Engineering Mechanics
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Module – 08
Lecture - 03
Simple Harmonic Motion – III

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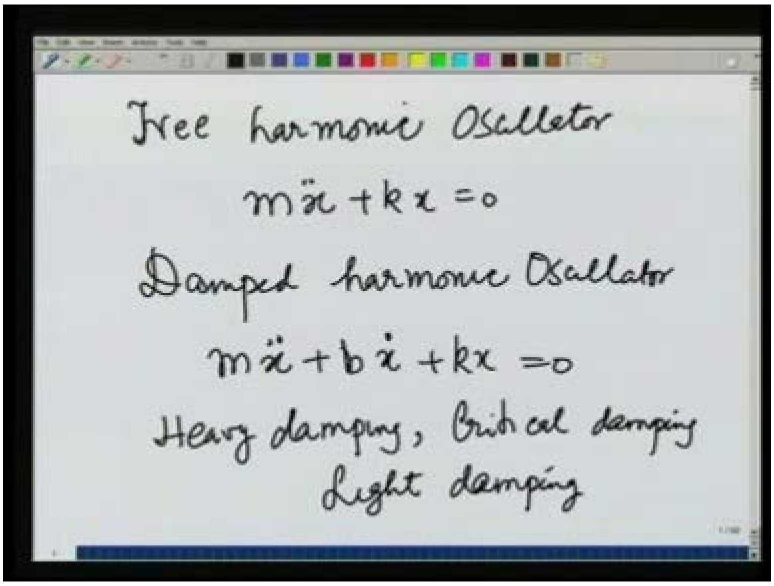

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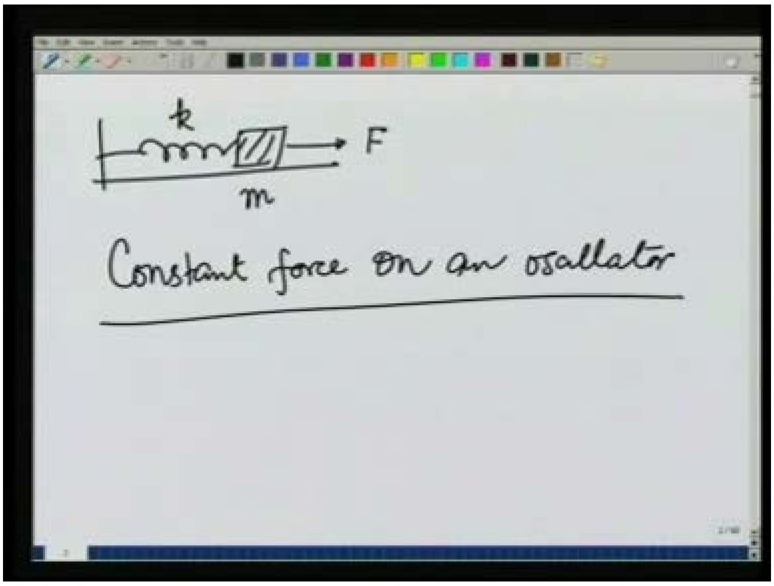

In the previous 2 lectures on harmonic oscillator. We looked at free harmonic oscillator for which the equation of motion was kx equals 0. Then, we looked at damped harmonic oscillator where there was some damping proportional to the velocity and the equation of motion pose like this. We defined the quality factor looked at this damped oscillator in the region of heavy damping, critical damping and light damping. In today's lecture we are going to look at another aspect of an oscillator.

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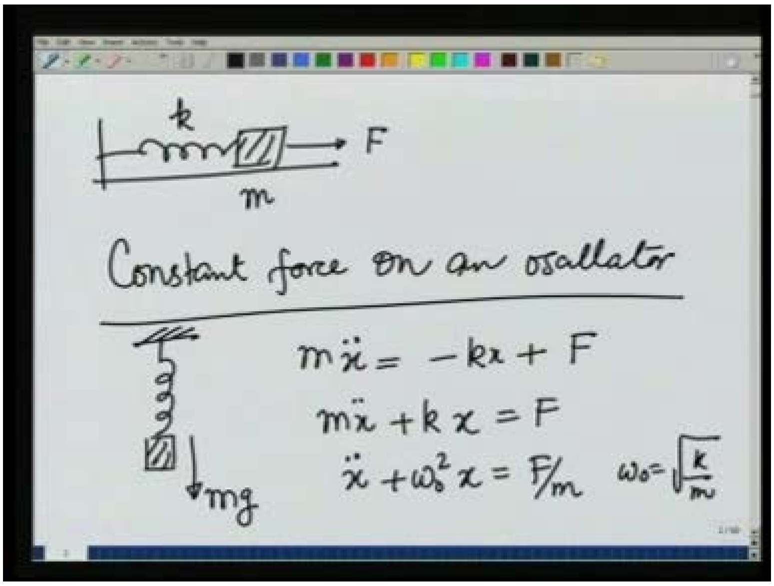

So, we are going to ask what happens. If I apply a force on the oscillator in addition, to it being subjected to force by the spring and its mass being m. The simplest case is that of a constant force on an oscillator and you already know the answer. In this case all that happens is, that the equilibrium point gets shifted. You know the answer of this I said because, you already know the example where if I take a spring put a mass on it and put it vertical in the gravitational field.

So, in that case this is being pulled by constant force mg. But still the equilibrium points shifts and still oscillates about the equilibrium point to see it mathematically. Let us, write the equations of motion mx double dot in this case is going to be equal to minus kx plus whatever that constant force that I have applied. And therefore, mx double dot plus kx is equal to F dividing by m. I can write this as, x dot x double dot plus omega 0 square x equals F over m where, omega 0 that is square root k over m.

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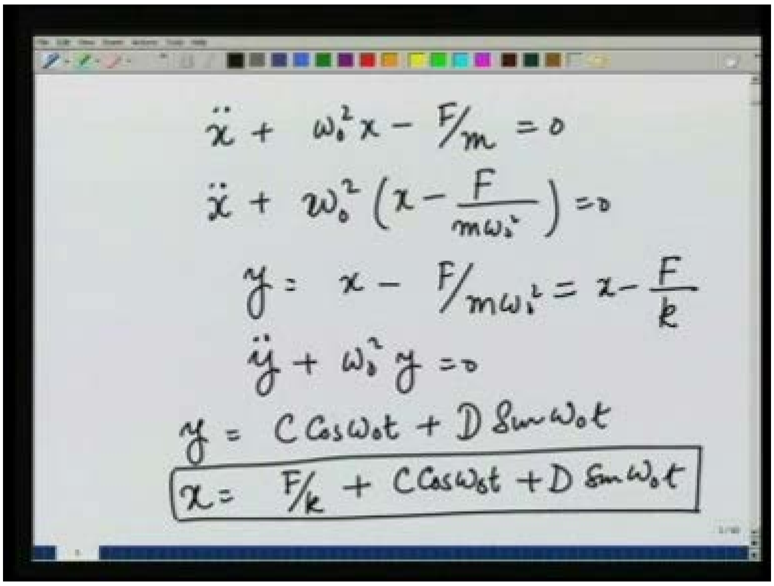

So, I bring a 4m to the left hand side and write this equation as x double dot plus omega 0 square x minus f over m is equal to 0 which can be written as, x double dot plus omega 0 square x minus F over m omega 0 square is equal to 0. Redefine y is equal to x minus F over m omega 0 square or equivalently a x minus F over k and your equation looks like, y double dot plus omega naught square y is equal to 0.So, I know, the solution for this in general is y is equal to C cosine of omega 0 t plus D sin of omega 0 t.

So therefore, x is going to be equal to F over k plus c cosine of omega 0 t plus D sin of omega 0 t. where C and D are determined by the initial conditions. So, all that is happening is x has a constant shift and then, it is oscillating about that point. More interesting case; however, is when we subject the oscillator to a force which depends on time. In general, a dependent periodic force can always be written as a Fourier Series like, Fn cosine of n omega t, where omega is the frequency of oscillation.

