Engineering Physics – II

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Module No. # 04
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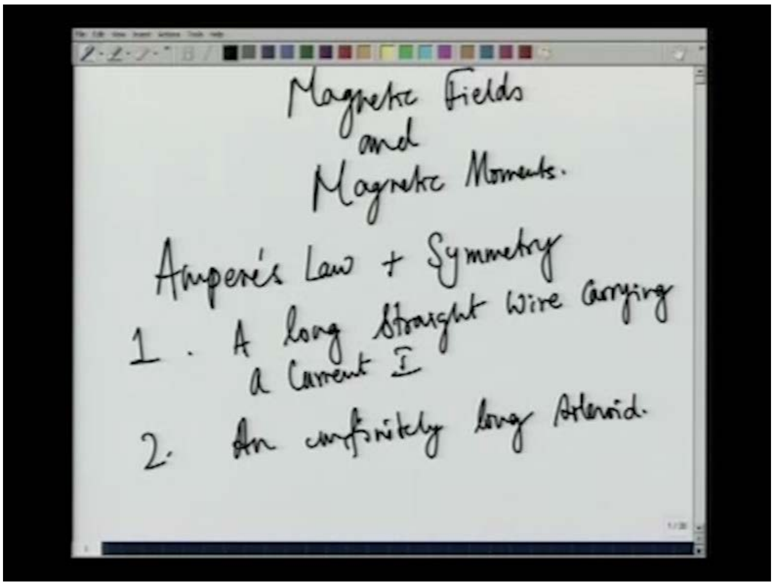

In the last lecture, we looked at a few examples of determining the magnetic field and what we did was to make use of Ampere's law and its symmetries. So, I started with Ampere's laws plus symmetry, and then we were able to determine the magnetic field in some simple configurations. What were the simple configurations? A long straight wire, carrying a current I, and also an infinitely long solenoid; I then ask myself the question, whether it is possible to actually determine the magnetic field in a slightly better way, rather than always appeal to the symmetries, because symmetries more often than not, are our constructions, they are not easily available in realistic situations, where you would actually like to determine the magnetic field.

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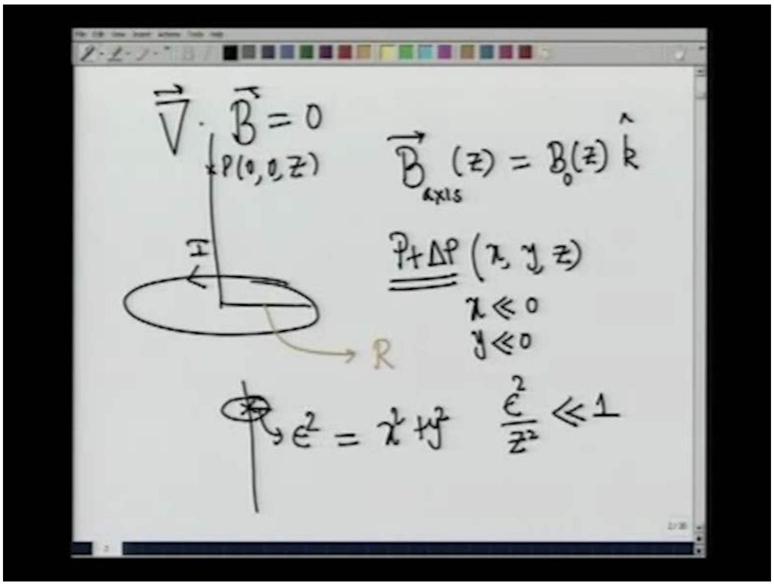

So, in order to do that, what I did was to invoke the divergence free expression, divergence B equal to 0. And I gave you an example, where this expression can be used to determine the magnetic field, even when it is difficult to evaluate the magnetic field exactly. And what is it based on? It is based on what we generally call perturbation theory or in your case it will be simply Taylor expansion.

I do not wish to spend too much time to repeat whatever I told you in the previous lecture, but let me just indicate what is it, that I did for the sake of continuity. So, remember, I have a circular loop of radius R. So, this has a radius R, it is carrying a current I and I am interested at a point P along the z-axis. So, the x coordinate is 0, the y coordinate is 0 and the z coordinate is not 0; in other words, this point P is at a distance z; in this case, above the plane from the center of the orbit, center of the loop.

Now, it is a very standard exercise for you people, what you do is to make use of the Biot-Savart law and then write down the magnetic field. Of course, I am not proceeding entirely logically, sequentially I mean, because I assume, that you people are already familiar with Biot-Savart law, which you are, because you have used that in your 12th standard. But I will certainly come to Biot-Savart law in a short while, we will formally derive that.

So, at this point P, we will write down the magnetic field. The magnetic field is along the axis, it is a function of z and what is this. This is simply, I will denote it as B naught of z into k. What you people have done is to actually evaluate this B naught of z by integrating the expression in the Biot savart law, but I am not interested in that. What I want to do is to consider a point P plus delta P, I am interested in this particular point; I move a certain distance delta P away.

Now, what is the nature of this moving a certain distance delta P away? This will have coordinates x, y and z, the same z. At any given value of z, I slightly move away, except that x much, much less than 0, y much, much less than 0. Whatever units you are considering, we wish to stay very close to the axis and I would like to ask, what will be the magnetic field at this particular point x, y and z.

So, in order to indicate that to you, what I have is, I have the point P and let me look at this thing, I want to look at a neighborhood point. In fact, given this point P, what I will do is I will draw an infinitesimal circle of radius epsilon, root x square plus y squared will be equal to epsilon squared. Epsilon squared equal to x squared plus y squared and if you feel like, you can also assert, that epsilon squared by z squared is very, very small; epsilon squared by z squared is much, much less than 1. And we would like to see, what it is? This is not an essential condition, but it will give you an idea of how to evaluate.

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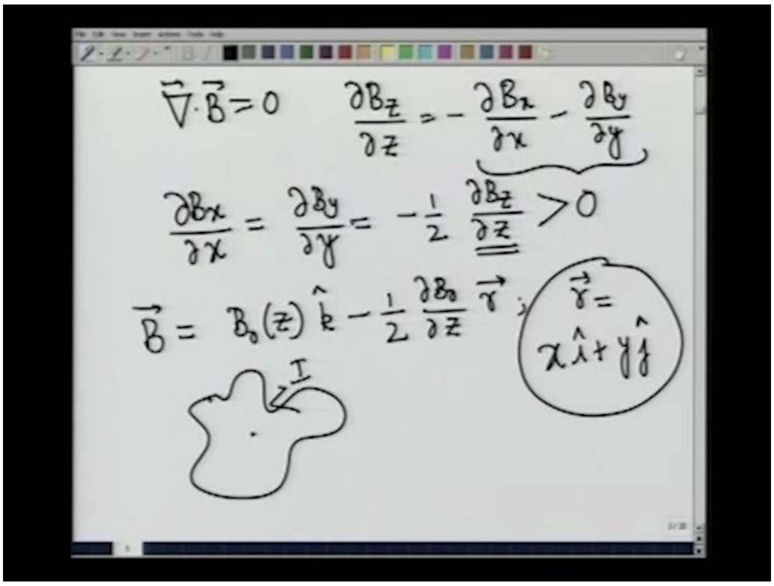

What is it that I did? What I did was to make use of the law, divergence B equal to 0; again, I made use of the symmetries. So, suppose, I open it up, what does it tell me? It tells me, delta B z by delta z is equal to minus delta B x by delta x minus delta B y by delta y. Please notice, that divergence B equal to 0 is a constraint, it has nothing to do with what kind of a current is producing in the magnetic field, it may be time independent, time dependent, it may depend on space in any manner, but this has to be identically satisfied. There is no violation allowed for this particular law, therefore this is valid everywhere.

