Engineering Physics – II
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Module No. # 03
Lecture No. # 04

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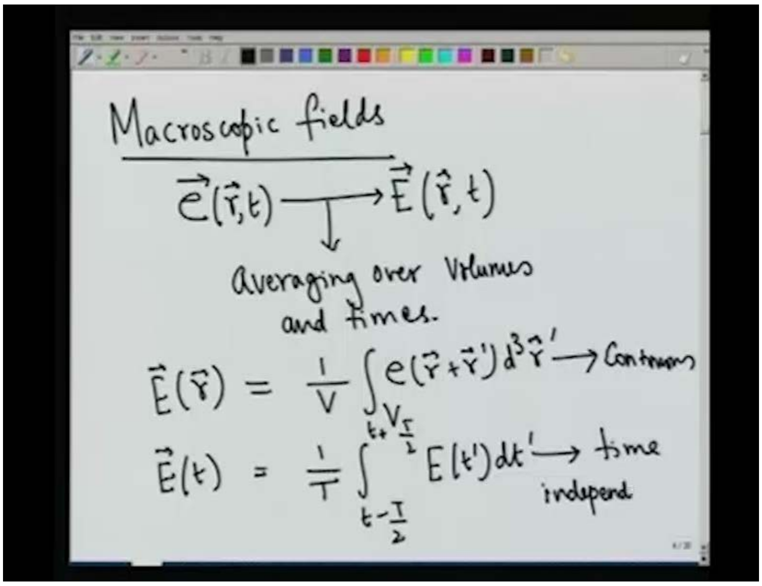

So, in the last lecture we introduced a new concept namely, macroscopic fields. So for notational convenience, I denoted the fundamental field which will produce by any charge distribution by lower case letters the so called small letters. So, I introduce the microscopic or the fundamental field I gave it the notation small e, and then I said I will make a distinction between the microscopic field and the macroscopic field. So, this of course is a function of (r,t), my capital E is also a function of (r,t), but I obtain the macroscopic field from the microscopic field by averaging by averaging over volumes and times. I spend quite some time describing how I go around averaging. So, let me repeat it shortly. If I want to calculate E of r, what I do is to replace the value of the true field at that point by the average field, the field average over a volume surrounding that particular point.

So, I will write it as 1 over V e of r plus r prime that is my radius d cubed r prime and that is over that volume - physical volume V. That is how we defined the special average. In a similar manner we can also define the temporal average, where we said that E of t will be given by 1 over T, where T is suitably chosen, so t minus T by 2 t plus T by 2 E of t prime dt prime. So, clearly what I mean by the capital E of r the macroscopic field or the macroscopic field as a function of time depends on the volume that I choose in order to average. This is entirely at my disposal and for our studies when we speak of averaging, we average over several molecular dimensions.

So, when I speak of a small physical volume that already contains sufficiently large number of molecules, such that my electric field will be independent of time, and the spatial field will be a continuous function of space. So, even if small e is discontinuous, this will be continuous and this for us will be time independent. If we read that then we are in a position to do macroscopic electrostatics also although the underlying microscopic fields are varying very rapidly in space and equally rapidly in time. Now, whenever we speak of time dependent electric fields, we imagine them to be produce by currents and they automatically produce the magnetic field.

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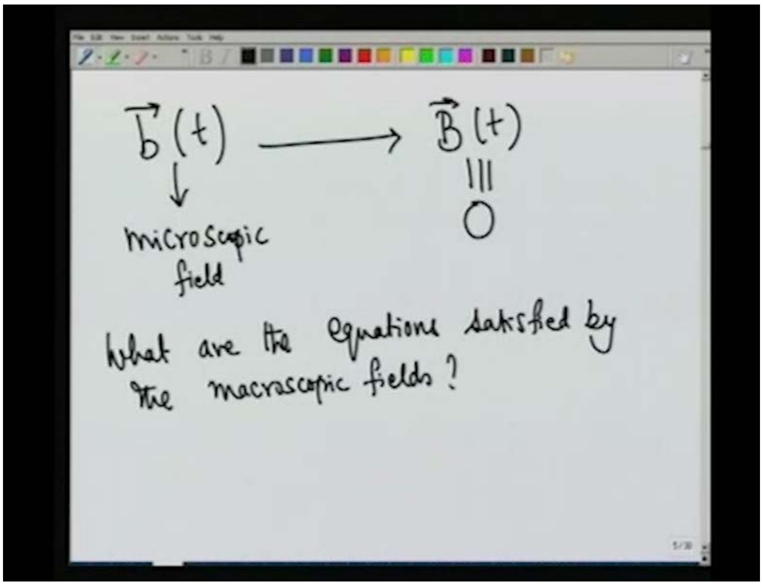

So, in a similar manner I can introduce the microscopic magnetic field. So, this is the microscopic magnetic field. This I average over several periods in order to get the macroscopic magnetic field. And as I argued in the last lecture, very similar to what I said of the time dependence in the macroscopic electric field, this can be chosen to be identically equal to 0. Please notice that the microscopic field is non-vanishing, it is a highly fluctuating, rapidly varying value function of time; at one instant it is positive, at the next instant it is negative, may be over a period of 10 to the power of minus 14 seconds or some such thing. But if I am going to coarse grain and I say that I am only able to see the mean field over let us say a micro second then the positive and the negative cancel each other, I get capital B is equal to 0. So, now it is with these macroscopic fields that we do electrostatics, and of course we will forget all about the macroscopic magnetic field.

Once we have done that the natural question that we have to ask is what indeed are the equations satisfied by the macroscopic fields. So, the question that we ask is, what are the fields equations satisfied by the macroscopic fields? This is the question that we are asking and the answer to that is simple.

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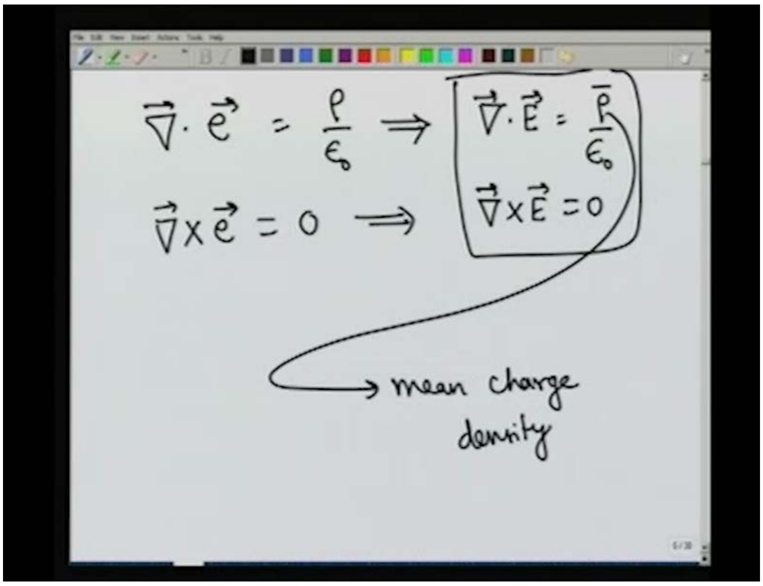

The basic equations are given by divergence e equal to rho by epsilon naught. Now, I am going to average the left hand side over space and time. I am going to average over the right hand side over space and time. This tells me that divergence e is nothing but rho bar by epsilon naught where rho bar is the average charge density.

In a similar manner, I have another important constraint equation; curl of e is equal to 0, well if the microscopic field is 0, it is taking an identical value - curl of e is identically equal to 0. This implies that curl of E also is equal to 0. So, these two conditions are inherited except that what we have to consider is not the true charged density, but the time averaged space averaged mean charge density. And (( )) ask you what this mean charge density is as to how it gets developed when I play some material in an external field or in the vicinity of a charge is a question which which is going to take our attention quite a lot in this particular lecture. So, let me proceed a little bit slowly. Now, whenever I speak of such a material in an external field, there are broadly two categories that we have to look at. So, we are interested in the material response.