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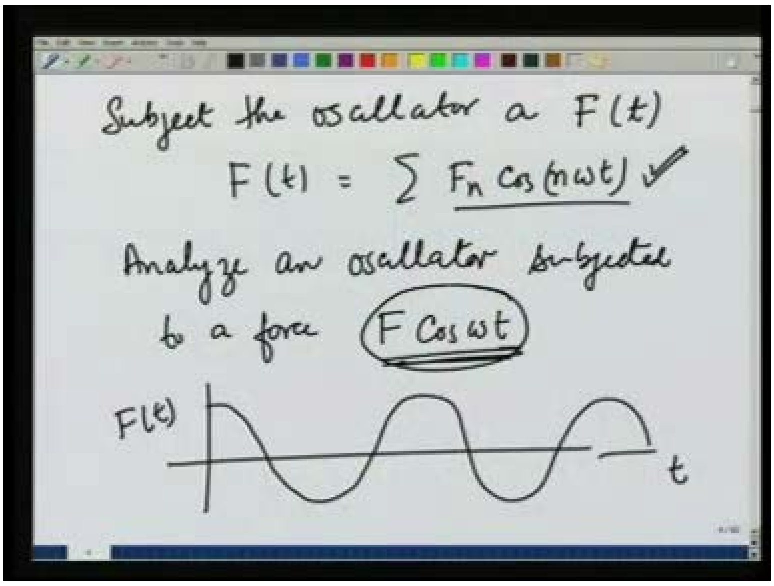

Therefore, it is best that we analyze an oscillator subjected to one of these components to a force F cosine of omega t, where omega is the frequency of the force. So, I am subjecting the oscillator to a force which varies like this with time. If I know the response of the system to F cosine omega t in general, for any periodic force I can find the response by superposing the solutions for each of these omega. In fact, I will assign you a problem later for 2 such forces being applied. For the time being we focus on one frequency force being applied to the system.

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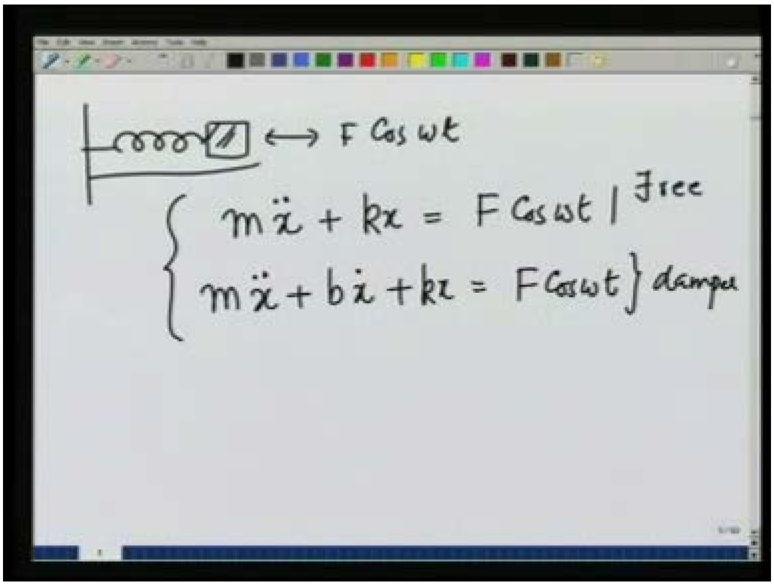

So therefore, the system I am looking at is a spring mass at the end of it and it is being subjected to a force F cosine of omega t. The equation of motion of the system is going to be mx double dot plus kx is equal to F cosine of omega t If it is a damped oscillator then, the equation is going to be mx double dot plus b x dot plus kx is equal to F cosine of omega t. This is a free oscillator and this is a damped oscillator.

So, we want to look at the solutions of this, these 2 equations and see how they vary when, omega is change from one frequency to another in relation with the frequency of the oscillator itself which is omega 0 is equal to square root of k over m. Let me, first discuss the case a free oscillator because, this will bring out the phenomena of resonance.

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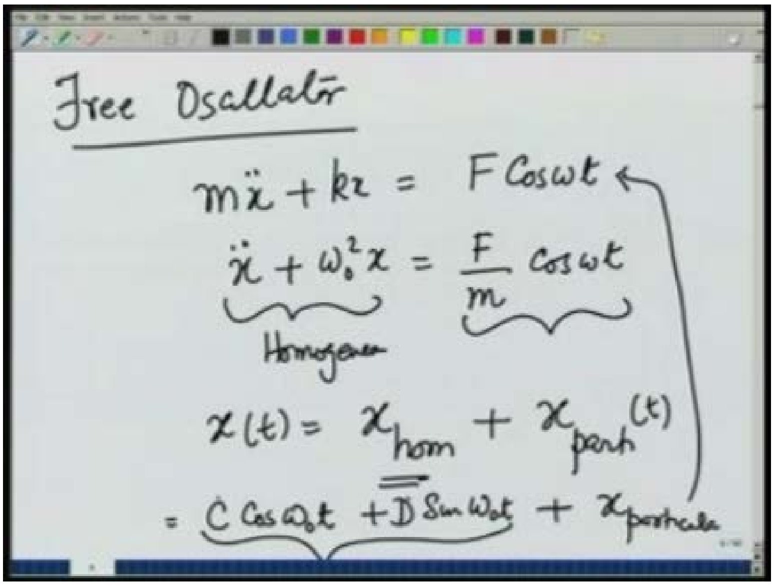

So, for a free oscillator the equation is mx double dot plus kx is equal to F cosine of omega t which I can rewrite as by dividing by m plus omega 0 square x is equal to F over m cosine of omega t. This is an equation which has a homogeneous part and a driving term. So, the general solution of this equation is going to be xt is equal to x homogeneous plus x particular I need the homogeneous part to satisfy the initial conditions.

Because, it is only the homogeneous solution that has a freely fixable coefficients. So, this I can write as c cosine of omega 0 t plus D sin of omega 0 t plus x particular. The particular part of the solution depends on the driving term and we want to find that. You see this homogeneous part has 2 constants C and D which will depend on the initial conditions. So, we find that for a Free Forced oscillator for which the equation of motion is x double dot is equal to omega naught square x is equal to F over m cosine of omega t.

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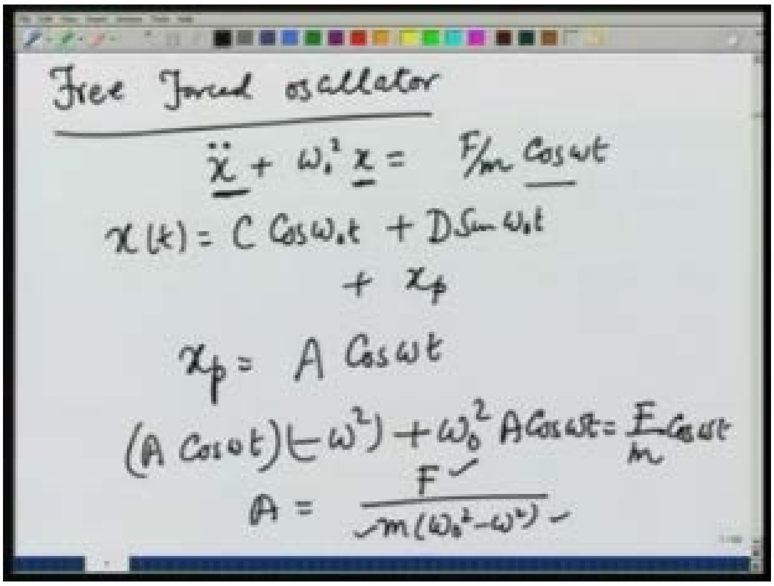

So, the solution is xt is equal to C cosine omega 0 t plus D sin omega 0 t plus x particular. Now, it is very easy to see from these equations that x particular is going to be of the form some constant A cosine of omega t. Because, x is substituted here gives me cosine omega t and second derivative with respect to time is also cosine omega t. When, I substitute this here I find that I have A cosine of omega t multiplied by minus omega square plus omega naught square A cosine of omega t is equal to F over m cosine of omega t.

So therefore, since this equation is satisfied at all times I get A is equal to F divided by m omega 0 square minus omega square. So, you see for the particular solution there is nothing arbitrary all the quantities are fixed and therefore, the particular solution x particular is equal to F divided by m omega 0 square minus omega square cosine of omega t and the complete solution of this, equation x double dot plus omega 0 square x is equal to F over m cosine of omega t is xt is equal to C cosine of omega 0 t plus D sin of omega 0 t plus F over m cosine of omega t.

