Engineering Physics - II
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Module No. # 04
Lecture No. # 03

So, in the last lecture, we discussed the motion of steady currents in conducting wires, and had a fairly long study of the dissipation phenomenon, which manifests as ohm’s law and joule heating. So, we also made some interesting observations about interfaces between two conductors, contact potentials etcetera, but the emphasis was on the relation between the electric field or the voltage difference that is the source of the current and the current itself.

And the physics underlying it was that, there are underlying frictional forces because of which, we need a constant force in order to sustain constant velocity; some kind of a terminal velocity, drift velocity. What we shall now do, is to look at the same currents again, they will be time independent and for our purposes, the charge densities will not play any role at all. We are interested only in the current, and we want to study the magnetic fields produced by these currents.

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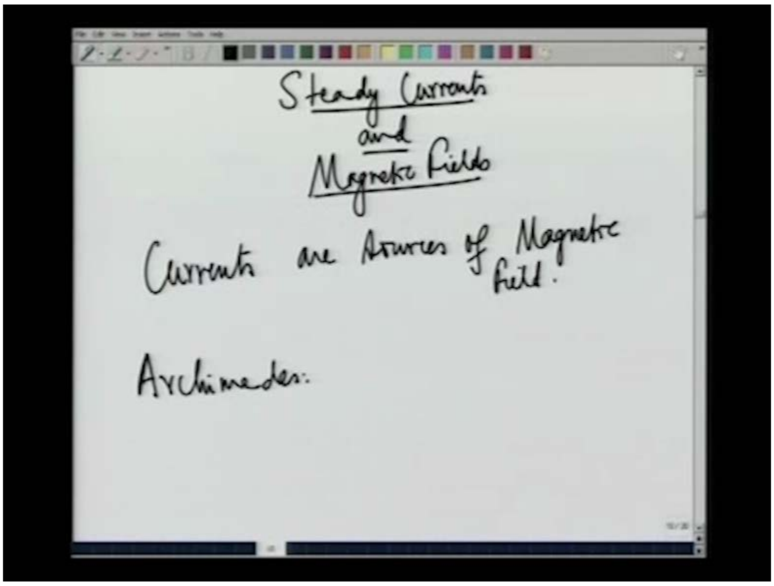

So, the important thing is that currents are sources of magnetic field and this observation goes back to the fundamental work of (( )), who actually saw the deviation of a magnetometer in the presence of a current. However, we need not know anything about the currents, in order to have a feeling for the magnet; magnets are known to us independently, magnets were known to the Greeks as lodestones.

So, they were these peculiar stones which would orient themselves or if you made fine needles out of them, they would always orient themselves with respect to north, south. Except that it is not the geographic north poles, there would be a certain angle with respect to the north pole. And the navigators have always used these lodestones for their navigation, because it always showed them the direction irrespective of the north, south direction irrespective of the direction in which their ship was navigating. So, they have had an immense importance.

And even before people had any inkling of the physics underlying magnetism; in fact, to be sure even today, with all the advances in science and technology with advance in quantum mechanics. We still do not claim, we still cannot claim that we have a complete understating of the phenomenon of magnetism that is a different subject matter all together.

Going back to ancient times 2000 years ago 302500 years ago, even then when people did not know much about the physics of magnetism, people were putting magnetic fields to good use. In fact, the legend has it that the great Archimedes; Archimedes knew a whole lot of things we all know that, he knew how to intricate the area under a parabolic curve, he is reputed to have made parabolic mirrors which focus sunlight and bend the sights of enemies ship.

He knew the laws of parabolic motion, if I throw a stone in earth's gravitational field, I know that it follows a parabolic motion, Archimedes was aware of it. And the legend is that Archimedes knew the use of magnets, so well, that he got powerful magnets excavated; obviously, people did not know how to make these magnets, like electro passing electricity electro magnets and things like that.

He could actually get very powerful magnets and the legend is that those magnets would pull out the nails of the enemy's ships or it would pull out the armor from the enemies and the enemies could all be dumbstruck. They would not know how to react and he was able to defeat the enemies.

It is immaterial to us whether there is a substance of or a grain of truth in these legends or not but the fact of the matter is that human kind, we as a humans have always been fascinated by the magnetic field. I am giving this some kind of fairly longish introduction, because the more associated with magnetic field is something which is immense, because of it is great power of attraction it is almost magical is that.

So, even now we say that the person has a magnetic personality, if a person is very influential if the person can say sway people's opinion or if the person can hold the attention of the audience a great orator let us say. We say that the person has a magnetic personality and for better or for worse, this has paid people to make speculations scientific and unscientific about the nature of magnetism.

So, as a kind of digression, let me spend a few minutes discussing the kind of lode people have had, one is that people like faraday intrinsically believe, that magnetism must be closely related to electromagnetic waves for example. Because at some stage you will study what is called as a faraday rotation, which will tell you that the polarization of the light will get changed, when it enters a magnetic medium. That was an eminently scientific highly intuitive; in fact, an excellent speculation, but on the other hand there has been speculation of the other kind it has persisted over the ages. And it is impossible to actually worth safe for the velocity of the statement.

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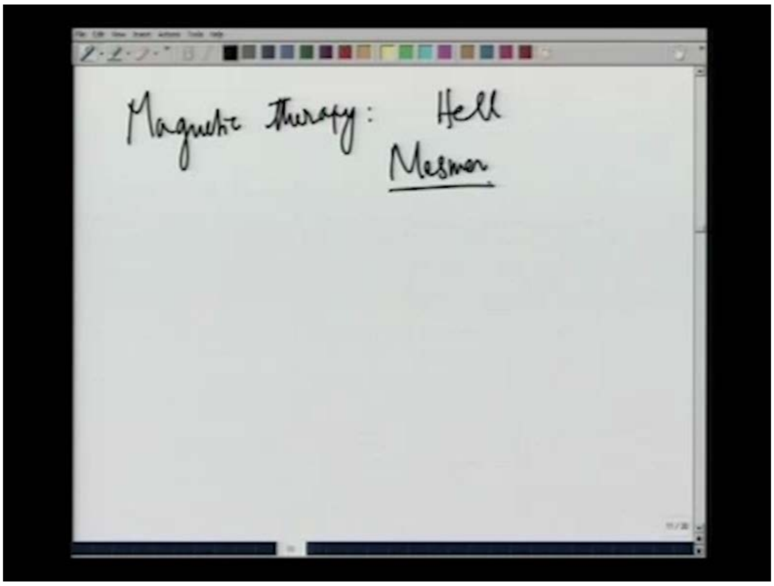

And what is that that is the famous magnetic therapy, I am sure if you walk around your town wherever you are staying, you will even today find boards people who say that they can perform magnetic therapy. And it is not a phenomenon, which was introduced today it has been there for several centuries and some of the names associated with them is a priest called hell. In fact, this is his title surname his real name was Maximilian hell or some such thing.

And his idea was taken by the great hypnotist, Mesmer and mesmerism is another word he was a psychologist Mesmer was a psychologist. Mesmerism another word which is there even today and Mesmer believed that he could actually cure people by the use of magnets. Mesmer was of course, in trying to understand the claims of this gentleman hell. So, Mesmer believed that there is a certain magnetic fluid or a magnetic field inside the human body and as long as it was in balance in harmony, then we are healthy but if the harmony gets subset then we lose our health.

And what these magnets external magnets do is to restore that kind of a harmony that was Mesmer's theory. But Mesmer was a bright person he found that whatever effect he could have on his patients, through magnets, he could have them even without the magnets. So, he was able to probably soon discard the theory, but then the lode itself has persistent.

In fact, if you look up the net it says that there is something 150 million dollar market for these magnetic equipment, magnetic medical equipment, magnetic belts, magnetic bracelets magnetic probably insoles which you put inside the shoes and all that. And many people actually believe that it can ward off evil effects, perhaps both physically or even in the sense of beyond physics in a supernatural sense.

So, probably it is all the more a reason why we should try to have as complete an understanding of magnetic field as possible as complete an understanding of the magnetic phenomena as possible. Purely with a scientific spirit is that and as we all know as always our starting point will always be Maxwell's equations. And remember I told you at the beginning of this course that Maxwell's equation, that is the electro dynamic phenomena as described by Maxwell's equations have been verified to enormous accuracy. In fact, there is no known scientific theory, which has been verified to that such a great accuracy.

In fact, electromagnetic theory is a prototype it is a model, it is a motivation for trying to build all other theories like for strong interaction, nuclear forces for gravitational forces so on and so, forth. So, perhaps we should really spend some amount of time trying to understand this almost mysterious phenomena called the magnetism.

But before we proceed it is always a good idea to ask ourselves, what are the typical strength of the magnetic fields that we encounter, what is the range of the magnetic field. We have raised and answered such a question for example, when we looked at electro electric phenomena, I gave you estimate of how much coulomb, we pick up when we walk on a for instance; when we are discussing electrostatics. And what will be the typical charge sitting on a charged conductor etcetera, etcetera.

