Engineering Physics – II

Professor V. Ravishanker
Department of Basic Courses
Indian Institute of Technology, Kanpur

Module No. # 04

Lecture No. # 05

Let us continue our discussion of the concept of magnetic moment. In order to do that, what I did was to take a slight deviation from the usual treatment, which you find in the books, and I started looking at a set of discrete charges. Let me, briefly repeat what is it that I was discussing in the last lecture.

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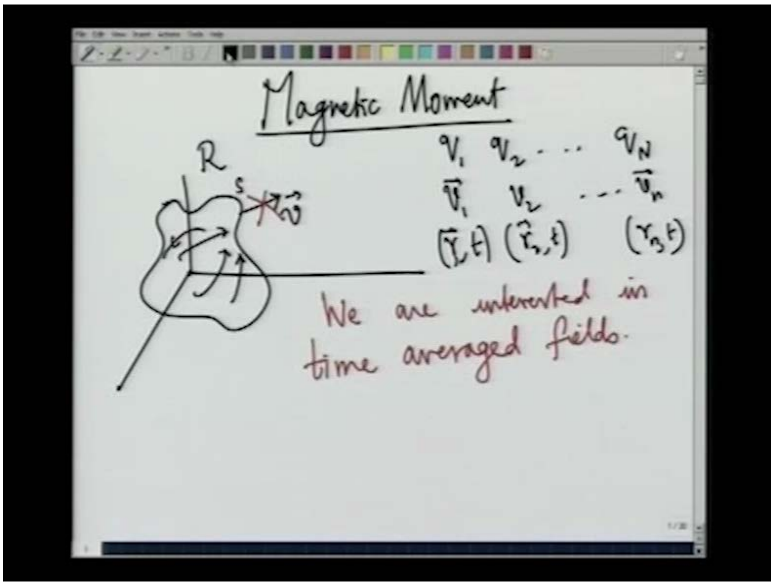

What we did was, we chose a certain region in space a finite region in space; and we agreed to choose the origin of our coordinate system anyway here, and we let the particles move in this region in space. The particles can move in any manner what so ever, they are all charged particles. So, they will have carrying been carrying charges q 1, q 2 etcetera q N, N is very, very large for our purposes.

And we said that at any given time, at any given position, they will have velocities v 1, v 2 etcetera v n. At some time, and at some position (r 1, t), (r 2, t) etcetera. So this completes the configuration. We did not put any restriction on v 1, v 2, v n, except that we insisted that, all of them moving must be moving only within this region that is on the surface S, there should be no normal component. So, the velocity of this kind at any given time is completely forbidden.

So, we have a system of confine charges, which are moving in a certain region and they obviously, constitute a current. There is a current density at any given point in any small given volume there is a current that is flowing. And then, what we can do is to actually write down the expression for the vector potential. Except that the position keeps on changing with time, the velocity keeps on changing with time. Therefore in general, there is a time dependent current density, there is a time dependent charge density therefore, we are not looking at the static situation.

In order to circumvent that problem I told you that, we are not interested in the field at any given time, but we are only interested in the value of the field averaged over a certain time. So, let me write that explicitly, we are interested in time averaged fields. Since, we are interested in time averaged fields and that will be expected to be time independent. Because over one period all the particles come back to their original position is that ok, at least very close to its original configuration.

Therefore, what we shall do is to write down the expression for a coming from each of these individual charges and then, proceed to do the averaging. So, let us start with that. As I told you, this is not a part of the usual treatment, but it is worth it, because we will be able to see the beautiful connection between the angular momentum and the magnetic moment. And we all know, from modern physics that the electrons, protons etcetera also possess what is called as an intensive magnetic moment. Therefore, this will be a very nice bridge to also appreciate, although not understand at this particular stage, what happens in quantum mechanics fine.

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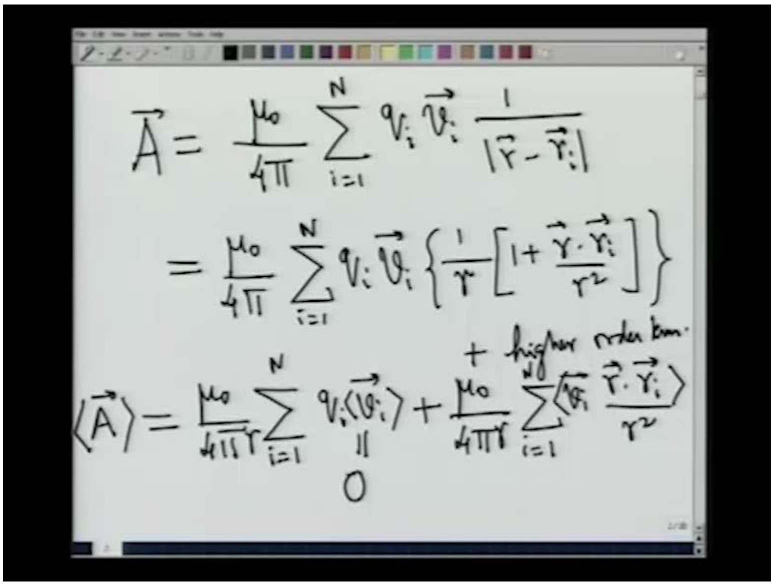

So, what will be my expression for A? My expression for A is a fairly simple expression now. It is not anymore difficult or less difficult than the expression for this scalar potential that I wrote, and that is simply given by mu naught over 4 pi summation i equal to 1 up to N. As I told you, N is a very, very large quantity, otherwise you will not be able to make the approximation of charge density and current density.

The charges carried by individual particles, their velocities at any given time at any given position I am suppressing that (( )) and then, I have minus r 1 minus r i cubed squared not the cubed, so this is the expression that I have. I think I made a mistake here, you should actually be 1 over mod r minus r i (Refer Slide Time: 04:45), because what I have is strictly the analog of the electrostatic law right. So, there should be only 1 over mod r minus r i. Remember the corresponding electrostatic statement would have been phi is equal to epsilon naught, actually 1 over epsilon naught and q i mod r minus r I, you have an extra v i sticking around. As I told you, we are interested in the field far away from the configuration.

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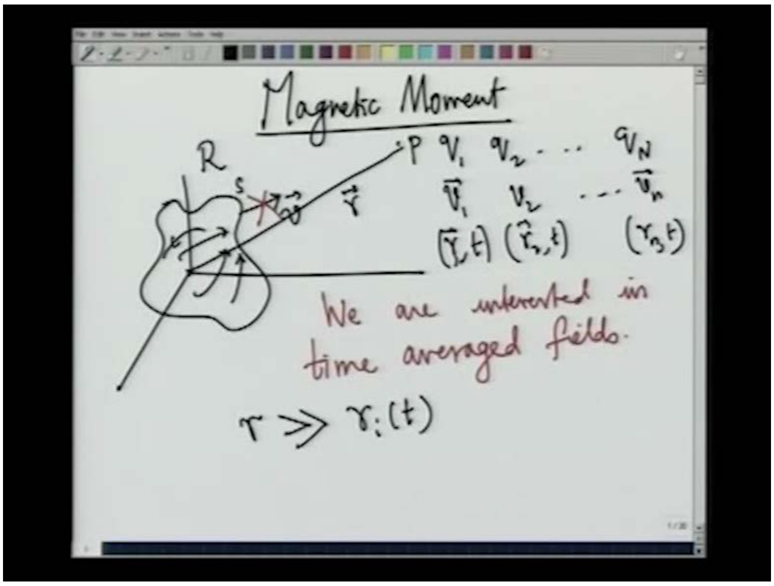

So, what is that I am saying, I am going to choose a point p somewhere here, and I am interested in the field at this particular point (Refer Slide Time: 05:20). And we already remark that, this r must be much, much greater than r i at all time t, r of course is a fixed point, that is not going to change with time; because I take my detector and I ask, what is the magnetic field at that particular point. So, under this approximation, we are going to take r much, much greater than r I, so we made a binomial expansion. I will not repeat the details of the calculation here.

