for this as soon as I close the circuit, is that ok? Now, the question is, find the current and the displacement current everywhere. Show that the displacement current allows for the completion of the circuit. So, if the charges flow, whatever it is, you do that, so that it allows for the completion of the circuit and this should adequately illustrate for you, whatever is it, that we wanted to say about the displacement current.

What I now wish to do is to pause for a while and ask myself, what is it that we have done in the last 24 or 25 lectures? We started with the idea that electrodynamics involves charges in currents. So, in some sense, if I were to write down the expression for the Coulomb force, the Coulomb's force tells me, what is the effect of one charge on the other charge and that was given by the inverse square law.

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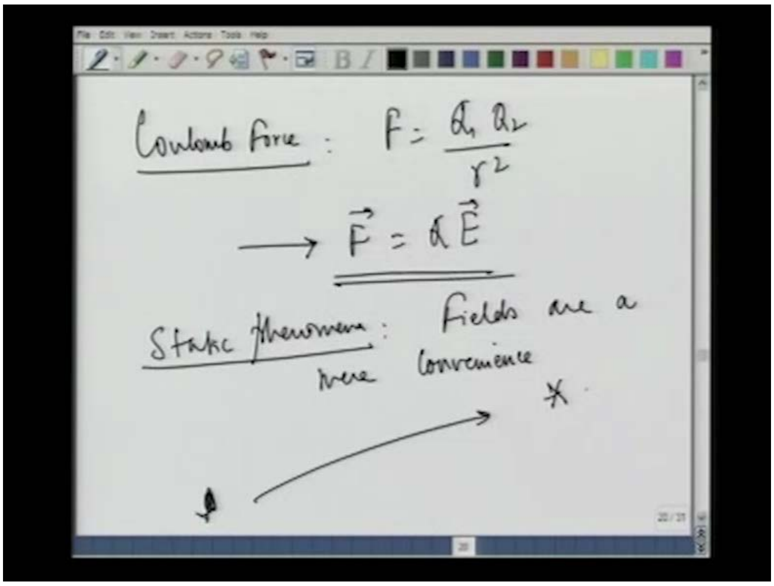

So, I wrote Coulomb force F is equal to Q 1 Q 2 by r square, let us not worry about the signs. So, what we are saying is that if I have a charge Q sitting somewhere, Q 1 sitting somewhere and if I bring a charge Q 2 at a distance r, then this charge Q 1 exerts a force given by Q 1 Q 2 by r square on the other charge of course, in around the direction, which is the line joining the 2 charges.

Then, we ask ourselves, what happens if we have a large number of sources? Then, we said, that there is a principle of superposition and we said that find out the addition of all the forces. So, from here we are able to move to the concept of a field, there we wrote F is equal to Q E. This was a force acted upon by one body on another body, whereas here, when we wrote F is equal to Q E, we have the concept of a field, which is acting on a source. We do not ask where are all the sources located, we only ask the question, a local question, namely, what is the electric field at this point and given the electric field at this point, please tell me, what is the force acting on the charge particle and that is given by F equal to Q E. And the way we proceeded the, using the principle of equivalence starting from the Coulomb law to the concept of a field was that the field was a convenient mnemonic, it was a convenient way of dealing with whatever is happening, it was not an independent quantity.

Now, whatever we did with Coulomb force is also true for the Lorentz force. We have the Lorentz force expression v cross B. then, of course, you have the Biot-Savart law, you put them together. So, as long as we are dealing with static phenomena, so long as we are dealing with static phenomena, what are static phenomena?

Stationary static charges and stationary currents fields are a mere convenience; fields are a mere convenience. In principle you do not have to introduce the concept of a field at all, you write down any elementary current loop, every elementary charge and find out the forces between them, and you can work out the whole thing. Why should I bother about the induction of the concept of a field?

But on the other hand, if you go back and look at the history of electrodynamics, invariably, without an exception, Faraday, Maxwell, Fresnel, Frizo, all these people seriously believed in the concept of a field; one person who very, very seriously believed in the concept of a field was Faraday because people believe, that there should be an all pervading medium. If there is no medium, it is very difficult to ask and answer the question, as to how a body sitting at one particular point can influence the body at another point.

