**Engineering Physics – II**

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**Module No. # 03**

**Lecture No. # 01**

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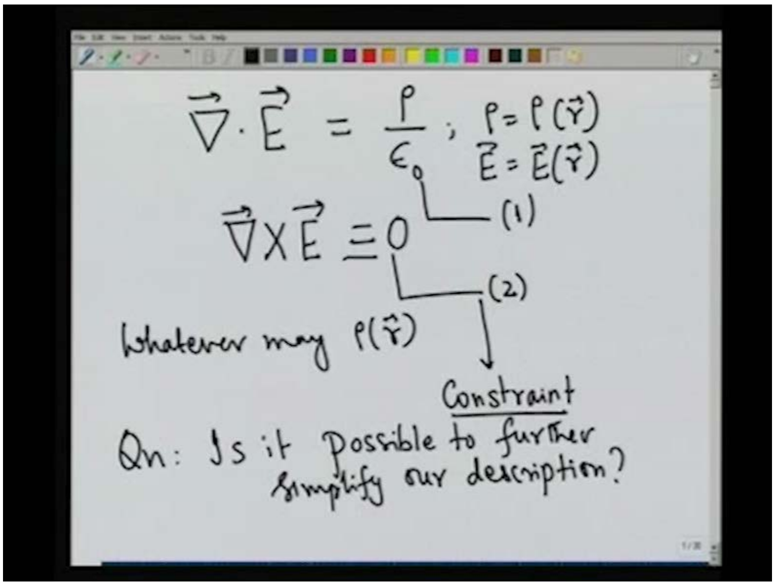


In the last lecture, we found that all the electrostatic phenomena can be described in the following set of equations. The first equation is of course gauss's law which tells us that divergence E is equal to rho by epsilon naught. Of course, rho is a function of coordinate and so is E, E is also a function of the coordinate. The second equation which supplements gauss's law which enforces that the phenomena that we are looking at is electrostatic, the charges are not moving is the additional condition on the electric field namely curl E is equal to 0. We were able to write this equation by looking at the explicit solution of the first equation and making use of the principle of super position. So, let me repeat.

All of electrostatic phenomena can be understood in terms of these two equations. Then we involve free charges, then we involve bound charges, dielectrics, conductors whatever there may be. So long as we are looking at phenomena which are electrostatic these two equations are sufficient. Before we proceed to discuss other things, we should notice that these two equations has slightly different character. In order to emphasize that let me call this as equation 1 and let me call this as equation 2. The first equation tells us how the charge density as a source creates the electric field. The charge density is the source for the field that is the electric field. So, these all a differential equation in order to obtain the solution for the electric field.

However, the second equation is not the solution of any source. It tells us that irrespective of what the charge density is. So, let me write it down. Whatever may be rho of r. It might be a point charge, it might be a dipole, it might be a quadruple, it might be a continuous charge distribution. Whatever it may be the solution for this particular equation, divergence E equal to rho by epsilon naught should satisfy an additional condition and that condition is given by curl of E equal to 0. In other words, while equation number 1 is a relation between the field and its source, equation number 2 is a constraint that is additionally required in order to fix the electric field completely. So, we say that this is in the nature of a constraint. So, this constraint tells us that this has always to be satisfied as an identity, therefore if you feel like I can put an additional line here which emphasizes that curl E equal to 0 is an identity.

Now, once I demand that curl of E must be equal to 0, it automatically follows that all the components of the electric field cannot be independent of each other. Please remember, what I am saying is that irrespective of the nature of the source, there is this constraint. All the components cannot be independent of each other. That means the natural question that we ask is if it is possible to further simplify the description of electrostatics. So, let us ask ourselves a following question. Is it possible to further simplify our description? And we have to provide an answer to this. This is the question that we are asking ourselves. What do I mean by this question?

Since there is a constraint which relates different components of the electric field, curl of E equal to 0. I ask myself whether it is possible to introduce lesser number of fields, my electric field is a vector field it has 3 components E X, E y and E z. Now, I ask myself whether it is possible to introduce less number of fields, a simpler field from which E can be determined and curl of E is automatically satisfied. The answer to this question is something that you know from your mechanics and from your vector calculus. So, I am not going to delve into it in any great detail, but let me just repeat it for you for the sake of completeness, for the sake of clarity, and what does it tell us?

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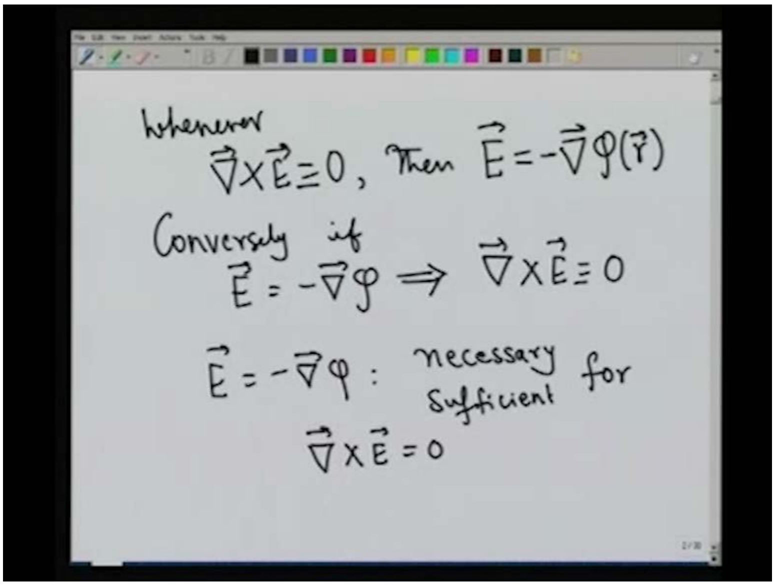


What you have learnt from your mechanics and vector calculus is that whenever curl of E equal to 0, here you can imagine that E is any vector field. Whenever curl of E equal to 0, then we can always write E to be the gradient of a scalar field, then E can be written as the gradient of a scalar field. And because I have a future application in mind, because that scalar field will be identified to be the electrostatic potential, let me introduce a minus sign that does not causes any harm. I can write it as the gradient of a scalar field and this scalar field is nothing but the electrostatic potential. So, this is the statement which I am not going to prove. I am going to I am only stating it. So, you can recall whatever you have studied from your mechanics or go back and look at the books. So, what the theorem tells you is that whenever curl of E is equal to 0, what do you mean by equal to 0? It is identically equal to 0 everywhere all over the space without any exception then E can be written as minus gradient phi of r all over the space.

On the other hand, there is something that you can very easily verify and that is conversely. Conversely if I (( )) to write E equal to minus grad phi. That is I do not state that curl of E equal to 0, but I give you that E equal to minus grad phi. Conversely if E equal to minus grad phi then this implies that curl of E equal to 0. How so, because the curl of a gradient is identically equal to 0. So, the essential statement that we are making is that E equal to minus del phi is necessary and sufficient for curl of E equal to 0. So, without any loss of generality, we can always replace our electric field by a suitable potential whose negative gradient will give me the electric field and do all our calculations, and these condition automatically satisfies the constraint that curl of E equal to 0. So, it is a good simplification, it is an excellent simplification, because now instead of dealing with three components of the vector field we are dealing with a single scalar field phi of r. So, this is a significant improvement in our analysis.