What is it give you, It is a very, very simple expression, mu naught over 4 pi i equal to 1 up to N q i v i is what I have, I am going to pull the 1 over r out of the expression and then, let me open a bracket here, that will be 1 plus r dot r i by r squared. This is exactly the expansion that you did, when it came to electrostatic potential and q i r i is what we interpreted as the electric dipole moment, let us not forget that.

The only difference between the expansion in electrostatics and in this magneto statics is that, you have the extra v i sticking around. So, we have to deal with it carefully. So, you have done this, and let me write down the higher order terms indicate that, there are higher order terms; higher order term means, of the order r i by r cubed and they are very, very small, because r i by r are very, very small.

So, let me write it as a sum of two terms that is nothing but mu naught over 4 pi summation i equal to 1 up to N q i v i, I can pull the r out, because r is a fixed point. And then, what is it that I have, I have mu naught over 4 pi; this is a very important term because this will be giving the leading order contribution i equal to 1 up to N v i r dot r i divided by r squared, r squared could be pulled out if necessary. This is what you are going to get, there is an r sitting here (Refer Slide Time: 07:43).

Now so far, we have not made any approximation, except the binomial thing we have not made use of the averaging idea. Now, as I told you what I am interested is, not in the vector potential itself, but its average. And this is the notation that I introduced (Refer Slide Time: 08:02), the charge carried by the particle is not going to change.

So, I am going to perform a time average here and I am going to perform a time average here, what is the time average that I am interested in? Let me come back here (Refer Slide Time: 08:18), I wait for a time such that, each of them completes one particular 1 full circle one full period.

Of course, if it is moving radially, it can get reflected and come back never mind. So, we want it to complete 1 full cycle. And if we did that on the completion of 1 full cycle, this is equal to 0, because the particle has come back to its original position. If you feel like, I can write dr by dt from 0 to t, but it is a same thing, let me indicate that here.

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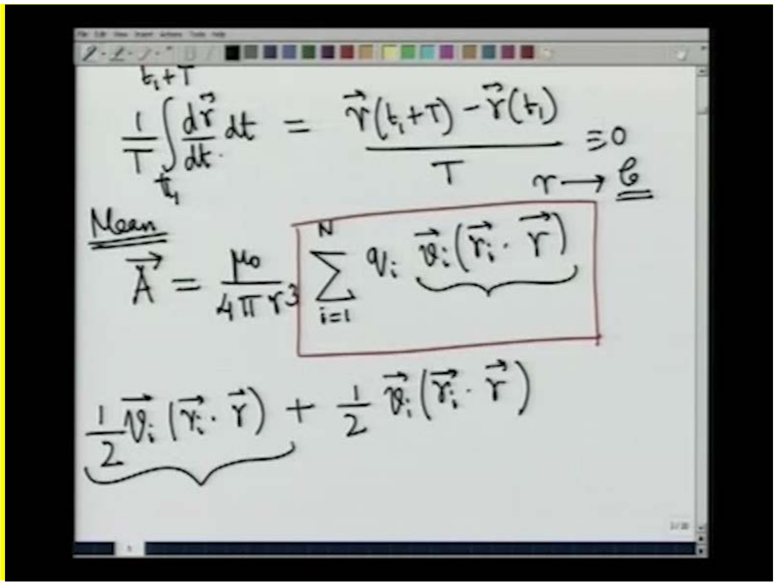

So, I have dr by dt from t 1 to t 1 plus T, (( )) the capital T is the period, I am going to do this and I am going to divide it by T, this is the definition of my average velocity; this is nothing but r t 1 plus T minus r t 1 divided by T identically equal to 0, because the original configuration has been restored, that is the statement that we want to make.

So, in other words, unlike the distribution of the charges, where the leading order term actually contributes to the electrostatic potential, and that was your monopole term, the contribution coming from the net charge of the system. The leading order contribution to my vector potential A is now given by, mu naught over 4 pi r cubed, let me pull all the powers of r out and then, I have i equal to 1 up to N q i v i into r i dot r, such a simple and a nice expression.

I have already indicated to you that, I am only going to take the means mean values; I do not want to keep on introducing this angular bracket at every point. Therefore, hence forth it is understood, we are interested only in the mean vector potential, correspondingly the magnetic field that I calculate will also be the mean magnetic field etcetera etcetera, everything will be mean (( )) appropriate period average values.

This expression is a very beautiful expression and in order to extract the physics out of it, I have to separate the total time derivative term let me explain that to you. In this integral, I put my r actually I could have replaced r by any quantity let me call it as c. So, long as that c return to its original value after a period, the mean value of c will be equal to 0; in other words, any quantity that can be written as a total time derivative, what is my velocity? My velocity is a total time derivative. So, its not just velocity anything that can be written as a total time derivative, its mean value should be equal to 0.

For example, suppose I looked at the force, force is dp by dt, so it is a total time derivative, the momentum will repeat after a full cycle approximately therefore, the average force acting on the particle will be equal to 0. But if you look at this expression v i r i dot r, my velocity is a total time derivative, but r i is not a total time derivative; therefore, I have to separate that part of this expression which is going to give me the total time derivative, and separate that part of the expression which cannot be written as a total time derivative. Once we did that, it is a very, very trivial thing to write down the relation between the vector potential and the magnetic moment, we are going to define the magnetic moment.

Therefore, in order to do that, let me consider the expression in the box and look at that separately. So, what is it that I have, so I will do it in two steps, so that there is no confusion about what we are doing; I have v i into r i dot r let me look at the quantity half of v i r i dot r plus half of v i r i dot r, this is purely for the sake of convenience, it is a kind of trick. So, I write v i r i dot r as, half of v i r i dot r plus half of v r r i dot r. And let me concentrate on the first term, why I need a factor of half, it will be very clear to you in a second.

