Engineering Mechanics
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Module – 01
Lecture - 04
Equilibrium III

We have been looking at equilibrium of bodies and the previous lecture, where we looked at certain elements and what kind of forces do they apply.

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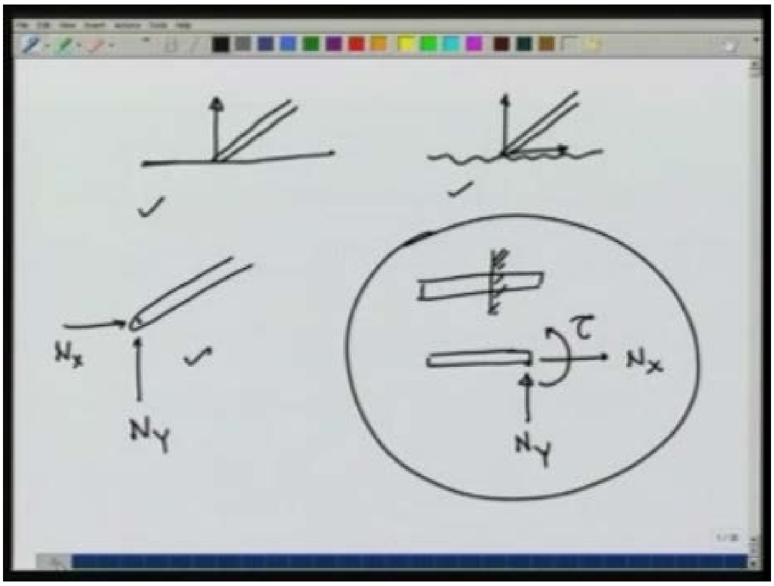

For example: we looked at smooth surface, rough surface and we saw that on a particular body a smooth surface applies a normal force on a particular body rough surface applies a normal force; as well as it is capable of applying a frictional force. Then, we looked at a hinge joint and saw that this can apply force both in the x and the y direction. And looked at fixed or welded joints and we saw that, this is capable of applying a force in the y in the x directions as well as it can create a torque. We had solved examples using this this and this. Now let me solve an example using a fix joint.

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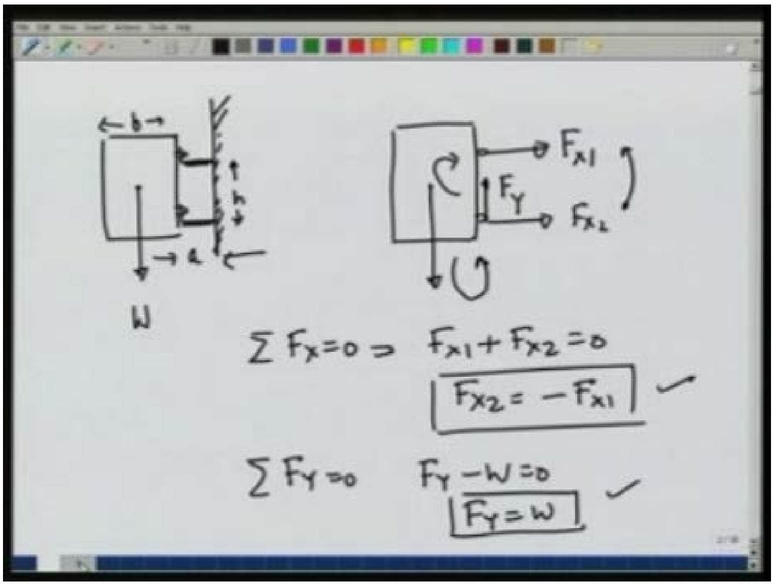

For this I take a household example of a gate which is on 2 supports like this. Let the weight of this gate be w, let its width be b let the distance between the supports be h and let this distance from the support wall to the point where a gate is supported be a. We want to find the forces that are being applied by the wall on the supports, assuming that the entire weight of the gate is bond by the lower support. So let us see, how the gate is being supported. If I look at the gate it has a weight pulling it down. Then, the entire weight is being supported by the lower support. So, it has a force Fy pulling it up in addition, there will be forces in this direction.

So, let us call this Fx 1 and Fx 2 these are the only forces that are there on the gate. Then, the equilibrium conditions, the summation Fx is 0 gives me Fx 1 plus Fx 2 is equal to 0 or Fx 2 is equal to minus Fx one. Similarly, the condition, summation Fy is equal to 0 gives me that Fy minus w is 0 or Fy is equal to w. You may ask at this point how come these 2 force Fx 1 and Fx 2 are generated. It is because, Fy and w give a couple in this direction and to oppose this, there must be horizontal forces generated at the joint to counter balance this couple. So, we have got in these 2 answers. Let us now, look at balance in the torque.

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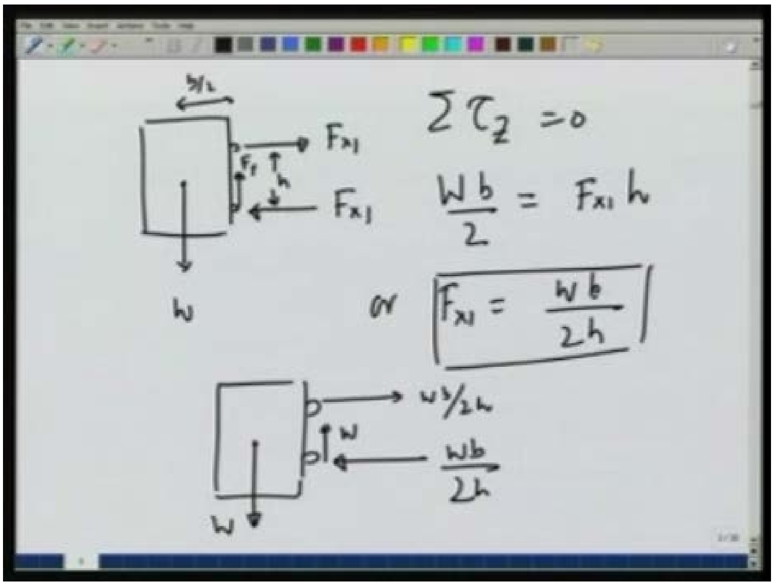

So, I have this gate there is the force Fx 1 and now we have already figured out, whether the force in the opposite direction of the same amount Fx 1 they are separated by distance h, there is a force Fy another force w separated by distance b by two. So, summation tau z is equal to 0 gives me w times b over 2 that is a couple due to Fy and w should be equal to Fx 1 times h or Fx 1 is equal to w b over 2 h. So, the forces acting on the gate are, w w this way w times b over 2 h and this way w times b over 2 h these are the net forces acting on the gate. How about the supports themselves? If I look at the support this is the wall.

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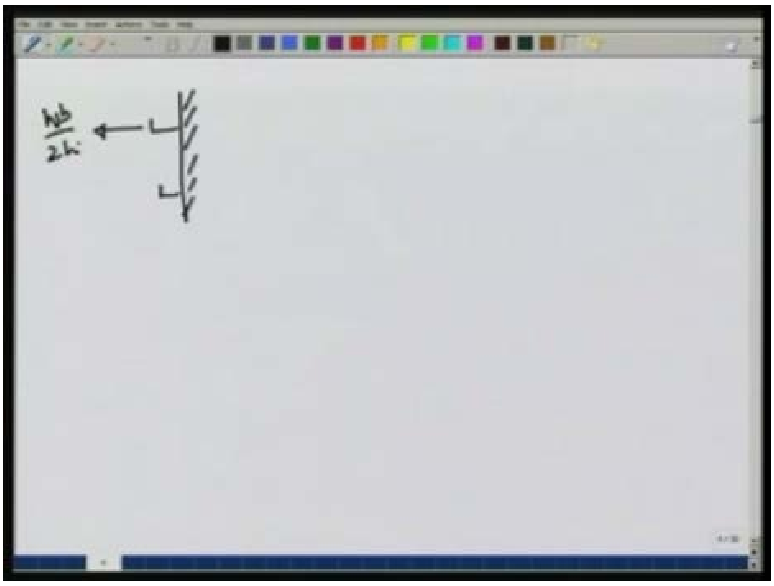

This is 1 support, this is the other support this support the upper support is pulling the gate in by force Fx 1 it is wb over 2 h and therefore, gate must be pulling by Newton's third law the support in this direction with wb over 2 h. Similarly, on the lower support the gate is being let us, look at this pushed.

