Engineering Physics - II

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Module No. # 03
Lecture No. # 06

In the last lecture, we looked at the electrostatics in the presence of conductors. Now, what we shall do is to continue our studies of electrostatics in media, but with dielectrics.

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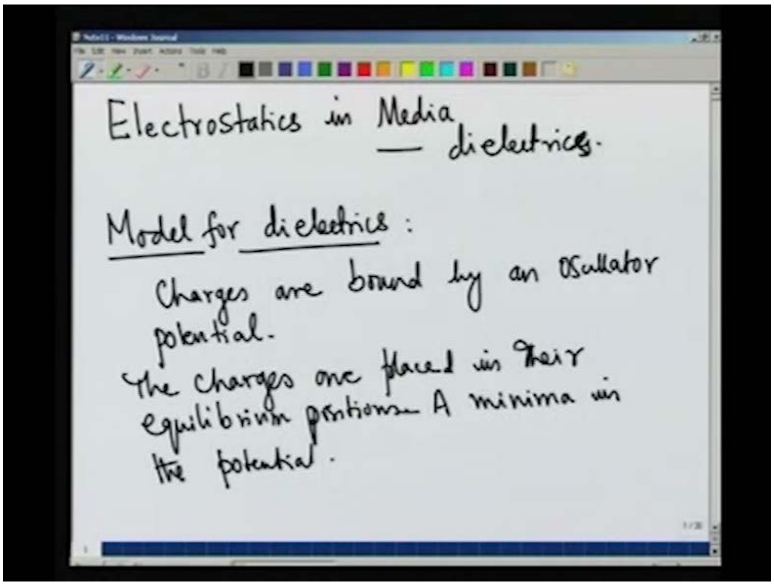

Electrostatics in Media
— dielectrics.

Model for dielectrics :
Charges are bound by an oscillator potential.
The charges are placed in their equilibrium positions. A minima in the potential.

So, we are going to study electrostatics in media namely dielectrics. You people are already familiar with the concept of a dielectric. In fact, you have calculated the capacitance, how the capacitance changes in the presence of a dielectric and so on and so forth. So, what I shall do is to try to give you a perspective as to, how a dielectric has the properties it acquires the properties that it has from a sort of microscopic viewpoint. In the case of conductors we argued that there is a reasonable supply of free electrons, rather there is a supply of reasonably large number of free charges, which can move freely to readjust their positions; they can move over macroscopic distances such that they can decrease the energy of the system. The electric field is completely made to vanish within the body of the conductor.

So, that was the defining property of the conductors. And we are able to derive a large number of properties such as all the free charges or the excess of charge resides on the surface of the conductor; the surface of the conductor should be an equipotential surface so on and so forth.

What we now want to do is to look at materials, where there is no supply of such electrons, there is no charge particle, which are actually move over macroscopic distances; but the charge particles only readjust their positions slightly to a certain extent. May be over a molecular length or a few molecular lengths and that is what distinguishes a dielectric from a conductor.

At the very beginning when I introduce the concept of dielectric and a conductor, I told you that it is a matter of qualitative distinction, it is not quantitative distinction. I even gave you the numbers as to under what circumstances we call a material a conductor or we call a material a dielectric; do not forget that I gave you the number in terms of resistivity.

So, if you remember that you can appreciate that, there is a rather smooth transition from dielectrics to conductors through what are called as static discharged materials. Static discharged materials we do not study in this particular course I have mentioned it for the sake of completeness.

Now, let us get to our studies. The first thing that I would like to do is to model conductors, rather dielectrics as follows. As we said in a dielectric, the charges are not free to move, but certainly they can be displaced. In order to make it somewhat quantitative, we shall imagine that charges are bound by an oscillator potential, let me explain that.

We have already argued that, if you look at average charged densities, (( )) charged densities then, rho is independent of time that is the definition of electrostatics. Although, the electrons may be moving very, very fast in atoms or in molecules, we are going to look at the average charge density. What is the average? The average is then over time, the average is then over reasonably small, but reasonably large compared to molecular dimensions of space if you did that my charge is a constant.

If my charge density is a constant that is it is not varying as a function of time that means the charges must be sitting in their equilibrium positions. So, the statement is that the charges are placed in their equilibrium positions. Obviously, these are stable equilibrium positions. So, at this point I have minima in the potential, not a maxima; a maxima would give you an unstable equilibrium, we are going to look at equilibrium positions.

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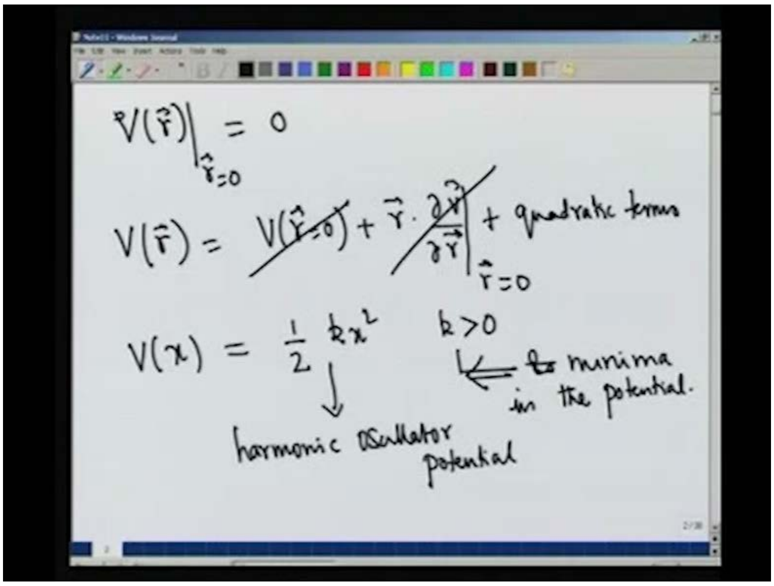

$$V(\vec{r})\Big|_{\hat{r}=0} = 0$$

$$V(\vec{r}) = \cancel{V(\hat{r}=0)} + \cancel{\vec{r}\cdot\frac{\partial V}{\partial \vec{r}}}\Big|_{\hat{r}=0} + \text{quadratic terms}$$

$$V(x) = \frac{1}{2}kx^2 \qquad k>0$$

harmonic oscillator potential

~~to~~ minima in the potential.

Now, if I imagine that the particle is sitting in the equilibrium position I can ask, what happens if I displace it slightly away? So, what do we do, let us look at the energy; without any loss of generality let us take the potential energy at equilibrium position, let me choose that to be r equal to 0 to be 0. What we are now going to do is to make a Taylor expansion of my potential around this particular point.

So, if I look at r close to r equal to 0 which I have taken to be the equilibrium position, I get V r equal to 0, which is equal to 0 plus r dot delta V by delta r, that is what I am going to get plus quadratic terms plus quadratic terms. And obviously, this derivative is to be evaluated at r equal to 0. Since, we are looking at an equilibrium position and we have chosen the 0 of the potential such that, at r equal to 0 it is 0 potential. What do I have? V of r has only quadratic terms, this cancels and this cancels.

Now, let me imagine that I am moving along the x direction. So, let me look at only along the x direction, the argument will generalize to other directions also. This will obviously I have the form half k x squared, where k greater than 0. How is it that k is greater than 0, because this is implied by the minimal minima in the potential.

So, the statement that we are making is that, if I slightly displace my charged particles from its equilibrium position, then in the leading order it experiences a harmonic oscillator potential. It vibrates around the equilibrium position with a frequency given by root k by m, and this will be the basis of all our discussion of dielectrics today.