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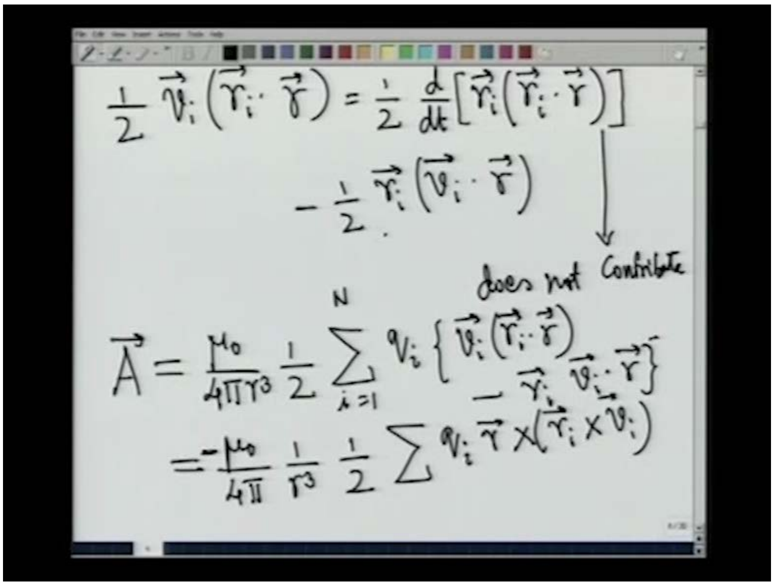

Therefore, I will write half of v i r i dot r is equal to half d by dt r i into r i dot r. So, this d by dt is going to act on the whole expression. Is this equality correct? Of course, it is not correct, because we know from the rules of differentiation that, d by dt is going to act on everything, that is going to be there.

So, it is going to act on the first r i, also on the second r i of course it is not going to act on the fixed position r, the point at which you are going to calculate the magnetic field. Therefore, I have to subtract this spurious contribution, which will be minus half summation there is no summation, because I am looking at a particular single charged particle minus half r i v i dot r.

So, we have added a total derivative and subtracted the unwanted term. What is it that I am saying, I have my half d by dt r i into r i dot r; since all the positions are going to repeat over a certain period, the first term does not contribute does not contribute to my expression for the vector potential, that is what I argued in my previous page.

Therefore, with this very, very simple elementary vector derivative, what is the expression that I have? I have mu naught over 4 pi r cubed I picked up an extra factor half and then, let me write everything now, i equal to 1 up to N let me correct this expression otherwise, it will not be visible properly i equal to 1 up to N q I, I had removed the factor half of a vi r i dot r. So, I will write that first v i r i dot r minus r i v i dot r, this is my expression for my vector potential.

What is it that we have to do now? We have to take one more small step and what is that, we have to remember the formula for a triple cross product; I will leave that as an exercise for you people to figure out I will say it in words. I will not write it explicitly, a cross b cross c is b into a dot c minus c into a dot b. Let us, remember that vector identity, I am going to make use of that and I am going to write an expression now.

I will write it as mu naught over 4 pi 1 over r cubed that is the expression that I have. And then, I have a factor of half summation q i that is what I am going to get. I have to write it as a cross b cross c that is the formula that I am going to use, and what will that be given by, that will be simply given by q i r cross r i cross v I, I think I have missed the minus sign, so let me supply that here (Refer Slide Time: 16:48). Well this is a simple exercise verify that.

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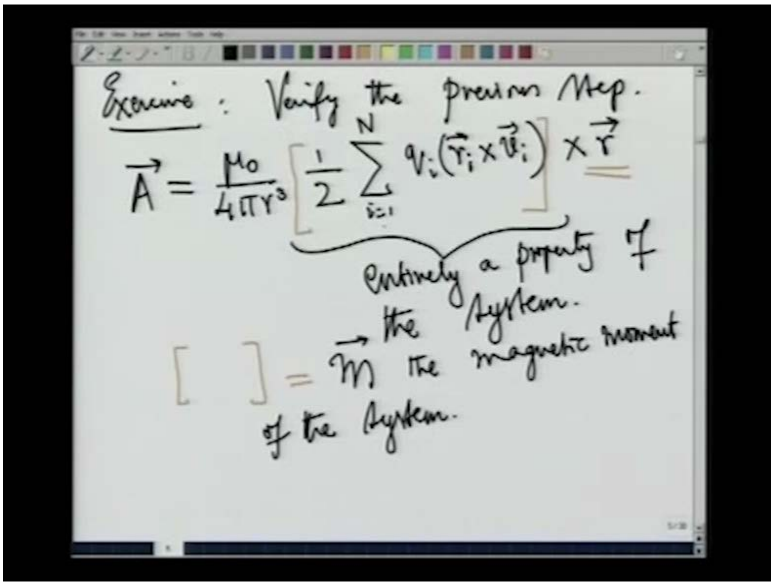

So, exercise for you is, verify the pervious step then how can I write my A? My A can be simply written as mu naught over 4 pi r cubed 1 over 2 that is what I have here. Then, let me invert the sign and write it in a slightly different fashion I have I still have my q i i equal to 1 up to N that is the expression that I have, and then r i cross v i cross r, this is my expression.

I want to collect what is your there is in the parenthesis, so let me put that here. I can put this bracket, because i equal to 1 up to N (( )) over r i cross v I; and this reference point r of course, has nothing to do with the system I have chosen it somewhere away from the system, it has nothing to do with actually the position of any of these particles.

Therefore, the statement that we are making is that, this is entirely a property of the system; it is entirely a property of the system, so let us give it a name and let us call it as the magnetic moment. So this, whatever is there in the bracket, the parenthesis will be called the magnetic moment of the system, so the magnetic moment. This is the magnetic moment of the system and the leading order contribution to the vector potential comes from the magnetic moment.

So, if you are sitting far far away from a current distribution, what this expansion tells you, this expansion tells you that you do not have to know the details of the current distribution, it is sufficient for you to give me the magnetic moment. And of course, the big question is, how do I get the magnetic moment? Though, I have to go and look at each of these individual charged particles, the answer is no. Let us be patient in order to do that let me repeat the expression.

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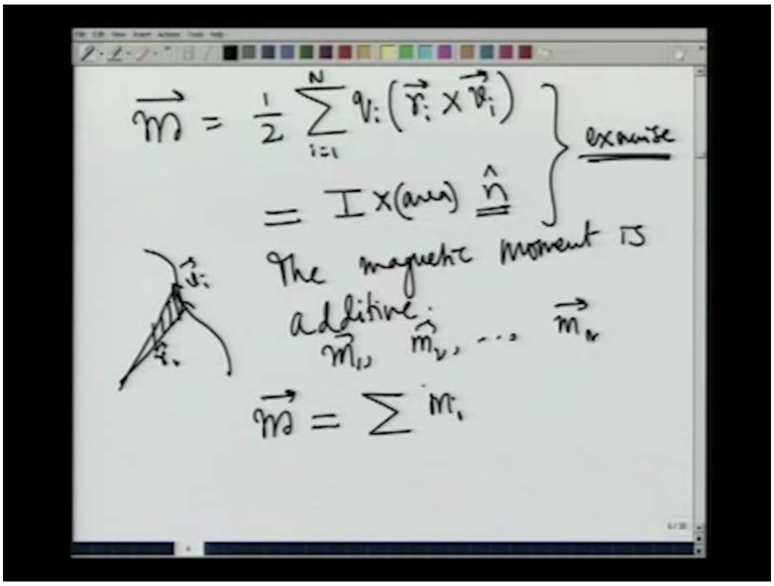

So, I have the magnetic moment, which is given by half summation i equal to 1 up to N q i r i cross v i. You people have done a lot of angular momentum in your dynamics course starting from your 11 standard 12 standard etcetera; and there must be ringing a bell, because r cross b is essentially the angular momentum carried by each particle, except that my mass is not appearing anywhere here.