In a similar manner, we should ask ourselves what is the range of the magnetic field that we observe in this universe. So, what I will do is to start with the smallest and go on to the largest magnetic field. The smallest magnetic field is of course, found in the cosmos the largest magnetic field is also found in the cosmos but when we come to the terrestrial domain at the level of the human beings again we find a reasonably good spectrum. So, let me start illustrating various strengths of the magnetic field.

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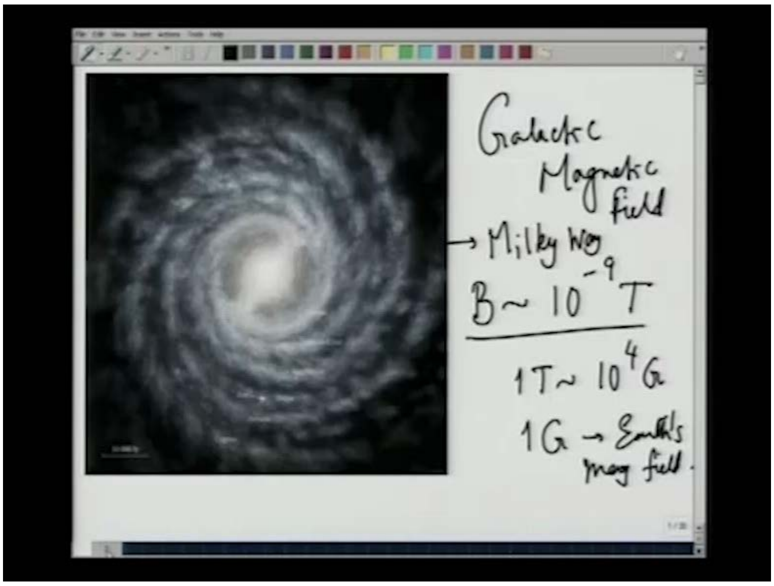

Now, the first example of the magnetic field is actually the magnetic field along the galactic surface. So, this is the picture of our milky way, you can see these spiral arms going round and round and round. So, what we do is to move along the, so called surface of the milky way, you did not deep into the milky way like we ask, what is the magnetic field on the surface of the earth is that. So, you go to the outer rim of the milky way and if you move, astronomers have been able to make careful measurements and they say that the magnetic field is of the order of 10 to the power of minus 9 Tesla.

So, in order to give you a sense of units, we should remember that 1 Tesla is about 10 to the power of 4 Gauss. So, Gauss is another unit of the magnetic field and it is named after the famous Gauss and why it is called Gauss and why is gauss a natural unit for us, because the earth's magnetic field is of the order of a Gauss. Earth's magnetic field I am going to come to that in a minute is that right.

Therefore 1 Gauss is 10 to the power of minus 5 Tesla. So, here is 10 to the power of minus 5 factors, which means the magnetic field in a galaxy is typically 100000 times smaller than the magnetic field on the surface of the earth. So, it is quite remarkable that actually people have been able to measure this; now comes magnetic field which is of slightly higher order by a factor of ten, five to ten or so.

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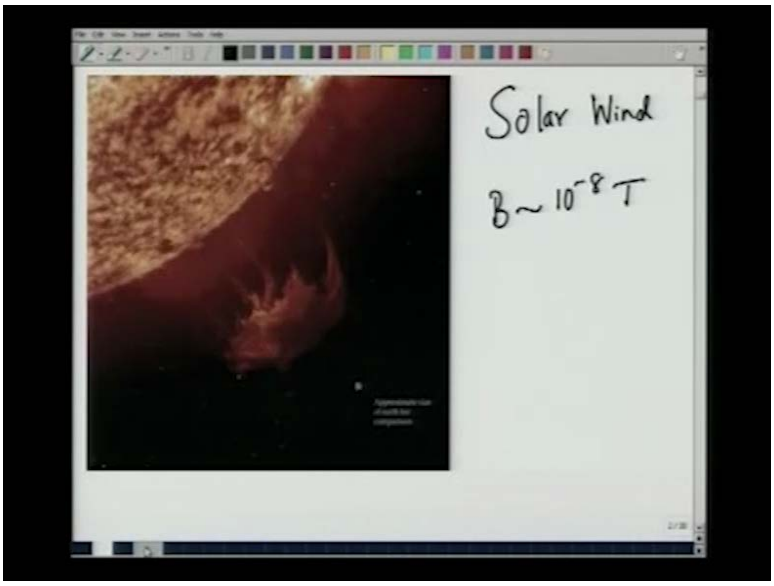

So, and these are the magnet1ic fields carried by the solar wind and I also told you the importance of solar wind in three lectures earlier. When we were discussing the currents produced by a convective flows and things like that; anyway let us not spend too much time on that. This is a kind of artist's impression of the solar wind, how the flares keep coming up. Now, as I move a little bit I had and if I ask are there magnetic fields are which are slightly larger than that and that is the interstellar molecular cloud.

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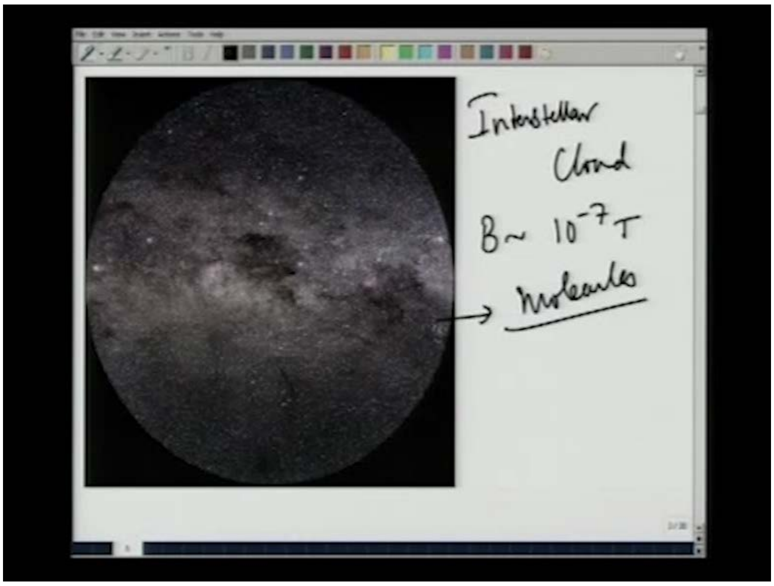

So, you see all these dust particles which are seen, shown as dots in this particular figure or molecules. And there is a large number of magnetic molecule oxygen etcetera, etcetera is that; and their strength is slightly higher about two orders of magnitude then on the surface of the earth. It is 10 to the power of minus 7 Tesla, thousand times smaller than the magnetic field on the surface of the earth.

And there is a whole lot of people especially chemist, who study this interstellar magnetic field by looking at the spectral lines emitted, because of the magnetic flips of the atoms is that because of this magnetic field.

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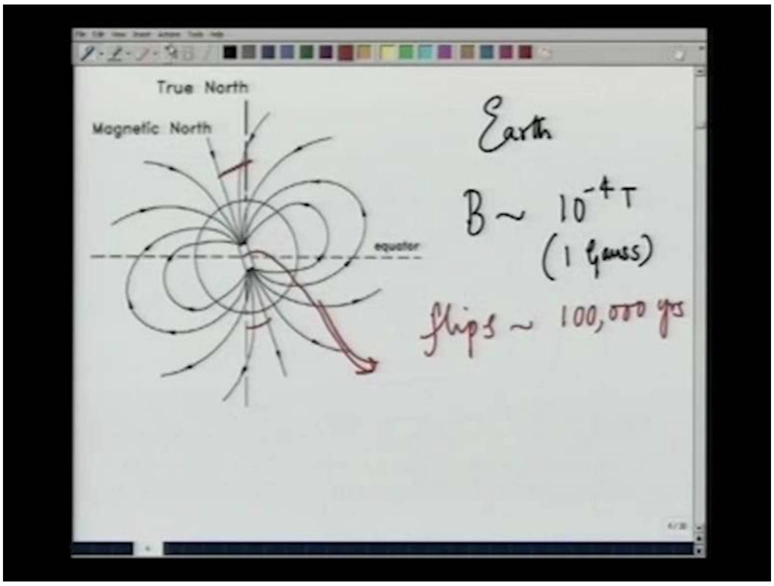

Of course, as I told you we have the earth we has a field of about 10 to the power of minus 4 Tesla. And we all know that the earth's magnetic field has a very, very important role on the overall behave of the earth on the life on the earth and also on the human beings themselves. We all know that the magnetic north is slightly inclined with respect to the geographic north, so that is showing here. And the interesting thing is that if we imagine that the earth is some kind of a bar magnet, because there is a dipole field that is shown here. How is this dipole field produced, this dipole field is produced by the currents the molten core, which is consisting of a large number of magnetized particles, magnetized metals is that it is a liquid core.

