

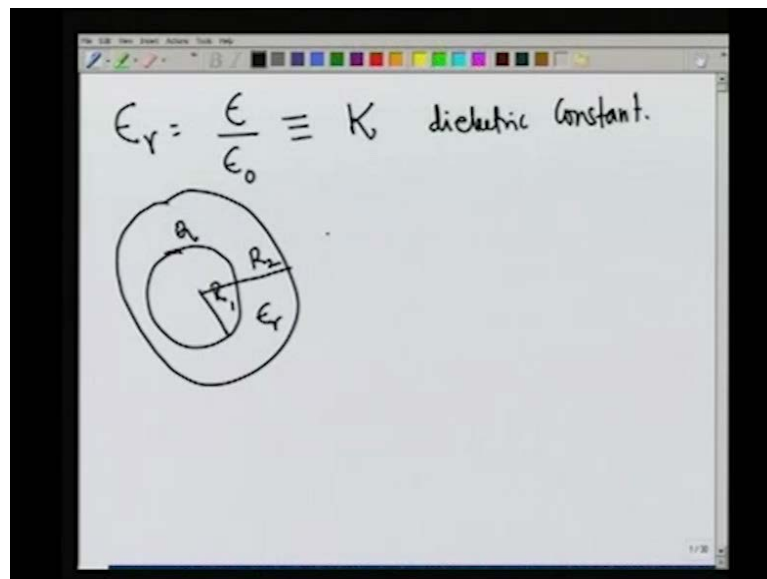
**Engineering Physics – II**  
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**Module No. # 03**

**Lecture No. # 07**

In the last lecture, we introduced the concept of the permittivity in a medium and I also introduced the related concepts of polarization and the susceptibility. So, in order to motivate ourselves, we gave a rather simplistic picture in terms of a linear restoring force and showed how the effects, because of the readjustment of the position of the charges can all be summarized in a single constant namely the permittive of the medium. Now, the ratio of the permittivity in free space and the permittivity in a medium is denoted by epsilon r r k, so let us remember that.

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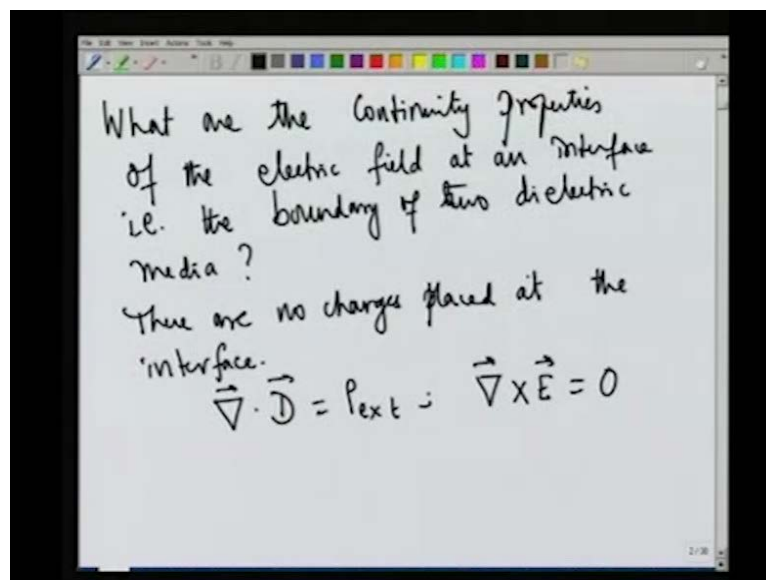
So, epsilon r is nothing but, epsilon by epsilon naught and many times in books, it is also denoted by the constant K so called dielectric constant. So, if you people remember I displayed a table, where I showed a large number of values for the value of K; for air, it is very close to 1, 1.006 are such thing, goes all the way up to something like 80 up to

water. And there are a large number of materials for example, paper has a dielectric constant around 2 so on, and so forth.

And then we also looked at a simple example, which I left as a problem for you of a dielectric spherical shell surrounding a metallic or a conducting sphere. So, what is the problem that I gave you? The problem that I gave you was of a conductor of radius  $r$ : let me call it  $R_1$  on which I placed a charge  $Q$  obviously, all the charge sits on the surface; this was enclosed by a spherical dielectric. So, this is of radius  $R_2$ , this had a value  $\epsilon_r$ ,  $\epsilon$  by  $\epsilon_0$ ; and I asked you to find out the electric field, the polarization, the susceptibility induce surface charge, all these qualities for this very, very simple example.

I hope that you have solved this problem, because what I now wish to do? Before I get into discussing the applications is actually to state, whatever you have found here as a general result and then proceed to discuss a few applications. Now, what are the general results that we want to discuss? It is a fairly simple thing to do, because we have looked at a similar situation, when we also studied the conductors.

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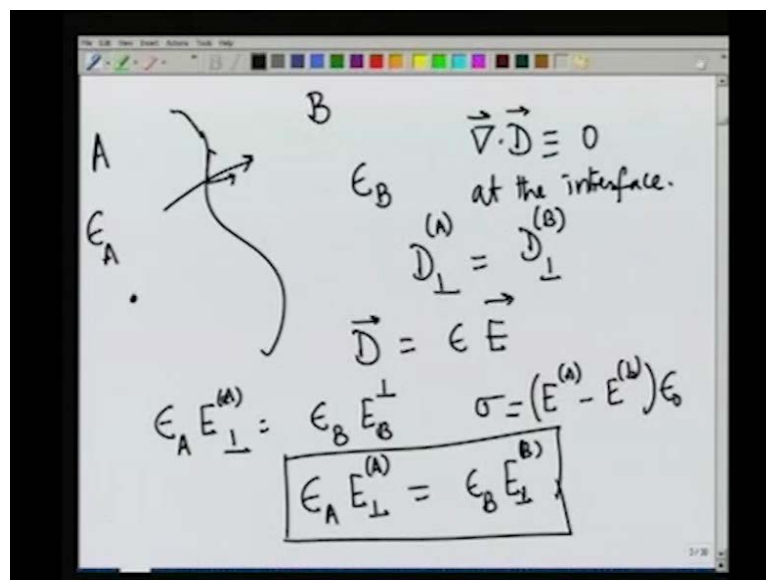


The question that we are asking is, what are the continuity properties **what are the continuity properties** of the electric field at an interface; or that is, the boundary **of a** of two dielectric media that is the question. So, we have post the question very, very precisely, all that remains is to answer this question. Except that I have to make one

more statement before I proceed. So, what we are going to say is that there are no charges placed in the media placed at the interface. In fact, the anywhere close to the interface let us say, we have not placed any external charge.

Now, the answer to that is fairly simple, if you remember that there are two governing equations. The first governing equation is that divergence D is equal to rho external; and the next governing equation is curl of E equal to 0. I am going to employ both these equations in order to settle this question. I have given you a physical input namely, that I do not place any external charge density near the boundary that is something that is given; so, let me illustrate that.

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So, imagine that this is the boundary separating two media. So, you have the medium A, you have the medium B. The medium A has a dielectric permittivity epsilon A; B has a dielectric permittivity epsilon B. And we are asking, how the components of the electric field change, when I go from medium A to medium B.

