Engineering Physics – II

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Module No. # 03

Lecture No. # 02

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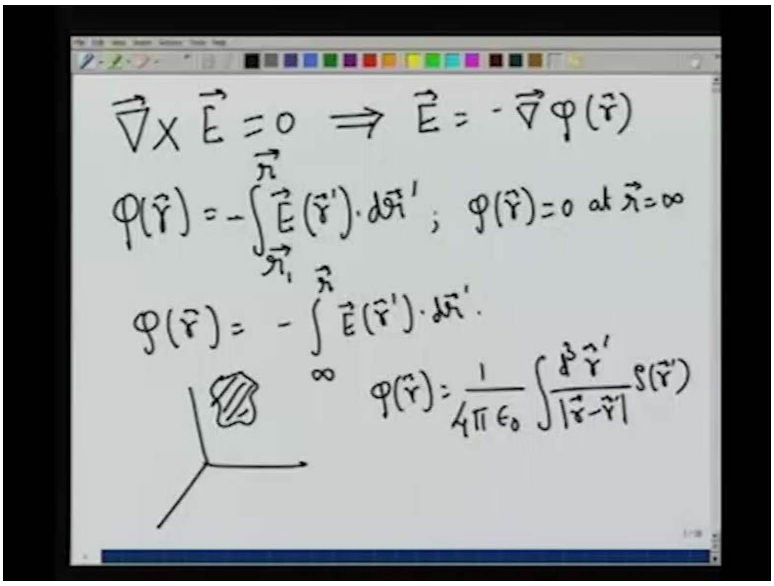

In the last lecture, we introduced the important concept of a potential because we use of, made use of the fact that curl of E equal to 0 implies that E can be written as minus gradient of the scalar fields phi which is your potential. Now, if I were to write down the solution for phi, I would write it as phi of r is nothing but integral E of r prime dot d r prime where the n point is fixed that r. Of course, I have to put a minus sign here, because E has been written as minus gradient of i.

There is an ambiguity about the initial point which we can choose to be any point and let me denote it by r 1. If I had an arbitrary vector field V, this particular integral on the right hand side would have depended on r 1; it would have depended on r and it would have depended on the curve, however because curl of E is equal to 0. We know that this is independent of the curve. Therefore, this r 1 is an inessential initial point. It only serves to define the zero of my potential.

The value of my potential at some particular point r 1, and with respect to that, we could determine the potential everywhere else. In other words, the Potential is defined everywhere up to an additive constant. So, once we remember that, I also made a statement that we can without any loss of generality, choose my potential to be 0 at r equal to infinity. So, phi of r equal to 0 at r equal to infinity. So, if we did that, we can immediately write my phi of r to be minus integral infinity to the point r into E of r prime dot d r prime.

We also wrote an explicit expression for the potential namely: if you are given a charge distribution, so, what do I mean by that? Here is my coordinate system. Let us say that my charge is distributed in this particular region. Then my phi of r is simply given by1 over 4 phi epsilon naught integral d cubed r prime over mod r minus r prime into rho of r prime.

So, it is essentially a matter of choice whether you want to determine the potential directly given the charge distribution or one would like to determine the electric field and then perform this line integral, and the point that I was trying to make in my last lecture, at the end of my last lecture was that, if you have a rather complicated char distribution, for example, in this case, I did not specify the nature of the char distribution. There may be no spherical symmetry; it might be behaving in every complicated fashion as a function of r. It may not be represent able as a simple function of r. Then, it is much better to determine the potential first and then determine the field, because determination of the potential requires only determination of one scalar field, whereas the determination of the electric field determines requires the determination of three fields.

On the other hand, if there is symmetry like a spherical symmetry or cylindrical symmetry, then it is better to determine the field first. The direction and the magnitude of the field gets easily determined by Gauss’s law. Come back, do the line integral which is very easy to perform and then you can determine the potential.

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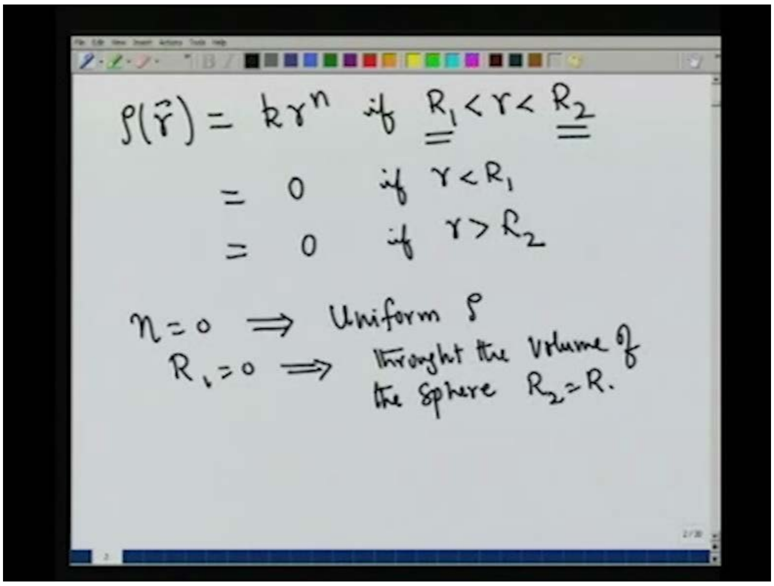

I was illustrating the later situation where there is a potential and the problem that I gave you was a very interesting potential namely: r of, rho of r is nothing but k r to the power of n if rR1 less than r less than R 2. So, what we have is two spheres of radius R 1 and radius R 2. So, in the space between the two spheres, in that annular region, in that spherical shell, my rho of r has a behavior which is given by k r to the power of n.

On the other hand, if r is less than r 1, so, if r is less than r 1, then it is equal to 0 and likewise this was equal to 0 if r is greater than R 2. What I asked you is to determine the field given this by making use of the Gauss's law. I will not work it out; I will leave it as a problem for you, and once you get the field, all that you have to do is to integrate. The line integral is a fairly simple thing to evaluate because E and the unit vector r are parallel to each other, because this is a spherically symmetric phase, and therefore, the field id radically inward or outward everywhere depending on the sign of the value of k.

The next thing that I asked you is to look at the special case. There are there are two special cases which I was trying to list - one was n equal to 0 corresponding to uniform rho; rho is Uniform in the annular region, and the next one corresponds to R 1 equal to 0 which corresponds to uniform rho through, throughout the volume of the sphere.

So, we can set r equal to R 2 equal to R. These two are familiar examples. So, you can specialize to this particular case check whether your answers agree with the standard answers which are given in any text book, and then, you can to try to work out the consequences. Now, in order to proceed with my illustration, what I shall do is to consider another example and that is of cylindrical symmetry. That example is very, very close to the example that I gave in the case two spheres. In fact, I am going to write down exactly the same formula except that the notation r means a slightly different thing.

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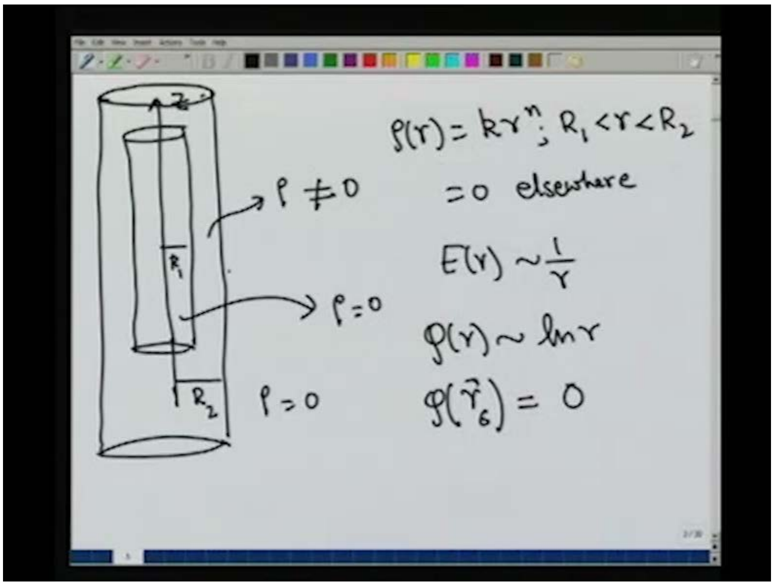

So, what do I mean by that? What I shall do is to consider, let us say z axis here, and let me consider an infinite cylinder. What do I mean by infinite cylinder; I mean it is a very, very long cylinder and I am looking at distances from the axis of the cylinder such that the length of the cylinder is very large compared to the distance. We have explained these things in great detail.