Let us not worry about that, but I want to give you a very, very simple problem. This is a problem, which is actually worked out in your book. What is the meaning of this r cross r i cross v i? Physically speaking it is related to angular momentum, I will come to that in a short while, but looking at it in terms of the current; suppose there is a current loop which is here I am sitting here, so this is my position r i and this is my v i. You people can see that, when I look at r cross v, actually I am looking at the area per unit time, this is the area per unit time (( )) by the current, that unit time coupled with q will give me the current, this half is necessary and that is the exercise.

So, what is the exercise for you, this is actually nothing but the current flowing in this loop into area into unit vector corresponding to this area element. For example, if this were a circular loop in the x y plane, then N would be along the z axis so on, and so forth. So, please take this as an ex exercise.

In fact, this is the expression that you people use, extensively in order to find the magnetic field, you will solve many, many problems square loop, circular loop, ellipsoidal loop; and there are two square loops sitting like this on the top of each other etcetera, etcetera, let me not give those problems. But this is the idea behind the magnetic moment, you know the area, you know the current, then all that you have to do is to put the element. Even, if the loop is not in a plane as I have already told you, all loops can be built out of infinitesimal circular loops in different planes, the magnetic moment is additive.

So, the next important statement is that, the magnetic moment is additive; additive therefore, if you have currents corresponding to magnetic moment m 1, m 2 etcetera. Say that let us say that, there are m loops corresponding to m 1, m 2, m n. Then, the net magnetic moment is nothing but summation m i. So, long as we are doing the proper dipole approximation. So, long as we are sitting far away, this can be employed and this is in fact, the analog of the additivity of electric dipole moment etcetera, etcetera and this follows (( )) the principle of superposition.

So, we got a geometric picture of what the meaning of the magnetic moment is. Now, let me return to this expression and argue how actually you do not have to know the individual charges provided your currents are well defined.

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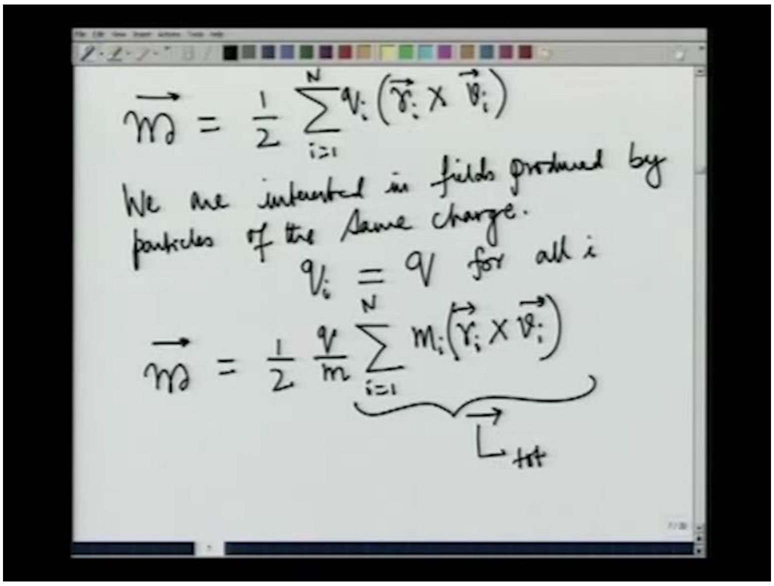

So, let me repeat my expression for magnetic moment, as I told you this is nothing but, half summation i equal to 1 up to N q i r i cross v i. We are interested most of the time in the fields produced by a definite type of charge. For example, in a conductor, the electrons carry all the currents, in a semi conductor it might be the holes.

So, what we shall do is, to collect all charged particles of a given species. So, let me write that explicitly. We are interested most of the time, in fields produced by particles of the same charge of the same kind actually same charge. Even in accelerator for example, you are having particles of the same kind, either the electrons or the (( )) or the protons whatever.

In a conductor, it is invariably electron therefore, we are saying that all q i are the same q for all i. So, what do I do? I think simply pull the q i out of the summation. In the case of an ionic conductor of course, it is a little bit more complicated, because you have the positive ions and the negative ions. In that case, I will split my magnetic moment into sum of two terms, magnetic moment coming from the electrons, magnetic moment coming from the ions, and this analysis can be repeated.

So, the q can be pulled out, not only can the charge be pulled out, they all carry the same mass. So, if you allow me that I can now write, m is equal to half q by m, I have divided and multiplied by m summation i equal to 1 up to N m i r i cross v i. Of course, our text books are (( )) of telling us that, if all the particles carry the same charge to mass ratio it can be pulled out. But it is very rarely that we find particles two distinct particles which will carry the same charge to mass ratio.

So, let us argue physically practically saying that, we are interested in one particular species of charge. And this is recognized simply to be the angular momentum carried by the particle the total system. So, this is the L total, because m 1 r 1 cross v 1 is nothing but r 1 cross v 1, m 2 r 2 cross v 2 is r 2 v 2 is r 2 cross v 2 so on, and so forth. This is the total angular momentum. And now, at last I am home I have found a very beautiful expression for the magnetic moment, provided the carriers are all of the same kind, and what is that?

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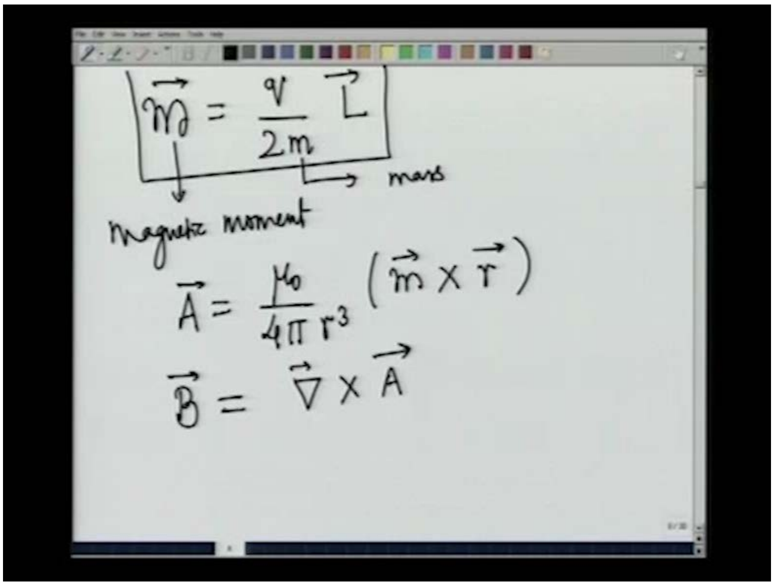

And that is, m is equal to q by 2 m into L, this is something which you should not forget. Once you are given this expression, please notice how to make a distinction between this **(( ))** m and this m, this is my mass and this is my magnetic moment this is my magnetic moment, there is no confusion about that; they are related to each other simply by the total angular momentum carried by the system. From this, it should also be clear to you that, in general the magnetic moment that you evaluate depends on where you choose the origin of the system. That is something that you have to remember, because if I shift my origin, my angular momentum also changes.

