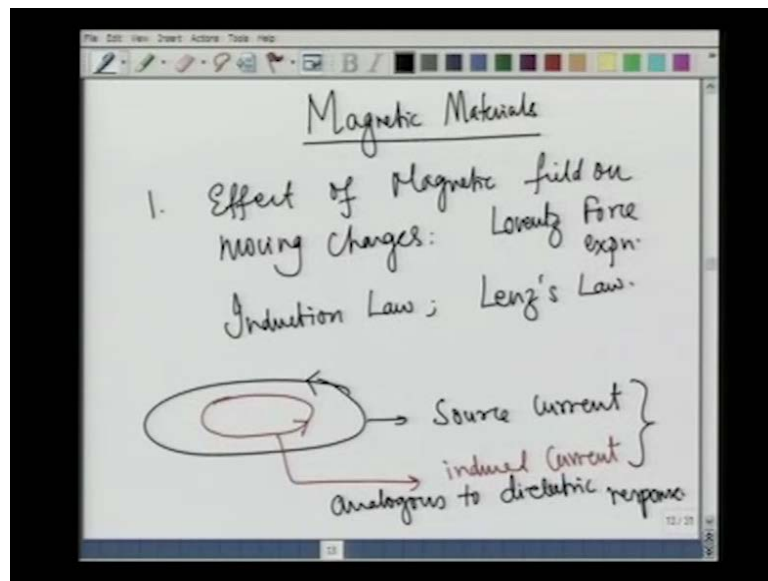


**Engineering Physics - II**  
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**Department of Basic Courses**  
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**Module No. # 04**

**Lecture No. # 08**

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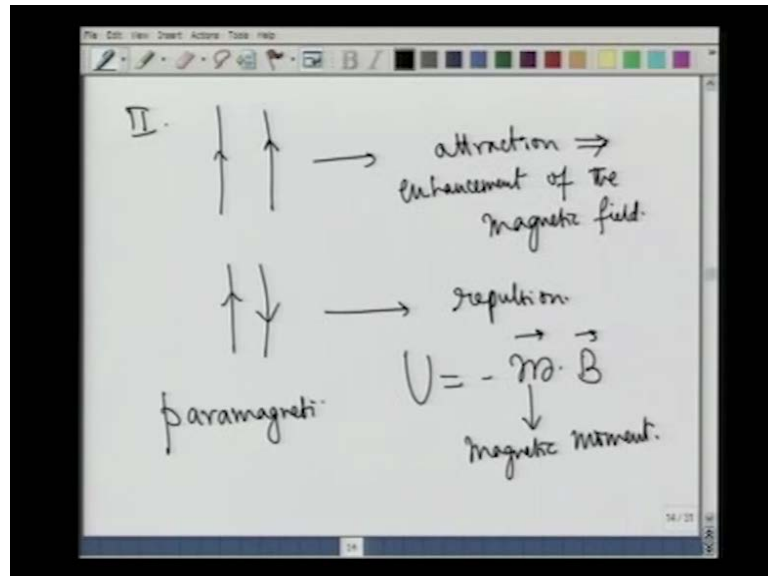


So, in the last few lectures, we looked at two very interesting phenomena, one is the effect of magnetic field on moving charges, Lorentz force expression and also induction law, Faraday's law of induction, also called Lenz's law. And we argued, however we may look at it, from the view point of the Lorentz force, from the view point of Lenz's law, we said, that if the currents are induced by an external magnetic field, the currents so produced will produce their own magnetic field, which will oppose the original magnetic field.

So, what is the pictorial representation? The pictorial representation is so let me say that this is the source current or inducing current, then the induced current, which I will show inside. So, this is moving anti-clock wise, this will start moving clock wise. So, this is the induced current, either because of the induction law or because of the Lorentz force expression, and that moves in an opposite direction. Therefore, it tends to oppose the

original magnetic field. In other words, this is an exact analog of what happens in a dielectric medium.

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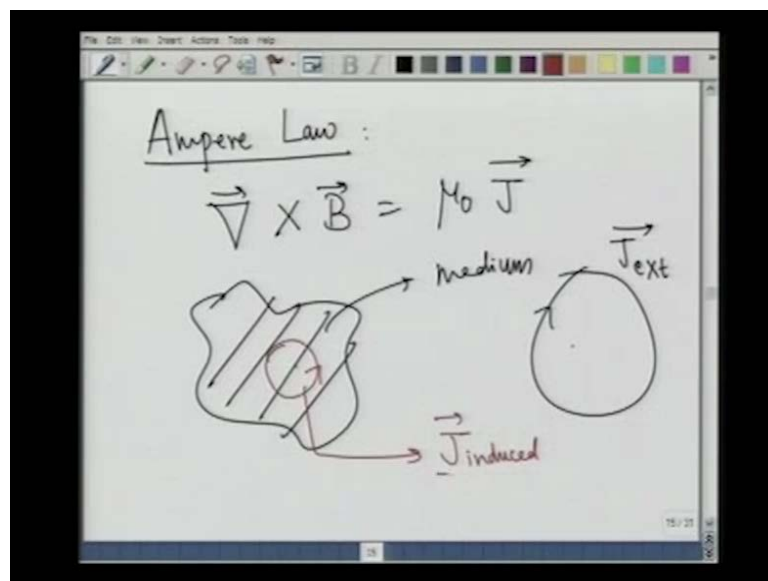
So, the analogy, analogous to dielectric response, but on the other hand, we said, that if the currents are already existing, so that is the scenario number 2. If the currents are already existing, so let me imagine, that there are 2 parallel wires, I am looking at a section of a large circuit, let us say, you can also look upon it as 2 magnetic moments, then these attract. So, here you see, they tend to enhance the magnetic field.

So, attraction implying enhancement of the magnetic field, very well, but on the other hand, if you have anti-parallel currents, then there is repulsion. Whenever there is repulsion, they tend to move away. So, the minimum energy configuration corresponds to attraction and that we were able to quantify by writing  $\vec{m} \cdot \vec{B}$ . Let us not forget this is the magnetic moment and there is no analog corresponding to this phenomenon. And this is what we said will lead to paramagnetism, can lead to. Whether in reality we encounter the earlier situation, which is diamagnetism or we will encounter paramagnetism or something even more complicated that is for the experimentalists, people who observe phenomena, to tell us. And of course, the phenomena are much, much richer than what I have told you just now, but we shall try to understand both this situations.

A complete understanding as to how and when a material can exist diamagnetism or how and when some other material will exist paramagnetism, is actually properly the subject matter of quantum mechanics, it is not easy at all. However, you people have had some exposure to chemistry, where you study paired electrons, unpaired electrons, etcetera, etcetera. You are also familiar with the concept of the electron spin from your chemistry course and I myself introduced the notion of an intrinsic magnetic moment, 4 lectures earlier.

So, what I will try to do is to motivate, whatever is, how diamagnetism comes about and how paramagnetism comes about plus, but please do not be under the illusion, that these are explanations, these are only some kind of motivations to give you a glimpse of what the phenomenon is, but the diamagnetism and paramagnetism complete understanding is much, much beyond this course, we have to remember that.

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But before we do that, let us set up the formalism to describe all these things. So, let us start with the basic Ampere law, let us start with the basic Ampere law, which states, that curl of B is equal to mu naught J, this is universally valid. What we now want to do is to adapt this equation into a phenomenal logical equation. Writing d equal to epsilon e is a phenomenal logical equation; writing Ohm's law, J equal to sigma E is a phenomenal logical equation in a similar manner when we have the presence of a medium.

So, let me imagine that there is a medium and let me imagine, that there is a current loop. Here in free space, I want to study, what the effect of this current loop? It produces a magnetic field on this medium is right, but on the other hand, I do not have the luxury of solving for the motion of all the charged particles in the medium. Therefore, I want to capture the response of the medium with minimum number of parameters.

Of course, since I do not know how to calculate the currents, I cannot actually derive an expression for the parameter. Therefore, what I will do? I will do an experiment and I will make a table of these parameters, this is the idea. So, here is a J source and let me call it as J external because it is to the medium, this is purely for analogy. If you want, you can also put a J external inside that does not concern us.

And now what I will do is I will draw some figures here schematically and this I will call as J induced. The induced current density depends on the external current density and the properties of the medium, exactly like we introduced the induced charge density. The induced charge density for example, in electrostatics always was of the opposite sign of the external density. The so called free density, you have the J induced. Therefore, if I were to rewrite Ampere's law, what is my magnetic field? B, that this total magnetic field. So, let me write it again.