What Faraday, Maxwell or Fennel argued was that if there is a shaking or a wobbling here, it creates a disturbance in the medium. The disturbance propagates and it comes and hits the test particle or the test current and that is what is felt as the force.

So, the concept of a field was something very, very important from an intuitive view point. But the way we have developed our subject, it appears more like an appendage, it appears like a convenience and not an intrinsically important quantity. But then, if you remember, what has happened in the last two lectures, that is not true because we have been arguing that a time dependent magnetic field can produce an electric field. It actually, it will always produce an electric field and a time dependent electric field can produce a magnetic field.

You see, it is a matter of new point. So, you take a new point, you work out the logical consequence and you ask, whether experiment tells, can actually verify, that experiments can actually realize that whether nature respects such an argument. So, somebody can come and tell me, if I have a time dependent magnetic field, there should be a time dependent current somewhere else. How will you produce a magnetic field? You need a current loop and if I change the current in the current loop, the magnetic field will change.

In a similar manner, somebody can come and tell me, that if you have a time dependent electric field, that means, there is a time dependent rho, that is a particular view point, but, but that two point does not give us any insight. In order to get an insight, let us explore the opposite view point, let us for the time being assume that the fields can exist without a source.

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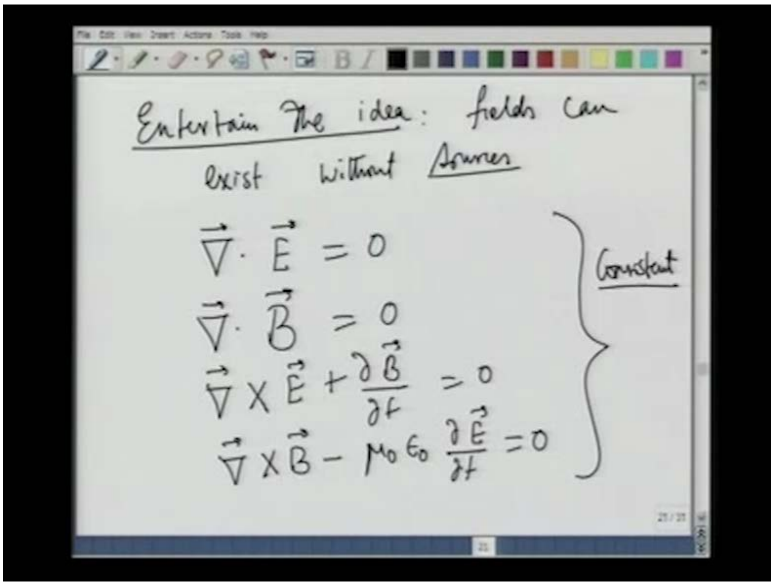

So, let me write that. Let us entertain the idea; let us entertain the idea. What it requires is imagination and boldness and of course, the willingness to confront our conclusions against with experiments and see, whether there are ideas are meaningful or not? What is the idea that we want to entertain? We want to entertain the idea that fields can exist without sources, material sources. So, this w is not visible. So, let me write it direct without sources.

In other words, although I derive my equations by looking at charges and currents, now I rise my equation to a higher pedestal and ask whether these equations makes sense even when there are no material charges and currents, is that part ok? So, we are going to write Maxwell's equations in the absence of sources; by source I mean charges and currents. If I did that, I will get a remarkable sets of equations, divergence E equal to 0; divergence B equal to 0; curl E plus delta B by delta t equal to 0 and curl B minus mu naught epsilon naught delta E by delta t equal to 0.

Let me repeat the viewpoint that I have been propounding. I said that delta B by delta t is the source for the electric field and what is that electric field? It is completely transverse, there is no divergence part. By the same token, mu naught epsilon naught delta E by delta T is going to produce a magnetic field, which is divergence free. Therefore, these equations are consistent, no rho. There was divergence E equal to 0. Therefore, you produce a curly electric field and a curly magnetic field; that is what we are saying.

What we want to now do is to explore these equations and ask, whether the consequences of this equations can be actually seen experimentally? How do I see this experimentally? The way I see this experimentally is obviously, by looking at non trivial solutions of electric and magnetic, for electric and magnetic field.