So, this is a cylinder of radius R 1. Now, what I will do is - I will consider another concentric cylinder which is a cylinder of a larger radius. So, let me write it here. So, this is a concentric cylinder coaxial; it is a same axis that is there. So, this is a cylinder of radius r 2. What I now do is to feel the space between the two cylindrical surfaces with a char density. So, rho is not equal to 0 here, and of course, in this region rho equal to 0. In the outer region, rho equal to 0. So, this is an example where my rho is existing only in between the inner cylinder and the outer cylinder. So, let us employ the same formula again.

So, I have the radius R 1 and I have the radius R 2 the distance of the surfaces of the cylinder with respect to the axis. Therefore, again I will write rho of r is equal to k r to the power of n if R 1 less than r less than R 2 and equal to 0 elsewhere. So, you see the problem is paralleling the earlier problem of spherical symmetry. You have to construct a Gaussian surface involving cylinders and not spheres in this particular case. The only care that you have to take is that you cannot choose my potential to be 0 at infinity, because I have taken the cylinder to be infinitely long.

What we have to remember is that my field goes by 1 over r for large values of r. As I go further and further away from the axis, my potential therefore goes like logarithmic of r and we know that logarithm r is an increasing function of r. Therefore, it is impossible to choose the potential to be 0 at infinity. If I did that, my potential would be divergent at all points in the finite regions of space that we are interested in and we would be not be able to do any calculation. Therefore, it is convenient to choose the potential to be 0 with respect to some other point.

And1 convenient point perhaps is to choose the surface of either the inner cylinder or the outer cylinder. I will leave it for you people to choose the, the, way you want it to be. Once you did that, you can determine the potential in the inner region, in the intermediate region and in the outer region. The important point that you have to notice is that the choice of your potential dictates the value of the potential in the inner region.

What do I mean by that the choice of the 0 of the potential. I have to fix the value of the potential at some particular point. So, let us say that there is a special point which I shall call as r s; at which, I will say this is equal to 0. Once I did that, there will be Uniform Potential between 0 and R 1 for the value of r and that will give us what the potential. The value of potential inside the inner cylinder s, and then of course, the potential will be a function of r as we come to the inter space between two cylinders and also when we go outside.

So, I will request all of you. I will urge all of you to solve this problem as well. Now, having worked out these problems, we have discussed a various problems. I first looked at the dipole; then I introduced the problem of the quadrupole. I have introduced char distribution between two concentric spheres; I have introduced char distribution between two concentric cylinders. Now, let us determine two matters of principle and explore the meaning of the potential a little bit more.

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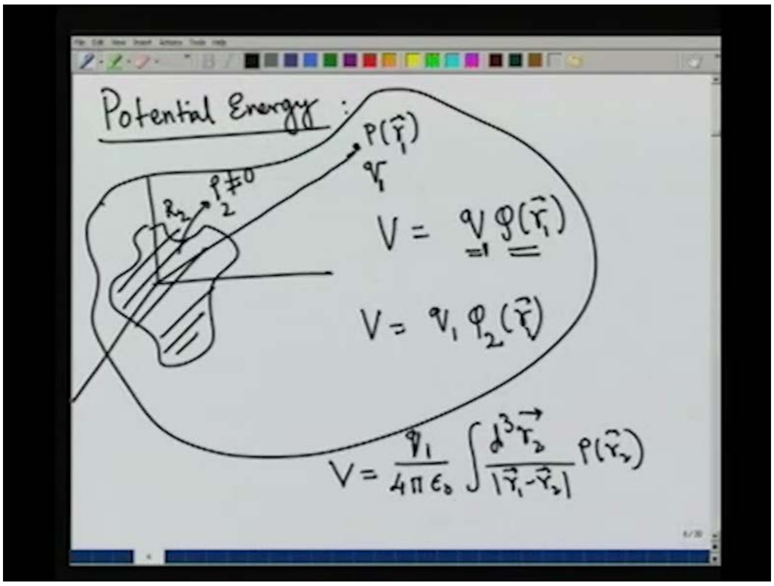

Now I am not interested so very much in the potential as much as the potential energy. So, let me some spend some time on the concept of potential energy and let us see what we can say about that. You see, suppose there is a certain char distribution. So, without any loss of generality, I will choose the char distribution to lie in this region. So, let me call it as r; the region is r and the charge is non-vanishing in this particular region; obviously, we are interested in finite char distributions. Now, here is a generic point p with a coordinate r.

What I am going to imagine is that, I am going to bring a test charge q at this particular point and I ask what is the potential energy of the test charge q, because of the field produced by rho which is not equal to 0 in this particular region. The answer to that has already written by us and we know that the potential is simply given by q into phi of r. Now, although it might appear to be sort of repetitive to you people, I am going to spend some time belaboring the same point and that is the following. This potential is the interaction energy between the charge q and an external field phi of r.

What do I mean by external field? By that I mean, this field phi of r is produced by the charges which are sitting in this region. q is not to be compared to be1 of those charges. So, if you feel like, I could have constructed a larger region. So, let me indicate here. I could have constructed a larger region like this and I could have said there is a charge density in this particular region. In which case, q would have been a part of the system, no, I am not doing that.

What I am interested is in the interaction potential energy between the charge q and the rest of the charges which are localized in this particular region and we are writing V equals to q phi of r. Now, in order to make it notationally clear, I want the notation to exhibit whatever I want to say I will charge call this charge as Q 1 and I will call this rho as rho 2. By that I mean, it is in a different region, the second region. This is the first region. If you feel like, my region is also r 2. Therefore, this will be my V can be written as Q 1 phi 2 into r. I hope everyone understands what I mean.

What I mean is that the charge Q 1 is seeing the potential see produced by the distribution 2. Of course, the potential is produced at the location of the charge Q 1. That is a statement that we are making. So, perhaps, I can even make this point r 1. I will make the point r 1. Charge Q 1 is always located at r 1.

So, once I write this potential, since I know how phi 2 is produced by the char distribution, I can now write my V to be 1 over 4 phi epsilon naught d cubed r 2; d cubed r 2 is this region r 2. The volume contain in this region r 2 and we are going to write mod r 1 minus r 2 rho of r 2 is what I have, and of course, I have to multiply it by Q 1 which is located at r 1. So, we have been able to write a nice expression where the notation makes it clear the kind of physics that I want to do.

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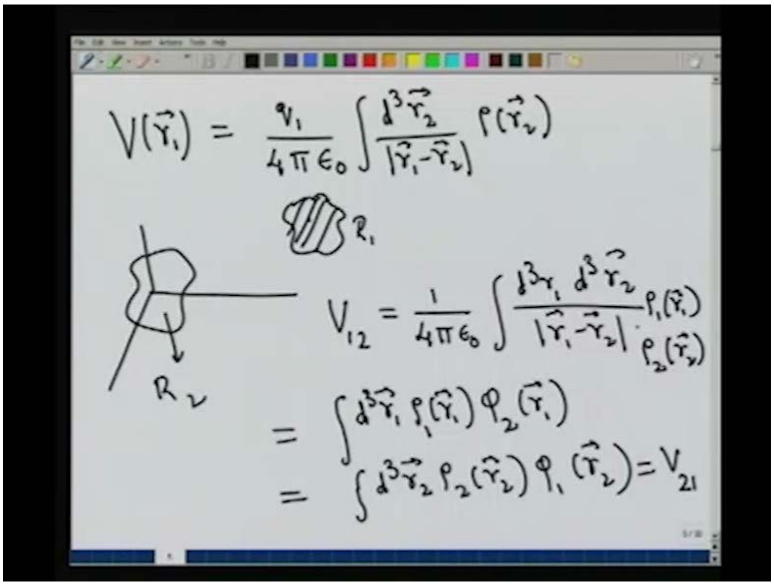

So, let me repeat that expression V of r 1 which is the location of the charge Q 1 is simply given by Q 1 over 4 phi epsilon naught d cubed r 2 mode r 1 minus r 2 into rho of r 2. This is the first step in our calculation. Now, we can generalize our situation a little bit more. What we can do is to let me draw the figure again. I have my char distribution in this region r 2 and I have a single charge here. This single charge let me spread it around in this particular region; let me call it as a region r 1.