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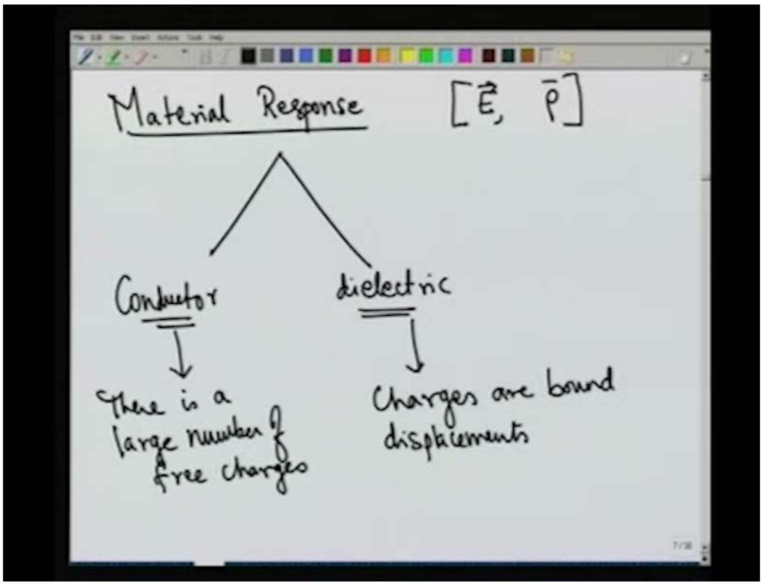

The material response is given in terms of the capital E. Let us remember. It is given in terms of capital E and the effective charge density - the mean charge density rho bar. Now, if I am interested in the response we all know from our 12th standard that we broadly categorize materials into two different sets. The first set is what is given by a conductor and the next set is what I call as a dielectric or an insulator. In fact to be sure there is a whole lot of other kinds of material like semiconductors, pyroelectrics, liquid crystals, superconductors, so on and so forth. Similarly, n response to a magnetic field, I can look at a ferromagnet, antiferromagnet then I have a ferry magnet, etcetera; paramagnet, diamagnet, etcetera, etcetera.

Here I am interested in electrostatics, therefore let me restrict myself to response to the electric field. Let us ignore for the time being all other different kinds of exotic materials. We shall try to consider only the broad class of these two materials namely, conductor and dielectric. So, how do I distinguish between a conductor and a dielectric? The text book definition is very, very straight forward. It says here there is a large number of free electrons, there is a large number of free charges to move. So, if I were to take this material and if I were to put it in an external field, there are charges which start moving and they keep on moving until they attain an equilibrium position. Only after the attain in equilibrium position that is all the charges come to rest then can we speak of the electrostatics.

What happens in the dielectric case? In the dielectric case, the charges are there, but they are not entirely free to move. Here the charges are bound. Now, the charges are bound does not mean that the system does not respond, they are allowed displacements that is the most important thing. They are allowed displacements, where they move from their original equilibrium position to a new equilibrium position.

So, what is the qualitative distinction between a conductor and a dielectric? In a conductor we assume that there is a sufficient number of charges, and these charges are free to move as much as they want. They can move all along the conductor and they can do so until they attain their equilibrium position in new minima in the presence of an external field. Whereas in the dielectric you are allowed only some small displacements, perhaps 1 or 2 molecules, perhaps a fraction of a molecular distance, but here it can go over many, many hundreds of molecules or many, many hundreds of atoms that indeed is the qualitative difference between a conductor and a dielectric.

Having said that we realize that what we are not doing is to give some kind of a mathematical definition. There are free charges here and there are no free charges there, because we know that real world is not all black and white, but there is a lot of grey area. So, I need a practical criterion in order to distinguish an insulator from a conductor. This practical criterion of course, depends on what kind of measurements I do, what kind of sensitivity I have in order to measure the conductivity or the resistance let us say. So, it is a matter not only of the definition it is also of a matter of what kind of daily situations which I encount the encounter, what kind of equipment that I have. So, keeping all that in mind; let me give now a practical criterion, a practical distinction between a conductor and a dielectric. However, before I do that I have to introduce the concept of a resistance or a resistivity. I am not going to spend too much time defining that. We all know what it is.

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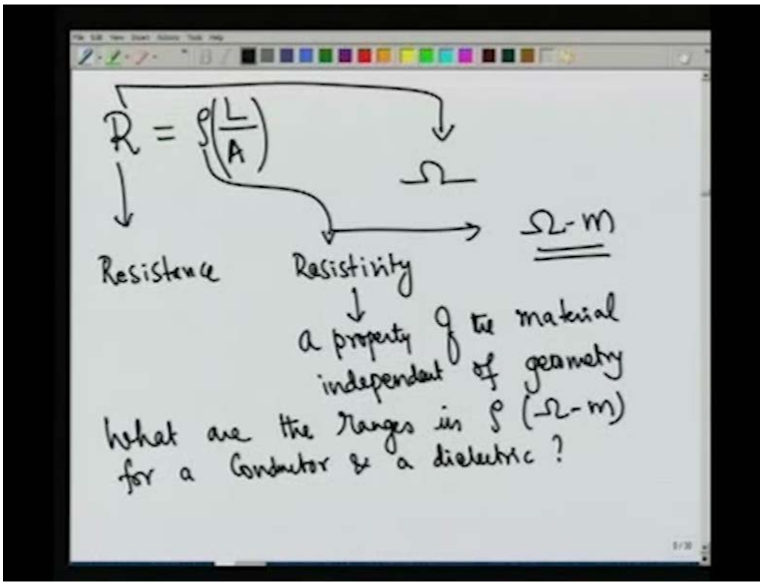

So, the statement that I want to make is that if I am going to define the resistance of a material, you may try to imagine that my dielectric has an infinite resistance, whereas my conductor has very little resistance very close to 0, so let me quantify that. We all know that the resistance is related to resistivity by the relation rho L by A. So, this is my resistance, this is my resistivity, this is the length of the object over which I would like to measure the current and A is simply the area of the cross section we know that. Now, that means what I call as a resistance is actually a property of the material and also the geometry. The length that it current has to traverse and the area of the cross section; whereas this is a property entirely of the material - a property of the material independent of geometry.

So, everyone knows that resistance is measured in the units of ohms. So, this is measured in unit of ohms, L by A is nothing but 1 over length, therefore my resistance is measured in unit of ohms-meter. Old fashion people sometimes use ohm-centimeter, but SI unit is what we are all time going to use, it is measured in the unit of ohms-meter. Now, I ask myself, suppose I have a multi-meter and I start measuring resistances, when shall I say that I have made a cross over from a conductor to a dielectric? That is the question that we are asking. So in other words, what are the ranges in rho in units of ohm-meters for a conductor and the dielectric? That is the question. So, let us answer this question.

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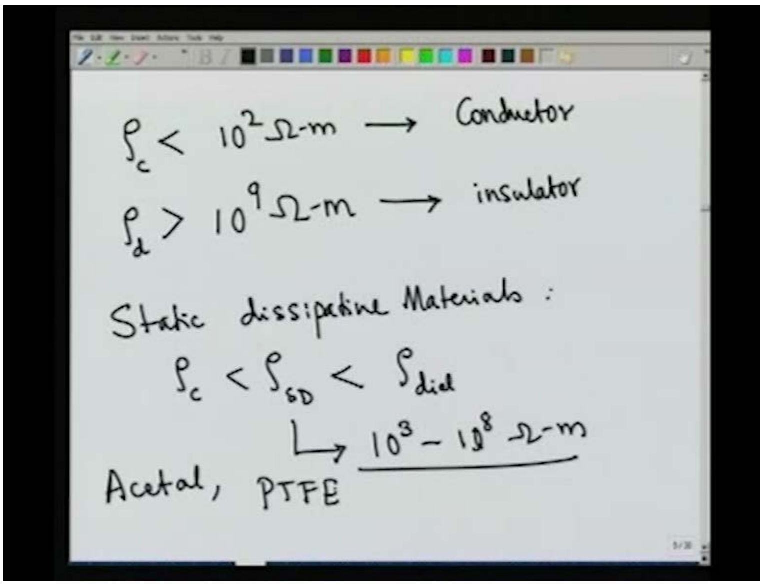

As I told you this is entirely a matter of criterion, and by convention one says that if the resistivity of the material is less than 10 to the power of 2 ohm-meters then this is a conductor. So, we are saying that if the resistivity is less than 10 to the power of 2 ohm-meters then this is a conductor. Now, what about a dielectric? Here I am putting an upper limit on rho. If my rho exceeds 10 to the power of 2 ohm-meters, be sure that it will not be called as a conductor.

Now, for an insulator or a dielectric, I am going to put a lower limit. So, I am going to say that rho should be greater than a certain number and that the convention is 10 to the power of 9 ohm-meters. So, if the resistivity is greater than 10 to the power of 9 ohm-meters then this is called an insulator. So, we see that we are jumping by 7 orders of magnitude in order to make a distinction between what I would call as a conductor and what I would call as an insulator, and over this 7 decades, factor of 7 on the logarithmic scale, there is a grey area and we have to ask ourselves what about the materials which fall in to this particular category.

Now, the answer to that is quite interesting. What the engineers do is to introduce another set of materials, another category and these are called static dissipation materials dissipative materials. So, this is for the conductor; this is for the dielectric. What is the range of the resistivity for these static dissipative materials? So, obviously rho c is less than rho static dissipative materials less than rho dielectric or insulator, and this rho is the lies in something like 10 to the power of 3 to 10 to the power of 8 ohm-meters roughly.