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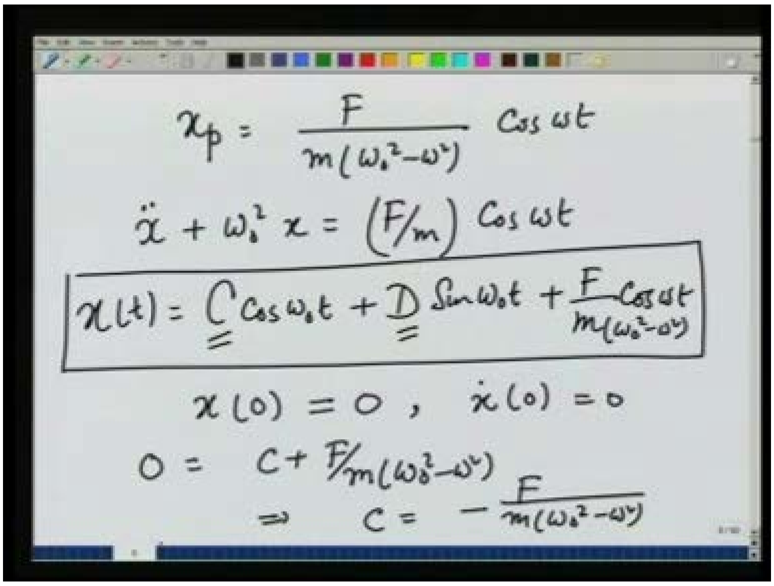

I have given you the full solution of a free oscillator subjected to an oscillating force coefficient C and D; obviously, depend on the initial conditions. Let us, now take an oscillator on which I apply start applying this force such that initially, it is at 0 position at it is a equilibrium point and it also starts with 0 velocity x dot is also 0. And I want to determine coefficients C and D accordingly, when I substitute x 0 equals 0 I get 0 is equal to C plus F over this is omega 0 square minus omega square F over m omega 0 square minus omega square. Or C is equal to minus F over m omega 0 square minus omega square.

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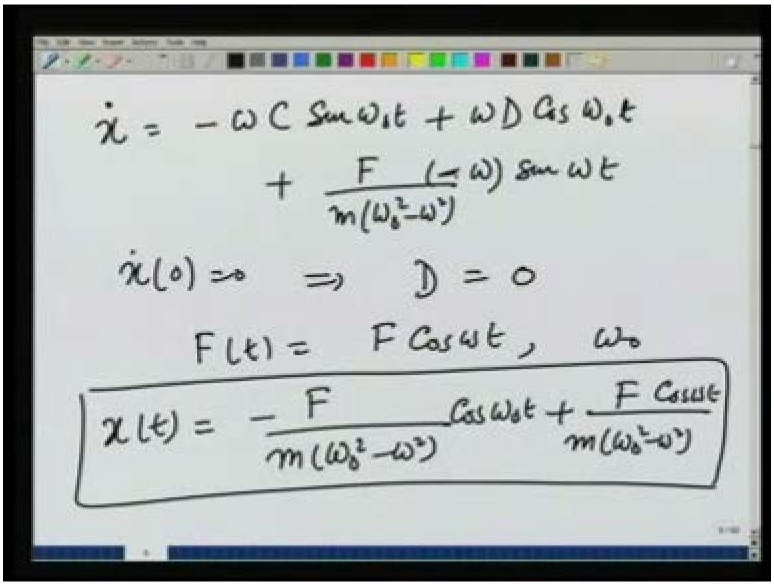

Similarly, when I take its derivative x dot will give me minus omega C sin omega 0 t plus omega D cosine of omega 0 t plus F over m omega 0 square minus omega square times omega sin omega t with a minus sign here. And since, I know x dot at 0 is 0 it gives me D is equal to 0 So therefore, if I take an oscillator subject to it a force Ft equals F cosine of omega t and its natural frequency is omega 0.

Then, the general solution starting with 0 velocity from the equilibrium point is going to be C which was minus F over m omega 0 square minus omega square cosine of omega 0 t plus F over m omega 0 square minus omega square times cosine of omega t. This is the general solution. Let me, write it clearly on the next page so, general solution for a freely Oscillating particle with natural frequency omega 0 subjected to the time varying force which is F cosine of omega t is xt is equal to let me, take this out m omega 0 square minus omega square cosine of omega t minus cosine of omega 0 t.

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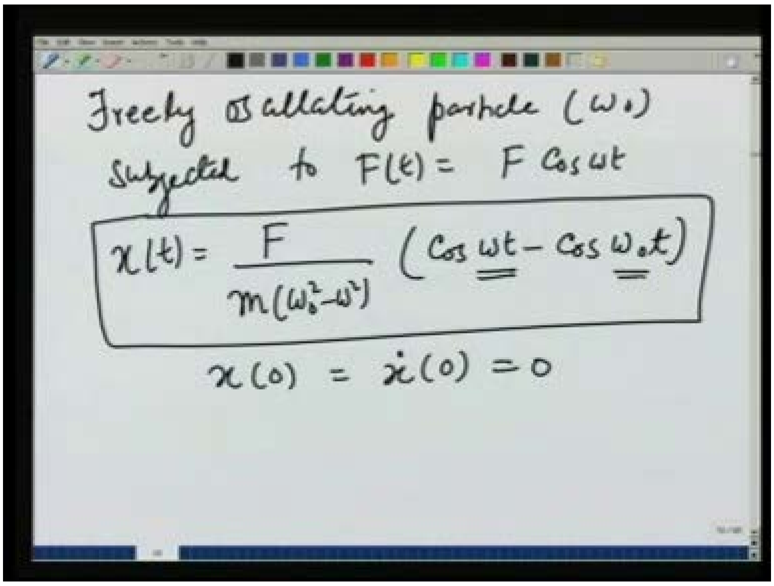

This is the general solution, this is under the condition that x 0 at time 0 and velocity at time equal to 0 are 0. How does the solution look? It has 2 frequency components omega t and omega naught t. So it is obviously, not harmonic when it is a super position of two different frequencies.

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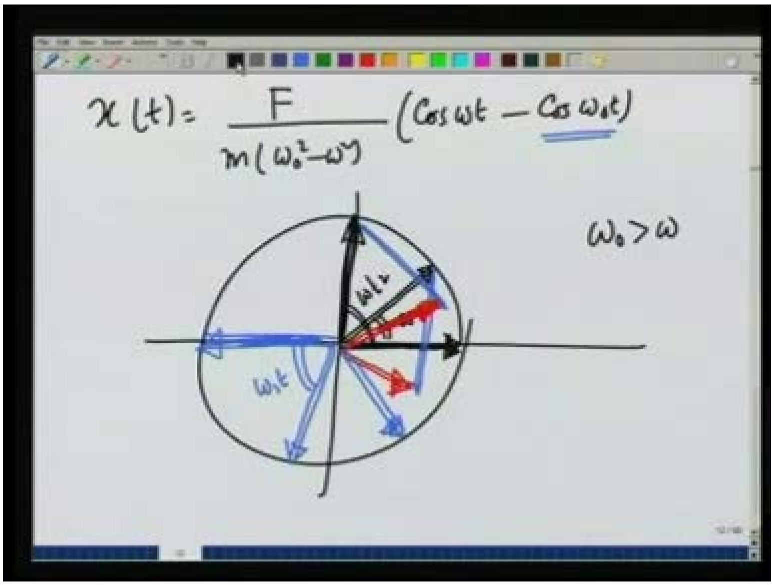

So, If I want to make this on a Phasor diagram which gives me analysis of this So, I am looking at the solution xt is equal to F over m omega naught square minus omega square cosine of omega t minus cosine of omega 0 t. Suppose, I would look at it t equal to 0 at t equal to 0 the term F over m omega naught over omega square cosine omega t looks like this. And at the same the other term let me, draw it with the blue color is in the opposite direction this term is this 1. After time t this term goes to this Phasor moves by an angle omega t.

Let us, assume right now that omega 0 is larger than omega in that case so, applied frequency is more that the natural frequency. This Phasor moves will move slightly more may be it comes here by omega 0 t. So, to get the net displacement I add it to this 1 and let me, show by red the net displacement is going to be like this. After some more time say t two this fellow may come here let us, say this is omega t 2 this will move much more. So, this would come say, let us say here.

So, at that time this the net displacement would be somewhere here let me, show it by red it'll be somewhere here. You see not vary as the amplitude change it is also move. So, it oscillates; obviously, between these 2 extremes, but the amplitude keeps on changing. If I want to see this displacement and plot it on displacement verses time plot it would look something like this. It starts the amplitude goes up comes down starts again goes up comes down goes up comes down and so on.