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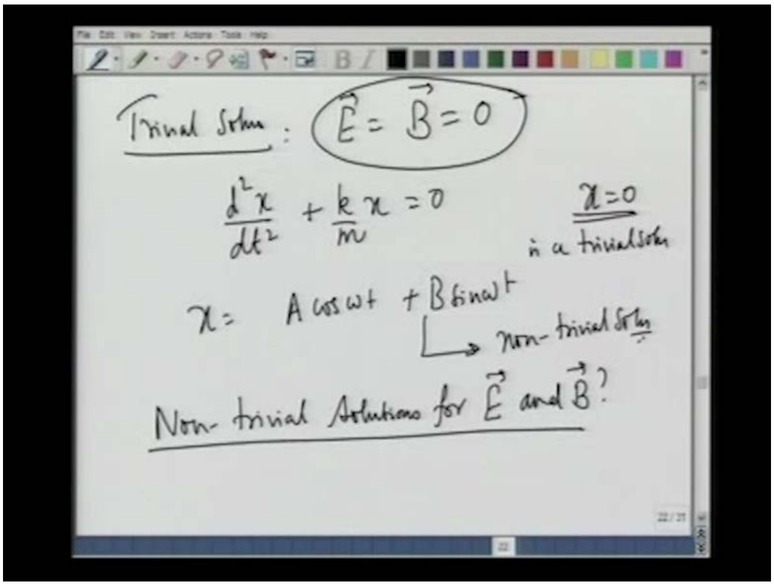

Please notice, that this is a set of homogeneous equation, homogeneous. What does homogeneous mean? There is either E or B in every one of these expressions and all homogeneous equations have a trivial solution, and that is what, E equal to 0; B equal to 0. So, the trivial solutions are E equal to 0; B equal to 0, nothing to do. So, you plug in 0 in the differential equation and you get that. What is an example?

Let us look at the harmonic oscillator, you will write d squared by x by dt squared plus k x is equal to 0, this is a homogeneous equation. Now, this equation has 1 trivial solution, x equal to 0, correct, x equal to 0 for all time. So, we are sitting there, there was no solution. But what is the non trivial solution? The non trivial solution corresponds to x equal to A cross omega T plus B sine omega T, which will give you the oscillation.

So, x equal to 0 is a trivial solution and this trivial solution will never allow you to measure the spring constant. How will you measure the spring constant? You have to displace it slightly and ask how it oscillates. Whereas, the solution x is equal to A cross omega T plus B sine omega T is a non trivial solution, correct. x is change in as a function of time, but then it is oscillating periodically, it is executing a simple harmonic motion and this is something, that you can verify. So, in a similar manner I am not interested in the trivial solution, but I am interested in non trivial solutions, non trivial solutions for E and B.

In other words, we want to mimic the non trivial solutions for this harmonic oscillator problem, simple harmonic oscillator problem and then, I want to see, whether I can write a similar thing and then, whether it can be seen experimentally or not, that is what Maxwell did and let us explore that slowly, step by step. In order to explore that, let me do a little bit of dimensional analysis and let me ask away myself, what is it that happens?

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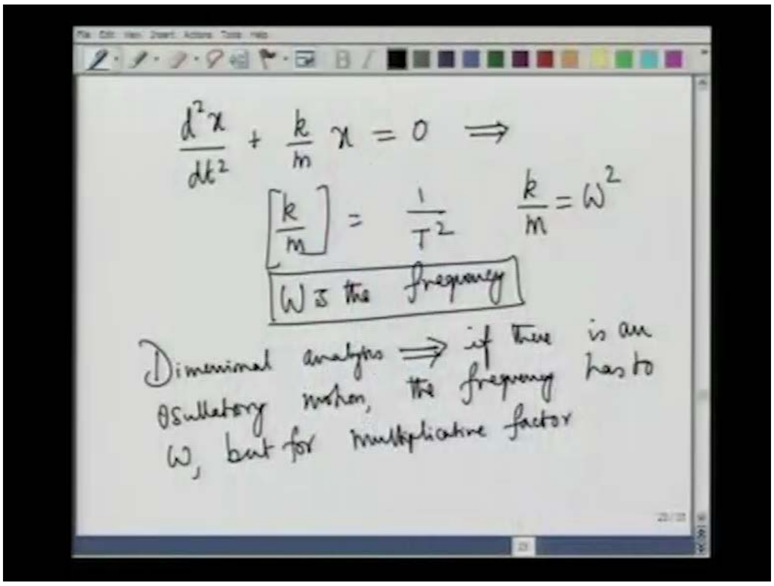