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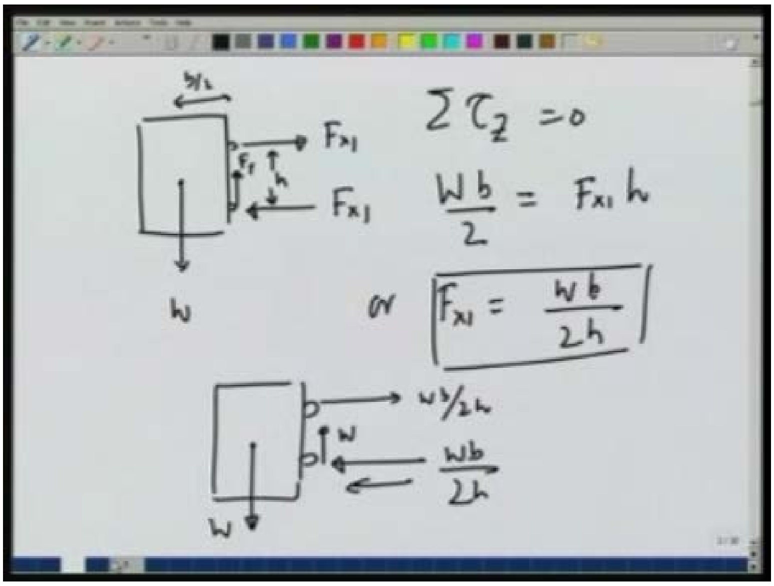

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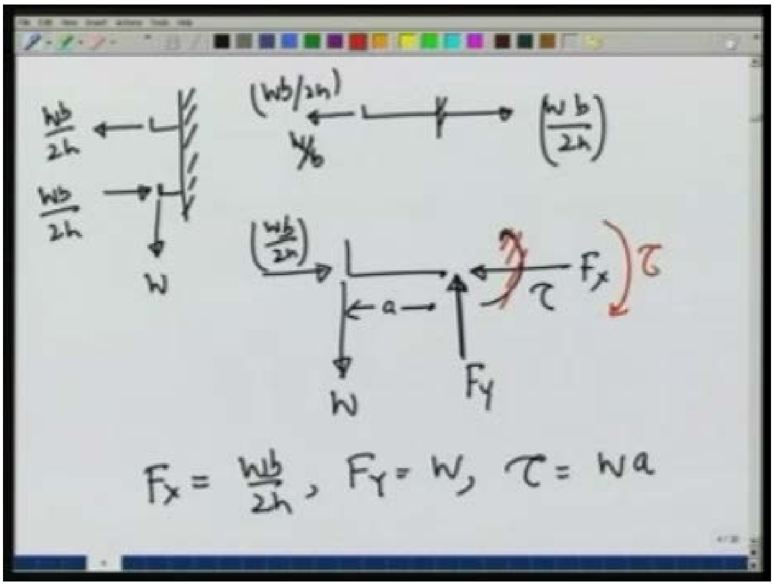

Here it is being pushed, gate is being pushed this way. So therefore, there will be a force pulling pushing the support in with the force wb over 2 h. On the lower support there are more forces, the lower support is providing a vertical force w to the gate and therefore, there must be a force w pulling it down. Now, we have to ready to calculate forces and moment generated on both the supports inside the wall.

So, for the upper support the only force is this way wb over two. So, the fixed support is capable of providing another force wb over 2 h it will pull it in with the same force wb over 2 h. For the lower support situation is slightly more complicated this distance is a, there is a force w down, there is a force wb over 2 h this way. Therefore, at the wall with respect to this point there would be a force to balance the vertical force Fy to balance the horizontal force there will be a force Fx and there will be a torque tau .Let us see, how what these values are.

So, straight away Fx is going to come out to be wb over 2 h Fy is going to come out to be w and the torque is going to come out to be w times a, but not in this direction. Because, this is a same direction as the w is providing the direction could be this way. This is the torque generated at the support. So, this is one example of how a fixed support is able to support the applied forces from outside both, vertical and horizontal as well as a torque applied from outside having done all this. Let me now; solve 2 or 3 examples for you from the book of Meriam.

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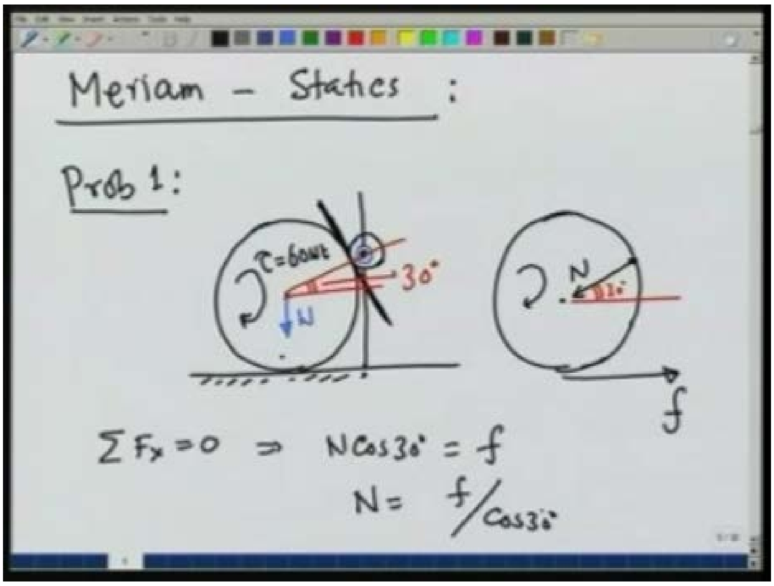

As a first problem let me take the problem is like this, we have a wheel on a rough surface and it is touching roller which is free to move and it is fixed to the support like this. We apply a torque tau is equal to sixty Newton's meter on the wheel such that the wheel does not move. We want to find what is the force applied by the wheel on this support. Let me show the support in blue here, if you want to calculate the force on the support at this point weight of the wheel is some w you will see it does not really matter since, the wheel does not move. Therefore, it is a static equilibrium problem there is a torque like this Wheel would have a tendency to rotate in counter clockwise direction and therefore, there would be a frictional force generated like this.

The other force would be provided where the roller is touching the force. What about the direction of this force. Since, roller is free to move there cannot be any force in the horizontal direction like this. If they there were a force this roller will start moving. Therefore, the only force is in the direction perpendicular to the surface in this direction. We have given that this angle is thirty degrees so, here this is angle 30 degrees. If we now apply the equilibrium conditions summation Fx is equal to 0 gives us. Let this force be N cosine of 30 degrees is equal to f and therefore, the normal reaction N is going to be f divided by cosine of 30 degrees.

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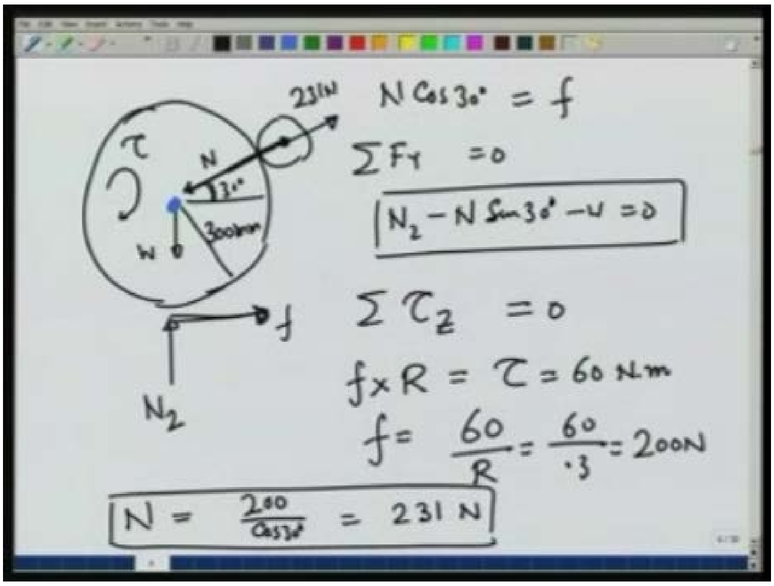

Similarly, let me make the picture again. Here is the weight w, here is f, here is normal reaction N, here is a torque tau this angle is 30 degrees. So, you found that N cosine of 30 degrees is equal to f. Summation Fy is equal to 0 would give me if there is a normal reaction N 2 here, N 2 minus N sin 30 degrees minus w is equal to 0 that is the other condition. And third condition for the torque is summation tau about z is equal to 0. Let me take, the torque about the centre of the wheel right here shown by blue. If we do that the torque due to normal reaction N, due to this force vanishes that is the advantage of taking torque about this point. And therefore, I would have f times the radius of the wheel is equal to tau applied which is 60 Newton's per meter Newton's meter and therefore, f is equal to 60 over R the radius R is given to be 300 millimeters.