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The image shows a whiteboard with the following handwritten equations and annotations:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_{tot}$$

$$\vec{J}_{tot} = \vec{J}_{ext} + \vec{J}_{induced}$$

Below the second equation, there are two vertical arrows pointing downwards:

- Under  $\vec{J}_{ext}$ , there is a vertical arrow pointing to  $\vec{J}_{free}$ , which is labeled "Stimulus" below it.
- Under  $\vec{J}_{induced}$ , there is a vertical arrow pointing to  $\vec{J}_{bound}$ , which is labeled "Response" below it.

So, perhaps I should switch over to black color, but let me write this equation, this will be  $\mu_0 \vec{J}_{tot}$ , but what is my J total? My J total is nothing but J external plus J

induced, fine. Here is a little bit of jargon, which text books on electrodynamics like to use. If you have an external current you are free to manipulate it, you can put any voltage source, you can keep changing the current, you can keep changing its direction, you can keep changing its magnitude, you can make it an A.C. current, you can make it a D.C. current. So, therefore, these are also called free currents, this is called free.

Now, what about the induced current? Induced current can only respond to the free current and it cannot flow anywhere it wants because the current is restricted to the medium, it cannot flow anywhere it wants. Therefore, this is called the bound current.

So, you have the stimulus and you have the response, this stimulus is free. So, let me call it as a stimulus and this is your response and the response cannot be made in to a free current. It is bound, it has to go round and round in a circle within the medium itself. There are some loops, therefore, instead of using words like J external and J induced it is customary in electromagnetism books to use the words J free and J bound and we will use them interchangeably. Very good.

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The image shows a whiteboard with the following handwritten content:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_{\text{ext}} + \mu_0 \vec{J}_{\text{ind}}$$

Qn: What would be the field if there were no medium.

$$\vec{\nabla} \times \vec{H} = \vec{J}_{\text{ext}} \quad ; \quad \vec{\nabla} \cdot \vec{D} = \rho_{\text{ext}}$$

$$\vec{\nabla} \times \left( \frac{1}{\mu_0} \vec{B} - \vec{H} \right) = \vec{J}_{\text{induced}}$$

↳ induced Magnetic field

Now, once we have that, let me rewrite that expression. I have curl of B is equal to mu naught J external plus mu naught J induced, that is what I have. Let us recall what is it that we did in the case of electrostatics? In the case of electrostatics we introduced a field called D, I think the displacement field, which would be sensitive only to the external

field, that is, we ask ourselves, suppose there were no medium what would have been the field, that is the question that we ask.

Question - what would be the field if there were no medium, well, that is given a name and that is called  $H$ , the  $H$  field, that is the name given, except that when you define your  $H$ , you already realize, that this  $\mu_0$  is a kind of a heavy baggage, especially if you people remember the detailed discussion, that we had on the what the physical meaning of  $\mu_0$  and  $\epsilon_0$ . I argued for you people, that neither  $\mu_0$  nor  $\epsilon_0$  has any intrinsic physical significance; they cannot be measured independently, which is the reason  $\mu_0$  is a definition in S.I. units. But what is measurable is a peculiar linear combination product  $\epsilon_0 \mu_0$ , which has the dimension of  $1/\text{velocity squared}$ ; we will be encountering that very shortly.

Therefore, what we want to do is to absorb  $\mu_0$  in my definition. So, we shall now introduce a field  $H$ .  $\text{curl } H$  is equal to  $J_{\text{external}}$ , this is exactly the analog of writing  $\text{divergence } D$  is equal to  $\rho_{\text{external}}$ . Remember I got rid of  $\epsilon_0$  there, here I get rid of  $J_{\text{external}}$ . Now, what we do is to transfer this expression  $J_{\text{external}}$  to the left hand side and what am I going to get? I can write it  $\text{curl of } 1/\mu_0 B$ , you people can see why I am dividing it by  $1/\mu_0 B$  minus  $H$ ; let me look at this expression.

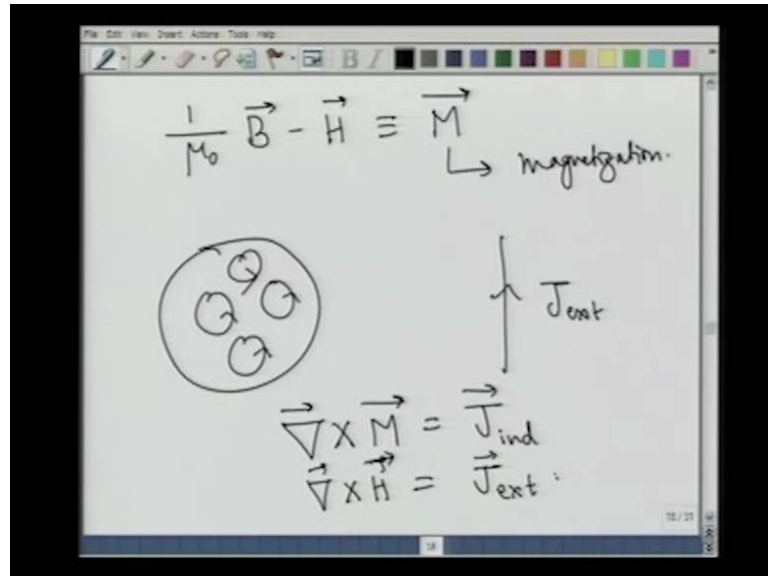
So, first of all I divide the left hand side and the right hand side by  $\mu_0$ . So, I will get  $J_{\text{external}}$  plus  $J_{\text{induced}}$  gets multiplied by  $1/\mu_0$  and what is this quantity, this is  $J_{\text{induced}}$ . Let me explain the physical motivation behind this kind of an introduction.  $J_{\text{external}}$  is the source of a magnetic field, which I call  $H$ , effectively. Forget about  $\mu_0$ ,  $\mu_0$  is an inessential constant, which we can manipulate.

There is a  $J_{\text{induced}}$  and  $J_{\text{induced}}$  produces its own field. Therefore, if you feel like, this is what I will call as the induced magnetic field, except that I cannot use the notation  $B$  because of this factor  $\mu_0$ . Therefore, let me be a little bit more careful, let me erase this and instead, let me say in words, induced magnetic field in units of  $1/\mu_0$ .

So, you have the external current, which produce the magnetic field  $H$ ; the induced current, which is producing another magnetic field  $1/\mu_0 B$  minus  $H$ . Of course, if you go back and look up the books, they introduce a lot of complicated jargon.

H is called the magnetic field, B is called the magnetic induction, let us not worry too much about that; we call it as the B field and the H field, this is the induced magnetic field and we want to give a name for that and the name given is the magnetization.

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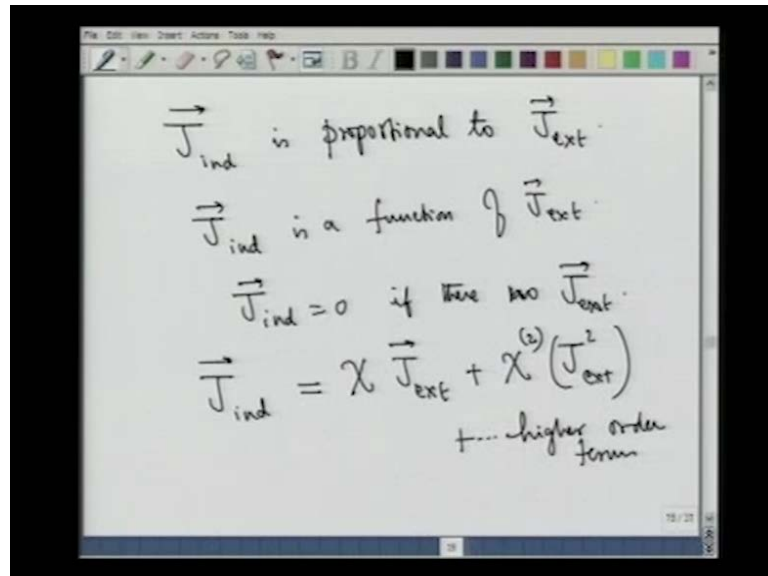
So, we will say,  $1/\mu_0 B - H$  identically equal to  $m$ . So, this is the magnetization, which is induced, how? So, I have a medium here and there is an external current and because of this external current, induced currents are set up. Each current loop is like a magnetic moment; therefore, it is inducing the magnetization. Therefore, this is what is called as the magnetization. So, what do I do? I collect all the equations together and I can write down a nice expression.