Before I do that, let me return to the oscillator problem and what is it that I wrote? I wrote d squared x by dt square plus k by m x equal to 0. Now, if I did a dimensional analysis, this tells me, that k by m, the dimension of k by m is what, can somebody tell me, is 1 over t squared because x and x have the same dimension. So, what do you do? You look at it and you define a new variable, namely k by m is equal to omega square; omega is the frequency.

I hope all of you appreciate what I am doing. In other words, in this problem, if I am interested in the dynamics, the oscillation, let me imagine, that I do not know how to solve this differential equation, but dimensional analysis tells me, that dimensional analysis implies, that if there is an oscillation, oscillatory motion, the frequency has to be omega, but for some multiplication factor. In fact, there cannot be any multiplicative factor because x and x cancel each other.

In other words, omega is your clock, reference clock with respect to which I can measure everything else, 1 over omega if you feel like. Therefore, what I should be doing is repeat that in the case of my set of equations, but before I do that, I have to do a little bit of jugglery because what I have is a set of coupled equations, whereas what I had done in the case of oscillated was an equation in one variable.

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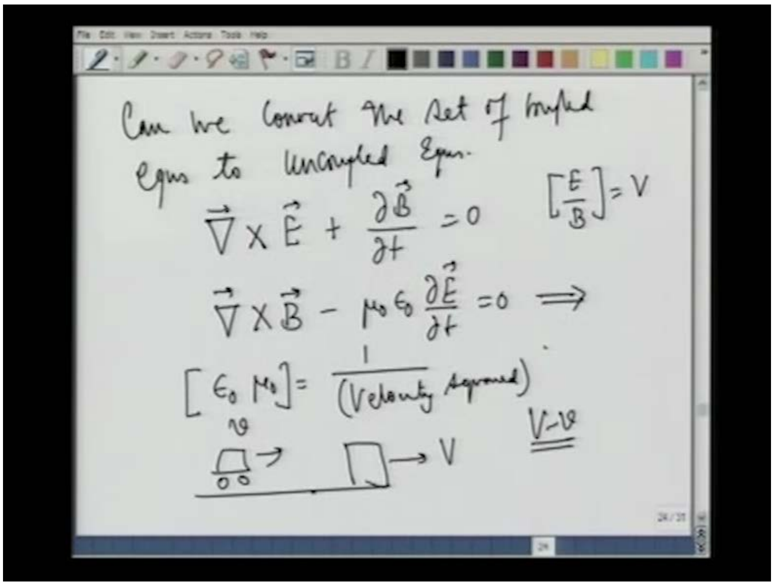

So, the 1st question, that we ask is, can we convert the set of coupled equations to uncoupled, that is, I want an equation involving only E, I want another equation involving only B, I want to eliminate that. How do I do that? In standard all that you have to do is to take a repetitive derivative.

So, let me show how to do that. I have my equation curl E plus delta B by delta T equal to 0 and I have the other equation, curl B minus mu naught epsilon naught delta E by delta t equal to 0. Let us not forget our dimensional analysis, I will do in 2 steps, actually I will do it twice. My dimension of E by B is velocity, remember the Lorentz force expression, E has the same dimension as V B, therefore E by B has the dimension of V. Therefore, this implies, my dimension of epsilon naught mu naught is 1 over velocity squared.

If I have a 1 over velocity squared, what is that velocity? You know, that velocity depends on the frame of reference. If there is a train moving with a velocity v with respect to the platform and I sit on a, sit on a car and start following it with a velocity V, with respect to the platform the velocity is v, with respect to the car the velocity is V minus v, but epsilon naught is a number, mu naught is a number and you determine them experimentally. Of course, you define mu naught to be 4 pi into 10 to the power of minus 7 or whatever, and then you determine epsilon naught.