In other words, I do not have a point charge in the region r 1; I have an extended charge. Therefore, I have to calculate the potential at every point r 1 in the region r 1 and I should perform a integration, and of course, every point r 2 in the r 2 is going to contribute. I am not going to spend too much time insulting your intelligence by trying to derive the expression. Let me write straightaway. We all know that in that case, my potential, the interaction Potential I can write it as between 1 and 2. 1 is a char distribution which is sitting here; 2 is another char distribution that is sitting here. This is nothing, but1 over 4 phi epsilon naught integral d cubed r 1 d cubed r 2 divided by mod r 1 minus r 2 rho1 of r 1 rho 2 of r 2.

Please notice how careful I have been. I am saying there is such a certain char distribution rho1 at r 1. There is certain char distribution rho 2 at r 2. They are two independent char distributions and the interaction energy is given by this. Of course, this can be written equivalently in yet another form. So, this can be simply written as d cubed r 1 rho of r 1 phi 2 of r 1, why, because integral d cubed r 2 rho 2 of r 2 divided by mod r 1 minus r 2 with the factor1 over 4 phi epsilon naught is simply going to give me this expression, and because of the symmetry, we see that this is also equal to d cubed r 2 rho 2 of r 2 phi 1 of r. So, we are saying this is the same as V 2 1.

The energy of the charge distribution 1, because of the 2 is the same as the energy of the char distribution 2 because of the1. Therefore, depending on the situation, depending on the nature of the problem, you can either use this expression, this expression or this expression in order to define the or evaluate the potential; all of them are equivalent expressions. At a later time after a while, I am going to work out a few examples where I am going to use different expression in different situations, but now, let me continue my theme in order to rewrite my potential in yet another different form.

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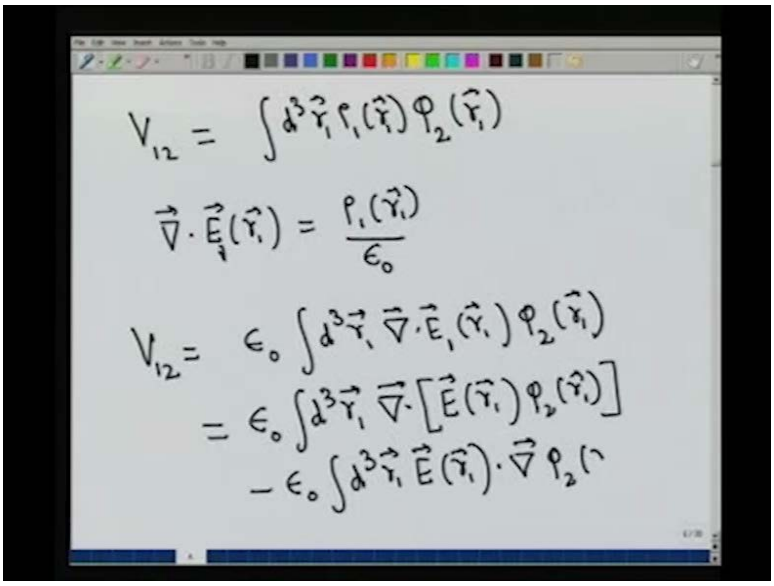

So, let me start with the expression for V 1 2. What was my V 1 2? My V 1 2 was written as integral d cubed r rho1 of r phi 2 of r. I have removed the notation r 1 because it is understood. Therefore, if you want, I can even keep this. Now, I know that there is a certain charge density rho 1, and whenever there is a charge density rho1, it produces its own electric field. So, we ask the question - is it possible to eliminate all references? Please notice what I am saying, is it possible to eliminate all reference to the charge and write the potential entirely in terms of the field?

That is the question that I am asking. This in fact is an important question, because at a later time when I discuss electromagnetic field, we are going to argue that the energy resides in the field even in the absence of the sources. So, as a kind of warm up, as a kind of preparation to whatever we are going to study, let us see in after another 6 or seven lectures.

Let us ask how I go about writing the expression V 1 2 entirely in terms of the fields. The answer to that is very, very simple, because I know that by the Gauss's law divergence E of r 1; obviously, the divergence is with respect to r 1. The derivative is with respect to r 1 is nothing but rho1 of r 1 divided by epsilon naught. This is my Gauss's law. That is how I started the discussion of the whole subject.

So, what I shall do is to replace rho1 of r 1 by epsilon naught divergence of r 1. If I did that, what do I get? I get V 1 2 is simply given by epsilon naught integral d cubed. So, this is r 1 which is sitting here, d cubed r 1 divergence of E1 of r 1 because this is the field produced by what I call the char density labeled 1 E1 of r 1 into phi 2 at r 1.

So, 1 and 2 clearly tell us that there are two distinct different charge distributions that we are interested in. Now, whenever there is a differential operator of this particular kind and whenever there is a volume integral, what do we do? We write the integrand as a total derivative, convert it into a surface integral and then look at the other term. So, let me do that. So, I am going to write this expression as epsilon naught d cubed r 1 if what I have. I will write a gradient outside, divergence outside and write E of r 1 phi 2 of r 1 and close the bracket, so in doing that I have added an extra term.

Therefore, I should write down the difference between the two terms, and what is that term going to be? This is nothing but minus epsilon naught d cube r 1 E of r 1 dot gradient phi 2 of r 1. This is a very very neat expression. So, what have I done? I have written my potential energy, the interaction potential as a first step. By eliminating rho, I have a electric field sitting here I have the gradient of I sitting here which is nothing but the electric field again and then there is total divergence jump.

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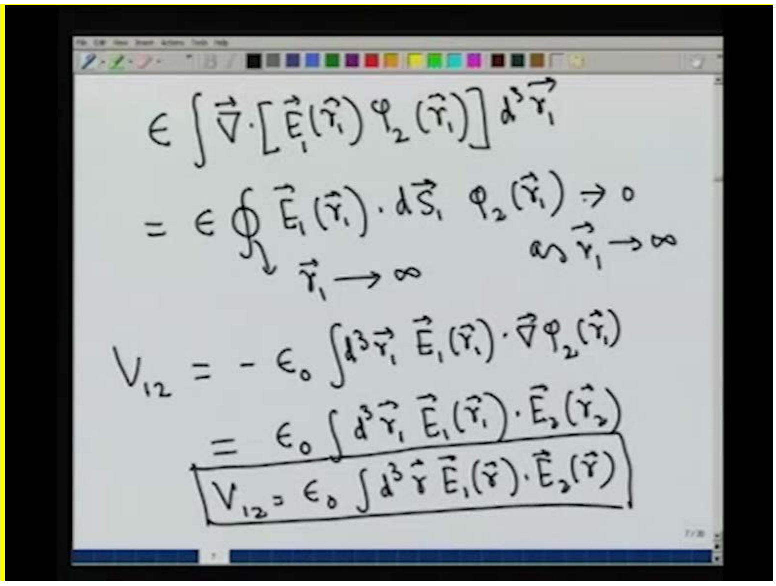

Now, let me examine each of these terms and say what it means. What was the first integral? The first integral was simply given by epsilon naught divergence E of r 1 phi E1 phi 2 of r 1 into d cubed r 1. This integration is over the whole space, but then mister Gauss comes and tells us look here, you have the divergence of a vector field. By electric field is a vector field, now phi as a scalar field; a scalar multiplying a vector gives me a vector.

Therefore, by Gauss divergence theorem, this can be written as a surface integral. Therefore, this is nothing but integral a surface. What is that surface? E1 r 1 dot d s1 into phi 2 of r 1. It does not matter how we split these integrand and the integrating variable. This is what I have, and since the integration is over all space, this surface integral is evaluated in the limit r 1 going to infinity. Please remember that we are discussing matters of principle; we are not looking at the some idealized case.

