

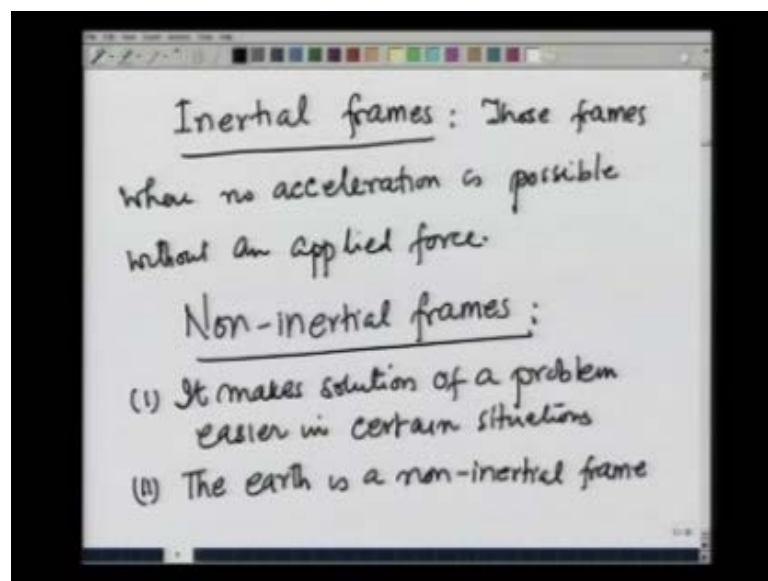
Engineering Mechanics
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Module - 09
Lecture - 01
Motion in Uniformly Accelerating Frames

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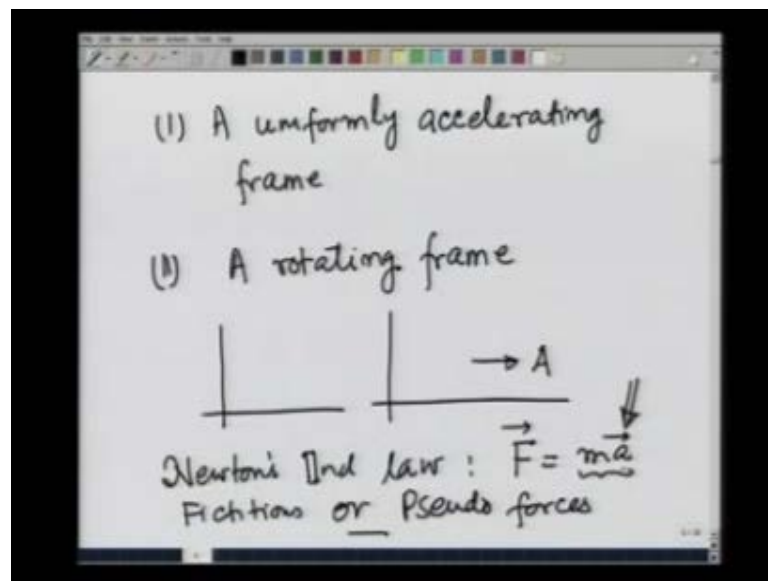
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In the problems that we have been solving so far, in dynamics we have been working in inertial frames by inertial frames we mean those frames of references, where no acceleration is possible without an applied force. As I have commented in the past this of course, is an idealization. On the other hand, it is sometimes useful to work in non-inertial frames where you would see that particles are accelerating without an applied force. The reason for working in non inertial frames are 2 force number 1 and it makes solution of a problem easier in certain situations and 2 as I already commented the inertial frame idea of inertial frame is an ideal 1 and most in real life the frames are non inertial and sometimes their effect is measureable and for example, the earth because of its rotation is a non-inertial frame and therefore, it has effects that are measurable and that are significant.

So, for these 2 reasons we will now work on how to solve problems in non-inertial frames. We will be considering 2 kinds of frames.

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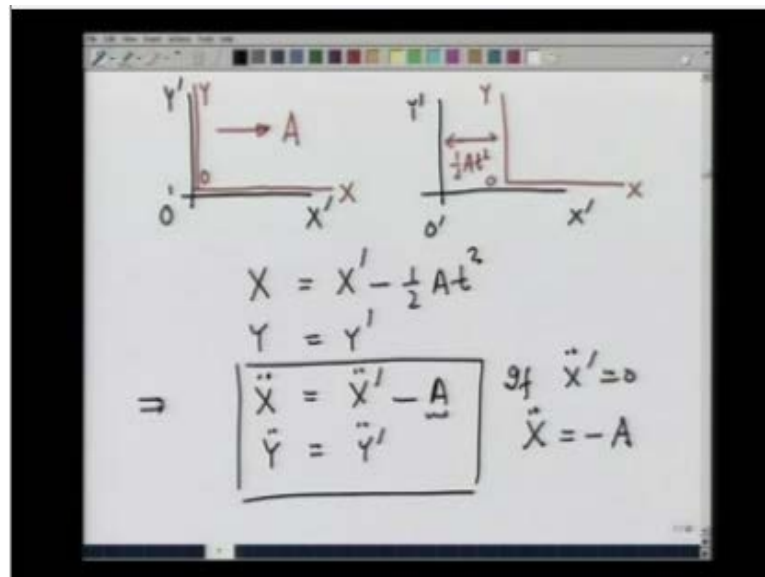


1 a uniformly accelerating frame and 2 a rotating frame. How an acceleration can arise in such frames without an apparent force? Can be seen if you see a form a frame which is uniformly accelerating frame. Suppose, you are in a car which is accelerating in a particular direction with acceleration A, as it is moving you will be seeing other things outside car accelerating backwards although there may be no force on those, but you will see them as if they are accelerating backwards and therefore, you see qualitatively how

sitting in a non-inertial frame or an uniformly accelerating frame you see things accelerating positive, but without any other any force being applied on them.

We want to solve problems in these frames applying Newton's second law that states that force equals m times the acceleration therefore, if you see things accelerating without any apparent force to account for this apparent acceleration you have to introduce certain forces and we call these forces the Fictitious or Pseudo forces. For example, if you are sitting in a uniformly accelerating frame you will have to imagine as if there is a force pulling things backwards opposite in the direction in which you are accelerating and that would be an a fictitious force which is equal to m times a.

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Let us see this quantitatively. Suppose, there is a fixed inertial frame let me call this X prime, Y prime with origin at O prime and let us have another frame which at time t equal to 0 is coincident with these axis X, Y with origin O, but this frame is accelerating to the right with an acceleration A. If that is the case, you can write the transformation equations as after time t what you would see is that the original frame is fixed where it is. But the accelerating frame has moved forward by distance of 1 half A t square in time t and therefore, you can write that X would be equal to X prime minus 1 half A t square Y equals Y prime and if there is a Z axis Z equals Z prime; Therefore, for a particle being observed in the 2 frames, X double prime would be equal to X prime or X double dot

that is the acceleration and the accelerating frame would be X prime double dot minus A . Y double prime would be equal to Y prime Y double dot would be Y prime double dot.