Obviously, the question that we are asking is what happens to the longitudinal component and what happens to the perpendicular component of the electric field that is the question that we are asking. The physical input that I am giving you is that, if I consider a region close to the interface, I have not placed any external charge. So, in other words, at the interface my divergence D equal to 0 at the interface.

Now, what do we know from the fact that the divergence  $D$  is vanishing at the interface? We immediately conclude that the  $D$  perpendicular on either side of this boundary, they should what? Match each other, they cannot change. So, in other words  $D$  perpendicular A is equal to  $D$  perpendicular B. This is the conclusion that follows from the condition that, divergence  $D$  is identically equal to 0 at the interface.

But on the other hand, we have the constitutive equation which relates my electric field to the  $D$  field the so called displacement field. So, let us remember what we are saying; what we are saying that,  $D$  equal to epsilon A  $E$  and either side what do I mean by that?  $D_A$  is epsilon A  $E_A$  let me write it neatly and  $D_B$  is epsilon B  $E_B$ .

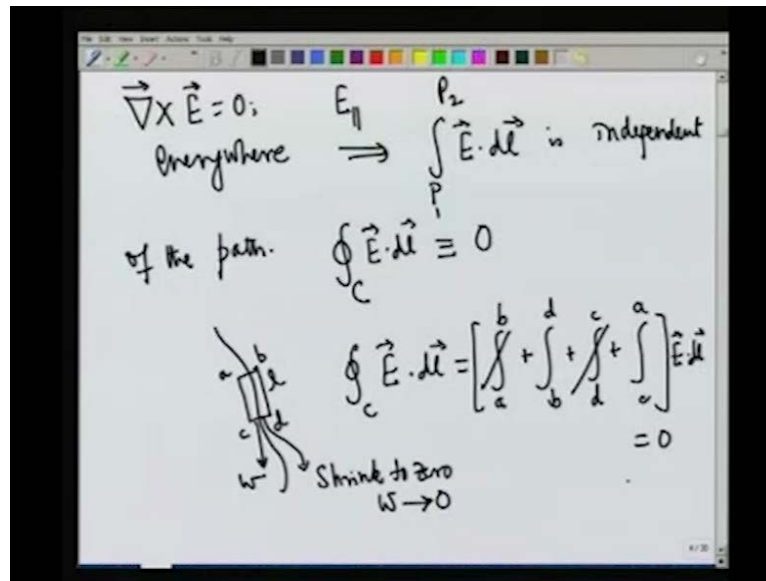
So, make use of the constitutive equation compare it to this equation; we conclude that, epsilon A  $E_A$  perpendicular is equal to epsilon B  $E_B$  perpendicular. So, although the perpendicular component of the  $D$  field is continuous across the boundary, the perpendicular components of the electric field need not be continuous across the boundary; and of course, it would be continuous if and only if, epsilon A equal to epsilon B.

No surprise at all, because if I were to place a charge here, we know that there is a certain surface charge density which is induced at the boundary; and therefore, when we crossover from this side to that side other side through the boundary, we have to sense the surface charge density; and that is what is given by this equation.

So, what I would like you people to do is to actually, look at this equation and find out, what the induced surface charge density is; and what is the result that you will use in order to obtain that? The result is fairly simple; the surface charge density at the boundary is simply given by  $E_A$  minus  $E_B$  divided by into epsilon naught, let us not forget that.

So, if you combine these two equations, you will immediately find the induced surface charge density. So, let me sort of display it by enclosing it in a box let me rewrite it. We are saying that, epsilon A  $E$  perpendicular on the **conductor's side** dielectric side A is equal to epsilon B  $E$  perpendicular B, so let me put a box. What remains now is to find out the problem conditions on the tangential components.

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In order to find out how the tangential component of the electric field behaves, we will now appeal to curl of E equal to 0. So, this gives information on the so called tangential component or the parallel component. Remember curl of E equal to 0 everywhere, this is quite independent of what? Whether I place a charge or not, it is independent of all kinds of sources and all kinds of fields, this is indeed the defining condition for the electrostatic nature.

And we have already found out that, curl of E equal to 0 everywhere implies that, integral E dot dl is independent this line integral is independent. Let us say that, I am evaluating it by from a point p1 to p2 is independent of the path. This is something that you people have studied even in your mechanics course and we used this property in order to define the potential and potential energy, do not forget that.

Now, because it is independent of the path it is also clear that, integral E dot dl along any closed curl is identically equal to 0. In fact, it is the theorem in vector calculus that, curl E equal to 0 everywhere implies, integral E dot dl for all closed curl is 0 and the converse. So, what I wish to do is, to make use of this condition and obtain a relation between the tangential components.

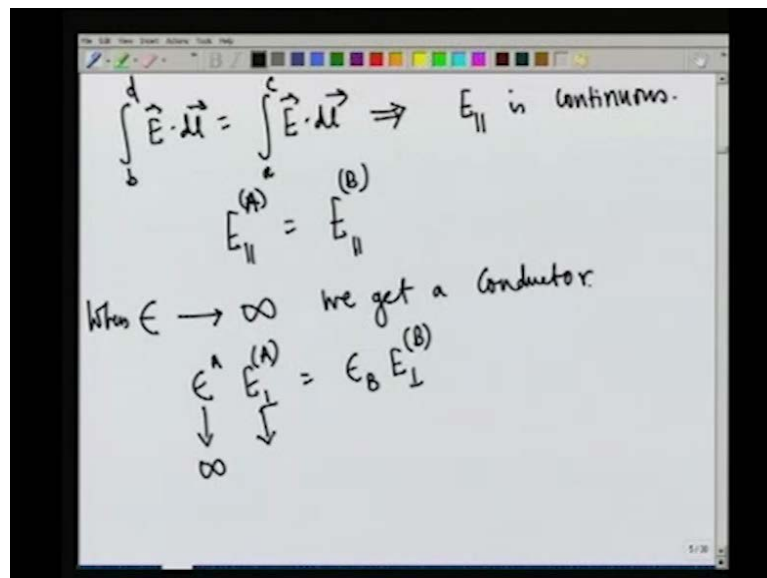
In order to do that, let me show the interface again and let me draw a rectangular loop. Now, let me label the adjust let me call it as a b c d; and since, I am interested in the

property at the surface obviously, the distance the side a b and c d are going to be very **very** small, that is what we are interested in.

So, what we have is that, the line integral along this closed loop is equal to 0. So, how do I write that? So, I write integral c E dot d l I have shown you my curl; this is nothing but, a to b plus b to d plus d to c plus c to a. So, let me write it in this particular fashion, E dot d l and this is equal to 0. What I now do is to shrink this side, I keep the length intact, but this side I will shrink to 0; that is I will make the width smaller and smaller shrink to 0.

So, if you feel like, if the side b d has a length l and if the side c d has a width w, I want to take the limit w going to 0. If I did that, you people can easily check convince yourselves that, this integral a to b and c to d are going to drop out; because the length is becoming smaller and smaller these things are going to drop out. And we conclude that, integral b to d E dot d l is the same as integral c to a E dot d l or in other words my **d ok**.

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Let me repeat that, b to d E dot d l is equal to integral let me go back (Refer Slide Time: 12:34), since I have written it from b to d this will be from a to c. So, this will be from a to c E dot d l whatever the points b d and d c may be; and therefore, this implies E parallel is continuous. Because, whereas b to d is evaluated in the medium B, a to c is evaluated in the medium A, we write E parallel A is equal to E parallel B.