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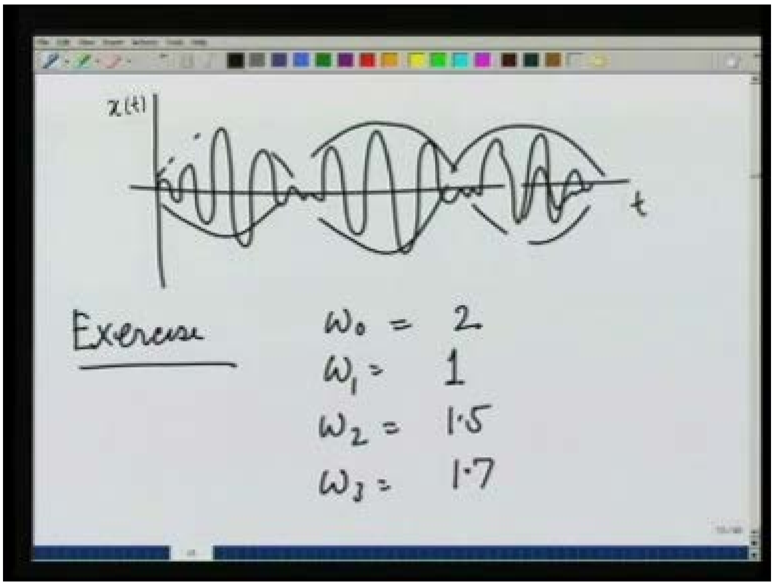

So, the amplitudes sort of change is the time I have given you a general sort of behavior when these 2 frequencies are super imposed. What I would argue to do is to an exercise choose an omega 0. Let us, say some number 2. Choose an omega equal to 1 choose an another omega 1.5 choose another omega 1 7 and so on. And try to plot these I would give you a general feature that as you approach closer and closer to 2 you would see that, this amplitude thing goes really far out in then goes down. But I would really like, that you solved you know the answer solve these problem for 3 different omegas and plot it and see at the how the amplitude varies.

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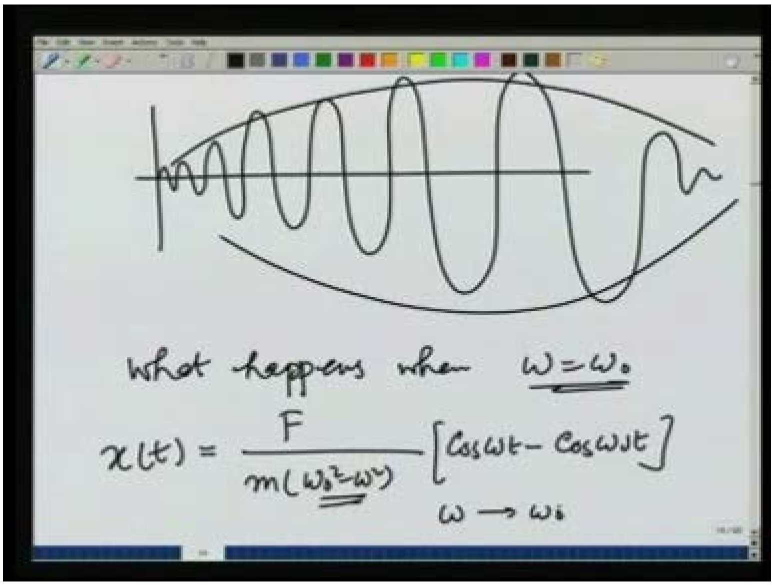

So, what would be very interesting question to ask now, is what happens? When, omega equals omega 0 recall, that the solution is xt equals F over m omega 0 square minus omega square cosine of omega t minus cosine of omega 0 t I cannot substitute omega equals omega 0 directly here. Because, when I divide it I assume omega and omega naught were not equal. So, I take the limit omega going to omega 0. So, let me take omega to be slightly smaller than omega naught and substitute it here.

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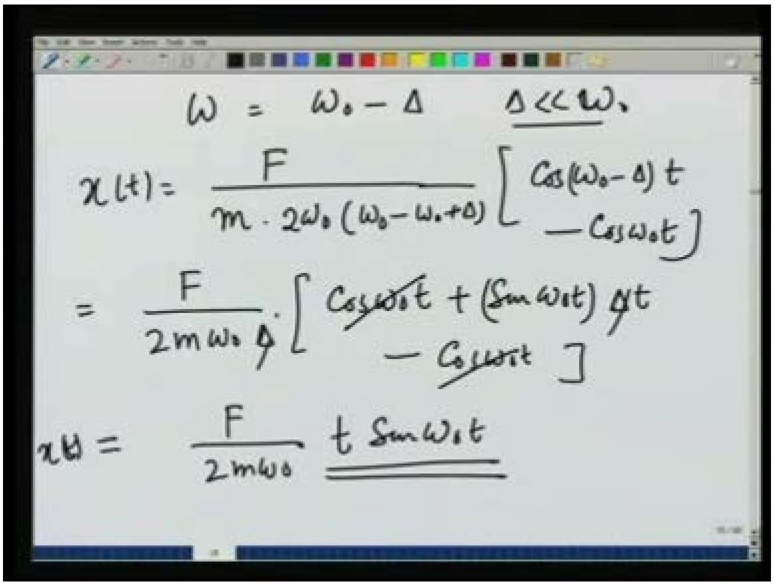

So, xt in this case would be F divided by m times 2 omega 0 because, this delta is very small times omega 0 minus omega 0 plus delta times cosine of omega 0 minus delta t minus cosine of omega 0 t. If this comes out to be F over 2m omega 0 delta times cosine of omega 0t cosine of delta t and delta being very a small I will take that as one plus sin of omega 0t times sin of delta which I will take as delta t because delta is very small minus cosine of omega 0 t.

In fact delta finally, I am going to take the limit of going to 0 this cancels. So, this answer comes out to be this delta will then, cancels with this delta F over 2m omega 0 times t sin of omega 0 t. Would you observe? You observe, that x increases with time linearly. If I want to plot this it would look something like this xt I am plotting as Ft over 2m omega 0 sin of omega 0 t. Remember, when I take delta equal to 0 this is an exact solution.

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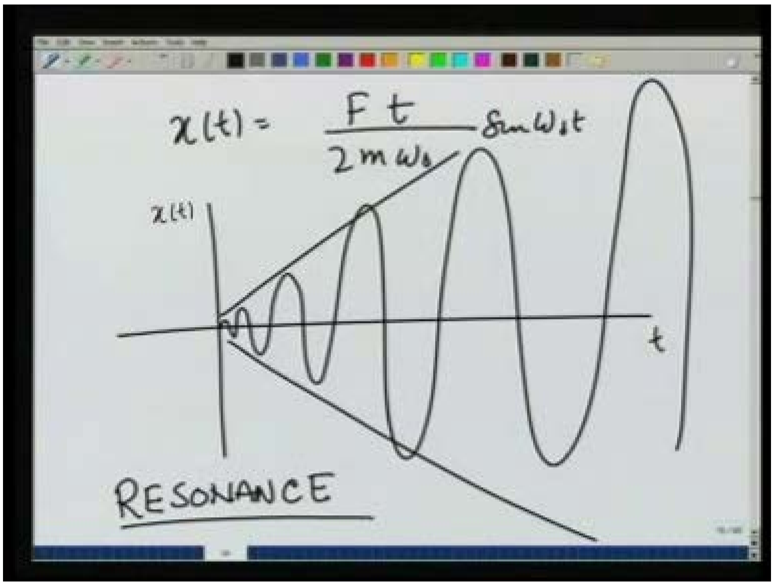

So, it will look something like this you see, this part goes up linearly this is the phenomena of resonance. So, when omega the applied frequency is exactly equal to the natural frequency the oscillations grow linearly with time and finally, the amplitude becomes very large. In fact, so large that the oscillator may break. I am doing this phenomena here for free oscillator.

Because, later we will see when we put damping in resonance does not mean that the amplitude becomes large and keep keeps becoming larger and larger rather it shows the maximum amplitude having done free oscillator and shown you the phenomena of a resonance. Now, let me go to forced, but damped oscillator.

In this case the equation of motion by of which you by now experts is going to look like mx double dot plus bx dot plus kx is equal to F cosine of omega t dividing by m I get x double dot plus b over m I have been writing as gamma x dot plus omega 0 square x equals F over m cosine of omega t.

