Engineering Mechanics
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Module – 03
Lecture No - 02
Properties of Surfaces – II

In the previous lecture, we had defined quantities like the.

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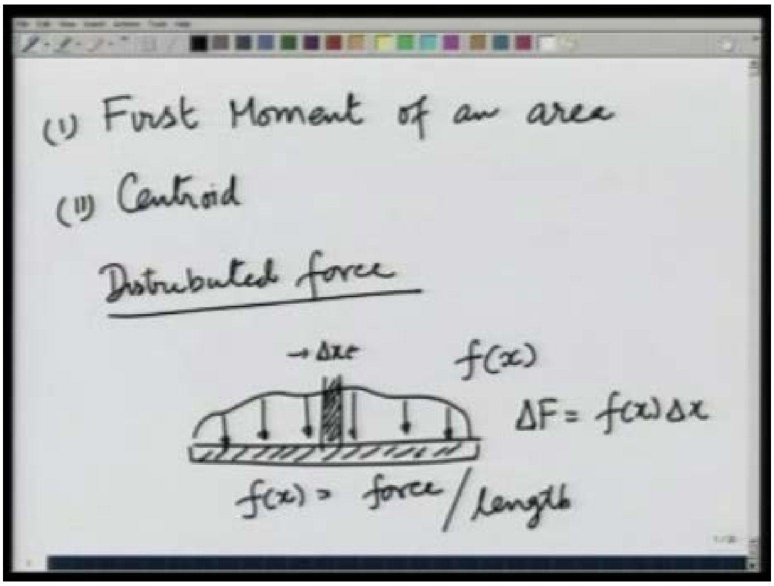

First moment of an area, and centroid mathematically, and said that these are related to problems in mechanics. In this lecture we are going to solve some problems using these concepts as I had indicated towards the end of my previous lecture. On this topic the place, where we use these concepts is where we have a distributed force, what do we mean by that?

For example, if I have a beam and there is some mass on top of it, so that it applies a force on the beam. The force may be described by a function f x. So, that if I take a small section here delta x length, the force on this section delta F is equal to f x times delta x. So, f x is nothing, but force per unit length. It is in dealing with such distributed forces that the concepts developed in the previous lecture are going to be ending.

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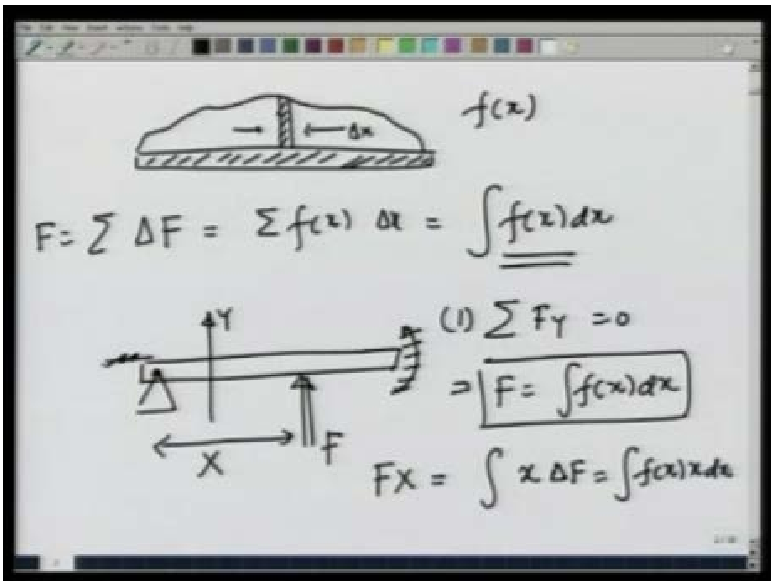

So, the question we ask is given this distribution of force f(x). What is the total force on the system? And where effectively is it acting. Let me explain that will further. The total force is going to be summation of that delta F that is acting on a small section of length delta x. So, this is going to be summation f(x) delta x, which in the limit is going to be integration f x d x. That is a net force and when we say where effectively is it acting. That means, what moment or torque should I apply to this beam in order to keep it in equilibrium. For example, if I have this beam fixed at this point or let me put a pin joint here, what torque should I apply here in order that this beam is in equilibrium or equivalently? At which point should I apply this net force F that I have calculated above here.

So, that the effect of this force both the torque as well as the net force is nullified to do that 1. We require that summation F y, where y is this direction be 0, and that gives me the net force F should be equal to f(x) d x. The second equilibrium condition is that the torque about this pin joint vanish and that requires that the distance of the force that I am applying, call this X be such that it nullifies the torque generated by this force f(x). So, F times X should be equal to summation x delta F, which is nothing but integration f x x d x. So, on this beam whether the force distribution f(x).

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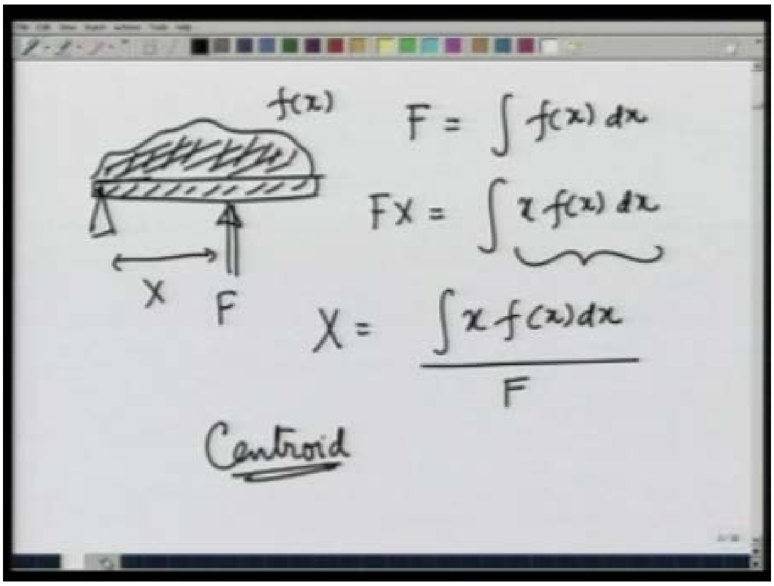

Then I have the net force F that I am supposed to apply this is about this pin joint to equilibrate. The beam is going to be f x d x and I should have F times X, where X is a distance at which the force is being applied equal to integration x f(x) d x, which is nothing but the moment generated by the force distribution F X. And therefore, X equals integration f(x) f(x) d x over F, and this by definition is the definition of centroid.