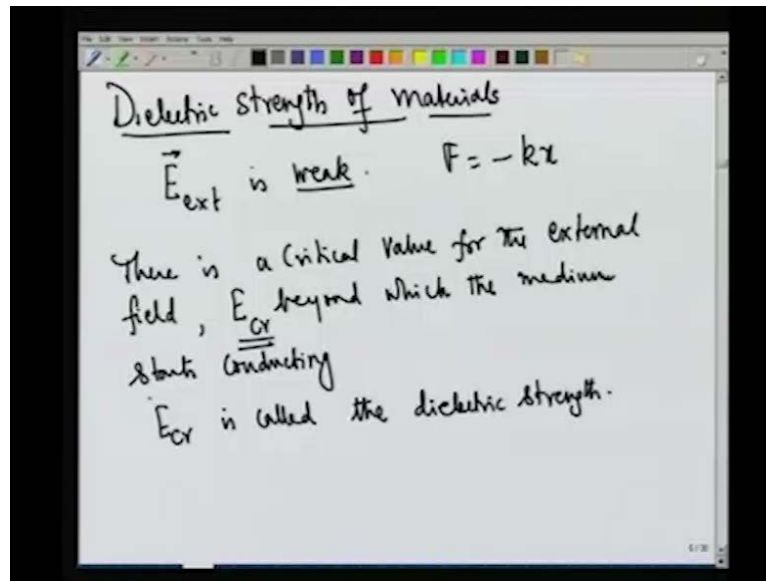
So, A and B refer to my medium A and medium B. So, this concludes in a very nice fashion, the properties of the electric field when you go from one medium to another in general; the perpendicular component of the electric field suffers a discontinuity, because of the induced surface charge; whereas, the parallel component or the tangential component remains continuous, because curl of E equal to 0 is a very, very robust condition, independent of the nature of the sources, the charges that are producing the electric field.

So, this concludes a discussion of the boundary condition. I would sort of round it off by making only one more statement namely; although, I am looking at the interface the boundary between an insulator and a conductor. We should not forget that, in the limit epsilon going to infinity, we get a conductor. So, when epsilon goes to infinity, we get a conductor; therefore, in writing these relations we can actually take one of the epsilons to go to infinity. For example; we wrote the relation  $\epsilon_A E_{\perp A} = \epsilon_B E_{\perp B}$ , I enclose that in bracket, so let me do that.

Now, let us say that, epsilon A actually refers to what, a conductor obviously, this goes to infinity. Let us say the medium B is a dielectric, the right hand side is a non vanishing finite quantity in general; whereas, the left hand side epsilon A is going to infinity. So, the only way that this equality can be maintained is that, this goes to 0 which simply tells me that, the field inside the body of a conductor inside the interior of a conductor vanishes and this is a result which we are all familiar with.

So, we have a fairly good picture of, what a dielectric is, what a conductor is, what the properties of the fields at the interface **of the** of any two dielectrics are. However, before I go on to discuss the application; I have to introduce yet another parameter. The dielectric properties of a medium are not exhausted by simply giving by dielectric constant. Please remember, that the word dielectric constant simply means relative permittivity.

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It is not exhausted by that, there is yet another parameter namely, the dielectric strength. So, unless we do not give the value of this material, then our discussion of the electrostatics and dielectrics will be incomplete; and in fact, if we did not pay attention to this particular quantity, we would be in trouble. We would make a draw erroneous conclusion and physical situations or we might build an apparatus, which will never work which will break down very easily.

So, what is this dielectric strength of materials? The concept is fairly simple; if you go back and analyze the way I introduce my dielectric constant, the idea was that the applied field is weak. So, what we are saying is that, all our previous discussion was based on the fact that,  $E_{\text{external}}$  is weak. How weak is it? It should be sufficiently weak such that I can always assume that, there is a linear restoring force.

So, I want to say that,  $F$  is equal to minus  $kx$ ; and we know Hooke's law, the way we have written right now is valid only for sufficiently small displacements. If the displacements become very, very large, then there are nonlinearities. And if the electric field is even stronger what happens? Then actually because of the very strong external field, the electron the charge particle will be able to overcome the electrostatic attraction coming from the positive charges in the material, they may actually start move that is the material can become a conductor.



So, what we are saying is that, there is a critical field **there is a critical field** value for the field, what do I mean by that? External field, I have to be careful. So, let me call it as  $E_{critical}$ . So, long as  $E$  is less than  $E_{critical}$  then everything is fine, the material continuously remain in an insulator. But, the minute  $E$  exceeds this  $E_{critical}$ , the material shall change its property; it will not continue to remain an insulator, it will not be a dielectric, it will become a conductor we know that.

For example, a person walking on a carpet we will pick up a lot of charge, so that when you go and try to touch the knob, there is a spark every time there is an electrostatic discharge. And remember, we had a fairly long discussion on the hazards and applications of electrostatic discharge actually there is a dielectric break down.

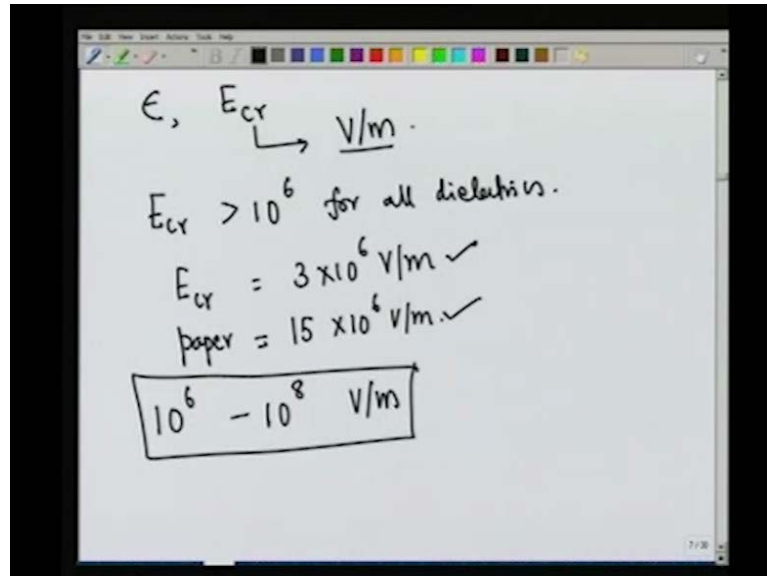
The medium instead of insulating starts conducting. So, if  $E$  exceeds  $E_{critical}$ , we will alter the properties of the medium. So, we can no more characterize the response of the medium simply in terms of the so called dielectric response. Therefore, this  $E_{critical}$  is very, very crucial.

So,  $E_{critical}$  if you feel like stands for both critical and crucial. In fact, the same thing happens even when we look at lightening; we know that, air is a very, very poor conductor of electricity. In fact, it is a dielectric it is almost like free space. Yet when a lightning strikes we know that, very, very heavy currents of the order of 20,000 amps kilo amps flow through very narrow channels in air, because of enormous voltages that are involved. What is the voltage that is involved, when a lightning strikes? It is of the order of a million volt. So, when there is something like a million volt, it can break the resistance of the air it can condemn convert it into a dielectric medium and very powerful currents can be set in.