So, what is this velocity, which is a pure number? If you have a velocity, which is a pure number, that means, you should be a property of the medium. For example, what is the speed of sound in air? You say 330 meters per second; the speed of sound is the property of the medium, why? The speed of sound is simply given by whatever that ratio, the compression coefficient divided by the density. As the air becomes more and more incompressible, the velocity becomes larger and larger, is that right. That is a reason why people like Maxwell or Fnol, introduced two concepts, epsilon naught and mu naught, I do not want to go into that.

If somebody wants to teach you a course on relativity, they would do that, but in any case, I have found a quantity, which is constructed entirely out of the parameters, epsilon naught and mu naught, which I have fixed from the other experiment. Therefore, just as in the case of simple harmonic motion where the spring constant gave you a unit of time, if these equations have nontrivial solutions, these would give you a natural unit of velocity, which has absolutely nothing to do with the motion of the earth, motion of the sun, nothing. Nature itself is giving, nature herself is giving, therefore that is something that we would like to explore.

After having done that what I will do is I will do curl of this equation and I will do curl of this equation, there is what is called as a repeated derivative and then I will combine these 2 equations with 2 other equations, what are the other 2 equations? Divergence E equal to 0; divergence B equal to 0 and ask myself, what is it, that I get? I will leave it as an exercise for you people because what is curl of curl e, everyone knows.

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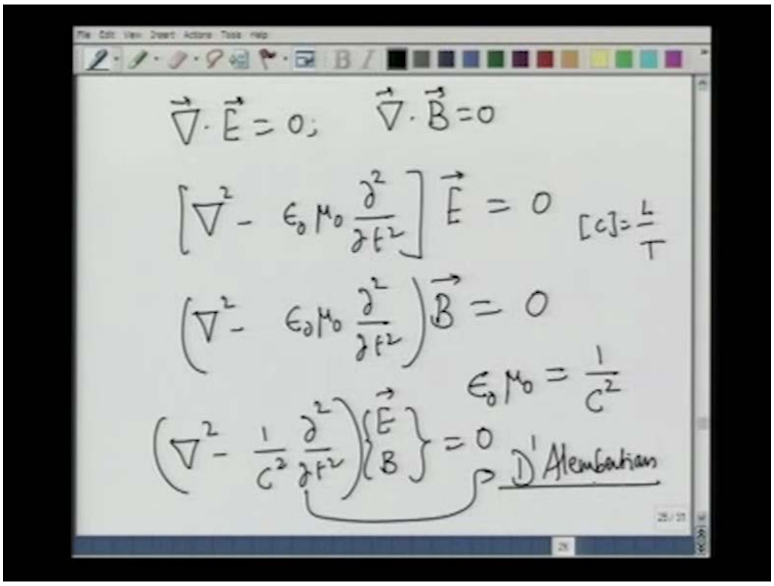

So, please do not forget, that divergence E equal to 0, divergence B equal to 0, combine that with the previous equation. If you did that you will get two very beautiful equations and what are they, del square, let me do it in 2 steps, epsilon naught mu naught d squared by delta t squared E equal to 0 and del square minus epsilon naught mu naught D squared by dt squared B equal to 0.

Every single component of the electric field satisfies this equation, every single component of the magnetic field satisfies this equation and as I told you, epsilon naught is a number, mu naught is a number, epsilon naught mu naught has the dimension of 1 over velocity squared. So, now, let me introduce a notation, epsilon naught mu naught is equal to 1 over c square, where c has the dimension of velocity, c has the dimension l by t. Then, this equation becomes, I am writing a very, very famous equation, del squared minus 1 over c squared, del squared by t square operating on either E or D is equal to 0. This equation is very, very important equation and whatever is in the curly bracket is called D'Alembation E x, E y, E z, B x, B y, B z.