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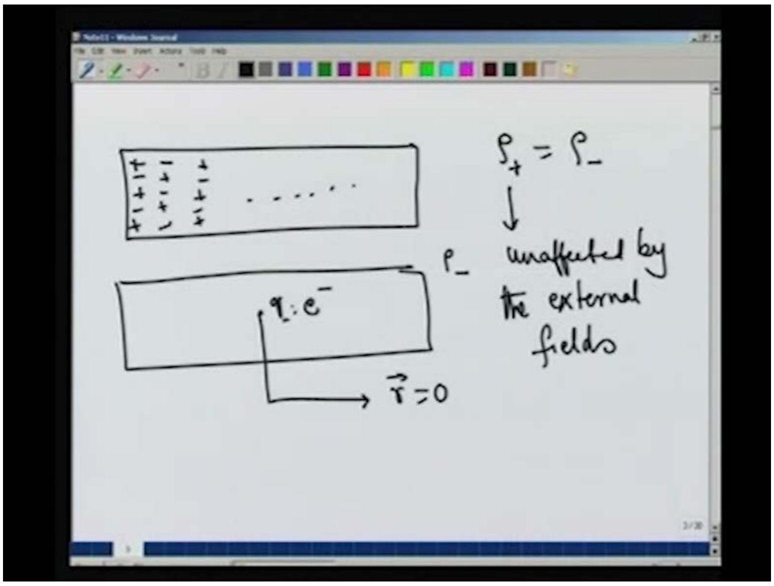

So, let me imagine that I have a dielectric (( )) I do not put any extra charge on it on the whole it is electrically neutral; and let me show how the charge distribution corresponding to the negative charges are there. So, may be I can do something better, my charge distribution is schematically shown in this particular fashion etcetera, etcetera, etcetera.

And what we are saying is that? Rho plus is equal to rho minus, rho plus is the mean charge density coming from the (( )), rho minus is the mean charge density coming from the electron distribution. They are both the same implying not only over all charge neutrality, but also the charge density itself vanishes at every point that is what we are interested in.

Now, let me imagine that this charge distribution inside the dielectric slab has actually being obtained by a super position of two slabs on each other; one of them is a uniform positive charge density, another is a uniform negative charge density. And let us look at a typical negative charge density. So, I am now going to look at only rho minus, there is an identical picture for rho plus. So, let me say that there is one charge q minus sitting here, which is actually electron, let me imagine that it is sitting here.

We are obviously interested in the time averaged position of this charge density need not be a single electron, it could be a collection of electrons also, because I am looking at a sort of physically small space, but larger compared to the molecular dimensions, let me call it as an electron for the time being. Now, if I look at that because it is stationary it is sitting in the equilibrium position that is the statement that we are making. Now, without any loss of generality I can take the position corresponding to r equal to 0.

What we now want to do is to study the response of this system to an external electric field. In particular, I am interested in the response of the electrons and not of the positive charges, because as we all know the positive charges are rigidly held on to the lattice, they are much, much heavier compared to the electrons.

Therefore, the response is small, although the experience the same force, because of their large inertia, they do not move much. So, we shall always assume that, rho plus is unaffected by the external fields. And we are imagining that, only rho minus is going to get affected, which is the reason I am looking at this distribution.

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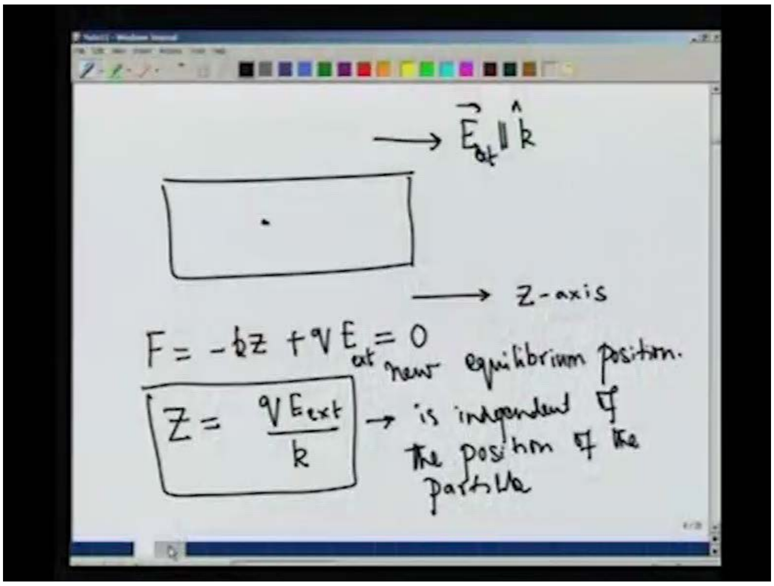

$F = -kz + qE_{xt} = 0$ new equilibrium position.

$$\boxed{Z = \frac{qE_{ext}}{k}} \rightarrow$$ is independent of the position of the particle

Now, the minute I am going to apply an electric field in order to make it concrete let me show it again. So, this is my dielectric slab and let me say that apply my electric field in this direction and let me take this to be z axis, so this is my z axis this is my z axis. And I am looking at one particular charge particle, whose equilibrium position I shall denote it by 0. In the absence of the external electric field, there was no net force acting on the particle.

However, the minute we apply an electric field, the electric field displaces my charged particle as soon as it gets displaced, it experience two forces. The first force is of course, the external electric field, the other one is the restoring force. Now, in order to make notation very precise, so that it depicts the physical situation that we have in mind; let me add a subscript here E external I have applied an external electric field.

So, if you want you can imagine that this external field is produced by placing this dielectric between a parallel plate capacitor consisting of positive and negative charge densities surface charge densities. So, if I want to write down the net force acting on the particle this will now be given by minus k x, where x is the displacement from the equilibrium position plus q E.

We are again interested in the time average distribution. So, although the charge particle might oscillate around the point, because of these two forces, we are interested only the time averaged equilibrium position. Therefore, we are now going to set this equal to 0 which defines the new equilibrium position.

Before the application of the external electric field, my equilibrium position was given by r equal to 0, now I have applied an electric field; and therefore, the equilibrium position has shifted. This is something that you people are familiar for example, in a spring mass system in a gravitational field, it comes down and stays in the new equilibrium position. Except that, since I have declared that my electric field is along the z direction, I have to make a change here I will replace this x by z; it could be along any direction I will take my z direction to be along the electric field.

So, if I said this equal to 0, my new equilibrium position will be simply given by z equal to q E external by k. In other words, the shift in the equilibrium position is essentially dictated by the strength of the external electric field, and also the stiffness of the spring how big or how small this k is.

Now, we are all familiar with a large number of dielectrics water, mica, glass, etcetera wood etcetera, etcetera; and if you take them and place them in an electric field, nobody has seen the material either expand or contract, unless one makes what a very, very precise careful measurement. This model is telling us that, the z shifts if the z is going to shift that means the charges are going to move. So, the shape or the size of the body should change however, those changes are very, very small and that effect is what is called as electrostriction. We are not going to get into that, we only want to use the fact that there is a slight displacement. So, let us forget about how the shape of the body changes, may be we will return to that subject later.

At this point what I want to do is to ask, what the impact of this on my charge distribution is. So, what have we done, what we have done is to apply a uniform external electric field and we find that for every particle for every charged particle, because we have taken E external to be uniform, the equilibrium position has shifted by a quantity z.

So, this z is independent the position of the particle, because we have taken E to be uniform, this is independent of the position of the particle. So, we are saying that, the new equilibrium position shifts by a quantity z, for all the charged particles.

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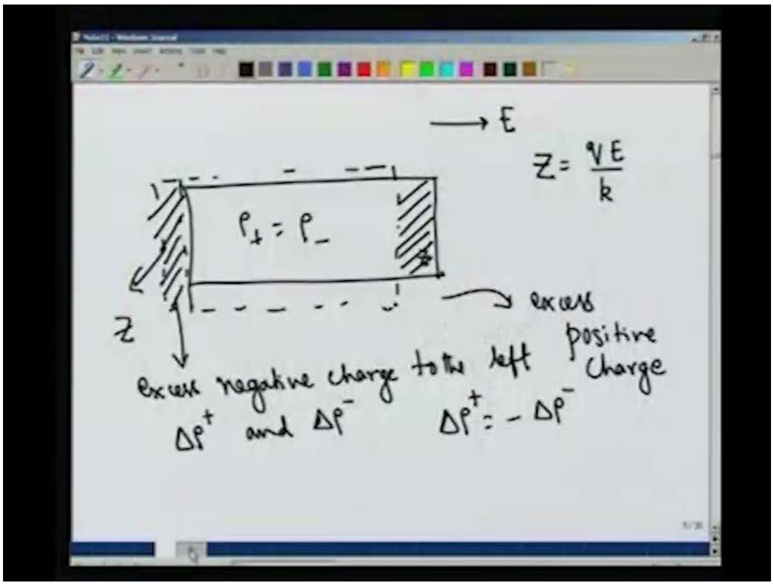

Now, with this I want to make a very simple picture, which actually almost completely explains what we mean by a dielectric. So, imagine this to be my dielectric slab, that part which corresponds to the positive charge density. Now, I have applied an electric field in this particular direction, if I apply an electric field in this particular direction, my charge particles the negative charged particles are going to move in a direction opposite to that of the electric field. How do they move? The equilibrium position shifts by a quantity, let us for not forget z is equal to q E by k, E is along the positive z direction, q is negative therefore, there is a shift.