However, there is a very special case when such a thing will not take place, there will be no shift we will come to that; but that is not come from classical mechanics, it comes from something else in fact, even come from classical mechanics, we will wait that for in a short while.

This is something that I am going to return to after a while, but let me complete the formal part. Therefore, I am writing A is equal to mu naught over 4 pi r cubed m cross r. What is my magnetic field? My magnetic field is simply given by curl of A. So obviously, the leading order contribution in the magnetic field goes like 1 over r cubed and not as 1 over r squared, unlike what happens in the case of electrostatics.

The next one is a very, very trivial thing to evaluate the curl, I am not going to spend any time, let me just write down the answer, and I will leave it for you people to evaluate it.

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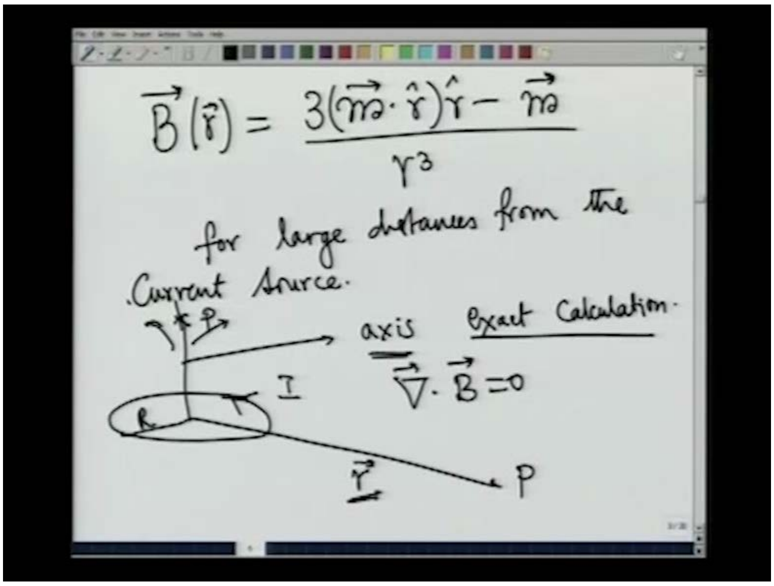

Therefore, my B of r is simply given by 3 m dot unit vector r into unit vector r minus m divided by r cubed, for large distances from the source current source. You should contrast this beautiful expression, from the earlier calculations that you did. Let me return to that problem, you had a current loop, and then you evaluated the field at this particular point p, this was carrying a current I, when you were trying to evaluate it exactly, you were only able to evaluate it along this axis, only here could you do an exact calculation.

Then, with a little bit of luck and insight, we were able to employ divergence B equal to 0 to infer the nature of the field in the immediate neighborhood. If I had asked you please give me the field at this point, right please give me the field at this particular point (Refer Slide Time: 29:20), you will be in deep trouble, if you try to actually employ the exact expression the Biot-Savart law, but this expression will immediately tell you how to evaluate it approximately.

And if this distance is very, very large, then you can ignore higher order corrections and the expression is essentially given by B of r is 3 m dot r r minus m r cubed. And what is my magnetic moment? It is the current into the area pi r squared, where r is the radius of the loop; look at the beauty of this expression. Now obviously, it is very tempting for me to evaluate this expression close to the axis. And ask, how well it agrees with my divergence of B, so that is something that we have to do.

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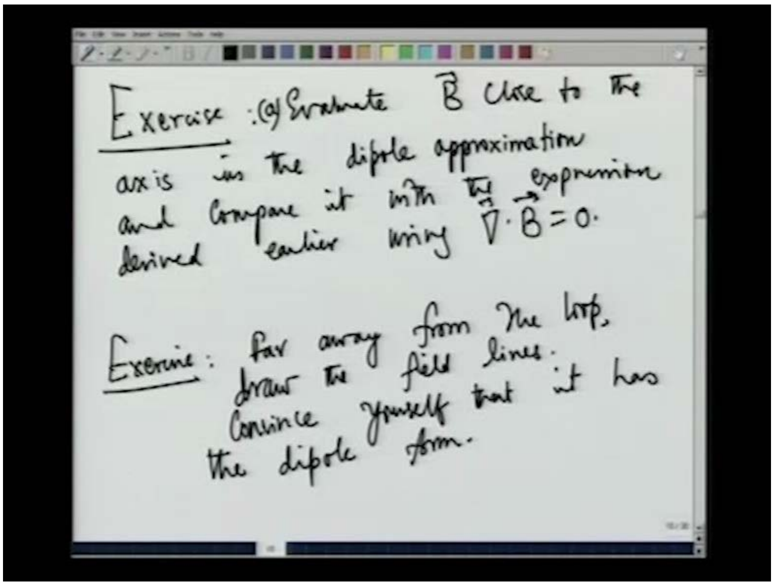

Obviously, I am not going to do that, you will do that evaluate B close to the axis in the dipole approximation, this is also called the magnetic dipole moment dipole approximation, because we are taking the first moment of the current. In the dipole approximation and compare it with, what you got the expression derived earlier, earlier using divergence free nature.

So, please compare when you are doing a order by order expansion, you should not really matter, you should get the same answer. Of course, the dipole approximation will also give you a higher order term that is one thing. My exercise two is that, at least far away from the loop draw the field lines. And convince yourself that you (( )) looks like a dipole field, convince yourself that it has the dipole form; as if there is a north pole and a south pole, a fictitious north pole, a fictitious south pole, which is producing this field.

So, we have come a full cycle we have understood the meaning of the magnetic moment. Now, we shall ask ourselves under what condition will this magnetic moment be independent of the origin that is the next question.

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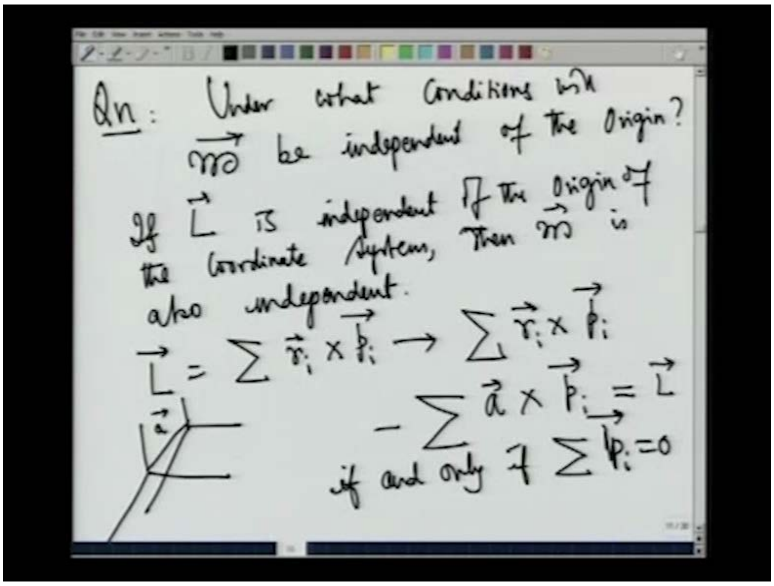

So, let us ask our self a question, under what conditions will the will the magnetic moment be independent of the origin; the answer to that is (( )), if L is independent of origin, where you are going to choose, your origin, origin means what of the coordinate system then, m is also independent. But under what condition is L independent of the origin? We know that, from your electrodynamics, so what is it that I have? My L is nothing but summation r i cross p i.