But in our case we know that there is nothing that tells us how to choose the x and the y-axis because there is a complete cylindrical symmetry. I have a straight y, which is carrying the, current, axis and there is a loop here. So, who tells me how I should choose x-axis and the y-axis. Therefore, by symmetry I can look at the right hand side and I can assert, that delta B x by delta x is equal to delta B y by delta y; delta B x by delta x is equal to delta B y by delta y, and what are they? This is nothing but minus half delta B z by delta z, because that is the only way the equation above can be identically satisfied.

Now, once we realize this statement, delta B x by delta x equal to delta B y by delta y, that means, we know, that the field is radial, is that right? The field is radial in the plane perpendicular to the axis. Now, whether it is radial in the sense, that the field lines come inside or the field lines diverge out depends on the sign of delta B z by delta z. But then, we know, as we move further and further away from the loop on the axis, delta by B z decreases. Therefore, delta B z by delta z less than 0, this is always greater than 0, therefore the field is spreading out radically, spreading out.

And what will be my field? My field will be simply given by B naught of z into k; that is the first term along the axis. Now, I switch on the correction term, I will write delta B naught by delta z, because everything is evaluated along the axis, that is what we are interested in into r, where r is written in the cylindrical coordinate system, is that right? So, what is my r? My r is simply given by x I hat plus y j hat that is what I have. Probably, I should have used the notation rho, but never mind about that, I have defined it carefully and that is what we did.

So, you see, with luck if you are able to determine the magnetic field in some approximation, in some limiting case, then you can be trying to use divergence B equal to 0 to determine the magnetic field in the immediate neighborhood, but I can give you a slightly different problem. Suppose, my loop was like this, it is carrying a current I and it has a complicated structure. So, what do you do? You take a battery, connect a wire and just throw it on the floor and it will occupy some complicated shape, is that right?

Now, I ask you, what is the magnetic field at any given point? Well, if you want you can choose a point in the plane of the loop, is that right, and then you can construct an axis, but then we do not know how to integrate the Biot-Savart law. So, all these problems become exercises in numerical integration; you have to evaluate them numerically.

But on the other hand, I would like to have a systematic technique of evaluating the magnetic field even analytically, is that right? And the 1st step in actually evaluating these quantities analytically is actually by introducing the notion of a magnetic moment, please remember, if you are given an arbitrary charge distribution.

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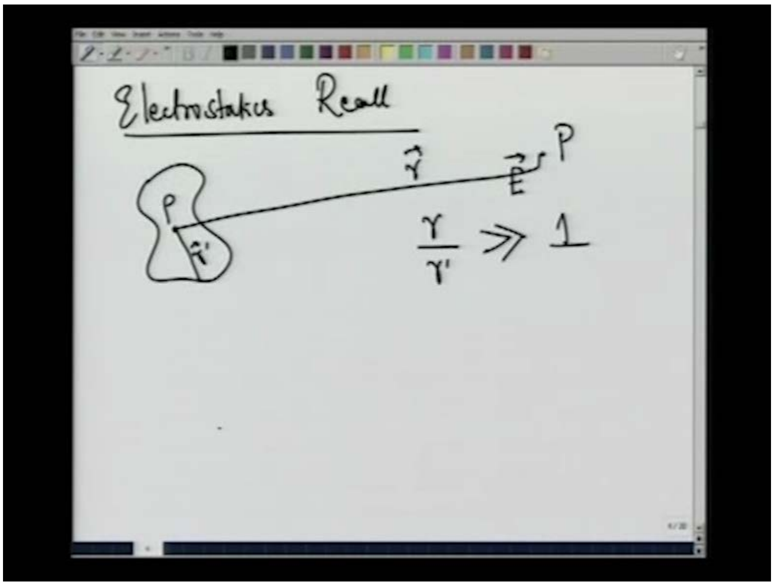

So, let me go back to the electrostatics, electrostatics recall. Suppose, I give you an arbitrary charge distribution density rho and suppose you are sitting at a point P far away, and you are interested in the electric field at this particular point; what do you mean by far away? So, here, the distance is something like r prime, is that right, and this distance is r. So, what are we saying, r by r prime much, much greater than 1, that is the statement, that we are making; it is much, much greater than 1.

So, you take any point in the distribution, it looks rather closed to any point within the distribution is, that compared to this we are far, far away from the source, then what is the approximation that we did. The approximation that we need was in the 1st instance. Assume that all the charges concentrated at this point, that is the monopole term. Then, you assume, that this rho is constituted of equal number of positive and negative, then you subtract the monopole term. Whatever will be the remaining, will be total charge is equal to 0. Then, we assume that it is a dipole; then there is a correction to that, that is the quadruple. I did discuss this problem with you at some length in some 7th or 8th lecture, is that ok.

Now, what we wish to do is to mimic, that accept, that the case of the magnetic field is a little be it more complicated than the electrostatic field, I will let you know in a short while. Therefore, I am not going to introduce the analogs of electric quadruple moment to you, but I will introduce the analog of the electric dipole moment, what that will be called as the magnetic dipole moment or simply the magnetic moment, and in order to do that, it is convenient, it is advisable, that we develop a certain amount of machinery.

Let us proceed slowly because at the end of the lecture you will see, that magnetic moment is a very, very important quantity, it is not simply a convenient definition. Electrons possess an intrinsic magnetic moment; we would like to understand their properties. So, let us proceed slowly. In order to proceed, it is always good to go back and look at the fundamental equations and by this time you people are all familiar with the notion, with the idea behind these lectures. We want to constraint the (( )) or at least, appreciate various parts of the (( )) of the electrodynamics as it is based on Maxwell's equations because Maxwell's equations adequately, completely and correctly summarize all known electrodynamics phenomena.