So, when we want to make use of dielectric materials to build devices, switches capacitors, etcetera **etcetera**; we should always take great care to see that, whatever field is contained, whatever field is produced, or whatever the field may be to which the material is exposed that should not exceed this  $E_{critical}$ . Therefore, what are we saying, there is a critical value for the electric field  $E_{critical}$ , beyond which the medium starts conducting; and this  $E_{critical}$  is called the dielectric strength. So,  $E_{critical}$  is called the dielectric strength.

I am not very clear as to what is the notation used by the engineers. Please find out from your teachers. So, let me not use any particular notation and let me continue to use this symbol E critical at this particular point.

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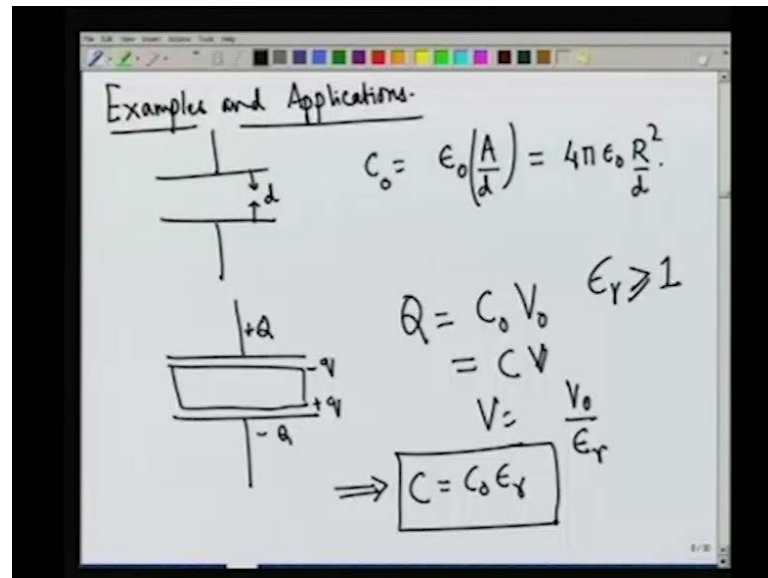
So, what we are saying is that, the electrostatic response or the response of any medium in an electrostatic situation is characterized by two numbers, epsilon and E critical. And as the notation suggest, this is given in units of volts per meter. Before we proceed, we should quickly ask ourselves as to, what are the typical values of this E critical; obviously, we do not want E critical to be very, very small in fact, it should be reasonably large.

So, let me give a few numbers. The first statement is that, E critical is greater than 10 to the power of 6 for all dielectrics, mean common dielectrics which we encounter. For air, E critical or the dielectric is something like 3 into 10 to the power of 6 volts per meter. So, it is a reasonably large quantity. If you look at paper, which is another dielectric medium this is roughly five times, so this is something like 15 into 10 to the power of 6 volts per meter.

In a similar manner you can go back and ask find out from the books, what are the dielectric constants for water, for mica, for glass so on, and so forth. So, if I want to actually insert these dielectric materials into capacitors for example, to introduce increase the capacitance then, we better be aware of these particular numbers.

So, typically the statement that I would like to make is that, the dielectric strength varies from counting like 10 to the power of 6 to 10 to the power of 8 volts per meter. So, rather than give a table at this particular stage, what I shall rather do is to give you a few applications, a few examples, where you can see how both epsilon and E critical naturally play a role. So, with this brief introduction today let us get into applications.

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So, let us start with examples and applications. The first example that I would like to discuss is the change in the capacitance, because of the insertion of a dielectric medium. Now, **you know** the capacitors come in a very, very variety of ways; if you go to any electric station, where lot of power is generated there will be capacitors as huge as cars; whereas, if you open up your radio or your computer, there are going to be capacitors in your micro circuits in your small, small circuits and they will be very, very small. So, tiny that you might like a lens actually to look up; so capacitors come from very, very huge devices to almost micro devices.

Now, the basic idea behind a capacitor whose physics we discussed in the previous lectures is that, they can store charge. And what is it that we have? As a prototype, I will always consider parallel plate capacitor, although many times there are also cylindrical capacitors which are used, but that is also a parallel capacitor, except that you have two concentric cylinders.

So, let me take two parallel plate capacitor, a parallel plate capacitor; and we know that, the capacitance of this system is simply given by  $\epsilon_0 A/d$ , where  $d$  is the distance between the two plates. Now, if this is taken to be a circular plate, both of them are taken to be circular plates; this will be  $4\pi\epsilon_0 R^2/d$ , where  $R$  is the radius of each of these plates.

Now, what happens, when I introduce a dielectric medium **in between the two capacitors** in between the two plates? The answer is very, very simple, let me show it schematically first and then argue it quantitatively later. So, let me say that, I have introduced the medium then what happens? Let us say the upper plate is charged with positive charges, the lower plate carries a negative charge. Now, the dielectric medium gets polarized and the charges introduced are I will show by small  $q$  minus  $q$  and plus  $q$ . So, that the energy between the two plates is reduced. That is the most important thing.

And we have stated in many, many ways please do not forget. What I said, that the electrostatic condition cannot be separated from what with thermo dynamic equilibrium. Electrostatic equilibrium and thermo dynamic equilibrium go hand in hand, we cannot forget that.

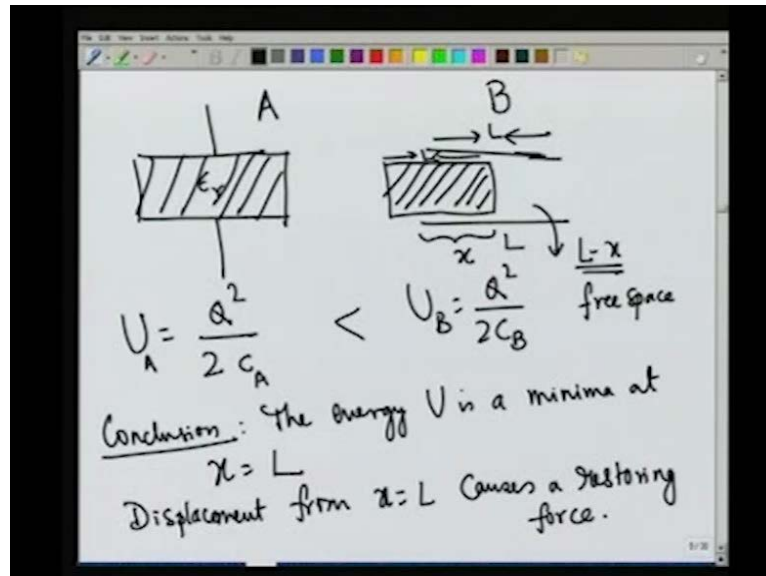
So, the energy is decreased and because the energy is decreased, the capacitance is going to increase. So, what is the statement? So, if I write  $Q$  is equal to  $C$  naught  $V$  naught,  $C$  naught and  $V$  naught are the voltages which are there in free space. And this is my  $C$  naught,  $\epsilon_0 A/d$  is equal to  $4\pi\epsilon_0 R^2/d$ .