So, you see that the real world consists not just of conductors and insulators, we have these static dissipative materials, and what is the property of these static dissipative materials? The charges are free to move, but not over large macroscopic scales. But they are also not as confined as bold as the charges in the dielectric, and these are the materials which are made of a variety of objects. So, if you go and look up the books or ask your electrical engineering professor in your college, he will tell you that some of the materials are what are called an as acetol, this is one of the material thought of plastics or organic materials. And then you have a substance called poly uretharimide, then you have what are called as PTFE's, is that ok, poly tetra fluoro urethane etcetera, etcetera, PTFE, all these materials are what are called static dissipative materials.

Whereas, of course we know the examples of conductors; aluminum, copper, silver is the best conductor. We know the examples of the insulators. What are they? You have wood, you have water, etcetera, etcetera. So, what we have done is to give a practical criterion, a quantitative criterion to improve upon our qualitative notion of a good conductor or a dielectric or whatever and this is indeed what this particular slide is going to tell you.

Now, what I shall do is to actually exhibit for you people, two tables which gives you the idea of what kind of resistivity of the materials is. Because otherwise, mean although I gave you some examples, I did not give you the precise resistivity's and that will give you an idea of what is a good conductor, what is a bad conductor. We speak of weakly dielectrics, we speak of very good conductor, we speak of very bad conductor, so let us get a feeling for that by looking at the following table.

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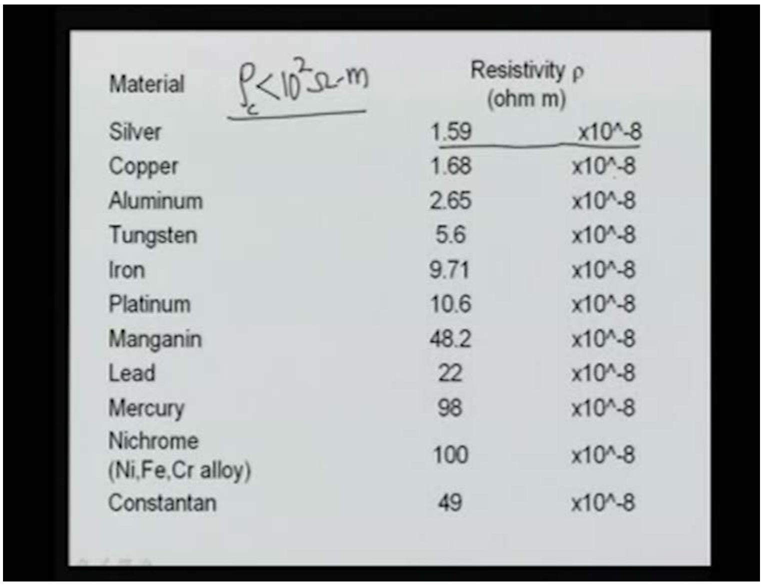

Material	Resistivity ρ (ohm m)	
Silver	1.59	x10^-8
Copper	1.68	x10^-8
Aluminum	2.65	x10^-8
Tungsten	5.6	x10^-8
Iron	9.71	x10^-8
Platinum	10.6	x10^-8
Manganin	48.2	x10^-8
Lead	22	x10^-8
Mercury	98	x10^-8
Nichrome (Ni,Fe,Cr alloy)	100	x10^-8
Constantan	49	x10^-8

Now, in order to do that let me start with this particular table, I think I will make it full screen. Now, look at this particular table. I said that we shall agree to call a substance to be a conductor provided my rho was what less than 10 to the power of 2 ohm-meter. This is the criterion that I gave for a conductor - rho conductor. Now, look at the value of conductivity for silver. It is turning out to be 1.59 into 10 to the power of minus 8. In other words, the resistivity of silver which is an excellent conductor is 10 to the power of 10 times, 10 to the power of 9 to 10 to the power of times or 10 times smaller than the threshold conductor.

In a similar manner, copper is very close to silver, aluminum is close to very silver which is the reason why we use copper or aluminum wires for our transmission. And of course, as you keep coming further and further down, you find that there are alloys, you have mercury etcetera, they are all in the 10 to the power of minus 8 range and they are going to give us this good conductors. This is the category of good conductors; in fact all of them are excellent conductors.

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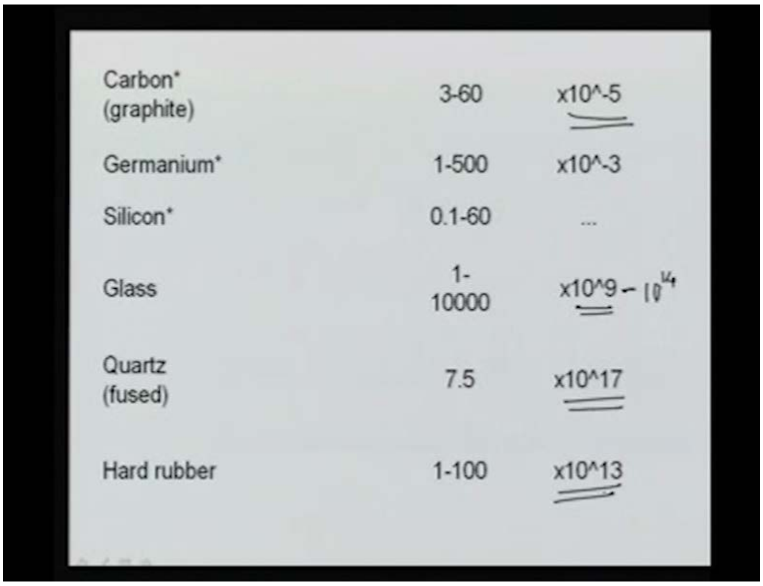

Now, as I keep going down further down the table. Now, you see that I have reached carbon which is 10 to the power of minus 5. It is still 7 orders of magnitude less than the threshold, so it is a conductor. But it is certainly not an excellent conductor, because the losses will be more. Come to germanium then it is 10 to the power of minus 3. Come to silicon it has gone up to 0.10 and 0.6; it is already hovering around the border line between a conductor and a non conductor. I cannot call it as an insulator. And then let us take a big jump, let us go to glass. Glass is of the order of 10 to the power of 9, no it is not that. Depending on the nature of the glass, it can go from 10 to the power of 9 to I have 1, 2, 3, 4, 5, 10 to the power of 14 or 10 to the power of 15 ohm-meters. You see this is indeed a very, very bad object as for as conductivity is concern, we can only call it as an insulator.

In a similar manner quartz is an excellent insulator, because it is of the 10 to the power 17. And rubber which is used for insulation everywhere that is also a very, very bad conductor. In fact it cannot be called as a conductor at all, because the resistivity is of the order of 10 to the power of 13, whereas there I placed an upper limit of something like 10 to the power of 7. I am not going to show you the table of all these smart materials, whatever I was telling you PFTN things like that, you may look them up the table.

So, the basic idea that we are trying to say is that was the resistivity keeps on increasing, we make a smooth transition from the so called good conductors to poor conductors, from poor conductors to static dissipative materials, and from static dissipative materials to insulators, and each of them has a role to play in technology. That we will discuss after I have introduced dielectrics and things like that. But right now we are all rather fine ground when we speak of a good conductor or whatever. So, we just now saw a table of different materials and their resistivity's which displayed for us values ranging from something like 10 to the power of minus 8 to 10 to the power of 17. So, we went through from very, very good conductors to very, very good insulators.

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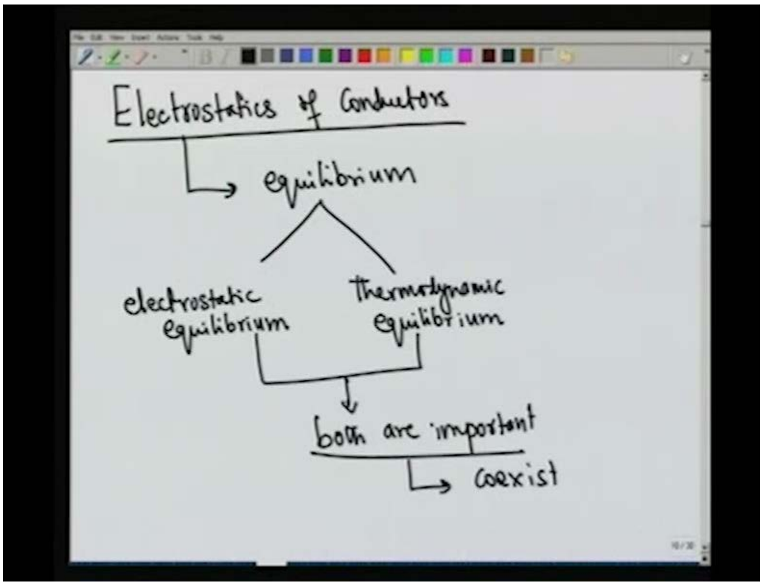

Now, what I shall do is to concentrate on the electrostatics of conductors. So, the rest of this lecture and perhaps a part of the next lecture will be on the electrostatics of conductors. So obviously, when I speak of electrostatics of conductors I will have in my mind a very good conductor. We are going to assume that charges are free to move, because the resistivity is very, very small. So, they keep on moving until they neutralize the external field, and we want to study what are indeed the properties. Now, every time when we speak of a conductor, we can imagine that if I apply an electric field the currents starts moving, and there is a dissipation etcetera, etcetera. That is not what I am interested in. I am interested in the electrostatics that is when there is equilibrium.