Therefore, the net force is the area of this force distribution curve and the point at which effectively this force acts is the centroid of this area formed by the beam, and this force distribution curve. That is how we use the concept of the first moment or the centroid, I must point out that when the total force capital F is applied at the centroid, no other force is near to support. The beam that is in that situation the force applied by the pin joint will be 0, as such in the case of f(x) being the gravitational force the centroid gives a position of the centre of gravity. Let us take some examples.

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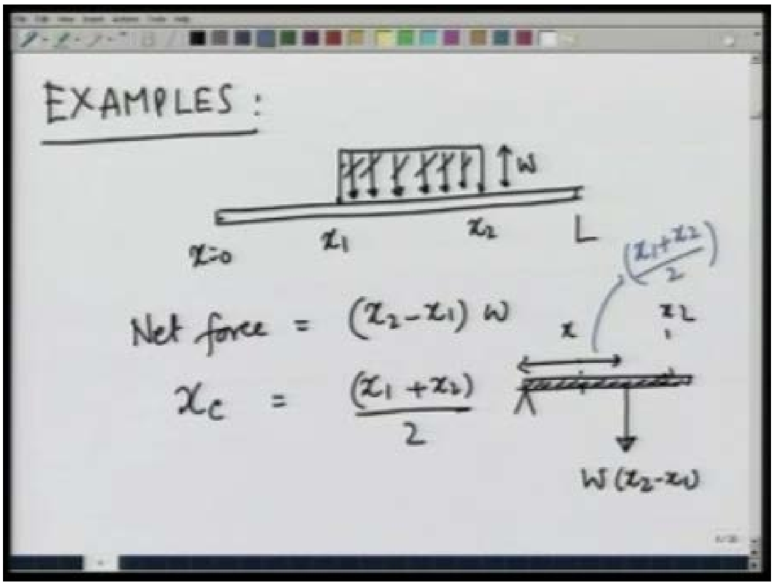

Suppose, I have a beam, let us call this point x equal to 0 of length L and there is a force distribution of the form of a rectangle from point x 1 to x 2. In that case, one can easily see, suppose this magnitude is w. One can easily see that the net force that the supplies is the area of this rectangle, which is going to be x 2 minus x 1 times w, and where does it act. It acts at the centroid of the area formed by this force distribution and the beam, and x centroid is nothing but x 1 plus x 2 divided by 2.

Therefore, I can replace this entire force or represent this entire force like this. This is a beam, this is where it is hinged, the net force is of the amount w x 2 minus x 1 acting at a distance, this is x 1, this is x 2 at a distance. Let me write it with blue this is x 1 plus x 2 divided by 2. That is one example next we consider triangular loading in that I have a beam of some length L.

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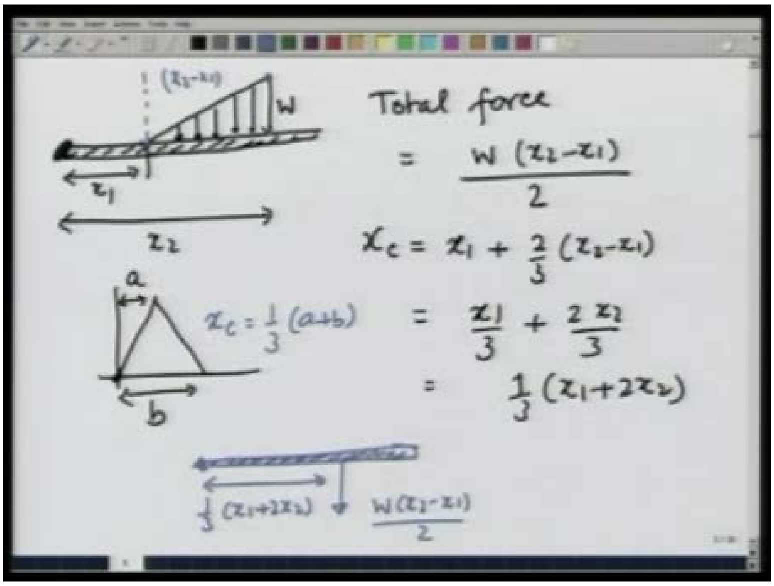

And it is loaded like this where the maximum load is per unit length is w load starts at a distance x 1 and goes all the way up to x 2, and what we want to figure out is how much is a total force acting on the beam, and where effectively is it acting? So, total force is going to be the area of this triangle, which is w x 2 minus x 1 divided by 2 and where it acts is at the centroid of this triangle.

Recall from the previous lecture, that if I am given a triangle, then with respect to 1 of this corners. If this distance is a and this is b, then the centroid x c is given as 1 third a plus b. In the present case, the 2 points are at distance of x 2 minus x 1 both the points are at a distance of x 2 minus x 1 from this corner. And therefore, the centroid x c is going to be at a distance from this point. The hinged here x 1 plus 2 third x 2 minus x 1 or this comes out to be x 1 over 3 plus 2 x 2 over 3 is equal to 1 third x 1 plus 2 x 2.

So, if I were to look at this load effectively, how it is working? If this is a beam then the load can be effectively replaced by a force of w x 2 minus x 1 divided by 2 acting at a distance of 1 third x 1 plus 2 x 2 form this point. This is another example of how we apply the concept of first moment and the centroid in mechanics. Next I will consider 1 more example where the loading is trapezoidal.

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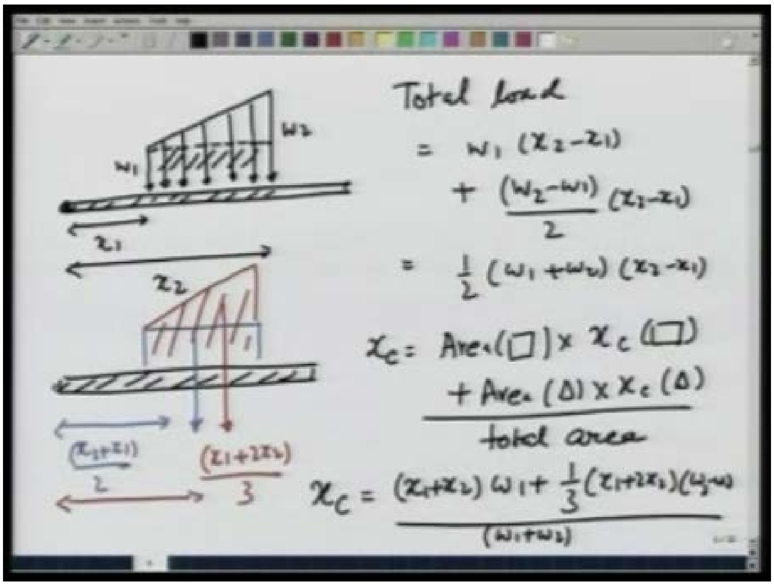

That means the loading starts at x one, but, it has a finite amount w 1, then it goes up to w 2 per unit length and distance x 2 along the way. This is how the load is distributed again with respect to this point where the beam is hinged. I want to find out what is a total load and where effectively is it acting. So, total load is going to be the area of this trapezoid I can for later references divide this area into 2 areas 1, which is corresponding to this rectangle of height w 1 and width x 2 minus x 1,and another 1 the triangle.