Therefore, we are necessarily looking at fields which die as we go to infinity. Finite char distribution produce fields which go to 0 as r goes to infinity. My potential can either go to 0 or a constant depending on the choice of the origin. Therefore, this integral goes to 0 as r 1 goes to infinity. Therefore, this integral is identically equal to 0.

The first integral cancels, it becomes 0 because of the gauss divergence theorem and because of the, of the, electric field. Therefore, my potential energy between the 2 char distributions is simply given by minus epsilon naught d cubed r 1. I had E1 of r 1 dot gradient phi 2 of r 1. The gradient of the potential at the point r 1 produce by the charge density rho 2, the distant physical system, but then, we all know what gradient of phi 2 is. This is nothing but epsilon integral d cubed r 1 into E1 of r 1 dot E 2 of r 2, my r 2 which I the integrating variable is a dummy.

Therefore, I can always replace1 dummy by another dummy. I will write V 1 2 is equal to epsilon naught d cubed r E1 dot of r dot E 2 of r. So, this is the another famous expression, popular expression much often used expression in order to determine the potential energy between two distinct char distributions. So, we can write the potential energies entirely in terms of the char densities or you can write the potential energy in terms of the char density multiply by the potential. There are two options there, and the third options are to write the potential energy entirely in terms of the fields. All this expressions in electrostatics are equivalent to each other. We can use any of them depending on the convenience of the situation, but later you will find that these expression acquired a special meaning. That is something that you have to keep in mind at this particular point. Therefore, it is good actually to solve the problems that I give you using all these techniques to the extent possible.

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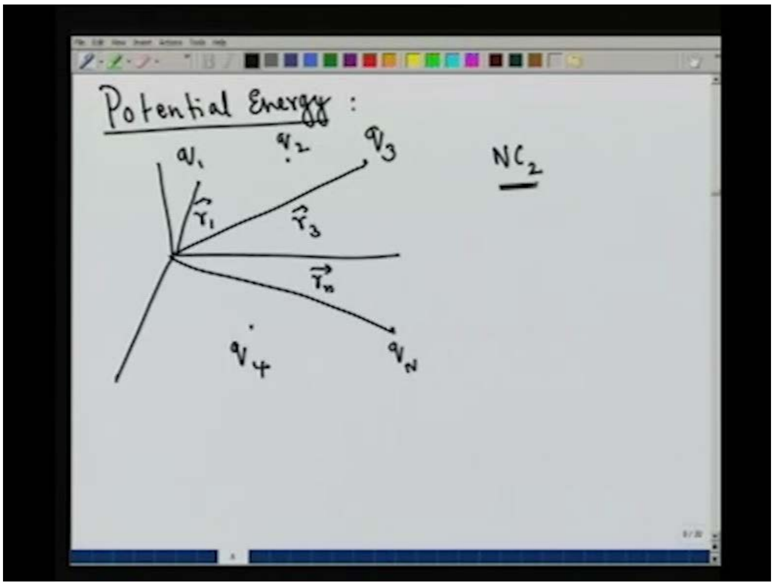

Now, having done this let me look at yet another concept of potential energy. This is a more encompassing concept. So, now, we are going to ask what is the total potential energy of a given charge system. This is a quietly more generalize idea and let me start by illustrating that with the simplest of the examples. What is the simplest of the examples? Let me take a coordinate system and let me locate points - r 1, r 2, r 3, r 4, r n. So, there is a charge Q 1 here, Q 2 at r 2, q 3 at r 3, q 4 at r 4, q n at r n. So, this is r 1; this is r 3 and this is r n. From mechanics, we know that the total potential energy of the system is written by the sum of the potential energies of different pace. Since there are n charges, there are n choose 2 pace and we have to write the sum of potential energies of all the space.

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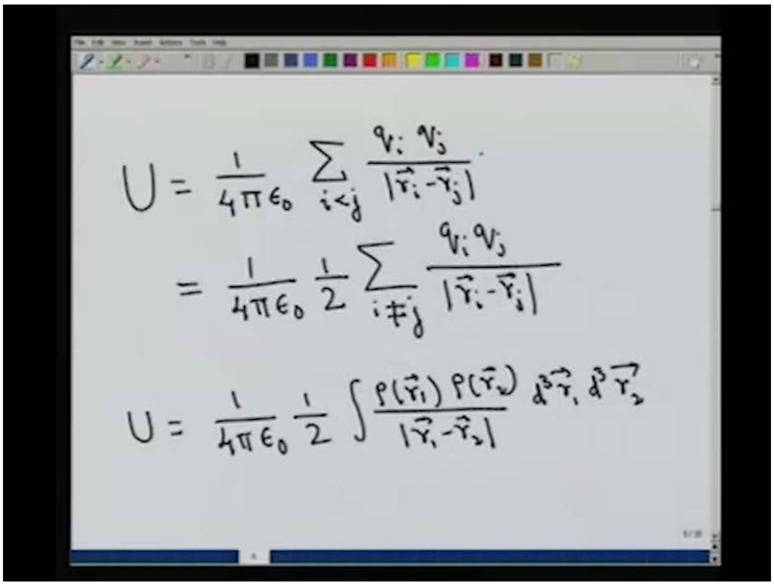

In order not to get into any confusion, I will use a different notation now. I will write the total potential energy as U. So, how does it look like? The potential energies is simply given by1 over 4 phi epsilon naught and I will write I less than j q I q j over naught r I minus r j. You know that the sum of the kinetic energies of the m particles is these terms are a conferred quantity in electrostatics so long as of course we ignore the magnetic field. This is what I have.

Now, I had put I less than j because I do not want to double count, but many a time we do not want to make such a distinction, because they could be ordering by charges. So, I simply want to make a statement I not equal to j, but then, there will be double counting. So, the way to avoid double counting is to re write it as 1 over 4 phi epsilon naught. I put a factor of half here and I write I naught equal to j q I q j over naught r I minus r j.

So, here, we are asking the question as to how much where we are going to do in order to what bring in and assemble n charges at points - r 1, r 2, r 3, etcetera, r n. That is the question that we are asking and we have got this expression. This is the total energy of the system. Now, it is again straight forward to write down the generalization when I have a charge density. So, how would I go about writing the char density? My q I will be replaced1 char density; q j will be replaced by another char density. Therefore, it might appear that the generalization is the straight forward. What do we do? We simply write it as1 over 4 phi epsilon naught. I have a factor of half. Now, my q I and q j will get replaced by the appropriate char densities multiplied by the small volume elements. So, I will write rho of r 1 rho of r 2 mod r 1 minus r 2 d cubed r 1 d cubed r 2. This is the expression that I have.

So, it appears that just as we were able to write down the interaction energy between two charge densities, but I did not ask how each of these charge densities was constituted. What I have done is to look at the interaction between amongst an individual chargers, I have written it as a discreet some smeared each of these charge densities to occupy finite regions in space. I am not asserting that they are occupying disjoint regions in space, and I got the expression1 over 4 phi epsilon naught half rho of r 1 rho of r 2 mod r 1 minus r 2 d cubed r 1 d cubed r 2.

Every single step here appears to be completely straight forward; however, this is an example which tells us that we should exercise great care while going from the discrete limit to the continuum limit because this expression means quite something different compared to the expression that we have derived earlier. Now, in order to illustrate that, again let me make use of Gauss’s law and let me proceed step by step.