Here, we will write this in meters. So, this is going to be 60 over point 3 is equal to 200 Newton's. So, we have found that the frictional force to support or the frictional force required. So, that the wheel does not move even after applying the torque is going to be 200 Newton's. And therefore, the normal reaction N near on the roller is going to be 200 divided by cosine of 30 degrees which you calculate it comes out to be 231 Newton's. So, the force by the roller on the wheel is 231 Newton' s and therefore, on the roller there will be a force by the wheel of the same amount 231 Newton's and that is your answer.

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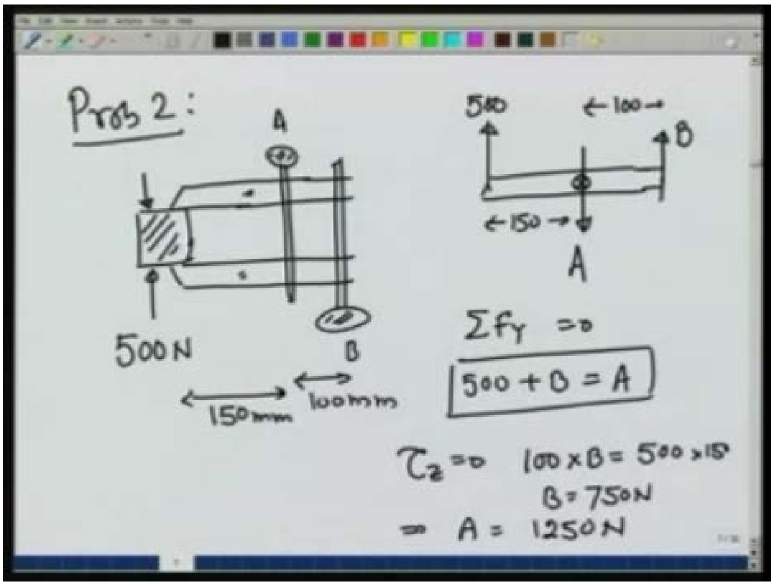

Let us solve one more problem and this is suppose, we have a block of wood or steel and it is being clamped by tightening the screws on these 2 clamps. So, here is one screw A, here is the other screw B and they are being tightened in opposite directions. They are tightened to the extent that the force on this block on both the directions are compressive force is of the amount 500 Newton's. If this distance is 100 millimeters and this distance is 150 millimeter. We want to find the forces applied by the screws on these clamps.

Assuming that, the forces applied by the screws are in the along the screw direction. So, let us look at the upper clamp. On the upper clamp there is going to be a force of 500 Newton's in the direction opposite to the force it is applying on the block and the screw would apply a force like this, let us call it A and the other screw applies a force in the opposite direction B.

You can see right away I made the forces acting in opposite directions. Because that is how I would balance the torque and the forces. Since, this is essentially a 1 dimensional problem I can straight away write Fy is equal to 0 and that gives me 500 plus B is equal to A. This distance is 100 millimeters and this is 150 millimeters. Let me now balance torque about this point the point in the middle where A screw is. So, tau z is equal to 0 gives me 100 times B is equal to 500 times 150 or B equals 750 Newton's and this implies A is equal to 1250 Newton's as the other problem. Let me solve 1 more problem and that is a problem where we support a big drum.

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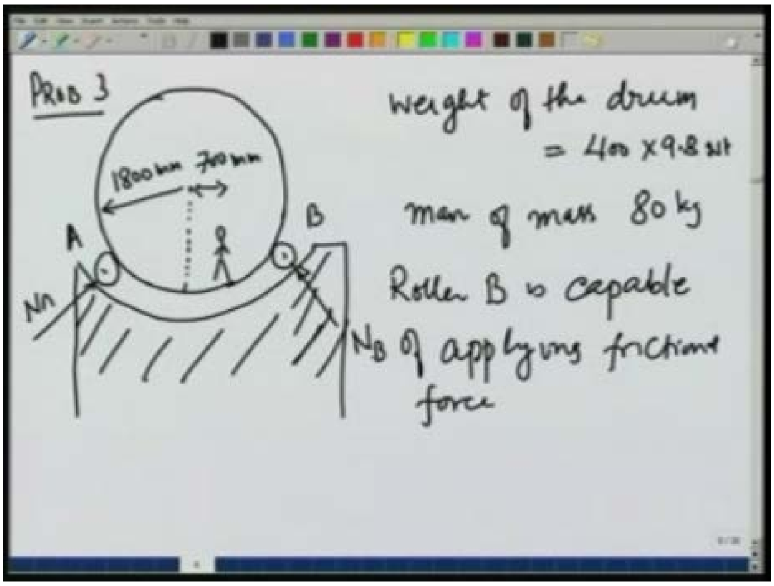

This is problem 3 on cylindrical rollers the radius of the drum is given to be 1800 millimeters. Its weight is given to be 400 times 9 point 8 Newton's that results a 400 kilogram drum. A man of mass 80 kgs starts walking on the drum and when he reaches here, which is 700 mm distance from the centre the drum starts moving. Just begins to rotate on these rollers A and B. The B roller these rollers are also cylindrical is capable of applying a frictional force roller B is capable of that is why, it is only after the man moves certain distance that the drum starts rolling.

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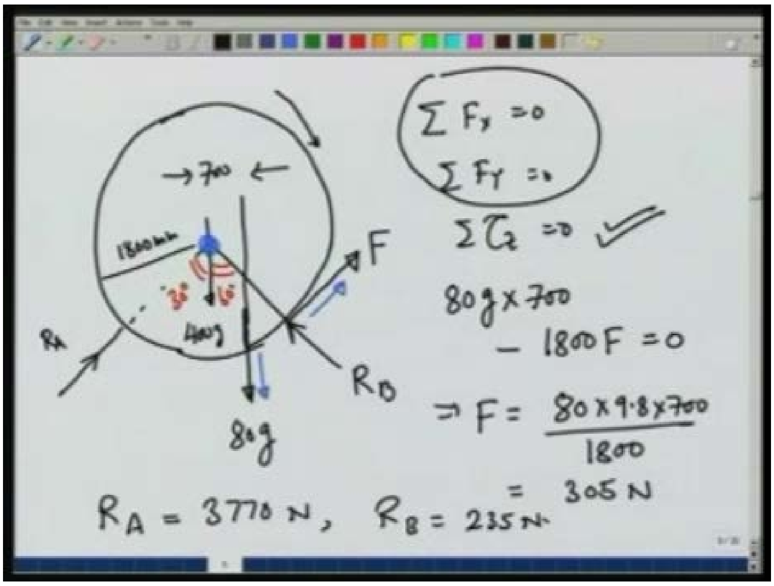

We want to find the reactions on A NA and reactions on NB to solve the problem. Let us, make this drum and see what all forces are acting. There will a force RA acting here, there will be a force RB acting here, a man is at this distance 700 millimeters there will be a force of 80 g acting this way and there is a force of 400 g acting this way.

The angles are given to be 30 degrees and 60 degrees here. In addition since, the drum has a tendency to rotate like this there will be a frictional force let us call it F, at roller B which is capable of applying a frictional force. Now, we have written all the possible forces that are there on the drum and let us now apply our equilibrium conditions. So, if we write summation Fx equal to 0, summation Fy equal to 0 and summation tau z is equal to 0. If we want to find F that we can do write away. So let us, first apply the torque equation to find F for this let us take the centre of the drum as our point about which we take the torque.

In that case, the only torques that will be coming into picture would be torque due to this weight and torque due to F. If we do that we will find that torque equal to 0 gives us 80 g times 700 minus 1800 which is the radius times F is equal to 0. Notice that I did not divide by 1000 to convert 700 and 1800 into meters because I am using the same units mm on both the forces and this gives me force F is equal to 80 times 9 point 8 times 700 over 1800, which comes out to be 305 Newton's having determined the frictional force F it is easy to find RA and RB by taking the force conditions. I will leave that exercise for you, but give you the answer which comes out to be RA is equal to 3770 Newton's and RB comes out to be equal to 2 three five 2 Newton's. You may have noticed that so, far we have been doing problems which are essentially 2 dimensional.