Now, this equilibrium is of two kinds; one is what I will call as electrostatic equilibrium and on the other hand, since I am dealing with macroscopic bodies with macroscopic motion, there is also thermodynamic equilibrium equilibrium. For our studies, we have to keep in mind, we should never forget that both are important both are important for us and both coexist. For macroscopic bodies, it is impossible to have electrostatic equilibrium. What does electrostatic equilibrium tell us? It tells us that the net force on the charge vanishes therefore, they are all at rest.

Thermodynamic equilibrium tells us that it is going to be stable, is that ok? So, both of them coexist for us, so it is going to be a stable thermodynamic equilibrium, stable mechanical or electrostatic equilibrium. So, we want to study how indeed conductors respond to each other, how indeed charges distribute themselves on the surfaces of the conductors, how the charges in the conductors respond to external electric field. This is what we want to do and let us do that systematically step by step.

Now, there are two approaches for us in order to study the electrostatics of conductors; one is to write down the electrostatic equation and look upon it as a boundary value problem, another is to remember what I said just now, treat it also as a problem of thermodynamics. At the minute there is a thermodynamics, in thermodynamical equilibrium we can took over a large number of those results and use them, and how do I know that there is thermodynamic equilibrium? I know that by experience.

So, what we shall do is to not merely appeal to Maxwell's equations that we have written; divergence is equal to rho by epsilon naught and curl E equal to 0. We shall also appeal to our experience. We shall formulate all the properties of the conductors, and then go back and reconcile, and show how the thermodynamic equilibrium is compatible with electrostatic equilibrium which actually follows from the Maxwell's equations. So that way we will not lose sight of the physics that we want to do, and we will not get lost in the mathematical jungle of what, the boundary value problems. So, I would like to avoid that.

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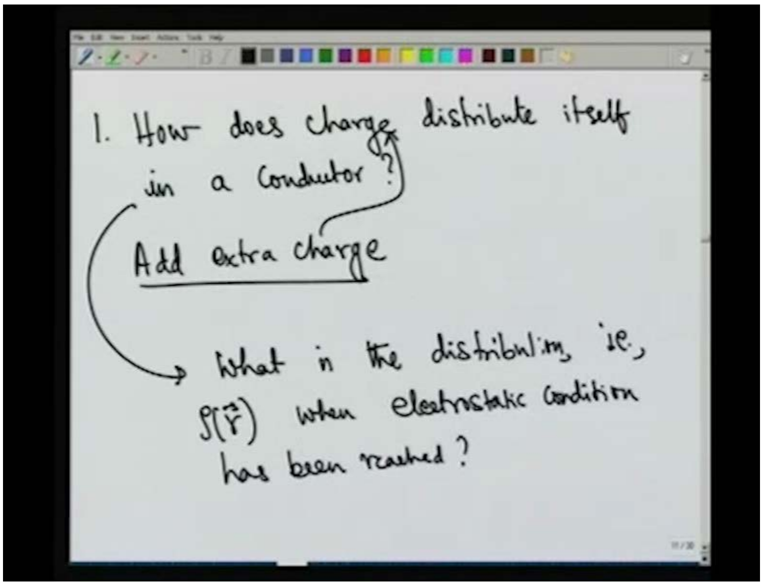

Now, what I shall first do is to look at charges on the conductors. So, there are many, many aspects that I am going to consider. So, the first question that I am going to ask is, how does charge distribute itself in a conductor? That is the first question that we shall ask. In asking this question, we are not going to ask how long it takes for the charge to distribute itself. That is beyond us. We only want to know what the equilibrium electrostatic condition is. Now, what are the charges that we are interested in? Obviously, my conductor has lot of charges, but the conductor is over all neutral. And what is the neutrality? If I look at any small space - physically small space, but large compared to molecular volumes, there is an equal amount of positive and negative charge. Therefore, not only is it all over all neutral, rho equal to 0.

So, the first statement that we want to make is that if you do not add any extra charge rho equal to 0 everywhere. Now, what we shall do is to add add extra charge. This is what I am interested in when will speaking of the charge distribution. So, I am going to add an extra charge. How do I add an add an extra charge? Well the same old thing, rub, let us say (( )) against Silicon or whatever, it gets charged, bring it in contact with a conductor and all the charge gets transformed to a transferred to a conductor and we are asking how does this excess charge distributes itself in a conductor. We are not putting the conductor in any external field. The external field is all 0. It is simply as if I have bodily picked up some charge and placed it on a conductor, and I am asking, what is the charge distribution?

Now, the first thing that we have to notice is that the charge is going to move until it attains an equilibrium position. So, we are asking, how does charge distribute itself in a conductor? So, let me make it more precise. What is the distribution that is that is the charge density rho of r when electrostatic condition has been reached? That is the question that we are asking has been reached. By the way it takes pretty little time of the order of 10 to the power of minus 15 to 10 to the power of minus 14 seconds. So, please do not worry about, how long it will take and all that for good conductors that is what is going to be. We are asking what the charge distribution is. Now, the answer to that curiously comes from what we knows to be known to be the ohms law. Let me remind you of what ohms law is an ohms law does not refer to electrostatics.

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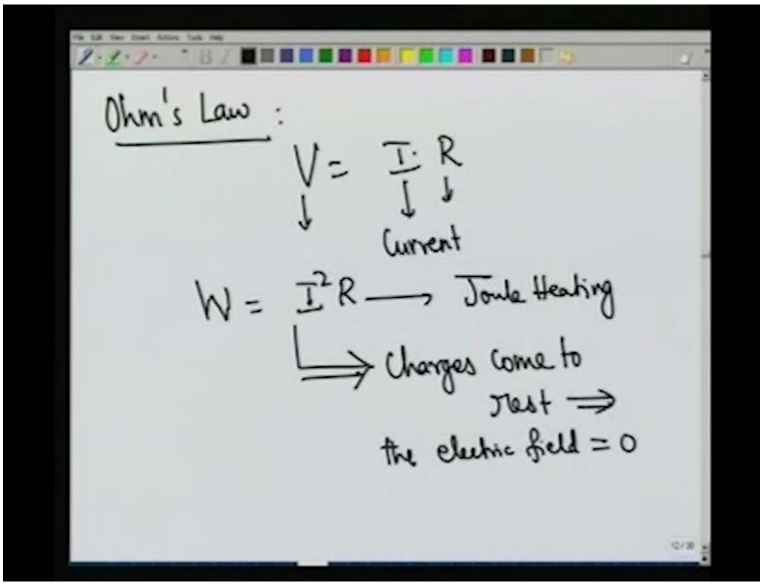

However, let us remember what ohms law is and then come back to electrostatics. Ohms law tells us that if there is an electric field there is a current. So, it tells that V equal to IR. Create a potential difference at the two ends of the conductor, this is my potential, and given the resistivity there is a current. That is what we know. And a current does not refer to electrostatic situation, ohms law does not tell me just that much, it also tells me that the power dissipated is given by I squared R. So, imagine that there is a potential difference, imagine that there is an electric field in the conductor, what would happen? Mister Ohm comes and tells us have you produced an electric field. In that case the charges shall move given by the formula V equal to I R.

Now, the minute charges start moving, Joule comes and tells us there is a corresponding Joule heating. So, this is the famous Joule heating. All resistances emit heat, is that right, because of the friction and because of the Joule heating you dissipate power at the rate I squared R, and where does the energy come from. The energy has to necessarily come from the kinetic energy of the charges. This means that unless I keep on continuously supplying energy from outside, like it happens when I connect the two ends of a wire to a battery, eventually the charge comes to rest. This means that charges come to rest.

Now, let me go back. Because the charge has come to rest, there is no current. And because there is no current, resistance of course is always there it is a property of the material and the geometry of the material, my V is equal to 0, and what is my V? The potential difference is nothing but the value of the electric field multiplied by the distance over which we are looking at sorry divided by the distance. Therefore, this implies the electric field is 0.