So, you see acceleration in the accelerating frame has decreased by an amount A , if X prime double dot is 0 that is; the acceleration in the fixed inertial frame is 0 you see that X double dot is equal to minus A . That is you will see, things moving past you accelerating backwards and the way we explain it as I said earlier is by introducing a Pseudo force let us see, how that comes along?

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The image shows a whiteboard with the following handwritten equations:

$$\ddot{X} = \ddot{X}' - A$$

$$m\ddot{X} = \underbrace{m\ddot{X}'} - mA$$

$$= F_{\text{applied}, X} - mA$$

$$\ddot{X} = \frac{F_{\text{app}} - mA}{m}$$

Below the equations, it is written: $F = F_{\text{applied}} - \underbrace{mA}_{\text{friction pseudo force}}$

So, just talking in this 1 dimensional case, X double dot which is the acceleration in the accelerating frame is equal to X prime double dot minus A . If I multiply both side by m mass this is $m X$ prime double dot minus $m A$ this we call is a true force F applied because X prime double dot is the acceleration in the inertial frames. So, this is real F applied force in the X direction minus $m A$ and therefore, the acceleration in the accelerating frame I write as F applied minus $m A$ divided by m .

This entire thing is like a new force which is calculated by F equals F applied or external minus $m A$ which is a fictitious or a pseudo force introduced to account for this kinematic effect. The kinematic effect is because, I am sitting in a frame which is

accelerating I will see things accelerating passed me for no apparent reason. So, reason I attach is, I introduce a force and now I have got my equation of motion.

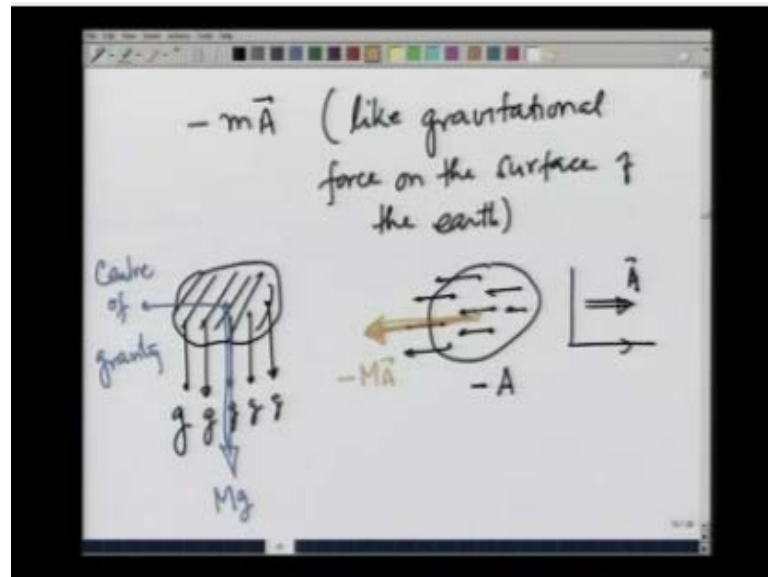
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The image shows a whiteboard with handwritten equations. The first equation is $\ddot{X} = \frac{F_{x(\text{applied})} - mA}{m}$. The second equation is $\ddot{r} = \frac{\vec{F}_{\text{applied}} - m\vec{A}}{m}$. Below these equations, it says $m\vec{A} = \text{pseudo force}$ and *Solution gives \vec{r} in the acc. frame as a function of time*.

And that is in the accelerating frame how would describe the change in the X coordinate as F in the X direction applied minus m A and that should solution of divide by m. So, solution of this should give me X as a function of time. In general, let me write r double dot in the accelerating frame as F in the accelerating frame F external or F applied minus m A vector, where A is the direction of acceleration of the uniformly accelerating frame divided by m.

This is going to be my equation of motion in the accelerating frame. I emphasize again the force that we have introduced m A is a pseudo force it does not exist its nature, but we feel as if things are accelerating pass. So, introduce this force and solution of this gives solution gives r in the accelerating frame as a function of time. One comment about this pseudo force that we have introduced.

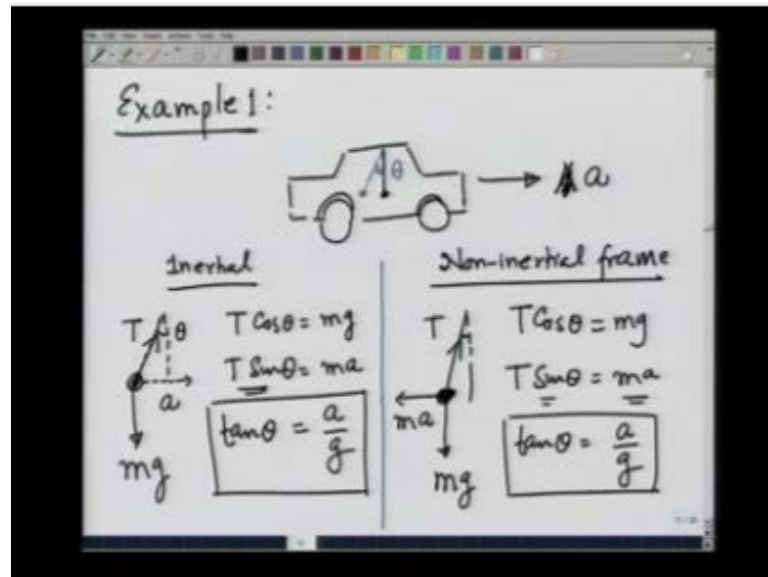
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Minus $m\vec{A}$ it is like gravitational force on the surface of the earth it is a there is direction is slightly different. Recall that, if I have a body on the surface of the earth all parts of it are pulled towards the earth by a uniform acceleration g . Similarly, in this case if I observe an object from a frame which is moving say towards the right with the acceleration A all the parts of this body are accelerating backwards with the same acceleration A . So, it is as if the body is an effective gravitational field with gravitational acceleration minus A . Just like the force out here the net force in this case acts on the centre of gravity which we call the net force is Mg and its act acts at the centre of gravity in exactly the same manner out here also.

Since, the force is uniform acceleration for each particle it will also be acting on the centre of gravity towards minus A direction and the total force of the MA minus MA , but it will work at the centre of gravity. So, that is the sort of similarity between the pseudo force observed in a uniformly accelerating frame and the gravitational force. The way we can use this fact that things in an accelerating frame appear to be being pulled by a gravitational kind of force we can use them to solve problems in much easier way in certain situations and this I will best illustrate through examples.

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Let us take the first example where, whether I solve in an inertial frame or non-inertial frame it does not matter if I take a car and I have a pendulum in it and the car is accelerating to this side with an acceleration A let me use a what could be the position of the pendulum in this car? You can almost feel that the pendulum is going to be like this if the car is accelerating, but why it should it be with this angle here theta. We solve this problem both in the non-inertial and the inertial frame.