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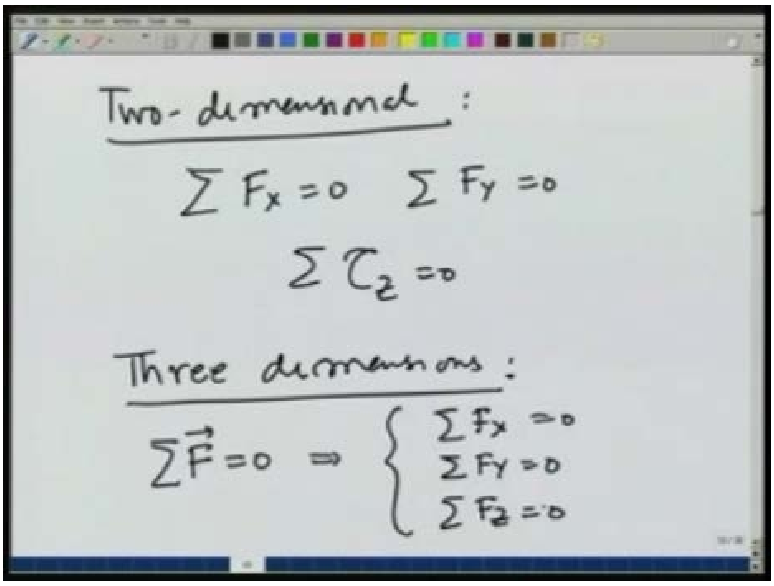

In the sense that we have been looking at equilibrium conditions at Fx is equal to 0, summation Fy is equal to 0 and summation tau z is equal to 0. So, we have been talking about 2 force components x and y and 1 component of the torque and that is in z direction or the torque about z axis. Now, let us generalize this to 3 dimensions. In 3 dimensions; obviously, the conditions that summation F vector is equal to 0 becomes summation Fx is equal to 0, summation Fy is equal to 0 and summation Fz is equal to 0.

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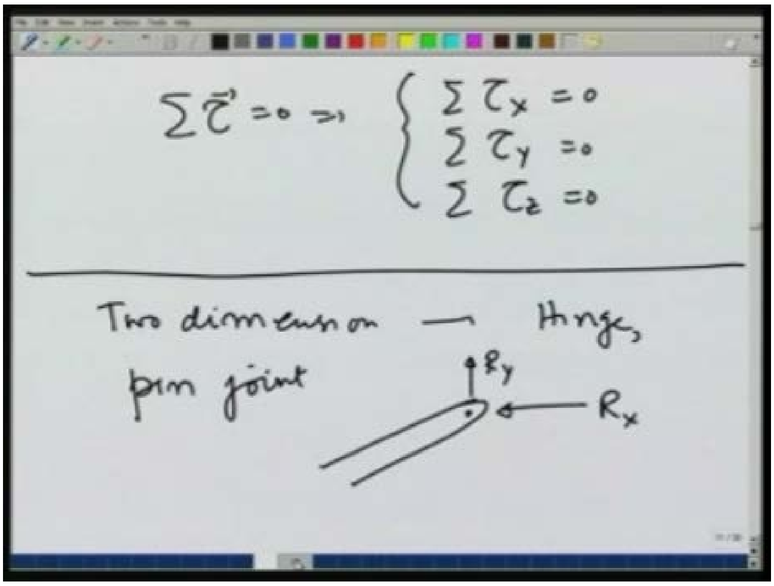

Similarly, the condition that summation tau vector is 0 implies that summation of all torques along the x axis is 0, summation tau y is 0 and summation tau z is 0. Similarly now, that we are talking about 3 dimensional cases. The engineering elements also now we have to consider and view of three dimensionality and see that they can apply forces in all three directions.

Thus remember in 2 dimensions there was a hinge or a pin joint which was like this and it was capable of applying a force in x and y directions. Sometimes they given as reactionary force Rx and Ry.

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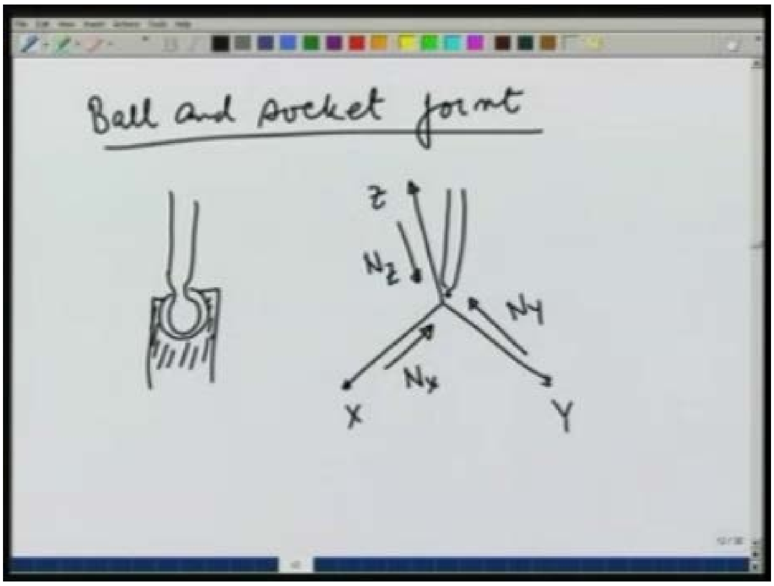

In three dimensions a similar joint is called a ball and socket joint which is nothing but, a socket. In which a ball fits and it can rotate in any directions. So, the ball and socket joint let me just show it, just like this is capable of giving force in x y and z all 3 directions. Let me make it like this, let us say this is the z direction on a socket joint it can provide a force in x direction, it can provide a force in y direction and it can provide a force in z direction. This is the three dimensional counter part of the pin joint or hinge joint of 2 dimensions.

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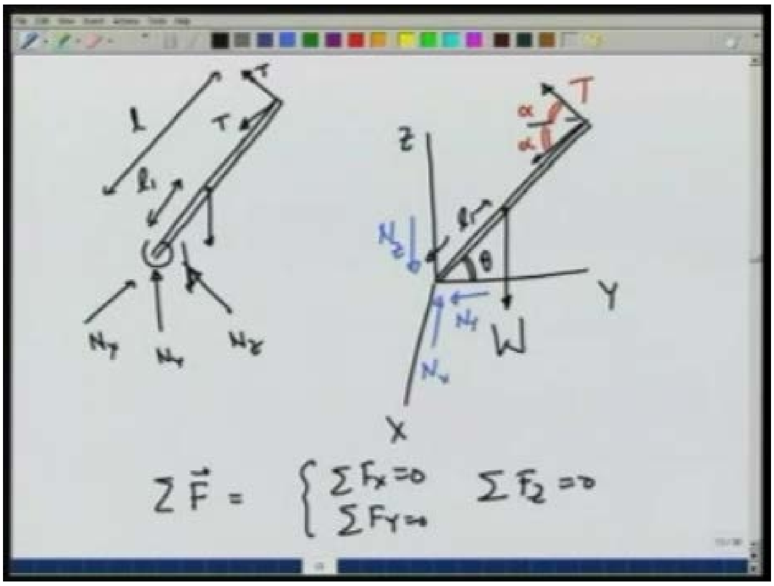

Let us solve a problem using this concept suppose, a person wants to lift a very heavy load and what he does is digs a hole in the ground and puts a pole in it. A long pole he hangs the weight somewhere in the middle let us say, at a distance of l 1. Let the length of the pole be l and then, he applies forces horizontally by 2 ropes in 2 directions like this. What is the tension in these ropes that is we want to know and what are the reactions here in x, y and z directions Nx, Ny and Nz.

So, let us first choose our axis x, y and z. We choose our axis in such a manner that the pole over y axis and in the y, z plane x, y and z. Let the pole be the y, z plane over the y axis making an angle theta from the y axis. The load is at a distance l 1 and let the weight be W. The ropes are in the x, y plane and let they make each of them an angle of alpha form the y axis they are in the x, y plane. We want to find out the tension in the rope and the forces Nx, the force Ny and the force Nz at this point. I am making all the forces towards the origin, but does not matter. So let us now, consider the equations for equilibrium summation over F gives summation Fx is equal to 0, summation Fy is equal to 0 and summation Fz is equal to 0. Let us consider forces in all three directions.