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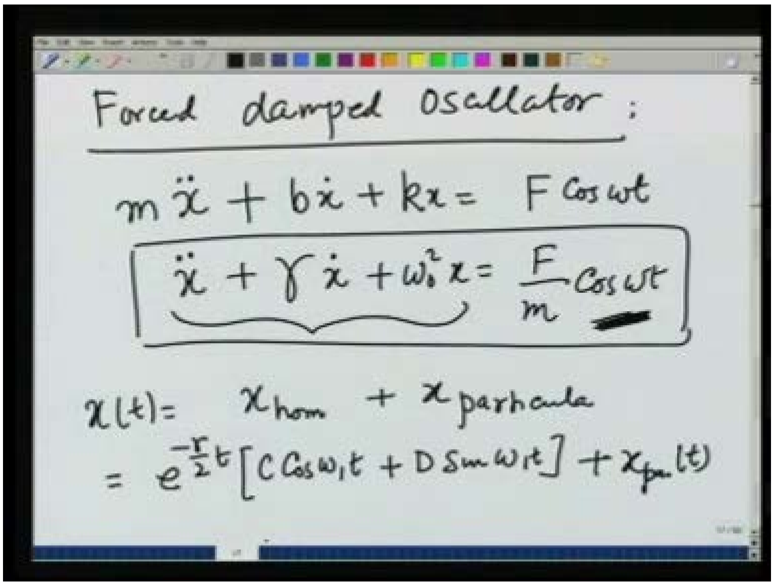

Again, as in the case of free oscillator I would have the general solution as the some of the homogeneous part solution and a particular solution. So, I am going to have xt as equal to x homogeneous plus x particular, where the particular solution belongs to what force I am applying and the homogeneous solution has goes to independent constants.

In general this therefore, would be e raise to minus gamma over 2t this we have solved in the previous lecture C cosine of omega 1t, where omega 1 a slightly different form omega 0 plus D sin of omega 1t this is the homogeneous solution plus x particular t. So, let me rewrite the solution for a Forced Damped Oscillator we have xt is equal to e raise to minus gamma over 2t C cosine omega 0t plus D sin of omega 0t plus x particular.

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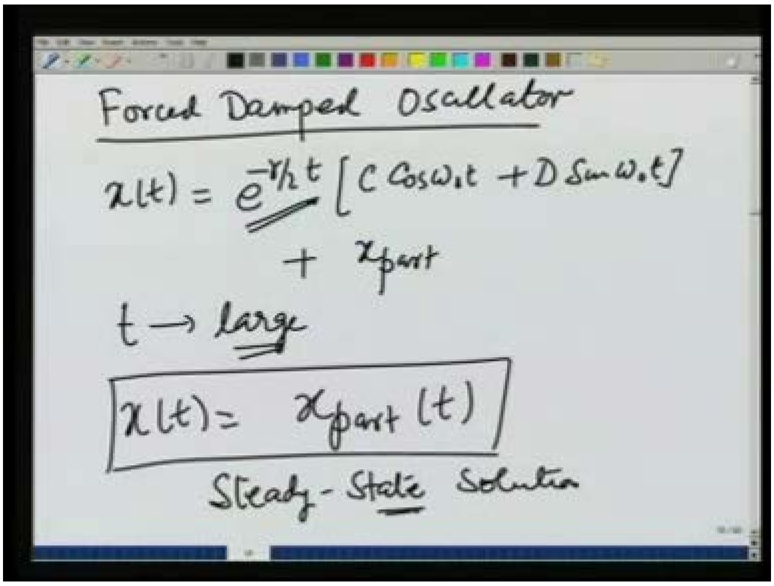

As t becomes large we see that this term would go to 0.Therefore, in the limit of large t I am going to have only x particular solution this does not get damped out and this is known as steady state solution. Because, by now the its own part has died down and the only solution is because, the oscillator is being driven and it achieves a steady state where because of the force it keeps on moving in a very steady manner. And therefore, this is known as steady state solution.

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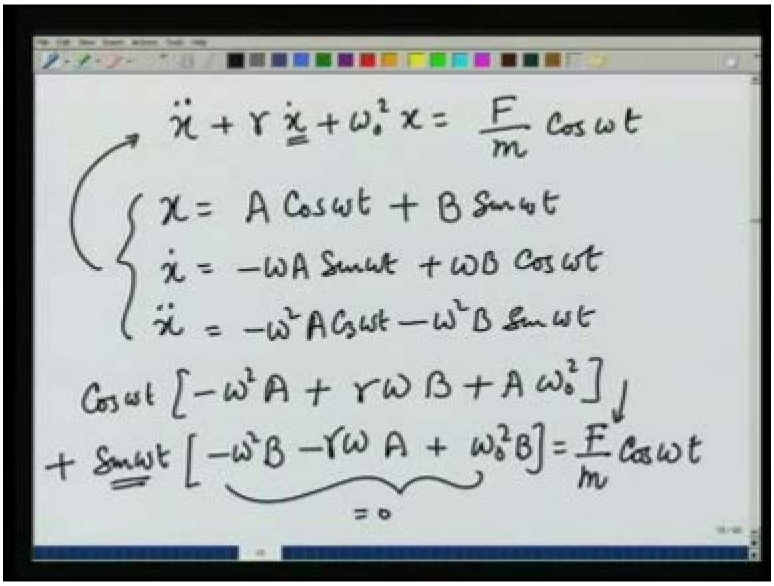

For the time being we will focus on this solution so, the equation that we have solving is x double dot plus gamma x dot plus omega 0 square x is equal to F over m cosine of omega t and I am looking for its particular solution. Because, in the limit of large time that is the only solutions so in other words, I am looking for steady state solution. I could try a solution like x equals some amplitude times cosine of omega t.

However, you see that when I take its first derivative to put in this term I get a sin omega t term Therefore, in general I should look for a solution of this form A cosine omega t plus sin omega t. So, you see right away that the solution is not in phase with the applied force as phase differs. Let us, see now what the general solution is. So let us, put this in here x dot then would give me minus omega A sin of omega t plus omega B cosine of omega t.

When, I substitute in x double dot is equal to minus omega square A cosine omega t minus omega square B sin of omega t when, I substitute these 3 in this equation I am going to get and collect cosine omega t and sin omega t terms together.

I am going to get cosine omega t minus omega square A from this term I am going get plus gamma omega B and from the last term I am going to get plus A omega 0 square plus collect the sin omega t terms and I am going to get minus omega square B minus omega gamma omega A plus omega 0 square B is equal to F over m cosine of omega t. Since, on the right hand side there is no term of sin omega t therefore, this term must be 0. And this term minus omega square A plus gamma omega B plus A omega naught square must equal F over m.

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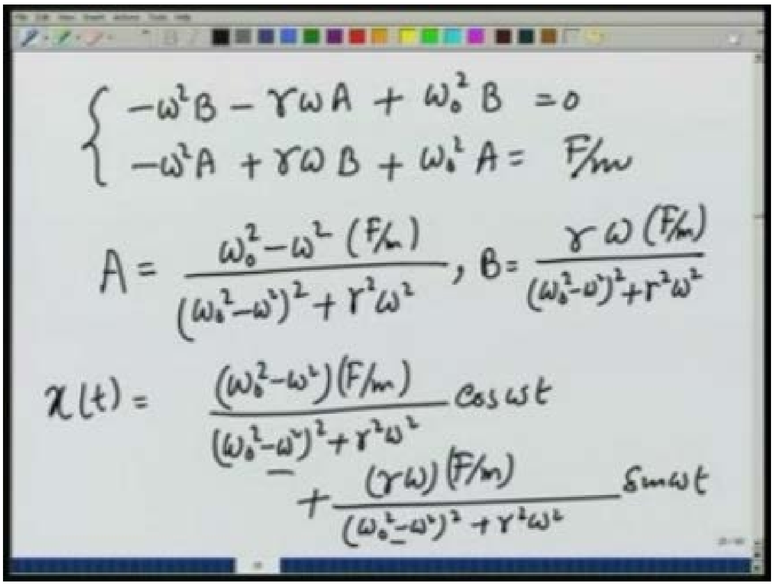

So, we have minus omega square B let us, see minus gamma omega A plus omega naught square B is equal to 0 and we have minus omega square A plus gamma omega B plus omega naught square A is equal to F over m. When, you solve these 2 equations when I leave it for you as an exercise you are going to get A is equal to omega naught square minus omega square over omega naught square minus omega square a square plus gamma square omega square and B is equal to gamma omega over omega naught square minus omega square square plus gamma square omega square.

So, therefore, the solution steady state solution is going to be omega naught square minus omega square or times F over m also, times F over m F over m over omega naught square minus omega square square plus gamma square omega square cosine of omega t plus gamma omega F over m over omega naught square minus omega square square plus gamma square omega square sin of omega t.

So, Look at, these 2 terms I will take square root of the denominator out and write the solution as xt is equal to F over m over the square root of x omega naught square minus omega square square plus gamma square omega square times omega naught square minus omega square over square root of this term cosine omega t.