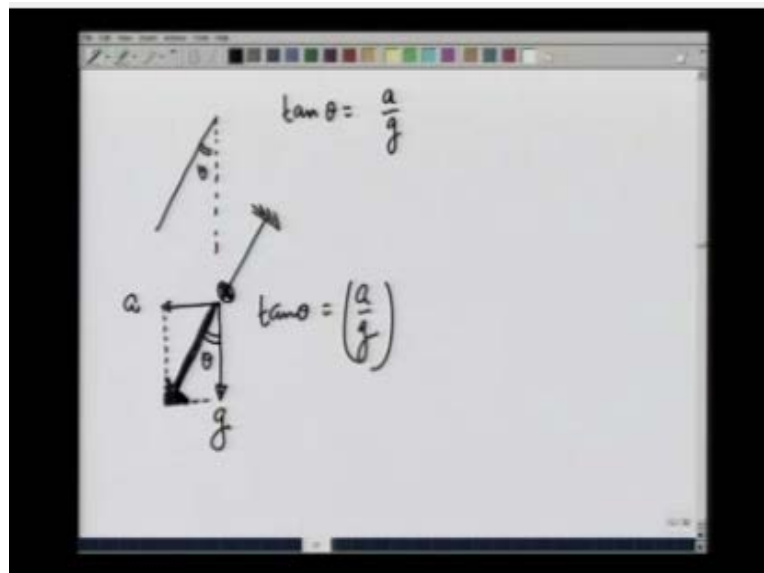
In the inertial frame that is frame from outside the free body diagram of the pendulum in this position is that on the bob there is a tension T force mg pulling it down and as a result of these 2 forces the bob is accelerating to the right with an acceleration a because the car is accelerating and the bob is stationary in the car. If this angle is theta then you can see that, $T \cos$ of theta is equal to mg because the ball is not moving up and down and $T \sin$ of theta is equal to ma because the bob is accelerating towards the right with an acceleration a and that gives you tangent of theta equals a over g .

What if I look at the same problem in non-inertial frame and what frame do I choose, I choose the frame which is attached with the car and moving to the right with it. In this frame you see the bob in equilibrium under the force mg tension T and a fictitious force ma to the left. Notice that when, I am sitting in the car the bob on the pendulum is in equilibrium position because, with respect to the car frame it is not moving at all the three forces are to be balanced for equilibrium and that gives you $T \cos$ of theta

equals mg and $T \sin \theta$ is matrix. Notice the difference in the inertial frame $T \sin \theta$ gives an acceleration towards the right. In the non inertial frame $T \sin \theta$ balances the fictitious force with the final result of course, being the same that tangent θ is a over g .

So, you see two way of looking at the same problem and of course, I should get the same final answer same effect. As I commented earlier that I can think of this, the force fictitious force in an uniformly accelerating frame as a gravitational force is illustrated beautifully by this example.

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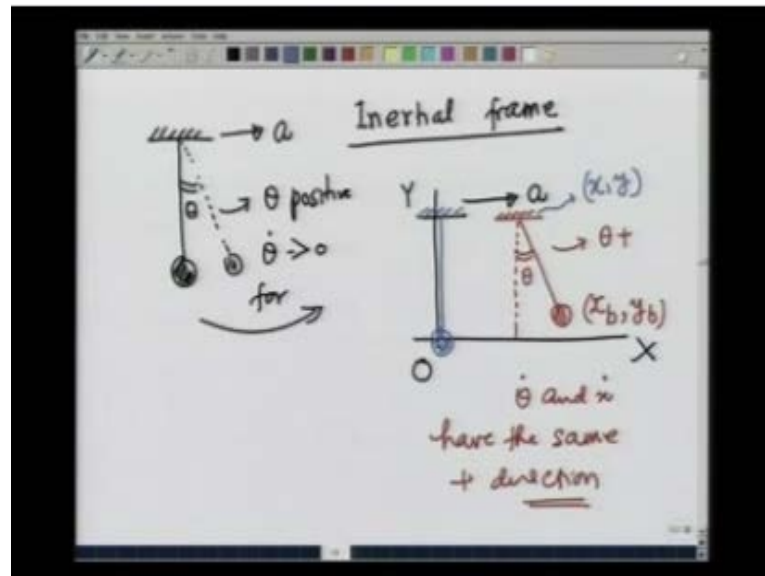


Because now, the pendulum is at an angle θ from the vertical and that tangent θ is given by a by g . If I look at it from the gravitational field point of view the pendulum is actually in a gravitational field which is the sum of these 2 gravitational fields. Acceleration g this way and acceleration a this way; so, the net gravitational field is a sum of these 2 and that is in this direction which is shown here by the big arrow. And this is at an angle θ with tangent θ equals a over g any pendulum in its equilibrium aligns with the gravitational field.

So, no wonder that the pendulum is aligned with this in its equilibrium position that is the explanation for this. So, we can see that thinking of this field in an the pseudo force or pseudo field in a no-inertial frame is like a gravitational field is meaningful and gives you the right answer. Using this now, we will solve a slightly more complicated problem

which will show you that actually going to non-inertial frame sometimes makes life very very easy.

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The problem that we solve now is suppose, there is a pendulum and its support is suddenly accelerate to the right with an acceleration a which way would the pendulum move and by how much angle that is a question that we want to answer. First, let us solve this problem in the inertial frame attached to the ground. I am solving it for two reasons; 1 you will see how we would solve a difficult problem and two when you compare the solution in an inertial frame compare to the solution in a non-inertial frame you will see the effectiveness of solving problems in a non-inertial frame sometimes.

So in this case, when this accelerates to the right let us assume that the pendulum also swings to the right with this angle being θ I am choosing it to swing to the right. So that, this way is θ positive and $\dot{\theta}$ will be greater than 0 for pendulum to this swinging to the right. This I do because, now I am going to fix my frame like this X and Y and initially let the pendulum I plotted with blue be here, with the bob at the origin and as the pendulum is its base accelerates. Its pivot point is accelerated to the right with a ... we are assuming that after sometime t the pendulum would look like this, with it having rotated by an angle θ to the right with respect to vertical.

As I commented earlier I choose this direction as θ positive because, then $\dot{\theta}$ and \dot{x} have the same positive direction and that is important otherwise, I will may I

may have to change sign along the way. Let me write for the bob position its coordinate as x_b and y_b and for the support let me write x and y . So, let me make this picture once more what we have is this.

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The image shows handwritten equations on a whiteboard. At the top, two equations are boxed: $\ddot{x}_b = -\frac{T \sin \theta}{m}$ (labeled (1)) and $\ddot{y}_b = \frac{T \cos \theta}{m} - g$ (labeled (2)). Below these, the relationship between the bob's position and the support's position is given as $\frac{1}{2} a t^2 = x_b - l \sin \theta \Rightarrow a t = \dot{x}_b - l \cos \theta \dot{\theta}$. This leads to equation (3): $a = \ddot{x}_b + l \sin \theta \dot{\theta}^2 - l \cos \theta \ddot{\theta}$. Similarly, for the vertical direction, $y = y_b + l \cos \theta \Rightarrow 0 = \dot{y}_b - l \sin \theta \dot{\theta}$, which leads to equation (4): $0 = \ddot{y}_b - l \sin \theta \cos \theta \dot{\theta}^2 - l \sin \theta \ddot{\theta}$.