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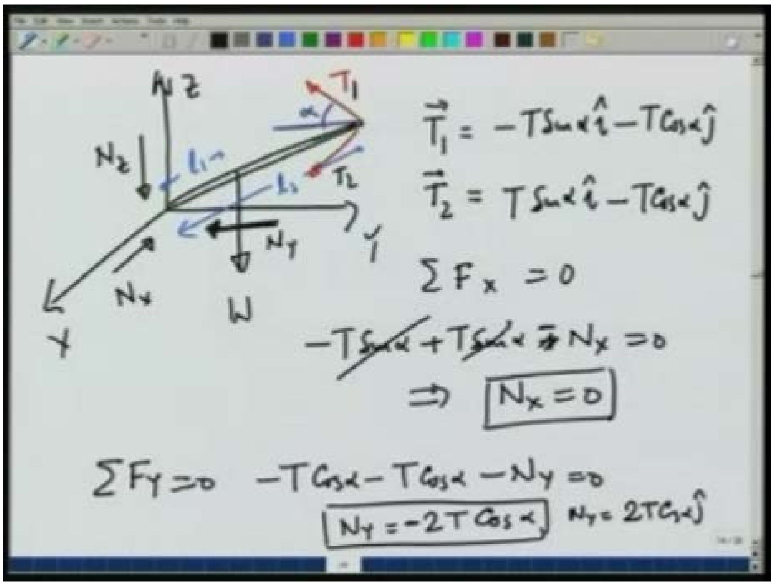

So, let we make the picture again X, y, z; this is how the pole looks. I am making a slightly tilted there is a weight w act in downwards. This length it is l 1 this entire length is l 2 there is normal reaction Nx, Ny and Nz and there are these 2 tensions in x, y plane making angle alpha from here. So, if you, if I would write the force let me call this tension 1, let me call this tension 2. T 1 vector is nothing but, minus T sin alpha I minus T cosine of alpha j because, the tension T 1 has component in negative x direction and negative y direction. Similarly, tension T 2 is equal to T sin alpha I minus T cosine alpha j.

So, these are the 2 forces from arising from tension. The weight acts in the vertical direction. So, summation Fx is equal to 0 gives me minus T sin alpha plus T sin alpha plus Nx is equal to 0 or this implies this cancels the Nx the normal reaction in x direction is 0. It is a nearly matter actually this should have been negative here. Similarly, summation Fy is equal to 0 implies that minus T cosine alpha minus T cosine alpha minus Ny is equal to 0 or Ny is equal to 2 T cosine of alpha with a minus sign. So, Ny has a magnitude of 2 T cosine alpha. And the negative sign in front indicates that it is in the direction opposite to what has been shown in the figure here, going towards the origin. So, Ny is actually 2T cosine alpha j.

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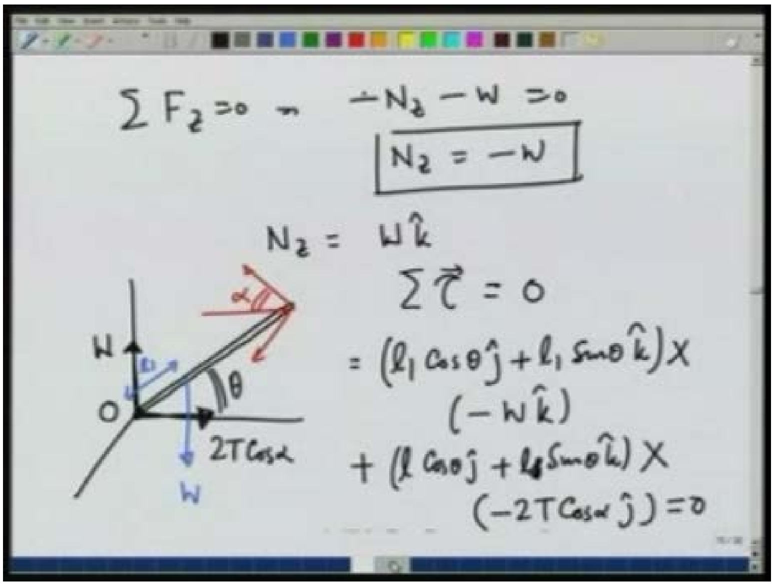

The third condition a summation Fz is equal to 0 gives minus Nz minus w is equal to 0 or Nz is equal to minus w. Again showing that, Nz is equal to w and in the direction opposite to what has been shown and therefore, Nz is equal to w time’s k. So, what we have determined on this rod or on the pole that has been sort of put in pore in a socket joint is that there is a force on this due to the ground, due to the hole on the ground which is equal to 2T cosine of alpha there is a normal reaction in the z direction which is equal to w there is a force w pulling it down. There is no force in the x direction and then, there are these 2 tensions in the x, y plane at working at an angle alpha like this, this angle is given to be theta. Let us now take the condition summation tau is equal to 0. To calculate tau let us take the origin, to be at the origin of the xyz system o.

So, that this comes out to be l 1 cosine of theta j plus l 1 sin of theta k that is when l 1 is this distance. So, distance where w is acting is l 1 cosine theta j plus l 1 sin theta k cross product of minus wk. The other forces are tension. So, that would be plus l cosine of theta j plus l sin of theta k cross. Since, the 2 forces are acting at the same point I can write the torque for their summation which will be equal to minus 2T cosine of alpha j and this should be equal to 0 this condition would determine the tension T. So, let us work this out the first term is going to give me j cross k which is I l 1 cosine theta times w.

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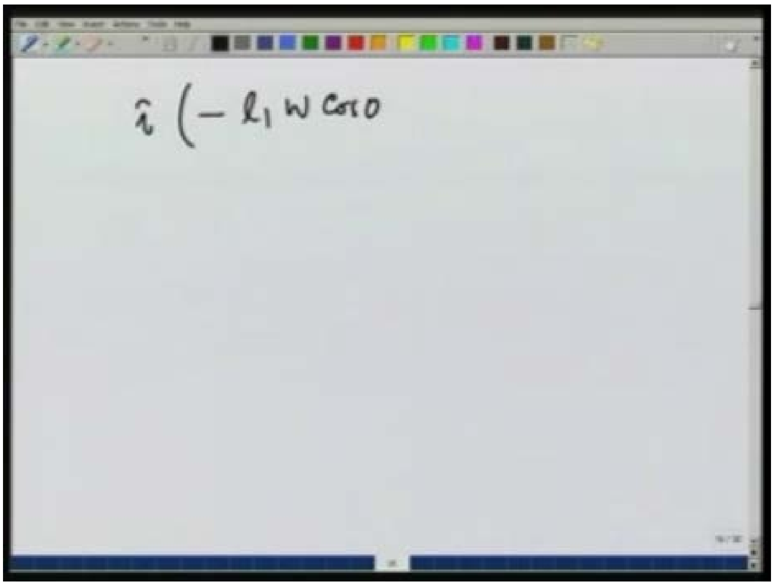

So, first term gives me I minus l 1 w cosine theta.

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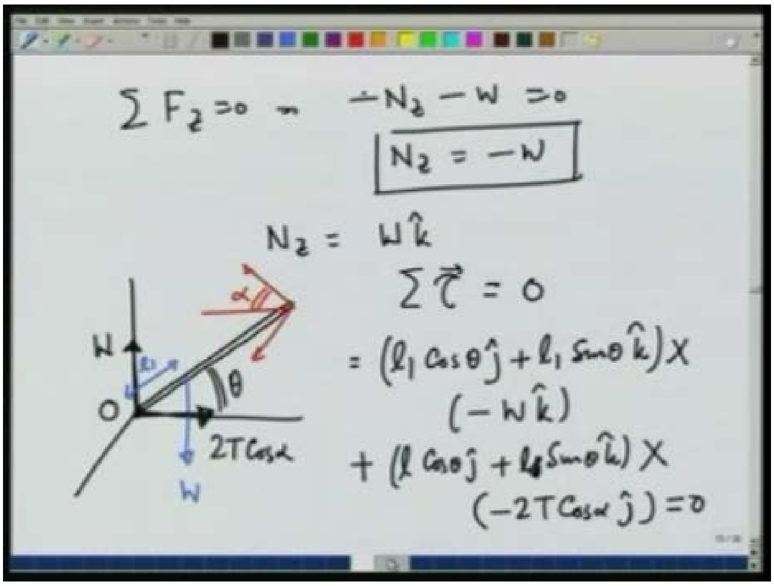

The second term again j cross j is 0. So, k cross j k cross j is minus i. So, it gives me plus 2T l cosine alpha sin theta.