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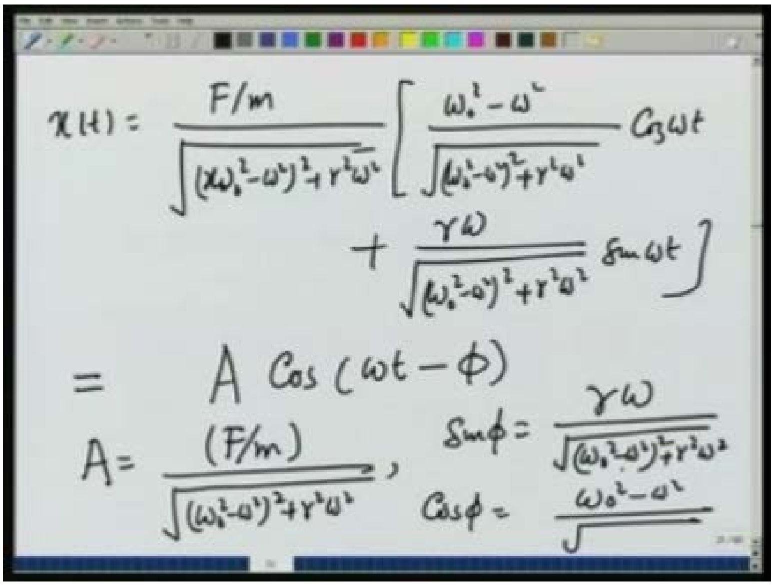

Let me, write this once omega naught square minus omega square plus gamma square omega square plus gamma omega over square root of omega naught square minus omega square square plus gamma square omega square sin of omega t. And this can be written as, some amplitude times cosine of omega t minus phi where the amplitude of the motion is F over m divided by square root of omega naught square minus omega square square plus gamma omega square and sin phi is gamma omega over square root of omega naught square minus omega square plus gamma square omega square square here.

Cosine phi is omega naught square minus omega square over a square root of this whole quantity. And tangent phi is gamma omega divided by omega naught square minus omega square let me, rewrite these things on the next page. So, when I solve for x double dot plus gamma x dot plus omega 0 square x equals F over m cosine of omega t.

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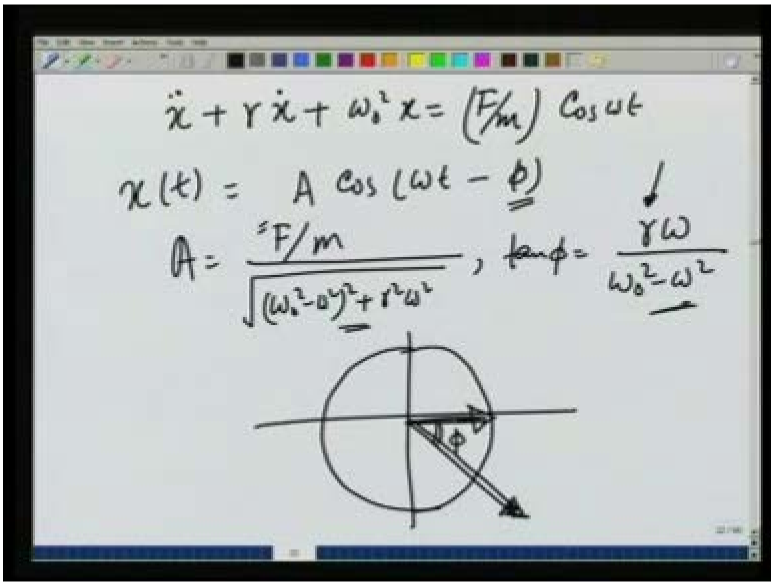

I will get a solution xt which is equal to amplitude times cosine of omega t minus phi, where amplitude is F divide by m over square root of omega 0 square minus omega square square plus gamma square omega square and tangent phi is equal to gamma omega over omega 0 square minus omega square. Notice, that the displacement of the harmonic oscillator is now lagging behind the applied force it is omega t minus phi.

So, it response slightly later the amplitude depends on the frequency and obviously, the force how much force is applying? And how much is lags behind depends only on the frequency? And how much damping is there? More the damping, more the lagging behind which is sensible form physical point of view. So, if I look at the Phasor Diagram at time t equal to 0 if this is the force the displacement obviously, the magnitude is going to be slightly different.

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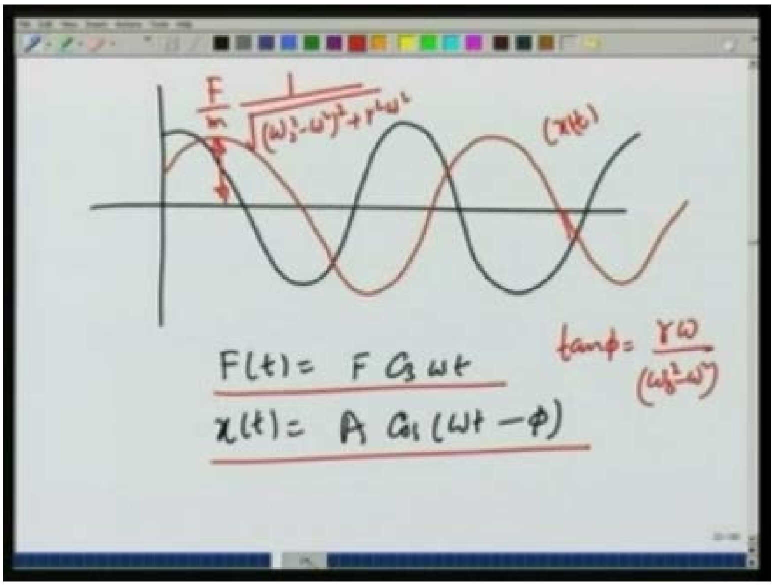

But if I will just look at the Phasor relationship is going to lag behind by this angle phi. If we translate this into the graph for displacement verses time The applied force is of the form cosine omega t Ft is of the form F cosine of omega t. whereas, xt is of the form some amplitude cosine of omega t minus phi.

That is going to be slightly behind. So, it catches up with this displacement slightly later. So, this displacement is going to look something like this. This is xt where this amplitude is F over m 1 over square root of omega naught square minus omega square square plus gamma square omega square and this phase angle is given by tangent phi equals gamma omega over omega naught square minus omega square.

Notice, that the steady state solution does not depend on the initial conditions the initial conditions whatever, they did to the motion have died out compare this with the situation that we solved earlier for an undamped oscillator. The initial conditions affected the motion all the way, but here the die out because of the damping term. And the steady state solution only depends only on the applied force. The solution has a nice geometric interpretation terms of Phasors.

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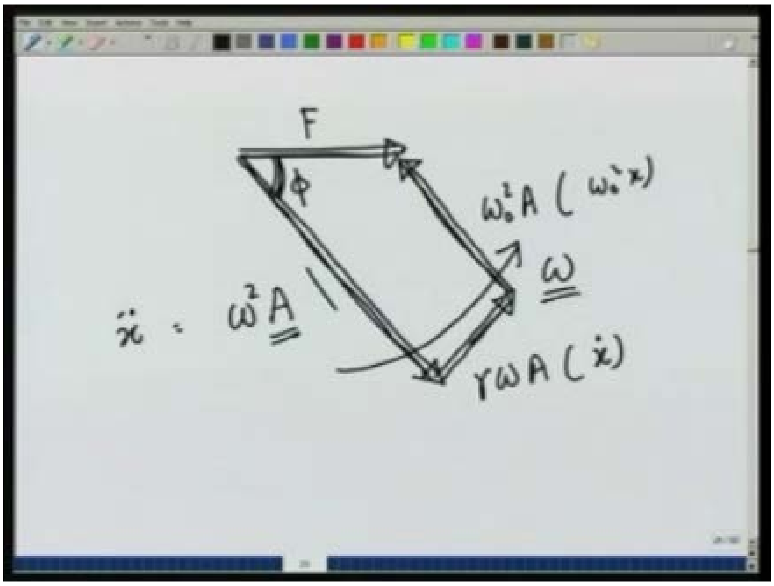

Let us, see at time t equal to 0 I am applying a force which is in horizontal direction the net displacement is behind this by an angle phi this is net displacement in this direction. So, this gives you a force which is omega square A the velocity is in this direction. So, this is term which is x double dot the velocity is this direction which is gamma omega A this is proportional to x dot and the spring's own force is in the direction opposite to the displacement is in this is in this direction which is omega naught square A this is the term omega naught square x.