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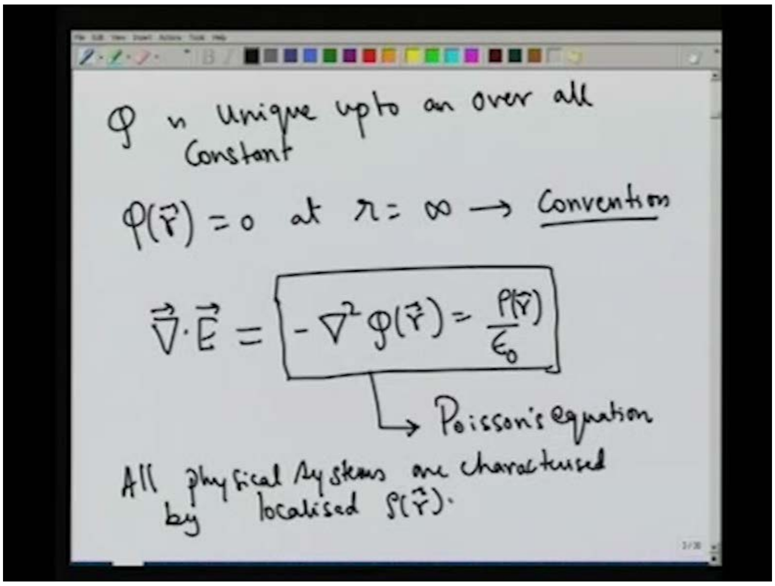


However, we should ask ourselves what the ambiguity or what the uniqueness in phi is. After all a is given by the gradient of a scalar field, so there should be some freedom in choosing the phi, the answer to get is also known to you. We know that phi is unique up to a constant an overall constant. What do I mean by constant? That constant is a number all over the space. I have to make a correction here, because my phi is not a vector field, it is a scalar field; phi is unique up to an overall constant. And without any loss of generality that can be fixed for example, by demanding that phi of r equal to 0 at r equal to infinity. So, this is the convention that we shall employ almost everywhere. So, once I employ the convention that I will choose the potential to be 0 at r equal to infinity then my potential is going to fix, my electric field uniquely to the relation E equal to minus grad phi.

Now, once I rewrote my electric field in terms of the potential field, all that remains is to solve the equation for the electric field in terms of the source except that I substitute E equal to minus grad phi for the electric field. So, what happens to this equation? My gauss's law becomes a second order partial differential equation, this becomes minus del squared phi of r is equal to rho by epsilon naught. In solving this equation, we are not directly solving for the electric field, we are solving for the potential. However, if I know the potential, I know the electric field, all that I have to do is to take a gradient and therefore, this equation is the fundamental equation which governs electrostatic phenomena. Because it is fundamental, it is given a name, it is a very important equation and it goes by the name Poisson's equation.

You people have already studied gravitation in your mechanics. Exactly the same arguments go over there also, because you start with the 1 over r squared force, and then you into you have the principle of superposition, and then what you do is to build any mass density through individual particle masses whatever they are. So, you can eventually set up an equation like this, the only difference is that there my rho will represent not the charge density, but the matter density. In this case rho can be positive, rho can be negative, but there rho is always positive and the force is always attractive, but otherwise the equations are identical. Therefore, Poisson equations are fundamental to both my gravitation the classical Newtonian gravitation and also electrostatics. In fact there is nothing much to distinguish between them mathematically except for the fact that the charges occur in two species. That is something that we have learnt at great detail in our earlier lectures.

Now of course, if you share at this particular equation, let me introduce this r here. We know that all physical systems are characterized by rho which are localized, so let me make that statement. All physical systems are characterized by localized that is the most important thing, rho of r, localized distributions. Now, I have to explain to you what I mean by localized distributions and the answer is very clear.

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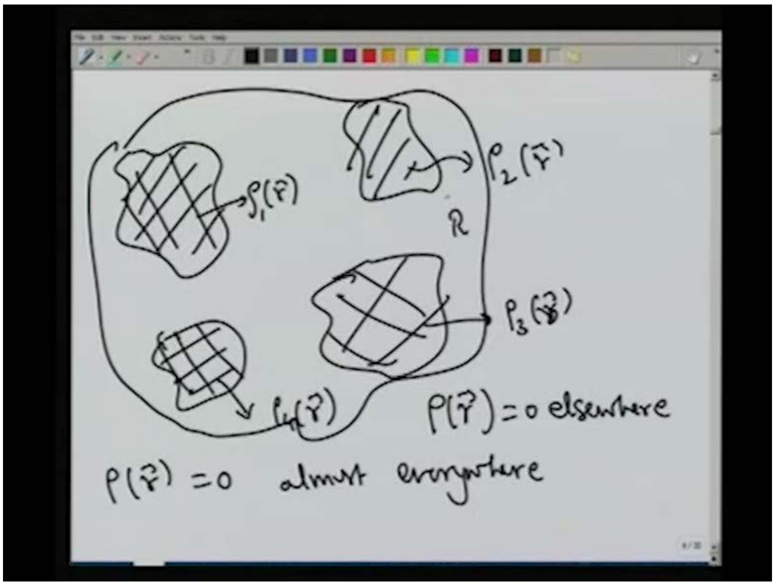

However, big your charge built up may be it is going to occupy a certain finite region in space. So, what I am going to say is that there is a certain region in space let us say which I denote like this, and my charge density is going to occupy this particular region. And even if I have a more complicated situation, there will be other finite regions. A finite number of finite regions in space, is that right? Finite region number of finite regions in space, where the charge densities will be sitting? May be this contains a certain charge density rho 1 of r, this contains another charge density rho 2 of r, this contains another charge density rho 3 of r, so on and so forth. So, let me call this rho 4 of r. But the fact of the matter is that so long as we are not looking at any mathematical idealization. I am looking at only physical situations, the truly physical situations. The only physical distributions that I am going to get are localized or distributions which are finite in number that is what we have.

But however the space is infinite that means that rho of r equal to 0 elsewhere. In fact a stronger statement that we can make is that rho of r equal to 0 almost everywhere almost everywhere. Therefore, rho of r not equal to 0 only in certain finite region in space. If I do not want to be very careful, I can actually enclose all these in a big space and call this are region r, but still it is only a small finite region. Therefore, rho of r equal to 0 almost everywhere. So, what is it tell us?

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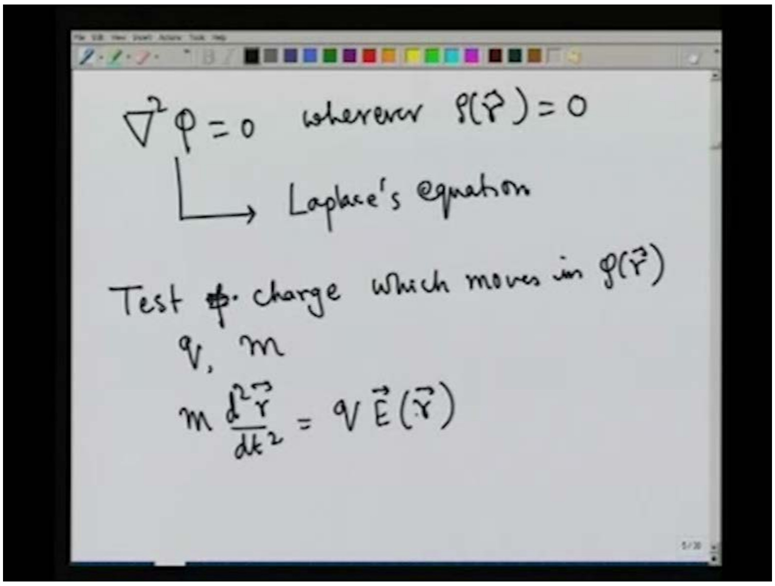


It tells us that almost everywhere my del square phi equal to 0. What do I mean by almost everywhere? Wherever rho of r equal to 0 and rho of r is non-vanishing only in a certain finite region in space, therefore this equation is also of importance, because those regions might actually correspond to charge distributions on the conductors. They will specify certain boundary values for the solution of this phi. This equation is also important and this goes by the name Laplace's equation. So, if you feel like what one can say is that the problem of solving electrostatics is in some sense equivalent to solving del squared phi equal to 0 with some specified boundary conditions at different points or different regions in space. Therefore, Laplace's equation is also a very, very important equation and we will realize that when we start discussing dielectrics and conductors that we will come in a few lectures late.