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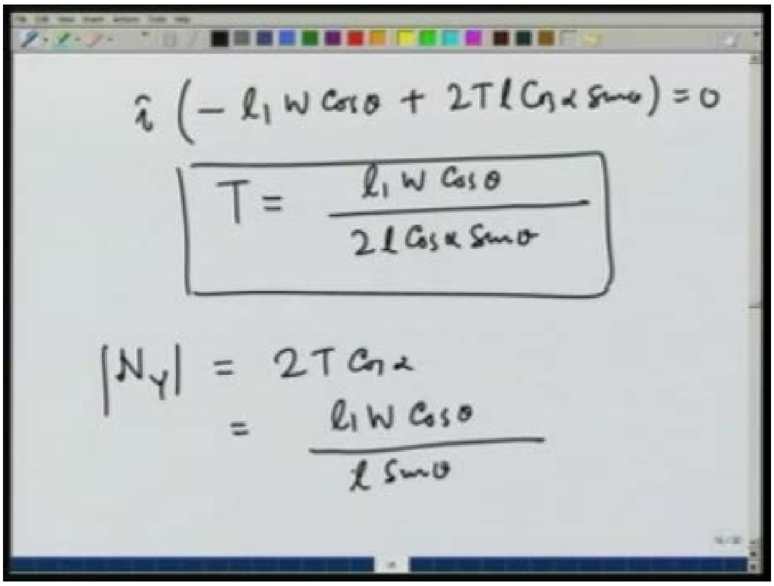

So, this gives me plus 2Tl cosine alpha sin theta. This is also in i direction and this is equal to 0 and that gives me T is equal to l1 w cosine theta divided by 2l cosine alpha sin theta. Once T is calculated you can calculate Ny which is equal to 2 T cosine of alpha in magnitude which will come out to be l1 w cosine theta over l sin of theta and that is your answer.

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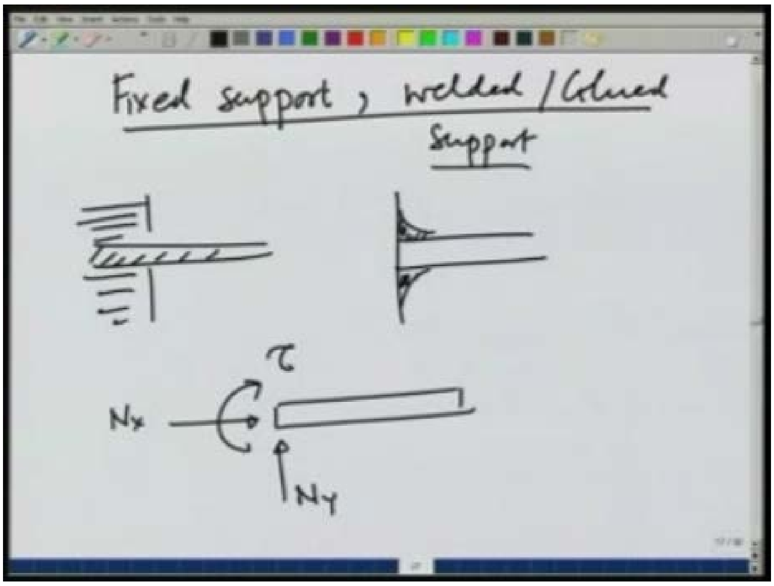

So, we considered a ball on socket joint, recall now that in 2 dimensions there was also a fixed support or a welded or glued support. Let me just remind you what it was, it was a support that was either built in a wall or it was something that was welded or glued here. And these supports put support to provide a force in x direction in y direction as well as a torque tau. The three dimensional generalization of this is going to be again a beam either fixed in a wall or something which is glued or welded at its corners.

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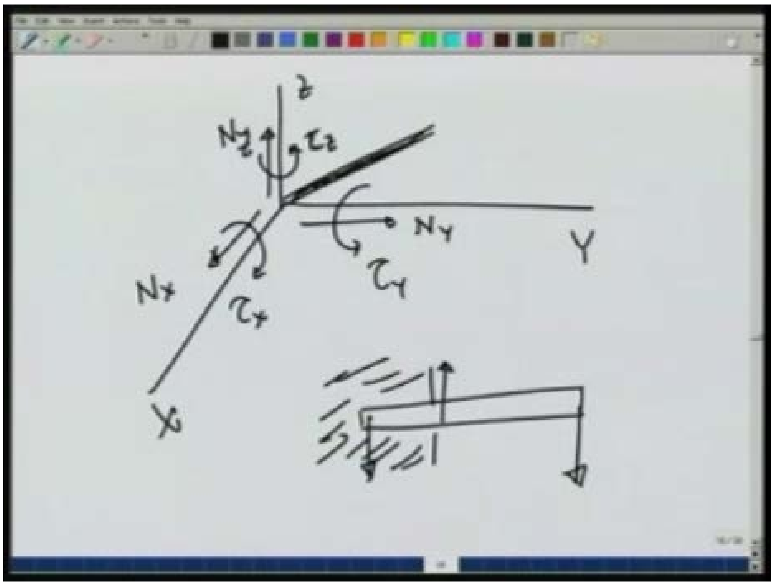

But, now it can provide given x, y and z directions and suppose, this is the fixed supported it can provide a force in x direction, it can provide a force in y direction and in addition it can provide a force in z direction. Since, it is fixed in the wall using arguments that we use in say two dimensions it can also provide a torque about the x direction. It can also provide a torque about the y direction and it can also provide a torque about the z direction.

This would come out using arguments that we used earlier for the 2 dimensional case in that when we pushed a built in support or fixed support in a wall like this, there were forces generated here and these provided a torque and a couple and a force. You generalize 2, 3 dimensions and you will get the answer that a a a fixed support in 3 dimensions can provide torques and forces along and about all the 3 axis. Hopefully, by the analysis carried out. So, far you would be able to now get an idea of what kind of force and what kinds of torque can a new element, engineering element provide.

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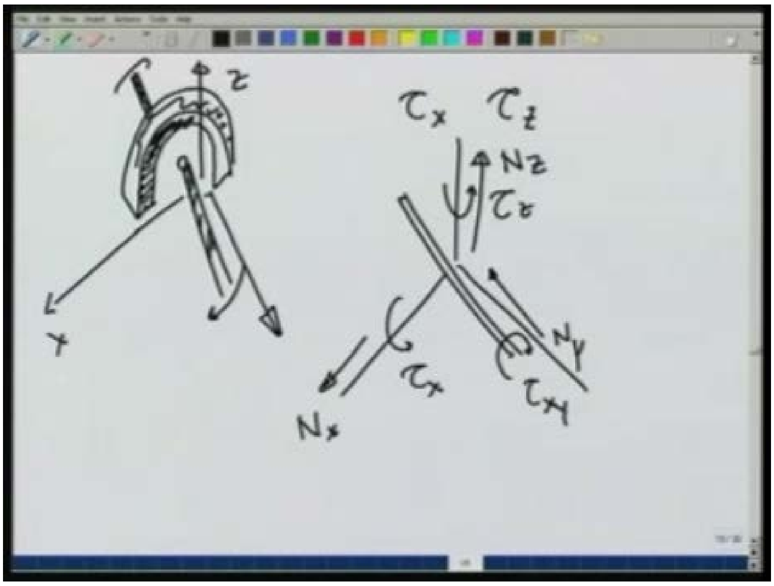

For example: suppose I were to give you a support like this with some thickness through which I put a shaft through hole here. Now, you will see if I rotate the shaft like this it cannot turn .So, if I take this to be x this to be y and this to be z direction let me be little more careful let me put y perpendicular to the support like this, then, you can see that about the x axis I cannot rotate it. So, support gives a tau x about the z axis I cannot rotate it. So, it gives me tau z; However, I can rotate it about the y axis and therefore, this kind of support would give me on the shaft a tau x a tau z, but no tau y.

On the other hand, I cannot push it along the x direction and therefore, it gives me a normal reaction along the x axis I cannot push it along the y direction if the shaft is fixed. So, it gives me a suppose I could push it along the y direction then, there is no force in this direction, but it I cannot push it along the z direction. So, it gives me a force in the z direction. In addition if it were also fixed along the y axis, if the thing if the shaft was completely fixed it would also provide a force in y direction. If it was not free to rotate it will also provide a torque along the y direction.