So, you see and this should end up at the tip of the force F. The net force is balanced all the time the amplitude A and the phase angle phi should be such that this Vector Diagram closes on itself. The net motion now, is that this entire assembly moves, rotates counter clockwise with an angular frequency omega. That is the geometric way or Phasor way or vectorial way of looking at damped harmonic oscillatory motion as far as getting the solution of damped harmonic oscillator which is forced is concerned we are done.

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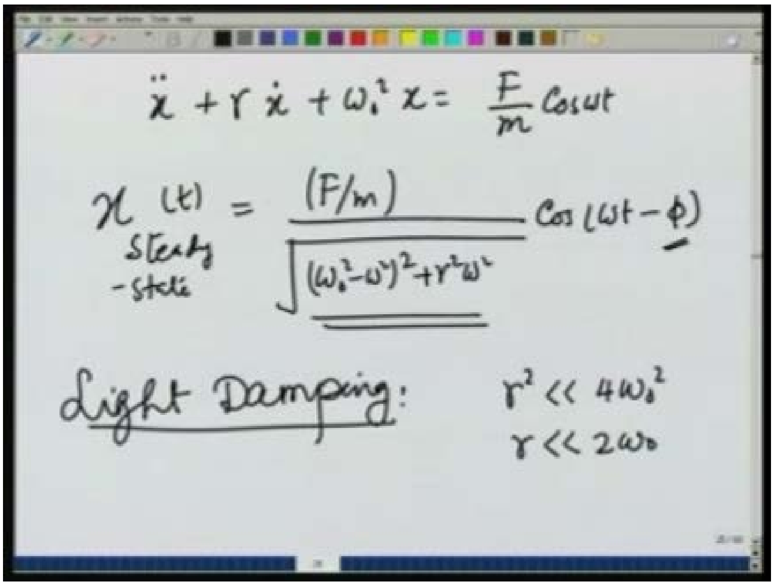

Now, we solved for x double dot plus gamma x dot plus omega naught square x equals F over m cosine omega t and got a steady state solution. Let me, write it is steady to emphasis state solution as the function of time it looks like, F over m divided by square root of omega naught square minus omega square square plus gamma square omega square cosine of omega t minus phi.

This is the complete solution. The motion the displacement lags behind the applied force by of phase phi and it has an amplitude that depends on the force and the applied frequency. What remains to be done now, is to analyze different situations. Let us, first case take the case of light damping and see what happens in this case remember, this case is where gamma square is much less than 4 omega 0 square or gamma is much less than 2 omega 0.

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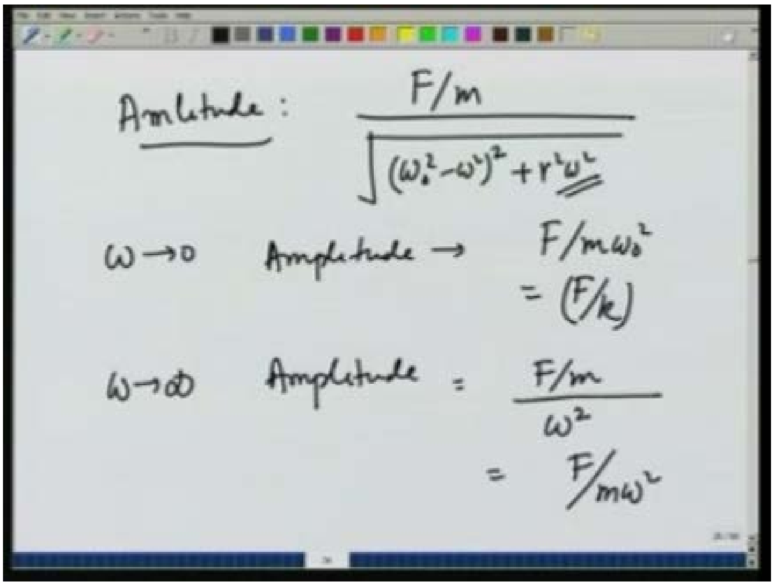

So, the amplitude in this case is F over m divided by square root of omega naught square minus omega square plus gamma square omega square. When, frequency goes to 0 the amplitude goes to F over m omega naught square equals F over k that means, at 0 frequency I am applying just a constant force and the mass is displaced. When, frequency becomes large the amplitude of the motion goes as F over m divided by this is square omega square.

Or F over m omega square I have neglected this term in comparison with omega square. So, it is clear that if I go to plot the amplitude of the motion with respect to the applied frequency of the force it will have a value F over k at omega equal to 0 and it'll decay as F over m omega square far away.

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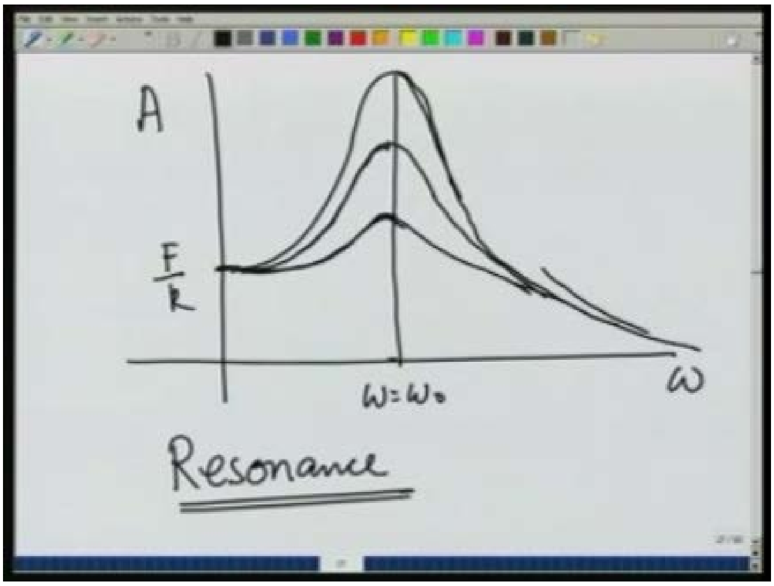

In between its behavior for light damping is something like this where the maximum occurs at omega equal to omega 0. If I increase gamma it will be something like this if I increase gamma further, will be something like this If you increase gamma the maximum shifts slightly to the left slightly to a value lower than omega 0. Of all practical purposes you can say that it is peaking at omega 0 and then far way it decays like this.

So, F the driving frequency if the frequency of the force that I am applying is equal to omega 0. The amplitude of oscillation becomes maximum and this is again reflection of phenomena of resonance Recall, that when there was no damping at frequency driving frequency equal to the natural frequency the amplitude was growing with time it was not stopping. Here it becomes maximum and this is again a phenomena of a resonance. We will see later that as a at resonance the power dissipated by the system and therefore, power given to the system is also maximum.

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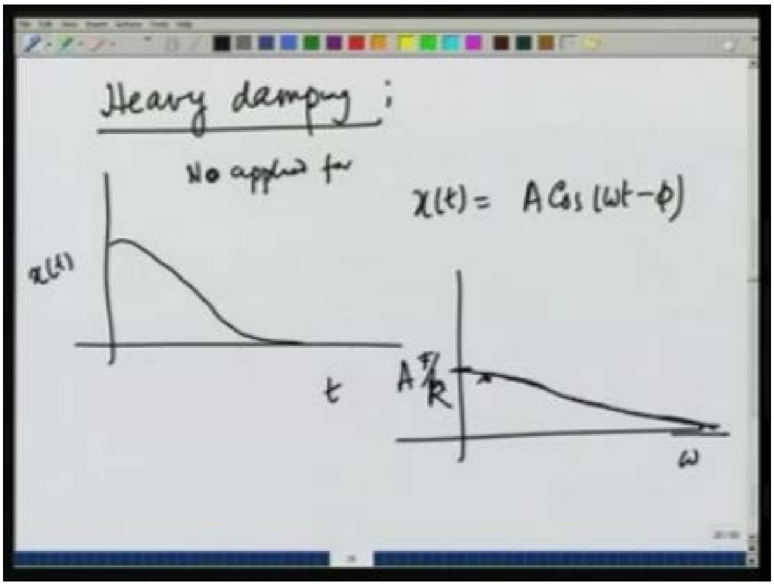

What happens? If I go to heavy damping. If you recollect the previous lecture we recall that in heavy damping without force no applied force the motion finally, came to a stop like this, but in this case no matter what the damping value is the motion will always be there oscillatory. Because, xt solution in steady state is some amplitude cosine of omega t minus phi.