So, the total are is going to be the area of the rectangle, which is w 1 x 2 minus x 1 plus the area of the triangle, which is going to be w 2 minus w 1 divide by 2 times x 2 minus x 1, which comes out to be a area of trapezoid, which is nothing but, 1 half w 1 plus w 2 times x 2 minus x 1. So, this is the net load, which is working on this to calculate where it acts. I am going to use a an observation that we made last time I know, that this rectangle where load acts right in the middle at a distance of x 2 minus x 1 oh x 2 plus x 1 divided by 2 on this point.

Similarly, the triangular loading, which we just calculated in the previous slide acts at x 1 plus 2 x 2 divided by 3 distance from this point. So, I can take these 2 loads and then calculate, what will be the effective centroid for this entire area, and that we know from our previous lecture is going to be x c equals area of the rectangle times x c of the rectangle. This making it symbolically plus area of the triangle times x c of the triangle divided by the total area, total area which is the net force we have already calculated. We also know the positions of the centroid of the rectangle.

We also know the position of the centroid of the triangle, and we know the area of both. Therefore, we can calculate x c, you do a quick calculation and the answer you get is x c equals x 2 x 1 plus x 2 times w 1 plus 1 third x 1 plus 2 x 2 w 2 minus w 1 divided by w 1 plus w 2. Let me write it again.

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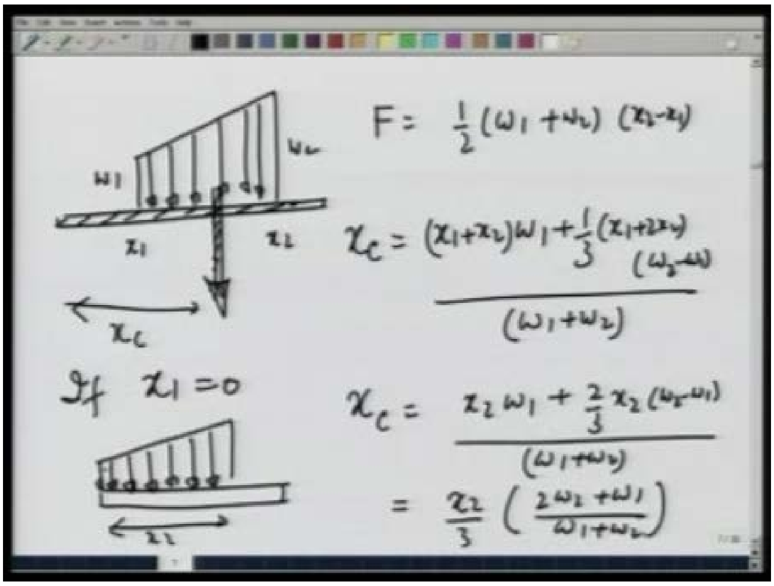

So, what we are considering is this beam which is loaded from point x 1 to point x 2 with this trapezoidal load, then the total load force comes out to be 1 half w 1 plus w 2 times x 2 minus x 1. And the point at which it acts, is at a distance x c which is equal to as I wrote in the previous slide x 1 plus x 2 times w 1 plus 1 third x 1 plus 2 x 2 times w 2 minus w 1 divided by w 1 plus w 2. If I take x 1 to be 0 that is the load starts right here and goes up to x 2, then the centroid comes out to be x 2 w 1 plus 2 thirds x 2 w 2 minus w 1 divided by w 1 plus w 2, which is nothing but x 2 over 3 2 w 2 plus w 1 divided by w 1 plus w 2 that is the example for trapezoidal loading. Next let us now solve a problem with a particular loading.

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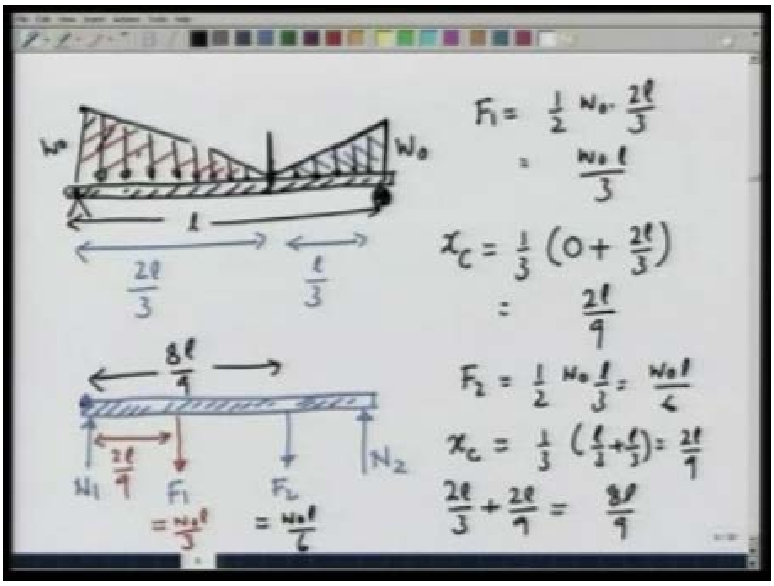

So, suppose I have a beam of length l, it is on a roller on this side and on a pin joint on this side, and it is loaded with triangular loading like this where d is playing w 0, w 0 this length being 2 l by 3 and this length. Therefore, being l by 3, and I wish to calculate the reactions at the pin joint and at the roller.

So, for that what I need to do is take this beam and make a free body diagram. We will assume that reaction at this point is N 2 reaction at this point is N 1, and the 2 forces due to the loading? The forces due to the loading, I will split them into 2 1 due to this triangle to the left and 1 due to this triangle to the right. I will take the triangle on the left acting like this at a distance, which we have to calculate and similarly, the triangle to the right also applies a load. Let us call this F 1 and F 2 at a distance from this point.

Now, what we did earlier I can take F 1 to be acting at the centroid of this first triangle, F 1 is going to be equal to 1 half w 0 times 2 l by 3. So, this is going to be w naught l by 3. And where does it act, it acts at the centroid of this triangle. The centroid in this triangle x c is going to be equal to one third a. In this case is 0, this is a point where a is and b is 2 l divided by 3 this is acting at 2 l by 9.