So, what is the expression that we are going to write? We are going to write curl of  $m$  is equal to  $J$  induced; curl of  $H$  is equal to  $J$  external, and until this point all that we have done is to identify the physical effects in terms of the current, in terms of the field and then, give them names. What we have to now do is to establish a relation between the induced current and the external current; the induced field and the external field.

Once we do that in terms of the properties of the medium through a right parameterization, it is only then, that we can claim, that we have actually set up Maxwell's equations in medium. And what we are going to do is to introduce the analog of what? The dielectric constant. What is the first step that we are going to do? Let me

write down an equation, I am going to assert, that  $\vec{J}$  induced is actually is proportional to  $\vec{J}$  external.

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It is not for me to assert, that  $\vec{J}$  induced is proportional to  $\vec{J}$  external, it is for the experimentalist to verify, whether  $\vec{J}$  induced is proportional to  $\vec{J}$  external or not. All that I know from my previous analysis is that my  $\vec{J}$  induced is a function of  $\vec{J}$  external, function of  $\vec{J}$  external. And one thing I know for sure, for most materials except the ferromagnetic materials is that  $\vec{J}$  induced equal to 0 if there is no  $\vec{J}$  external.

And I also expect that if my field is small enough, that is, my external current is small enough, the induced current is also small enough. We do not want a butterfly effect where a butterfly is flapping its wings here and suddenly there is a storm elsewhere. That is a subject matter of chaos, which people study, which you will know in your dynamics courses, non-linear dynamics, we do not want to look at such a phenomenon. Therefore, we want to look at a situation where a small stimulus, a stimulus of small magnitude will actually give rise to a response of small magnitude. And experimentalists do find that such a thing is possible.

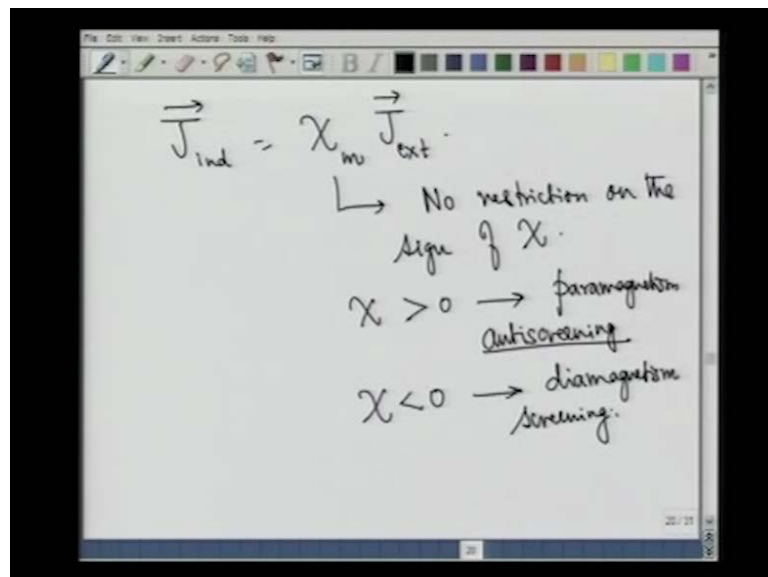
Therefore, what I can do is to make an expansion for  $\vec{J}$  induced in terms of  $\vec{J}$  external. So, the leading order term, let me call it as  $\chi \vec{J}_{ext}$ . So, then, let me introduce a notation called  $\chi^2$ , then this will be what?  $\vec{J}_{ext}^2$ , how the vector comes about let us not worry that, plus higher order terms, etcetera. What we are saying is that



in the leading order, my  $J$  induced will be proportional to  $J$  external. Then, there will be a term, which is proportional to  $\chi J$  squared; then, there will be a term proportional to  $J$  cubed, so on and so forth.

But what we say is that if we are going to look at the small fields. How small a field is? It may be half a tesla, may be 1 tesla and if you look at a host lot of materials like water, bismuth, silicon, all kinds of compounds of metals, all of them invariably, actually satisfies this relation, this quantity;  $\chi^2$  is very small, is very small for most materials and for reasonably small magnetic fields.

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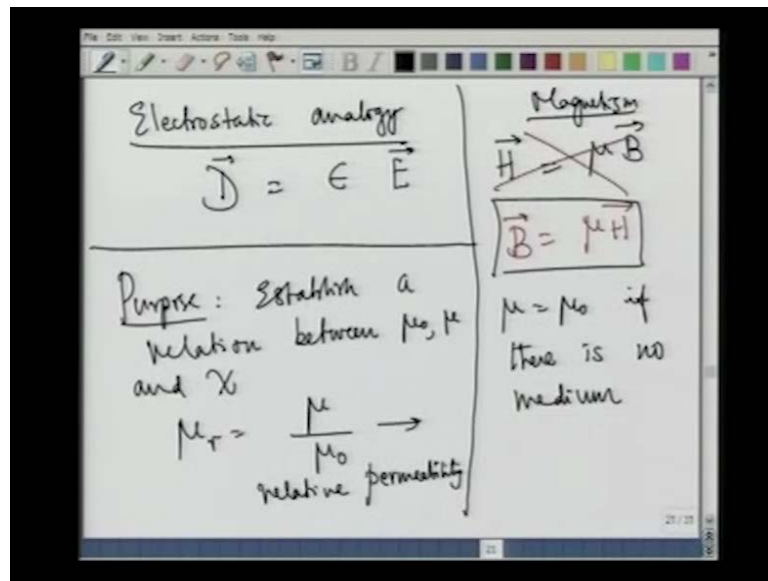
So, with this introduction what I will now do is to write  $J$  induced is equal to  $\chi_m$ ; everything has something to do with magnetic field;  $J$  external. What about the sign of  $\chi$ ? If the material is diamagnetic,  $\chi$  will be negative; if the material is paramagnetic,  $\chi$  will be positive. So, no restriction on the sign of  $\chi$  is that your  $\chi$  can be greater than 0, in which case I have paramagnetism, why?

My current, external current was in some direction, it produced the magnetic field. The system in its response is also producing currents in the same direction; therefore the magnetic field is getting enhanced. Instead of screening, what we have is anti-screening; what we have is anti-screening. It is not always screening, that takes place. But on the other hand, if  $\chi$  is less than 0, coming from Faraday's law and Lorentz force

expression, we have diamagnetism and we have screening. The screening is, of course, imperfect, if I have a perfect conductor, then there would be very good screening.

There is a class of materials, which you people have heard of, what are they? Superconductors, superconductors are perfect diamagnets; they screen the external field completely. Of course, superconductors are much more than that, we will not get into that at particular point; so, we have a diamagnetic screen.

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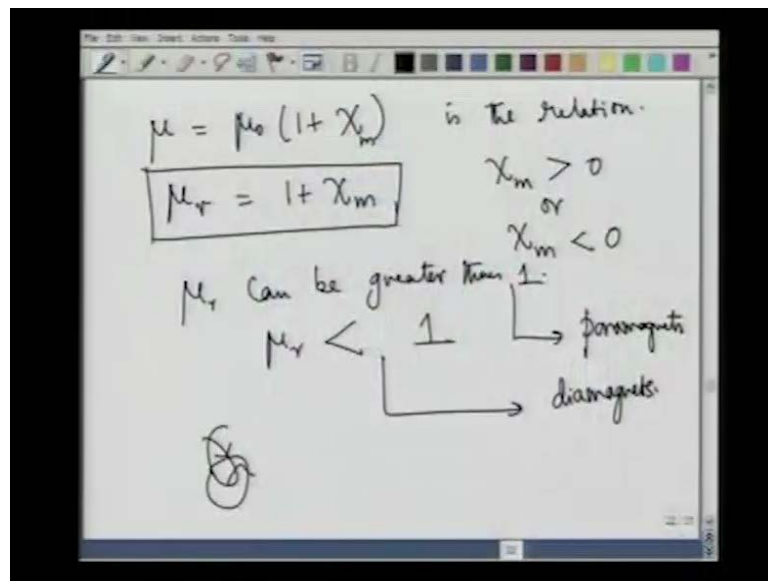
What I have to now do is to actually introduce a notation analogous to D and E. So, back to electrostatic analogy; back to electrostatic analogy. I had D equal to epsilon E, remember D is the field due to the external current and E is the total field. If the founding fathers of magnetism had employed a consistent notation, they would have written H is equal to mu B, but unfortunately that is not what they did, they did not employ this. They wrote B is equal to mu H. In fact, I already sneaked in this notation in a short while; is that part clear? Please notice, that instead of writing H equal to mu B, which would have been analogous to D equal to epsilon E, we are writing B equal to mu H, is that ok? We are writing B equal to mu H. And of course, if there is no medium mu equal to mu naught, mu equal to mu naught if there is no medium, is that right?