Frame at T equal to 0 this is the way the pendulum is and after time t the base has moved here the pendulum has moved this way by an angle θ . This distance is $\frac{1}{2} a t^2$ because, the base or the support of the pendulum is moving with an acceleration a this is x_b , y_b and y of the support does not change let the tension in the string be T .

Let us write the equations of motion for the ball. I am going to have \ddot{x}_b equal to minus $T \sin \theta$ divided by m that is my equation one. I am going to have \ddot{y}_b equal to $T \cos \theta$ over m minus g that is my equation 2 because, in the bob in the vertical direction is $T \cos \theta$ component and a g component. How many unknowns do I have? I have unknown x_b , I have unknown y_b , I have unknown θ and I have tension T four unknowns.

So, I need two more equations. Those equations are provided by the relationship between x_b , y_b and the coordinate of the support. The support has moved by distance $\frac{1}{2} a t^2$ and that is equal to x_b minus if the length of the pendulum is l it is going to be minus $l \sin \theta$. Similarly, I am going to have y of the support equal to y_b this is y_b plus $l \cos \theta$ this is my equation three and equation 4. I have gotten 4 equations,

four unknowns and I can solve for them. So, let us do that. Let me rewrite the equations once more and then we will solve for the problem.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, two equations are boxed: $\ddot{x}_b = -\frac{T \sin \theta}{m}$ (labeled (1)) and $\ddot{y}_b = \frac{T \cos \theta}{m} - g$ (labeled (2)). Below these, the equation $\frac{1}{2} a t^2 = x_b - l \sin \theta$ is shown, which is differentiated to $at = \dot{x}_b - l \cos \theta \dot{\theta}$. This is further differentiated to $a = \ddot{x}_b + l \sin \theta \dot{\theta}^2 - l \cos \theta \ddot{\theta}$, labeled (3). Similarly, the equation $y = y_b + l \cos \theta$ is differentiated to $0 = \dot{y}_b - l \sin \theta \dot{\theta}$, which is further differentiated to $0 = \ddot{y}_b - l \cos \theta \dot{\theta}^2 - l \sin \theta \ddot{\theta}$, labeled (4).

So, I have x_b double dot equals minus $T \sin \theta$ divided by m , I have y_b double dot equals to $T \cos \theta$ divided by m minus g , I have $\frac{1}{2} a t^2 = x_b - l \sin \theta$ if I differentiate this once it gives me $at = \dot{x}_b - l \cos \theta \dot{\theta}$. If I differentiate it once more it gives me $a = \ddot{x}_b + l \sin \theta \dot{\theta}^2 - l \cos \theta \ddot{\theta}$ and let me call this equation number 3.

Similarly, $y = y_b + l \cos \theta$ gives me $\dot{y} = 0$ equals $\dot{y}_b - l \sin \theta \dot{\theta}$. Differentiating it once more I get $0 = \ddot{y}_b - l \cos \theta \dot{\theta}^2 - l \sin \theta \ddot{\theta}$ let me call this equation number 4. So, I have equation number 1 here, we box them this is equation number 1, equation number 2, equation number 3 and equation number 4. Let me substitute for x_b double dot in terms of T and y_b double dot in terms of T in equation number 3 and 4 and what do I get?

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I am going to get $x\ddot{\theta}$ which is $-\frac{T \sin \theta}{m} + l \sin \theta \dot{\theta}^2$. Let me confirm that, $l \sin \theta \dot{\theta}^2 - \cos \theta \ddot{\theta}$ is equal to a and for the y equation I get let me again see this $y\ddot{\theta} - \cos \theta \dot{\theta}^2 - l \sin \theta$.

So, I am going to get $y\ddot{\theta}$ which is $\frac{T \cos \theta}{m} - g - l \cos \theta \dot{\theta}^2$ is equal to 0. Let me call, this equation number 5 and equation number six. Multiply equation number 5 by $\cos \theta$ and multiply equation number 6 by $\sin \theta$ and add. If I add, if I do that I get the first term $T \cos \theta$ minus $T \sin \theta \cos \theta$ plus $T \cos \theta \sin \theta$ that cancels; Second term, $l \sin \theta \dot{\theta}^2 \cos \theta - l \cos \theta \sin \theta \dot{\theta}^2$ that cancels. So, I get $-g \sin \theta$ these 2 terms; give me $-l \cos^2 \theta + \sin^2 \theta$ that is $l \ddot{\theta}$ is equal to $a \cos \theta$.

In other words, $l \ddot{\theta}$ is equal to $-a \cos \theta - g \sin \theta$ that is my equation for θ . I have done my job I solve this equation and I can get all the answers let us do that.

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Handwritten derivation on a whiteboard:

$$l \ddot{\theta} = -a \cos \theta - g \sin \theta$$

$$\ddot{\theta} = \frac{1}{2} \frac{d}{d\theta} \dot{\theta}^2$$

$$\frac{l}{2} \dot{\theta}^2 = -a \sin \theta + g \cos \theta + C$$

at $\theta = 0, \dot{\theta} = 0$

$$\Rightarrow C = -g$$

$$\boxed{\frac{l}{2} \dot{\theta}^2 = -a \sin \theta - g(1 - \cos \theta)}$$

So, the equation that we have gotten is $l \theta$ double dot is equal to minus $a \cos$ of θ minus $g \sin$ of θ . Let us put θ double dot equals $\frac{1}{2} \frac{d}{d\theta}$ of θ dot square and integrate it to get $\frac{l}{2} \theta$ dot square is equal to minus $a \sin \theta$ plus $g \cos$ of θ plus the constant C ; However, at θ equal to 0 then it started θ dot is equal to 0 and that gives me C equals minus g and therefore, $\frac{l}{2} \theta$ dot square comes out to be minus $a \sin \theta$ minus $g(1 - \cos \theta)$ that is the solution for θ dot square.

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Handwritten derivation on a whiteboard:

Diagram 1: A horizontal bar with acceleration a to the right. A pendulum is suspended from the bar. The angle θ is measured from the vertical.