So, this distance is 2 l by 9 and F 1 is equal to w naught l by 3. Similarly, for the other triangle the force F 2 is going to be the area of the triangle, which is going to be 1 half w naught l by 3, which is w naught l by 6. So, this force is equal to w naught l by 6 and it acts at the centroid of the right hand triangle, the centroid with respect to this point, the point in the middle is going to be at a distance of one third of l by 3 plus l by 3, which is one third of 2 l by 3, 2 l by 9. So, the distance from this point the corner is going to be 2 l by 3 plus 2 l by 9. So, the force here acts at 8 l by 9. We are now read y to solve the problem.

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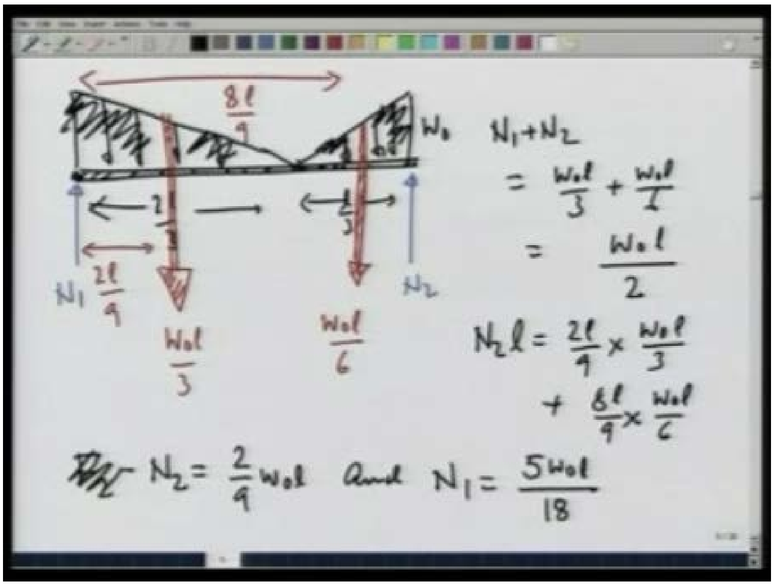

So, what we have done effectively is took this beam which is loaded like this with this being w 0 this distance being 2 l by 3, this distance being l by 3. Then, we said the reactions, at these point are going to be N 1 and N 2. And the 2 triangles replaced by 1 force acting downwards, which is w naught l by 3 acting at a distance of 2 l by 9 and the other force acting here whose magnitude is given by w naught l by 6, and it is acting at a distance of 8 l by 9.

So, I can forget about these triangles now, and instead focus on these forces and do my calculations as we are doing in the beginning of the first few lectures of this course. A beam is loaded with these 2 forces and they are these are 2 reactions simple N 1 plus N 2 being equal to w naught l by 3 plus w naught l by 6, which comes out to be w naught l by 2. Then, we balanced the moments about this point all the distances are known and when we saw.

So, it is going to be N 2 times l is going to be equal to 2 l by 9 times w naught l by 3 plus 8 l by 9 times w naught l by 6, when I solve the 2 equations I get F 2 N 2. Sorry, N 2 to be equal to 2 9 w naught l and N 1 to be equal to 5 w naught l over 18. So, that is how we have used the concept of centroid and finding out where effectively the force given by a distribution acts, what is its moment? And then applying the regular point force diagrams to calculate the reaction forces and soon.

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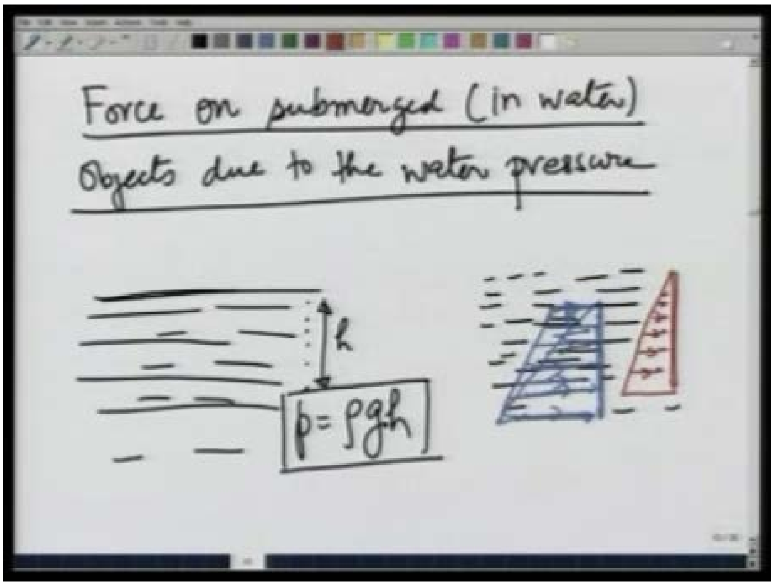

Next, I want to consider a very specific problem that of a force on submerged. Let us say in water objects, due to the water pressure. This is where the concept developed. So, for are going to be very handy. Now, we have seen that if I have or we know if we have water, then it applies pressure according to how deep we are? The pressure at depth h is rho where rho is the density of water g, which is the acceleration due to gravity h and due to this pressure. There is force acting on any object that is submerged in it.

For example, we will take a simple example, if I have a plane sheet here it will be experiencing a force here which will be given by the pressure at this depth and at lower depth the pressure is going to be lower and therefore, the force is going to be lower. So, the force varies like a triangular loading or. In fact, here it will not be triangular; it will be more like trapezoidal loading because there is pressure at this depth also.

So, the loading of force is going to be like this, if the sheet was right from the surface of the water then the loading would have been triangle, which is nothing but, a special case of trapezoidal loading. So, you can see that pressure provides a loading, which is changing linearly with depth and it has certain shape and we wish to now apply the concepts of centroid to find out where effectively does this pressure work, and what is its average pressure working on the sheet. So, let us take a sheet submerged in water.