So, you can do analysis like this and find out what all an engineering element, what all kind of forces and torques can it tolerate or provide for equilibrium having talked about elements. Let me now talk about a few cases of different forces that due to geometry their particular geometry give make certain conditions equilibrium conditions irrelevant.

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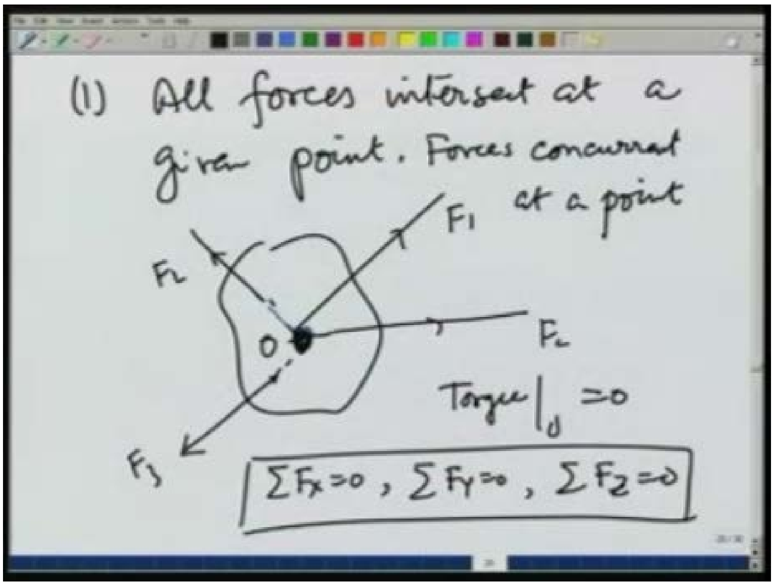

For example: let us take case 1 all forces intersect at a given point. So, suppose there is a body all the forces applied they may not be applied at the same point. So, F1 F2 F3 F4, but they all say intersect at a particular point let me show this point here like this they all intersect at this point. In that case the condition that torques be 0 becomes irrelevant. Because, if I take the torque about this point where they intersect the torque, about this point o would be 0 for all the forces. And therefore, the only equilibrium conditions are required are summation Fx is equal to 0, summation Fy is equal to 0 and summation Fz is equal to 0.

The forces that intersect at a particular point are known as forces concurrent on a point. So, for forces concurrent at a point the only equilibrium condition is summation Fx equals to 0, summation Fy is equal to 0 and summation Fz equal to 0.

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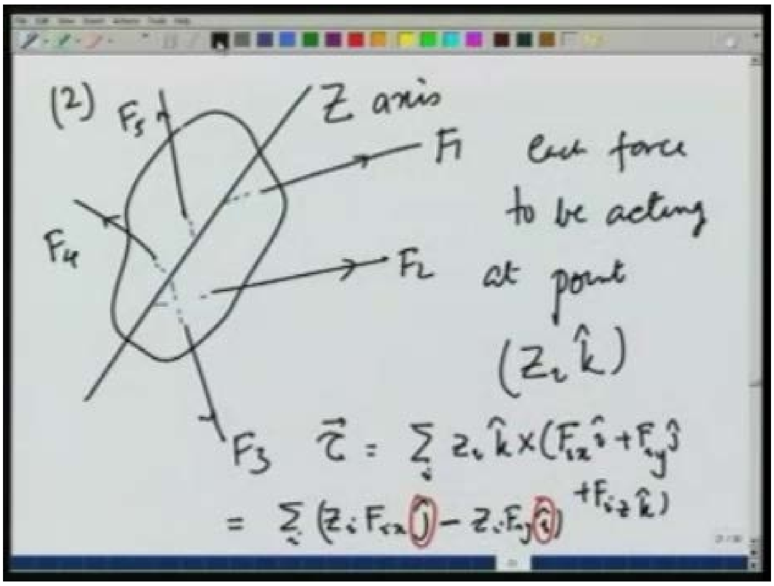

Let’s take look at another situation in which suppose the forces all intersect a particular line moment where they are applied, but they all intersect a particular line. And let us call that line the z axis for convenience, that line can be arbitrary chosen to be z axis. So, this is force F1, force F2, force F3, force F4, force5 and they are all intersecting the z axis or given line at 1 point or the other. Let us see what happens in this case, in such a case by the transmissibility of force vector. I can take each force to be acting force to be acting at point say, zi k that is at a distance zi in the z direction. And therefore, the torque by the forces is going to be summation i zi k cross Fi x component i plus Fi y component j direction plus F iz component in k direction.

And if I evaluate this since k cross k is 0, you will see this comes out to be summation I zi Fi x k cross i is j minus zi Fi y k cross j’s I with the minus sign. So, minus sin we taken care of. So, you see the only component of torques that such forces that are intersecting all intersecting 1 particular line which we take to be the z axis give torques in the direction only x and y. And therefore, we need not worry about the condition that summation tau z is 0 such forces are called concurrent on a line. So, forces concurrent on a line give me torque tau z equal to 0.

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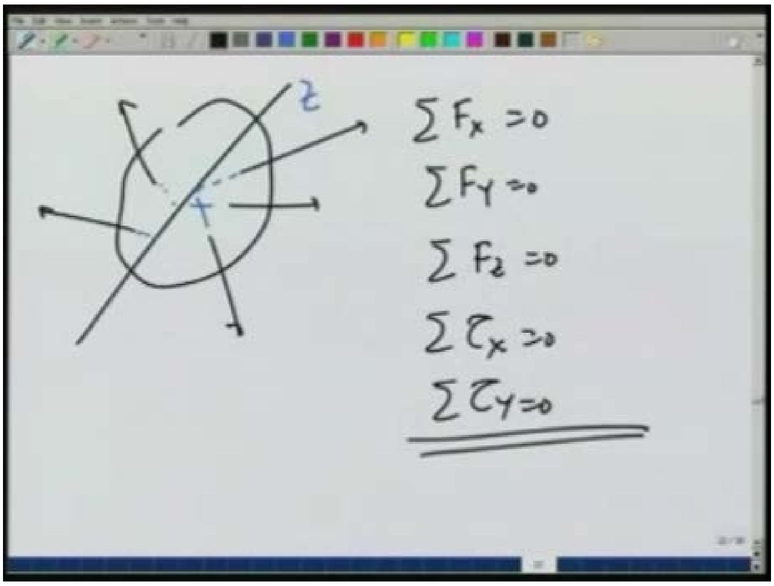

So, let us rewrite it. So, we have a situation where all the forces are concurrent on a line that is they all intersect 1 particular line which we are taking to with the z axis in that case of course, I have to satisfy summation Fx is equal to 0, summation Fy is equal to 0, summation Fz is equal to 0 and summation tau x is equal to 0 summation tau y is equal to 0 there is no equation for tau z because that automatically is 0. So, that is the another situation in which you realize all of a sudden the you realize suddenly that there is no component of torque in the z direction and therefore, I need not worry about equation.

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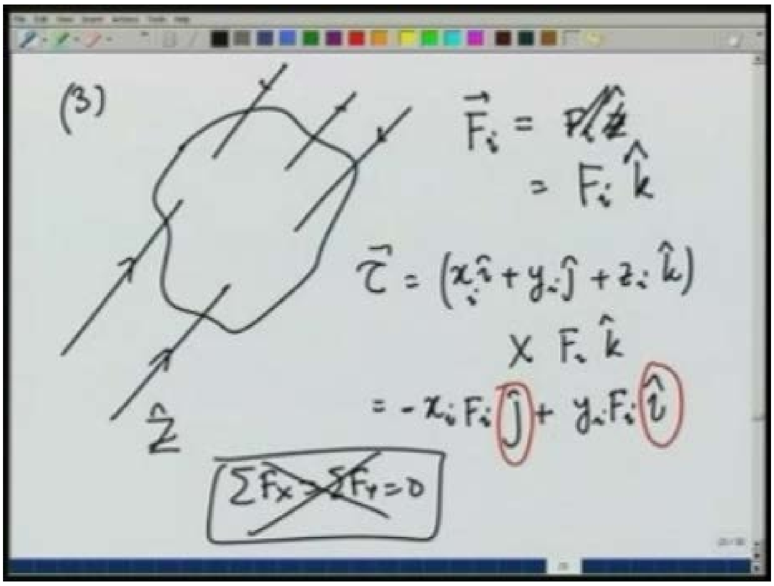

As a third situation let us take forces, which are parallel. Since, they are all parallel let us take that direction along which they are acting as a z direction again. So, that all forces Fi vector can be written as, Fi z let us write the unit vector that we have been using Fi k. And therefore, if I were to calculate their torque about any given point the torque which is xi i yi j plus zi k cross Fi k would give me again xi Fi i cross k is j with a minus sign plus yi Fi j cross k is i.