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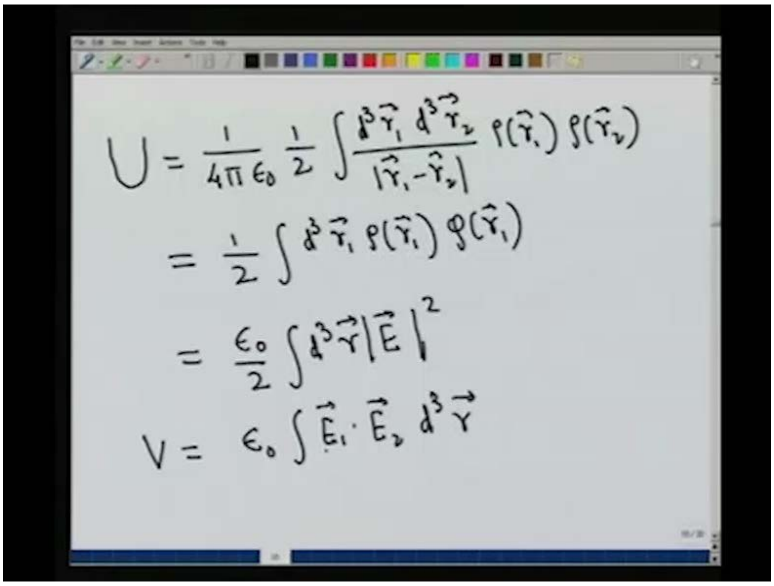

So, what was my expression for the energy density? I wrote it as1 over 4 phi epsilon naught; I have the factor half sitting here d cubed r 1 d cubed r 2. These are the volume elements mod r 1 minus r 2 rho of r 1 rho of r 2. Notice, earlier I was very careful; I used to write rho1 rho 2. I am not doing that. I have written rho of r 1 and rho of r 2. Now, we all know the expression for the potential produced at a point r 1 because of the charge density. This is nothing but half integral d cubed r 1. That is what I have rho of r 1 phi of r 1.

In writing this, we have written down the potential coming from all possible sources. I am not saying that one particular charge distribution is external to another particular charge distribution. Now, all that remains for us is to write down the expression for entirely in terms of the fields by eliminating rho. I did that just now using Gauss's law. If you did that, you will get epsilon naught by 2 integral. I can replace the dummy r 1 by the dummy r d cubed r E squared mod E squared into is what I am going to get.

So, let me remove this and put the bracket here. So, rho will be written as divergence E into epsilon naught, that divergence will be brought here; the gradient phi will be nothing but E. Therefore, I will get d cubed or mod E squared.

What is the important statement that I am trying to make? Let me illustrate. When I wrote the potential, I wrote it as epsilon naught integral E1 dot E 2 d cubed r. This was the interaction energy between two charge densities, whereas when I looked at the total energy of the system, irrespective of what came where and how it came. Without making a distinction between something external to the other, I got the expression epsilon naught by 2 d cubed r mod E squared; obviously, the two expressions cannot be the same.

V can be positive or negative depending on the relative science of E1 and E 2. For example, if you have two point chargers - one of them positive, another negative, we know what V is; it is minus Q 1 Q 2 divided by r the distance between them. If there are 2 positive chargers, V would be positive, but you define is always greater than 0. In fact, this is equal to 0 if only if my electric field is vanished in all over the space, that is, there are absolutely no charges. Therefore, we have to ask ourselves what indeed is the relation between U and V.

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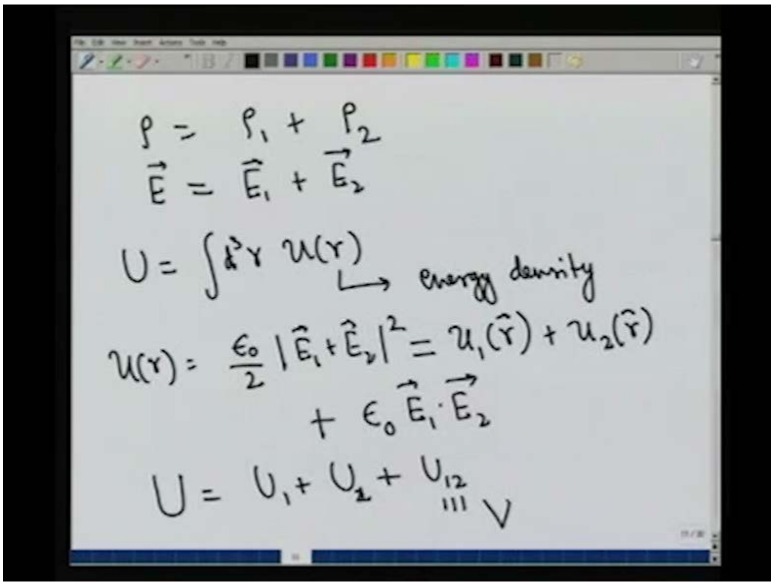

The answer to that is very, very simple. What we do is to simply write by to be rho1 plus rho2. So, depending on the system of interest, I split my rho to be rho1 plus rho 2. In which case, my electric field by the linearity of the Coulomb law will turn out to be E1 plus E 2. What am I doing? I am writing my U in terms of E. Therefore, if I look, if I write my, the total charge energy to be d cubeb r U of r, what is my U of r? This is my energy density. My U of r is nothing but epsilon naught by 2 mod E squared, which is nothing but mod E1 E 2 whole square E1 E 2 whole square.

And now, if I were to expand this, what am I going to get? I will get a U 1 of r. U 1 of r is the energy density because of the field, because of the charge density rho1; U 2 of r is the field energy because of the charge density rho 2, because I get a mod E1 squared plus mod U 2 squared. Now, comes the interesting term that is epsilon naught E1 dot E 2, and what does this expression tell me? Epsilon naught E1 dot E 2 was indeed V. Therefore, I can write my total energy U to be U 1 plus U 2 plus U 1 2, and U 1 2 is identically equal to V U 1 plus U 2 plus V.

So, please realize that depending on the context either I can ask for the, the, total energy of the system or I can ask for the interaction energy, they have distinct physical meanings; they have distinct physical interpretations. Sometimes we are interested in interaction energy which is always finite, which can be positive or negative, whereas the total energy is always a non negative, in fact, positive definite quantity for any charge distribution.

So, I think this has been a rather comprehensive introduction to the concept of a potential and a potential energy. What remains for us is to work out a few examples. So, let me start discussing those examples, and after that, I shall introduce the concept of a dielectric and a conductor and let us see how we can explore the physics even more. That of course is a rather complicated subject, because once we introduce the concept of a medium, we have to carefully introduce the concept of a time average, space average field, but that will come later. But now, let me start with some of the simplest examples.

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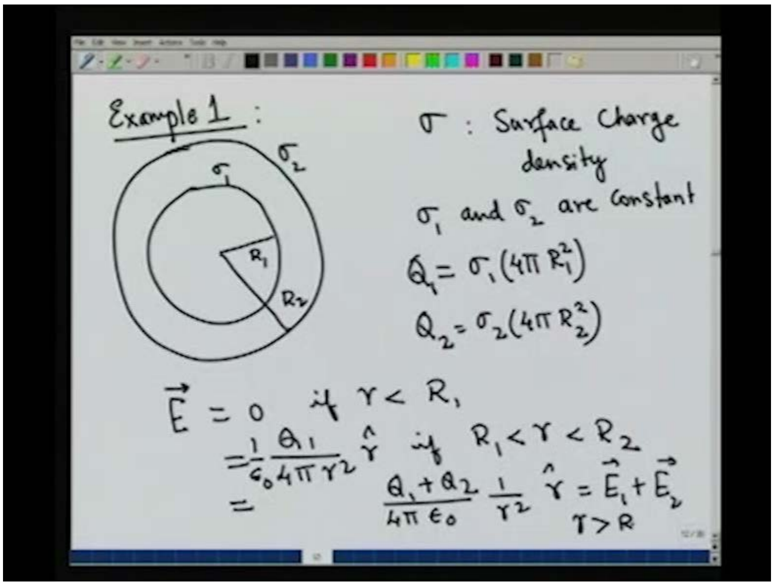

The simplest example that I will consider is the case of two concentric spherical shells of negligible thickness. So, I have a sphere of radius R 1 and there is concentric sphere of a radius r 2; this is a concentric sphere of radius R 2. The inner sphere has a charge density sigma1. Sigma is the surface charge density. So, this is the surface charge density and the outer sphere has a surface charge density sigma 2. So, what we are saying is that sigma 1 and sigma 2 are constant, either positive or negative. So, what about the total charges? Well, I know what my total charges. My Q 1 is nothing but sigma1 into 4 phi R 1 squared; 4 phi R 1 squared is the surface area of the inner sphere, and my Q 2 is nothing but sigma 2 into 4 phi R 2 squared.