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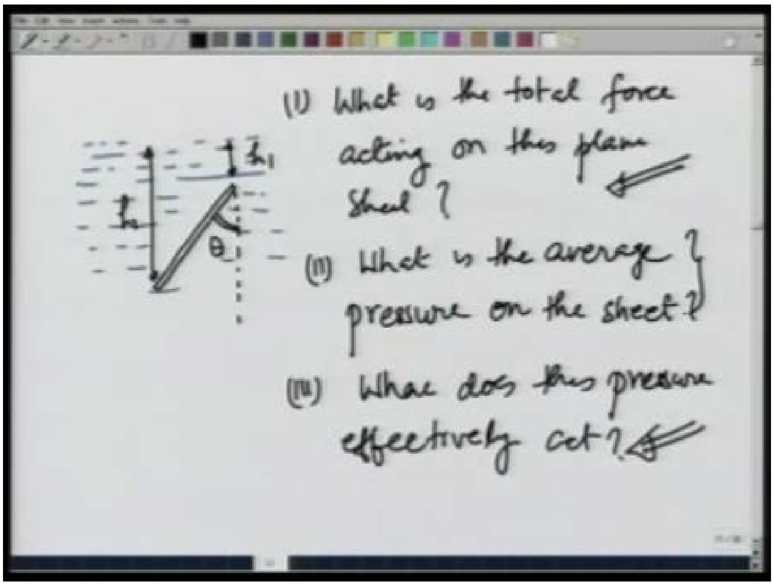

Let me show water with blue starting from depth h 1 to depth h 2. It is making an angle theta from the vertical, what we wish to find is, what is the total force acting on this plane sheet number 1? Number 2 what is the average pressure after all the pressure varies as you go from less height to less depth to more depth.

So, what is the average pressure on the sheet? And third where does this pressure effectively act? What we mean by that is where is it that I should apply a opposing force equal to the total force applied by the pressure. So, that the moment is also balanced somehow you can feel that centroid is going to be involved in this some area is going to be involved in this. So, let us work this out.

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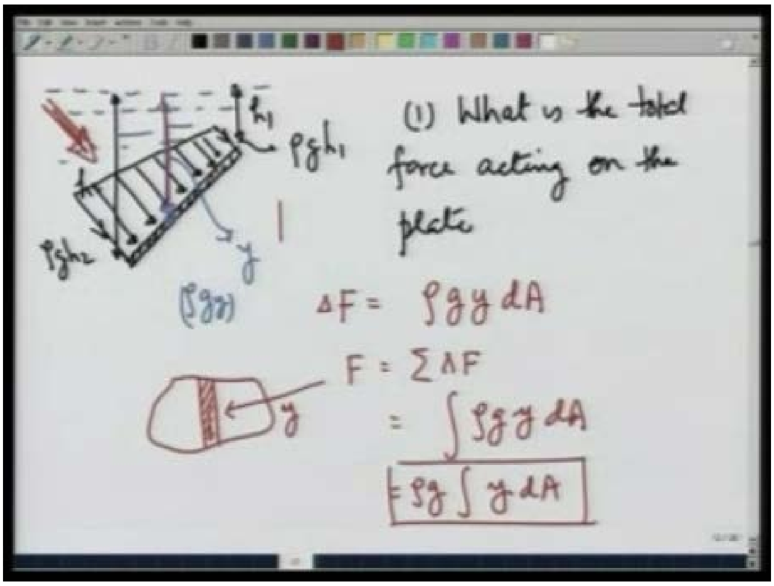

Let us take this depth to be h 1 this is h 2 and you can see that the pressure out here is going to be rho g h 1 and it increases. And this point it becomes more. So, the loading on the plate of the sheet is going to be of this form where this is rho g h 2, this is rho g h 1. Here, is the water surface on top and depth y on the top of the water surface, the force acting on a thin slice here is going to be rho g y per unit area.

First question we want to answer is what is the total force acting on the plate? That is easy to answer. Let me look at the plate from this side, then you will see plate could be of this shape. If, I take a particular area here and depth, and I am measuring depth from this side at depth y, then the force acting on this area is going to be rho g y d A y is being measured like this vertically down.

So, net force is going to be rho g y d A, and the this is sorry small force acting on this. So, net force is going to be delta F some rho over, which is nothing but integration rho g y d A rho and g are constants. It is y d A this is the net force acting, what about the average pressure? To calculate average pressure, let me again make this picture.

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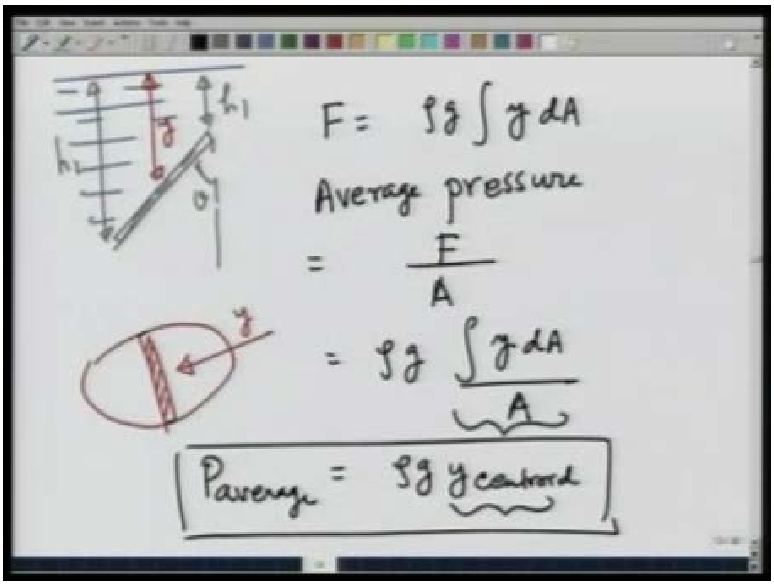

This is at depth h 1 depth h 2 at an angle theta this is water line and this is the plate, this is the area that we took at depth y from the top this is depth y. So, the net force F is going to be equal to rho g integration y d A, where d A could be a function of y because the shape of the plate or sheet could change according to how deep you are from water. So, this could be a function.

And the average pressure is going to be the net force divided by the total area, which is equal to rho g integration y d A divided by the area, but this precisely is the definition of the centroid of the plate. So, this is going to be rho g y centroid of the plate this is P average. So, the average pressure that a sheet submerged feels in the water is going to be rho g times the depth of its centroid. For example, if the sheet is rectangular.