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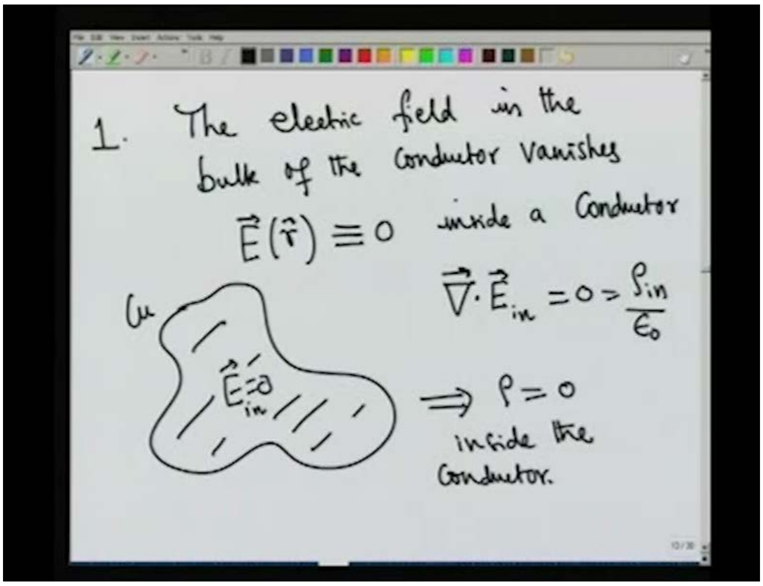

So, let me summarize that by making the statement as a statement of principle. So, the first statement that I want to make is that the electric field in the bulk of the conductor vanishes, E of r identically equal to 0 inside a conductor. So, let me show it in the following manner. Imagine this to be a lump of copper, imagine this to be a lump of copper or silver, in (( )) I put some extra charge on this, if I charge this my E equal to 0 in the interior, in the bulk of the conductor, in the meat of the conductor as some Americans would like to put it.

However, we know that Maxwell's equations are identically satisfied everywhere. Whether it is in the interior of the conductor or in outside the conductor, so we now conclude that divergence E inside the conductor is equal to 0. But what does Maxwell's equation tell us which we have derived the electrostatic equation. This is rho in by epsilon naught, what is my rho in? That is the volume charge density in the conductor, so this implies rho equal to 0 inside the conductor. So, by appealing to Ohms law which would otherwise have caused dissipation and because of the fact that we know that there is an electrostatic situation. There is an electrostatic equilibrium that there is a thermodynamic equilibrium. We have now been able to draw a very, very powerful, a very, very important conclusion namely, if I put some charge, if I charge a conductor then the charge does not reside in the bulk of the conductor. So, where does the charge reside?

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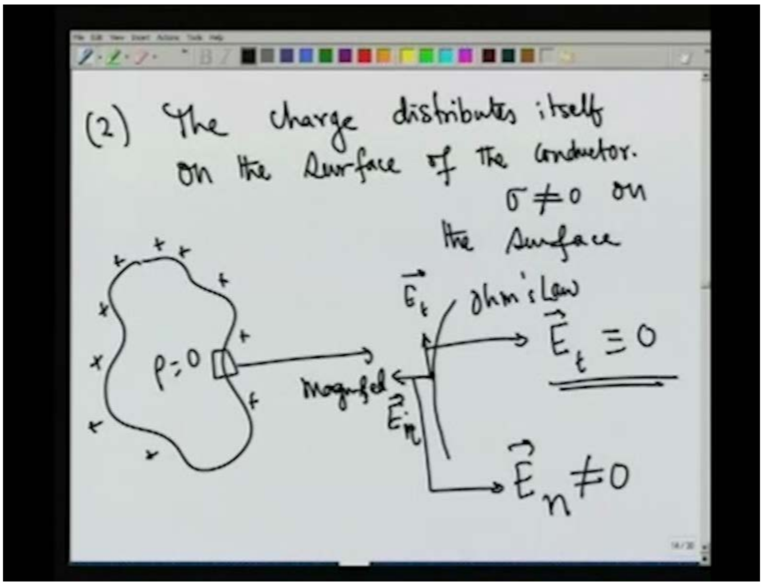

So, the next statement that we can make is as a corollary the charge distributes itself on the surface of the conductor on the surface of the conductor. Now, the next question is, how does the charge distribute itself on the surface of the conductor? So, we are saying that sigma not equal to 0 on the surface. So, if I were to draw a conductor somewhere here, there are lot of charges that are going to sit something like this, imagine this. If I have charged it positively, they are going to distribute themselves and inside rho equal to 0. Now, these charges are of course going to produce an electric field, so let me take this particular surface element and let me magnify that... So, let us say it looks like this. So, I have taken the surface element and I have magnified this and I am saying that it looks like this. It is not faithful to the original figure. So, let me change the direction the curvature. So, let us say that it looks like this, magnified.

Now, clearly at this particular point I can resolve my electric field as a normal component. So, this is my normal component and this will be my tangential component. I can always resolve my electric field in to normal and tangential component. The normal component brings you out of the conductor into the outside world, whereas the tangential component keeps you in a plane. There is a certain non-vanishing surface charge density that non-vanishing surface charge density necessarily produces an electric field. That is what we know. Charges are the sources for the electric field. And now we are asking the crucial question as to what is the contribution of the surface density to the normal component and to the tangential component? By the same token, by the same argument of the ohms law, so let me write it here, Ohms law dictates plus electrostatic dictates that E of t identically equal to 0. I am not going to write it down, but we let me just repeat it in words.

If there were a tangential component of the electric field, the charges was started moving. Because the charges are free to move in a conductor right. They can travel arbitrarily large distances. Once they start moving there would be a dissipation corresponding to I squared R, so they would quickly move, they would lose all their energy, they would find their new minima and at that point E t cannot be equal to non-zero, therefore we conclude that the tangential component of the electric field is equal to 0 and only the normal component only the normal component is non-zero; E n not equal to 0.

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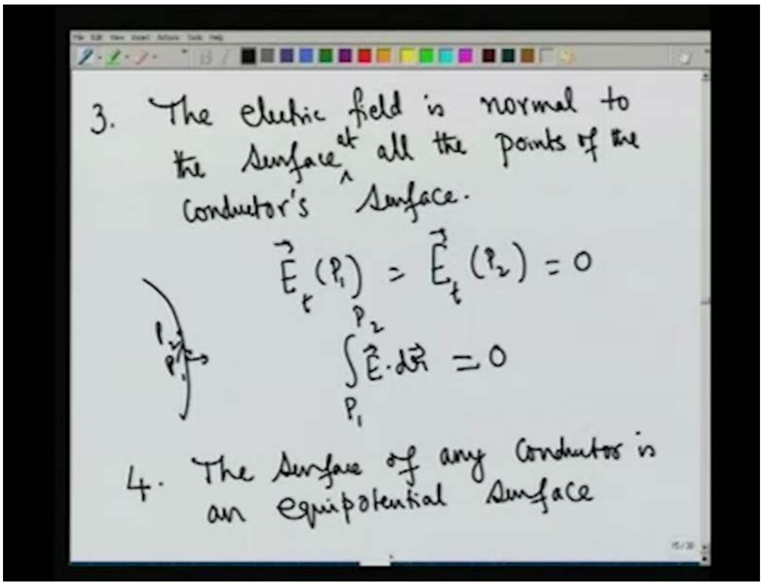

So, this is again another very, very powerful conclusion from exceedingly simple considerations, what are we say that is the next statement that we want to make. The electric field is normal to the surface at all the points at all the points on the conductor of the conductor. I mean the surface of the conductor - conductor's surface. Now, if there is no tangential electric field and suppose somebody moved the electric the charge particles from this point to this point, the electric field was 0 here, the electric field was 0 here, so let me call this p 1, this is p 2. So, E t at p 1 is equal to E t - the tangential component at p 2 is equal to 0. Therefore, integral p 1 to p 2 E dot d r, and the d r is along this particular line along the surface, this is equal to 0, and what does it tell us? This tells us a statement which we can again write as an important point that the surface of a conductor of any conductor is an equipotential surface. This is indeed remarkable. What we are saying is that?