This is a very, very simple example. Then we can go to more complicated examples later. I will give them as problems to you, but this problem has enough generality to illustrate all the principles that I want to tell you. Now, we can ask ourselves what is the total energy of this particular system, and then we can ask ourselves what is the interaction energy between the inner spherical shell and the outer spherical shell.

So, this is a each of them is a sphere of infinite I of 0 thickness, is that ok? And all the charges residing on the surface of each of these spheres, again Gauss's law comes to our great rescue; it allows us to write down what the fields are. So, let me write down the fields. My electric field is equal to 0 if r less than R 1. Everyone will agree with me. This is equal to Q 1 over 4 phi r squared r if R 1 less than r less than R 2.

In the region r greater than r 2, again by Gauss's law, let me write it at a further distance Q 1 Q 2 divided by 4 phi epsilon naught1 over r square r hat. I mean the factor1 over epsilon naught here. So, let me introduce. So, these are the fields and this is nothing but E1 plus E 2. E1 is the field due to Q 1; E 2 is the field due to Q 2 and we are looking at a point r greater than R 2. What is it that we find? We find that the total charge Q 1 plus Q 2 contributes. We cannot make a statement that some of the charges distributed in the inner sphere; some of the charges distributed on the outer sphere. No experiment on outside will be able to tell us. That is what it is.

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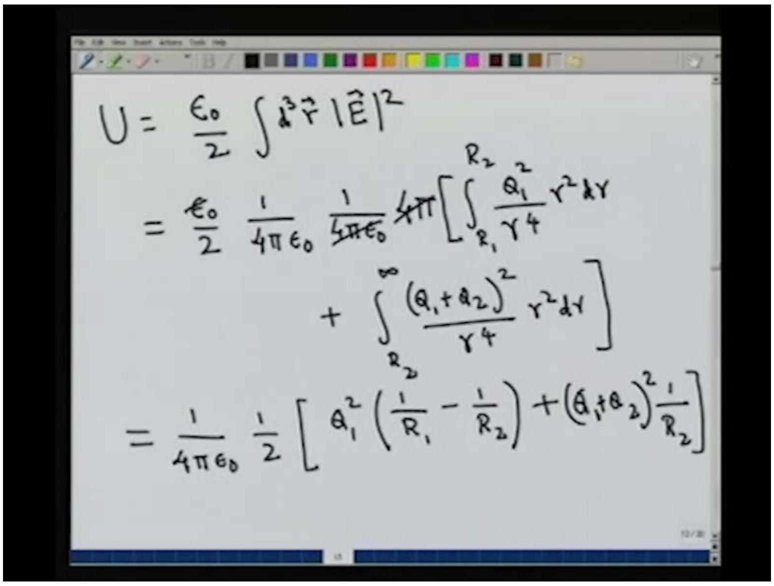

Now, the determination of U is a very simple thing for us because I have already obtained an expression for you. What was it? It was simply given by epsilon naught by 2 d cubed r mod E squared. I have an expression for E. Therefore, let me start integrating this expression straightaway and see what I am going to get.

So, let me proceed being bit slowly. I have got an epsilon naught by 2. My electric field always came with a factor1 over 4 by epsilon naught. Since I have mod E squared, I am going to write 1 over 4 phi epsilon naught, 1 over 4 phi epsilon naught, 1 over 4 phi epsilon naught squared. Since my field is spherically symmetric, it is radically outward. In this d cubed r, I can perform the surface integral independently. Therefore, my integral d omega is going to give me a 4 phi, and now, I am left with the radial integral to perform.

So, let me open a bracket here and start looking at the radial integral between 0 and r 1. So, what was my R 1? My R 1 is the inner sphere, so, in this particular region. So, what I have is - in this particular region, my field is identically equal to 0. Therefore, I am interested in the region R 1 to R 2. So, let me write r 1 to R 2. I had Q 1 squared divided by r to the power of 4 r squared d r. That is the expression that I have because the field is exclusively coming from the surface of the charge on the inner sphere and the, and the, star distribution on the outer sphere is not going to contribute.

Now, comes the next thing between R 2 to infinity both Q 1 and Q 2 contribute. Therefore, this will be Q 1 plus Q 2 whole squared divided by r to the power of 4 r square d r. This indeed is my expression for the total energy; this is finite. So, everything is well defined and we can evaluate the quantities precisely. So, if I were to do this, there is a 4 phi epsilon naught and a 4 phi epsilon naught which cancel each other. So, let me do that in the first place.

This is nothing but 1 over 4 phi epsilon naught. There is a factor of half. So, let me open the bracket and let me try to integrate this expression. So, let me write my Q 1 squared first. I have1 over r square d r between R 1 and R 2, all of you are great experts in solving this. This is nothing but 1 over r 1 minus1 over R 2; this integral is nothing but1 over R 1 minus 1 over R 2; d r by r squared is minus 1 over r. Therefore, the lower limit and the upper limit they will switch, this is the expression that I am going to get.

And what is the next expression? The next expression is nothing but Q 1 plus Q 2 whole squared by divided by 1 over R 2. The surface at infinity of course is not going to contribute because when r goes to infinity 1 over r goes to 0. So, this is indeed an expression in a very beautiful form because it is clearly exhibiting my U 1, my U 2 and U 1 2 which is nothing but V. So, let me look at the expressions again.

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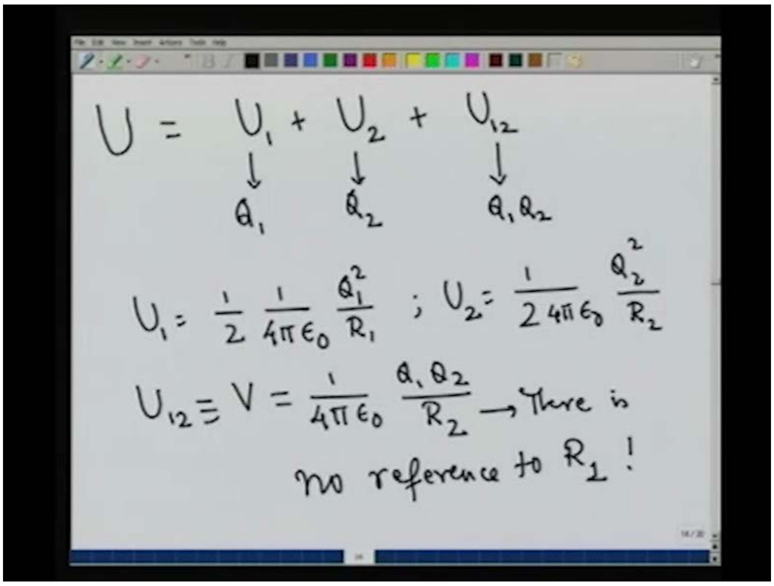

So, let me write my total energy to b U 1 plus U 2 plus U 12. How do I identify U 1 U 2 and U 12? U 1 comes exclusively from Q 1, that is, if I put Q 2 equal to 0, I get the contribution U 1. U 2 comes exclusively from Q 2. If I put Q 1 equal to 0 in the previous expression, that will be the energy of the outer sphere, the charge of the outer sphere. U 1 2 involves the cross term Q 1 Q 2. So, let me go back to my previous expression. Look at this and write down U 1, U 2 and U 1 2. If I look at Q 1 squared, I have1 over r 1 minus1 over R 2 Q 1 squared has plus1 over R 2. The R 2 term cancels.

So, let me come back here and let me write U 1 is nothing but half 1 over 4 phi epsilon naught Q 1 squared by r 1. That is what I have. The r 2 term cancels. What about U 2? Well, in order to write down U 2, let me again go back to the previous expression. I am going to put Q 1 equal to 0; Q 2 squared is what I have. I am going to get 1 over 4 phi epsilon naught half with a factor of R 2.

That is indeed the correct expression. Therefore, this is nothing but 1 over 2 1 over 4 phi epsilon naught Q 2 squared by R 2. Now, we have to write down the expression for U 1 2 which is the same as the V. The interaction energy between the 2 spheres 2 spherical char distributions. So, let me go back to this expression and what do I do? I ask for the cross term between Q 1 and Q 2. I get a 2 Q 1 Q 2 that cancels this factor of 2 in the denominator. Therefore, I will get1 over 4 phi epsilon naught Q 1 Q 2 divided by r 2.