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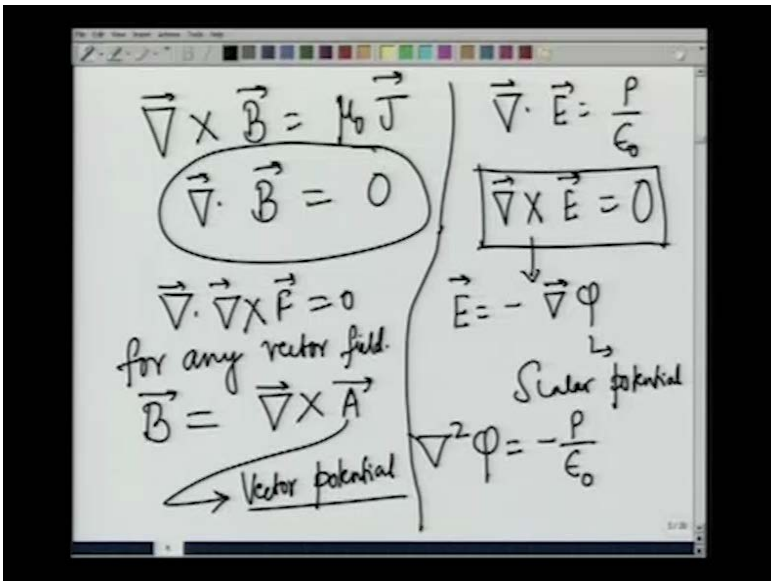

So, let me return to the basic equations and my basic equations are curl B is equal to mu naught j and divergence B equal to 0. So, for convenience's sake, I am going to write down the analogous electrostatic equations. These are the famous magneto-static equations; magneto-statics come when there are stationary currents, electrostatics arises when there are stationary or static charges. So, the analogous equations are divergence E equal to rho by epsilon naught and curl E equal to 0.

Now, the way we proceeded in order to discuss electrostatic was to look at this, which is a constraint irrespective of what the source is. Then, we solve divergence E equal to rho by epsilon naught, your electric field shall identically satisfy curl of E equal to 0. It is as my telling you, you do whatever you want, but you shall not spend more than 50 rupees, that is what the father tells the son; curl of E equal to 0 is that kind of a constraint. What did we do? What we did was to exploit curl E equal to 0 and rewrite E is equal to minus grad phi, where phi is the scalar potential, scalar potential.

Now, once I write the scalar potential, once I write E equal to minus grad phi, I do not have to worry about the constraint anymore. Because I will directly determine the scalar potential and I will calculate the gradient and the curl of a gradient is identically equal to 0, elementary exercise in vector calculus, which you people have done. So, what happens to this equation? This becomes del square phi is equal to minus rho by epsilon naught. Divergence of del is nothing but the Laplacian del square and we did discuss this electrostatics, electrostatics in media and all that. This is the famous Poisson's equation and we said, instead of solving for the electric field, please solve for the potential.

So, there, I said 2 fold advantage for you in solving for the potential. Number 1, we will automatically satisfy the condition curl of E equal to 0, electric filed is irrotational, it can only diverge, it will diverge only at those points where there are sources. Number 2, the other advantage is that instead of solving for three components, three quantities, the x component, the y component, the z component of the electric field, you are going to solve for a single quantity phi. In other words, the condition curl of E equal to 0 tells you that E x, E y, E z cannot be independent of each other. If they were independent of each other, curl E would not have been equal to 0, and the way they are dependent on each other is simply by writing E equal to minus grad phi.

So, what is our purpose? Our purpose is actually, to mimic whatever is on the right hand side, at the left hand side and see, whether we can get any convenient way of actually incorporating divergence B equal to 0 as the condition. Now, divergence B equal to 0 is slightly different from curl of E equal to 0. Curl of E equal to 0 is a set of 3 equations because the left hand side is a vector, whereas you have a single equation divergence B equal to 0. Therefore, it is actually possible to rewrite B in terms of a more elementary quantity. What was the elementary quantity in the case of the electric field? The scalar potential.

In a similar manner, I would be able to do that, except that it will not be a scalar. Anyway, if it were a scalar, how would I be able to write down divergence B equal to 0? All of you know under what condition divergence B will be equal to 0, and that is divergence of curl of f equal to 0 for any vector field, for any vector field. I remember telling you people, that this is the analog of A dot A cross B, A cross B is a vector perpendicular to both A and B. Therefore, A dot A cross B is the inner product between a vector and a vector orthogonal to that, perpendicular to that will be 0. The same thing goes through here, otherwise you simply plug the expression and you will find by the continuity of the f in the derivatives.

In the 2nd derivatives, this is identically equal to 0. Since this is valid, I should be able to write divergence B in terms of a more elementary quantity, which I shall call as a potential. Therefore, what I will do is to write B is equal to curl of A, just as E equal to minus gradient of phi. So, you took the derivative of the scalar potential in order to get the electric field. Here also, you take the derivative of this vector filed A in order to get B, and therefore, this is called the vector potential.

The scalar potential is a quantity, which transforms like a scalar on the rotations, whereas now I have introduced a quantity, which you have not been exposed earlier, which you have not been introduced earlier and that is, B equal to curl of A. Obviously, you cannot immediately jump up and say this is also going to give me some kind of an energy because energy is a scalar and the vector potential is not a scalar. So, we have to be very careful in dealing with A and phi.

So, in any case one - sided B equal to curl of A, I can forget about divergence B equal to 0, that is the physical statement, that there are no magnetic charges, that there are no magnetic monopoles, is mathematically realized by writing B as the curl of another vector field, namely the vector potential the problem is standard. Now, what we have to face before I go on to discuss the determination of the magnetic field itself and for that, let me return to the electrostatics.

You give me a scalar potential, I know how to calculate the electric field, what do I do? Simply calculate the derivative. In a similar manner, give me the vector potential, I know how to calculate the magnetic field, simply calculate the curl. But the big question is, given the electric field and given the magnetic field, is it possible for me to determine A and phi. Obviously, we have to do integration and therefore, in doing integration, there is always a problem of the uniqueness of the solution. How unique is the solution?

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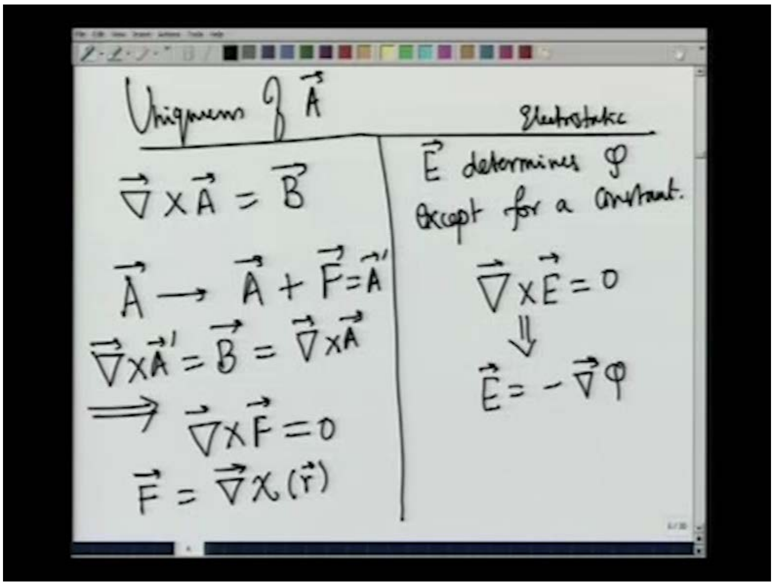

So, let us ask ourselves how unique is the solution? Uniqueness of A, so let me draw a line, again dividing the electrostatic with the magnetic field. So, this is my electrostatic case. In the electrostatic case, we know phi is almost determined, the only thing that is not determined is a constant, is that right? So, E determines phi, except for a constant and this goes very well with the principle of energy, namely. Only energy differences are measurable, absolute energies are not measurable, so E determines phi except for a constant. And what is it that you people do normally for all finite charge distributions? That is what exists in reality; in any case, you choose the potential at infinity to B equal to 0.