Since, all of them are going to shift by the same quantity z, what I can do is to now show another rectangular slab cylindrical slab, this is a rectangular cross section, this is a highly exaggerated picture. What I am saying? The negative charges are now going to shift by a certain quantity and this is indeed nothing but the (( )) all of them are going to shift by the same quantity z. Therefore, these two regions correspond to z and in this region, rho plus equal to rho minus, the negative charges have moved in this direction.

What do we conclude? We conclude that, there is excess negative charge here charge to the left, my positive charges have not moved at all; therefore, there is an excess positive charge here. Obviously, you can see that, if I denote them by delta rho plus and delta rho minus, these are the excesses delta rho plus is equal to minus delta rho minus, because the material itself has remained overall neutral.

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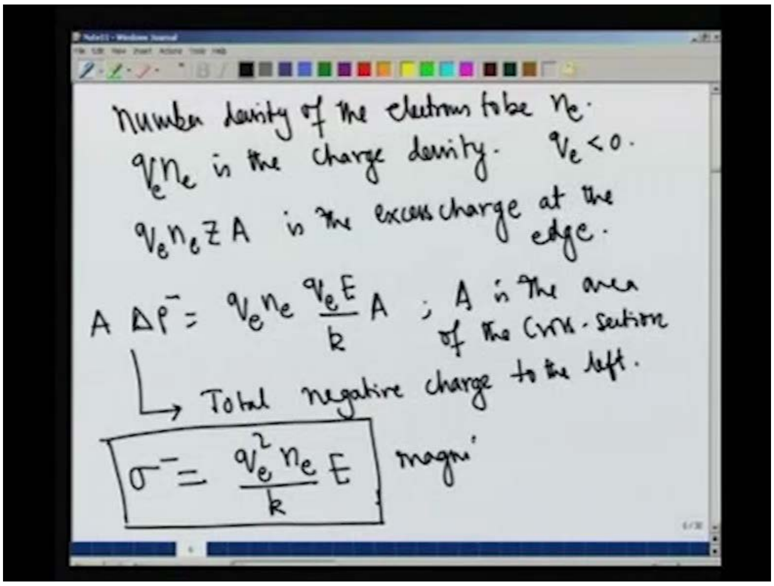

number density of the electrons to be n_e.

$q_e n_e$ is the charge density. $q_e < 0$.

$q_e n_e z A$ is the excess charge at the edge.

$A \Delta\rho^- = q_e n_e \frac{q_e E}{k} A$; A is the area of the cross-section

$\hookrightarrow$ Total negative charge to the left.

$\boxed{\sigma^- = \frac{q_e^2 n_e}{k} E}$ magni

Now, we can make a very simple estimate of what this rho is and what is going to happen. So, let me take the number density of the electrons number density of the electrons to be n e. So, the excess charged density is simply going to be what? q n e is the charge density, q n e is the excess charge density, q is negative. So, if you want I will put a q e and q e is less than 0.

Now, that much charge density has shifted by a quantity delta z. And as we said, we are not interested in the dimension delta z, that delta z is very, very small compared to the dimension of the system. So, let me come back to this picture. If this total length is l, we are saying that z is much, much less than l, mod z is much, much than l. So, we want to imagine that, almost all the charges are concentrated at the edge of the material; that is we want to replace the surface the volume charge density by the surface charge density.

How do I do that? If q e n e is the charge density, so I have excess q e n e that is the charge density, and what is the excess? The excess charge is simply given by z into area is the excess charge at the edge we want to replace this by a surface charge density. So, let me write again, my delta rho minus is q e n e; and what was my z, z was again q e E by k into A, this is your excess charge density, where A is the area of the cross section.

So, let me multiplied by A here which gives me the total charge. So, the left hand side is the total negative charge to the left. So, now I want to replace it by an equivalent surface charge density. Therefore, my sigma minus is nothing but q e squared n e by k into electric field. So, this is indeed my negatives (Refer Slide Time: 20:32).

The whole thing is of course of negative sign, because we have not bothered to (( )) track of (( )), this is the magnitude. This much of surface charge density is sitting to the left of the (( )). What we shall now do is to continue and see how much of surface charge density is sitting to the right and then ask, what is the net electric field, which is produced by the system?

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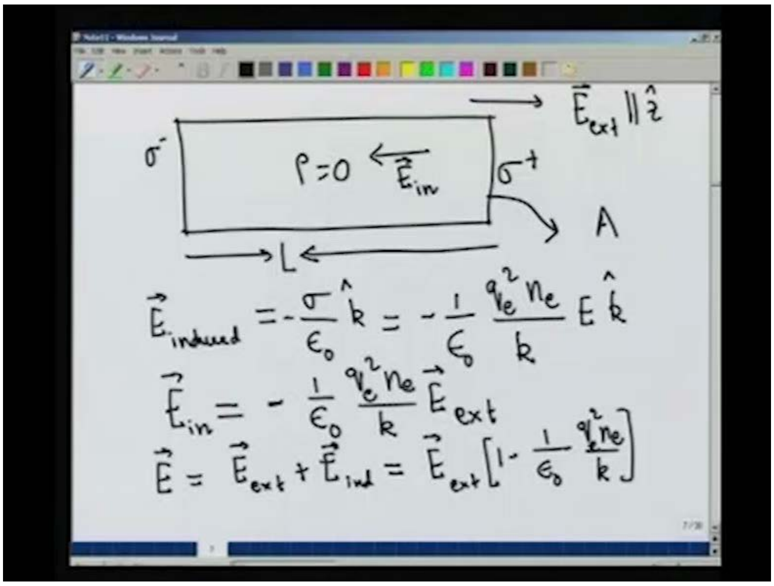

$$\vec{E}_{induced} = -\frac{\sigma}{\epsilon_0}\hat{k} = -\frac{1}{\epsilon_0}\frac{q_e^2 n_e}{k} E\hat{k}$$

$$\vec{E}_{in} = -\frac{1}{\epsilon_0}\frac{q_e^2 n_e}{k}\vec{E}_{ext}$$

$$\vec{E} = \vec{E}_{ext} + \vec{E}_{ind} = \vec{E}_{ext}\left[1 - \frac{1}{\epsilon_0}\frac{q_e^2 n_e}{k}\right]$$

Now, what is the picture that we have, the picture is a fairly simplified picture. So, my rho equal to 0 everywhere, there is a sigma minus and there is a sigma plus, my external electric field is along this direction which is taken to be the z axis; there is a sigma plus and there is a sigma minus here. And this material has a length l and the area of cross section is A, n e is the number density of the electrons, q e is the charge carried by the electron.

Now, if you take the area to be quite large in fact we are not going to look at very, very small pieces of dielectric, let us say something of a millimeter or whatever that is the kind of experiment that we do in the laboratory. Then this is as if, there are what? Two charge plates with uniform charge density, this is a dielectric medium.

So, we can imagine that there is a uniform charge density, because the charges move only along the z direction, they do not have any transverse motion. Therefore, I can ask what is my E induced? My E induced is obviously in the opposite direction, this is a very important thing; my E induced is in the opposite direction and this is simply given by sigma by epsilon naught along the minus k direction, is that right? Sigma by epsilon naught in the minus k direction.

But in the previous example we already wrote an expression for the sigma. So, what is this expression this is nothing but minus 1 over epsilon naught; and for sigma, let me read the expression from the previous thing (Refer Slide Time: 22:57) q e squared n e by k into E. So, q e squared n e by k into E into k, which I can write as minus 1 over epsilon naught q e squared n e by k into E external.