What we have to now do is to plug this whole expression and ask what is it that happens when there is a medium? I have to establish a relation between mu naught, mu and chi. So, what is our mission now, purpose? Establish a relation between mu naught, mu and

chi. I think I should not spend any time plugging in that, it is a matter of very simple algebra, even the babies do it.

So, what I will do is, I will simply write then expression, but one important quantity for us is the relative permeability  $\mu_r$ , which is  $\mu$  by  $\mu_0$ . So, this is my relative permeability and we want to write an expression for, I always want to use the expression for relative permeability because that is what is going to tell me, whether the system is paramagnetic or diamagnetic.

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So, let me write down the expression now. My  $\mu$  is nothing but  $\mu_0$  into  $1 + \chi_m$ , this is the relation; **is that part ok?**  $\mu$  is simply given by  $\mu_0$  plus  $1 + \chi_m$ . Therefore, my  $\mu_r$  relative is nothing but  $1 + \chi_m$ . So, just as you have the relative permittivity  $\epsilon_r$  relative dielectric constant here, you have the magnetic constant  $1 + \chi_m$ . But let us not forget,  $\chi_m$  can be greater than 0 or  $\chi_m$  can be less than 0.

In the case of electrostatics, when  $\epsilon_r$  was greater than 1 there was screening, but here, when  $\chi_m$  is greater than 0, there is anti-screening and why is that. So, because the founders of electrodynamics, they preferred to write the relation  $B$  equal to  $\mu H$  and not  $H$  equal to  $\mu B$ , is that part ok? That comes with a baggage  $\rho$  by  $\epsilon_0$  and  $\mu_0$  into  $f J$ , that is the reason why they did that.

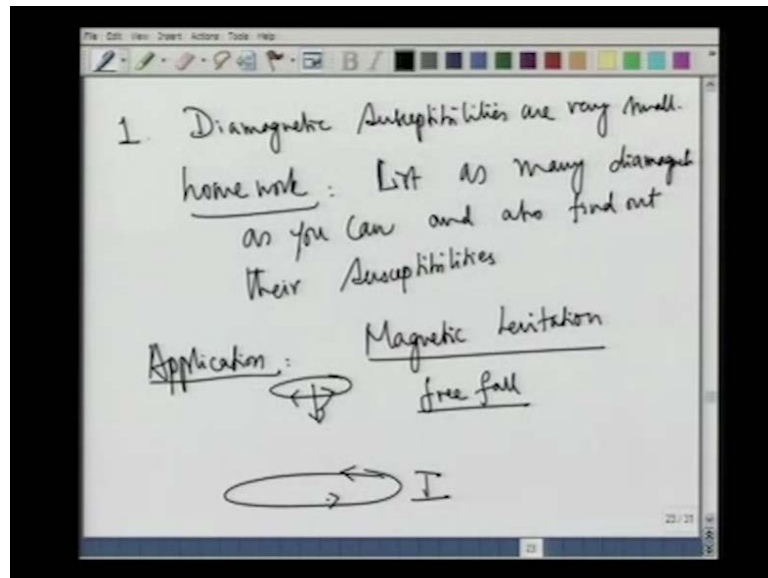
Therefore, my  $\mu_r$ , we can see, can be greater than 1 or less than 1, whereas my dielectric constant was always obliged to be greater than 1. So, greater than 1 corresponds to paramagnets and less than one corresponds to diamagnets. Before the advent of what we call as modern physics, atomic physics, people had been measuring the magnetic susceptibility for a large number of materials and many of them were showing paramagnetic behavior, which was completely incomprehensible, why is that so?

Now, remember, when I introduced the concept of a macroscopic field, I told you, that on an average the magnetic field should be equal to 0 inside any medium because if there is a current flowing in this direction, there will be another current flowing in this direction. So, the time average, the space average of the magnetic field vanishes, the magnetic current should be equal to 0.

Now, if the net current is equal to 0 and if I apply a magnetic field, both Lorentz force expression and Faraday's law of induction tell us, that the response can only be diamagnetic, is that ok. So, classically speaking, if you do not make use of the concepts coming from atomic physics, then it is very, very difficult to understand paramagnetism. So, paramagnetism was a great mystery until people understood that there is another source of magnetic field other than the body motion of the electrons, and what is that? That is the electron spin and Pauli was the first person to actually understand and explain that, therefore this is called Pauli paramagnetism.

On the other hand, diamagnetism itself was understood, explained very nicely by Landau in the context of quantum mechanics. So, if you go and look up a book in solid state physics or statistical mechanics, you will see beautiful sections on Landau diamagnetism and Pauli paramagnetism; that is of course, beyond the pale of our course, but never mind about that, I want to give you a little bit of introduction, is that ok? And that is the reason why I am telling you this, so we actually need paramagnets is that quantum mechanics in order to understand paramagnets. Therefore, what I will do, do is to show you nice figures.

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The first statement that I want to make is that diamagnetic susceptibilities are usually small, susceptibilities are small. I think good examples of diamagnets are what? Inert atoms, helium, neon, all these are diamagnets, probably elements like bismuth are also diamagnet. I do not want to get too much into diamagnetism at this particular point, all that I will do is to leave it as a homework assignment list as many diamagnets as you can, not just atoms; you can look at compounds, molecules, whatever, whatever, as you can and also find out their susceptibilities, they are all quite small actually. In fact, I will show you a table where there are large numbers of diamagnets.

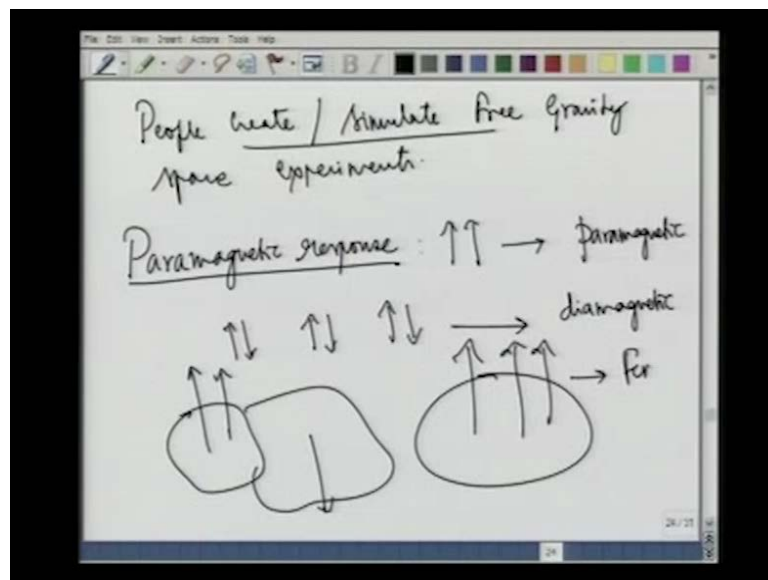
What is the application of diamagnetism? One of the most striking application is the so called magnetic levitation; in my institute, actually this is shown, magnetic levitation. So, what happens is the following thing, let me show it here.

You have a loop, which is carrying a very large current, it can be a solenoid and you have a smaller loop, which you let drop in the earth's gravitational field, free fall. Now, when it is falling, it is changing. Seeing a changing magnetic field, the magnetic field is of course, increasing. Therefore, what happens, it produces its own current, which opposes the original magnetic field. So, it produces its own current in this particular direction. So, if it is in this direction, this will be in the opposite direction. I am sorry about it, so let me make the correction. So, it produces a current in this direction. So,

eventually, what will happen is that the two forces will cancel each other; it will also cancel the gravitational field, the net field. So, this will give rise to levitation.

What do I mean by that? There is a gravitational force, which is acting downwards and there is a repulsive force between the two currents, which is acting upwards. Lorentz force expression, the two cancel each other and therefore, this ring starts hovering around. So, what should we do? Please go to your instructor who is teaching you Physics, request her or him to take to the lab and demonstrate this, is that ok? All that you need is a nice solenoid carrying a lot of current and it will be hovering around, and this is a quite a fantastic, quite a spectacular demonstration of the effect of diamagnetism.

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Well, it might be spectacular, but what is the use of such a thing? That is a good question that you can ask. There is a very, very interesting application and that is people create, rather simulate free space, free gravity space experiments, this is the most recent application of diamagnetism, that is taking place. There are lots of other applications also, but this is something, that I would like to tell you.