In a similar manner, I have to ask myself, how unique is A? Of course, you can always add a constant vector potential because curl of a constant is equal to 0. But the right question to ask is, are there more non-trivial ways of finding the vector potential? If that is so, then perhaps I can decrease the number of degrees of freedom in B. I hope you people appreciate whatever I am trying to tell you. You had three quantities of the, namely the three components of the electric field. We were able to express all of them in terms of a single quantity, namely the scalar potential. But now, what has happened is that we have three components of the magnetic field, three components of the vector potential.

Therefore, I want to see, if there is further simplification possible in determination of A. What is it that I have? I have curl of A equal to B. So, in asking for the degree of freedom in defining my A, what is it that I can do? I can replace A by A plus another vector field F. That is exactly what we did here, you replaced phi by another constant phi prime and then, you will find, that that phi prime has to be completely a constant another scalar phi prime, that phi prime will have to be a constant. Therefore, suppose I write A equal to A plus A and curl of A equal to B. So, let me call it as a prime. What is that I am insisting? I am insisting the curl of A prime is equal to 0 B, which is equal to curl of A.

So, what does it mean? This implies that curl of F would be 0. Back to the electrostatic condition, curl of E equal to 0; curl of E equal to 0. And what is it that we did? We wrote E equal to minus grad phi. So, in a similar manner, since curl of F equal to 0, I will write F is equal to some grad, I do not want to use phi because phi is reserved for the scalar potential, so let me use a Greek index and I call it as k of r. So, what does this analysis tell me?

This analysis tells me, that look here, suppose I want to find the vector potential corresponding to a given magnetic field B. So, suppose one of you produces one vector potential, another student produces another vector potential. This analysis tells you, that the 2 vector potentials can defer utmost by a gradient function. So, you have a freedom of choosing your gradient that is the scalar function, anyway. You want, you can write anything that you want, take that as a homework problem, and what is the home work problem? This is the following.

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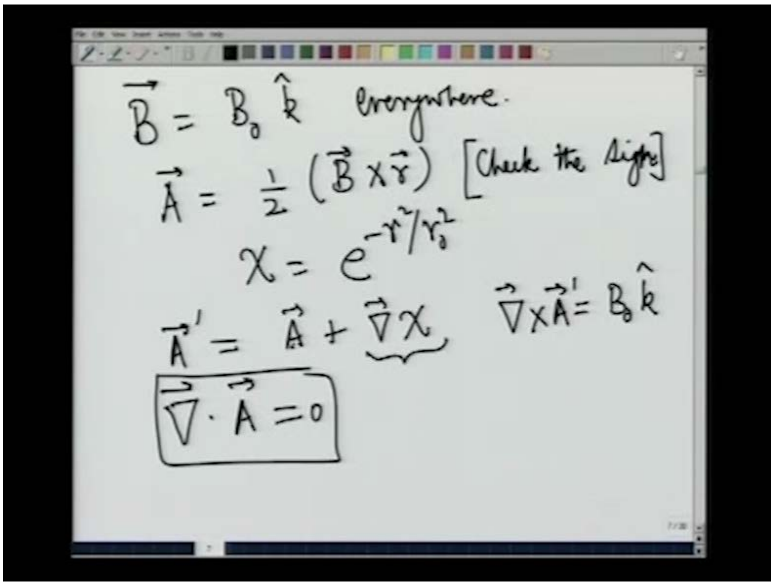

Suppose, I give you a magnetic field, which is constant everywhere; B equal to B naught k everywhere. Now, I want to find out a vector potential, which is going to give me this. Let me give you 1 vector potential and that is simply given by A equal to half B cross r. I hope I am right, if I am not right I might have committed a sign mistake. So, please go and check. So, here is the mini problem, check the sign, check the sigh, not sigh, sign.

Now, you can add to that any scalar function that you want. So, suppose I choose k equal to E to the power of minus r square by r naught square. You calculate the gradient; then you do the curl, that will all be 0. So, if k equal to E to the power of minus r square by r naught square A prime is given by A plus gradient k. What will you check, you will find, that curl of A prime will be equal to B naught k. So, we have a large freedom in choosing my vector potential. In the case of scalar potential, you could only add 1 constant, but here, you can you have a freedom in choosing the whole scalar function; k is an arbitrarily scalar function. Of course, since there is a gradient sitting, you should be differentiable at least once, there should be no discontinuities, is that ok?

So, once we are guaranteed that we can actually try to exploit this freedom in the choice of chi and we can simply find my A. What is the simplification that one does that come from experience, it is not all that simple, take my word at that. What one does is, always choose divergence A equal to 0. What is the statement that I am making? Suppose, I give you a vector potential and the divergence of that vector potential is not equal to 0, then you replace that vector potential by a plus gradient chi, then demand divergence of A prime is equal to 0; demand that divergence of A prime equal to 0. You will get an equation for the corresponding equation chi, solve for chi and divergence E would be equal to 0, fine.

So, once I use divergence A equal to 0 as a condition, then my equations become very simple and if you think that I am doing something of an, of some exercise, which is somewhat esoteric, which is out of way, please be patient because now, you will find an equation, which is very, very beautiful and which is very close to the Poisson equation, that I wrote. I know how to write down the solution for the Poisson equation, Coulomb's law by the principle of superposition. So, I will exploit that and I will also write the solution of the corresponding equation is that, that will give me the Biot-Savart law, so let us proceed.