In introducing the dielectric medium I have not changed the external charges at all; all that I have done is to introduce a minus  $q$  and a plus  $q$ . So, it is as if effective charge has decreased in both the places. So, how does it get affected? It will be written as  $C$  into  $V$ , the voltage has changed. And we know that my  $V$  is nothing but,  $V$  naught by  $\epsilon_0$ , where  $\epsilon_0$  is the permittivity of the medium, the relative permittivity of the medium. So, let me show denote it by  $\epsilon_r$ .

The voltage between the two plates has decreased has gone down by a factor of  $\epsilon_r$ ; and therefore, we conclude that  $C$  is nothing but  $C$  naught  $\epsilon_r$ . As I told you, many times people use the notation  $k$ ,  $k$  is identically the same as  $\epsilon_r$ ;  $\epsilon_r$  is always greater than or equal to 1, for a medium it is always greater than 1. Therefore, we increase the capacitance to the system well this is a very simple algebraic identity. But,

as I told you just a little time back, the effect of a dielectric is always to decrease the energy by producing what, partially screening charges.

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So, in order to appreciate that, what we shall now do is to look at the following situation. So, let us consider this capacitor and let us see the space between them by a dielectric medium. So, it is completely filled by a dielectric medium of relative permittivity  $\epsilon_r$ , this is my capacitor. Now, compare this situation with the following situation. What I will do is, let us say that this plates have a length  $L$ . Now, let me take it to be a square plate, if you feel like. What I will do is, I will not fill this space completely, but I will fill it only partially. So, I am filling this space not completely, but partially.

And let us say the medium has penetrate at the distance  $x$  into a depth  $x$  in the between the two plates and this distance  $L$  minus  $x$  is still free space. Even a child will answer as to which of them has over energy, the answer is very simple; the configuration A has lower energy than the configuration B, why? Because my energy is simply given by  $Q$  squared over  $2c$  whereas, here so let me call it as A in the configuration A. Here,  $U_B$  is equal to  $Q$  squared over  $2c_B$ ; and we know that,  $c_A$  is greater than  $c_B$ ; therefore,  $U_A$  is less than  $U_B$  that is the statement.

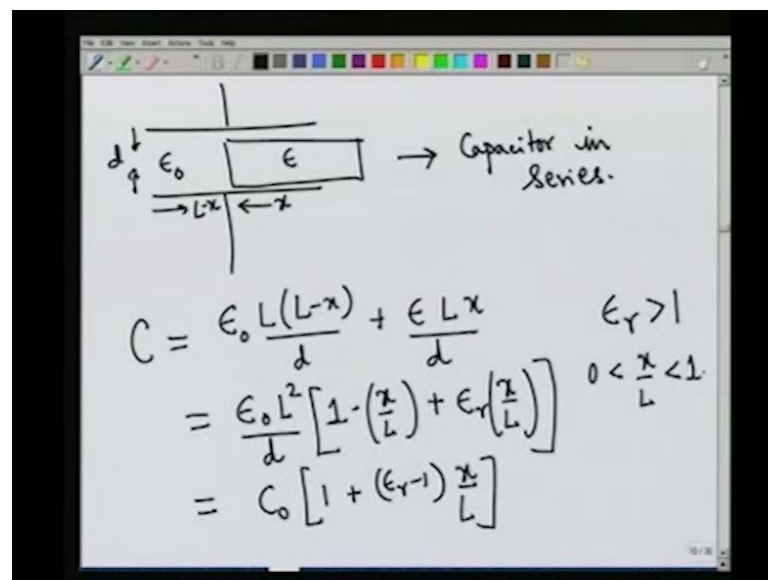
So, given the fact that, the energy is minimized, when the spaces completely occupied by the dielectric. So, what we are saying is that, if this length is  $L$ , so if this length  $L$  of the dielectric, so the energy is minimized when the spaces completely filled. If a part of it

were to come out then the energy increases. What do we conclude? The conclusion is that, if I were to plot the energy as a function of the displacement  $x$ ,  $x$  is the degree of penetration. So, what I will do is to plot as a function of  $x$ , the energy  $U$  is a minima at  $x$  equal to  $L$ ; when  $x$  equal to  $L$ , I get the configuration A; when  $x$  equal to  $0$ , I have pulled out the dielectric completely.

If the energy is a minima at  $x$  equal to  $L$ ; so, I have two capacitor plates and I have the dielectric medium. So, imagine I go and push it on one side, there is a restoring force I push it to the other side, there is a restoring force; which means we have to do work in order to remove the dielectric medium from in between the two plates; whereas, in order to send it in between the two plates, you do not have to do any work, there is a natural attractive force.

So, if you give me this expression in a minute I can tell you that there is a restoring force. So, displacement from  $x$  equal to  $L$  causes a restoring force because the energy is a minima at  $x$  equal to  $L$ . The evaluation of the expression for the energy is a very, very simple thing.

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So, let me illustrate that again here I am not going to work out everything, because this is a standard example you people can go and look at it. So, let me say that, my medium has penetrated a certain distance  $x$  and this distance is  $L$  minus  $x$ .

Well you can imagine this to be an arrangement of two capacitors, this is epsilon and this is epsilon naught, they are next to each other and the voltage is being applied from the top and the bottom. So, this arrangement is capacitor in series. I am not going to spend any time on this, because you people have solved any number of examples of this kind; and therefore, we can immediately write down the expression for the total capacitance.

So, my total capacitance is simply given by epsilon naught L, let me be careful L minus x divided by d, d is the separation between the two plates plus epsilon L x divided by d. This is my total capacitance. So, perhaps one would like to collect all the terms in a systematic fashion. So, what I shall do is to pull out epsilon naught L squared by d. So, in writing this expression I am imagining, what? Two plates which are squares in nature and they have a side L, the length of each side is equal to L; so, if I pulled out this epsilon naught L squared by d that would have been the capacitance in the absence of the medium.

So, if I were to collect terms, I am going to get 1 minus x by L plus epsilon by epsilon naught is nothing but epsilon r r k; I have pulled out L squared by d which is equal to x by L, this is the expression. So, let us be very clear about what we are doing; the statement that we have is that, epsilon r is greater than 1, 0 less than x by L less than 1, the geometry is very clear.

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$$U = \frac{Q^2}{2C}$$

$$U(x) = \frac{Q^2}{2C_0} \frac{1}{1 + (\epsilon_r - 1) \frac{x}{L}}$$

$$F = -\frac{\partial U}{\partial x} = \frac{Q^2}{2C_0} \frac{1}{\left\{1 + (\epsilon_r - 1) \frac{x}{L}\right\}^2} \frac{(\epsilon_r - 1)}{L}$$

Diagram: Two parallel plates with charges  $+Q$  and  $-Q$ , separated by distance  $L$ . A dielectric of thickness  $x$  is inserted between them. The electric field  $E$  is shown pointing downwards. The text "home assignment" is written at the bottom right.

So, if you feel like this can be simply written as,  $\epsilon_r - 1$  into  $x$  by  $L$ , nice expression. What then is the energy now corresponding to the situation? My energy is simply given by  $Q^2$  over  $2C$  and what is this? This is nothing but,  $Q^2$  over  $2C$  naught. So, that is the reason why I pulled out this  $\epsilon_r - 1$  and I will get  $\epsilon_r - 1$   $x$  by  $L$ ; so,  $1$  over  $1 + \epsilon_r - 1$  into  $x$  by  $L$ . So,  $U$  is **displaced** displayed as a function of  $x$ .