$$\boxed{\frac{l}{2} \dot{\theta}^2 = -a \sin \theta - g(1 - \cos \theta)}$$

Note: $\dot{\theta}$ cannot be positive

When $\dot{\theta}^2 = 0$

$$\theta = 0$$

$$0 = -a \sin \theta - g(1 - \cos \theta)$$

$$= -2a \sin \frac{\theta}{2} \cos \frac{\theta}{2} - 2g \sin^2 \frac{\theta}{2}$$

$$\boxed{\tan \frac{\theta}{2} = -\frac{a}{g}}$$

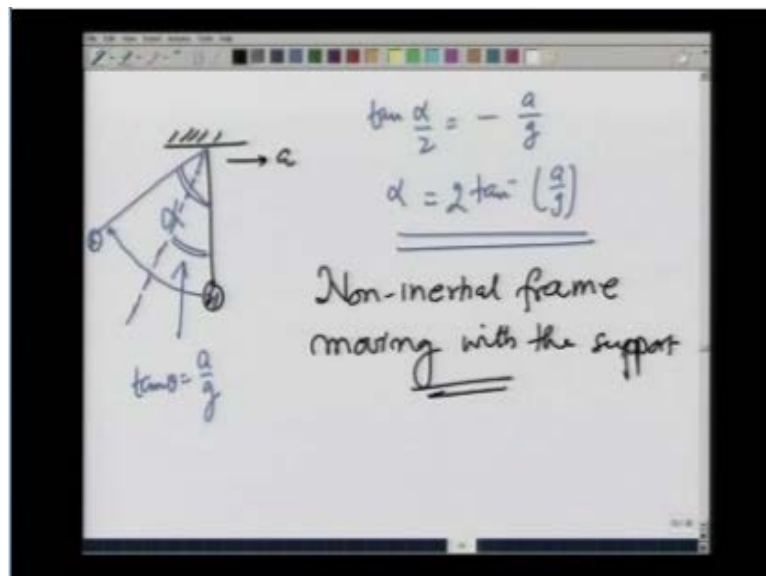
Diagram 2: A pendulum suspended from a pivot. The angle θ is measured from the vertical.

So, what we have got is when I have a pendulum had a support and suddenly the support moves with an acceleration a , I am going to have the angle of this change by $l \dot{\theta}^2$ is going to be equal to minus $a \sin \theta$ minus $g (1 - \cos \theta)$.

Note 1: θ cannot be positive if θ becomes positive the right hand side would be negative and that will give you $\dot{\theta}^2$ to be negative which cannot happen; therefore, the pendulum has to swing in negative θ direction it has to swing back. So, after some time its position would be something like this. It would have swung in θ in negative direction and where does it stop. So, question we ask where is $\dot{\theta}^2 = 0$.

One answer of course, is $\theta = 0$ which we point we started with and the second point is going to be $0 = -a \sin \theta - g (1 - \cos \theta)$ which I can write as $-2a \sin \frac{\theta}{2} \cos \frac{\theta}{2} - 2g \sin^2 \frac{\theta}{2}$ and that gives you before let us see, this $\sin \frac{\theta}{2}$ by 2 of them drops out these 2 drops out and that gives you $\tan \frac{\theta}{2} = -\frac{a}{g}$. So, $\dot{\theta}^2 = 0$ again when $\tan \frac{\theta}{2} = -\frac{a}{g}$.

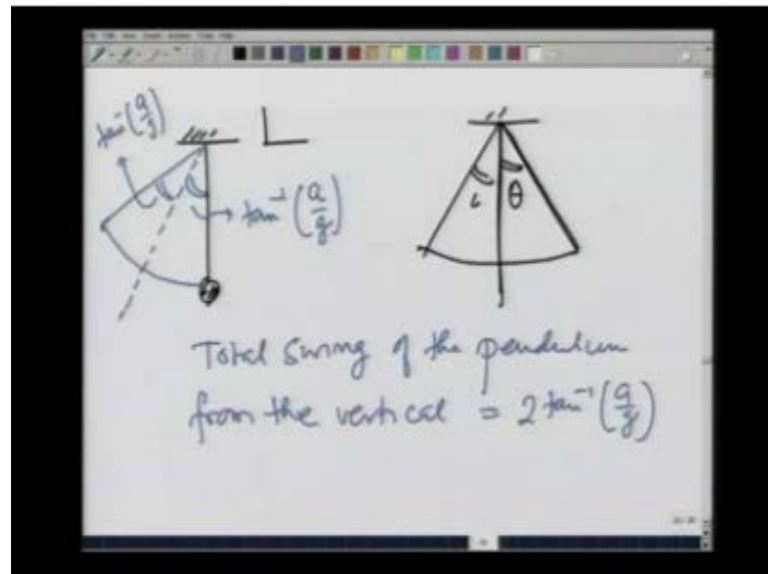
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So, here was a support which suddenly moved towards this with acceleration a and the final position is such that at a point such that this angle is α , then $\tan \frac{\alpha}{2}$ is minus a upon g or α just in magnitude is equal to $\tan^{-1} \frac{a}{g}$ times two. This has a beautiful interpretation remember earlier this was the angle which has tangent

theta equals $\tan^{-1}(a/g)$ that angle where the new equilibrium point of the pendulum was. So, in a way the pendulum has swung across the equilibrium point by as much angle as it was away from it on 1 side and this is a solution that we gotten by solving an a an inertial frame. Let us see now, how would the solution look in a non-inertial frame, non-inertial frame moving with the support.

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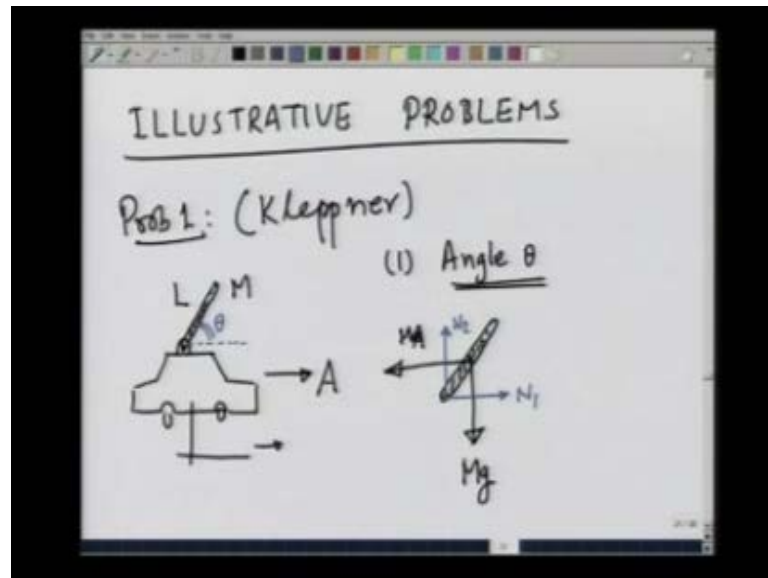
So, I look at this whole thing in a non-inertial frame moving with the support in that frame this is the equilibrium position of the pendulum such that this angle is $\tan^{-1}(a/g)$. So, in this non-inertial frame this is the equilibrium position. So, I am looking at the pendulum as if it is moved to 1 side from the equilibrium position by an angle $\tan^{-1}(a/g)$. Recall what happens, if I have a pendulum from the equilibrium position if it is left by if it is raised by an angle θ to 1 side what happens, when it swings back.

It swings exactly by the same angle to the other side. So, in this move the this frame accelerating frame the pendulum which swing across the equilibrium point exactly by the same angle as it was initially to the 1 side and that is going to $\tan^{-1}(a/g)$. So, the total swing of the pendulum from the vertical is equal to $2 \tan^{-1}(a/g)$.