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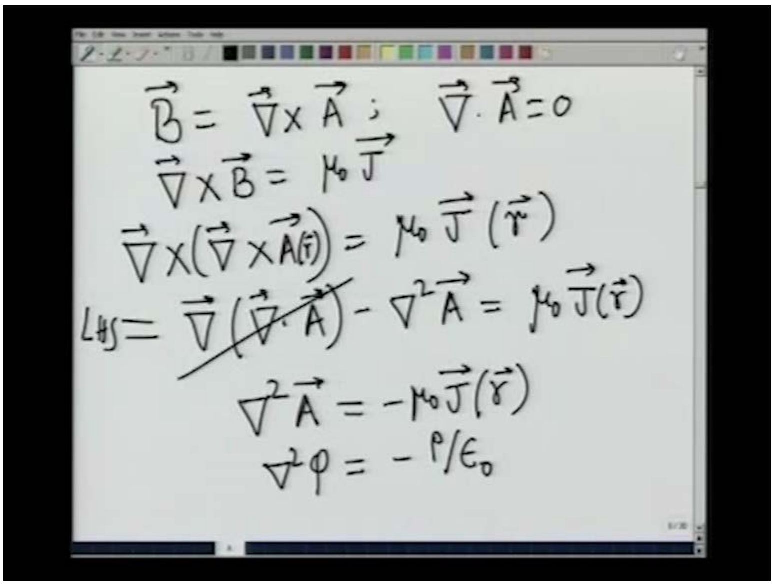

So, we have taken care of one of the equations, divergence B equal to 0 by writing B equal to curl A; by writing B equal to curl A. Now, I plug this in the source equation and what is that curl of B equal to, mu naught J. So, what I am going to do is to write curl of curl of A, this is equal to mu naught J and this is also a function of r. It is an elementary exercise, this is not difficult at all, open it up, calculate the determinant, what will you get? Let me write the answers straightaway for you.

This is gradient divergence of A minus Laplacian of A, that is the left hand side, that is equal to mu naught J of r. So, what do you do, you should understand the meaning of this equation. Suppose, you look at the x component of the vector potential, you calculate del square, del square by del x square del square by del y square del square by del z square. Operate each of them on x, add them up and on the right hand side, it will be mu naught J x of r; is that part ok, that is what I have.

But then, I told you, that divergence A equal to 0, that is the additional condition that I am employing; divergence A equal to 0 is the additional condition, that I am employing. Therefore, I can get rid of this equation and now, I have a set of 3 equations, del square A is equal to minus mu naught J of r.

And what do I need to compare this equation with the analogous electrostatic equation and that was del square phi is equal to minus rho by epsilon naught. So, electrostatics is characterized by a single Poisson equation, whereas the magneto-statics, stationery magnetic fields are characterized by three Poisson equations. What are the three Poisson equations?

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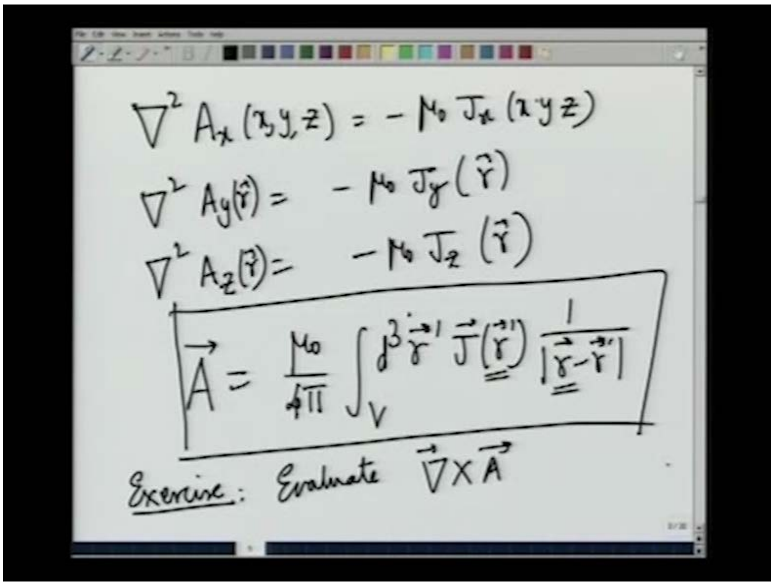

Let me write it for you, so that there is no confusion. del square A x, which will be a function of x, y, z will be minus mu naught J x x, y, z. del square A y is equal to mu naught J y; del square A z is equal to minus mu naught J z. So, you can make it a function of r; function of r. And each of these equations is independent. Since, I know how to solve del square phi equal to minus rho by epsilon naught and this equation is no different, all that we have is to replace rho by J1 or epsilon naught by mu naught phi by one of the components of A. I can write down my solution and what will my solution be?

My solution is simply given by mu naught over 4 pi integral d cube r prime j of r prime 1 over mod r minus r prime, that is what I have. So, I write down the solutions for A x, A y, A z individually, bring them together, make it into a vector equation and this indeed is my solution.

And what is the integration over? The integration is over the region r prime. So, this is integration over the region r prime, that volume integral. And it is in this region r prime, that my current distribution is non-vanishing, you are not worried about contribution coming from points r prime, where there is no current because current is the source of the magnetic field. Therefore, at last we have been able to write down an integral solution for the vector potential A. And what is it that you do now?

Well, if you are given the vector potential as I tell you, calculate the curl of the right hand side. So, we are going to calculate the derivative with respect to r, it is not going to act on r prime, it is going to act only on this quantity r. Everyone knows how to calculate this and that will give you the magnetic field straightaway, that is the idea. I will not actually work out the curl of this A in order to give you an expression for B. I will write it as an exercise, I will give it to you as an exercise, is that ok?

So, let me write down the exercise. Evaluate curl of A, I will of course, write the answer for you in a second, but before I do that, let us be very careful, let us be slow, let us actually recall, let us remind ourselves what the meaning of this integral is, whatever is in this square. And it is also good for us to remember, that although the solution looks the same, my current densities are not the same as the charge densities. There is no question about that because we are looking at steady currents.

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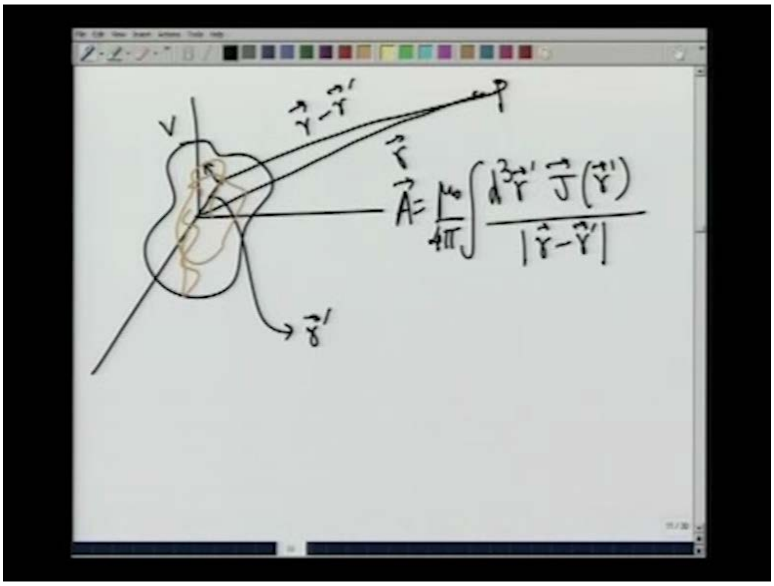

So, let me spend a time drawing a picture. This is my coordinate system and let me imagine, that this is my region volume and there are currents, which are flowing, I indicate them like this. I am doing it with a dual purpose, I want to remind you the, what the meaning of a steady current is as an approximation to time, time dependent currents is that.