There are 6 quantities, 3 components of the electric field vector, three components of the magnetic field vector, all of them satisfy the condition, same equation del square minus 1 over c squared d square by dt square equal to 0 operating on E or B. Therefore, one thing is for sure, if there are nontrivial solutions, those non trivial solutions will propagate with this speed c. I do not ignore the solution, I will write it down in a short while. In fact, it is not going to be very different from your oscillator solution; it will propagate with the speed c.

Now, I invite you people to go back and substitute for epsilon and c in your S. I. units. If you did that, you will find that c is equal to roughly 3 into 10 to the power of 8 meters per second; that is the number that you want to get. You know, if you go back and look up your optics book, the 1st person to try to measure the speed of light was Newton. So, what he did, he called one of his friends, asked him to stand at the other end of a very, very long road. So, this man had a source of light; those days they were all obviously, there were no electric lights, so probably they lit a candle or a Petromax or whatever. So, the idea was that as soon as he lit the light, he would note the time and then the other person as he received the light at the other end, he would note the time, the time difference and the distance, they would give me this, this speed.

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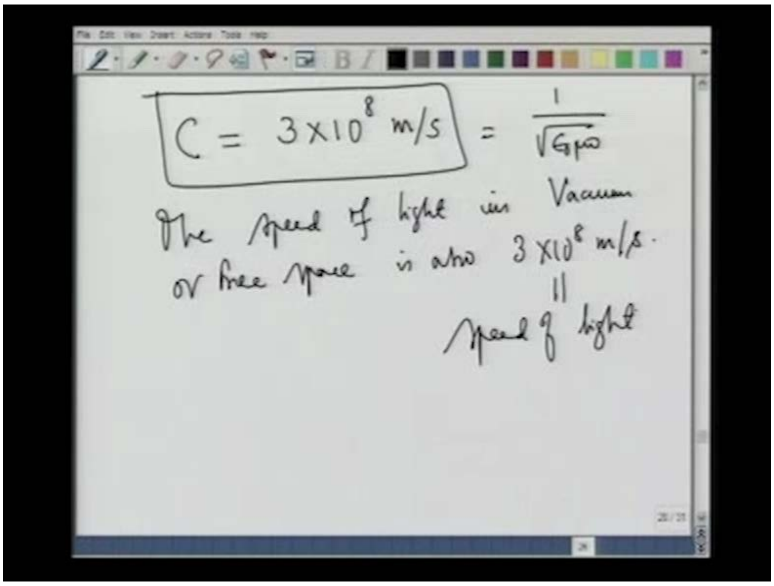

When Newton performed this experiment, they found, that they could not measure it and therefore, they rightly said, that perhaps light moves with an extraordinarily fast speed. The real careful measurements of the speed of light was done by the great astronomer Homer who looked at the eclipses of the moon of Jupiter and then, there were terrestrial experiments performed by people like (( )) and it turns out, that the speed of light in vacuum or free space, that is what those people used to call, is also 3 into 10 to the power of 8 meters per second.

So, what Maxwell asked himself was, whether this is a coincidence or whether there is a deep relation between light and electromagnetic phenomena? Please remember, if you go back and look at your lens, formula, presumed formula, reflection, refraction etcetera, etcetera, all that you do is to make use of the idea, that there is a certain wave; nowhere do you make use of a single electromagnetic phenomena.

So, there is nothing that suggests that by light propagation, by properties of light have anything to do with electromagnetic waves. But suddenly, Maxwell sets up an equation, he finds, that there is a quantity equal to 3 into 10 to the power of 8 meters per second and where is it coming from? It is coming from 1 over root epsilon naught mu naught and this is the speed of light.

And Maxwell is asking himself, is this a coincidence in which case there is nothing much to be read or is this a deep connection in which case I understand light. What people thought of light as some funny propagation is actually a propagation of an electromagnetic disturbance, perhaps in the medium called ether or may be free space as we understand today, but it is this insight which one would like to check. And Maxwell took a giant leap of intuition, of imagination and decided, one should be able to identify light as a particular electromagnetic phenomena. And perhaps over dramatizing that because there is something very interesting and this observation goes back to Faraday.