Now, I would like you to appreciate the power of having solve this problem in a non-inertial frame while solving in the inertial frame I have to do work with many many equations 4 or 5 equations and had to manipulate a lot of them. On the other hand, when

I went to non-inertial frame the problem could be solved in 3 or 4 lines and that is the power of using non-inertial frames in certain problems having established that non-inertial frames do provide. Particularly in this case uniformly moving non-inertial frames do provide method of solving problems in an easier way. We will now solve a few examples; most of them will be from the book of Kleppner.

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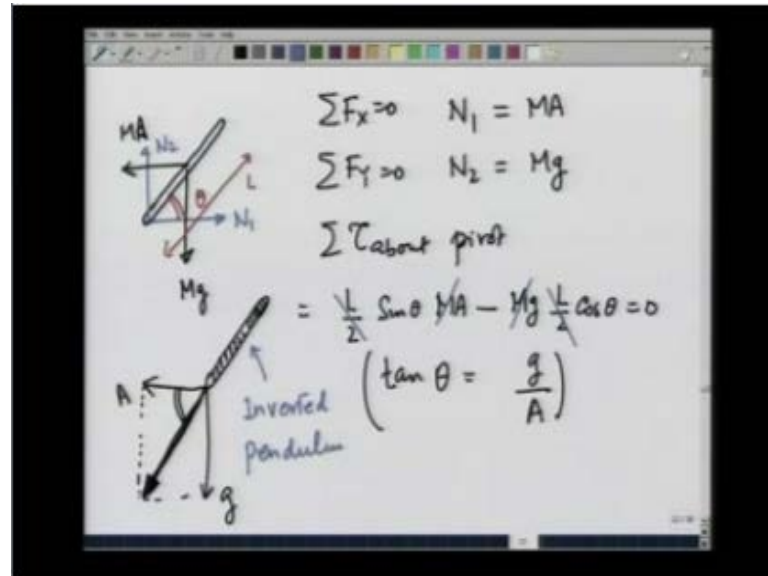
So, we are going to solve illustrative problems. Problem 1: this is taken from the book of Kleppner whose reference I have already given you. If I have a car moving with acceleration A to the right and it has a rod on the pivot of length L and mass M we want to find at what angle from the top of the car would it balance and what happens, when it is moved from that balance point slightly. So, let us first calculate angle θ at which the bar is balanced. We have already established that going to an uniformly moving frame makes problems easy.

So, we will be solving this problem in a uniformly moving frame which is attached to the car and therefore, accelerating to the right with acceleration A . When the rod is in equilibrium in this uniformly moving accelerating frame it is under equilibrium it is in equilibrium under the forces of Mg and has I have already argued. I can think of the fictitious force MA like an effective gravitational force which is working in the direction opposite to the direction of acceleration of the frame and it also acts on the centre of

gravity. So, it is under the influence of the MA to the left and there are normal reactions at the pivot let us call this N 1 to the right and N 2 vertically up.

So, this rod is under equilibrium in equilibrium under these forces MA, N 1, N 2 and Mg. Let us go the next page and show this free body diagram of the rod again,

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It has a force Mg acting downwards a force MA working to the left normal reaction N_1 working the right normal reaction N_2 working to the vertically up and summation F_x equals 0 gives me N_1 equals MA . Summation F_y equal to 0 gives me N_2 equals Mg and summation torque about pivot gives me if this angle is theta and the length of the rod is L it gives me L over 2 sin theta times MA minus Mg times L over 2 cosine of theta equal to zero. M drops out L by 2 drops out and that gives me tangent of theta equals g over A .

So, the rod is at an angle theta such that tangent theta equals g over A from the horizontal. What direction is this? Recall earlier I had told you if there is an acceleration and there is g then the effective gravitational field is this one. The sum total of the 2 at this angle which is tangent theta equals g over two. So, it is not surprising that the rod is in equilibrium precisely aligned up with the direction of the effective gravitational field it is like an inverted pendulum right. It is the pendulum you can sort of put pointing up and that will also be in equilibrium; however, inverted pendulum is in unstable equilibrium if you just disturb it slightly from the equilibrium point it takes off. So, this rod should also be in an unstable equilibrium. Let us see, that in the next part of the problem.

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(b) What happens to the rod when it is disturbed from its equilibrium position

$$I \ddot{(\theta + \phi)} = I \ddot{\phi}$$

$$= MA \frac{L}{2} \sin(\theta + \phi) - Mg \frac{L}{2} \cos(\theta + \phi)$$

$$= \frac{ML}{2} \left[A \sin \theta \cos \phi + A \cos \theta \sin \phi - g \cos \theta \cos \phi + g \sin \theta \sin \phi \right]$$

$\phi \rightarrow$ very small

So, in part b we see what happens to the rod when it is disturbed from its equilibrium position. So, the rod is at angle theta from the horizontal or from the top of the car and what we do now, is disturb it slightly by an angle phi what happens, then naturally there is a force MA working to the left, Mg pulling it down and if I write the equation for theta plus phi about the pivot point. Here let us see, what that equation is looks like. So, that equation is going to look like. I where I is the moment of inertia of the rod about the pivot point theta plus phi double dot which is nothing but, I phi double dot theta is a given a fixed angle is equal to phi is going this way positive.

So, it is going to equal to MA L by 2 sin of theta plus phi minus Mg L by 2 cosine of theta plus phi. Let us expand this, when we expand this we are going to get ML by 2 can be taken out. A sin theta cosine phi plus A cosine theta sin phi minus g cosine theta cosine phi plus g sin theta sin phi. Let us take phi to be very small.

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$$I \ddot{\phi} = \frac{ML}{2} [A \sin \theta + A \cos \theta \phi - g \cos \theta + g \sin \theta \phi]$$

$$\tan \theta = \frac{g}{A} \Rightarrow \cos \theta = \frac{A}{\sqrt{g^2 + A^2}}, \sin \theta = \frac{g}{\sqrt{g^2 + A^2}}$$

$$I \ddot{\phi} = \frac{ML}{2} \left[\frac{A \cdot g}{\sqrt{g^2 + A^2}} + \frac{A \cdot A}{\sqrt{g^2 + A^2}} \phi - \frac{g \cdot A}{\sqrt{g^2 + A^2}} + \frac{g \cdot g}{\sqrt{g^2 + A^2}} \phi \right] = \frac{ML}{2} \sqrt{g^2 + A^2} \phi$$

So, that sin phi is almost phi and cosine phi is 1 and therefore, I get I phi double dot is equal to here, let us look here A sin theta ML by 2 A sin theta plus A cosine theta times phi minus g cosine theta plus g sin theta times phi. Recall tangent theta is g over A and therefore, cosine theta is equal to A over a square of g square plus A square and sin theta is g over cosine theta is A over g over square root of g square plus A square. Let us substitute that and then we get I phi double dot is equal to ML over 2 A sin theta is g over a square root g square plus A square plus A cosine theta is A over square root of g square plus A square phi minus g cosine theta is A over square root of a g square plus A square plus g sin theta is g over square root of g square plus A square phi.

This term cancels and you get this is equal to ML over 2 square root of g square plus A square times phi. And therefore, the equation when, the rod is disturbed from its equilibrium position is I phi double dot is equal to ML by 2 square root of g square plus A square phi or I phi double dot minus ML over 2 I a square of g square plus A square phi equal to 0.