Let us imagine that the currents are all moving in this region. Now, since the current density is confined to this region v for all times, so this boundary is fixed; this boundary is fixed. Of course, there is a very special case, where all of them move along this loop that would be called as a loop of current. But let us not worry about that, you can see, that as far as this boundary is concerned, at least at this boundary, the velocities can only be tangential, it cannot be normal because if it were normal, it would have escaped away. It can be normal inwards, but not normal outwards. So, v perpendicular outside is always equal to 0, which can be inward. Now, that means, the charges are forever confined in this region v.

Now, they go, they collide with each other, they keep going round and round and round and round and therefore, if I write for a sufficiently long time, then each charge will complete 1 particular circle. So, over that period, if I introduce the means, v bar will be equal to 0 over that mean. If V bar is equal to 0, the corresponding statement is that J bar is also equal to 0 over the period of revolution and mind you, I am interested in a steady current. That means, the same pattern should be there, is that ok? It should look the same. Once an average over my period of time is that, so by the same token after a while the charge particle has to come back.

So, suppose this is the loop and there was an electron here, what do I mean by the steady current? I want to say, that if I look at the period, time period, that the electron takes to complete 1 loop, I will measure the current, at times scales, larger than that, like it happens in case of atoms, is that ok? Then it is equivalent to uniform charge density, which is moving along this particular loop, is that right?

In any case, if I look at the position of this loop charge particle after at certain time t, it has to come back. Therefore, not only is v equal to 0, the mean field of r is also equal to 0. In order to show the mean values, either one employs this bar or equivalently this is more convenient, this is equal to 0 and this is equal to 0, we are interested in the time averages. In fact, this is the precise way of defining the steady currents, I have spent some quite time, quite some time for you when I discussed the distinction between a microscopic field and a macroscopic field, just then I was introducing the medium for you people. So, let us remember that. So, these two are very well kept in mind before we proceed.

Having done this now let me repeat this figure. So, I have my region enclosing the volume v and there is a current here. So, there is, imagine a charge particle, which is moving and the coordinate vector is r prime for this particular point. I am interested in a point P whose coordinate vector is r and of course, if I were to draw these lines, this will be r minus r prime, this is the configuration.

So, when I wrote that integral, what was the integral? d cube r prime J of r prime divided by mod r minus r prime, that is what I wrote. Forget about mu naught over 4 pi, this r prime is going to be in this region, this r prime is going to be in this region, whereas my r is somewhere here. Of course, I am not asserting for you, that my r should always be outside the current distribution because my solution is valid everywhere.

But when I am integrating over r prime, I am only looking at those points at which the current density is non-vanished, mean, basic, elementary fact, but it still worth spending that much time because we want to study, what is it that happens to the electric field? So, mu naught by 4 prime, fine.

What you have to now do is in 2 steps? 1st I have to define for you the Biot-Savart law from this, rather derive for you in some approximation. This is actually a Biot-Savart law for all practical purposes, but anyway. And then, I would like to indicate for you a method of determining A and hence, be at very, very large distances, is that part right? So, let us do that in steps.

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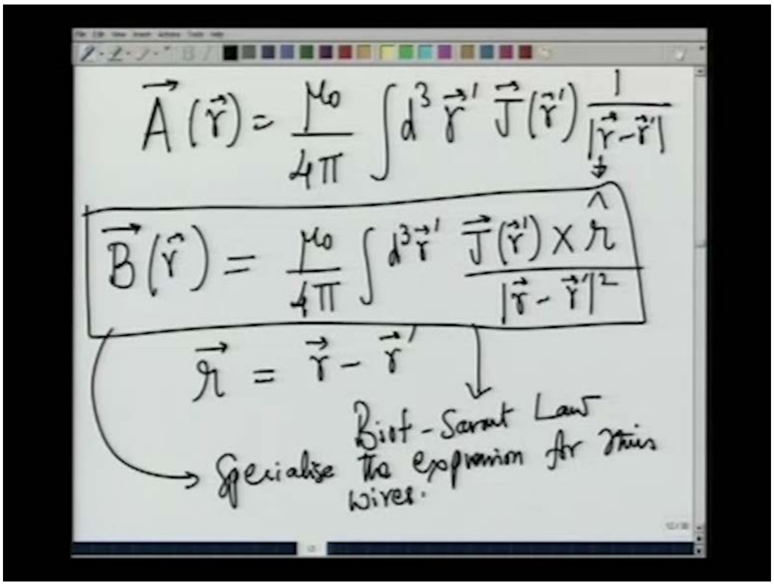

What is it that I wrote? I wrote A is equal to, as a function of r mu naught over 4 pi integral d cube r prime J of r prime 1 over mod r minus r prime. If I want the magnetic field, what will I do? I will simply calculate the curl of the above expression and the curl is going to act only on this variable r and not on r prime. r is the variable we have actually integrated over r prime, that is exactly what we have to do.

What will be the expression for the magnetic field? The expression for the magnetic field is simply given by mu naught over 4 pi. Of course, I will have d cube r prime J of r prime cross, now there is a certain amount of cumbersomeness in this rotation, so for once I will introduce another unit vector r and this will be mod r minus r prime whole square, this is the expression. And what is my vector r? My vector is nothing but r minus r prime. I did not want to write this and then put a unit vector on that particular object. Therefore, my magnetic field is simply given by this expression, I am getting this expression is not more difficult or less difficult, then getting the expression for the electric field starting from that of the potential, there you calculate the gradient, here you calculate the potential above the curl.

So, let me put it in a nice box for you people and this is the famous Biot-Savart law and this is actually the expression, that you people use in order to evaluate the field from the currents. In fact, this is not the form in which you use Biot-Savart law in order to evaluate the currents because your currents were running through thin filaments, thin wires. You had a surface current or you have a copper wire of negligible thickness. So, what I would like to do is to specialize this expression, specialize the expression for thin wires or filaments. So, let me do that and that is done very, very easily, there is no problem at all.