Now, I shift my coordinate system therefore, this will go to summation r i cross p i minus summation a cross p i. So, I had my coordinate system and I move it by a position vector a, this a is a constant vector, this can be pulled out. Therefore, this is equal to L, if and only if summation p i equal to 0 that is you should be sitting in the center of mass of the system say frame of the system.

So long as, the total momentum carried by all the particles is equal to 0, then there is no problem, the center of mass should be addressed, then your magnetic moment will be independent of the origin of the system. And we know that, the angular momentum in the center of mass frame is what is called as the spin.

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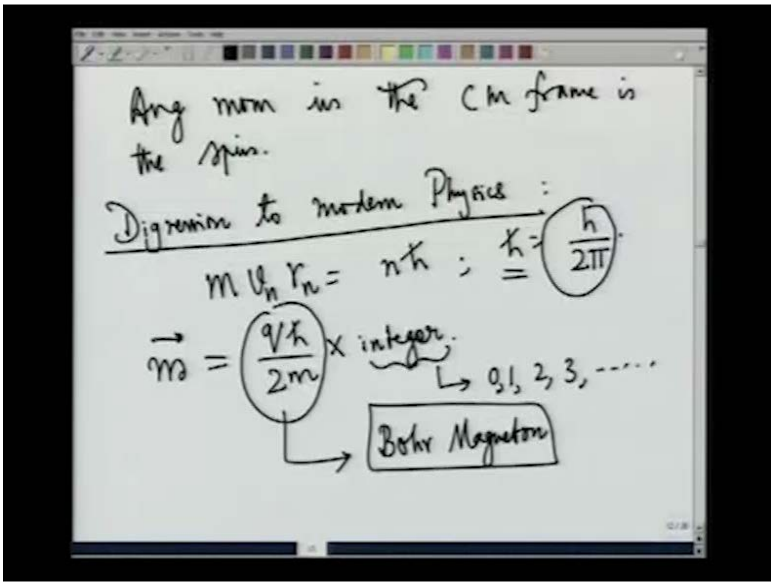

So, let us remember that, angular momentum in the center of mass frame is the spin. And whenever you have the spin, the magnetic moment is the same irrespective of where you look at it from. Of course, under rotations it will behave like a vector, there are three components which transform among each other, cos theta, sin theta etcetera etcetera; but you simply shift translate the origin of the coordinate system nothing is going to happen fine.

What I now want to do is to take a deviation from whatever so called classical magnetic moment that we are studying; and remind you that, in your modern physics class that is something that you are going to study here in your course here, and you have also studied in 12 standard. You have learnt that electrons actually possess, what is called as an intrinsic spin. So, let me take a few minutes to remind you, what is it? That you studied in your modern physics.

So, let us take a digression to modern physics. For our purposes, we need only two ideas, one is the Bohr model which you people are all thoroughly familiar with, and another is the idea of Pauli, who introduced 2 degrees of freedom in order to fill the atoms in order to understand the periodic table. So, let me spend a few minutes and explain what it is to you.

Now, one thing that Bohr thought us that is very interesting is that, my angular momentum cannot be continuous; it has to be a discrete multiple, let us not forget that, he wrote the famous equation m v n r n is equal to n h bar. What is h bar? h bar is nothing but h by 2 pi, where h is your famous Planck's constant, this is something that I alluded to when I was discussing the standard of resistance I was telling you that, nature provides us with a natural unit an intrinsic unit of angular momentum, and that is your Planck's constant. h bar is what occurs more frequently in our studies of fundamental physics, that is why we have introduced a special notation.

So, if you remember that, let me write down the expression for angular momentum again, magnetic moment again. I will introduce this h bar, then it will look like q h bar by 2 m, that is what I have into some integer. If I were to take an electron, and ask what is the magnetic moment? If the electron is sitting in the lower state the S state, the angular momentum is equal to 0.

So, what are the values integer means 0, 1, 2, 3 etcetera, it is a dimensionless in number, obviously it is dimensionless, all numbers are dimensionless. Therefore, there is a natural unit for us to measure the magnetic moment, and this is called q h bar by 2 m, in the units of h you will write it as q h over 4 pi m. And in the honor of Bohr, who introduced this for the first time, this is called Bohr magneton; and it is the measurement of this Bohr magneton.

When you actually apply the magnetic field and see this splitting that, actually verifies the quantum theory whatever modern physics that we are going to have. And this is an expression which you can actually workout, and you can find out what its value is. So, q h bar by 2 m has the same dimension as the magnetic moment that is a first thing that we have to notice.

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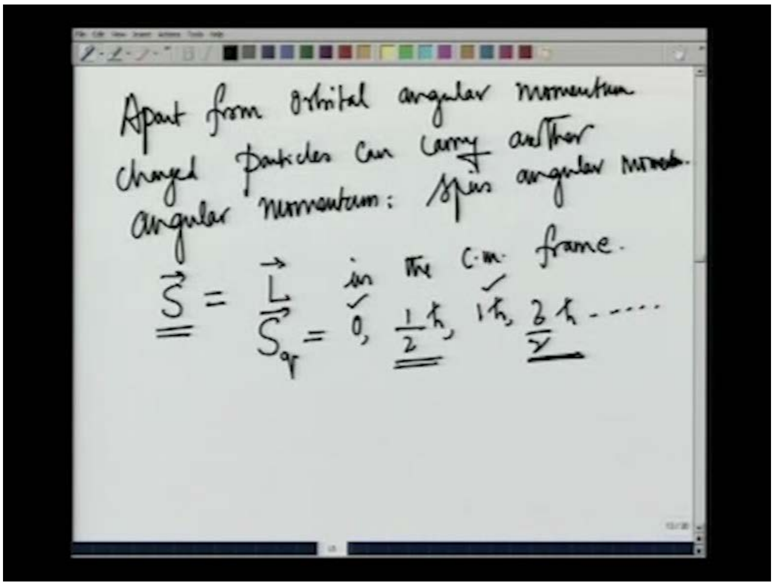

However, interestingly what physics tells us is that, apart from orbital angular momentum charged particles can carry another angular momentum, and what is that called spin angular momentum. Now, you will come and tell me that I am wasting your time, what is the spin angular momentum, because just a minute back I introduced the concept of a spin angular momentum; and I said, let me introduce a notation my spin is nothing but L in the center of mass frame.