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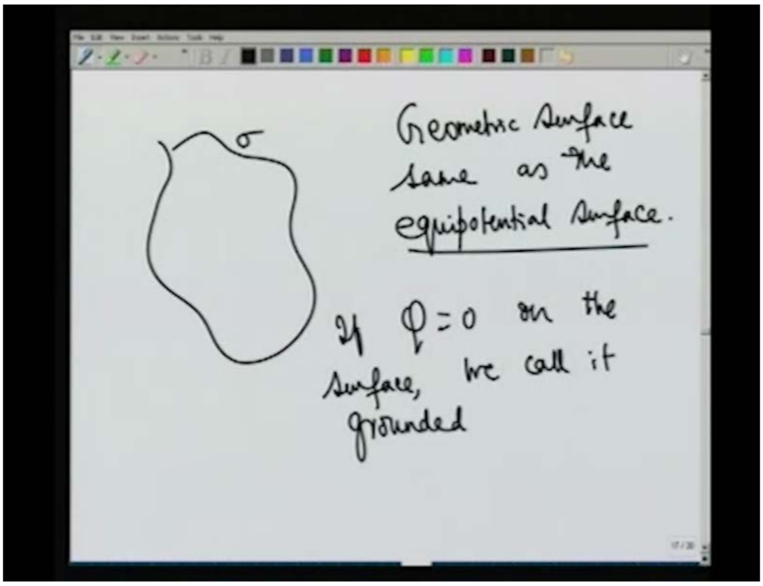

If I give you a conductor like this, and if I place a charge on it, sigma is such that sigma is such that the geometric surface and the electric equipotential surface coincide with each other. The geometric surface same as the equipotential surface. Those of you who are mathematically inclined will immediately realize, what the import of the statement is, how you can exploit this particular statement, all that you will do is forget all about the conductor and if you want to solve Maxwell's equations you will say that on this particular surface my potential have this particular value.

Please mind that we are not going to assert that this potential is equal to 0, because as I have already told you the potential is defined to be 0 at infinity. Therefore, unless I connect it to the point at infinity or if I chose one of the conductors to be at 0 potential, unless I connect the another conductor to this conductor by a wire or some such something, this will be at some other value of the potential, is that ok? So, what we are asserting is that it is an equipotential surface; we are not saying that the value of the potential is 0. If the value of the potential is equal to 0, we say that such a conductor is grounded. So, we have a precise mathematical notion.

If phi equal to 0 on the surface, we call it grounded, because in all electrostatic applications earth is taken to be at 0 potential. It is taken to be of an infinite extent, it is taken to be at 0 potential. So, if phi equal to 0 that means you have earthed it, you have grounded it, we say that it is grounded. It is taken to be at 0 potential otherwise this is what we have.

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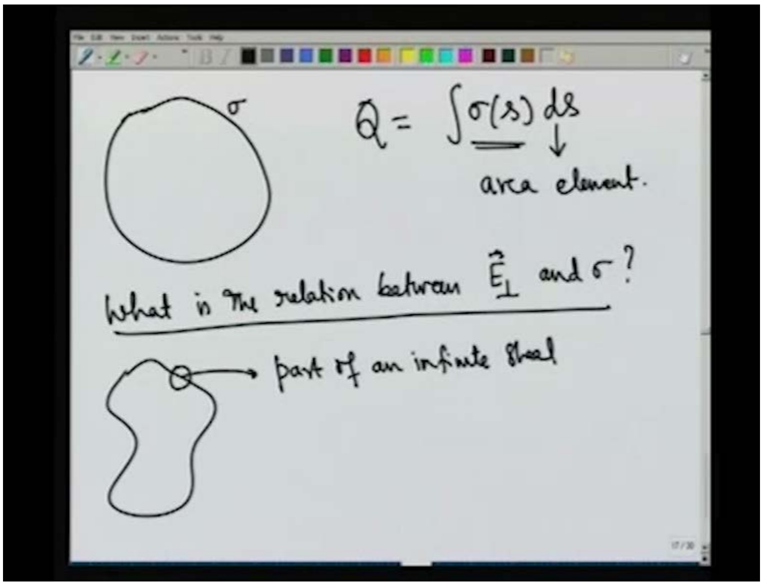

So, because all the charge resides on the surface of a conductor, so let me show it here. So, there is a certain sigma. All the charge resides on the surface of the conductor; the total charge Q is nothing but sigma of s ds which is the area element. Now, clearly the perpendicular component of the electric field at any point should depend on sigma.

So, the next question that we are going to ask is what is the relation between E perpendicular and sigma? That is the question. Now, in order to answer that what we shall do is to consider a very, very small area. So, let me imagine that this is my conductor. So, again I am going to look at the small area, I am going to magnify. Without any loss of generality I can imagine that this is a part of a sphere. I can approximate it by a certain disc of a sphere. And we know or even better I can do something better than that. So, let me change. So, I can take it to be the part of an infinite sheet. Let me explain myself. I am interested in the value of the electric field just outside the surface. So, let me draw another figure and let me explain to you before we get confused.

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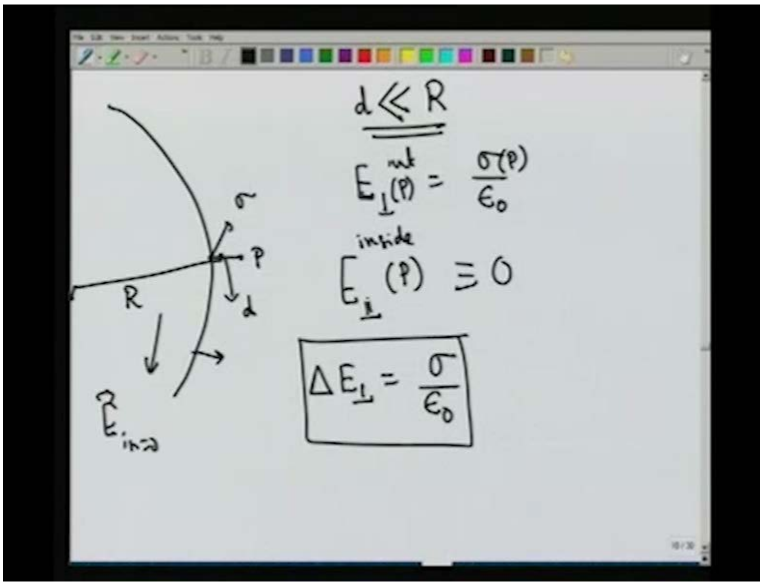

So, I have a surface like this; I have magnified my surface. And I am interested in the value of the field at this particular point. So, let us say that I have magnified little bit more. So, let me say that this is the point at which I want the field. And let us say that the distance of the point from the surface is d. The statement that I want to make is, at this point this surface has a certain radius of curvature. So, let me denote it like this, certain radius of curvature. We are saying that d is much, much less than R. That is the statement that we want to make.

If d is much, much less than R, if I bring my electroscope, if I bring measuring apparatus, it will never sense the curvature in this picture. It will see a straight wall therefore it was as if you are trying to measure the field due to an effectively infinite sheet of charge. So, at this particular point, if my value is sigma, what is the value of the electric field? Please remember, my electric field here E in is equal to 0, and here my electric field is perpendicular. I can happily make use of the Gauss's law and write down the value for the electric field, and what is that? E perpendicular out will be nothing but sigma by epsilon naught. All of us know how to find out the electric field due to an infinite sheet of charge. So that is exactly the approximation that I am making. I am saying that I am insensitive to this curvature.

If I want to see this curvature in this equipotential surface or the physical surface of the conductor, I should go to distances comparable to that. But this distance is very, very small, therefore I will say that E perpendicular outside is nothing but sigma of epsilon naught. If this is at a point p, this is also indeed at a point p. So, you see that there is a discontinuity in the electric field. How does the electric field behave? My E perpendicular inside at any point p identically equal to 0, my E perpendicular outside is given by the sigma p by epsilon naught. Since I have put the perpendicular, I do not have to put the vector direction here. So, let me erase that and let me rewrite this particular part.

So, the perpendicular component outside, just outside is proportional to sigma of p by epsilon naught, inside it is equal to 0. Therefore, the discontinuity in the electric field signifies the existence of what, a discontinuity in the charge distribution namely, that there is a certain charge distribution which is spread along that particular surface. Therefore, you can write that delta E perpendicular is nothing but sigma by epsilon naught. This indeed is a very famous statement. So, the discontinuity in the perpendicular component of the electric field gives you the sigma or conversely the sigma at that particular point will give you the discontinuity in the electric field. Of course in this particular case, all the contribution is coming from E out, because E inside is identically equal to 0.