So, a certain fraction of the applied electric field becomes E induced, except that it is in the opposite direction, please note the occurrence of the minus sign. Because in the electric field the positive charges move in the direction of the electric field, the negative charges move in the opposite direction. Therefore, E induced tries to oppose whatever is happening with the external field; that is how the system actually tries to minimize its energy, we should never forget that we are looking at a static situation, we should never forget that.

So, if this is indeed E induced all that remains for me is to write down E total. So, what is E total? My E total I will just call it as E is simply given by E external plus E induced; and this is nothing but let me pull my E external out 1 minus 1 over epsilon naught q squared n e by k.

Please notice in writing this thing, we have 1 unknown parameter k otherwise, you can try to find out what the number density is the charge of the electron is known so on, and so forth. Therefore, we understand the response, we are trying to appreciate what the response of a dielectric is to be a model which has a single free parameter k namely the spring constant.

Now, as I already told you this expression contains almost everything that I want to know about the dielectric. So, all that remains for me is to actually rewrite this expression in terms of what we shall call as the conventional symbols, let us do that.

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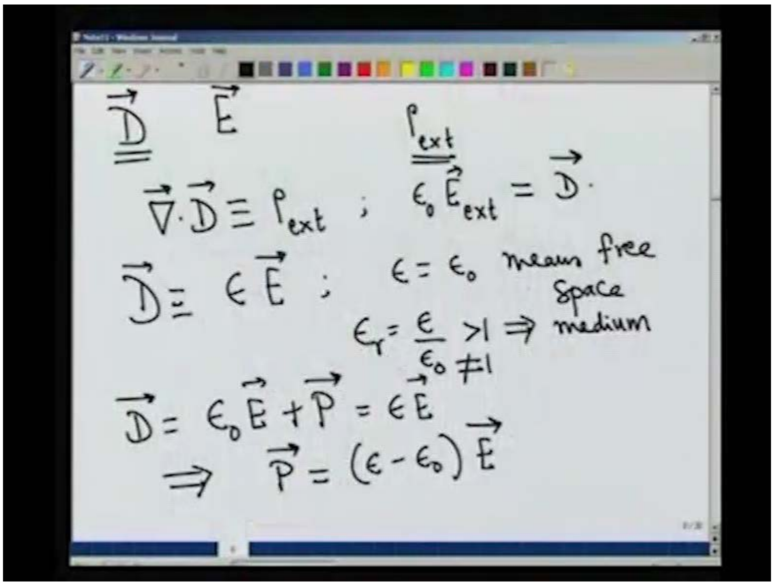

$\underline{\underline{\vec{D}}}$ $\vec{E}$ $\underline{\underline{\rho_{ext}}}$

$\vec{\nabla}\cdot\vec{D} \equiv \rho_{ext}$; $\epsilon_0 \vec{E}_{ext} = \vec{D}$.

$\vec{D} \equiv \epsilon \vec{E}$; $\epsilon = \epsilon_0$ mean free space

$\epsilon_r = \frac{\epsilon}{\epsilon_0} > 1 \Rightarrow$ medium

$\neq 1$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E}$

$\Rightarrow \vec{P} = (\epsilon - \epsilon_0)\vec{E}$

In order to do that, what I have to do is to remind you whatever you people have studied in what your 12 th standard or you have already studied in your classes in your engineering. We do not deal with E external, we do not deal with free constant, but we deal with what is called as the displacement field D, which is due to the external charges. And we deal with the field E, which is indeed our total electric field.

So, let me introduce this standard notation, the conventional notations and then compare it with the expression that we have derived from our model, and try to understand what is the physical meaning of each of the terms in the conventional expression. So, let us say that, there is a rho external; rho external is the charge density which is not there in the dielectric. It is something external that has been brought that is, you should imagine that you have not ionized your dielectric or anything.

However, this rho external can be (( )) sitting either outside the dielectric or inside the dielectric, we do not have any prejudice regarding that. But for the time being you can imagine that, this rho external is sitting outside the dielectric; and we are asking for what is the response of the dielectric medium, the dielectric medium is overall neutral, let us have that picture.

So, if you did that in the absence of the medium, I would have got an electric field. So, that information is captured by the symbol D, how do we do do that, my D is defined by the equation my D is defined by the equation rho external, please notice that I do not write rho external by epsilon naught. Therefore, my D is not the conventional electric field, but it is something different. So, if you want to make comparison with our notation, my E external whatever I use to call is what? It is related to D by the following notation epsilon naught E external is D that completes the picture. Divergence, D is given by rho external.

What is the next thing that we have to do? The next thing that we have to do is to relate D to E. I argued that, the total induced electric field is proportional to the external electric field; the external electric field is of course proportional to D, my total electric field is a sum of induced plus the external.

Therefore, my total electric field is proportional to D that is summarized in the next equation and it is written as, D equal to epsilon E D equal to epsilon E and this is the next definition whenever I write three horizontal lines, it is the definition. And epsilon equal to epsilon naught means frees space epsilon equal to epsilon naught means free space, if epsilon differs from epsilon naught then we know for sure that, there is a medium.

So, the deviation of epsilon from its epsilon naught therefore, let me introduce another variable this is the so called relative permittivity, which is epsilon by epsilon naught greater than 1 means medium. Actually, I should be careful it should be not equal to 1, but very soon I shall argue that, epsilon naught cannot be less than 1, it can only be greater than 1.

So, we have introduced D we have introduced E from your books you know that, there is yet another quantity namely the polarization. So, let me also introduce that quantity and what is that, we now rewrite D equal to epsilon naught E plus P, but we have already defined this to be epsilon E. Therefore, P and epsilon cannot be independent of each other. So, if you compare these two expressions, this imply that P is nothing but epsilon minus epsilon naught into E.

So, these indeed are the constitutive equations for electro dynamics or electro statics in the presence of a medium. Please notice that all the relations are linear, and I have been able to argue that, all the relations are linear by modeling my charge particles to be held by a simple harmonic force by a Hooke’s law. You people should go home and amuse yourself by putting an un harmonic term, minus k x minus k prime x cube if you did that you will get a non-linear term that in fact is the basis of what is called as a non-linear medium, which is very, very important in the presence of strong electric fields; that is something that you should certainly do. I do not have the time to get into it, but I have essentially defined all the quantities for you.

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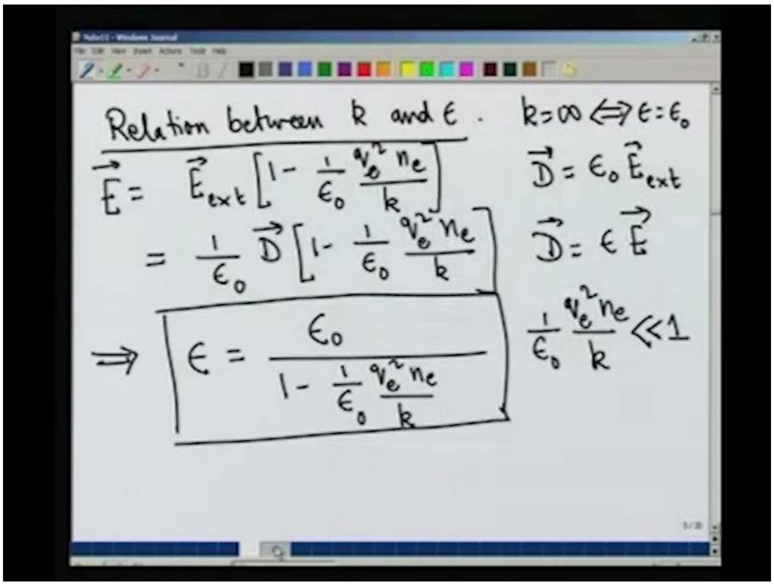

Relation between k and ϵ .

$$\vec{E} = \vec{E}_{ext}\left[1 - \frac{1}{\epsilon_0}\frac{q_e^2 n_e}{k}\right]$$

$$= \frac{1}{\epsilon_0}\vec{D}\left[1 - \frac{1}{\epsilon_0}\frac{q_e^2 n_e}{k}\right]$$

$$\Rightarrow \boxed{\epsilon = \frac{\epsilon_0}{1 - \frac{1}{\epsilon_0}\frac{q_e^2 n_e}{k}}}$$

$k = \infty \Leftrightarrow \epsilon = \epsilon_0$

$\vec{D} = \epsilon_0 \vec{E}_{ext}$

$\vec{D} = \epsilon \vec{E}$

$\frac{1}{\epsilon_0}\frac{q_e^2 n_e}{k} \ll 1$

Now, all that remains for us is to actually go back and look at our expression for E compare it with the expression for D, and write one single expression in terms of the unknown parameter. In other words, the question that I am asking is you have to write down the relation between k and epsilon. Clearly you people can see that, k equal to infinity means epsilon equal to epsilon naught. If the spring is so stiff that my charge particle does not move at all that means the system does not respond; if the system does not respond, as far as electrostatic is concerned that medium has absolutely no role to play, it is as good as free space that is the statement.