So, if you look at them they are going round and round, because of the carioles forces the rotation of the earth etcetera, etcetera it produces a field. The most interesting thing is that, this magnetic field flips, what we call as the north pole today will not be the north pole tomorrow, what we call as the south pole today, will not be the south pole tomorrow. In fact, it will flip exactly by 180 degrees the north will become the south, the south will become the north and that is suppose to cause major really drastic, in fact devastating climate changes, climatologist are always worried about it.

So, do I and you worry about that, well in order to give an idea of that the flips the magnetic flips take place. Once in about 100000 years when I say 100000 years I do not mean exactly 100000 years, it could be of the order of 100000 years, it could be 200000 years it could be slightly 100000 years. And people who study earth's magnetism over millions and millions of years, they tell us that we have reached the tip of the flip and very soon may in the next few 1000 years or so, probably there is going to be a flip in the magnetic field and we do not know what will happen at that particular time.

In fact, there are also people who believe that the extinction of the dinosaurs is associated with the flip in the magnetic field so, called magnetic reversal. So, in any case you see, the generation of the magnetic field by the earth is that the, so called dynamo whatever has a very, very important consequence not only in terms of our understanding of physics but also for people who want to know life the fate of life on the earth itself, so are also interested. Now, we move on to the surface of other objects.

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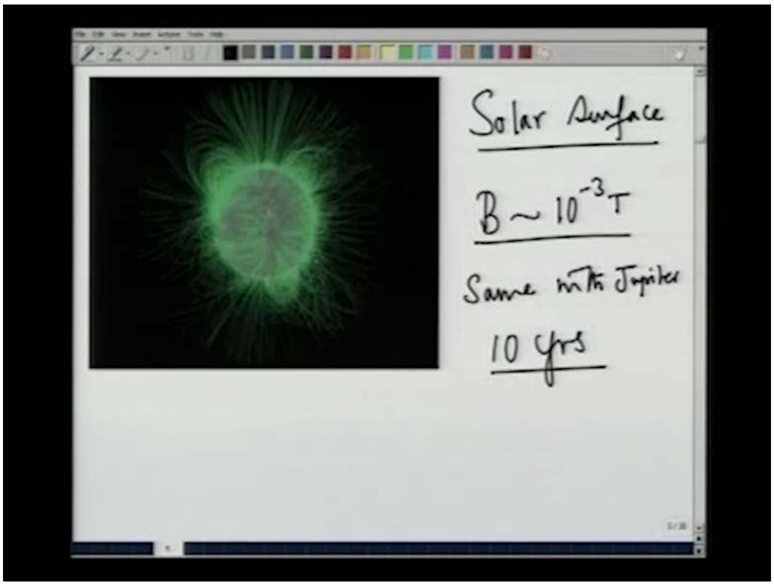

So, we had the earth incidentally Jupiter and sun have the same magnetic field which is about 10 to the power of minus 3 Tesla. So, it is about 10 times the magnetic field produced by the earth is that and what about the magnetic field of the sun does it also flip. Interestingly yes but the flipping is over much shorter duration and that is about 10 years. There is a famous liverier cycle you know for the flip of the magnetic field and that is known through the motion of the, so called sun spot appearance of the sun spots. And this is again a very, very active area of research for people who study solar dynamics people, who study the sun.

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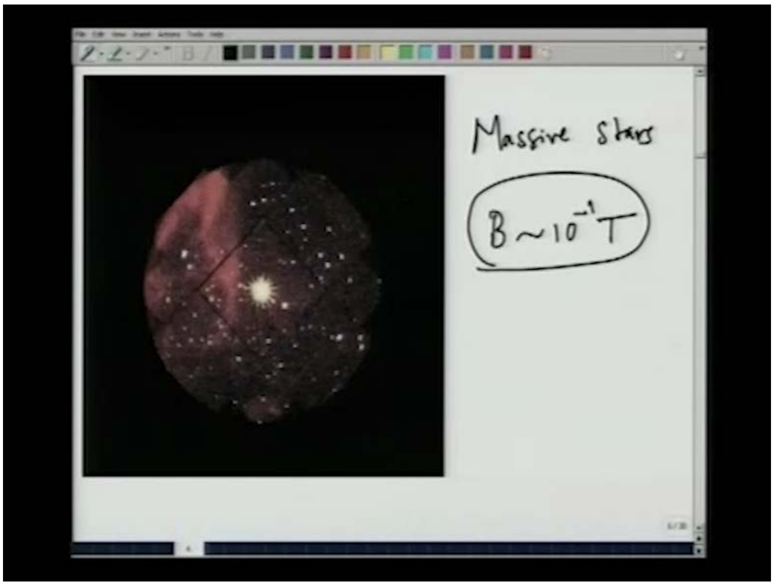

So, we have come a long way over 6 decades, six orders of magnitude from 10 to the power of minus 9 to 10 to the power of minus 3, is that. We have reached a magnitude which is one order more than that on the surface of the earth; now I will start making a rapid transition to the other fields.

Sun is a moderately size star it is not too heavy a star but if you look at reasonably massive stars four to five times or whatever pre-supernova stars. So, called pre-supernova stars, their magnetic field on their surface is much larger it is about 10 to the power of minus 10 Tesla. Please remember 1 Tesla is a fairly strong magnetic field if you have a 1 Tesla magnetic field in your laboratory you can do very, very good experiments and about 10 to the power of minus 3 Tesla about 100 Gauss. 10 to the power of minus 2 Tesla is what you find in your toy magnets, which you put on the refrigerator for pinning your memo and all that all your notes whatever you should not forget etcetera.

So, the massive stars produce a magnetic field of about 10 to the power of minus 1 Tesla and as I told you these magnetic fields are detected by looking at the atomic transitions from one magnetic level to the another magnetic level.

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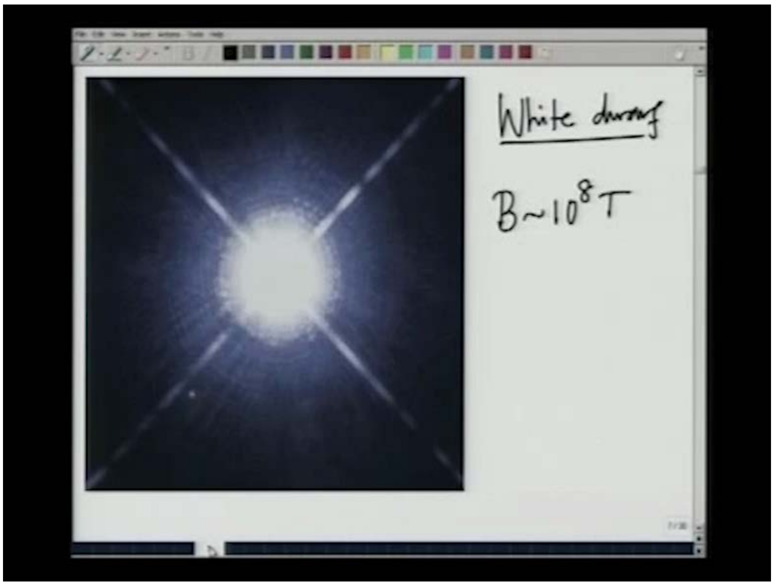

Now comes white dwarf now you see I am really making a big jump and they produce really huge magnetic fields power of of the order of 10 to the power of 8 Tesla. They are really very very large we do not know how to produce such a strong magnetic filed in the laboratory.

The strongest magnetic field that you produce in the laboratory today, time dependent or stable magnetic field is of about the order of 30 Tesla. If you want a transient magnetic field magnetic field which will last for a short duration you can produce about 100 Tesla they are called pulsed magnetic fields but here you have a white dwarf which is about 10 to the power of 8 Tesla.

There are also the famous (()) and neutron stars which will produce magnetic fields of very high order. I will leave that as an assignment for you.

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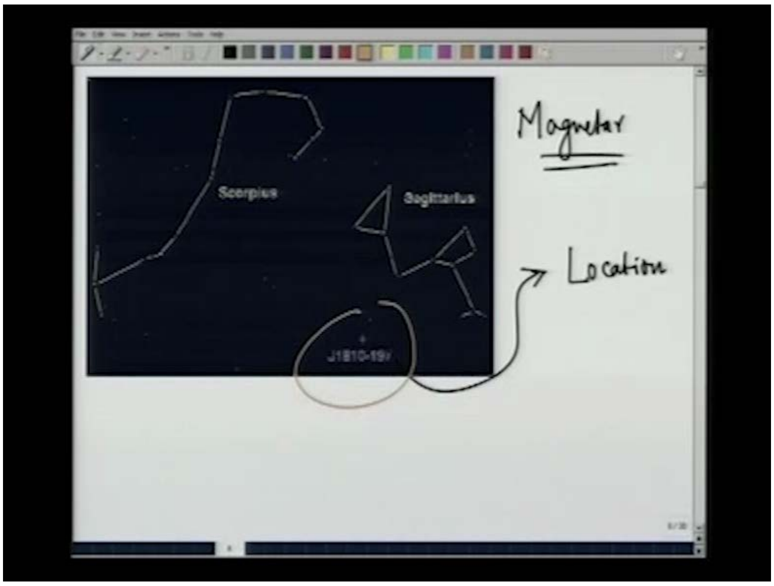

And after that comes a very, very special kind of star celled the magenta, which were discovered only recently about 15, 20 years ago and here is the picture which shows you the location of the magenta.