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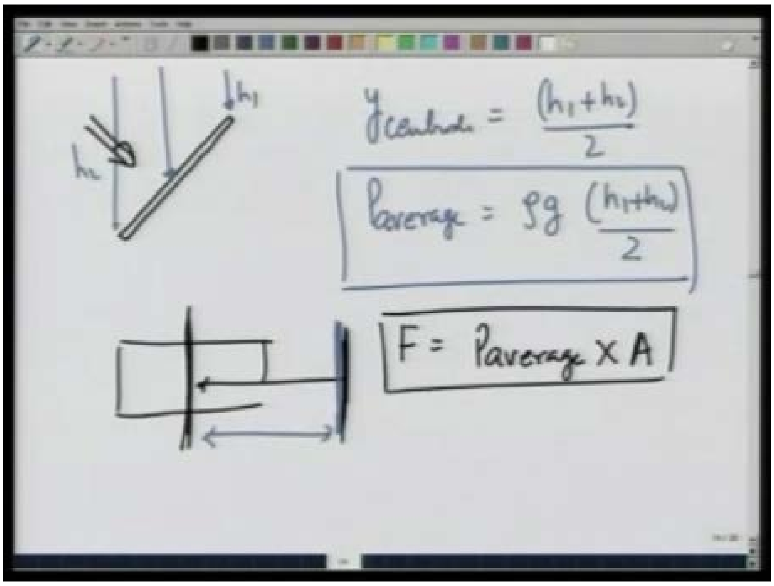

So, that it looks, if I look at it from this side it looks like this, then I am measuring depth from this side. Then, the centroid is going to be along this line and from the top of the water surface. Let me show that by blue this is a top of the water surface this depth is going to be right in the middle.

So, this is h 1, this is h 2 y centroid is going to be in the line at the middle, which is going to be h 1 plus h 2 divided by 2. And therefore, P average is going to be equal to rho g h 1 plus h 2 divided by 2. This is minute the centroid of the plate that we talking about the plate, which is feeling the force, this is going to different form another centroid that we are going to calculate now. So, this is the average pressure acting on the plate, which is given by the depth of the centroid of the plate. Next, we calculate where does this force act? So, net force F is going to be P average times the area of the plate and the question you want to answer is where does it act? For that we look at the force distribution in the plate.

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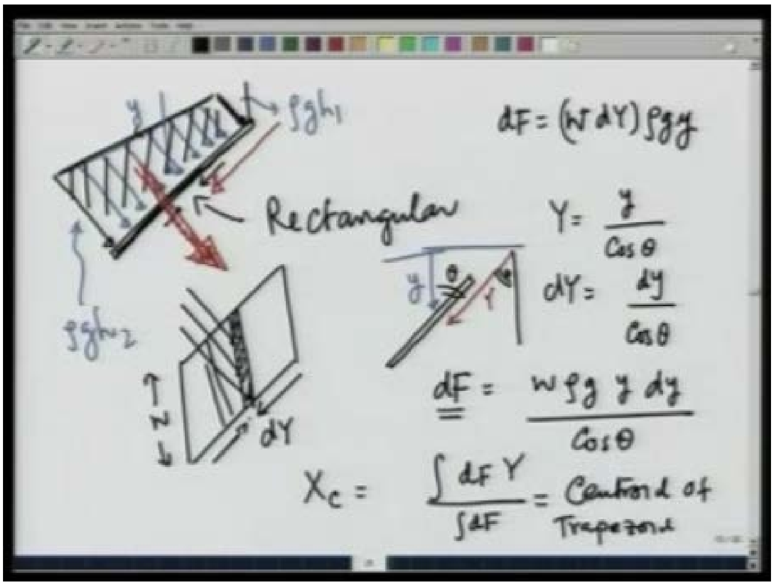

So, this is the plate and the way the force varies is it is trapezoidal, here it is rho. Let me write it with blue, rho g h 1, at this point it is rho g h 2 and somewhere in the middle it acts. So, loading is like this, I would like to clarify one thing out here that, this loading that we are taking to be linear is going to be true only in the case when the sheet is rectangular. So, now we are, we are restricting our self to a very specific shape of the sheet which is rectangular.

So, if I make it like this, this is the way sheet is and the force is acting on this like this, this is how the pressure is working. If this were not rectangular the force at different points would vary according to how the area changes. So, now the force depends only on the depth because a area for each step as a constant. So, then loading becomes linear. So, now we are being restrictive to rectangular sheets.

So, let us see now the force acts at the centroid of this trapezoidal loading how much of force, if this width of the sheet is w the force d F is going to be w and let me call the width along the sheet d capital Y. So, d F is the w d capital Y times the, this is the area pressure rho g y, what is the relationship between y small y and capital Y? Capital Y we are measuring along the sheet. So, let me make a picture again, if we have the sheet here this is a top surface of water, we are measuring small y like this and capital Y like this.

So, it is clear that we have capital Y equals y divided by cosine of theta because this is theta and this is theta. So, d capital Y is equal to d y over cosine of theta and therefore, d

F is equal to w rho g y d y over cosine of theta, what about, what about the force acting at certain point that is going to be given by X c? Which is going to be integration d F times capital Y divided by unit force. This by definition is the centroid of this loaded area, the load area. So, this force acts at a point at a distance from here, which is the centroid of this load, which is a centroid of this trapezoid. This we have already calculated in this case let me make the picture again.

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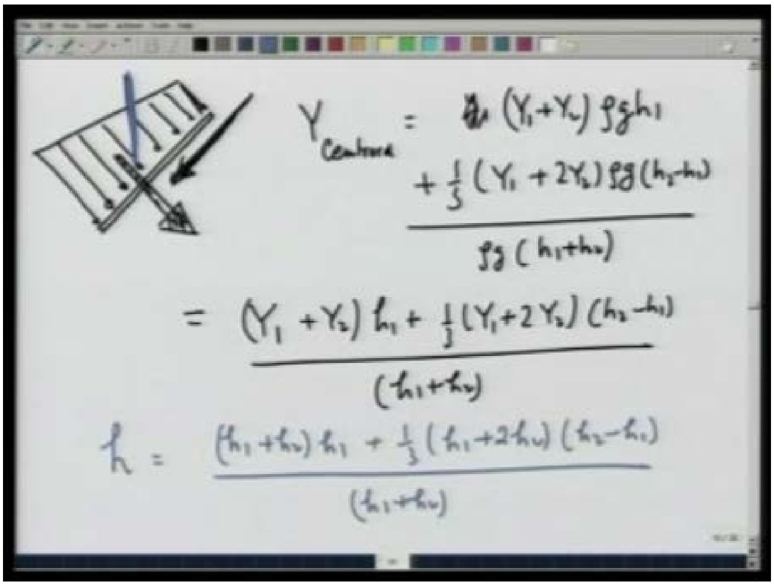

This is a sheet where the load here, where the load here, which varies linearly for the rectangular sheet and it acts at the net force at the centroid of this place, and this is given by. Let us calculate Y centroid which is going to be y 1 capital Y 1 plus Y 2. I am using the formula that we derived earlier for the centroid rho g h 1, which is the pressure here at this point plus 1 third Y 1 plus 2 Y 2 rho g h 2 minus h 1 divided by rho g h 1 plus h 2.