So, in order to write down a relation between k and epsilon, let me go back and look at the expression for E (Refer Slide Time: 31:18), I wrote E equal to E external into 1 minus 1 over epsilon naught q e squared n e by k, so that is what I have to do. So, E I wrote was E external into 1 minus 1 over epsilon naught q e squared n e by k. It is not easy to remember this expression, which is the reason why I went to the previous page, this is my expression.

And I wrote the relation between E external and D, let us not forget that D is nothing but epsilon naught E external. So, this is nothing but 1 over epsilon naught D into 1 minus 1 over epsilon naught q e squared n e by k. However, our defining relation between D and E is given by what, D equal to epsilon E. So, please compare the left hand side and the rights hand side look at this equation, these two together imply that, epsilon is nothing but epsilon naught divided by 1 minus 1 over epsilon naught q e square n e by k.

So, my permittivity in a medium change just by a quantity epsilon naught divided by 1 minus 1 over epsilon naught q e square n e by k. So, my permittivity in a medium changes by a quantity epsilon naught divided by 1 minus 1 by epsilon naught q e square n e by k. And as I told you, if k becomes very, very large this term becomes a very small perturbation on 1 epsilon is roughly equal to epsilon naught, this is the thing about this expression.

Now, what is the point that we want to make, this expression we have already seen is arising because of the displacement of the charge. But we have said that the displacement is very, very small that means 1 over epsilon naught q e square n e by k, which is a dimensionless quantity should be much, much less than 1.

So, let me make that statement 1 over epsilon naught q e square n e by k is obviously much, much less than 1. And whenever we have such a situation, it is good to replace the expression in the denominator by an expression in the numerator by a binomial expansion. So, let us do that.

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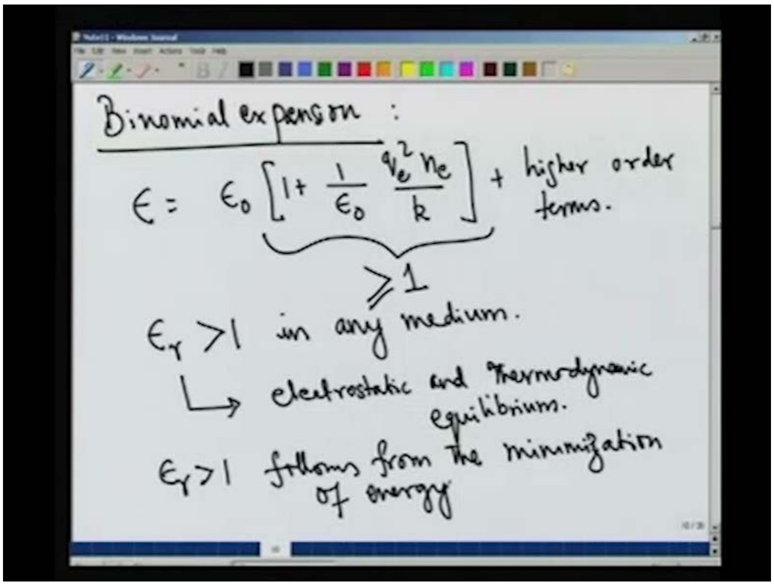

Binomial expansion :

$$\epsilon = \underbrace{\epsilon_0 \left[1 + \frac{1}{\epsilon_0} \frac{q_e^2 n_e}{k}\right]}_{\geq 1} + \text{higher order terms.}$$

$\epsilon_r > 1$ in any medium.

$\hookrightarrow$ electrostatic and thermodynamic equilibrium.

$\epsilon_r > 1$ follows from the minimization of energy

So, perform a binomial expansion perform a binomial expansion, and what does it give me, it gives me epsilon equal to epsilon naught 1 plus 1 over epsilon naught q e square n e by k plus higher order terms, which are small. How small they are, we will come to that later.

Now, you people will appreciate why I have written in this particular form, although that result will also follow from the previous expression. The important point that you have to notice is that, (( )) the bracket the parentheses is always greater than or equal to 1; it is equal to 1, if and only if q e equal to 0 that is there are no charge particles, or n e equal to 0, which is the same thing; or k is equal to infinity that is the spring is very, very stiff. Otherwise, this expression is always greater than or equal to 1 therefore, this tells me that epsilon r is greater than 1 in any media medium.

This is a result which is independent of the nature of the medium. So, long as it is a dielectric even more importantly, so long as you have an electrostatic condition that is the only assumption that we have made; it is not even depend on the details of the kind of potential that we wrote, epsilon are be greater than 1 in any medium. So, this condition is necessary and sufficient for both electrostatic and thermo dynamic equilibrium.

Please remember that we are looking at a macroscopic system. So, when I say electrostatics, I am looking it at a certain temperature at a certain pressure. And we are saying that, the system is in equilibrium, I do not produce any current is that right. So, it is electrostatic and thermodynamics equilibrium, because epsilon r greater than 1 follows from the minimization of energy minimization of energy. My system responds in such a way so as to minimize the energy in the medium.

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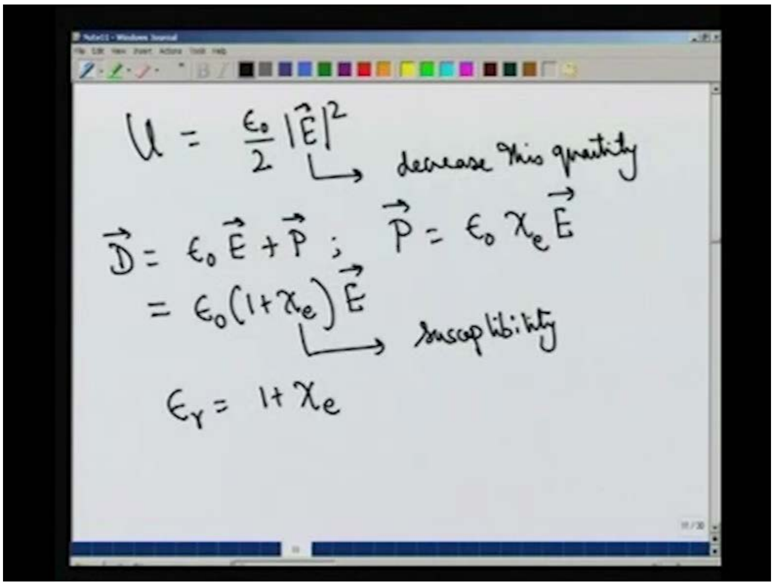

$$U = \frac{\epsilon_0}{2}|\vec{E}|^2 \xrightarrow{} \text{decrease this quantity}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \;;\quad \vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$= \epsilon_0(1+\chi_e)\vec{E}$$

$$\chi_e \xrightarrow{} \text{susceptibility}$$

$$\epsilon_r = 1+\chi_e$$

Indeed what is the expression for energy? My energy density u was nothing but epsilon naught by 2 mod E squared from the macroscopic thing. So, if you look at it, epsilon if I if I want to minimize this, I have to decrease this decrease this quantity by re arrangement of charges and that is exactly what the system is doing by produce charges of the opposite polarization.

So, now you appreciate why we have used words like polarization etcetera, etcetera. So, for completing our description of notation, let me also write one more expression which I forgot to write earlier, I wrote D is equal to epsilon naught E plus P. One also write, P equal to epsilon naught chi e E I want to separate this epsilon naught factor therefore, this will be epsilon naught 1 plus chi e into E.