So, you have the scorpion here you have the Sagittarius here, in the sky and it is not clear actually this can be seen through a optical telescope, if you had a clears kind of powerful telescope is that. So, this is the location and what is the magnetic field produced by the magenta; magentas are extraordinarily strong magnetic field producing stars. We still do not understand their physics entire level, how they are able to produce them.

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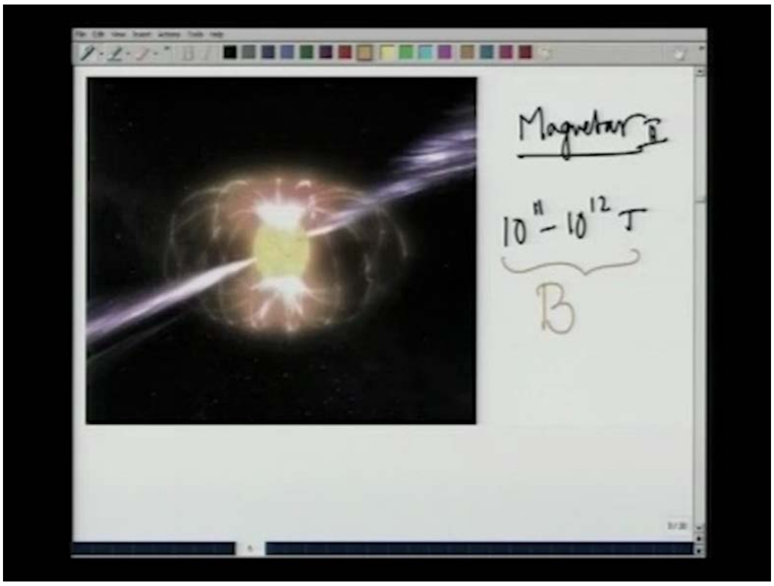

And this is the artist's impression of the magnetic lines of force, which are produced it extends over large distances is that. And you find that the magnetic field is of the order of 10 to the power of 11 to 10 to the power of 12 Tesla. So, somewhere between 10 to the power of minus 9 Tesla to 10 to the power of 12 Tesla; so we have jumped over the order of magnitudes twenty orders of magnitude, which is enormous by any standard is that that means, we know for sure that magnetic field plays a crucial role in the dynamics of the universe from the very, very small to the very, very large; I do not have pictures but I can give you some detail. So, let me start begin with that.

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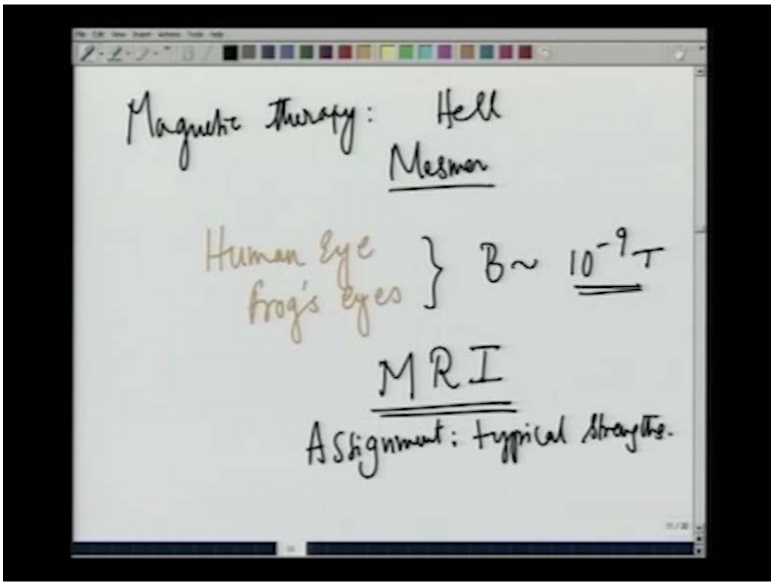

Suppose I start looking at the magnetic field produced in the human body, you know people are able to measure the magnetic fields produced by very, very small currents in the human body. So, one example is the human eye or you could also look at a frog's eye it is easier to do experiments with frog's eyes, they are more easily available. In fact, these also produce magnetic field of the order of 10 to the power of minus 9 Tesla nano Tesla.

So, whatever is the field somewhere in the galactic range is also the field produced by the motions of the ions in the eyes, when light falls you know the ions move they have to carry the signal to the optic nerve etcetera, etcetera; it is of the order of 10 to the power of minus 9 Tesla.

And forget about the hype about Mesmer and hell and magnetic therapy so, on and so, forth. We do know that magnetic fields do play a very, very important role today, a technological role. A (()) tool for us in actually finding out if there is any problem with us and that is the famous magnetic resonance image.

So, we have come the full circle from myth to reality, from wild speculation to real application, I would not like to spend any time on magnetic result and imaging I will leave it as an assignment for you people, to find out the typical strengths. This is some kind of a very brief panoramic introduction to the kinds of magnetic fields, that we encounter within the human body on the surface of the earth, on the surface of the planet, on this star like sun, slightly heavier stars, (()) neutron stars etcetera, etcetera.

The point is that almost all of it can be understood in terms of Maxwell's equations, so let us return to the Maxwell's equations. As I told you we are interested only in steady magnetic fields not time dependent magnetic fields, if there is a time dependent magnetic field there is an equally interesting phenomenon, namely induction which faraday actually discovered in his famous experiments.

And of course, if you have both time dependent electric field and time dependent magnetic field, that will give rise to the, so called electromagnetic radiation, which was speculated, which was hypothesis by Maxwell, when he wrote down the Maxwell's equations. That is how far we will go in this particular course, what we will do is to return and start with the simplest of the cases, namely time independent magnetic fields.

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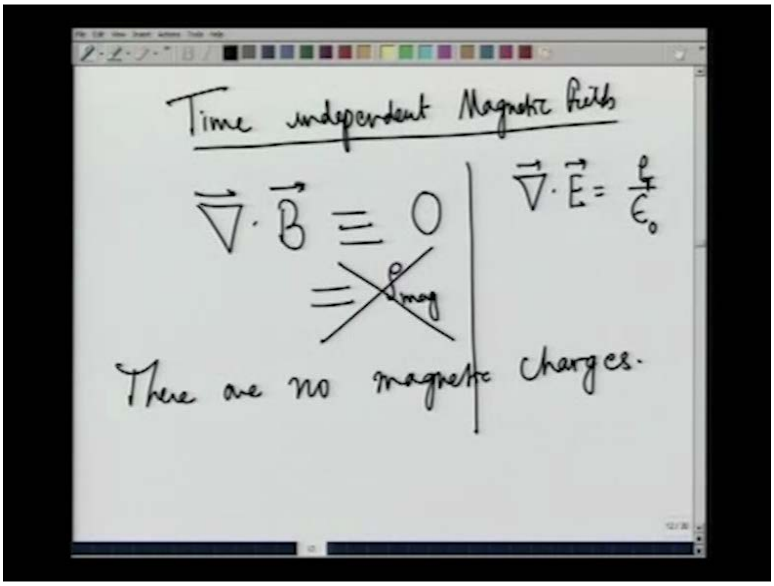

All of you are experts in solving problems involving time independent magnetic field such as magnetic field inside a long solenoid solenoid you will immediately give me the answer. Or magnetic field in a finite length solenoid but along the Z axis you will give me the answer or I have a current carrying loop of a certain area, you will immediately compute the magnetic moment and you will give the magnetic field in the dipole approximation.

So, perhaps it is not very useful for me to spend time on those examples but rather I would actually look at the physics the beauty and the symmetry of what you have the underlying interesting physics, whatever is there is expressed by Maxwell's equation. So, in order to do that, let us write down the two basic laws and start exploring it is its outcome manifestations.

So, the first of the equation is the celebrated equation divergence B equal to 0, probably to contrast this equation with the corresponding electrostatic phenomenon. Let us remember that the electrostatic equation is given by rho by epsilon naught, where rho is the charge density. So, if you want I will put a Q here to remind a charge density.

Now, for all practical purposes, if we did not know anything about the current is producing magnetic field, so on and so, forth. And if I showed you a magnet a lodestone you would have imagined that each lodestone is made of a tiny magnet and that tiny magnet should be the analog of the electric charge. Therefore, there should be a magnetic charge.