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$$I \ddot{\phi} = \frac{ML}{2} \sqrt{g^2 + A^2} \phi$$

$$I \ddot{\phi} - \frac{ML}{2I} \sqrt{g^2 + A^2} \phi = 0$$

$$\phi = A e^{\lambda t} + B e^{-\lambda t}$$

$$\lambda^2 = \frac{ML}{2I} \sqrt{g^2 + A^2}$$

And the solution of this you have already seen in many many examples, of the form phi equals A e raise to the lambda t plus B e raise to minus lambda t, where lambda is lambda square is ML over 2 I square root of g square plus A square. So, we can see with time the solution is going to grow phi is going to grow and therefore, the equilibrium is unstable problem number 2.

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Prob 2

$$I \ddot{\theta} = \frac{MAW}{2} \cos \theta$$

$$\ddot{\theta} = \frac{1}{2} \frac{d}{dt} \dot{\theta}^2 = \frac{I \dot{\theta}^2}{2} = \frac{MAW}{2} \sin \theta + C$$

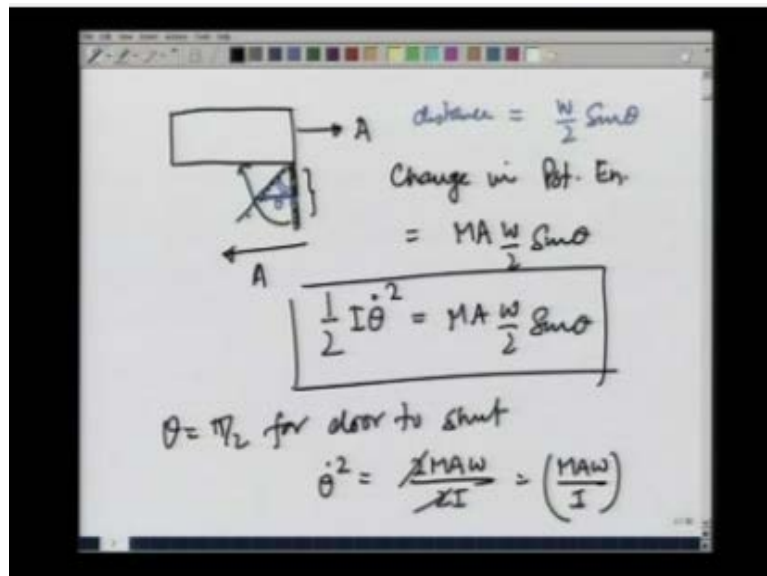
$$\dot{\theta}^2 = 0 \text{ for } \theta = 0 \Rightarrow \frac{I \dot{\theta}^2}{2} = \frac{MAW}{2} \sin \theta$$

We do suppose I have a car whose door is opened at the right angle to the car and suddenly the car accelerates to the right with an acceleration A. I want to find as the door

swings back when, it has swung by an angle theta what is its angular speed. As I said earlier is best to go to an accelerating frame in this yes and you will see that the door hinge here a sort of moving in a gravitational field with a force MA being applied as its centre of gravity.

At an angle theta the force acts as a centre of gravity on this side MA. If the width of the door is W then the distance from the hinge to the force if it has moved by an angle theta here is W by 2 cosine of theta. Therefore, the equation of motion of the door in the accelerating frame is going to be I where I is the moment of inertia of the door theta double dot is equal to MAW over 2 cosine of theta. Again writing theta double dot as 1 half d over d theta theta dot square we get I theta dot square divided by 2 as equal to MAW over 2 sin theta plus a constant, but theta dot square is equal to 0 for theta equal to 0 and therefore, I theta dot square over 2 is equal to MAW over 2 sin of theta and that gives you the angular velocity of the door at an angle theta when, the car starts accelerating towards the right.

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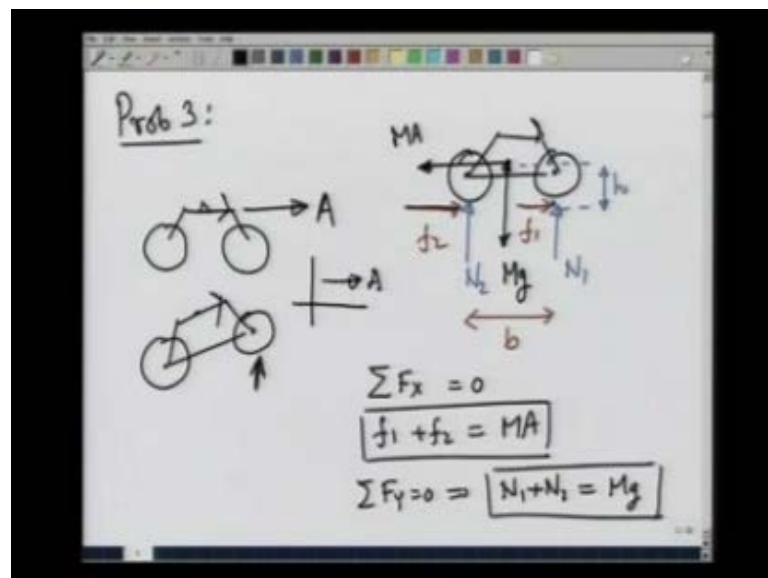


This is a nice interpretation of this and you will see that really the interpreting of this force fictitious force MA like a gravitational field makes sense, When this is moving towards the right with acceleration A we can consider this door to be in a gravitational field with gravitational acceleration A by the time the door has swung by an angle theta a centre of mass has come down by this much distance here. And that distance that it has

travelled along the direction of minus A is nothing but, the W over $2 \sin$ of theta and therefore, change in its potential energy is equal to MA which is a force times W over $2 \sin$ of theta. It is by that much that its potential energy has decreased and therefore, the increase in the kinetic energy should precisely that MAW over $2 \sin$ of theta and that is your answer.

By the time door shuts that is it comes all the way by then theta equals pi by 2 for door to shut. So, its shuts at an speed theta dot square which is equal to $2 MAW$ over $2 I$ 2 cancels, equals MAW over I that gives you the angular speed with which the door shuts. There is another example of solving the problem using accelerating frames as a as a 3 problem.