So, this chi e is called susceptibility, which is positive by the way from our expression. Chi e is the susceptibility and you people can easily see that, my epsilon r is equal to 1 plus chi e. So, all that remains is to try to actually get a feeling for the value of chi for the value of epsilon r by looking at various materials that is something that I am going to do in a few seconds.

However, you know that today we know much better than what people like Maxwell knew, people did not have the atomic model of matter, people were actually trying to understand electrodynamics properties through the elastic properties of matter. So, they wanted to understand electros magnetic interactions through what Young's modulus or coefficient of linear expansion spring so on, and so forth. But today, we know it is the other way round.

We know that, the electron is held by some kind of a Hook's law, because deep down there is an electromagnetic interaction and that manifests us what some kind of a harmonic force, if I actually expand the (( )) equilibrium position. And you people have already studied a little bit of modern physics and then, the second part of this course you will know a little bit more of modern physics.

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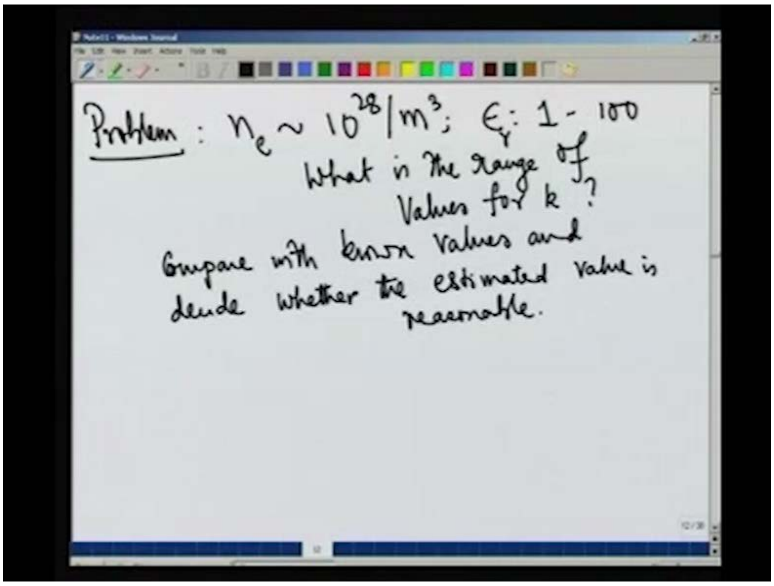

Problem : $n_e \sim 10^{28}/m^3$; ϵ_r: 1 - 100

What is the range of Values for k ?

Compare with known values and decide whether the estimated value is reasonable.

So, what I invite you just to look at the following problem before we continue. So, what is the problem let me phrase it now, so this is the first problem for you to solve. So, I want to make only an order of magnitude estimate, take n e to be of the order of 10 to the power of 28 per meter cubed, we know the value of what? The charge of the electron, we know the value of epsilon naught.

So, you ask yourself that, if my epsilons vary from 1 to 100 what is the range of values for k? Obviously, if epsilon equal to 1, k equal to infinity and it starts from k equal to infinity and comes down. And you should ask yourself, whether such a value of k is reasonable that is the whole thing you know a little bit of modern physics; you know for example, that the binding energy of the electron is what 13.6 electron volts. If you take other atoms not hydrogen, but let us say beryllium or even calcium, the binding energy becomes smaller. So, let us say it is of order of an electron volt.

And if you go and ask your chemistry instructor, who studies vibrational modes, they will tell you what the vibrational modes are. What is the binding energy for the charges, which have a vibrational mode, and that is what we are looking at; compare the number that you are going to get, and compare with whatever your instructor or book gives you, and they should tell you that whether the model that we have built is robust or not. If we do not do that then, all this will be only up in thin air that is not of much of use.

So, what is it that you have to do? Compare with known values if you do not know where to look for, ask your teacher and decide whether the value you get the estimated value is reasonable or not. So, this is the way one touch tests various models and I think, it is a worthwhile example. Now, I made a statement that you should look at epsilon in the range 1 equal to 100; obviously, when a write epsilon I mean what, the relative permittivity we are not interested in the free space permittivity, that is simply a number coming because of a peculiar choice of the units the great S I units.

So, we are I said that it goes in the region, where range 1 to 100. So, we might as well ask ourselves what indeed is the range of the values of epsilon for real materials. So, let us look at a table and let us compare all this numbers it is a fairly long table, but I am going to concentrate on a few of the well known materials. And perhaps, you will get an idea of why I gave you the range epsilon equal to 1 to 100. So, let us do that.

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Material	Min.	Max.
Air	1	1
Amber	2.6	2.7
Asbestos fiber	3.1	4.8
Bakelite	5	22
Barium Titanate	100	1250
Beeswax	2.4	2.8
Cambric	4	4
Carbon Tetrachloride	2.17	2.17
Celluloid	4	4
Cellulose Acetate	2.9	4.5
Durite	4.7	5.1
Ebonite	2.7	2.7
Epoxy Resin	3.4	3.7

This table is not a very precise table as it is very clear from the very first line; it says air as an epsilon equal to 1. So, it has a value which is approximately equal to 1. And in fact one should be careful it says see, these are giving values between minimum and maximum; because these are all macroscopic materials, the composition changes they had they can have different values of epsilon corresponding to for example, different temperatures or different doping etcetera, etcetera.

So, we are looking at the values of epsilon r. So, it says it has a value 1 that is really approximate, it should actually be greater than 1 slightly greater than 1. So, I should write slightly greater than 1. And you people should actually go back look up a hand book, where precise values are given and that you should compare and convince yourself what you mean by epsilon is equal to 1.

But anyway, this is not a bad statement it is not a grossly incorrect statement as far as our laboratories are concerned. Because we are going to treat air to be what a as good as a free space for almost all our electrostatic experiments. So, relative to air or free space we are now going to look at a large number of materials.

Amber is the most famous material because amber is what started the subject matter of electro statics, you see my epsilon is going to be around 2.5, 2.6, and 2.7. So, that is going to already cause considerable screening. Asbestos fiber has something of the range 3.5. And interestingly, there is this material called Barium Titanate, which has a minimum of 100. So, the strength of your electric field falls by a factor of 100, D equal to epsilon E that is what we are rating. It falls by a factor of 100 that is the minimum by which it falls whereas, it can fall by a factor of 100 cubed. You see, Barium Titanate probably can be used as an excellent material for electrostatic screening, if you cannot get a faraday cage or whatever.

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Ethyl Alcohol	6.5	25
Fiber	5	5
Formica	3.6	6
Glass	3.8	14.5
Glass Pyrex	4.6	5
Gutta Percha	2.4	2.6
Isolantite	6.1	6.1
Kevlar	3.5	4.5
Lucite	2.5	2.5
Mica	4	9
Micarta	3.2	5.5
Mycalex	7.3	9.3
Neoprene	4	6.7

So, there are other materials like for example, Beeswax, Celluloid etcetera, etcetera. So, let me go on to the next one, Alcohol has a fairly large value of epsilon 6.5 to 25; you should certainly do an experiment and see and ask your chemistry teacher as to why it has such a large number.

Mica is a favorite sample to study the electrostatic properties that has a value of 4 to 9, Glass indeed is a very, very well known dielectric medium. And Glass of course comes in many, many species. So, here we have what Glass ranging from 3.8 to 14.5. The Pyrex Glass of which you make test tubes has something like 4.6 to 5.