Therefore, we would have like to write this is equal to rho into magnetic, probably there will be some mu naught let us not worry about that we would have expected this but such a thing does not take place divergence B is identically equal to 0; whatever the source of the magnetic field may be, whatever the nature of the magnetic field may be, time independent time dependent. Dependent on space, dependent on time falls off rapidly, falls off slowly never mind whatever it may be divergence B is identically equal to 0 and what is it tells you, it tells you that there are no magnetic charges.

So, just now I have illustrated to you what it means, there are no magnetic charges. In fact, it is possible to make it a little bit more plausible, easier, quantitative by actually rewriting this equation as in integral equation let me do that.

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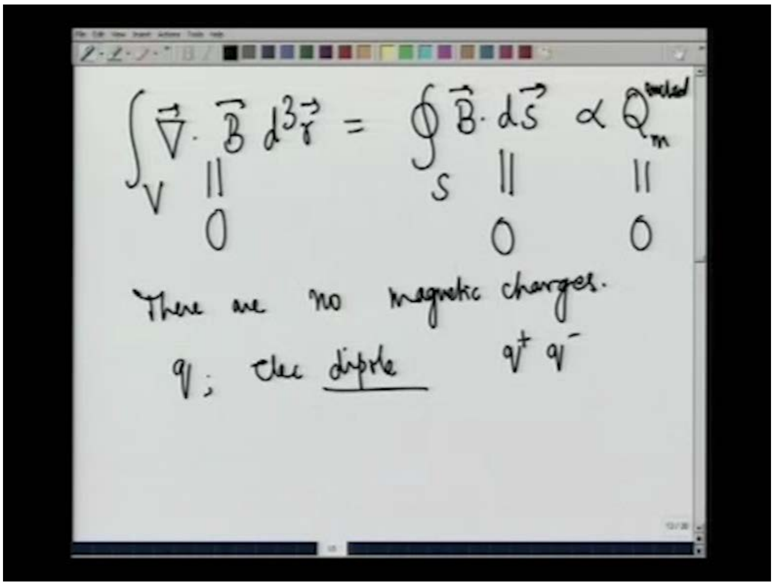

So, integral divergence B d cubed r I am going to integrate over a certain volume this we know can be written as a surface integral. It is a surface that encloses the volume integral B dot d s right and if there were a magnetic charge this would have proportional been proportional to Q magnetic enclosed is it right Q magnetic enclosed.

But then Mister. Maxwell tells us that divergence B is identically equal to 0, 0 integrated over a volume gives you 0. Therefore, this is equal to 0 independent of any surface, therefore this is 0 independent of how big how how large the volume enclose this. So, this is another statement of the fact that there are no magnetic charges very well.

If there are no magnetic charges what indeed are the sources of the magnetic field, we have to ask ourselves that question. Let me repeat in the case of electrostatics you had the electric charge q, given the electric charge q you could develop you could produce very complicated field configurations for example, you could produce a dipole. Now dipole is not an independent entity, the electric dipole is not an independent entity, because it is nothing but an assembly of q plus q minus both of them having the same magnitude a positive charge and the negative charge but the same magnitude.

And the dipole moment is simply given by e into d the charge into d though d is the distance between the two charges. So, in other words all electric fields are produced from a primary source. Namely the static electric fields are produced from a primary source namely the electric charge but when it comes to the magnetic field we do not possess, we are not given such a magnetic charge. Therefore, we cannot build a magnetic dipole out of a magnetic charge, because it is not there. But on the other hand, the field produced by a bar magnet, the field produced by the earth. In fact, it is predominantly a dipole field, therefore we have to ask ourselves what is the primary source of the magnetic field.

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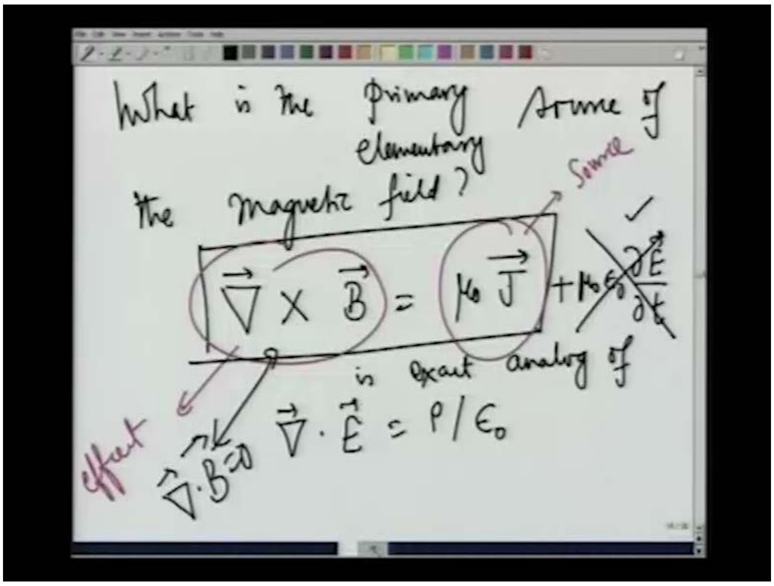

What is the primary source elementary source of the magnetic field; this is where the famous experiment of a large number of people comes right. (()) etcetera, etcetera you will find that in books on history of electricity and magnetism. All that is succinctly summarized by a single equation, qualified by a single equation due to Maxwell and that says curl of B is equal to mu naught J, where J is the current density. Please remember this is not the complete Maxwell's equation for the production of a magnetic field. In fact, there is another term, which is mu naught epsilon naught delta E by delta t.

A changing electric field can also produce a magnetic field, this term was actually postulated and experimentally verified by Maxwell but then this is a time dependent electric field as I told you we do not admit any time dependent source, admit any time dependent field. Therefore, you are going to get rid of that J is completely independent of time, it can utmost depend on space, then the governing equation is given by curl of B equal to mu naught J.

By curl of B equal to mu naught J, is the exact analog is the exact analog of divergence E equal to rho by epsilon naught. Suppose there were no current but you had a time dependent electric field then curl of B is equal to mu naught epsilon naught delta E by delta t is the exact analog of the induction law.

So, there is a peculiar symmetry between the electric and the magnetic field and that has given rise to a large number of speculations etcetera, etcetera, which physicist even today worry about that is the famous electric and magnetic duality. Those of you who are interested in physics will certainly encounter that let me not get into that.

At this point the statement that I would like to make is that curl B equal to mu naught J is the equation that governs the production of the magnetic field. Remember the right hand side this is the source and the left hand side this is the effect; this is the cause, this is the effect. When I solve the equation curl B equal to mu naught J this is a differential equation first order differential equation. When I solve that there is still some amount of freedom left for me in the solution for B and that freedom unphysical freedom is removed by what imposing the condition divergence B equal to 0.

So, this is to be conjoint put together with curl B equal to mu naught J, put the right boundary condition that all fields should fall off to 0 as R goes to infinity, then your problem as a physicist or an engineer is solved it is simply a problem of carefully solving this differential equation for various configurations. Now that we have said curl B equal to mu naught J

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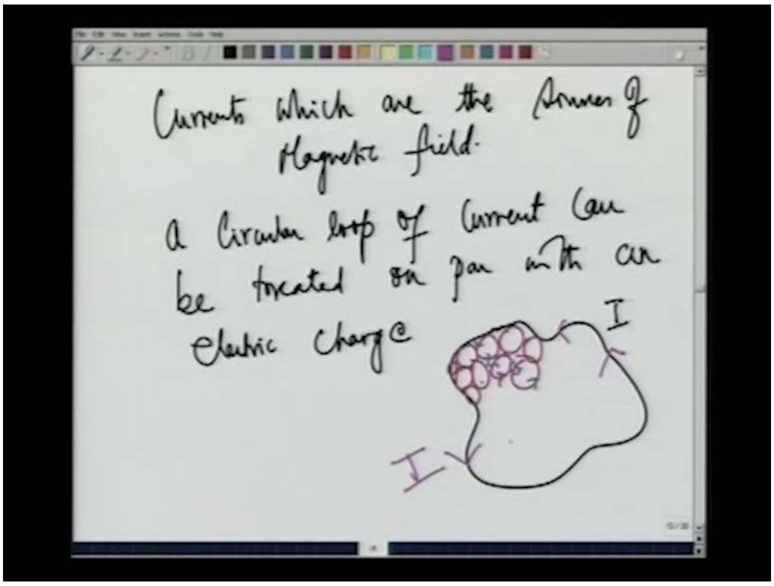

We have identified and we have answered the question and we have identified the source, what is that it is the currents, which are the sources of the magnetic field. This took 1000 of years for people to realize magnets were known 1000 of years ago but this realization that actually magnetic fields are produced by currents there are no magnetic charges was only sometime in the 19th century.

So, if you ask me what is it that a ferromagnetic right, ordinary magnet is made of I will tell you that there are tiny, tiny currents except that you should not imagine; those tiny currents to be the currents that flow in the wire they are much more complicated. They are intrinsic magnetic moments which we will come to at a later stage or perhaps you will study that in your second part of this course, when you do the atomic physics.