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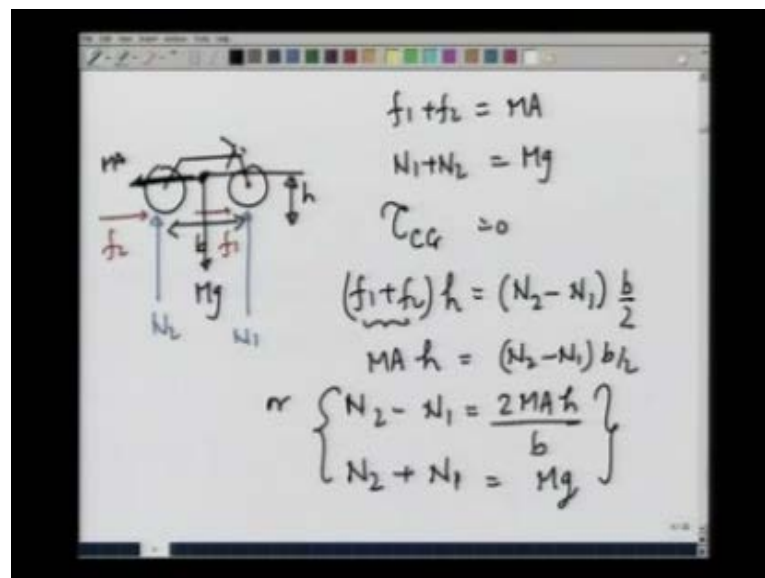
I take a problem that you must have seen sometime, that if you have a motor cycle and the rider accelerates it suddenly at a very high acceleration, then you see it tends to topple the upper the front wheel sort of lifts off the ground. And we want to calculate as to at what acceleration does it start lifting of the ground. I am again going to look at this problem from a frame which is accelerating with the motor cycle towards right with A.

If it does that, then the motor cycle is in equilibrium it is not moving in the that frame and if I look at motor cycle, it is in equilibrium under the following forces; that, the force N_2 on the back wheel normal reaction N_1 on the front wheel there is a frictional force f_1 in the front wheel. There is a frictional force f_2 on the back wheel there is the fictitious

force working on the centre of gravity towards the left MA and at the centre of gravity there is a force gravitational force Mg .

Let us, take the height of the centre of gravity from the ground to be h and let us take the distance between the 2 wheels to be b . We wish to calculate as to at what acceleration does the front wheel start getting off the ground. So, in this accelerating frame the motor cycle is in equilibrium under these forces. Let us, write the equilibrium conditions. You get summation F_x is equal to 0 which gives me f_1 plus f_2 equal MA . Summation F_y equals 0 gives me N_1 plus N_2 equals Mg and the torque about any point should be 0 let us take for convenience the torque about the centre of gravity. Let us make the picture again for you in the next page;

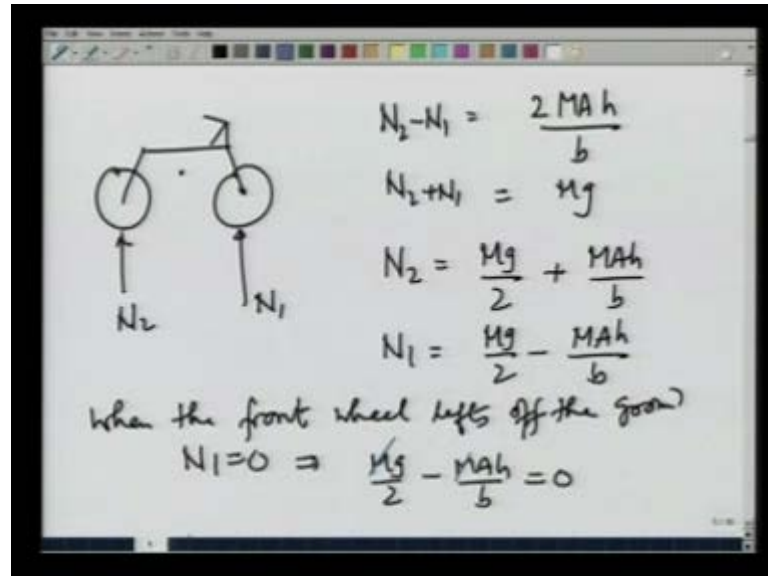
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Here is the motor cycle with distance b here, centre of gravity being at height h from the ground there is normal reaction N_2 , N_1 there is friction f_2 , f_1 and there is a force at the centre of gravity MA and Mg and we have already found that f_1 plus f_2 equals MA and N_1 plus N_2 equals Mg and balancing the torque about CG equal to 0 gives me, f_1 plus f_2 which is a counter clockwise torque times h equals N_2 which gives you clockwise torque minus N_1 minus. Because, N_1 gives you a counter clockwise torque times b by 2. If I assume that the centre of gravity lies right in the middle of the 2 wheels and therefore, substituting f_1 plus f_2 equals MA h equals N_2 minus N_1 b by 2 or N_2 minus N_1 equals

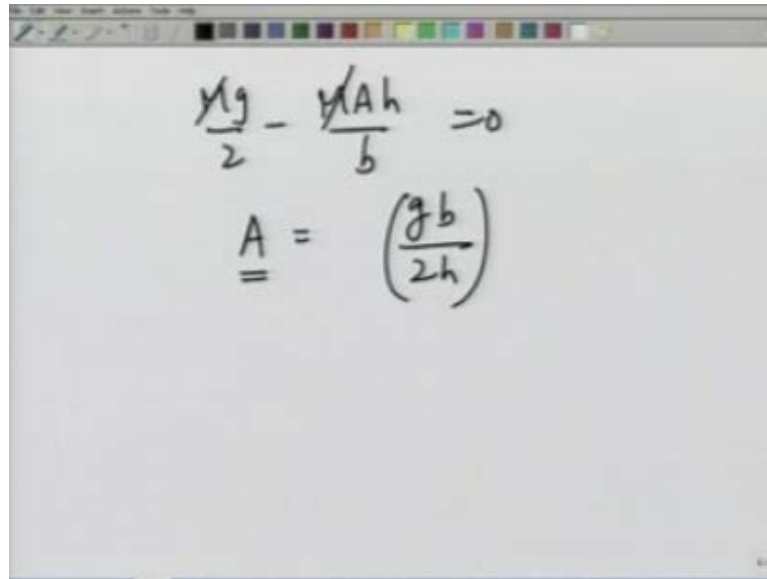
MA h times 2 divided by b and I already know that, N2 plus N1 equals Mg. I have got these 2 equations to solve for N2 and N1 let me show again.

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As a motor cycle is a centre of gravity here is reaction N1, here is reaction N2 and we have found that N2 minus N1 is equal to 2 MA h over b and N2 plus N1 is equal to Mg and that gives you N2 equals Mg over 2 plus MA h divided by b and N1 equals Mg over 2 minus MA h divided by b. When, the front wheel lifts off the ground N1 equals 0 and that gives you Mg over 2 minus MA h over b equals 0. M drops out and therefore, A equals we go to the next page,

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The image shows a whiteboard with two equations written in black marker. The first equation is $\frac{Mg}{2} - \frac{MAh}{b} = 0$. The second equation is $A = \left(\frac{gb}{2h} \right)$.

You have Mg over 2 minus MA h over b equal to 0 . M has dropped out. So, A equals gb over $2h$. If the acceleration is gb over $2h$ the front wheel would start lifting off the ground. So, if you keep acceleration below this the front wheel does not go up. Similarly, if you decelerate exactly opposite what happens, the rear wheel would start coming off the ground that I will leave for you as a problem. To conclude this lecture what we have done today is looked at solving the problem in an uniformly accelerating frame.

In the coming lectures we will go to another kind of rotating, another kind of non-inertial frame which is a rotating frame our earth is one example, of that and see the consequences of that.