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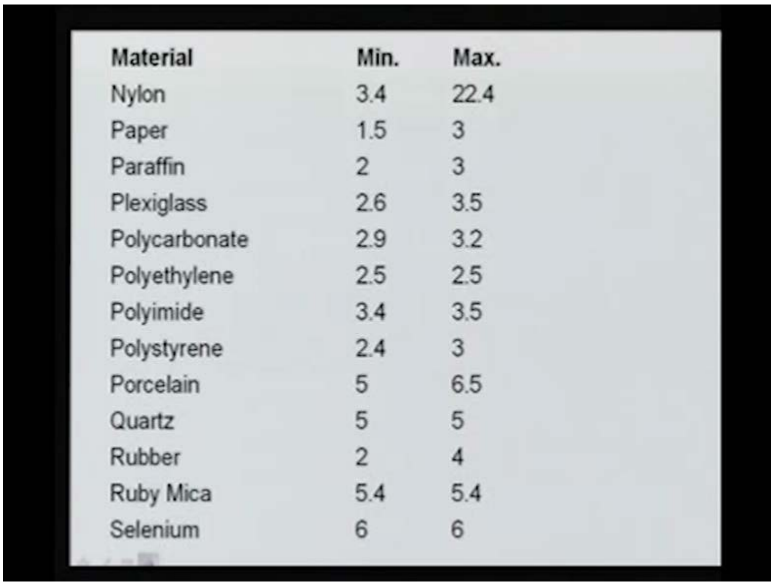

Material	Min.	Max.
Nylon	3.4	22.4
Paper	1.5	3
Paraffin	2	3
Plexiglass	2.6	3.5
Polycarbonate	2.9	3.2
Polyethylene	2.5	2.5
Polyimide	3.4	3.5
Polystyrene	2.4	3
Porcelain	5	6.5
Quartz	5	5
Rubber	2	4
Ruby Mica	5.4	5.4
Selenium	6	6

And then of course, there are certain other very, very important objects for us let me come to that.

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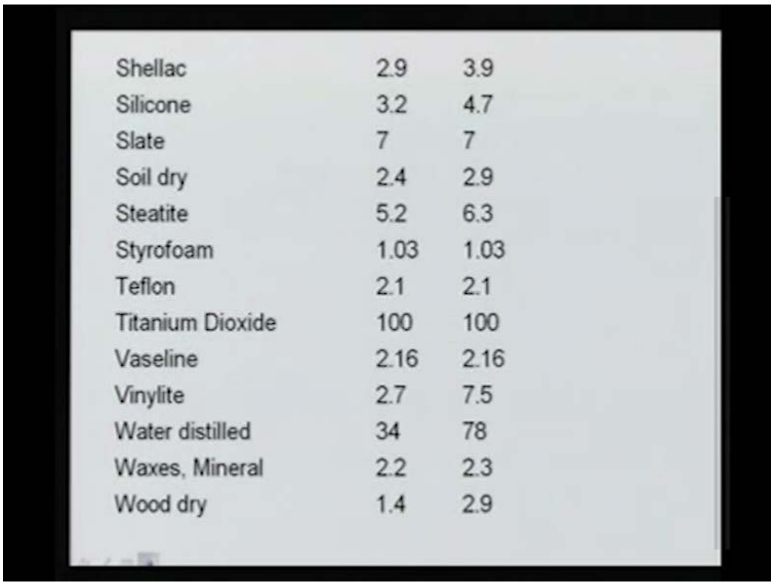

Shellac	2.9	3.9
Silicone	3.2	4.7
Slate	7	7
Soil dry	2.4	2.9
Steatite	5.2	6.3
Styrofoam	1.03	1.03
Teflon	2.1	2.1
Titanium Dioxide	100	100
Vaseline	2.16	2.16
Vinylite	2.7	7.5
Water distilled	34	78
Waxes, Mineral	2.2	2.3
Wood dry	1.4	2.9

And then come back to the other one, what is that, that is water look at water, the value being given is from 34 to 78. In fact, the common value quoted for water in most of the books is that epsilon is equal to 80; and by the way, it is indeed a very, very interesting physics problem to find out has to how water gets epsilon is equal to 80. And then there is a variety of Plastic, Synthetics whatever you see, you have Teflon, Styrofoam, Steatite and these numbers are important; because they are what parts of many, many electronic and electric components, we know that ok.

So, let me go back to the previous slide, Quartz has an epsilon of 5, Rubber has an epsilon of something like 2 to 4. And mind you, we gave a value of resistibility for of a Rubber and it was an enormously of the large number of the 10 to the power of 13. Paper, which is actually used in many, many capacitors in order to increase the capacitance, has an epsilon ranging between 1.5 and 3.

Polystyrene has between 2.4 and 3, so that should give you a fair idea; so, Barium Titanate which where the value went all the way up to 10 cubed. You see, that the values are all sort of close to 10 and occasionally going to 50 or 80 or 100, which is the reason why I asked you to estimate the value of k in the range epsilon 1 to 100.

It would be an excellent thing actually, if you could also study the chemical properties of this substance. And try to reconcile the value of epsilon, which comes from a macroscopic measurement with the value of k, which is a microscopic estimate; if you did that one can say that, you have developed a better appreciation. A better tool to find out, whether you have appreciated whatever modern physics tells us. It is in fact not necessary to do very complicated physics quantum mechanics or anything; this itself will give you a fair idea of what the relation between epsilon and k S.

Let me repeat, epsilon is a macroscopic phenomenological parameter in the absence of anything I introduce it by hand. Now, I am trying to relate it to a microscopic parameter and I am trying to tell you that, it is not difficult to understand what that microscopic parameter by actually looking at what our chemist colleagues give us. So, those of you, who are interested in pursuing for example, electrostatics electronics, semi conductor physics, even being engineers; you people should not ignore this question you should actually proceed and then try to make an estimate.

So, it may not be a big problem mathematically speaking, but certainly it is quite an interesting problem from the view point of the subject matter of making a model. So, what we shall now do, is to proceed to continue with our studies. I have given you a table of what all these materials and I have given you this problem. Now, yet another problem probably that I should write, because I do not want to write is to obtain the expression for it is a very simple thing, but you should do it; chi e in terms of what is our parameter q e squared n e by k, which has the dimension of epsilon naught.

So, obtain the expression, so we have a reasonably good understanding of what permittivity is, what susceptibility is what polarization is and what else, also to understand why materials have that particular value.

Most importantly we have argued that, the dielectrics screened the electric field partially, it always tends to decrease the energy of the carried in the electric field. And therefore, we also understand why epsilon is greater than 1. Please do not think that, this is a trivial statement, when it comes to electro dielectrics and electro static response, epsilon will always greater than 1.

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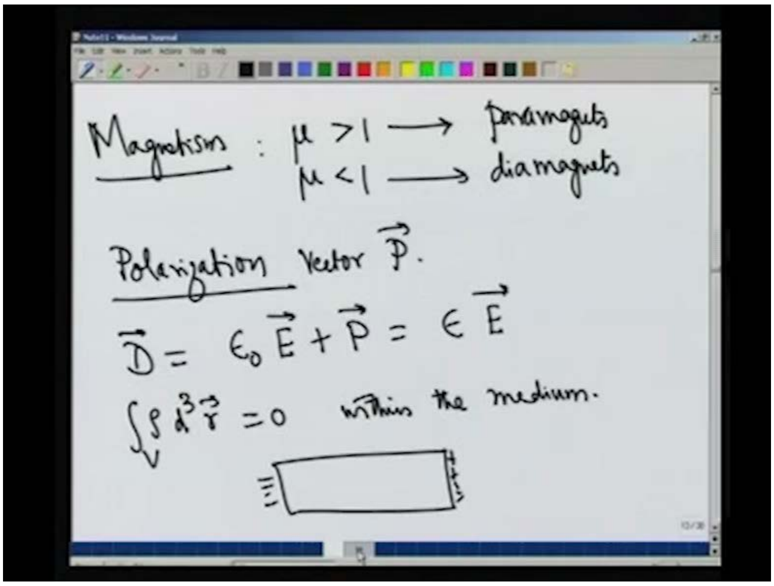

Magnetism : $\mu > 1 \longrightarrow$ paramagnets

$\mu < 1 \longrightarrow$ diamagnets

Polarization Vector $\vec{P}$.

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E}$$

$\int_V \rho \, d^3\vec{r} = 0$ within the medium.

However, you people know even from your 12 th standard that the same thing is not true in magnetism. What happens in magnetism? In magnetism, you have the corresponding magnetic permeability, mu can be greater than 1 mu can be less than 1. This correspond to paramagnets, this correspond to diamagnetism.

There are two different kinds of responses; the paramagnets tend to increase the strength of the magnetic field, diamagnets try to decrease the strength of the magnetic field; whereas, my epsilon always tends to decrease the strength of the electric field. Therefore, there is a certain inherent asymmetry between the electrostatic response and the so called magneto static response. This is very important. Because books in electro dynamics tell you that after teaching the electrostatics, they say in order to grow magnetism, you replace this by this quantity, this by this quantity, replace electric dipole moment by magnetic dipole moment, it is not that simple.