But the important thing is that currents are the objects which are the sources of magnetic field. And if you gave me the piece of information, I would say that a circular loop of current can be treated on par treated on par with a with an electric charge. For example, if you gave me a loop like this carrying a current I, what will I do, I will actually split it into large number of loops. This is some kind of a limiting procedure correct and each of them can be taken to be carrying a current. For example, this is a carrying a current this is carrying a current, this is carrying a current, this is carrying a current (Refer Slide Time: 33:35) etcetera, etcetera.

So, this will be carrying a current and all the currents are anticlockwise and what is the statement that I am making, the statement that I am making is there is a loop which is carrying a current I. This loop can be built from a large number of infinitesimally small very, very small loops of currents is that. Do not think that there is a gap there, that gap should also be filled with a loop then what happens the current flowing here and the current flowing here they cancel each other what will be left is actually the current in the outer loop is that right.

So, all finite loops can be built from infinitesimally small and infinitely large number of small current loops. Therefore, by the principal of superposition, we should be able to build any magnetic field if I know the field produced by a small circular loop of current.

And that is a problem, which you people are all familiar with therefore, what I will do is to actually work out that problem or probably I will simply write down that solution. But before I go on to do that I should actually perhaps spend a little bit more time on the Maxwell's equations themselves, rewrite into the integral form the famous Amperes law is that.

And after that show how Amperes law is analogous to Gauss's law and see how simple problems involving symmetries can be solved, we shall do that and only after that, we shall return to the problem of fields produced by this circular loops.

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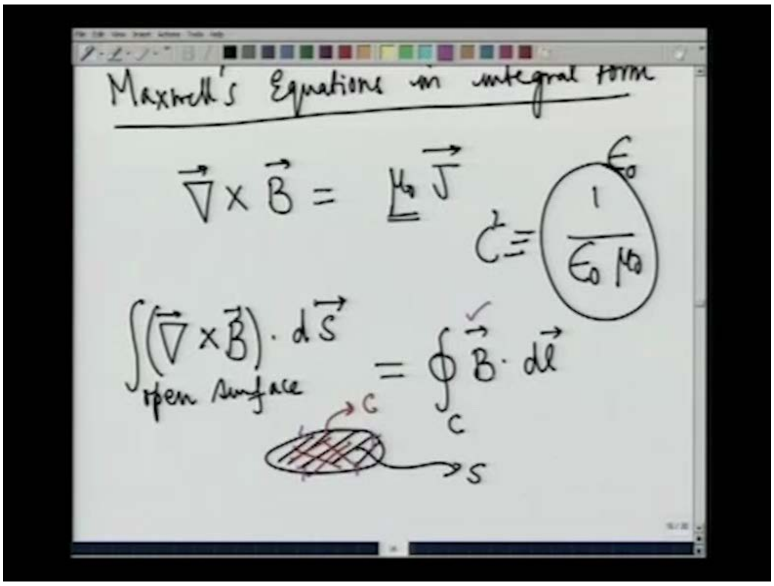


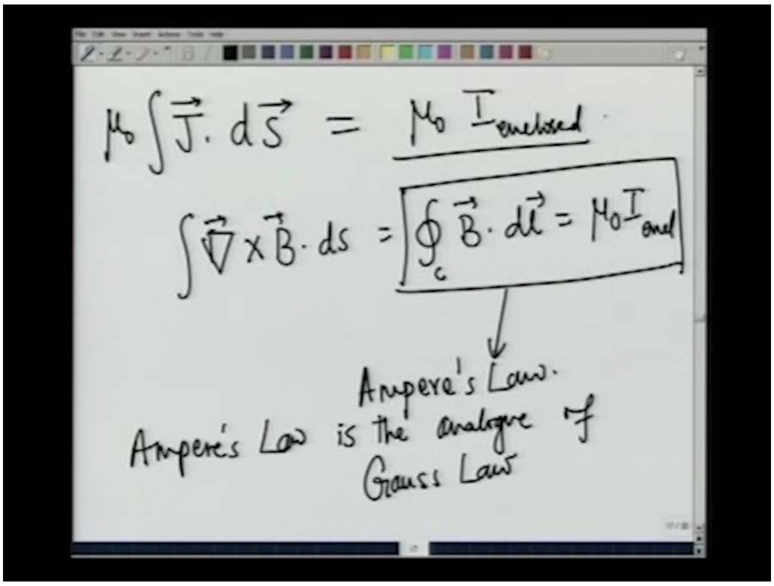
So, what we shall now do is to look at Maxwell's equation in integral form, Maxwell's equations. In fact, that is what you are more familiar with then the differential version. I have the equation curl of B equal to mu naught J, I am sure there is nobody among you who will get confused about the relation between the current density and the current. Please do not get confused, I have spent a long, long time your teachers have spent a long, long time describing for you what it means. So, let us not spend too much time; however, what I would urge you people is to remember that this constant mu naught, which is also called the magnetic permeability have some value is that.

I have already discussed that 10 to the power of minus 7, whatever Newton parameter squared etcetera, etcetera but then please remember neither mu naught nor epsilon naught are individually measurable. They are the artifacts of our choice of units what is measurable is simply the quantity 1 over epsilon naught mu naught which has the dimension of the speed of light, so as speed squared. In fact, 1 over epsilon naught mu naught is identically equal to C squared we should remember that.

It is rather unfortunate that the units that we employ in order to study electrodynamics is not completely consistent it is not minimal, we use more quantities than we can determine. It is not something wrong to do but it can be of a nuisance value which is the reason why I am reminding you again of this particular thing.

Now, if you give me that curl of B equal to mu naught J, what I will do is I will now evaluate the surface integral of this equation. So, whenever I say that there is a surface integral, I am better careful about that because, we have to ask whether it is an open surface or a close surface, answer is that this is an open surface, we are not interested in a close surface.

Now, remember your stokes theorem, which you actually proved in the third or the fourth lecture it tells you that this is nothing but the closed integral that is the reason why I am writing the circle C B dot d l. So, suppose I am integrating this over a circular surface. So, this is my surface then this boundary is my curve, so you have two options either you calculate the curl of B all over the surface in this region. And then evaluate it value and evaluate it is integral the surface integral or forget about it you simplifying the magnetic field along the rim of the loop. Is that right? You simplifying the magnetic field along the curve, that encloses the loop or integral B dot d l, the fact that B is a vector field forces the conclusion, that curl of B dot d s is integral B dot d l along the close surface.



So, we have taken care of the left hand side; now what about the right hand side my right hand side is already; obviously, given by let me use a proper color the right hand side is; obviously, given by integral J dot d s let me put a mu naught let us not forget that.

Now, this is of course, the current flowing per unit area across that surface of area A and that is nothing but the total current right total charge per unit time. Therefore, this is nothing but mu naught I enclosed very good. So, now, we have got an integral version of Maxwell's equation, which tells you that curl of B dot d s is equal to B dot d l along any closed loop equal to mu naught, I enclosed in the surface which is enclosed by that loop and this is your famous Ampere's law.

And as I told you Ampere's law is the analog of Gauss's law, which tells us that the total charge enclosed is equal to the electric flux out of that surface; whereas, are here it tells you that the total current in the enclosed surface is nothing but the line integral B dot d l along the closed curve. So, once we are given this it is very easy to solve the problems, whenever there is a good symmetry. So, let me illustrate that for you give you a few simple problems and then go on to discuss the other cases.

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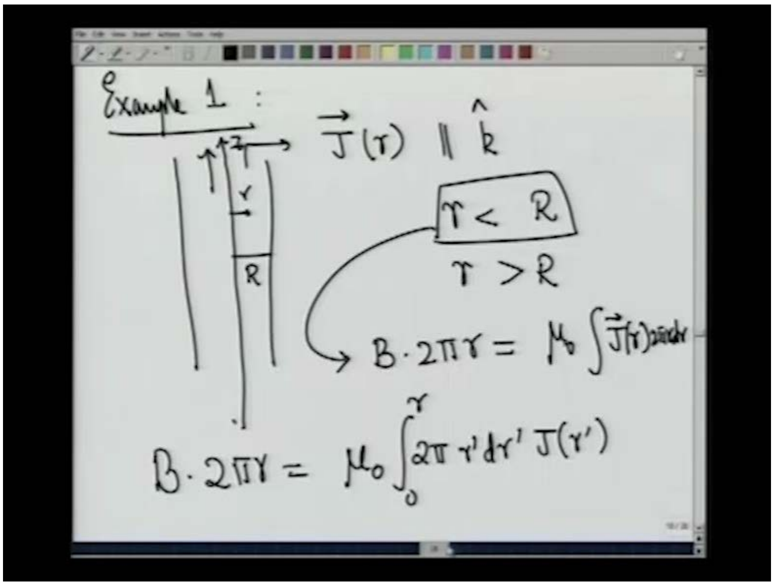

So, comes example one and that is you have a straight wire radius R it is carrying a current but the current can depend on the current density can actually depend on R therefore, I will write J is proportional to r dependent on r. So, the way I have shown the current I can say that it is parallel to the k axis let me take it to be the k axis or the z axis unit vector k always denotes the z direction. Now I am interested in finding the magnetic field all of you are familiar with the famous right hand thumb rule; namely if the current is flowing in the upward direction the magnetic field close themselves in the anticlockwise direction.