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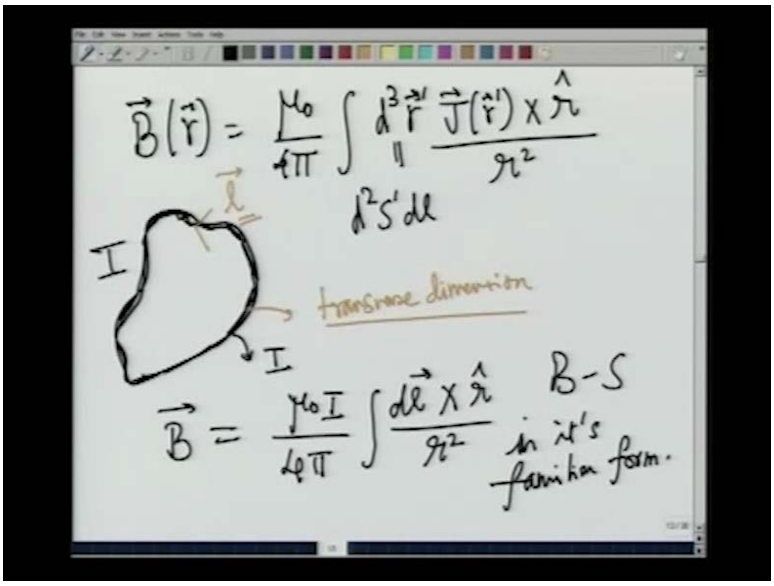

Let me write down the expression first. B of r is simply given by mu naught over 4 pi, which is the baggage we are carrying because of the peculiarity of the S. I. units I have. My integral d cubed r prime J of r prime cross unit vector r, actually I can write it as r square, I have already defined it.

Now, I want to imagine a situation, where I have a loop like this of negligible thickness and that is carrying a current, what do I do? d cubed r prime is the volume element, so let me make it thicker. So, let me make it thicker for you, let me make it thicker for you, so my current is flowing through this volume. What is my d cubed r prime? One is the transverse dimensions, so you have the transverse dimension and that is of negligible thickness and then you have the longitudinal direction, which I will call as the length here, unit vector length l. It keeps on changing, not the unit vector, I am sorry, but this is the other degree of freedom, the longitudinal thing, which I will cross d l, which tells you how the loop is curving.

But then, I know the definition of the current, I have to actually integrate over the transverse dimension, that is what I have to do. So, let me do that, it is of negligible thickness. When I am doing that, I do not have to worry about either the unit vector or r squared because it is of negligible thickness. So, if you remember that and if you remember the incompressibility, it is the same current, that flows everywhere. You cannot have a current I 1 here and a current I 2 here, it is the same current I, that flows everywhere.

What will be my B, magnetic field? My magnetic field will be simply given by mu naught over 4 pi. I write d cubed r prime as what? d 2 s d l, that is what I am going to write. There is the length element, but J of r prime d 2 s prime is nothing but the current and it is a constant quantity. So, I pull the current out. So, once I do that, what will be remaining is the direction of J and what is the direction of the current? The direction of the current density is actually following the curvature of this wire because there is no perpendicular motion. It is always tangential to the curvature of this loop, therefore I will write it as d l cross r divided by r squared; elementary geometry, pretty simple.

And now, all of you should be extremely pleased with yourselves, this is the Biot-Savart law in its standard form; let me call it as in its familiar form. Now, I have closed the loop of my argument in order to motivate the meaning of the divergence B equal to 0 and its utility, I actually worked out a solution making use of Biot-Savart law. But then, I was able to introduce the vector potential, write a set of 3 Poisson equations, whose solution we know from experience with electrostatics. I wrote down the, the solution, I did a curl and I re-did a Biot-Savart law. So, now, it is a tightly in it, well-defined way, well-defined way of actually appreciating what the solution for a steady current is.

But what is the most important difference between a charge density and a current density? We are not going to assert, that the mean value of the charge density averaged over any time or any period is equal to 0 or over a space. But the mean value of a position of a charge and the mean value of the velocity of a charge should be equal to 0 in order to get a steady current, in order to have localized current distributions and that is something, which we have not yet used so far, because this was some kind of a formal manipulation. I anyway assumed that J is independent of time, but if you want to really make sense out of that in a careful fashion, it is better to go back and rewrite the expression for the electric field, rather the vector potential in terms of elementary charges.

Now, this might appear to be a little bit out of way, the way probably the books, that you use in your engineering will teach you, but please believe me, this is a very, very important thing and especially those of you, who would actually like to pursue higher studies either in physics or in engineering, semiconductor devices, fabrication of nano devices, quantum devices, what I am going to tell you is going to lay a very firm foundation. So, please be patient. And in any case, it is a very, very beautiful topic to study. So, what I will now do is to go back to the expression for the vector potential and then, rewrite the solution for that in terms of the elementary charges, let me do that.

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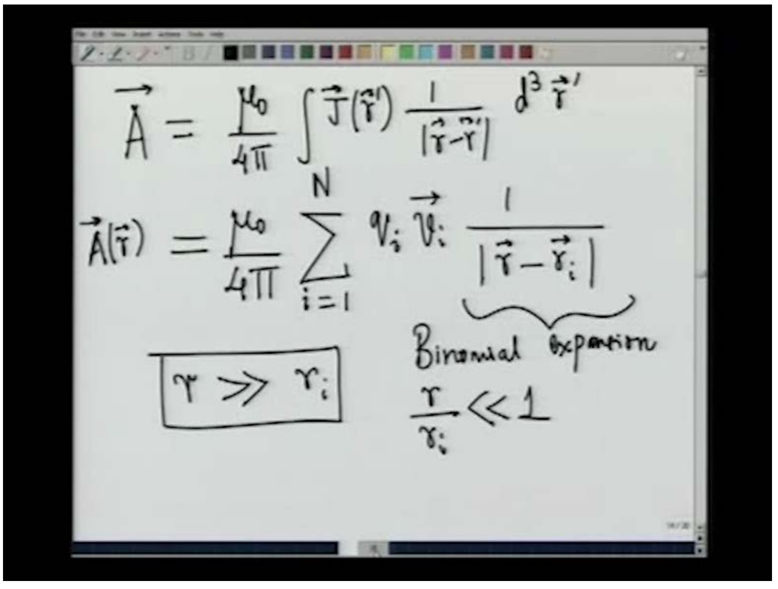

What is the expression for vector potential that I had? That was simply given by mu naught over 4 pi integral J of r prime 1 over mode r minus prime d cubed r prime. Let us remember the long story, that we said at the beginning; what is it? All charges are quantized, they are discrete; they come in discrete units of either the electron charge or the proton charge. If you have ionic conductors and the protons or may be the nuclei will be moving, if I have conductors involving electrons, the electrons will be moving. But deep down everything is discrete; it is only averaging, forging, that makes it appear to be a constant current. If I take a microscope, an electron microscope and look at the conducting wire very, very carefully, you will see a large number of them moving with gaps between them, alright, of the order of fraction of an angstrom or whatever.

Now, I want to exploit that, but what is a current density? Now, current density is occurring simply because individual charge particles are moving. Therefore, without going through the intermediate steps, which are quite trivial, you people can write it immediately. What I shall do is write it as mu naught over 4 pi, I go to the primary summation expression of which this integral is an approximation.

So, let us say, there are n charges, n very, very large, remember the number of free electrons for copper, that you have. We had large numbers per mole, 10 to the power of 22 per centimeter cubed or whatever of that order. So, n is a very, very large quantity, this is nothing but q I v I. The velocity of the individual charges r prime will be replaced by the position of that particular charge, mode r minus r i and this is your expression for A.