But, the surprise that comes from atomic physics is that, even when you consider a point particle an electron is a point particle let us not forget that, its radius is known to be less than 10 to the power of minus 17 centimeters. Even, when you consider a point particle and even when it is addressed, it can possess an intrinsic angular momentum, which is given by S. Quantum mechanics tells us that, S comes in the units of half integer multiples. So, what are the allowed values, 0, half h bar, 1 h bar, 3 by 2 h bar, so on, and so forth, these are the units of S.

Orbital angular momentum comes only in these units 0, 1, 2, 3 that is Bohr quantization. But what the experiments of Stern-Gerlach taught us, that is something that you will study at great length in your modern physics course is that, there are also half r integer multiples of h bar, which actually contributed angular momentum, and this does not come from any motion.

Why is it intrinsic? It is intrinsic, because this spin cannot be understood in terms of, so let me put a q here, cannot be understood in terms of orbital motion therefore, there is an intrinsic spin. If there is an intrinsic spin and if there is a charge, we immediately have the magnetic moment. Therefore, I should be able to write down the magnetic moment corresponding to this spin.

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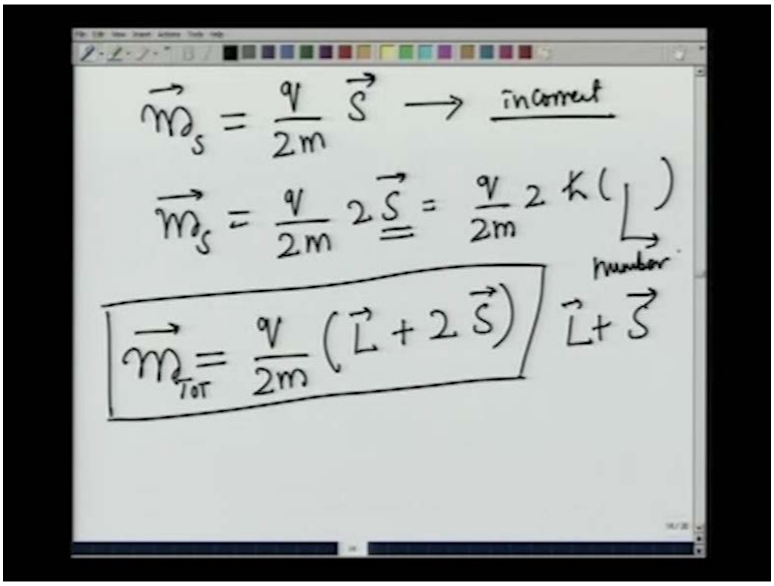

So, what is the magnetic moment corresponding to this spin, you might be tempted to write that this is equal to q by 2 m into s, but this answer is incorrect this answer is incorrect. And there is a very famous factor called g equal to 2 very, very important for us. Even, in making quantum devices technological devices, quantum hall effect depends on this it turns out that m s is actually given by, q over 2 m into 2 s, there is a factor of 2.

Now, let me introduce again the units of h bar. So, this will be q by 2 m into 2 into whatever h bar into a number. Now, imagine an electron going round and round and round; so there are two components of angular momentum, one come from its orbital motion, another coming from its intrinsic spin.

Let me give you a picture do not take the picture seriously, it is incorrect, but you can use it for imagination; the earth is rotating about its axis, that is the rotational motion that is the spinning motion, and as it is rotating, the center of mass of the earth is going around the sun. The earth going around the sun completing the revolution in 365 days is the analog of the angular momentum. The earth rotating about its axis with a period of 24 hours is the analog of the spin angular momentum.

Now, imagine that you are shrinking the radius of the earth, the radius goes to 0, the rotational velocity goes to infinity such that, the angular momentum remains the same, this is mathematically not allowed. However, if you could imagine such a thing, then that would approximate some kind of picture way to the notion of a spin that I (( )), is the part ok. Therefore, by the additivity of magnetic moment, how will I write the total magnetic moment, my total magnetic moment is given by q by 2 m L plus 2 s.

If I ask you, what is the total angular momentum, that will have been L plus s, because angular momentum is additive, magnetic moment is also additive, but the total magnetic moment is not proportional to the total angular momentum, but it is proportional to L plus 2 s, there is an extra (( )) and this is something that we have to remember.

You may be wondering why I am spending so much time on this, I am telling you this, because there are always surprises which physics throws on us. Let me complete the argument, we said there are no magnetic monopoles, divergence B equal to 0. And then we argued therefore, that all magnetic moment should be produced by currents; and we derived an expression for the magnetic field in terms of currents charges which are bodily moving; then I make the large distance approximation and I introduce the concept of a magnetic moment.

Now, I am appealing to modern physics and I am telling you that, even a particle addressed can have a non vanishing magnetic moment. So, a particle addressed has an effective current, it is not coming from any velocity, it is not coming from any motion, it is coming from fundamental physics quantum mechanics, and that can produce a magnetic field.

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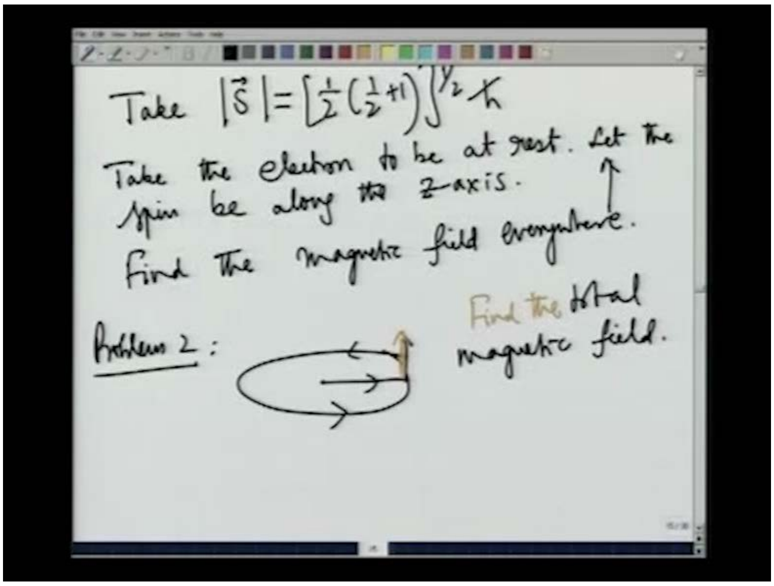

So, that means I have to give you another problem. Take the modulus of s to be half into half plus 1 square root h bar, h bar take this to be that, and take an electron at rest, take the electron to be at rest, this is for my electron, electron to be at rest; You can place the electron anywhere it does not matter, let us take the origin at the location of the electron. I am giving you s and let the spin be along the z axis, along the z axis, in chemistry this is what you called as the upstate. The electron is in the up direction, the electrons spin is in the up direction, find the magnetic field everywhere.

Since, this is an intrinsic spin, there is no question of computing higher order corrections like for a circular loop, and your dipole approximation will be exact for the electron spin. So, the magnetic field goes like 1 over r cubed even for an electron at rest. So, find the magnetic field everywhere, so this is problem number 1.