If you think it is a fairly simple expression, actually there is a small complication regarding the sign of  $x$  I would not get into it. I would rather advise you to plot this as a function of  $x$ , show that it is a minima, when  $x$  is equal to  $L$ , and it increases as we go away.

Now, if you give me this expression, I can immediately find out what the force is, what is my force? My force is simply given by minus  $\Delta U$  by  $\Delta x$ , because it is simply the gradient of the potential energy. Except that, this  $F$  **is the** this is the force, because I have calculated the total energy that is what I have. So, this is minus  $\Delta U$  by  $\Delta x$ ; and what is this quantity? This quantity is nothing but,  $Q^2$  over  $2C$  naught and I have  $1$  over  $1 + \epsilon_r - 1$   $x$  by  $L$  whole squared; then I have to differentiate this further I will get  $\epsilon_r - 1$  by  $L$ . This is the expression and this tells you the force with which the material is pulled in.

So, purely from energetic we have been able to argue that, a dielectric medium actually tends to get sucked in between the two plates. However, there is a small amusing thing at this particular point which you might like to think about, you can have a discussion among yourselves. I place the charge plus  $Q$  here, I place the charge minus  $Q$  here, and I evaluated the expression for the capacitance by taking the field to be perpendicular. So, that is how the field lines are.

Now, if the electric field lines are all perpendicular everywhere and let me say that, I have introduced a medium which I will show sideways, because we do not have space. So, I will again put a plus  $Q$  and a minus  $Q$  here; the induced charges minus  $Q$  and plus  $Q$  are also there and the field is again in this direction, except that it is decreased in intensity.

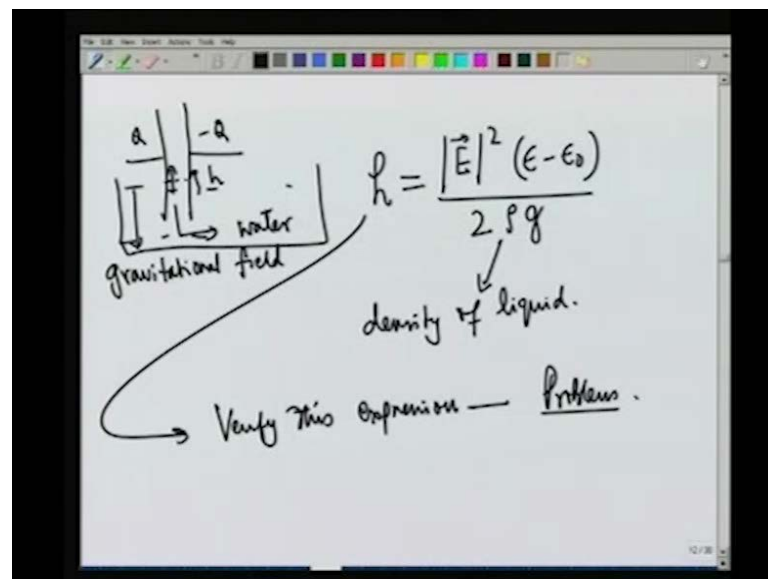
The restoring force has to necessarily act on the charges, because the force is of electrostatic origin. But then, if you look at the geometry of these two pictures, the forces



are all the time what? Perpendicular along the axis which connects the two rights; but on the other hand, this  $F$  is inside. So, let me draw a coordinate system, if this is my  $x$  axis and this if my  $z$  axis, my electric field is around the  $z$  direction; whereas, the restoring forces along the  $x$  direction. So, what I would like you people to think about and solve at least qualitatively is that, what is the origin in terms of the electric field for a force to be produced along the  $x$  direction?

Remember all physics all forces should be understood in terms of the induce charges and the original charges. So, obviously, there is something missing in these two diagrams. There is an approximation that, is giving you a wrong conclusion, which is showing an apparent incompatibility between the motion along the  $x$  direction and the field along the  $z$  direction. This I will leave it as a problem or a home assignment for you. So, please work it out because, at this point I do not want to get into it. This is a very nice topic for a discussion.

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The way you people verify this particular result experimentally is a very nice thing. What you just do is to look at a capacitor not the way I showed you horizontally. But, imagine that you have a plus  $Q$  and a minus  $Q$  let us say something like this and so, they are vertical; and imagine that for an electro medium, you have water in this. So, **the** there is an electrostatic attraction upwards, the restoring force and there is a gravitational field downwards.

If there were no gravitational field, if you imagine that you have taken a beaker of water and then put it in between two capacitor plates carrying charge plus  $Q$  and minus  $Q$ , you could have said that the water simply starts flowing up like a capillary effect. But, however the restoring force gets balanced by the uniform gravitational force. Therefore, the water attains a certain height.

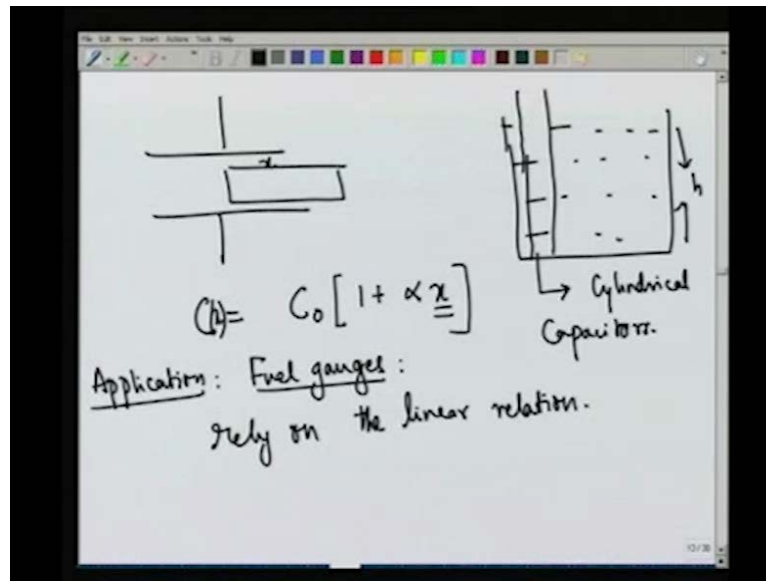
So, let me show a beaker here, the water attains a certain height; and the question is what indeed is the height extra height achieved by the water. So, rather than working out the solution I have already given you the restoring force; all that you have to do is to write down the expression, for what? The gravitational force acting on the water, when it rises to a height  $h$ . So, if you were to balance both the forces, you get a rather nice elegant expression for the height  $h$  to which the water rises.

In writing this expression, I am going to assume that  $h$  is a small quantity it is not as if it will reach to a very, very large number. When you do your experiments in your lab, you will realize and it has a nice expression. It is simply given by  $\frac{\epsilon_0 E^2}{2\rho g}$ , where  $g$  is the gravitational constant and this is the density of water of the liquid.

So, you could do your experiment with kerosene, you could do your experiment with petrol, you could do your experiment with water, they all come with different densities and different permittivities and you could verify this particular expression. As I told you, this follows by assuming that  $h$  is small.