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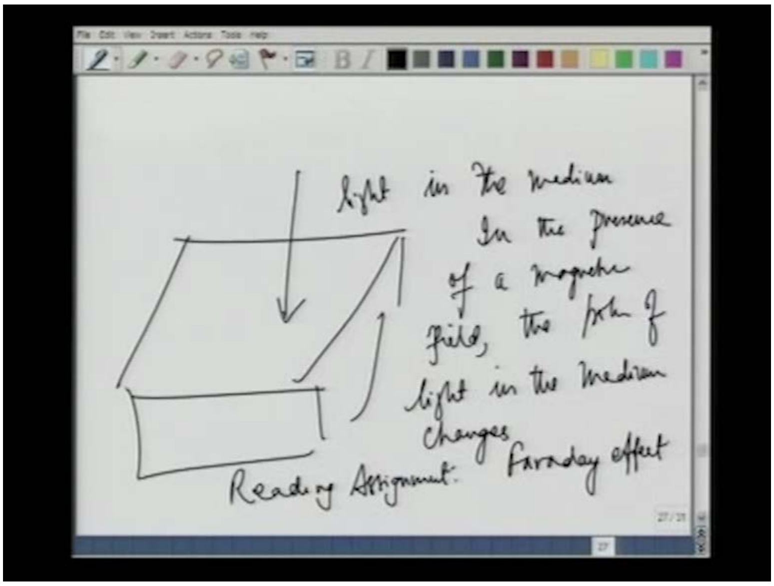

Let me give you this. Now, what Faraday did was to look at a class of materials, transparent materials and he studied the propagation of light in the medium and he measured the polarization of light in the medium.

One thing that he found was that you take this material and you switch on a magnetic field. When you switch on the magnetic field, this material is a magnetic material, so in the presence of a magnetic field, in the presence of a magnetic field, the polarization of light in the medium changes. So, depending on how we apply the magnetic field, the polarization rotates and Faraday was actually able to write an equation that is called as the Faraday coefficient. So, here is a very beautiful, what I call, as a reading assignment, read about Faraday Effect, which is not to be confused with Faraday's law of induction.

And as I told you, Faraday was a remarkably intuitive person of very deep insight; although he was not learned in the usual sense, his education was very minimal. So, Faraday said, that if a magnetic field can influence the polarization of light, then perhaps light should have something to do with the magnetic field.

For example, when we say there is a gravitation field, which is acting on a body, what is common to the sun and the earth, both of them carry a mass. So, a magnetic field cannot act on something that does not respond to a magnetic field.

So, Faraday had already speculated, that what we call as light should have something to do with electromagnetic phenomenon. Now, Faraday was not in full possession of all the electrodynamic equations; Maxwell was in full possession of all the electrodynamic equations. Therefore, we are able to further push the speculation and ask, whether electromagnetic waves are indeed what we call as light, are indeed electromagnetic waves.

In order to pursue that a little bit further, I have to do a little bit more analysis and let me do that slowly, step by step, so that you people appreciate what are all the properties of electromagnetic waves and whether they coincide with whatever we understand with light. Right now, we are only seeing the analogy with the speed, but we have to establish a little bit more. For example, I know, that light is transversely polarized, can I understand that.

What is that transverse degree of freedom? What is vibrating? That is something that is mysterious to us, but that should get settled at this particular point. Therefore, now I am going to look at a situation, which is a very, very simple, simplified situation compared to the original general equation, that I have written and we will start working out the consequence.

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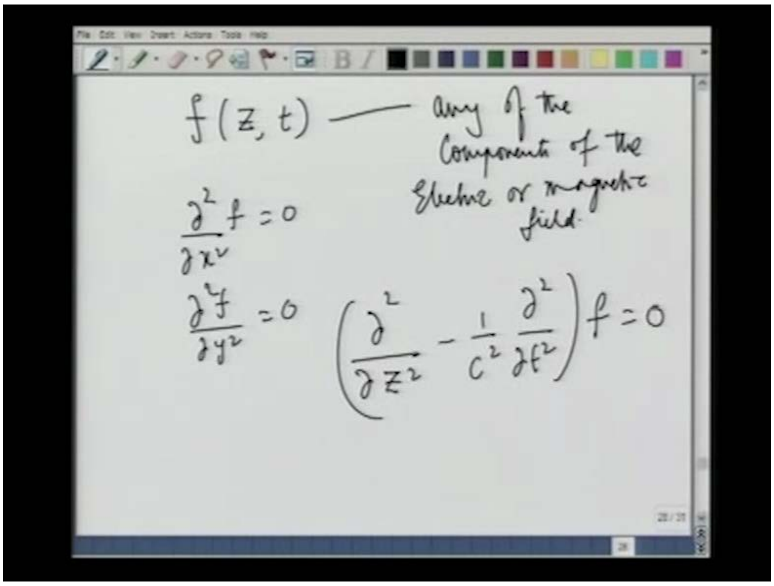