What I now want to do is to perform a tailor expansion of 1 over mode I r I r minus r I and get a very, very interesting expression for you in terms of the magnetic moment. As I have been telling you, magnetic moment is a fundamental quantity because what quantum mechanics has taught us is that there can be an intrinsic magnetic moment therefore. And we also know, that since the, we have closed currents, the magnetic moment should be in somewhere related to the angular momentum carried by these charges, this is what we want to exploit. Therefore, let me proceed slowly and let me start looking at the Binomial expansion or the Taylor expansion of this quantity.

What is the Binomial expansion that I want to do? The Binomial expansion, that I want to do is in the variable r, much, much greater than r I, these are the magnitudes. We are so far away from the charge distribution, from that particular point in the lowest approximation; probably it will look like a circular loop. If they are all moving in a plane and then we want to see the deformation in the loop, the deviation from the circularity, so on and so forth. So, we want to make an approximation in this quantity, r much, much greater that r I, that is, r by r i, much, much less than 1. So, we want to keep the needing order contribution that comes from f r; let me proceed to do that.

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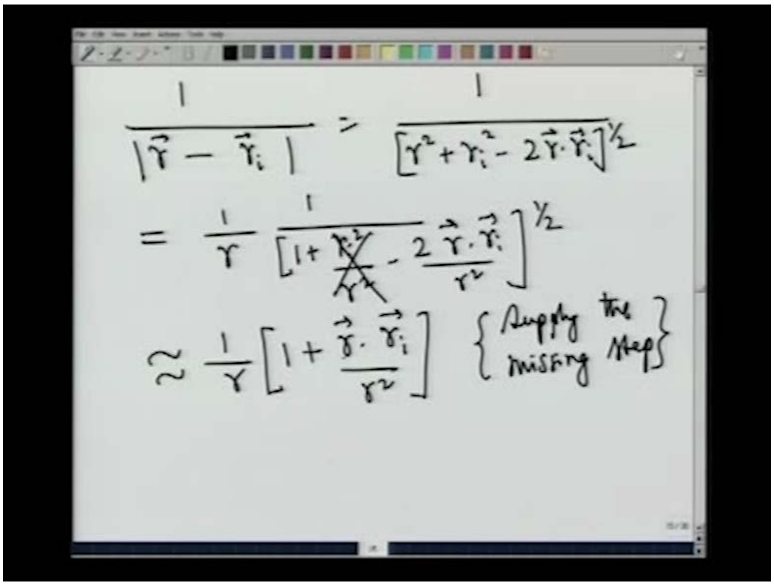

The Binomial expansion is straightforward. I have r minus, r I is nothing but 1 over r squared plus r I squared minus 2 r dot r i to the power of half. So, what do I do? I will pull 1 over r out and I will write it as 1 plus r I squared over r squared minus 2 r dot r I by r squared to the power of half.

I told you, that r I by r is a small quantity, r I squared by r I squared is an even smaller quantity. So, let us forget about that because I want to keep only the leading or the term. Those of you who are enthusiastic, who have the drive to compute higher order corrections, then you should not ignore this term that will give rise to the famous magnetic quadruple moment, magnetic couple moment, like electric quadruple, electric couple, so on and so forth.

So, what is this expression? This is approximately 1 over r that is what my Binomial expansion tells me, 1 plus r dot r I by r squared; supply the missing step, which is only 1 step that is the exercise for you. So, I am going to take this expression, 1 over mode r i r minus r I equal to 1 over r 1 plus r dot divided by r squared, plug it into the expression for A, that is what we did when in, in order to get the expression for the electric dipole moment. In fact, it is this r that gives you the electric dipole moment.

Q I r I divided by r squared into r, all right. E dot r by r squared, that is the expression for the scalar potential. In our case, of course it is going to be somewhat more complicated.

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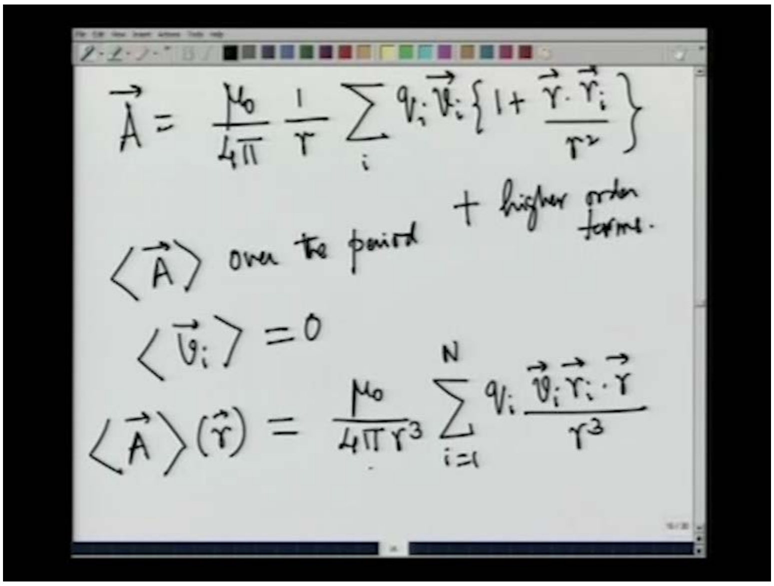

Therefore, let me plug that expression. So, my vector potential is therefore, given by mu naught over 4 pi. I have pulled out a 1 over r, which is independent of the location of all the particles and then I have q i v i, my velocity is of course, a vector quantity 1 plus r dot r I divided by r squared plus higher order terms, as our mathematician friends will always remind us to mention.

Please remember, when I am looking at discrete charges, the velocity is continuously changing with time; the position is continuously changing with time. Therefore, my vector potential will also be changing with time. If I were to calculate the curl, my magnetic field will also change with time, but I am not interested in that, I am interested in the magnetic field averaged over a time, the mean magnetic field.

So, what should I do? I should not calculate A as it is above, but this averaged over a time, the period. On the right hand side, again I should calculate the average. And what does the first term will give, give me? I will get v I, which is equal to 0. Therefore, if you are interested in time average A, which will be a function of r, which will be independent of time, that will be simply given by mu naught over 4 pi r cubed because a first term is not going to contribute.

I still have my charged particles equal to 1 up to n q i v i r i dot r divided by r cubed, this is my expression. The velocities will be 0 on an average, the positions will be 0 on an average, but nobody said, that v i r i will be 0 on an average. We have to worry about that and that evaluation requires simple trick involving vector algebra, not even vector calculus. You have to make use of A cross B cross C and rewrite this expression in a slightly different fashion, and that will give rise to your famous magnetic moment expression. In fact, you will write A equal to mu naught over 4 pi m crossed r by r cubed, that is the form in which we want to write.

And then we want to relate, that expression for magnetic moment to what, the angular moment carried by all these charged particles. But since we are running out of time, I will stop now, and I will continue in the next lecture.