So, this also has components only in x and y directions. So, I need not worry about the z component of the torque. Since, there are no components of the force in x and y direction in this case; therefore, I have summation Fx is equal to summation Fy is equal to 0 automatically satisfy I need not consider it

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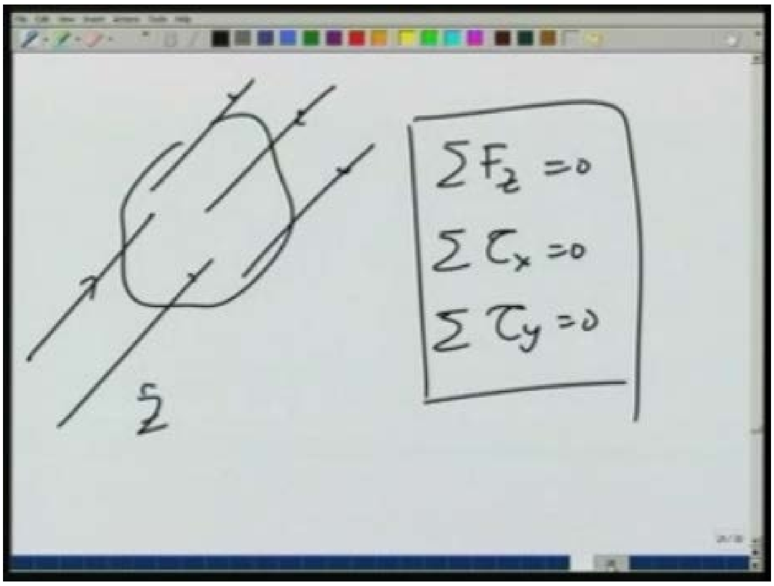

So in this case, when all the forces applied are parallel. And we take that parallel direction to be the z direction. In that case, the equilibrium condition is going to be summation Fz is equal to 0, summation tau x is equal to 0 and summation tau y is equal to 0 only 3 conditions. Of course, if none of these satisfy in general we have the condition general.

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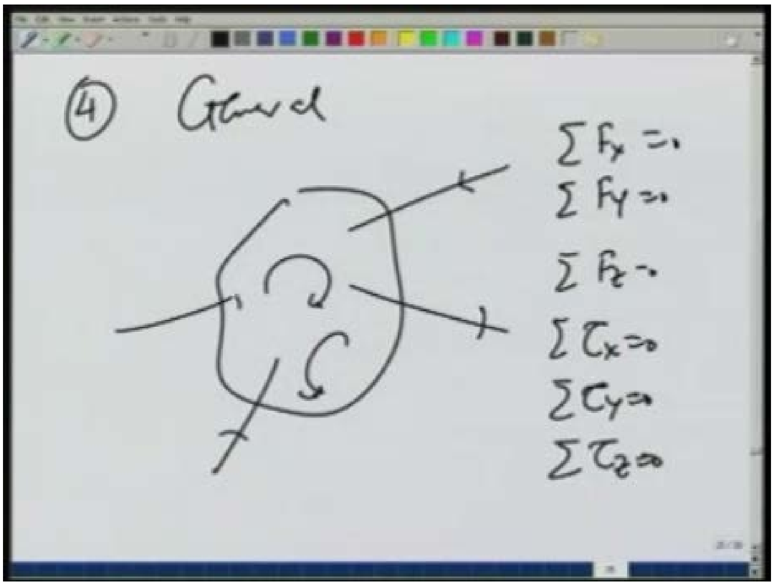

If there are all sorts of forces and all sorts of torques applied. In that case of course, summation Fx is equal to 0, summation Fy is equal to 0, summation Fz is equal to 0, summation tau x is equal to 0, summation tau y is equal to 0 and summation tau z is equal to zero. That is a most general condition, but what we covered earlier in 3 cases if the forces are concurrent or they are concurrent on a line or if they are parallel some of these conditions are automatically satisfied and we need not worry about them.

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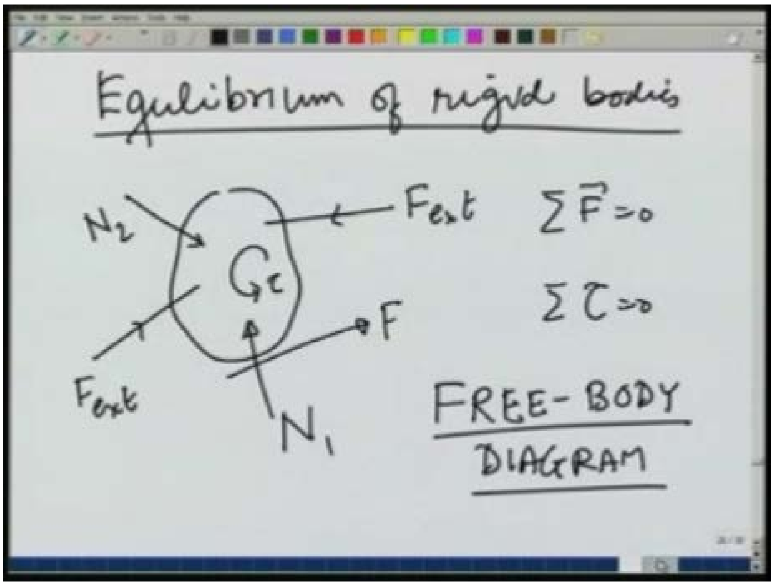

So, to review this equilibrium of rigid bodies, when I emphasize that they are rigid body because, we have not allowed any deformation. What we have done so far, is taken a particular body and saw what are the external forces applied on them and what are the forces generated normal reaction normal reaction 2, 1 may be a torque tau may be a force frictional force. What are these forces given by various elements by which it is held and then, we did the equilibrium condition summation F is equal to 0 and summation tau is equal to zero. Such a diagram where the elements which are holding the system we isolate the system and replace those elements holding system by the normal reactions of those elements or the torques provided by those elements is known as FREE- BODY DIGRAM. We have been using free body diagram. So far, but did not use this term now, I am introducing it.

So, free body diagram is the one where I take the body isolated and replace all the engineering elements that are holding it by their respective forces or torques provided.

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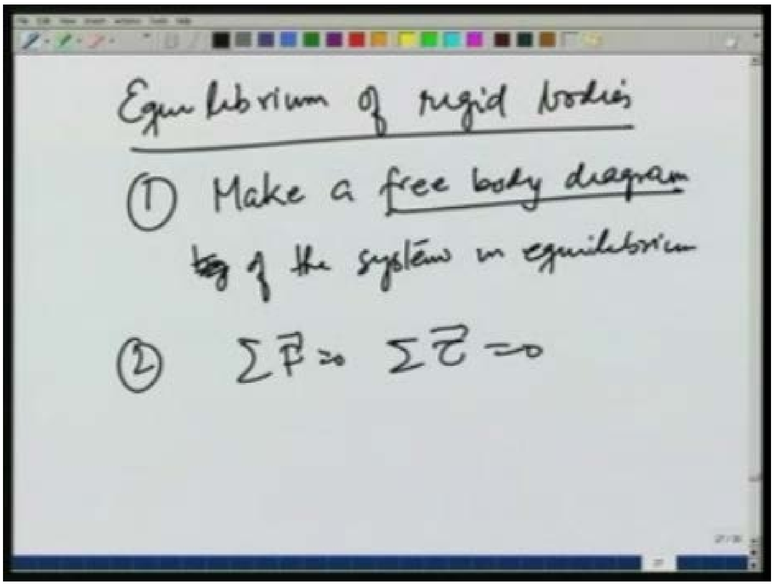

So, in considering the equilibrium body first thing we do is step number 1 equilibrium, make a free body diagram by isolating of the system in equilibrium and 2 apply the condition summation F is equal to 0 and summation tau is equal to 0 and solve it. So, this is a brief introduction to equilibrium of rigid bodies. In the next lectures, coming lectures we will be applying this condition to a very special engineering structure called trusses.