You know, we have satellites make rockets, which go to free space. At some stage, the rocket, which has an escape velocity, whatever, 7 kilometers per second, I think, once it escapes from the earth's gravitational field, imagine that it is launched to reach the moon, there is a navigation in gravitation free space; there is no force acting on you.

And you may like to study, what is the effect of such a free space, in gravitation free space on the human metabolism rate, etcetera, etcetera, etcetera. You may like to perform a whole lot of Physics experiments. So, what you do is you try to create a platform by making use of this diamagnetism, such that the effect of gravity is removed. Now, you can do a large number of experiments on that platform. Then, you would be actually trying to understand, what is happening in the far away space.

So, you can actually create or simulate gravity free space experiments and this is one of the nice applications. As I told you, this is not the only application, there are lot of other mundane applications, let me not get into that. Instead, what I shall try to do is to get into the next kind of magnetic response and that is the paramagnetic response. I told you, that when it comes to paramagnetic response, it was a great surprise for people before atomic physics came into existence. Rutherford, Bohr, Heisenberg, Dirac, Schrödinger. So, it is a good question to ask, are these paramagnetic materials some kind of exception or are they proliferating? Do you see whole lot of them? Well, the answer to that can be seen in this particular table; is that ok?

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Table 4. Magnetic data for the studied compounds of Co(ED), Ni(ED), and Cu(ED).

$\text{Co(ED)} \cdot 4\text{H}_2\text{O}$			$\text{Ni(ED)} \cdot 4\text{H}_2\text{O}$			$\text{Cu(ED)} \cdot \text{H}_2\text{O}$		
T/K	ZFC/PP	$\mu_e/\text{Bo}$	T/K	ZFC/PP	$\mu_e/\text{Bo}$	T/K	ZFC/PP	$\mu_e/\text{Bo}$
76	4328	3.76	76	2513	2.14	76	546	0.46
123	4097	4.09	123	2989	2.43	123	608	0.82
133	3552	4.07	133	2660	2.46	133	770	0.91
143	3487	4.09	143	2370	2.48	143	831	0.98
153	32626	4.12	153	2173	2.51	153	880	1.04
163	3142	4.14	163	1989	2.53	163	921	1.09
173	3254	4.17	173	1711	2.55	173	941	1.14
183	3204	4.19	183	1527	2.58	183	949	1.18
193	3153	4.22	193	1382	2.60	193	961	1.22
203	31204	4.27	203	1158	2.60	203	974	1.26
213	30799	4.29	213	1039	2.62	213	981	1.28
223	30394	4.31	223	981	2.63	223	981	1.31
233	30073	4.34	233	949	2.65	233	983	1.33
243	29709	4.35	243	903	2.66	243	941	1.35
253	29449	4.38	253	859	2.68	253	945	1.38
263	29246	4.41	263	813	2.69	263	929	1.40
273	29066	4.43	273	754	2.71	273	921	1.42
283	2774	4.46	283	702	2.74	283	913	1.44
293	2581	4.49	293	622	2.75	293	904	1.46
303	2369	4.51	303	543	2.76	303	892	1.47

Here, you can see, there is a table for a whole lot of materials, mu effective by mu b. There is a chi m in the units of 10 to the power of 6; these are very special compounds involving cobalt, water, hydrogen, etcetera, etcetera. Now, let us not, there is a cobalt compound here, there is a nickel compound here, there is a copper compound here.

Those of you have heard of what is happening in modern material science, I will leave it for you people to sort of fathom, what it means.

So, you people can see that they have reasonably large paramagnetic response. So, what is the large paramagnetic response that we want? So, let me highlight it right now. So, please look at this, this is the paramagnetic response that I have, otherwise it will be missing the wood for the trees. There is far too much data because temperature is given in all that; let us not worry about that.

So, you are shown in the units of 10 to the power of 6; that means, if there is a number like 892; that means, it is 892 into 10 to the power of minus 6 as you are keep on changing the temperature. But these numbers are not very small because here, you can see for example, that there is a number like 23208, 2.3208 into 10 to the power of 4 into 10 to the power of minus 6. So, there is a proliferation of these compounds, is that ok?

You have three compounds, already you have seen, you will see many more. So, paramagnetism is something, which people encounter routinely in many, many molecules, in many, many atoms, the atomic information I will give you in a short while. And you can see that in this particular table, it starts at something like 10 to the power of minus 3. This is actually 10 cubed and there is a 10 to the power, 10 to the power of minus 3 and all the way goes up to something like 0.1, 0.2. So, it is not a small thing at all. You are saying that the induced current is 20 percent of the original current, which is not a small number at all. So, this is one particular table of the interesting information that we have in paramagnetic material.



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The image shows a presentation slide with a table titled 'Table 1' and handwritten notes. The table lists elements with their atomic numbers (Z), symbols, and magnetic moments in Bohr magnetons. The elements are Cr, Mn, Fe, Cu, Eu, Gd, and Dy. The magnetic moments are 3.8, 5.9 (weak field), 5.9 (weak field), 1.7-2.2, (6.9), 7.9, and (5.9) respectively. Handwritten notes include 'Used in MRI' and a diagram showing a downward arrow pointing to 0, labeled 'magnetic moment'.

Atomic Z	Ion	3d	4f	Magnetic moment (Bohr magneton)
24	Cr	↑↑↑↑	✓	3.8
25	Mn	↑↑↑↑↑	✓	5.9 (weak field)
26	Fe	↑↑↑↑	✓	5.9 (weak field)
29	Cu	↑↑↑↑↑↓	✓	1.7-2.2
63	Eu	↑↑↑↑↑↑	✓	(6.9)
64	Gd	↑↑↑↑↑↑↑	✓	7.9
66	Dy	↑↑↑↑↑↑↑↓	✓	(5.9)

Used in MRI

↑ ↓ → 0  
magnetic moment

Now, comes the question, where does the paramagnetic information come from? And this indeed is really fantastic. I argued, that there can be permanent currents without the motion of the electrons because our permanent magnetic moment, an intrinsic magnetic moment, and that is shown in this particular table. So, this is chromium, this is manganese, let me use this color, then you have iron, then you have copper, then you have **eurobium**, Gd and I do not know what Dy is called, never mind about that. How do I know, that they have a permanent magnetic moment? You measure that in the units of Bohr magneton  $Dh \text{ bar over } 2 m$ , I already introduced it.

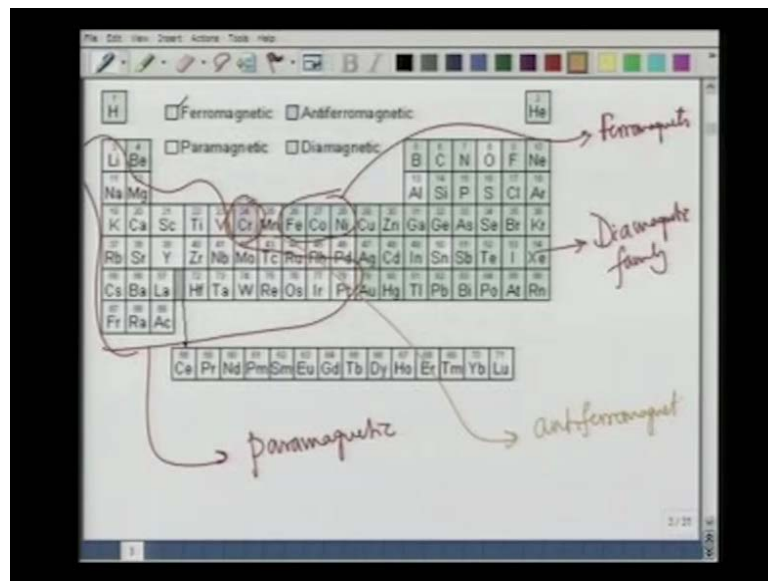
And you will find these are, there are always, these fellows, which are unpaired. If you look at copper, copper by itself would not have given you any paramagnetic response; chromium by itself would not have given you any paramagnetic response. What people did was to look at the ionized state.

Now, let me highlight that part. So, this is the important thing for me, you look at the ionized state in chromium, you knocked off 3 electrons. So, there are 3 electrons, which are unpaired, otherwise you would have had what, an up and a down electron. And the net magnetic moment would have been equal to 0, because the up electron has a magnetic moment downwards because its charge is negative; the down electron has a magnetic movement upwards.

So,  $m_1$  plus  $m_2$ , the magnetic moment is additive, that would have canceled each other. So, this would have meant 0 magnetic moment. Same thing is manganese, iron, copper, **eurobium**, etcetera, etcetera. So, what do you do? You send another atom or you send light, such that my electrons start getting ionized. Now, you have see 3 of them sitting unpaired, all of them in the upstate; here you have 5 of them in the upstate; these are again 4 or 5 of them in the upstate.