You work it out and this comes out to be Y 1 plus Y 2 h 1 plus one third Y 1 plus 2 Y 2 h 2 minus h 1 divided by h 1 plus h 2 that is a distance measured along the sheet. We can also transform now, this to at what depth this way. Let we make it with blue at what depth is it working? So, that depth h is going to be h 1 plus h 2 times h 1 plus one third h 1 plus 2 h 2 h 2 minus h 1 divided by h 1 plus h 2, which when worked out comes out to be h equals two thirds h 1 square.

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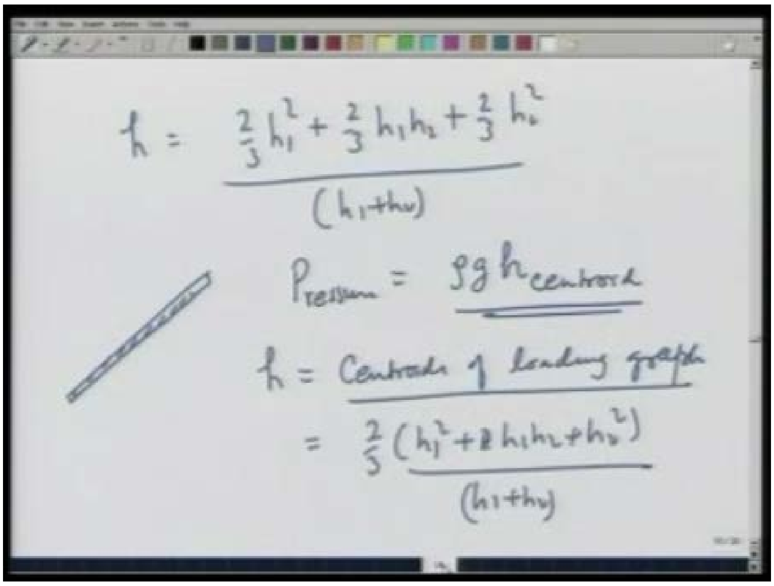

Plus two thirds h 1 h 2 plus two thirds h 2 square divided by h 1 plus h 2. So, the rectangular sheet that is submerged in water is an average force, which is given by average pressure, which is given by rho g and then depth h of the centroid of the plate. And it acts at a depth h equals the centroid of loading graph, which in the case of rectangular sheet comes out to be two thirds h 1 square plus two third h 1 h 2 plus two third h 2 square divided by h 1 plus h 2. To make you understand things in a very clear way I took a rectangular sheet. However, in general also or you can see that, if I had a plate which is of some other shape.

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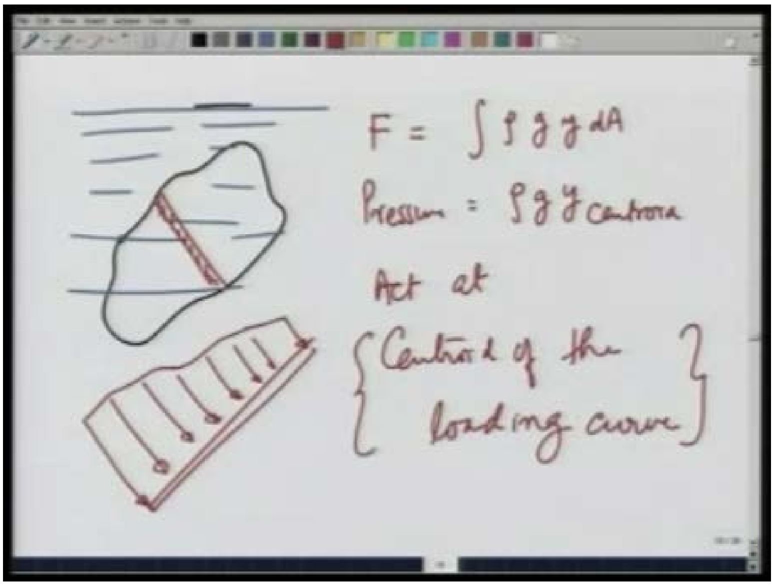

And it is submerged in water, which I make by blue this is width a sheet along the width loading in this case. If, I look at the sheet from side may not be linear, but vary according to how the area changes. In this case also if you calculate the force this we have already calculated will come out to be equal to rho g y d A and average pressure would come out to be rho g y centroid of the area of the sheet, and it will still act at the centroid of the loading curve. So, this is how you deal with pressure on a sheet submerged in water. Let us now do an example of this, as an example.

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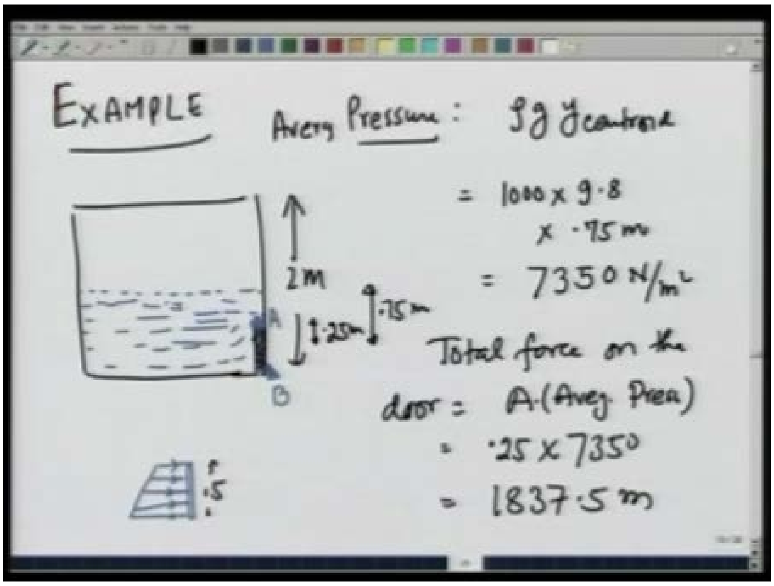

Let me take a water tank, which is 2 meters in height and let there be a door at the bottom, which is 0.5 meters by 0.5 meters a square door. This is hinged at this point A and is stopped by a verge or a block at B. We want to calculate the force, the reactions at A and B when water is filled up to 1 meter. To do so, first realize that this door is loaded by pressure like this, this is a square door so, the loading is trapezoidal.

Therefore, the net the average pressure on the door is going to be equal to the rho g y centroid of the area of the, at the door here. That is going to be since this is 0.5 meters is going to be 0.5 meters below A, 0.25 meters below A. And therefore, 0.75 meters below the surface of the water. And therefore, average pressure on this door is going to be rho, which is 1000 time g 9.8 times centroid, which is 0.75meters. That is the average pressure, which is going to be 77350 Newton's per meter square. Therefore, the total force on the door is going to be area times average pressure, which is going to be 0.25 times 7350, which is going to be equal to 1837.5 meters. So, what we know about this the, this door now is.