Having discussed this particular point, now what I shall do is to quickly go through the fact that this function phi has a natural significance of giving us the potential energy. That is something again that you have learnt premiere mechanics course through the conservative forces. So, let me not get into very great detail regarding them. Let me very briefly summarize them. So, what is it that we are going to do? What I shall do is to observe that if I am going to look at a test charge, this of course is the operative word, if I look at a test charge which moves in the potential field in phi of r. So, what I have is a certain rho of r which produces the potential field phi of r or equivalently the electric field of E electric field E and the test charge is going to move in that particular field. In moving so it is going to get affected. It is trajectory, its velocity is going to get affected by the electric field, but the test charge is not going to act on the charge density that is the reason why it is called a test charge. It is not going to act on the charge density. Therefore, it is going to be acted upon. My electric field shall continue to remain whatever it was even before I introduce the test charge.

So, let us say the charge is q and it is carrying a mass m, and what does Lorentz force tell us? Lorentz force tells us that the force is simply given by m d squared r by d t squared is equal to q into E at the location of the charge. And this electric field of course originates from this potential phi E is the minus gradient of i which has originated from the Poisson equation. Now, it is a very simple matter for us to work out the consequence of this. Remember curl of E equal to 0 that is E is a conservative force field in the language of mechanics that I have already learn.

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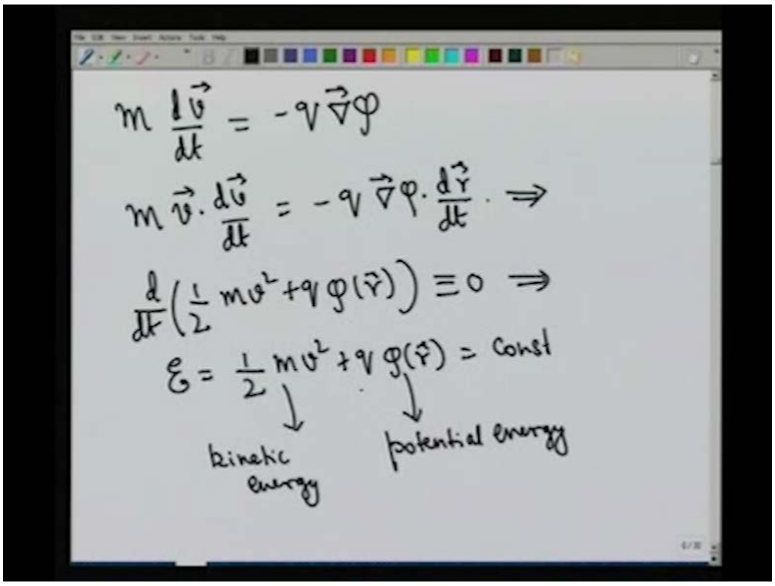


So, what I can do is to rewrite it as m d v by d t is equal to minus q grad phi. I have replaced d squared r by d t squared, the acceleration by d v by d t, and of course I have replaced my electric field by minus grad phi. If I were to dot both of them with the velocity, if I were to take the inner product, what am I going to get? I am going to get m v dot d v by d t is equal to minus q grad phi dot d v which is nothing but d r by d t. This is what I get.

Now, it is a very simple matter for us to check that this implies the conservation law. What is the conservation? Namely, that d by d t of half m v squared plus q phi of r identically equal to 0. This is identically equal to 0 that is my total energy of the system which is given by half m v squared plus q phi of r is a constant of the motion. This is a conserved quantity. Now, half m v squared is identified to be the kinetic energy of the particle kinetic energy of the particle and q phi of r is nothing but the potential energy. So, the potential energy depends on the potential produce by the source and also on the charge carried by my test particle. It can either be positive or negative depending on the relative sign between the sources that are produce in the field and the charge of the test particles themselves, this is a conserved quantity.

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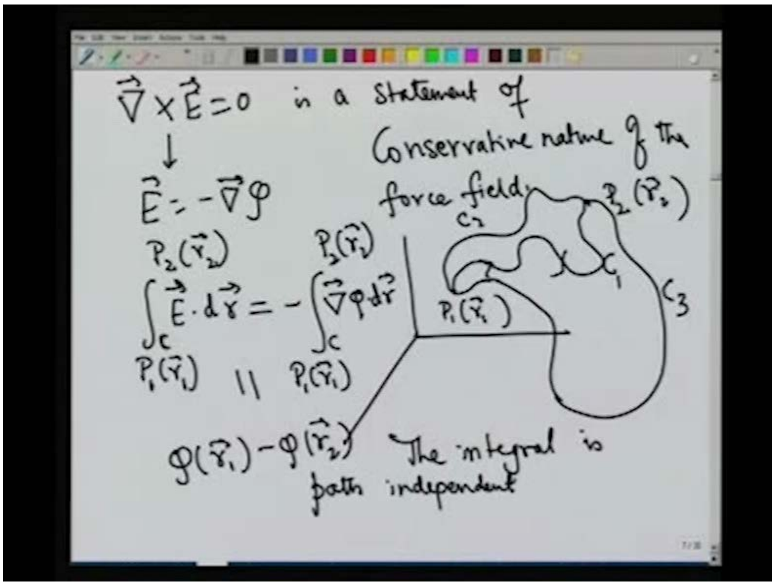


Now, I am not going to derive this equation for you. But what I would like you to realize, what I would like to emphasize is that this was possible, because curl of E equal to 0 is a statement of is a statement of what we call conservative nature of the force field. So, let me again state that all of you are very, very familiar with that. The statement that we are making is that if curl of E equal to 0 that is E is equal to minus grad phi. That is what we are writing. Then integral E dot d r, I am going to calculate the line integral of this function E dot d r. So, let us say that I start with a point p 1 whose coordinate is r 1, I go to a point p 2 whose coordinate is r 2 and I evaluate this curve. So, let me illustrate that here. So, this is my coordinate system, this is my point p

1 with coordinate r 1, this is my point p 2 with coordinate r 2 and I am drawing a curve like this. So, this is my curve. This is any arbitrary curve.

So, I would like to see what E dot d r is. As I move along this particular curve, as I move along this particular curve, what the value is between p 1 and p 2. But because E equal to minus grad phi, this can be simply written as minus whatever that curve is, p 1 of r 1, p 2 of r 2 is what I have into gradient phi dot d r, and as we are all familiar with the properties of the line integral of the gradient function. Remember there is a minus sign here. This is nothing but phi of r 1 minus phi of r 2. What am I trying to tell you? Both these integrals are evaluated along a particular curve. This is one particular curve, for example I could choose another curve. So, if I call this as c 1, this will be c 2, I could choose yet another curve which comes in this fashion, so let me correct it here and it comes here, this takes could call c 3.

And normally we would have expected that the value of this integral would depend on the initial point, the final point and the curve that is chosen. But this result clearly tells us that this line integral is a function entirely of the initial point and the final point, it does not matter to us which curve that we are going to check. In other words, the integral is path independent. The integral is path independent, and this is this feature that makes it possible for us to actually define the concept of a potential and then associated potential energy. Normally in mechanics we are all the time almost always dealing with conservative forces, unless I introduce something like a friction. But electrodynamics is much, much more complicated. Because as you all know that whenever there is a time dependent magnetic field that also induces an electric field. Here the source of the electric field is not the magnetic is not the charges, but it is the magnetic field. And how does it induce?