Let me give another problem for you, so that you appreciate what is happening. So, let me look at the loop here, my electron is going round and round like this, I have to average it over a period of course and let it spin be along this direction. So, let me introduce this line for the spin, it is always in the parallel direction. Find the total, let me use the black ink, find the total magnetic field. Obviously, in this case I have to do a time average calculation find the total magnetic field. Of course, you can do a whole lot of other calculations, even without the concept of a spin, these are all standard problems.

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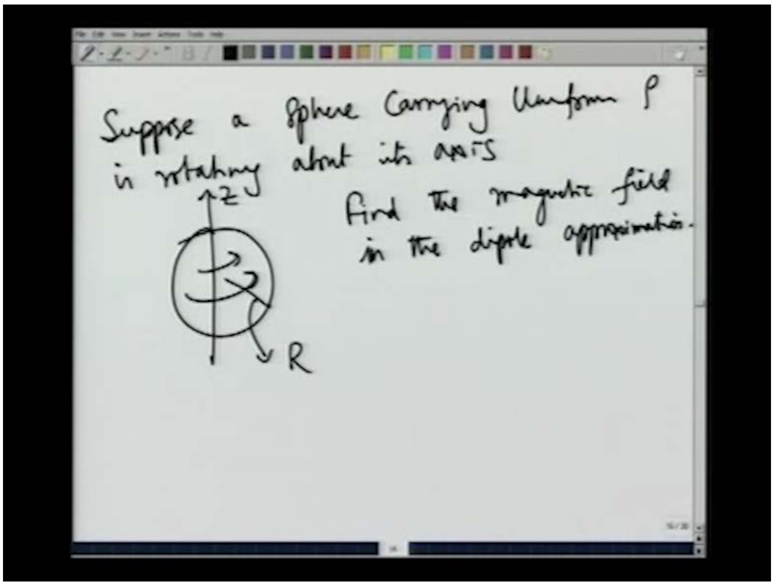

Suppose, a sphere carrying uniform charge density right is rotating about its axis, the axis is passing through the center. So, I have my sphere here, this is my z axis, and it is rotating. So, the radius of the sphere is capital R, suppose it is rotating about its axis find the magnetic field in the dipole approximation. So, I have already written for you, a equal to m cross r by r cube, B equal to 3 m dot r r etcetera etcetera find that, and now you see therefore, you have a very, very powerful tool to determine the magnetic field at large distances; not only that, quantum mechanics throws a surprise and it gives us an intrinsic spin and it was the genius of Pauli, who actually exploited to the maximum (( )).

To repeat, it not a meaningless digression that I have done, because modern technology all of electronics in fact, all of modern devices are driven by quantum physics. You have the nano devices, you have what are called as SQUID, Superconducting Quantum Interference Device. If you want to understand them as engineers and technologist, even if you are not physicist if you want to understand them, if you want to use them, if you want to fabricate them, you better know the basics of what is happening.

In fact, there is a whole subject matter called spintronics as a new revolution in technology which is emerging. Most of you might have actually heard of, what is called as quantum computation, it has been there for the past decade or so, and tomorrow if people build computers, which run on quantum mechanical loss, not classical computers which we use desktops, laptops or whatever. Whatever the basic ingredient is suppose to be the spintronics and what you exploit there is the intrinsic spin and the intrinsic magnetic moment in order to manipulate.

You replace the bit by what is called as a cubic, that is an extraordinarily fascinating study and therefore, what I have done is to give you a small glimpse, one ray of light which is coming from a dazzling sun, suppose everything is darkened that how it is looking like. So, you people should certainly go look up literature, look up the net, talk to your teachers and try to learn more about it, because these are indeed the most fascinating that are happening.

Let us, try hard to this particular subject. And now, move on to another very, very interesting topic namely, the effect of the magnetic field on the charge particles. So, what have we done so far?

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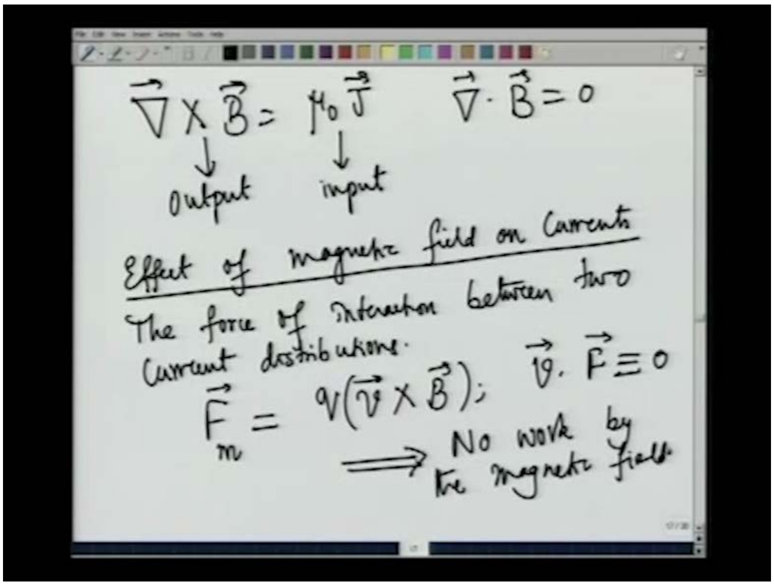

We looked at the equation curl B equal to mu naught J, divergence B equal to 0, we should always ask ourselves, what is the input, what is the output? If we do not know that, as they say input garbage, output garbage, we have to avoid that, this is the input and this is the output that is what we are trying to calculate. But then we know if currents are going to produce magnetic fields, the magnetic fields are going to act on the currents; just as if charges produce electrical field, electric fields going to act on the charges. And what is that given by? That is given by, the Lorentz force expression. Therefore, now I will keep the Maxwell's equation aside for a minute, and I will start looking at the Lorentz force expression.

So, we are interested in the effect of magnetic field and charge particles currents. Actually I have already discussed that, when I looked at the hall system or the cyclotron motion etcetera, etcetera, but my purpose here is to actually write down an interaction force and a interaction energy between two different currents, that is the idea. So, I am interested in the source of interaction between two current distributions.

In fact, it is this idea, which allows you to introduce the concept of mutual inductance, self inductance etcetera, etcetera as (( )) how the energy changes, and also the interaction between two magnetic dipoles. So, as the first step let me write down the Lorentz force expression, F m the magnetic force is nothing but q into v cross B. And what is the first conclusion that we draw from this? The first conclusion that we draw from this is v dot F identically equal to 0 implies, no work by the magnetic field.

So, suppose I have a highly complicated inhomogeneous magnetic field 1 bar magnet here, 1 bar magnet, some electromagnet there, earth magnetism; and suppose, I send a stream of electrons, I send a single charge particle, it will execute a complicated motion; but that motion will be constrained by the condition that the kinetic energy is always the same, magnetic fields do not do any work.

This of course should be interpreted with care otherwise, when I bring the magnet, how will it pull out the other magnet, how will I understand induction? So, that is where the intricacies of the interaction energy between two current loops come into picture. And that is how we prepare ground in order to understand induction; therefore I will take that up in the next lecture.