The analogy cannot be carried completely beyond a certain point. So, it is good to remember that, whatever we have argued is eminently physical and not at all trivial. Now, there is a last bit of formalism a little bit of jugglery that is remaining and that is regarding polarization, why is that? P is called the polarization vector. So, in order to appreciate this, let me return to my expression and see what the meaning of P. That again makes use of (( )) law, which is all pervasive in electrostatics, either to estimate the fields or to draw rich physical conclusions.

So, let me see what we have to do. So, let me start with this expression again, D equal to epsilon naught E plus P, which is actually equal to epsilon E, this is my expression. Now, the statement that we want to make is, whenever I put the material in a medium, it is going to polarize the system, net charge is equal to 0, but the charge density is not equal to 0.

How do I understand that, the statement that we make is that rho d cubed r is equal to 0 within the medium. That is very clear from the picture that I wrote for the dielectric medium. What did I do? I wrote a cylindrical slab and put some negative charges here, positive charges here, in an external field. What is the next thing that I can do, if the total charge is 0; try to find the total dipole moment. So, let me start evaluating that.

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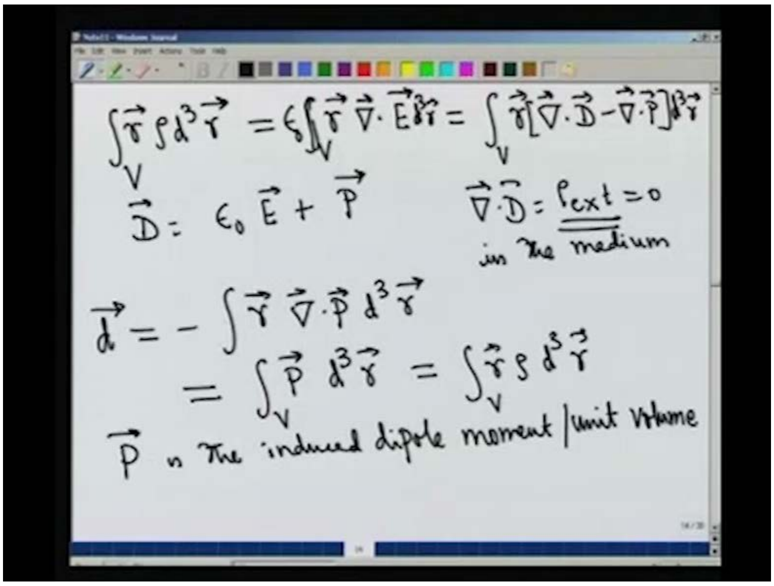

$$\int_V \vec{r}\,\rho\,d^3\vec{r} = \epsilon_0 \int_V \vec{r}\,\vec{\nabla}\cdot\vec{E}\,d^3\vec{r} = \int_V \vec{r}\,[\vec{\nabla}\cdot\vec{D} - \vec{\nabla}\cdot\vec{P}]\,d^3\vec{r}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} \qquad \vec{\nabla}\cdot\vec{D} = \underline{\underline{\rho_{ext}}} = 0$$

in the medium

$$\vec{d} = -\int \vec{r}\,\vec{\nabla}\cdot\vec{P}\,d^3\vec{r}$$

$$= \int_V \vec{P}\,d^3\vec{r} = \int_V \vec{r}\,\rho\,d^3\vec{r}$$

$\vec{P}$ is the induced dipole moment / unit volume

So, what I will now do is to look at rho r d cube r, and v refers to the volume of the medium, what is this expression? I do not know what my rho is, however I can relate rho to E. So, let me proceed slowly this is nothing but r, rho is divergence E by epsilon naught. Therefore, I will put a 1 over epsilon naught into divergence E. This is the fundamental expression for determining whatever quantity we are not going to alter this.

However, my electric field you know is dependent on (( )) to the relation let me repeat that, D is equal to epsilon naught E plus p. I think I made a mistake I am very sorry about that (Refer Slide Time: 53:15), what I should have is not 1 over epsilon naught, but epsilon naught into E, where divergence is equal to rho by epsilon naught very good.

So, if I compare the two expression epsilon naught E is nothing but D minus P. So, this I will write integral this the volume of the dielectric. So, divergence (( )) is nothing but epsilon naught is divergence D minus divergence of P, r divergence D minus divergence P d cubed r. But however, divergence D is rho external, and the rho external is placed outside the medium. So, let us not forget that, divergence D equal to rho external by definition, but I am interested in the induced charge density.

Therefore, within the medium it is equal to 0 in the medium. So, what are we left with? So, now therefore, we are left with the expression, let me introduce a symbol d for this, the total dipole moment, it is nothing but minus integral r divergence P d cubed r. Now, as I told you, I want to make use of (( )) theorem, and write this as an expression involving a surface integral plus a volume integral. I will use that as an exercise, because I have done such integrals ever so many times therefore, let me write the answers straightaway. So, I invite you to perform this integration, this will simply turn out to be P into d cubed r.

So, let me repeat this this is nothing but r rho d cubed r, where V is the volume of your dielectric medium. I will get another term corresponding to the surface integral, which vanishes. If you cannot prove that the surface integral vanishes, go back to the picture that I made with a cylindrical block of the dielectric that will provide you enough physical motivation to get the right mathematical reasoning.

Therefore, what do we conclude? We conclude that, P is the induced dipole moment per unit volume, and that explains the notation polarization. So now, we also appreciate the language called permittivity, because whenever there is a medium, epsilon is a measure of how much of electric field is permitted; actually it is an inverse measure therefore, by extending that epsilon naught is also called as the free space permittivity.

So, in this lecture, what we have done is to essentially gain an appreciation try to gain and understanding of the physical significance of the quantities like D, P, chi, which is introduced in the literature. Now, what we shall do is to take up a few applications in the next lecture.

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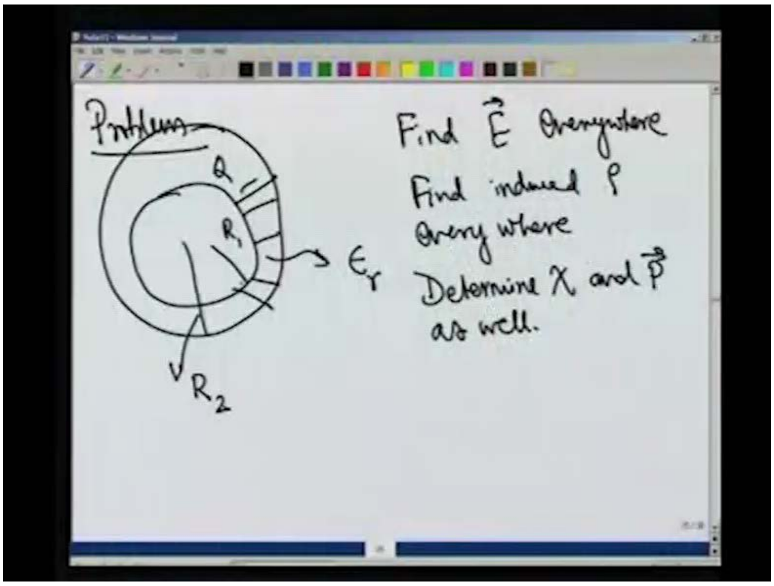
Problem

Find $\vec{E}$ everywhere

Find induced ρ everywhere

Determine χ and $\vec{P}$ as well.

However, before I do that, I want to give you one last problem to solve. It is a standard text book problem, and that is you take a conductor of radius R 1, put a charge Q on that, surround it by a concentric sphere. This is a spherical conductor; this is another spherical dielectric of radius R 2. The dielectric is in the region R 1 to R 2, this has the permittivity epsilon, the relative permittivity epsilon r. So, what is the problem? Find E everywhere, find induced rho everywhere, determine chi and P also as well.

As I told you, this is a text book problem which is found in many, many books; you have to determine the induced surface charge density both in the inner radius R 1, and outer radius R 2. Find out the direction of the E induced, find the total E that will give you some kind of a warm up for the examples that we are going to discuss in the next lecture, where I will consider as many application as possible; that will be exclusively for applications. So, we shall stop here.