So, in one shot, we have been able to determine the energy required to assemble a certain charge Q 1 on the surface of a radius, on the surface of a sphere of radius R 1. Then I asked what is the energy required to assemble another charge Q 2 on the surface of a sphere of radius R 2. They are two independent problems and we have U 1 and U 2. U 1 does not refer to U 2; U 2 does not refer to U 1, but now, I say that these two are concentric spheres, and I ask suppose there is already a certain char density, let us say corresponding to the inner sphere and there is a char density corresponding to the outer sphere.

What is the interaction energy and that turns out to be Q 1 Q 2 by r 2. Now, this expression for U 1 2, there is something funny about it and that is there is no reference, there is no reference to R 1. So, this is indeed surprising. Normally when we wrote down the expression for the interaction energy, we wrote q i q j divided by mod R I minus r j. It has to depend on the separation of the two spheres; it has to depend R 1 and R 2. That is what we would have expected. But however, when we looked at this expression, when we derived this expression, what are we finding? We are finding that it is a function only of R 2; it does not refer to R 1 at all.

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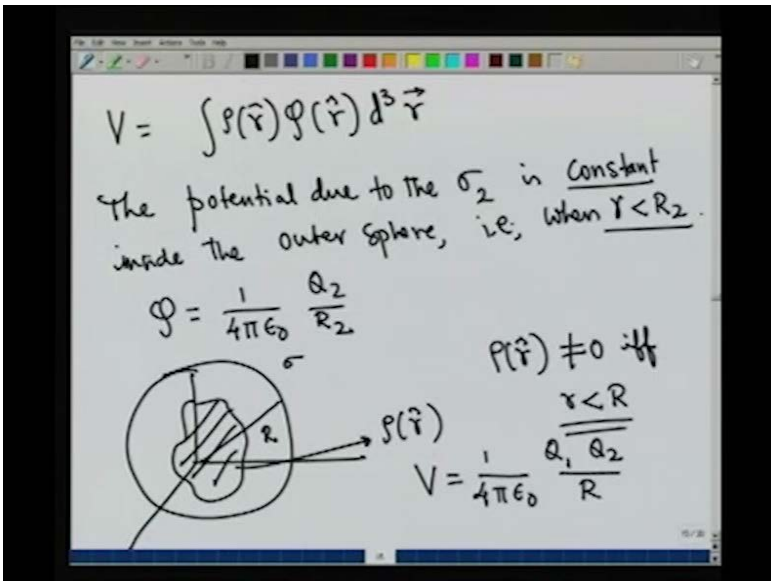

Now, is it possible to understand this expression. Well, there is nothing very surprising about that because remember V can also be written as integral rho of r phi of r d cubed r. So, in writing this expression, I am actually referring to the inner sphere rho corresponding to the inner sphere. And what is the potential due to the outer charge? The potential due to the surface char density sigma 2 is constant. That is the most important word is constant inside the sphere, inside the outer sphere, that is, when r less than R 2, when r is less than R 2.

Now, we fix them our potential to be 0 at r equal to infinity. This has to be a constant, and by the continuity of the potential, we know that phi is nothing but 1 over 4 phi epsilon naught Q 2 by R 2. This is a constant integration rho of r d cubed r is the same as integration sigma of s ds because it is defined on the surface that is going to give me the total charge Q 1, which is the reason why my interaction potential V is independent of R 1.

In fact, one can make a stronger statement and that is something that you have to notice. So, imagine the following situation. I have a sphere, I have a sphere of radius r and there is a uniform char distribution sigma on this. What I do is to take the interior of this sphere and I put a very complicated char density, I put a very complicated char density rho of r. The only restriction is that rho of r not equal to 0 if and only if r less than capital r.

Now, the above analysis clearly tells us that the potential is always given by V is simply given by. So, I have to employ notation, that is, let me write it down and in it 1 over 4 phi epsilon naught Q 1 Q 2 by r. So, sigma is going to give me a total charge Q 2. The total charge Q 1 coming from rho is given by Q 1. It is given by1 over 4 phi epsilon naught Q 1 Q 2 by r. It is completely independent on the nature of the distribution, of the shape of the distribution. Whatever the volume may be, it might be as complicated as you feel like, but still the potential is given by this.

Please remember this expression because it is expressions like this that are useful later when we try to define the concept of a capacitance for conductors. So, here, we have an interesting problem which tells us what the meaning of a potential is and how to distinguish between the total energy of the system and also the interaction energy.

Now, there is another classic text book example. I would like to work it out not so very much because of the mathematical complication. In fact, it is a very simple problem, but because of the richness of the physics that is involved in it and that is the following problem.

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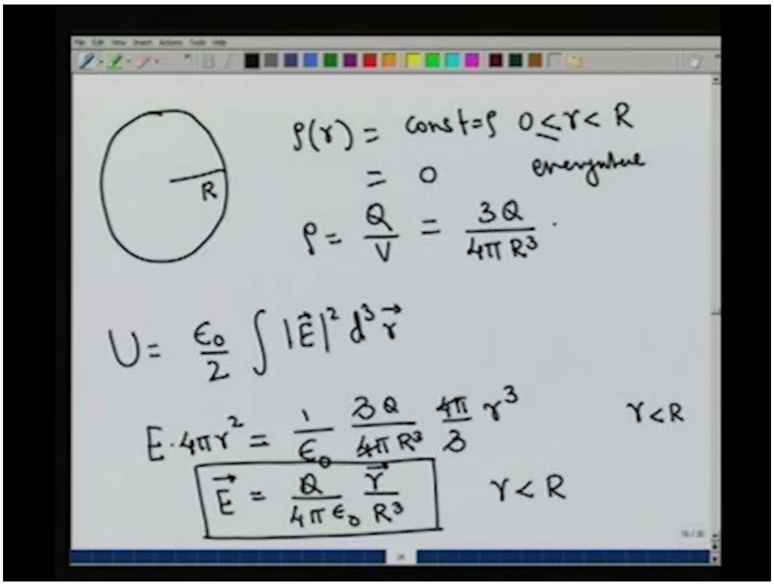

What we shall do is to consider a sphere of radius r, and now, I am going to look at a rho of r which is equal to constant if 0 less than r less than R and equal to 0 everywhere else. So, this constant is equal to rho. That is what we are saying some number rho; obviously, my rho is simply given by charge divided by the total volume and the volume is 4 by 3 phi r cubed. Therefore, this is nothing but 3 q over 4 phi r cubed; this is my rho.

Now, let me ask myself what is the total energy of this spherical charge, spherical distribution of the charge. You can imagine it to be some kind of a spherical drop let, is that ok? It is small drop. The answer is very simple. The total energy is simply given by epsilon naught by 2 integral mod E squared d r d cubed r.

So, this integration obviously is over the all space, whole space. However, in order to determine my E, I need the field in the interior region and I need the field in the outer region which I will derive by using the Gauss's law. So, let me start with the expression E into 4 phi r square as usual as the charge enclosed. So, how do I write that? It is total charge enclosed divided by epsilon naught. I have my o1 over epsilon naught. My rho is nothing but 3 q over 4 phi r cubed because 4 by 3 phi r cubed is my total volume 3 cube over 4 phi r cubed.

And now, I have to integrate it up to a point r because I am interested in the region r less than r and that is going to give me 4 phi by 3 r cubed. So, this is indeed the expression for the electric field. So, what do I conclude? I conclude that my electric field is nothing but all my 4 pi's cancel, 3 cancel. So, I have q over 4 phi epsilon naught; I have to be careful now. I am going to get r divided by r cubed.

So, in the interior region, my electric field is given by q r 4 phi epsilon naught r cubed if r is less than r. So, this you see is a parabolic potential, and if I imagine that the q is positive, if I bring in an negative charge, it gives you a confinement. My potential energy looks like r squared. The simple harmonic potential if you move away from the origin, the field of course equal to 0 at that point. There is a minima that r equal to 0.