There is yet another statement that you should remember at this particular point, this  $E$  is the field which is produced what? In the absence of the dielectric medium; since the height is very, very small  $E$  is practically unchanged. So, please take that as a problem. So, verify this expression, so this is your problem neat example. Having discussed this very simple case let us go back and look at the expression for the capacitance that we derived when the dielectric medium partially filled the space. So, let me draw the picture again.

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So, I have this parallel plate capacitor and the dielectric medium has penetrated partially up to a depth  $x$ . I wrote an expression for the capacitance in this particular case and that was  $C = C_0 [1 + \alpha x]$ . What is the statement that I am trying to make? We know what the value of  $\alpha$  is, let us not get into that I derived it just a few minutes back. The important statement is that, the capacitance is a linear function of your penetration  $x$ . Of course, the capacitance is maximum, when  $x$  is equal to  $L$ ; the capacitance is a minimum when  $x$  is equal to  $0$ .

Now, this very innocuous relation can actually be used and this is what brings us to the first of the examples or applications. The first application is what we call as the fuel gauge, all gauges are meters. So, look at our vehicles we have scooters, cars, etcetera and there is this needle which keeps showing how much of fuel is there; if you filled your tank the needle goes to the maximum.

Then, there is something called a reserve, it shows the fuel has gone beyond a certain critical value you better fill up your tank, because otherwise perhaps you will not be able to run our vehicle for more than another half a kilometer or so. And then of course, when you completely exhaust the fuel, the needle goes to  $0$ . How is it that, this fuel meter or the fuel gauges is working; and the answer to that is in this diagram.

So, suppose I have a fuel tank let me show it in this particular fashion and let me say that, it is filled by fuel up to a particular height  $h$ . So, this is the height  $h$  to be filled,

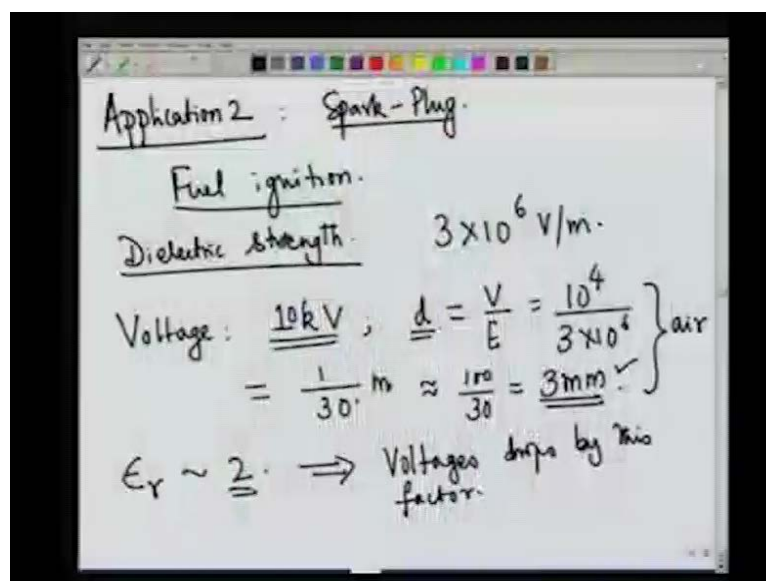
what we have to imagine is that, there is actually a thin capacitor somewhere a small capacitor which is sitting inside and typically these are what cylindrical capacitors.

So, I have two concentric cylinders and the inner and outer cylinders are connected to a voltage source; so that, their charge to plus Q and minus Q. That is what happens. You see that, the height of this capacitor will be the same as the height of the fuel tank. And as we start our vehicle and it start running, h decreases what happens as h decreases? As h decreases, h here is the same as x, my capacitance decreases.

So, what I do is to build the circuit, which keeps on measuring my capacitance as a function of x which I shall show here. And therefore, since there is a direct linear relation between the height h and the capacitance, I can immediately read off what the height is. So, the field gauges rely on the linear relation.

So, if you go up and open up your fuel tank and look for where the capacitor is, these are at as I told you long capacitors, but very little spacing of course, modern day fuel tanks may use something more sophisticated, the spacing between them will be of the order of a millimeter or a fraction or so. Therefore, that is how you build a fuel gauge in order to measure the height of the tank, it exploits what? The dielectric property of the petrol; by the way, petrol has a dielectric constant of something like I think about 2 therefore, this works quite well.

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Application 2 : Spark-Plug.

Fuel ignition.

Dielectric strength.  $3 \times 10^6 \text{ V/m.}$

Voltage:  $\underline{10 \text{ kV}}$ ,  $d = \frac{V}{E} = \frac{10^4}{3 \times 10^6}$  } air  
 $= \frac{1}{30} \text{ m} \approx \frac{100}{30} = \underline{3 \text{ mm}}$

$\epsilon_r \sim \underline{2}$ .  $\Rightarrow$  Voltages drops by this factor.

Having looked at this capacitance based fuel gauges, let us now on to next application namely spark-plug. One statement that, perhaps I should do make before I proceed is that, what the manufacturer does is actually to calibrate. So, he keeps on measuring capacitance because there are all kinds of corrections, finite lengths, finite size, etcetera etcetera; so, he calibrates h against capacitance, draws a line which not which need not exactly be c naught alpha and with this calibration the needle is made to show that particular reading.

So, now let us return to the next application namely spark-plug. Spark-plug is used for what, fuel ignition we all know that; and those of you own a scooter or a car you know that you frequently run into trouble with the spark-plug. The spark-plug has to be clean sometimes there is water on that, sometime there is an oil deposit and it refuses to work. And if you take this spark-plug to the mechanic, the mechanic fine tunes it.

Now, we want to understand the physics behind the fine tuning of the spark-plug. The basic idea behind this fuel ignition or spark-plug is the dielectric strength. As I told you, air has a dielectric strength of 3 into 10 to the power of 6 volts per meter; so, if you were to convert it into kilovolts never mind let us leave it as 3 into 10 to the power of 6 volts per meter.

Now, what does the mechanic do and what is the kind of fine tuning? If the field exceeds 3 into 10 to the power of 6 volts per meter, then what happens? There is a dielectric break down and then there is a spark is that right, the minute there is a spark that can actually ignite the fuel. So, let us imagine a hypothetical situation and see, what a mechanic may do in free space that is in air and what a mechanic might actually do in what, a medium that is what happens in the fuel tank. So, let us proceed slowly air we know has a dielectric strength of 3 into 10 to the power of 6 volts per meter. That is, if I can produce an electric field in some region of space, which exceeds this value E critical that is what we call 3 into 10 to the power of 6 volts per meter then, there would be a spark.