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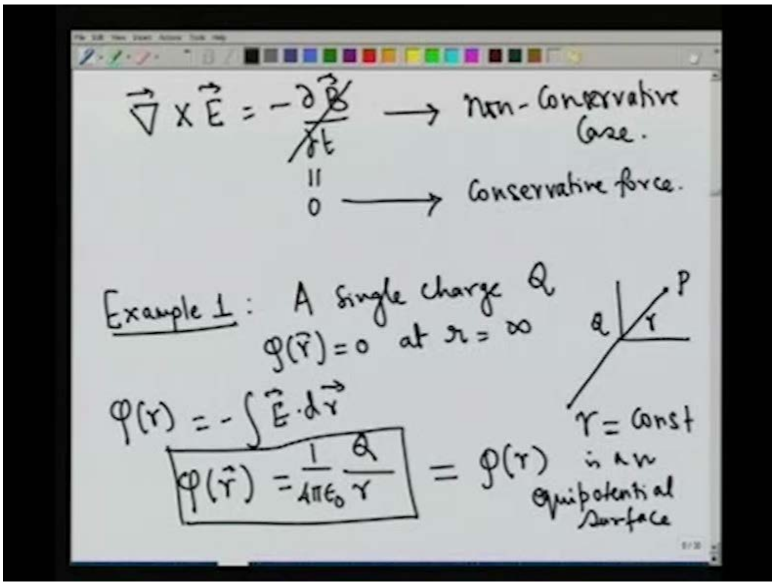


At a later time, I am going to write an equation curl of E is equal to minus delta b by delta t where b is the magnetic field. Now, what is happening here is that the change in the magnetic field, in fact the change in the flux induces a change in the EMF that is induces an EMF. That is the statement that we make in your induction. That is what you have studied in your 12th standard. Here you see that curl of E is not equal to 0, it is given by minus delta b by delta t. In that case my electric field will not be a conservative force field and the simple potential energy description will break down. Therefore, in that case the natural question to ask is, does it mean that there is no conservation of energy? The answer is no, there is indeed a conservation of energy. But this simple concept of a q phi is something that is going to break down. We have to add additional terms, in fact we have we have to add terms corresponding to the field energy itself. That is something that you have to remember. So, this corresponds to the so called non-conservative case.

However, at this stage we do not have to worry too much about this particular thing, because we are looking at static phenomena. Even if there were a magnetic field my b would be independent of time, therefore I would say that equal to 0, curl of E continues equal to 0 and then this will give rise to conservative force. So, now we appreciate what the role of the potential is. Let me repeat. The fact that curl of E equal to 0 allows us to describe electrostatic phenomena in terms of a single scalar field called phi whose gradient whose negative gradient will give me the electric field. Now, what is the solution for phi? The answer is very, very simple, if we appeal to the principle of superposition. So, what I shall now do is to look at various cases, I will toggle between application and the explanation, and let us write down the solutions for phi.

So, let me consider the first of the examples namely, the simplest of the example that is a single charge q. Let us ask ourselves what is the potential produced by a single charge q, and let us not forget that I am going to impose the condition that phi of r equal to 0 at r equal to infinity. As r goes to infinity my field shall behave in such a way that it goes to 0. The solution is very, very simple, because I can simply integrate. My phi of r is nothing but what; minus integral E dot d r. We all know what my electric field due to a point charges. Let me locate the point charge at the origin rather; let me choose the origin of my coordinate system at the location of the point charge. So, my q is sitting here.

So, if I impose the condition phi of r equal to 0 at r equal to infinity, I simply get phi of r is equal to q by r with a factor of 1 over 4 pi epsilon naught; this is what I have. So, I have phi of r is given by 1 over 4 pi epsilon naught q by r. It is only a formal thing that I have written it as a vector; actually my phi is independent of the angle. It is a function only of the distance of the point p from the origin from the location of the charge. In particular, if I were to move on the surface of a sphere of radius r. So, this is my r if I were to move on the surface of a sphere of radius r, what would happen? My potential would not change at all. If my potential is not changing at all, if my charge particle were moving on the surface of the sphere, there is no change in the potential energy. Therefore, there can be no change in its kinetic energy. This surface is what we call as an equipotential surface. So, r equal to constant. That is the definition of a sphere is an equipotential surface.

When I consider more complicated charge distributions, the equipotential surfaces become more complicated. We can see that because the field is radially outward, my electric field is actually always perpendicular to the equipotential surface. So, in some sense if I can draw equipotential surfaces corresponding to complicated charge distributions, I almost immediately get an idea of the nature of the electric field even without actually calculating the gradient of the scalar potential.

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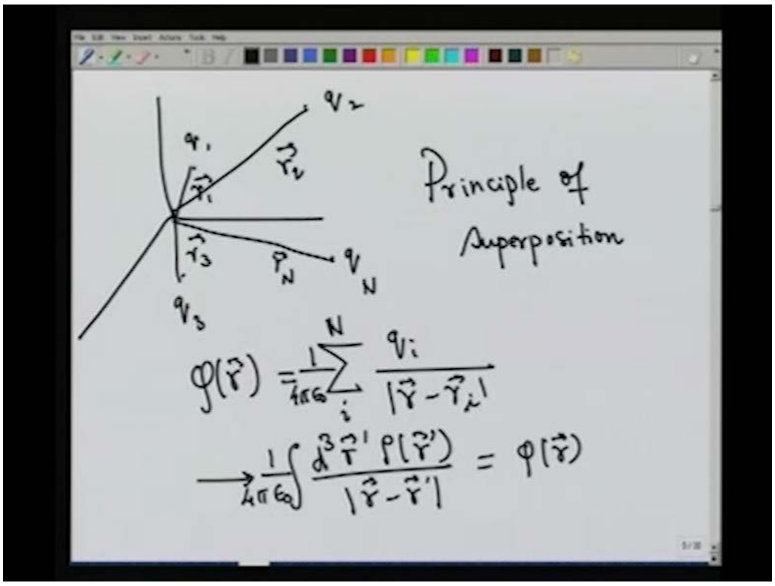


Once I do that I can immediately write down the potential due to a distribution of charge particles. So, let me look at this coordinate system, I am going to imagine that there are N charged particles, so this is q 1, this is q 2, this is q 3, etcetera, and let me call this as q N. There are N charged particles. This has a position vector r 1, this has a position vector r 2 the location, this is r 3 and in a similar manner this is r N. So, with respect to some coordinate system, the position vectors corresponding to q 1, q 2, q 3, q 4, q N is given by r 1, r 2, r N etcetera. And what we have to write down do is to write down the potential and that is very simple, because if we know how to write down the potential due to a single point charge. I know how to write down the potential due to this collection of point charges, because because of the principle of superposition the total potential is nothing but the sum of the individual potentials.

So, what we shall do is to employ principle of superposition. If we employ the principle of superposition my phi of r is simply given by summation i q i mod r minus r N. Please remember r N is a fixed quantity, each r is a fixed r i I am sorry I should not write r N, I should write r i here. r i(s) are fixed and r is my variable function, i goes all the way up to 1 up to N. This is my principle of superposition.

So, in some sense again the problem of determining potential is a trivial thing, because all that I have to do is to look at individual charges and add them up. And if now somebody comes and tells me look here, I do not give you individual point charges, forget all about your charge quantization etcetera, etcetera. I will give you a continuous charge distribution, no harm about that. We know how to actually compute the potential due to the continuous charge distribution. This simply goes to the relation d cubed r prime rho of r prime divided by mod r minus r prime. And I have to put a factor 1 over 4 pi epsilon naught here, I have to put a factor 1 over 4 phi epsilon naught here ,and this is the solution for the potential, because of the continuous charge distribution. d cubed r prime tells me that I have to integrate over the volume. That particular volume where my rho is non-vanishing, of course wherever rho vanishes there is nothing to integrate over and r is my variable. So, this is my phi of r whenever I have a continuous charge distribution.

So, in some sense we can claim that we have actually formally solved the problem of electrostatics. It is now entirely a matter of numeric's, probably with a good computer and a good programming to determine phi(s) and d(s) corresponding to all the complicated situations. But we know that physics is much richer than simply rushing to a calculator or computer and solving a problem. There is a much, much richer physics, because of the nature of the charges whether they are free to move, whether they are not free to move so on and so forth.

So, physics is not all about writing a formal solution of this particular kind, but actually appreciating simpler cases, abstract physical principles out of that and make statements which are sensible from the practical application viewpoint. After all, all our life is dominated by our dealing with conductors, with dielectrics, with c c circuits what are what so on and so forth, because I have to use these concepts and they are not yet defined here. First of all let us get a feeling for a few simple cases and then let us proceed to look at more complicated cases. So, let us do that. Before I do that I would like to show a few cartoons and a few pictures corresponding to various situations. So, let me start with the simplest of the situations, the point particle again. And let us see how the relation between the potential and the field can actually be demonstrated.