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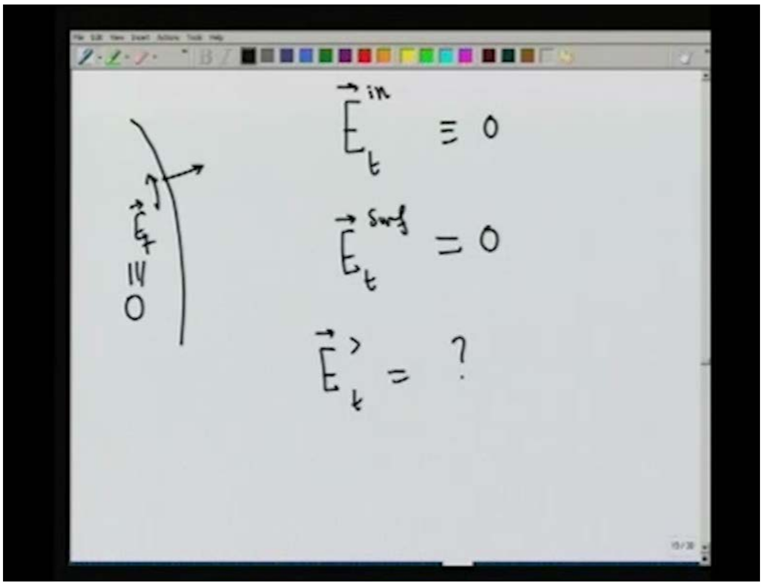

Now, comes the question of the tangential component of the electric field and let us proceed very slowly. Now, what is the statement that we are making? We know that within the bulk my E t is equal to 0. I know that. And I know that at the surface my E is perpendicular. However, the question that we are asking is, is my E t going to be continuous or just outside it is possible that it can actually pick up a non-zero value.

If my perpendicular component of the electric field could pick up a non-zero value between inside and outside, perhaps it is possible that the tangential component of the electric field is what, 0 inside the material, 0 on the surface, but non-zero just outside the surface, is that possible. That is the question that we are asking. So, let me state it mathematically. My E t inside identically equal to 0; E t on the surface, so this is on the surface is equal to 0. We already said that the potential is in surface is an equipotential surface. Now, I am asking what is E t just outside? I will denote it the greater. At a point just outside, what is this? Can this be a finite value or non-zero finite value? Or does it have to be equal to 0?

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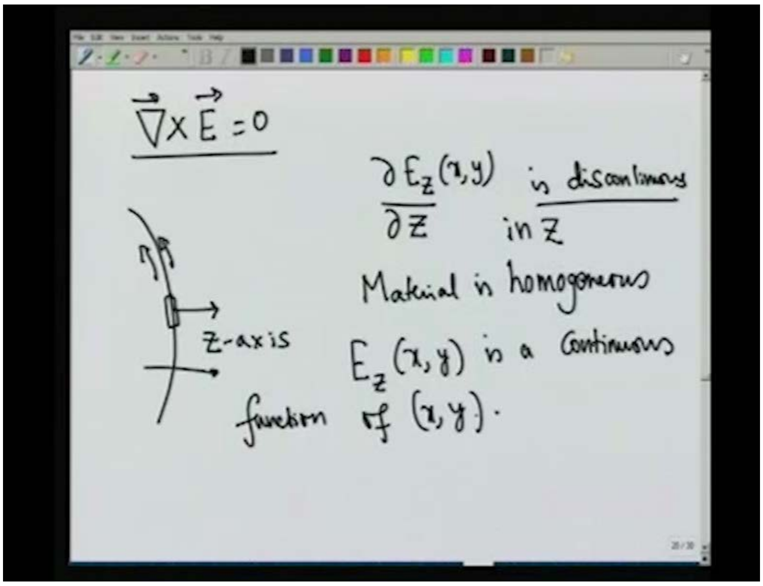

The answer to this question will come not from divergence is equal to rho by epsilon naught, but by the complimentary equation namely, curl of E equal to 0. Now, let me make use of it and show you that the tangential component is 0 just outside the surface. Now, again let me draw the surface and I have this perpendicular component, and without any loss of generality, let me take this to be the z axis. If I take this to be the z axis, the statement that we are making is that delta E z. Delta E z is a function of (x,y), because I am looking at this infinitesimal region, and when I am moving I am assuming that it is a part of a plane, delta E z of this delta E z by delta z is discontinuous. This is the same as saying divergence E is 0 inside and non-zero and non-zero on the surface.

However, when we speak of the tangential component, we are not interested in what happens when I go from this point to this point. I am interested in what happens when I go along this particular point along the surface. Now, what I this statement that I want to make is that - the material is homogeneous. I am not looking at the interface of the two conductors. We will perhaps worry about that later. I can have for example, an interface of copper and aluminum or copper and silver that will give rise to contact potentials. You people must be already aware of that. I am not looking at that. I am looking at a single homogeneous material.

What did I say? A ball of copper, a aluminum of some particular shape. If the material is homogeneous, we can conclude that E z (x,y) is a continuous function that is the important statement. It is a continuous function of (x,y). It is discontinuous function of z, so let me write. It is a discontinuous function of z, it is a continuous function of (x,y), this particular information together with the boundary condition curl of E equal to 0 will immediately tell us what happens to the tangential component. How shall we do that?

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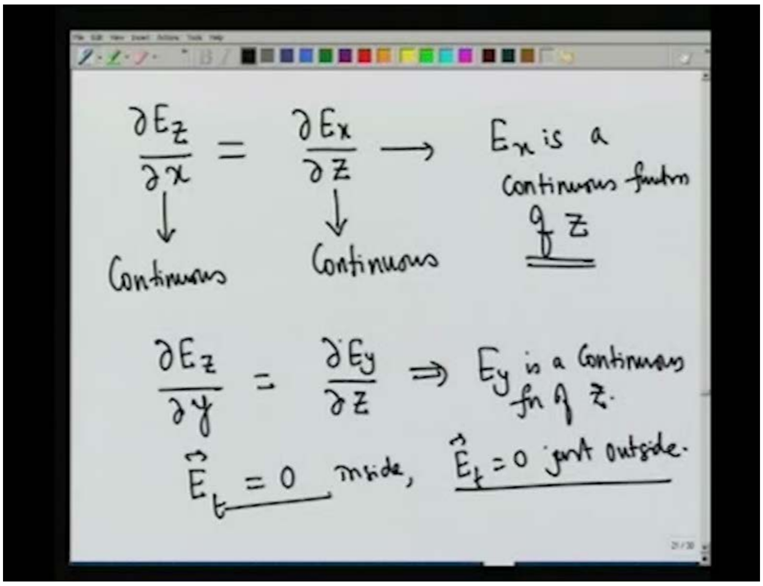

So, what we shall do is to look at delta E z by delta x. This is a continuous function of (x,y). But then by the curl free condition, this is nothing but delta E x by delta z. Because delta E z by delta x minus delta E x by delta z is nothing but the y component of the curl.

Therefore, this is the same. So, we conclude that this is continuous implies this is continuous. But continuous in what; whereas my E z was a continuous function of x let me write it in words. E x is a continuous function of z. That is the most important thing. In a similar manner, you can write down the other complementary equation delta E z by delta y, this tells me this delta E y by delta z which again tells me that E y is a continuous function of z. Therefore, you conclude that if E tangential equal to 0 inside, E tangential 0 just outside.

So, by appealing to the fact that curl of E identically equal to 0 everywhere, whereas divergence E is non-vanishing only on the surface of the conductor. And looking at the fact that only the perpendicular component of the electric field can survive on the surface, we conclude that E t equal to 0. Therefore, my tangential component is continuous across the surface whereas the perpendicular component is sensitive to the charge distribution.

What I would like to people to do is to (( )) over whatever I told you, and argue that this should be valid not only for a conductor but also for other arbitrary surfaces. That is a very well known result. You can look up your books on electrodynamics. I am not going to spend any time on that. Rather I will generalize the result as I need it when I look at the dielectric. But right now this is the statement that we have made, so let me summarize all the results that we have stated now, and then proceed to discuss the other very, very important property of a conductor namely that of a capacitor. That is where it is used in electrostatics. So, but let me summarize.

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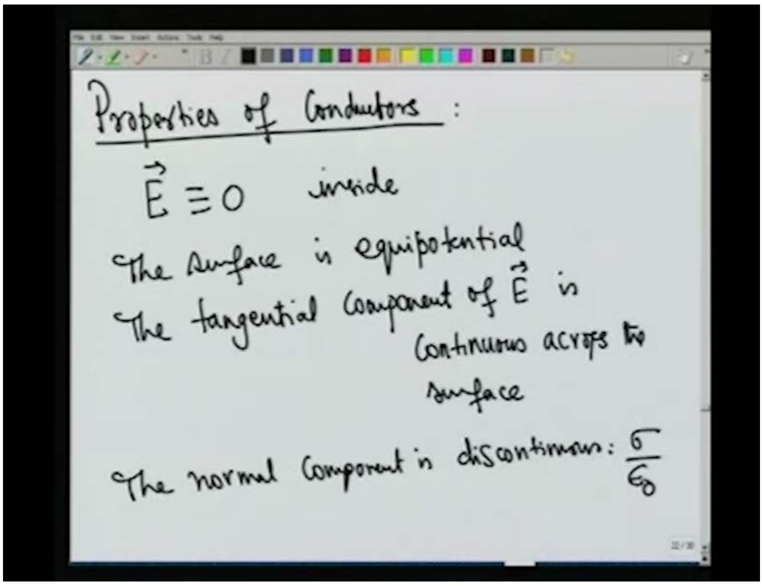

So, properties of conductors: This is a summary. So, what are the statements that we want to make? E identically equal to 0 inside, the surface is equipotential, the tangential component it is sometimes also called parallel component or the in plane component of E is continuous across the surface across the surface. The normal component is discontinuous and the discontinuity is simply given by sigma by epsilon naught; the discontinuity at every point is given by sigma by epsilon naught.