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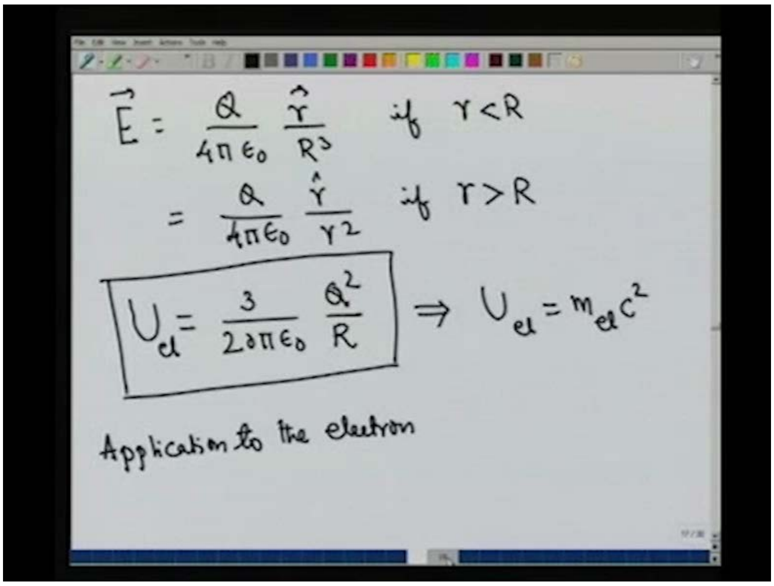

So, what is my field? My field is nothing but Q over 4 phi epsilon naught r by R cubed if r less than R and this is equal to Q over 4 phi epsilon naught r by r squared if r greater than r. Notice that the field is continuous at the point r equal to R. Now, we are in a position to evaluate this integral. So, let me not spend too much time evaluating the integral. We have already performed such integrals. Let me give you the result. So, believe me, go back and verify.

What I am going to get for you is nothing but 3 over 20 phi epsilon naught q squared by r. This indeed is the total energy that is required to assemble charge in a certain volume V, and what is that volume? That volume is given by a sphere of radius r and the total charge is Q. So, we have this much energy which is given to which is required in order to assemble that.

Now, all of you have heard of the name of Lorentz. Lorentz was a very bright man. He had already learnt relativity. So, we are introducing a concept of relativity right now, and what does relativity tell us? He tells us associated with every energy. There is mass, and associated with every mass, there is an energy. So, this means that there is a certain mass because there is a certain electrostatic energy.

So, in order to emphasis that, let me put E l; that means there is a certain electrostatic energy; there is an associated electrostatic mass. So, what do I do? I write U electrical is equal to m electrical c squared. So, this means that my mass, there is a certain mass associated with m electrical c squared, but then, I already know that there is a certain mass associated with a char distribution because we have not yet seen charge without any mass, electron, proton, neutron, etcetera, etcetera. Therefore, what we shall do is to consider a simple case. The simplest case is to consider the application to the electron. That is what Lorentz did. So, what we shall do is to look at application to the electron. So, what shall I do? I shall now specialize this application.

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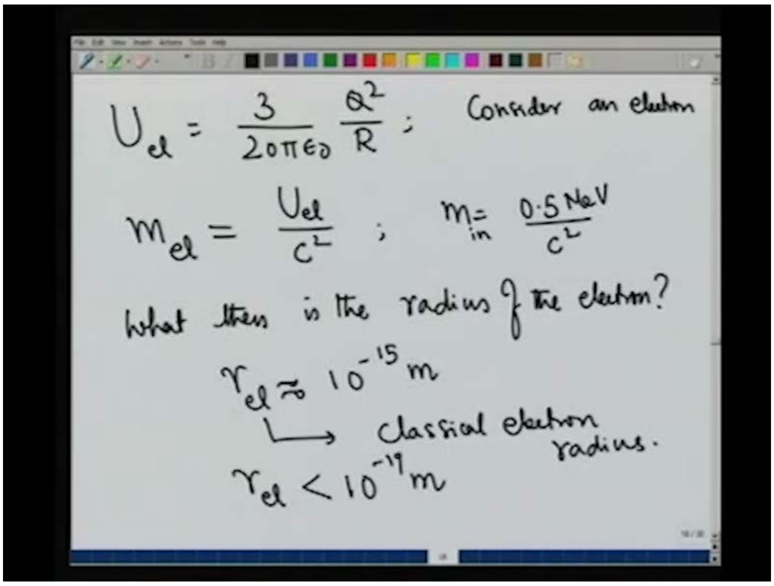

So, U electromagnetic was 3 over 20 phi epsilon naught Q squared by R. This is a generic expression. Now, consider an electron. What Lorentz says is that corresponding to this energy, there is an electromagnetic mass which is given by U electrical divided by c squared. It has a dimension of the mass, but on the other hand, if I consider an electron, it as its inertia the usual mass which is given by 0.5 m E V by c squared roughly, 0.5111, whatever, does not matter about that, 0.5111 m squared, and suppose I take an electron and put it in an external field, what happens? My electron moves because of this inertia.

When electron is moving, of course it is dragging this much energy along with it. The electro static energy is also moving Q squared by r. So, how many kinds of energies that we are going to write? How many of masses are we going to write? So, with a stroke of genius, Lorentz said let us indentify this inertial mass with the electromagnetic mass.

So, now, we are trying to do some physics which goes beyond electrostatics that we are doing. We want to argue that my electron has a mass because it has a charge, but the minute I do that, I know that my m is fixed; my q is fixed; epsilon naught is fixed. The only free parameter is r. Therefore, the question is what then is the radius of the electron? So, this is the question - what then is the radius of the electron?

Well, let us plug in the expression. I ask you to go back, look at the table and write down the expression. If you did that, my radius of the electron will turn out to be roughly 10 to the power of minus 15 meters. So, it is of that order 10 to the power of minus 12 to minus thirteen centimeters. So, up to order of 10 to the power of minus 4 to 10 to the power of minus 15 meter, and indeed if Lorentz is right, then we should be able to perform an independent experiment to see that my electron has indeed a radius of this particular kind and this is called classical electron radius.

Unfortunately this idea does not work; this idea does not work because I gave this information to you sometime back. We know that the radius of the electron is less than 10 to the power of 19 meters. That is what the experiments tell me. So, electron cannot be looked upon as a marble like a rigid marble, you know, where there is a certain mass distribution and there is a certain char distribution, if at all it has a radius, it is less than 10 to the power of minus 19 meters. Perhaps it is less than that, perhaps it is a point particle, and now, notice as my radius goes to 0, my energy goes to infinity, it blots up and this is the called self energy problem.

Now, I am mentioning this to you because it does not mean that these ideas are not useless. Actually since I have brought in relativity, I have brought in c and all that, this example goes to show the limitations of our classical mechanics. If you do a more careful analysis, we can argue that quantum mechanics must become important around 10 to the power of minus 15 meters. My electron cannot be describe in terms of simple electrostatic phenomenon. That is something that those of you are interested in physics will take up later. I need not worry about it at this particular point, but this is one illustrative point that I wanted to tell you.

Now, to summarize the last two Lectures, what have we done? We have introduced the concept of a potential because of the curl free nature of the field, and what we have done is to exploit Gauss's law and show how potential can be evaluated. I have also told you how when the charge distributions do not have a simple form; it is easier to determine the potential and then go on to determine the field. That is something that I have told you.

And then, we made a distinction between the total potential energy of a given char distribution and the interaction potential energy between two different charge distributions and we have worked out a simple electrostatic case. Now, if you want to proceed further, we should now give up, we should give up this kind of a toy model example and look at real physical systems. In real physical systems, we cannot keep on dictating. This is the charge distribution; that is the charge distribution. Once I have a charge distribution and I bring in another charge, the charge distribution rearrange themselves because of the interaction.

As to what kind of a rearrangement takes place. Whether the charges are completely free to rearrange themselves or they are, whether they are not completely free to rearrange, this goes by the subject of electrodynamics in a media. If the electric charges are not completely free to rearrange themselves because of other interactions. The electrons are bound to the lattice. They are called dielectrics. If the charges at least a fraction of the charges are free to move and rearrange themselves in an external field, those are called conductors.

So, the real importance of the concept of a potential this charge distribution comes when we discuss conductors and potentials indeed an interesting physics idea. Since we are running out of time, we shall explore that in the next lecture.