And as you keep on increasing, the number of electrons is, that, as you keep on number of increasing the number of electrons with magnetic moment in a particular direction, then the corresponding Bohr **magneton** increases and of course, you will find a very good paramagnetic response, all these are paramagnetic response. And you might be wondering, why I am showing you all these things? These are the ones, which are actually used in medical diagnostics, namely MRI, I will not get too much into detail about that, but you should know, that applications are there galore, not only in Physics, not only in engineering, even in medicine.

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Now, this is a kind of very nice table, which tells you, what the materials, which are paramagnetic. In fact, all the, this is the periodic table, what are the atoms, which are diamagnetic and what are the atoms, which are paramagnetic and we also have atoms, which are anti-ferromagnetic and ferromagnetic, which I will come to later.

Diamagnets are seen here for example, this is the blue color, so these are my diamagnets - iron, cobalt and nickel. And obviously, I am looking at, sorry, these are ferromagnets, I was wondering what mistake I am making, I will come to that later. The diamagnets are what are labeled in green and you see a whole lot of them, except for oxygen and strontium, you have the diamagnetic family including of course, hydrogen, beryllium and helium. So, it is sort of right hand side.

You people know from your chemistry, that hydrogen is a very peculiar atom, it can be placed anywhere in the periodic table, it can be sort of, moved from left to the right. So, the right hand side of the periodic table top is the diamagnetic family and if you look at the left hand side, this is the paramagnetic. So, paramagnetic materials compete with the diamagnetic materials in their number and then of course, you have the iron, cobalt, nickel family, which are ferromagnets. These are also ferromagnets of what are the, are the rare earth elements, we should not forget about them.

And then, one very, very interesting thing, which is shown in purple here, is that right, which is shown in purple. So, let me use this color for this, this is what is called as an anti-ferromagnet. What I will now do is to actually illustrate for you, what happens in the case of a diamagnet? What happens in the case of a ferromagnet? What happens in the case of an anti-ferromagnet and how the response comes about, by giving some nice diagrams? This gentleman will come in a short while when we are going to discuss electromagnetic waves. So, let us not worry too much about that.

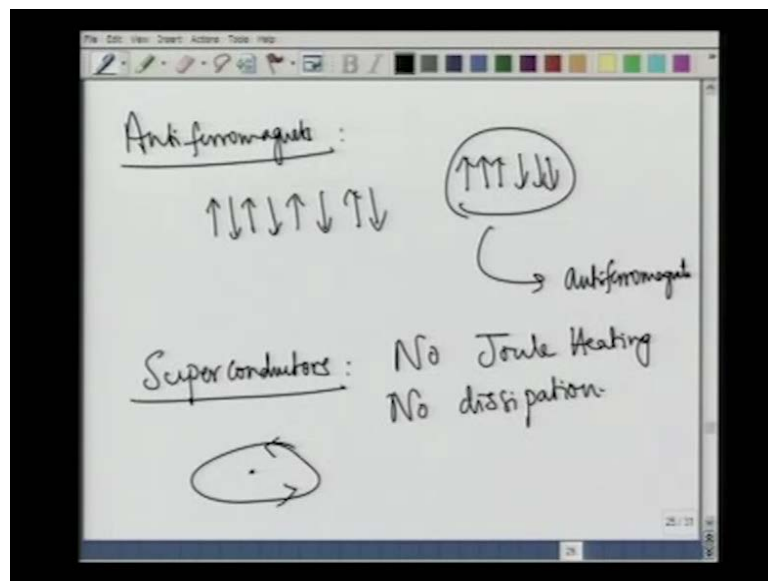
So, let us move on to study, how the paramagnetic response comes about? How the diamagnetic response comes about? And how that is ferromagnetic or anti-ferromagnetic? So, from those figures it is clear, that paramagnetic response comes if you have unpaired electrons; paired means up and down do not come together, this is paramagnetic. But on the other hand, if the electron comes in pace when you are feeling the atomic shells, this is coming from chemistry, this will be diamagnetic.

What about ferromagnetism? In ferromagnetism you do not have 1 or 2; you have whole domains, where these pins are all pointing out in the same direction. These are macroscopic domains, is that ok? Unlike paramagnets, there are only a few of them, these are macroscopic domains. So, there might be another domain, where it can be in the opposite direction, but when you actually apply an external magnetic field or when

you cool the system, etcetera, etcetera, then all the magnetic moments will align in the same direction and that will be, this is macroscopic, this will be a ferromagnetic and this kind of a giant magnetism, is that ok? Large dipole moment is what you find for examples in all those ferromagnetic materials – iron, cobalt, nickel, whatever you saw.

And the last one, the anti-ferromagnet is indeed an interesting thing; let me write it down, I would not have any time to get into that.

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What am I going to get here in the case of anti-ferromagnets? Here, it is a very peculiar situation; you will have an order like this – up, down, up, down, up, down, up, down. So, imagine a spin chain, is that ok. So, if you look at large areas, probably there will be no magnetic order, but then, if you look at small enough areas, you can also have situations like up, up, up, down, down, down, for example; this is the kind of situation, that you have. And therefore, this is what we will have give rise to anti-ferromagnets. There is a local order in the magnetic moment, but there is no global order and the high TC superconductors of which you have heard, they are very closely related to the, very closely related to anti-ferromagnets. So, this is a kind of scenario that we have.

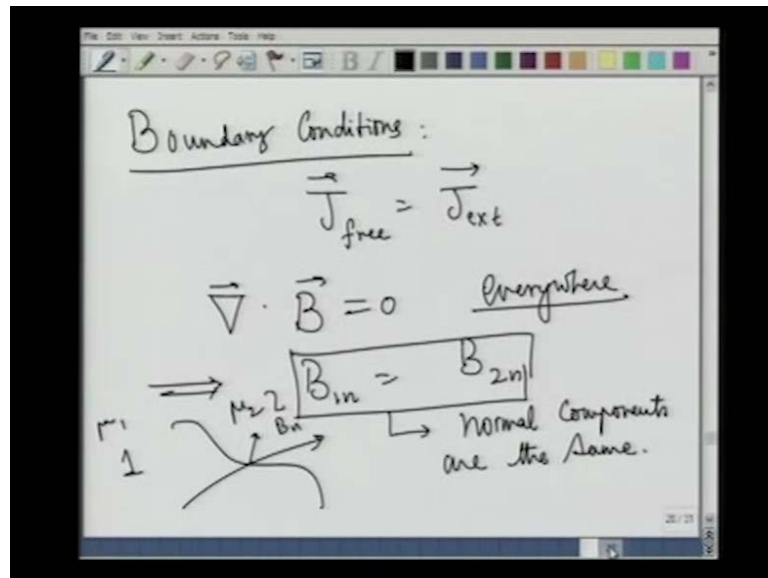
I am going to sort of wind up my discussion on the properties of these materials except that I should mention one more material, which we cannot discuss and they are superconductors. Superconductors, as I told you are perfect diamagnets and the characteristic of superconductors is that no joule heating, no dissipation, no inelastic

scattering. So, imagine that we are able to build electric lines, electric circuits with superconductors. No joule heating, so there will be no loss and then it would be fantastic because the efficiency would go up to 100 percent and that is what people are working at when they look at high TC superconductors.

What do I mean by saying there is no joule heating? Let me tell you, that there is a superconductor kept in, there are superconductors kept in various labs and they have created a super current and they have removed the source. Since there is no joule heating, the current keeps on flowing even without a voltage source, is that ok? Why does it keep moving? Because there is a velocity and there is a reaction of the wire, which makes it go round and round and people have been observing the magnitude of the super currents for the past, I do not know, 25 years, 30 years, 40 years; it has not decayed at all.

So, so long as you maintain the superconductor below the critical temperature and it remains a superconductor, you will find that it is always there. And how do you know, that this current has not decayed? Well, put a magnetometer at the center, measure the magnetic field at the center, is that ok, it will immediately tell you what the magnetic field produces. If it had decayed, what would have happened? There would have been no magnetic field at the center. So, no dissipation over very, very long time, last 30 years the currents have not decayed. Therefore, these are other class of very remarkable materials, fine. So, this was some kind of a pictorial introduction, figurative introduction to all these magnetic materials. Let us not worry too much about them at this particular point, but let us complete our study of magnetism in media by looking at another unfinished agenda, namely boundary conditions.