So, what I will do is, I will look at a function f and I will say it is a function of only 2 variables, z and t. This f can be any of the components of the electric or magnetic field because all of them satisfy the same equation and we are asserting that this depends only on one special variable. So, if you move along the x-y direction, that solution looks the same, it will be the same, it was like, for example, producing an electric field through an infinite sheet or an infinite line or whatever. It will be cylindrically symmetric, it only vary with the distance.

Now, I plug this into the differential equation obviously, del square by del x square operating on f is equal to 0 and del square f by del y square equal to 0 and I get a simpler equation, and what is the simpler equation, that I am going to get? That will be del square by del x square minus 1 over c square del square by del t square operating 1 f is equal to 0. This is the so called one-dimensional version of the wave equation, this is the wave equation. In fact, the original thing is a three-dimensional wave equation; this is the one-dimensional wave equation.

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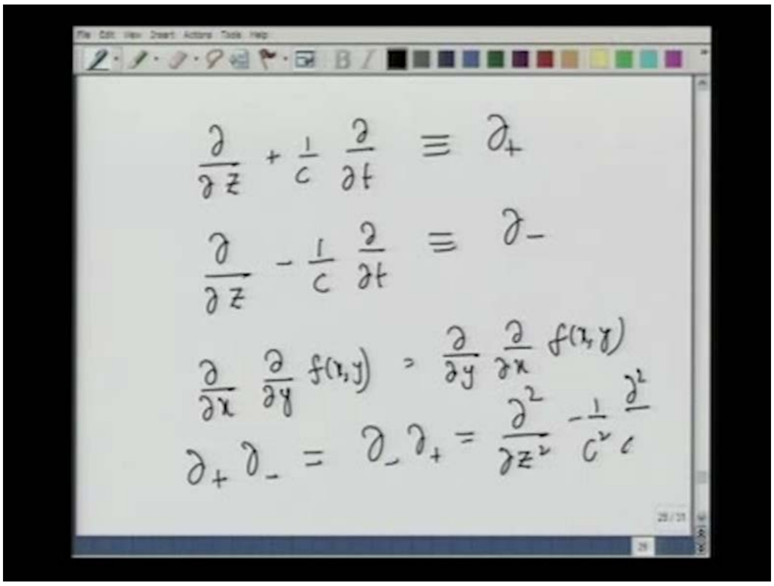

What do I do with this wave equation? What I shall do is to introduce 2 differential operators, delta by delta z plus 1 by c delta by delta t, let me call it as del plus, that is a short hand notation, you will understand why I wrote this plus.

Now, let me introduce the other quantity, minus 1 by c delta by delta t, identically equal to del minus and the partial derivative with respect to time, they can be exchanged for any function. Remember del by del x into del by del y f of x y is equal to del by del y del by del x f of x y, so long f is continuous and differentiable at that point.

So, if you remember that and if you multiply, you will find del plus del minus is equal to del minus del plus is equal to del square by del z square minus 1 by c square del square by del t square of a differential operator.

Now, that means, I am going to write an equation of the following kind, del plus del minus f of z comma t equal to 0. What I would now like to do is to explore the consequence and argue that f is equal to some function of z minus c t plus another function of z plus c t. This is what is going to tell me what the meaning of a wave is. After doing this, we are going to plug it in the Maxwell's equations, make use of all the 4 Maxwell's equations and establish the identity of light with electromagnetic waves and then, provide the experimental evidence for that and that is where Jagadish Chandra Bose plays a very, very crucial role and we will take that up in the next lecture.