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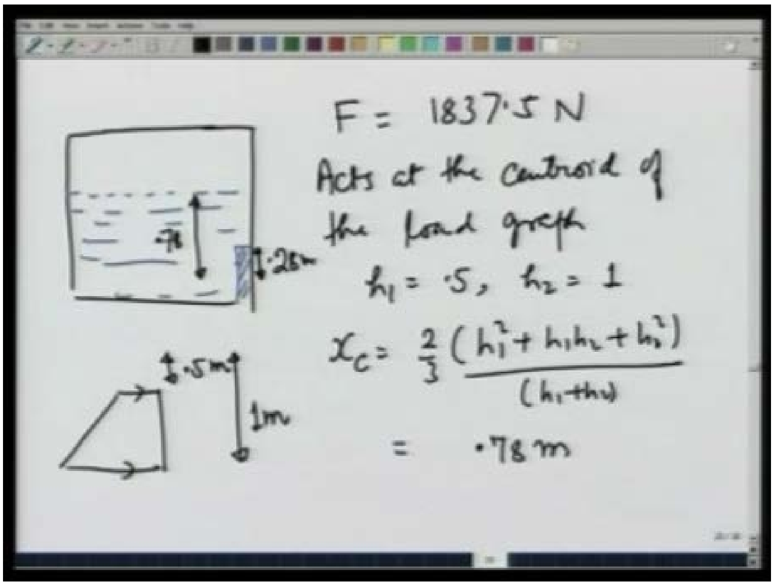

That when the water tank has a door is half filled the force on the door is 1837.5 Newton's, and where does it act? It acts at the centroid of the load graph, which in this case happens to be a trapezoid. This distance is 0.5 meters the depth of the other side is 1 meter. And therefore, I am given that h 1 is equal to 0.5 h 2 is equal to 1.

So, the loading x c is at 2 thirds h 1 square plus h 1 h2 plus h 2 square that we calculated earlier h 1 plus h 2 and this comes out to be 0.78 meters from surface of the water. So, from surface of the water this is at 0.78 meters or from the point A, it is at 0.28 meters. So, now we know everything about the door.

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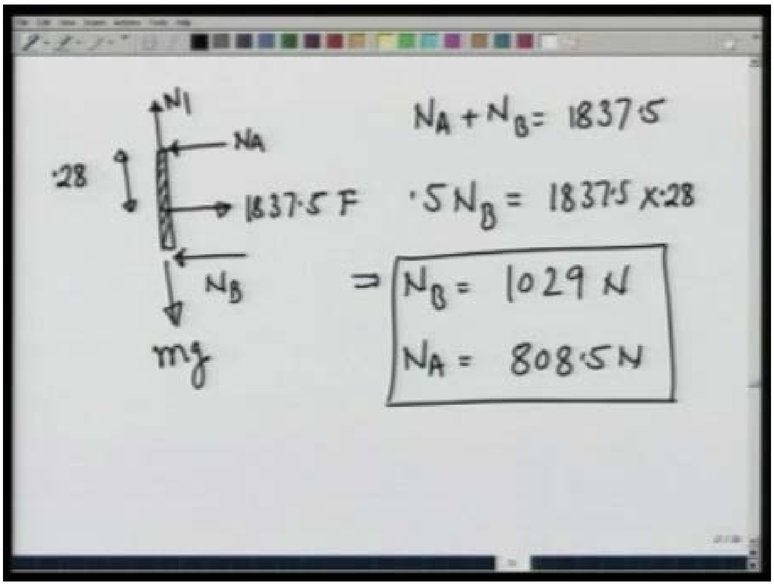

On this door a force of 1837.5 Newton’s acts at a distance of 0.28 meters from point A, and let there be a reaction at A of N A. Let there be a reaction of NB at B. Then, we know that N A plus N B is going to be 1837.5 and 0.5 NB is going to be equal to 1837.5 times 0.28 and that gives me N B equals 1029 Newton’s.

Therefore, N A equals 808.5 Newton’s that is the answer. If the door also had some weight there would have been a reaction at A, which would have nullified that weight mg. So, you see how we have calculated the forces due to water pressure on the door and consequent reactions on the hinge holding the door, and on the weight which is not letting the door come out. I leave a problem for you and that is in the same situation.

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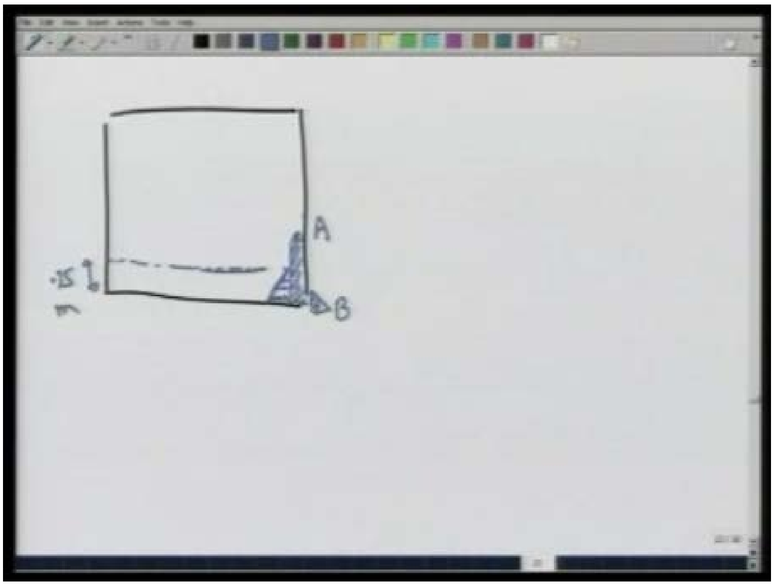

If this tank is filled only up to 0.5 meters. In that case I want you to calculate the force on A and the force at this verge be holding the door. This will be slightly different from the problem that we just solved because the surface of water is below this point and the loading is going to be triangular from this point onwards. I leave it for you.

So, in this lecture we have used the concept of centroid and the area of and the first moment, and the area of load verses distance curve to get some formulae to see where effectively a given force or distributed force acts? Then, applied it in the situations of where a beam was loaded or when a sheet was submerged in water in particular, we focused on the triangular sheet. In the next lecture we are going to work on a more slightly more, and more mathematical concept called the moment second moment of area and the product of area.