So, I can club that piece of information together with Amperes law you people should not think that I am doing something illegal sleight of hand, when I gave you this right hand thumb rule, by this time you people have had enough of experience with what the vector analysis. In fact, the right hand thumb rule follows from the original equation curl B equal to mu naught J. So, take that as an assignment, I will not even write it down but that is the situation.

So, I am interested let me choose this to be the z axis and this is a point r and I am interested in the magnetic field there are two possibilities r less than R and r greater than R. There capital R is the boundary or the surface of the conductor, what do I do make use of your right hand thumb rule, which actually follows from the curl B equal to mu naught

J. And make use of your integral formula and we will be able to write down the solution in no time let us write that down.

So, let me consider this case first r less than R what does the right hand side tell me it tells me that B into 2 pi r is integral B dot d l. The magnetic field is not going to change as I move along the circle, because all points on the circle are perfectly symmetric, there is nothing to choose one point over the other therefore, the magnitude of the magnetic field is the same everywhere.

That is what I have and Mister. Ampere tells us that you have to look at the total current which is enclosed in that. So, I will do J of r 2 pi r d r, so I should be a little bit more careful about this. So, let me rewrite this part B into 2 pi r is simply given by mu naught r 0 to r 2 pi r prime d r prime J of r prime.

You know where we get the factor 2 pi from the integral over the polar angle and the area element is simply given by 2 pi r d r. So, you do this integration this is the total current which is enclosed up to a point r bring 1 over r to the left hand side. So, what is the answer that we are going to get?

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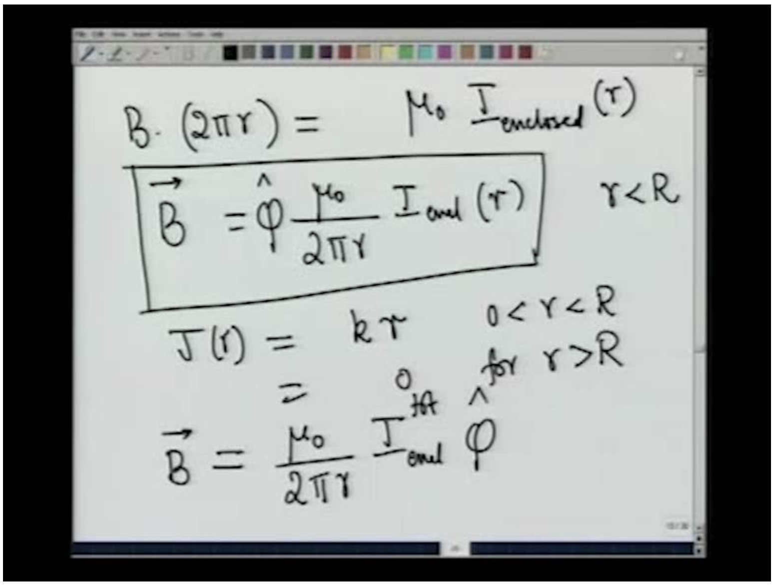

So, the answer that we are going to get, let me rewrite it again B into 2 pi r is simply given by mu naught 2 pi into I enclosed up to a point r. So, this factor of 2 pi should not be there because I wrote the total current. So, let me get rid of that, so I will put a mu naught I enclosed therefore, we conclude B is simply given by mu naught over 2 pi r I enclosed up to r, here is a simple problem for you. Suppose J of r is simply given by some constant k into r 0 less than r less than R equal to 0 for r greater than R plug this expression and write your answer in terms of the total current trivial exercise but it is worth doing that.

So, this is the answer for r less than R what happens if I look at r greater than R well the total current enclosed is independent of how far you are away from that therefore, B for r greater than the radius of the cross section of the wire is simply given by 2 pi r I enclosed total is that right. So, you see without doing any calculation variable to do that, except that I am not consistent with either of them, because I did not show you the direction I will show that here it is along the unit vector phi.

And remember the unit vector phi is orthogonal to the unit radial vector R one vector is going radically outwards, another is going round and round tangential to the circle, we did discuss this also when we studied the vector analysis. So, here we have a very single solution for a problem. If I had to try to solve the same from a differential version, then you would have had to perform an integration, which would have been much more complicated, but here we do not have to do anything because we have exploited the cemetery. So, since you people are all completely familiar with the use of cemeteries I have illustrated that here is a problem for you.

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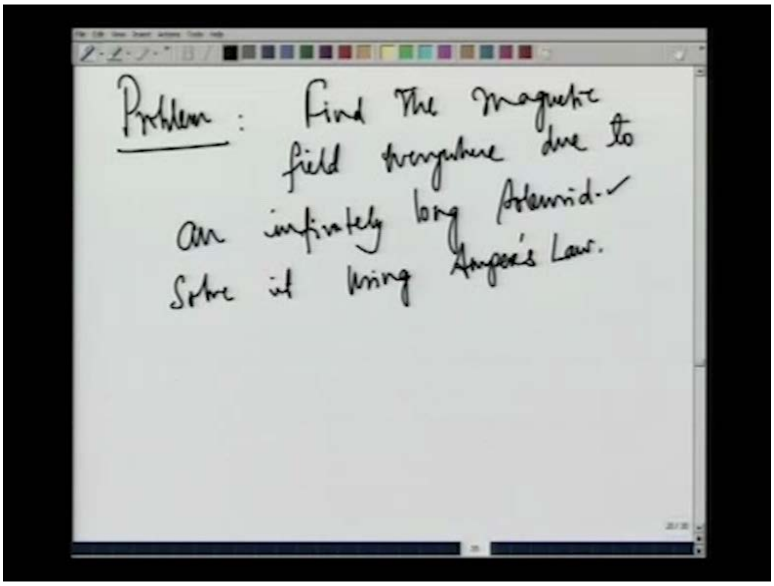

This is a worked out problem in all the books but never mind you should that find the magnetic field everywhere due to an infinitely long solenoid of course, it has n tons per unit length it carries a current I etcetera take this as a problem and you have to solve it using Ampere's law solve it using ampere's law.

You will have to construct a surface. In fact, a loop which is analogous to the pin box which you do in the case of the gauss theorem is that. And that would indeed be a fruitful exercise for you people to do, because you will appreciate the role of the cemetery in solving seemingly complicated problems. Now having done that there are yet a large number of problems, which one can actually solve but I do not want to do that, but what I wish like to do is to give you a feeling for how the relation divergence be equal to 0, can be used in a very nice manner to actually, find the magnetic field even in regions where it cannot be solved analytically, that is something that I would like to do and let me take it up slowly step by step.

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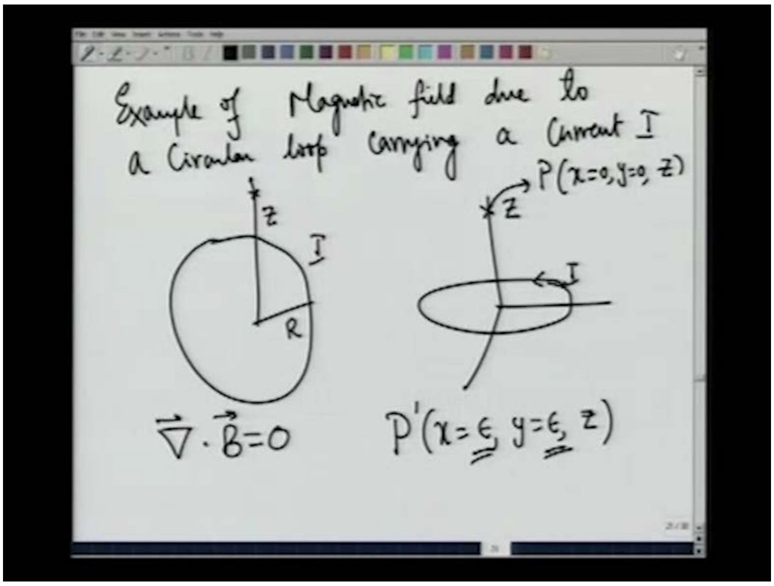

So, in order to do that, we shall consider the example of magnetic field due to a circular loop carrying a current I. So, I have circular loop radius R carrying a current I and then you are interested in the magnetic field imagine this line to be perpendicular to the plane at a distance z, I am sitting on the z axis I am interested in that. So, probably I should write it this way it will be much easier for you to picture. So, it is carrying a current I this is the origin of my coordinate system located at the central of the circle and I am interested at a point z.

Now, this is a fairly simple problem for you people to solve, we have solved it in many many manners but the question that I am going to ask you is slightly different I tell you you know how to solve this problem along the Z axis but can you solve this problem. If I choose a point slightly moved away from the z axis. So, I suppose I called it as a point P what are its coordinates x equal to 0 y equal to 0 and z.