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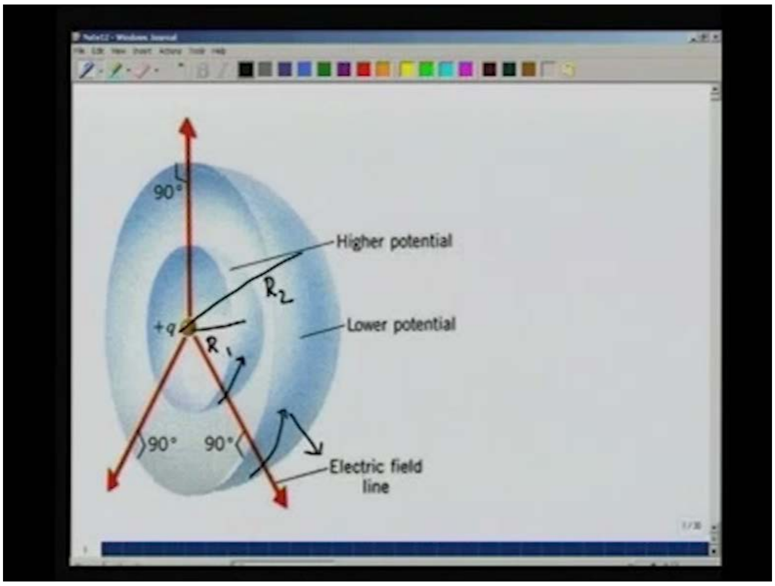


Now, this figure is a kind of a cartoon which tells you how the potential and the electric field behaves. I have rotated located a charge q at this particular point. So, I have located the charge q at this particular point, earlier I might have used an upper case letter plus q, but never mind about that. And what one has done is to show sections of spheres which are concentric. So, presumably the inner sphere let us say has a radius R 1, the outer sphere has a radius R 2, the sphere of radius R 1 is an equipotential surface, the sphere of radius R 2 is what is another equipotential surface.

I have chosen the potential to be 0 at R equal to infinity. So, as I go farther and farther outside the potential drops and that is what is indicated here. The inner sphere is at a higher potential compare to the lower potential, this sphere at a larger radius. So, the outer sphere is at a lower potential compare to the inner sphere, and of course, as you go very close to the charge the potential increases in an uncontrolled fashion. It goes like 1 over r, it diverges, never mind about that. And the red lines that I have drawn here, the brown lines they tell us that the field lines are intersecting this equipotential surfaces in a perpendicular fashion. The field lines are radially outside and therefore, if I were to move along this sphere, and if I were to move along this sphere I would not be changing my electric field at all. Is that ok?

The direction would change the magnitude would not change. Whereas, if I want to do some work what I should do is to actually come out of the surface, then the electric field would be able to do the work not if I were to move on the surface of the sphere. So, this is something that we are all completely familiar with. So, what we shall now do is to employ the same thing and look at a few more complicated situations. I have already written down the solution for the potential when you are given N charges. Now, let me specify it to a situation where there are only 2 charges, and I am going to write down everything explicitly, and we are going to look at the solution.

When I do that in one shot, I will be comparing two different physical situations; the first case is when there are 2 charges both of which are of the like sign, let us say positive, in fact I can make them of equal magnitude. The other situation is that when the first charge has the same magnitude as the second charge, but is of the opposite sign and that is your famous well known dipole. So, let us study both of them.

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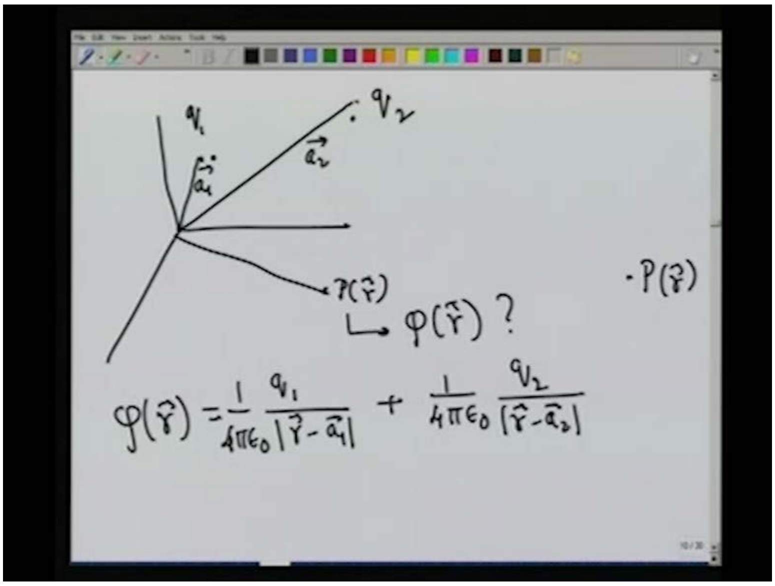


So, what I have is the following geometric situation. I have a charge q 1 here and I have a charge q 2 here. And let us say this charge the location is given by the radius vector a 1 and this is given by the radius vector a 2. These are 2 fixed charges. And you give me an arbitrary point E with a coordinate r, and I am interested in asking the question what is phi of r. So, this you see is a pretty simple problem, I do not have to do much about that. All that I have to do is to add this and add this. So, what shall I do? I shall write the solution to be my phi of r is nothing but q 1, I should not forget my factor 1 over 4 pi epsilon naught, q 1 over mod r minus a 1 that is what I have in the first situation, plus 1 over 4 pi epsilon naught q 2 over mod r minus a 2.

Now, if I were to ask what are the equipotential surfaces, without doing any calculation you can make a number of qualitative statements. If my r were very close to the charge q 2, then the dominant contribution to the field at this particular point would come from the charge q 2 and not from the charge q 1. Remember, the potential here will get a dominant contribution from this, because mod r minus a 2 is very close to 0. Therefore, this is diverging, whereas this is a finite number, however large it may be.

In a similar manner if I were to choose a point p which is very, very close to the charge q 1, the dominant contribution would come from this and it would not come from this. On the other hand, if my p is somewhere here which is roughly equidistance from these two points; that is mod r minus a 1 and mod r minus a 2 are comparable then both of them make comparable contributions and what is the final thing. If I were to choose my point p at this particular point let us say. That is my r is much, much larger than either a 1 or a 2 then what would we find? We would find that these vectors a 1 and a 2, the distances a 1 and a 2 could play a subdominant role, not a dominant role, not a leading order contribution, but a subdominant role in contributing to my potential.

So, these are the essential qualitative features that we should be familiar with. Therefore, if I realize that it is very easy for me to do the calculation. Let me not worry about the situation where the points are very close to either of the charges. It is not very illuminating to do the calculation when my point at which I am evaluating the potential is roughly equidistance, let me take a far off point and let me make a binomial expansion. So, let me repeat my expression phi of r again.

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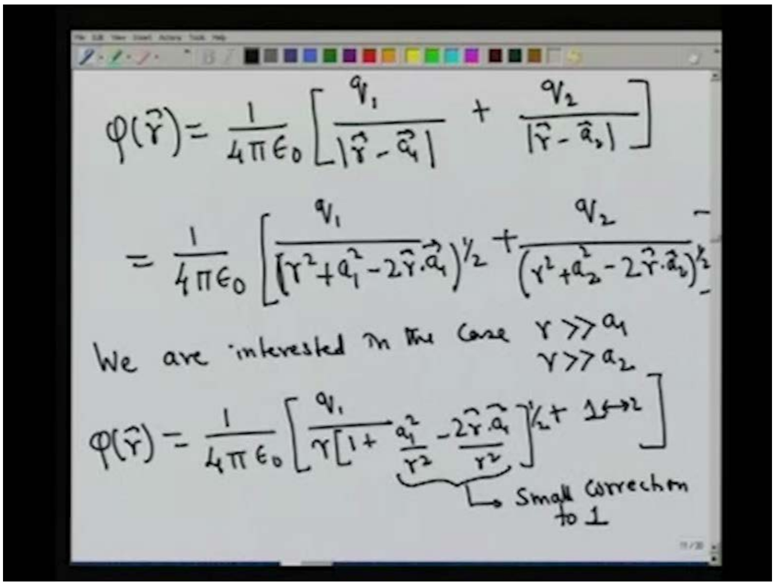


So, my phi of r is simply given by 1 over 4 pi epsilon naught, my charge q 1 divided by mod r minus a 1 plus charge q 2 divided by mod r minus a 2. All of you are familiar with this geometry from your 12 th standard course. So, I do not have to display any figure or any such thing for you. So, let me write down the solution. This is nothing but 1 over 4 pi epsilon naught divided by q 1. Let me expand the expression for mod r minus a 1 that is nothing but r squared plus a 1 squared minus 2 r dot a 1. This is the distance squared between the point r and the point a 1. Therefore, let me put a factor of half, because I am interested in the distance and not in the distance squared. And the next term will be simply given by q 2 divided by r squared plus a 2 squared minus 2 r dot a 2 to the power of half, so let me square.