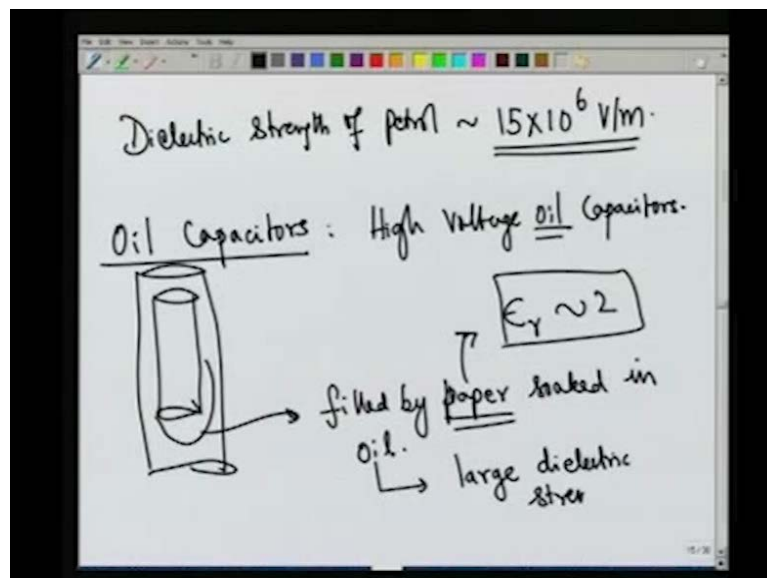
Now, imagine what happens in a car garage, the mechanic will actually apply a voltage of 10 kilovolts I have written down these numbers. So, that we do not have time to we do not have to waste time to look them up. So, if there is a voltage of 10 kilovolts, then in order to produce an electric field of 3 into 10 to the power of 6 volts per meter, I can

estimate what the distance is, distance is nothing but voltage divided by the electric field; and it turns out the spacing the gap in the spark-plug should be of the order of 3 millimeter not exactly 3 it is slightly more than that. And therefore, the mechanic may be very happy and the person who sees may be very happy, that there is a beautiful spark which is actually being seen.

Unfortunately, what happens is that when we are going to ignite the petrol itself, the situation is not that simple, why is that so? The point is that, if you are want to ignite petrol, petrol is a dielectric medium and as I told you, **the dielectric strength of the sorry** the dielectric constant of petrol is the of the order of 2, it could be 2.5 it does not matter. So, if the dielectric constant is let us say, twice the dielectric constant of air obviously, what happens? The voltage drops by this factor this implies that voltage drops by this factor.

Now, if the voltage drops by a factor of 2, you can see that the gap also falls by a factor of 2, but this is not the end of the story. There is yet another fact that you have to take in mind, and that is this expression for  $d$ , the gap between in this spark-plug was obtained by looking at a number 3 into 10 to the power of 6 volts per meter.

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But what is the dielectric strength of petrol? The dielectric strength of petrol is roughly five times the dielectric strength in air. So, that is roughly let us call it 15 into 10 to the power of 6 volts per meter. So, now we are really into something, because not only has

the voltage dropped, the dielectric strength has also dropped; therefore, the gap in the spark-plug has to be considerably smaller.

So, I have given you all the numbers, you people can actually estimate what the gap should be; the gap will turn out to be a fraction of a millimeter instead of 3 millimeter that we got it, it will turn out to be close to 0.3 millimeter and that indeed is the fine tuning. Over and above that of course, if there is an additional dielectric coating on the spark-plug, because of the dust or because of some other oil or because of water then again there is not going to be any discharge. So, these simple applications actually a sound basis in the physics of dielectric and even at this level, we can actually understand and appreciate how this systems work.

So, that is something is very nice about these things. As a final application, I want to look at yet another capacitor and these are what are called as oil capacitors. So, the dielectric medium is actually consisting of oil, but not exactly the way we want to look at it and in fact, the precise name is that high voltage oil capacitors. So, we want to actually build capacitors which operated at very, very high voltage.

So, let me repeat you cannot increase the voltage arbitrarily because, for a given gap in the capacitors for a given separation between the two capacitor plates, as you increase the voltage the electric field increases; and the minute the electric field exceeds my dielectric strength, there is going to be a break down; and therefore, the capacitor will not continue to work as a capacitor, it will get shorted. But on the other hand, we want to operate capacitors at very, very high voltage.

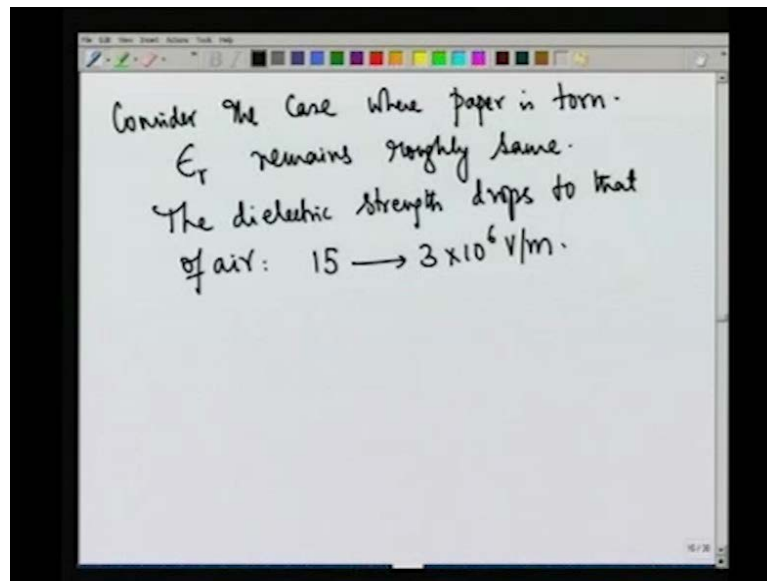
So, how do we do that? The trick is actually to introduce a dielectric medium, which actually decreases the voltage and one of the ways of accomplishing is through the oil. In this case, of course what one does is not to simply fill the whole space between the two oils or whole space between the two plates or the two cylinders by oil, but what one does is to do the following. Let me illustrate that. So, suppose I have a cylindrical capacitor, this space in between is filled by **water sorry** paper soaked in oil.

So, the space in between is filled by paper soaked in oil. This oil may not have a very large value of dielectric constant. Therefore, the dielectric constant comes entirely from paper my relative permittivity; and my permittivity is of the order 2, 2.5 that is what one

does. However, the oil itself has a very large dielectric strength this has a large dielectric strength.

So, oil contributes the dielectric strength, paper contributes the dielectric constant. And a combination of these two, actually allow you to apply a reasonably large voltage to work for example, the kilo volt range. Now, in order to illustrate let me consider a situation where my paper gets torn.

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So, consider the situation the case where paper is torn. If the paper gets torn, epsilon r remains roughly same **right**, a small tore in the paper is not going to change the dielectric constant of the medium; however, the dielectric strength drops to that of air. You tore there is a small gap and that is not necessarily filled by anything, because it was simply soaked in oil.

So again, if we have something like a factor of 15 or factor of 5 let us say for the dielectric strength, the voltage that the capacitor can take actually drops by a factor of 5. So, if you are going to look at all these high voltage oil based capacitors, it is very, very important that, such accidents do not occur, so drops to that of air. So, for example, if it were 15 it would go to 3 into 10 to the power of 6 volts per meter and then that would be a sad tale; because, you might assume a very large voltage, but then the threshold value has dropped by a factor of 5 and my capacitor breaks down. There are also other applications in fact numerous applications involving multi layered dielectric capacitors



so on, and so forth, but we do not have time to get into that. If you look up a nice book on electrical engineering, where electrical circuits are build **you know** and electrostatics is actually studied you will find a large number of them.

At this particular point, what we shall do is to conclude the discussion of dielectrics. There is a small point, which I did not bother to discuss, because you people are all familiar with that, that is the capacitor in series and capacitors in parallel. I will leave that for you people to read it by you yourselves. What we shall do is to go on to study other things, steady currents and magnetism from the next lecture.