So, what are we saying? What we are saying is that the minute you put a charge, the charge is going to arrange in such a way that the surface of the conductor is an equipotential surface. So, depending on the geometry of the conductor, there could be a depletion of charge in some region, there could be an accumulation of charge in some other region. That is something that we are going to study not in this lecture, but in the next lecture. That has enormous application in lightning conductor so on and so forth. But right now what we shall do is to now look at a system of conductors and the energy.

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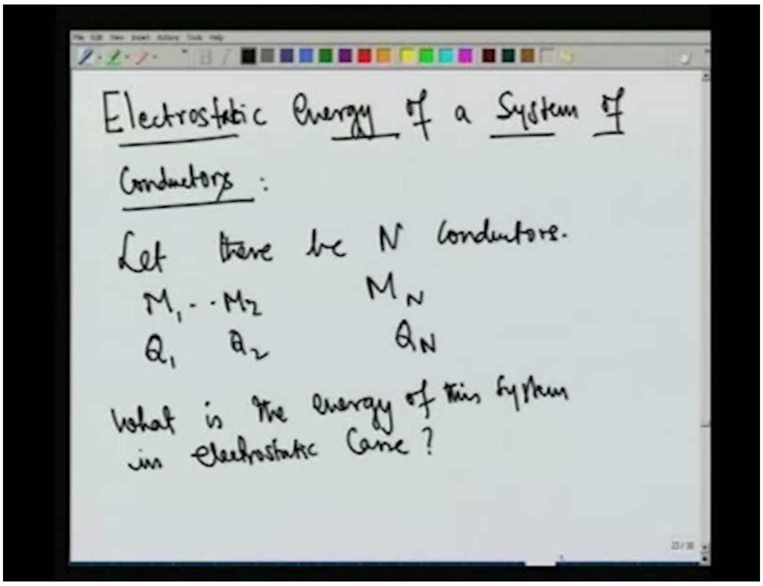

So, what we shall do is to ask the question as to what is the electrostatic energy of the conductors of a system of conductors. It is this question, this study which will lead us to the concept of the capacitance. The capacitance that we are normally used to we say very, very specialized notion, the most general notion is something which you will study at a later time, I will not get in to that. But you will get a hint of what I meant by the statement in a short while. So, what we shall now do is to imagine that there are N conductors. So, let there be N conductors. So, these conductors let me label them as M 1, M N material, whatever. They can come in any shape, any geometry, whatever it may be. And they are all located different points in space.

Let M 1 carry a charge Q 1, M 2 will carry a charge Q 2, M N will charge carry a charge Q N. Now of course, all the charge on M 1 will reside on the surface of M 1, all the charge that I put on M 2 will reside on the surface of M 2, all the charge that I put on M N will reside on the surface of M N, and each of them is going to produce an electric field. The electric field produced by M 1 acts of course on the distribution and also on M 2, M 3, M N. The electric field produced by M 2 will act on the rest of this one. Now, the minute the charges the charge on one conductor seize the electric field, because of the others it will redistribute itself. Therefore, our question is what is the energy of this system in electrostatic case? We are not interested in the dynamical case and that is something which we can answer very, very easily.

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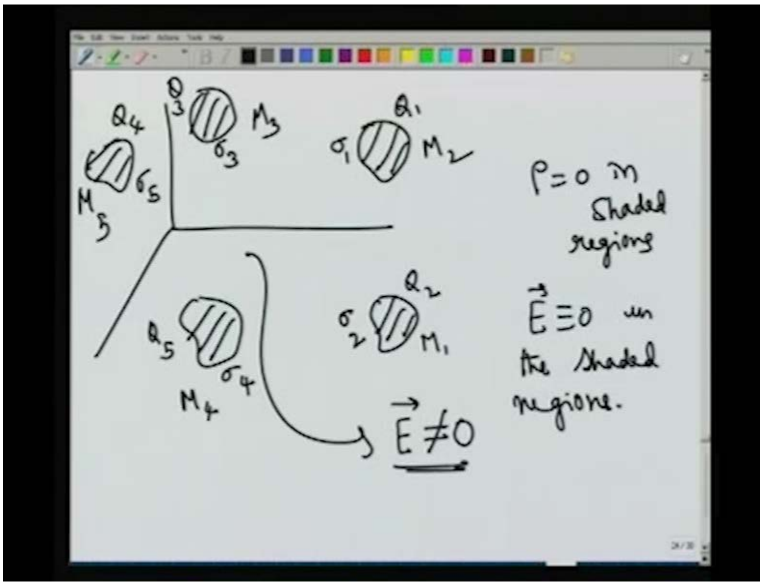

Now, let me first of all draw a picture and then show you how it looks like. So, this is another conductor, this is another conductor, so I have let us say 5 conductors M 1, M 2, M 3, M 4, M 5. This carries a charge Q 1, this carries a charge Q 2, Q 3, Q 4, Q 5. Now, correspondingly there is a surface charge density sigma 1 here, there is a surface charge density sigma 2 here, there is a surface charge density sigma 3, sigma 4 and sigma 5. In these shaded regions, electric field 0 in shaded regions, E identically equal to 0 in the shaded regions. Clearly, because we know that if it were there the charges would have started moving; in this region, obviously E not equal to 0. What we want to do is to relate the electrostatic energy of this system of conductors, is that ok, to the charge distribution on this conductors or equivalently the values of the potentials at which each of these conductors is held.

Now, the most important thing that you have to notice is that you either give me the charge or the potential. If you give me the charge I will fix the potential for you, if you fix me the potential I will fix the charge for you that is the statement that I want make. And in order to study that let us proceed it systematically.

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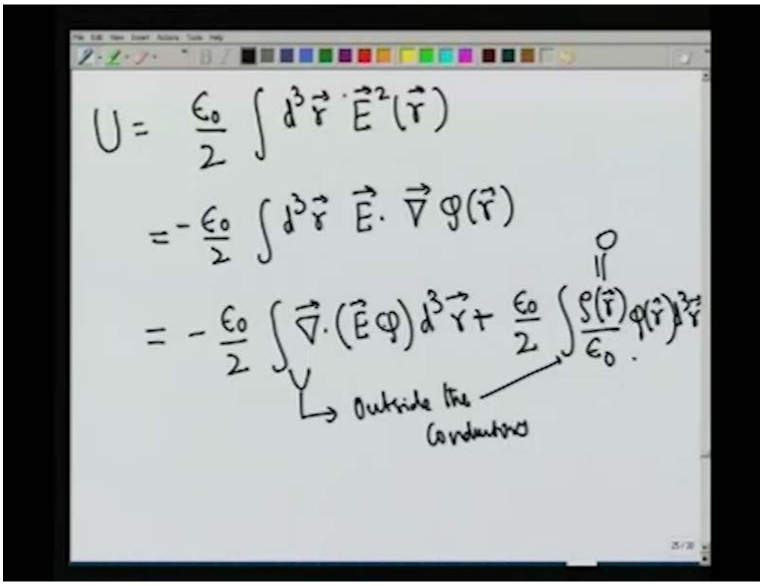

Now, what indeed is the total energy? My total energy we all know is nothing but epsilon naught by 2 integral d cubed r E squared. So, E squared of r. So, I am going to essentially mimic all the steps that I did, when I derived the expression for either the potential energy or for the total potential energy, interaction energy or the total energy. So, what we shall do is to write this as epsilon naught by 2 integral d cubed r E dot gradient phi. So, I have to put a minus sign and this is my… This potential phi is coming, because of the electric field produced by all the conductors - the charges on all the conductors.

Now, by this time we are all great experts in handling such volume integrals together the with the presence of the gradient. However, please notice that what I am going to do is not to kick out the boundary term, but only keep it. This will be written as minus epsilon by 2, we have to keep careful track of the signs and things like that divergence of E phi d cubed r. The next term is going to give me divergence E. So, I am going to write plus epsilon naught by 2 integral rho of r by epsilon naught phi of r d cubed r. Now, my integration is to be done in the volume outside the conductors. Because within the conductor my electric field is 0, but outside the conductor rho of r is equal to 0, therefore this term does not contribute only the first term contributes.

What I will do in this next lecture is to convert this term divergence of E phi d cubed r into a surface term and show how it can be fixed by giving the potentials on the surfaces of the conductors. That we shall do in the next lecture.