Now, I will move to a neighboring point P prime where x is equal to epsilon y equal to epsilon and I am still in the same side I have moved slightly away from the axis and then if I ask you to solve the problem you will not be able to do the integrations. The way you are able to integrate your expression curl B equal to mu naught J is that right or Biota savant law whatever.

Then probably some of you will tell no no tell me no no I will have to write a complicated program, that will take a long time. In fact, this can be guessed that propagate this epsilon is small enough is that, the magnetic field can be found out provided we use the relation divergence B equal to 0.

So, divergence B equal to 0 has an immense physical significance, namely the absence of the magnetic monopole but for us as practitioners of electricity and magnetism even. If you are not doing physics, even if you are not doing fundamental physics divergence B equal to 0 has a very good practical application, which you would actually like to illustrate now.

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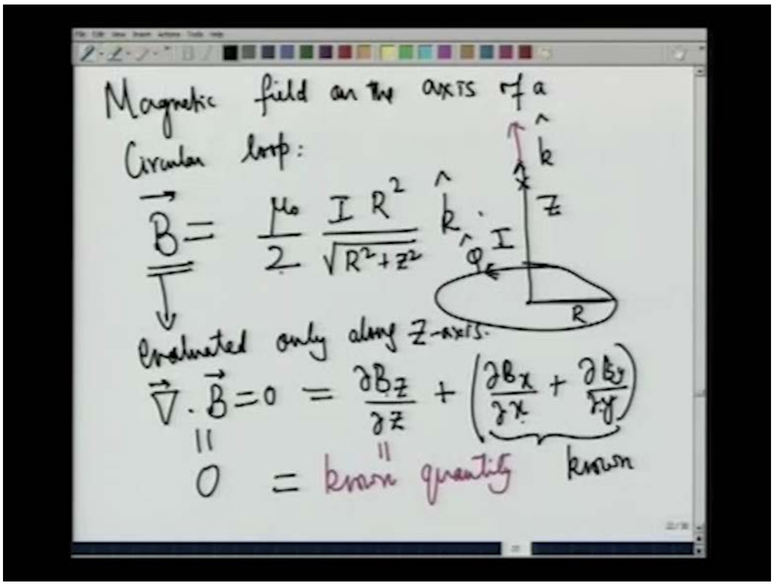

I have again illustrated the magnetic field produce by a circular loop, I would not like to spend any time in deriving this expression I know all of you can derive. So, look at the figure again here, you have a circular loop of radius R, the current is going anti clock wise manner along the phi direction. And I am interested in the field at any point on the z axis and the axis is passing through the centre of the loop. So, imagine the loop is in the X Y plane, the axis is passing through the centre piercing the centre and going up the z axis and I am interested in the field at any given point.

So, what you people will do is to make use of the Biota savant law and write the answer B is mu naught by 2 I R squared over root R squared plus z squared k. Before I proceed I should perhaps ask you people to make use of this, to find out the field produce by uncoils that will give you the field due to a solenoid. And if you make the length of the solenoid infinite of course, you are should be able to recover the other result.

So, take that as a problem but here the important point is that this magnetic field is not evaluated everywhere but it is evaluated only along the z axis. And my question is what happens if I slightly move away with respect to the z axis and there I want to make use of the fact that divergence B equal to 0.

Now, let me open this equation up what is this this is delta B z by delta z plus delta B x by delta x plus delta B y by delta y. And I am going to group these two terms together in a bracket parenthesis and I am going to write that. Now, please understand what is happen if I move along the Z the z direction; suppose I move along this particular direction, I know the magnetic field all along that direction, because B is equal to mu naught by 2 I R squared root R squared plus z squared k.

Therefore, this is a known quantity this is a known quantity the left hand side is known and what is the value of the left hand side, this is equal to 0. So, 0 is equal to a known quantity plus some quantities delta B x by delta x plus delta B y by delta y is that right therefore, this is also known, delta B x by delta x plus delta B y by delta y is nothing minus delta B z by delta z. And all of you know that the magnitude at the magnetic field decreases as you go away from the source; therefore delta B z by delta z is a negative quantity.

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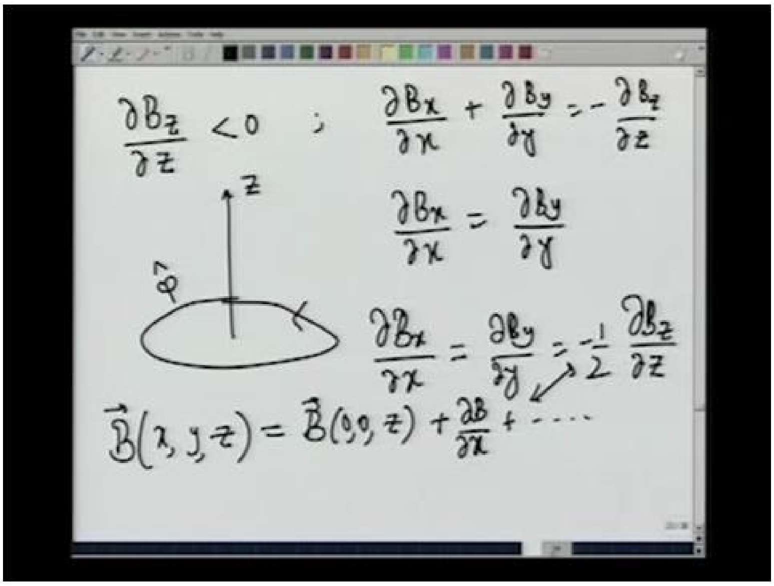

So, delta B z by delta z is less than 0 and I have delta B x by delta x plus delta B y by delta y is equal to minus delta B z by delta z. Again I will make use of my symmetry I had a circular loop here and there is a z axis the circular loop uniquely defines for me the axis perpendicular to the plane the z axis but for this circular loop, there is nothing to tell me what I should choose as the x axis or what I should choose as the y axis. Because the current is along phi and you can simply keep on going round and round without being aware of any way of choosing the x or the y axis. So, what do we conclude, we conclude that delta B x by delta x is equal to delta B y by delta y for this problem.

Therefore, I will write delta B x by delta x is equal to delta B y by delta y is equal to minus half delta B z by delta z. I think I have given you all the hints related to the solution of this particular problem, it would be very unfair for me to give the full solution I am sure all of you are raring to go and solve this problem. So, what is the next thing that we have to do, I am going to look at B x y z and I will make a Taylor expansion what is this B 0 0 z plus delta B by delta x etcetera, etcetera. I will ask you to couple with this particular equation and draw the following figure.

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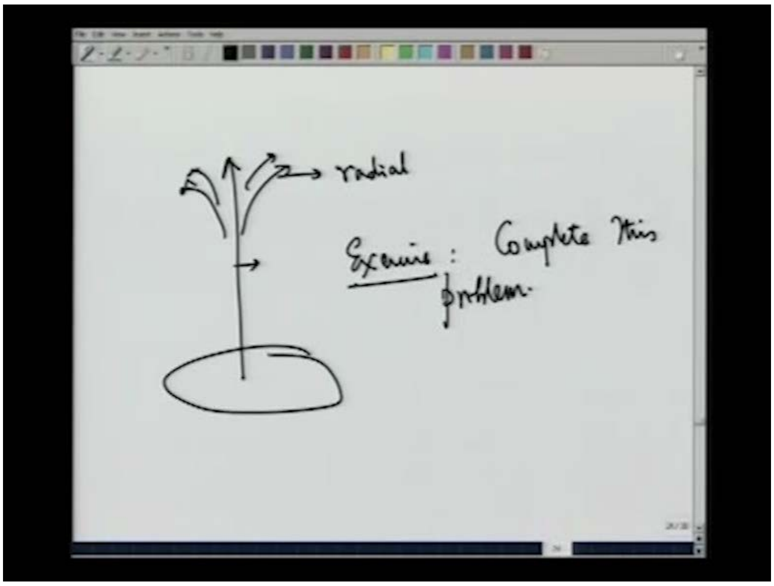

What is that if the magnetic field is around the Z direction the neighborhood points should be in this direction. Remember delta B z by delta z is negative therefore, delta B x by delta x is positive delta B y by delta y is positive. There is nothing to choose between X and Y therefore, the only way this can restore your cylindrical symmetry is that this should be radial, that is the transverse component of the magnetic field should be radial. So, the exercise for you people is complete this problem, if you did that you will understand why the field lines diverge in this particular manner as you go away from the z axis and here is your loop.

Now, I think probably I should stop at this particular point; the other important concepts for us are the magnetic moment, lamer precision these are very, very important for us. And magnetic moment allows us to actually find out the magnetic field at very large distances, even if I do not know the details structure of the current we will postpone it to the next lecture, where we will also start discussing the very interesting phenomenon of induction; so we will stop here.