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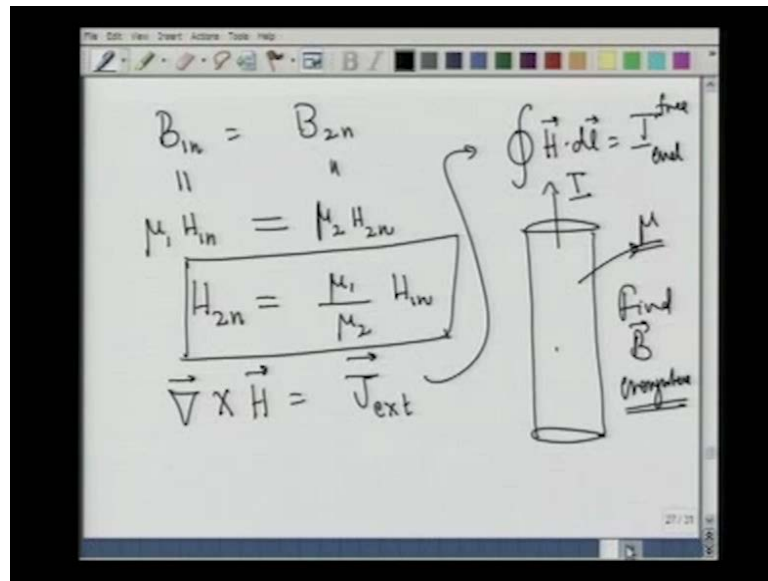
Obviously, I am now going to look at different media with different magnetic permeabilities, is that right, relative permeabilities. And I will ask, how should H change across the boundary and how should B change across the boundary?

As I was remarking one of my previous lectures, we have spent enormous time looking at these boundary conditions, we had 4 or 5 lectures on the vector analysis. So, in other words, we have reached a stage of maturity where I do not have to work out everything. Therefore, what I will do is to quickly summarize and there are many problems, which are there in your standard text books – **Griffiths, Purcell, Resnick and Halliday**, please go back and solve them. If I were to make a problem, they would be very similar to them, so there is not much point in spending too much time on them, is that ok; maybe I will make 1 or 2 problems as I go along. So, let me state the boundary conditions.

When I am stating the boundary condition, I would like to stick to the situation where J free is J external. It is always more convenient, so that I can immediately write down the conditions immediately. So, what are the equations at my disposal? The 1st equation, that at my disposal is divergence B equal to 0; divergence B equal to 0 everywhere, no magnetic monopoles medium or no medium, there are no magnetic charges. So, what does it imply? This implies that  $B_{1n}$  is equal to  $B_{2n}$ , normal components are the same; normal components are the same.

So, I have a boundary, this is medium 1, this is medium 2. So, this has a relative permeability  $\mu_1$ , this has a permeability  $\mu_2$ . And let me imagine, that there is a magnetic field, then you draw a normal component.  $B_n$ , it says, that the normal components are the same; it is continuous across the boundary, divergence  $B$  equal to 0; very good. Now, what happens if I look at the  $H$  field?

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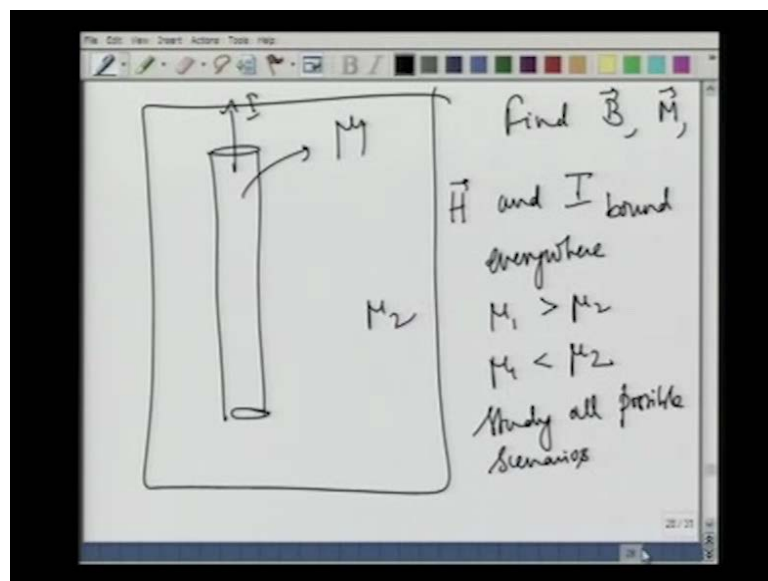
Let me rewrite it in terms of the  $H$  field.  $B_{1n}$  is equal to  $B_{2n}$ , but what is this? This is equal to  $\mu_1 H_{1n}$  and this is equal to  $\mu_2 H_{2n}$ ;  $B$  equal to  $\mu H$  and these 2 are equal. So you see, while the normal components of the induction field are continuous, the normal components of the  $H$  field, the so called magnetic field, they are not continuous. It suffers a discontinuity, is that part right? It suffers a discontinuity because  $H_{2n}$  is simply given by  $\mu_1$  by  $\mu_2 H_{1n}$ . If the normal component of  $H$  is suffering a discontinuity, you can work out the consequence. Whether there is going to be a bound current or not, I will leave it as a problem for you because this is entirely analogous to what you did in the case of electrostatics, but this is what you have. The normal component of  $H$  suffers a discontinuity in particular divergence of which is not equal to 0. Of course, there should be no current, but that is for you people to figure out.

What I now do is to look at the situation, where the free current is outside the medium, is that part clear? That is what I told you because it is very convenient for us to write down these equations. So, let me write down the other situation, where  $\text{curl } H$  is equal to  $J$

external. By the way, this equation can be used to write Ampere's law,  $\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{free enclosed}}$ , correct. So, you solve for this, suppose there is an external current, you solve for this. Even otherwise, if there is a free current in the medium, you solve for it, find the  $\mathbf{H}$ . And if you want the magnetic field, what do you do? You simply multiply it by the  $\mu$ . So, here is a standard problem, probably I should give you now. I am looking at a conductor with a magnetic permeability  $\mu$ . I gave, I showed you a table and I will tell you, that there is a current  $I$ , which is flowing inside this medium. I ask you what is the magnetic field?

Well, how do you do that? Make use of the Ampere's law and write  $\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{prime}}$ ,  $\mu$  inside the medium, multiply it by  $\mu$  you will get the magnetic field. Whereas, outside, you know, what you have to do because there is no medium there. So, you know how to find out the magnetic field? So, take it as an example. So, given a medium with permeability  $\mu$ , find  $\mathbf{B}$  everywhere; find  $\mathbf{B}$  everywhere. It is a very simple thing or if you want, I can actually make a little bit more complicated.

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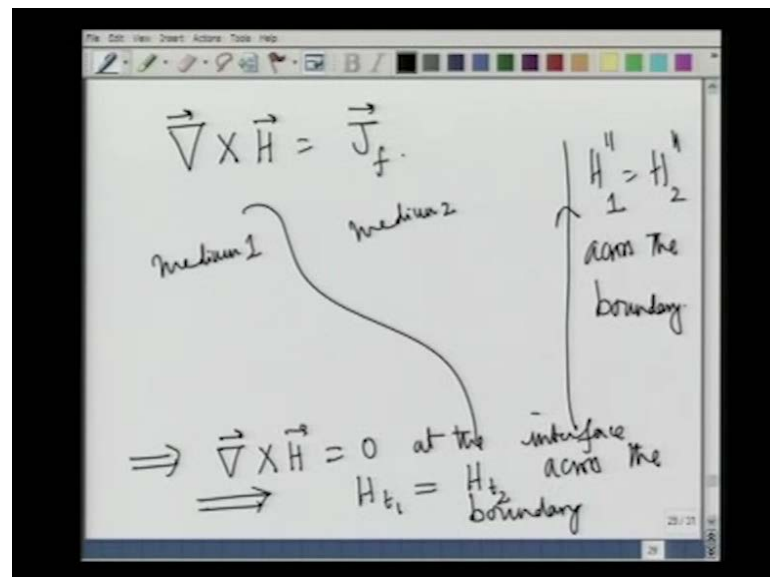
What I will do is I will take a wire, it is carrying a current  $I$ , it has a magnetic moment  $\mu_1$  and this whole thing is immersed in a medium, which has a magnetic moment  $\mu_2$ . So, there is a wire, you dip it in an electrolyte or the wire is surrounded by some medium, is that ok? Whatever, it might be embedded in another medium, which has a magnetic moment  $\mu_2$ . So, we ask, find  $\mathbf{B}$ ,  $\mathbf{M}$ ,  $\mathbf{H}$  and  $I$  bound everywhere, is that part



right? Make use of the boundary condition and find  $I$  bound everywhere and I think this problem to a certain extent succinctly illustrates all the relations that I wrote. And the interesting situation is of course, when  $\mu_1$  greater than  $\mu_2$ ,  $\mu_1$  less than  $\mu_2$ , is that right;  $\mu_1$  greater than  $\mu_2$ ,  $\mu_1$  less than  $\mu_2$ .