And where are we interested in the solution? We are interested in the limit, interested in the case r much, much greater than a 1, r much, much greater than a 2. If r is much, much greater than a 1 and a 2 then r squared is obviously much, much greater than a 1 squared and 2 r dot a 1, r squared is again much, much greater than a 2 squared and r dot a 2. Therefore, what I can do is to pull the r out and make a binomial expansion. So, what does the binomial expansion give me? My binomial expansions gives me phi of r is nothing but 1 over 4 pi epsilon naught, so the first let me write it in 2 steps, there is no harm in that. I will pull a a out and this will be let me open a bracket here 1 plus a 1 squared by r squared minus 2 r dot a 1 by r squared plus let me symbolically denote it as 1 goes to 2. That is wherever there is 1 you replace by 2 that will give me q 2 divided by r into 1 plus a 2 squared by r squared minus 2 r dot a 2 by r squared. And if I were to observe that this is a small factor, small correction to 1.

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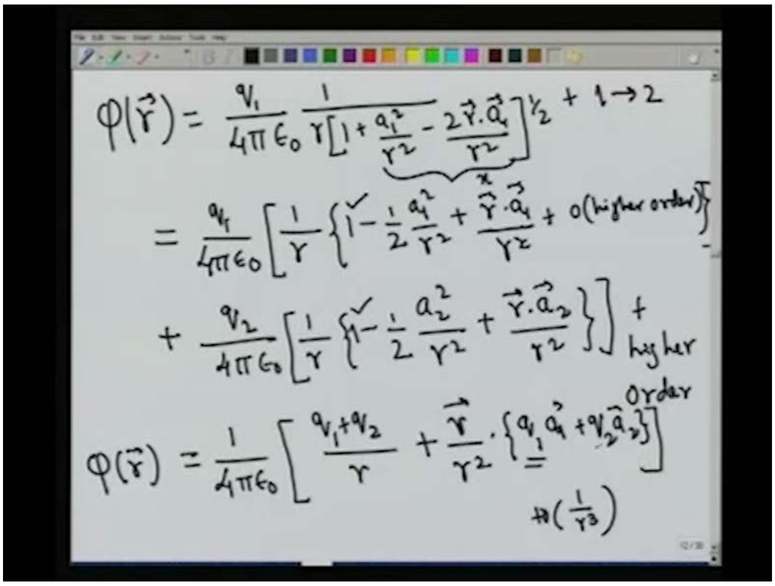


Notice that it is dimensionless, I can immediately perform the binomial expansion. I should not forget my factor of to the power of half. So, what is the solution that I am going to get? So, my potential is given by q 1 over 4 pi epsilon naught, I pulled out my factor r and wrote it as 1 plus a 1 squared by r squared minus 2 r dot a 1 by r squared to the power of half plus 1 going to 2. That is replace q 1 by q 2 and a 1 by a 2 and of course r remains the same and we have to make a binomial expansion. So, if I were to make a binomial expansion all of you are familiar with, how does it look like?

The first term is q 1 over 4 pi epsilon naught. Let me make the expansion for the first term. I have 1 over r and I have 1 plus let me call this as x to the power of half. So, 1 over 1 plus x to the power of half can be simply written as 1 minus half x which will give me 1 minus half a 1 squared by r squared plus r dot a 1 by r squared plus higher order terms. So, let me write it as higher order terms. So, this is what I am going to get. In a similar manner the next term is going to be q 2 over 4 pi epsilon naught, let me open the bracket again, 1 over r, this time it will be 1 minus half a 2 squared by r squared plus r dot a 2 divided by r squared, and again you have higher order terms.

So, by making a binomial expansion, we have been able to simplify our expression and now let me start collecting the terms. Now, I am going to arrange the terms in the decreasing order of contribution. So, you can easily see that there are powers of r. The first 1 is of the order 1 over r, the second term is of the order of 1 over r cubed, the third term is of the order 1 over r squared, because there is a r dot a sitting here. Now, there is a small notational error here, I had written r 1, so let me correct that and rewrite this part r dot a 1 by r squared. So, if I arrange them in the decreasing order of contribution, how does it look like? My phi of r looks like 1 over 4 pi epsilon naught, so let me open a big bracket here, the first term will be simply given by q 1 plus q 2 divided by r. So I have taken the contribution coming from these two terms.

So, in the leading order, the distances between the 2 charges are of absolutely no consequence to us or the distances of the 2 charges from the origin of the coordinate system are of no consequence to us, it is simply q 1 plus q 2 by r. And the next term is indeed of interest to us. So, how will I write that? I will write it in the following fashion. I will pull the 1 over r squared out, I am interested in these 2 terms, I will pull the 1 over r squared out. I will write my vector r dot here and write it as q 1 a 1 plus q 2 a 2. So, I have made another (( )) up in my notation, so let me quickly correct that. So, the correct thing is q 1 a 1 plus q 2 a 2, and then I am going to drop the next higher order term which is of the order 1 over r cubed we are not interested in that. This is order 1 over r cubed.

So, if you have a collection of 2 charges which are given with position vectors a 1 and a 2. And if I am interested in the value of the potential for far away, the leading order contribution is q 1 plus q 2 by r that is the famous monopole term. It is as if both the charges were situated at the origin and they produce the usual field q by r where q is q 1 plus q 2. The second term is indeed more interesting, what you get is a correction we has the given by the position vectors of a 1 a 2 waited by the charges q 1 and q 2. So, let us study both these situations independently.

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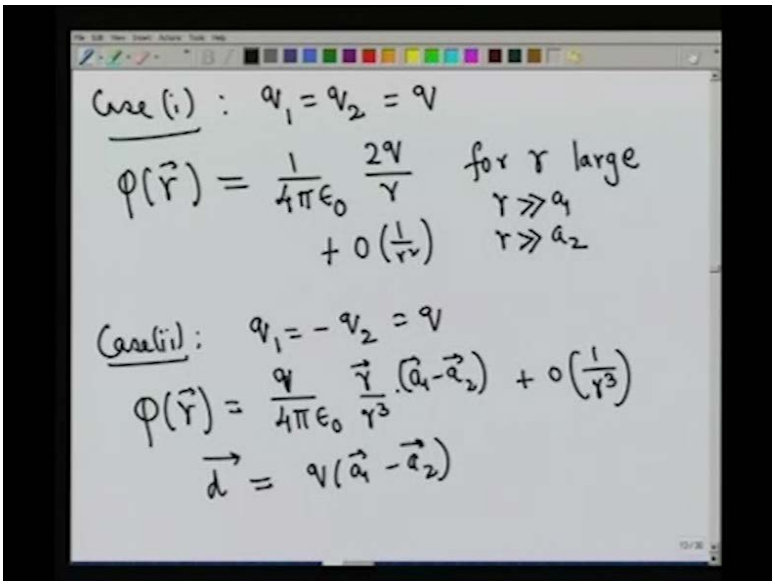

Now, let me let look at the situation which is case 1 namely, q 1 is equal to q 2 is equal to q. They have the same sign and the same magnitude. Then you can easily see that if I were to keep only the leading order contribution, this is nothing but 1 over 4 pi epsilon naught into 2 q by r for r large. What do I mean by r large? r much, much greater than a 1, r much, much greater than a 2. The next order correction is small compare to this particular contribution for sufficiently large r. Therefore, I can write higher order term 1 over r squared and that is a subdominant term that is not the dominant term.