You can also look at a situation, where for example, the wire is paramagnetic, the surrounding medium is diamagnetic, the wire is, this medium is diamagnetic and that fellow is paramagnetic so on and so forth. You can study various situations, so please study all possible scenarios.

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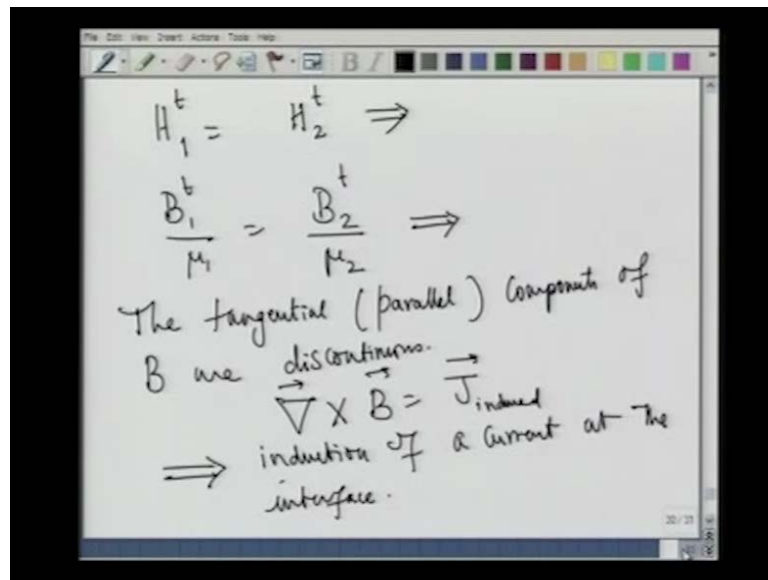


Now, let me return to my old problem of actually writing down, what the boundary condition of  $H$ . So, let me come back to this equation, curl  $H$  is equal to  $J$  external  $J$  free and for my purposes it will be  $J$  external. So, let me make a picture again. I have my medium 1, I have my medium 2. Now, I am imagining that there is a free current somewhere here. Typically, medium 2 you can take it as free space or vacuum or air and this can be taken to be some magnetic material.

Now, clearly,  $J_f$  is not sitting at the boundary, is that part clear? Therefore, this implies, curl of  $H$  is equal to 0 at the boundary interface. I, it is there at the interface again, we know how to evaluate that. So, what does it tell me? This immediately tells me, that the tangential component or the parallel component is the same across the boundary. So either you write  $t_1 t_2$  or you write  $H_{\text{parallel } 1} = H_{\text{parallel } 2}$  across the

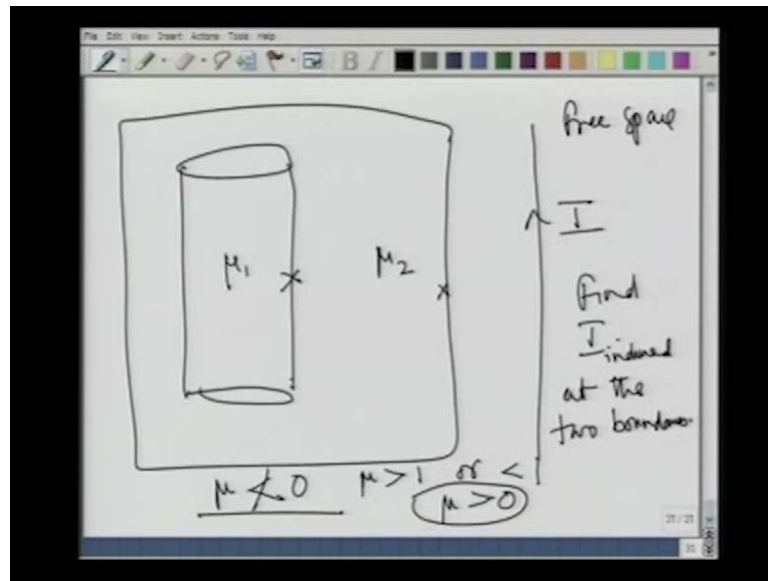
boundary; this is important for us. You know, that if you have two dielectrics and there is a medium and there is an external charge, then there is an induced charged density at the interface; that is something that we know. So, analogously I want to ask, what happens to the magnetic field?

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So, remember we are writing  $H_1^t$  is equal to  $H_2^t$ ; what does this imply? This implies, that  $B_1^t$  divided by  $\mu_1$  is equal to  $B_2^t$  divided by  $\mu_2$ , why? Because  $B$  equal to  $\mu H$ ;  $H$  equal to  $B$  by  $\mu$ . So, this tells you that the tangential component of the magnetic field is not continuous. The tangential component, equally, equivalently parallel component, parallel to what, spelling mistake, parallel components of  $B$  are discontinuous. But remember, the original Maxwell's equation, which tells you  $\text{curl } B$  equal to  $J$ , there is no  $J$  external at the interface. Therefore, there is a  $J$  induced. So, at the interface implies induction of a current at the interface.

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So, we can make a nice neat problem on that. What do we do? You take a conducting wire, is that, place it in a large medium. In fact, very much analogous to whatever I stated earlier, there is a  $\mu_1$ , there is a  $\mu_2$  and this is free space and let me send a current  $I$  here. So, what you can do is to find the induced current at this interface and the induced current at this interface. This is something, which is embedded inside this thing. Take them all to be infinitely long, so that you do not have to worry about what happens here and here.

So, the problem is find  $I$  induced at the two boundaries and that will give you an idea of what is going to happen in this particular case. So, we have completed the study of the boundary conditions; we have studied induction; we have studied Lorentz force; we have understood the kinds of magnetic materials, which are much, much richer than in the case of electrostatics.

There is one information that I want to give you before I wind up. I told you, that  $\mu$  can be greater than 1 or less than 1. I want to give you a problem, which I want you people to ponder about and that is  $\mu$  can never be less than 0, is that part ok?  $\mu$  can never be less than 0, that is something, that I would like you people to think about and come up with the answer.

Let me explain; in the case of electrostatics,  $\epsilon$  was always greater than or equal to 1;  $\epsilon$  equal to 1 was free space, vacuum;  $\epsilon$  equal to infinity was the limit of

the conductors, electrostatics. When it comes to magnetism,  $\mu$  can be greater than 1 or less than 1; greater than 1 is a paramagnet, less than 1 is a diamagnet. So, can it become so less than 1, that it will become negative? I am asserting for you people, I am giving you the piece of information,  $\mu$  can never be less than 0,  $\mu$  is always greater than 0. What is the physics behind that in the case of  $\epsilon$ , never less than 1? I had a thermodynamic argument, what was that the system has to be stable?

So, if this configuration has to be stable, they do not have to be, if they, they should not become unstable;  $\mu$  should always be greater than 0 and that I will leave as a problem for you people to think about. So, with this we sort of conclude our discussion of magnetostatics.

Now, what we have to worry ourselves is what happens if my current also becomes time dependent? What happens if my charge density also becomes time dependent and that will give a complete picture of Maxwell's equations, complete picture of electrodynamics phenomena and that throws in surprises after surprises.

Maxwell was obliged to introduce a new current, what was that current called? The displacement current and once he did that, the 2nd great unification in physics came about. And that is, until that time people believed, that optic was a different branch of physics, people did not realize, that what we call as light is nothing but a part of the electromagnetic spectrum. It is actually an electromagnetic wave, which is created inside. Maxwell speculated that probably what we call as light is nothing but an electromagnetic wave and this speculation was experimentally supported by 2 very great experimentalists. One of them we are all familiar from our text books, Hertz, but the other person, in fact, who did even more spectacular experiments far ahead of his time is our own countryman, Jagdish Chandra Bose.

So, we have a long way to go to appreciate what Jagdish Chandra Bose did. But before I do that, let me start switching on time dependent charge densities, let us return to the basics. As I told you, all of electrodynamics is on the firm foundation of a conservation law, what is that conservation of charge? We have now found no evidence for the violation of conservation of charge; it is as sacred as conservation of momentum conservation of energy. So, we will ask ourselves, whether the Maxwell's equations as

we have written today, are compatible with what? Conservation of charge, when there is a what, time dependent charge density.