However, the next case is really interesting and that indeed is ubiquitous. It occurs everywhere in electrostatics that is when q 1 is equal to minus q 2 equal to q. So, you have 2 charges of equal magnitude but opposite sign. Now, what we find is that the minute we write that the first order term drops out and the leading order contribution is what was subdominant in the earlier case. So, how does my potential term look like? Now, my phi of r has the character 1 over 4 pi epsilon naught, what is it that I wrote? I wrote N r divided by r squared dot q 1 a 1 plus q 2 a 2 which will be the same as a 1 minus a 2 into q. So, let me put my q here. This is how it looks like.

So, here you see that my field is behaving in a slightly different fashion except that again I forgot the factor cubed that is to occur here. Therefore, I am getting a 1 over r cubed. The leading order contribution is simply given by 1 over r squared and the correction is of the order 1 over r cubed. Now, there is a standard notation that one employs in order to describe such a situation. So what we shall do is to define the dipole moment corresponding to this charge distribution which is nothing but q into a 1 minus a 2. Please mind you that such a definition is possible if and only if the total charge is equal to 0. If I define this to be d into q into a 1 minus a 2, I now have a very, very elegant expression for my phi of r. This is nothing but 1 over 4 pi epsilon naught d dot r by r cubed.

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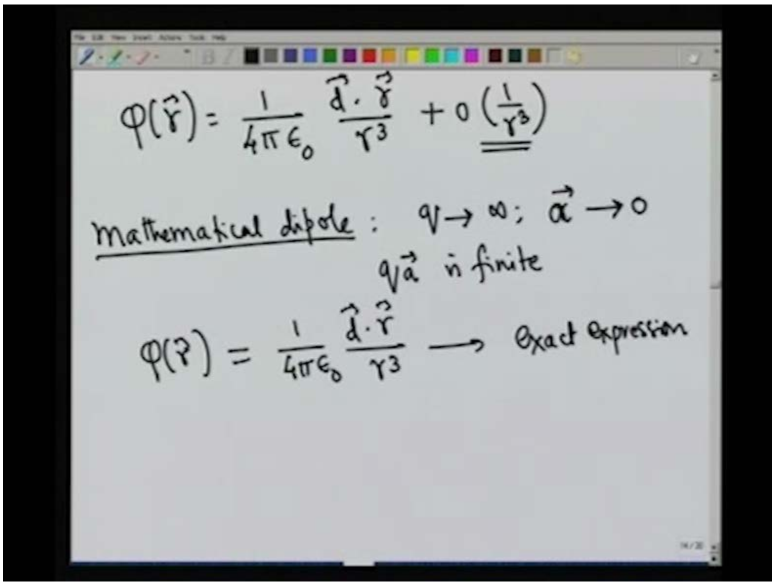


So, the potential is not spherically symmetric anymore. There is a 1 over r cubed term all right, but if we look at the numerator the potential depends on the angle between the position vector at which I am calculating and the direction of the dipole moment. This is something that you have to remember it cannot be spherically symmetric. However, there is a cylindrical symmetry. If I fix the angle between d and r, and move in a plane such that the angle is fixed then phi of r will be a constant. There is a cylindrical symmetry along the axis that connects the two charges - equal and opposite charges, but there is no spherical symmetry.

Now, I should not forget to add the term; terms of the order of 1 over r cubed, because if I were to keep higher order terms this would also contribute. Therefore, whenever we cannot ignore in the strictest sense, the terms of the order 1 over r cubed my system is called a physical dipole. So, what is a physical dipole? A physical dipole is obtained by looking at two equal and opposite charges separated by a certain distance a, let us say.

However, for purposes of convenience, I can also introduce what is called as a mathematical dipole. So that is of course an idealization like almost everything else. So, in the case of mathematical dipole which is an idealization what I will do is to take q go into infinity that is q 1 goes to infinity, q 2 goes to infinity, q 1 minus q 1 is equal to minus q 2, the distance between them let me call it as a a goes to 0, but q a is finite.

Now, what you people should do is to look at the higher order terms, all of you know how to do the binomial expansion, and verify that in this limit, all these higher order terms do not contribute. Then in that particular case phi of r is simply given by 1 over 4 pi epsilon naught d dot r by r cubed. So, plus no correction, this is an exact expression this is an exact expression. In reality we will never find a mathematical dipole, we always deal with physical dipoles, but then it is many, many times convenient to stick to this expression and not worry about the higher order contributions, and therefore that is something that we are going to do the same here.

So, I have discussed two cases; q 1 equal to plus q 2, q 1 equal to minus q 2, therefore it is natural to ask how the potentials surfaces, the equipotential surfaces will look like in each of these cases, and how the field lines will look like in each of these cases. In order to illustrate that what I shall do is to show you two figures which actually tells you what one should be doing. So, let us do that. These are the two figures.

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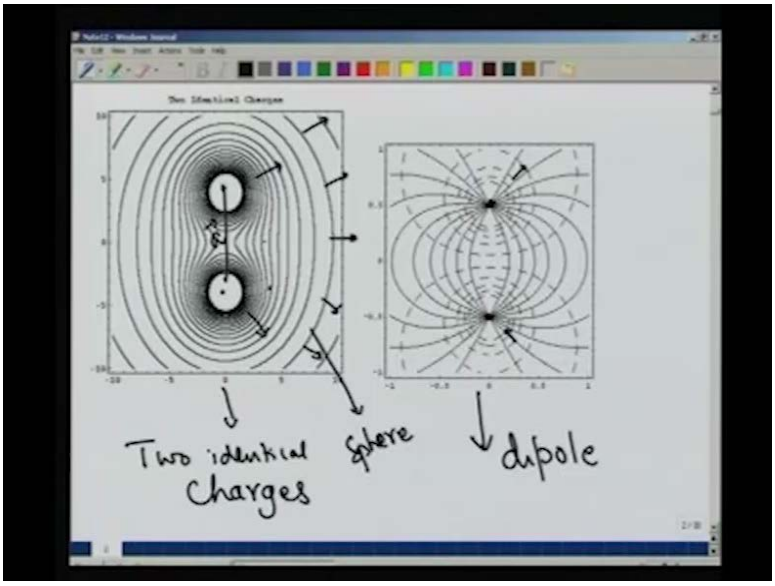

Now, this corresponds to two identical charges and this corresponds to the dipole. What are the essential features that we have to know? Both of them are not drawn to the same scale, so please do not try to make any other comparison. Very close to either of the charges, the equipotential surfaces are all spheres. Here also very close to either of the charges, the equipotential surfaces are all spheres. Here the equipotential surfaces are shown by broken lines, the field lines are shown by what, the close the continuous lines. In this figure, the field lines are not shown, but we know how to indicate the field lines, all that you have to do is to draw perpendiculars. This is not spherically symmetric.

So, if you are very, very close to the either of the charges, there is nothing much that happens. Is that ok? You see essential the field or the potential produce by one of the charges. So, there is not much of a distinction. The only distinction is that if I take this to be plus q and this to be minus q, the field lines will be divergent outside, and the field lines will be going inward; whereas, here the field lines will be going outward in both the cases. The interesting situation occurs when I look at intermediate distances, if I place a test discharge here, in this case it gets attracted by one charge, it gets repelled by one charge. So, you see the characteristic dipole lines in this case 2, whereas here the field lines are not going to converge, they are going to form some surfaces which are enveloping both the charges. They are not terminating on the charges, the way you would like to them to have, both of them are repulsive.

And as you go farther and farther away this is a very good approximation to a sphere. Of course it is not seen here. If I were to draw a very large sphere it would be a sphere, because in that situation you will forget all about the distance, let me call it as sphere between these two spheres, but here as you go farther and farther away you do not get a sphere, you get a characteristic 1 over r cubed. The surface becomes more and more complicated. Therefore, you see that indeed the behavior of a dipole is very, very different from the behavior of a identical charges sitting displace with respect to each other. In one case for large distances the leading order contribution is 1 over r, the so called monopole term or the single charge term. Here the leading order contribution is 1 over r cubed.

What I would argue people is to go back, write down the expression for the potential coming from this, write down the expression for the potential coming from the dipole, take the gradient and study the properties of the fields. Convince yourself as to how to compute the equipotential surfaces here and how to compute the equipotential surfaces. These are perhaps worked out in many, many books. But you should do it for yourself without looking at any book, and then only compare your results with the standard results that will go a long way in order to help you in understanding. Now, what I would like to do is to give you a few more problems in order to exercise yourself. I would have like to work it out, but we do not have that kind of a time. So, let me state a few problems for you to work out and let us go step by step. The dipole was obtained by looking at two charges equal and opposite. Actually one can construct what are called as higher order multi-poles; dipole, quadrupole, octupole and so forth.

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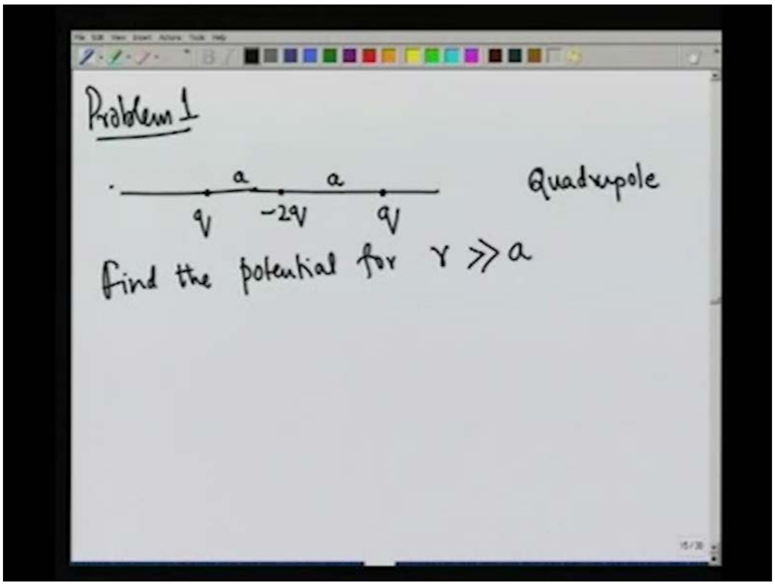


So, let me give an example of an octupole. An octupole is a system of two dipoles which are separated with respect to each other. So, what I will do is? I will take this line, if you want you can take this to be z axis. So, let me take this to be the origin and let me place a charge minus 2 q here; let me charge plus q here, let me place a charge plus q here. This distance is a, this distance is a, and this system is what is called as a quadrupole. This system is what is called as a quadrupole and you can easily see q and minus q forms one particular dipole, minus q and q forms another dipole. In one case the dipole is in this direction, in another case the dipole is in this direction, I have superposed it to produce a quadrupole. So, this is problem 1 for you people to solve find the potential everywhere; find the potential for r much, much greater than a, and you will see that the leading order contribution comes from 1 over r cubed. That is problem number 1.

Now, I am to give you two more problems which are quite interesting. And in these problems we are not going to use the principle of superposition, but we are going to employ Gauss's law, and what we are going to do is the opposite. Let me explain myself. Suppose I gave you an arbitrarily charge distribution which is very difficult to evaluate by using any class standard analytic techniques. Then it is always simpler to determine the potential everywhere, and then take the gradient and determine the field - the field components. But if there is a problem with a very nice symmetry like spherical symmetry or cylindrical symmetry, then we know that Gauss's law immediately allows us to determine what the field is. Once I know the field I can simply integrate it in order to get the potential therefore.

If there is a symmetry it is good to determine the field first and then determine the potential, if there is no symmetry if you have a complicated description then it is probably better to determine the potential, and then the field potential and then go to the field. So, in order to illustrate that let me give you the following problem.

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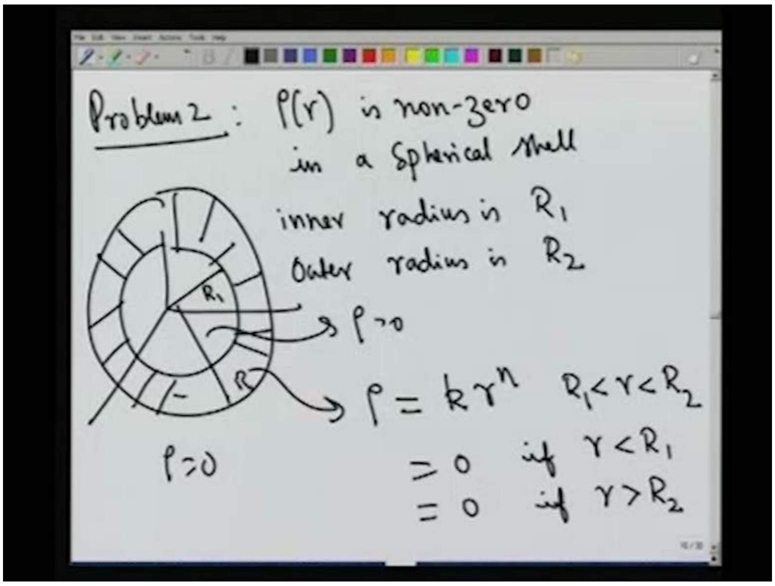


So, let me imagine a spherically symmetric case. So, this is problem number 2. Let me imagine a spherically symmetric case, where my rho of r is non-zero rho of r non-zero in a spherical shell in a spherical shell. So, whenever I speak of a spherical shell there should be an inner radius and an outer radius. So, inner radius is R 1, outer radius is R 2. So, how do I denote that? Let me try writing that, so you have a sphere, you have another concentric sphere, this is R 1, this is R 2, my rho is non-vanishing here. So, here rho equal to 0, here rho equal to 0, here rho is given by k r to the power of N that is the most important thing. Let me emphasis R 1 less than r less than R 2 equal to 0, if r less than R 1 equal to 0, if r greater than R 2. The problem that I am asking you people to solve is to make Gauss's law and solve for the electric field everywhere.

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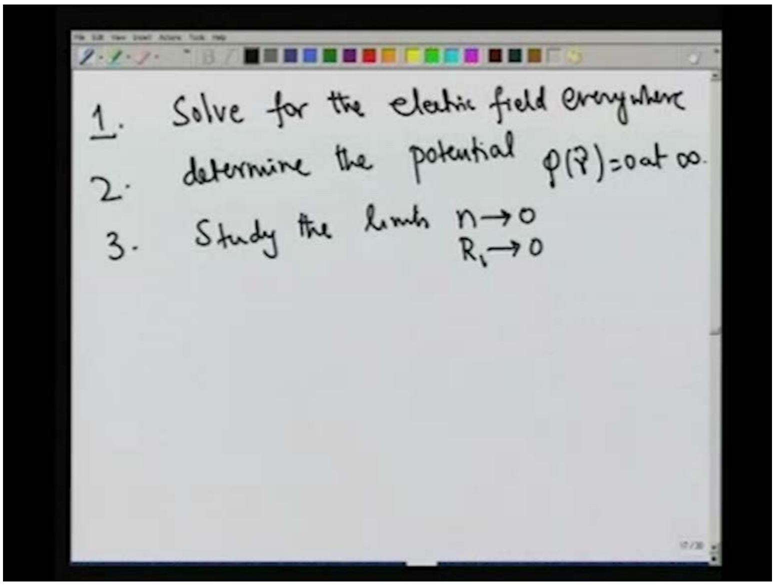


So, first solve for the electric field everywhere. Second determine this potential. And how do you choose your 0 of your potential? Choose phi of r equal to 0 at infinity. No problem about that. Once you do that you draw that and now you see you have two free variables, and thirdly study the limits thirdly study the limits N going to 0, R 1 going to 0 that corresponds to the well known case which is described in most of the books. Since we are running out of time, there is yet another problem that I would like to give you which I will give you in the next lecture, and then we shall continue to study the potentials. .