However, when the damping is heavy. Then, if I go to plot the amplitude verses omega it has no maximum it starts with F over m and just decays. So, for low frequencies the oscillator oscillates with large amplitude always less than F over F over k. And then, for larger frequencies the amplitude becomes less and less and less. This is sensible because, for low frequencies force has enough time to act on the system.

So therefore, displaces it however, as the frequency of the applied force goes up the system does not react it does not have enough time to react to the applied force and therefore, its amplitude is very small. So, in general if I combine the 2 that is light damping and heavy damping. If I go to plot the amplitude of oscillations with respect to frequency the curve looks like this is light damping.

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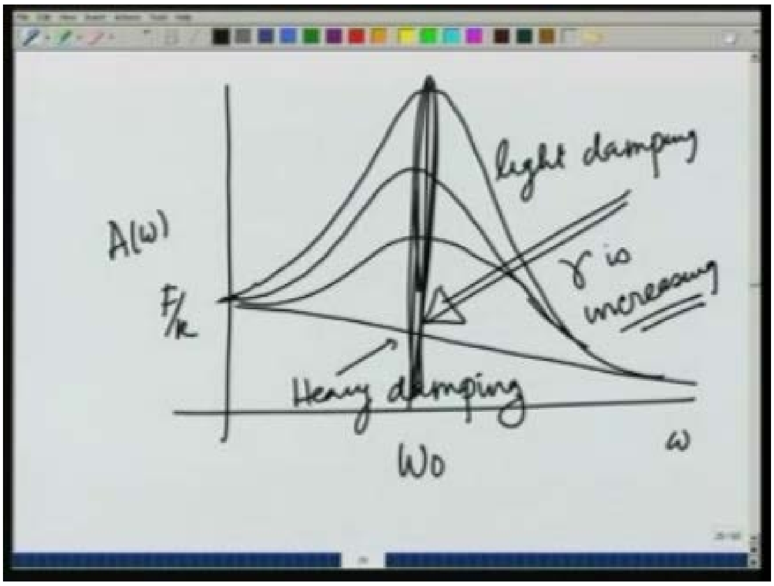

So, there is no maximum for heavy damping the curve just goes like this. Nonetheless the system does oscillate even in the case of heavy damping when, subjected to a an oscillating force. I can say that in this direction gamma is increasing. So, in light damping case I do have a resonance at this point omega 0 which really reflects that the maximum the amplitude becomes maximum at that point.

How about the phase of the system? Recall that, the system when subjected to this force lags behind the force by an angle tangent phi equals gamma omega over omega 0 minus omega 0 square minus omega square.

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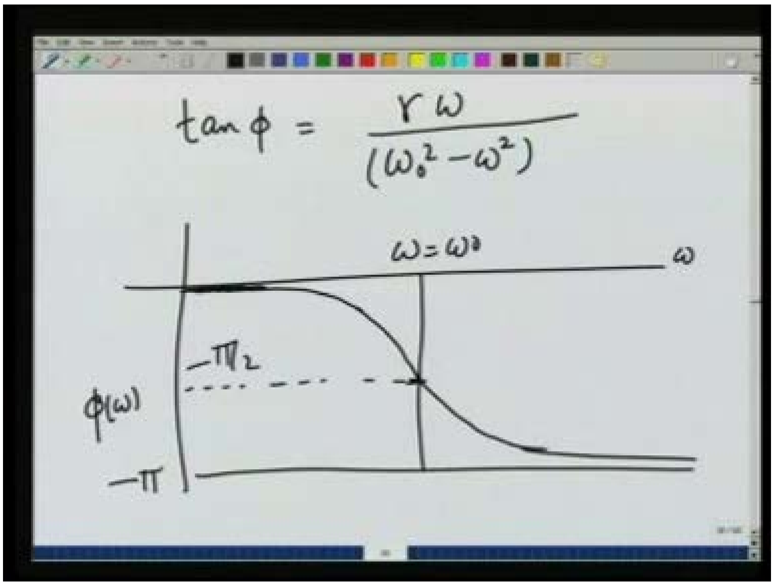

So, for very low frequencies if this is omega and I go to plot phi omega a very low frequencies the phase is almost the same as the applied force. So, the system is moving with the applied force then, at omega equal to omega 0 the phase lags behind the system lags behind by minus pi by 2 at omega equal to omega 0 this becomes 0. So, tangent phi is infinity.

So, when the frequency becomes large this is minus pi the force and the displacement are out of phase by pi. So, when the force is maximum, the displacement is minimum. So let us, see how the displacement and force are in relation with the each other at different frequencies.

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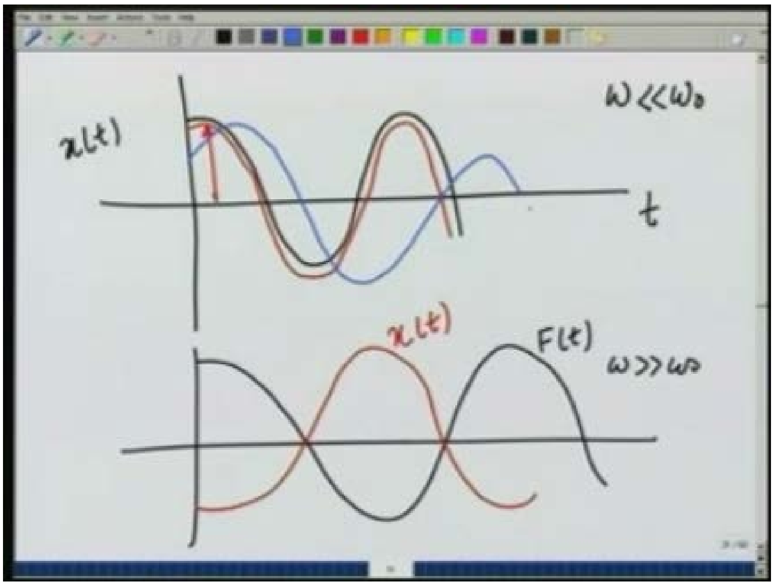

This is time xt let us, say this is omega much less than omega 0 a forces like this and in this case the displacement will also be following roughly the same graph I have not shown the amplitude correctly only the relative phases. On the other hand if, omega is much larger than omega 0 then, is the forces like this the displacement would be out of phase by pi this is xt black 1 is Ft.

So, In between the phases somewhere in between and so on. So, we have talked about the motion steady state motion in the case of light damping and heavy damping and seen how in light damping cases at omega equal to omega 0 the amplitude is maximum. How about power absorption in the system?

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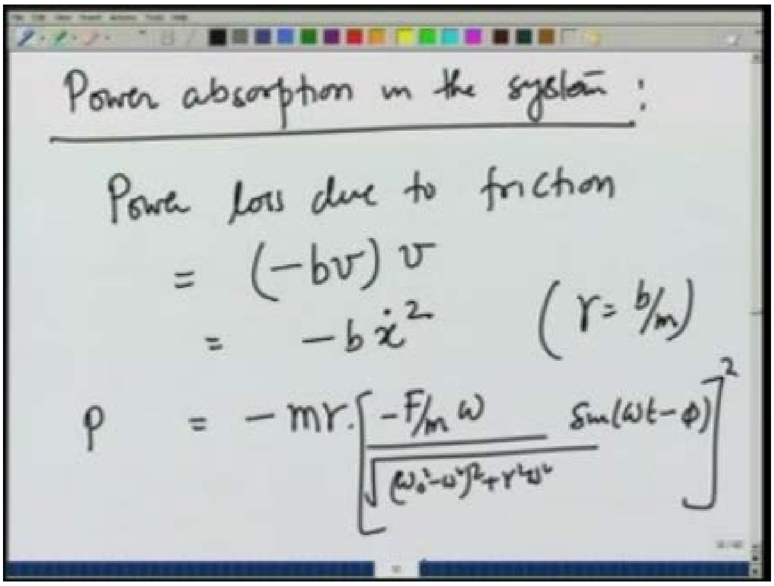

Since, there is friction in the system obviously, when the oscillators moving it is losing power due to heat and that power supplied due to friction and that power is supplied by the force being applied. Now, power loss due to friction is going to be the force minus bv times v which is equal to minus b x dot square.

Let us, put that in. So, power loss as the function of time is going to be minus b which is nothing but, m gamma remember gamma was defined as, b over m times x dot square which is F over m times omega with the minus sign divided by square root of omega naught square minus omega square square plus gamma square omega square sin of omega t minus phi square this is the velocity square.

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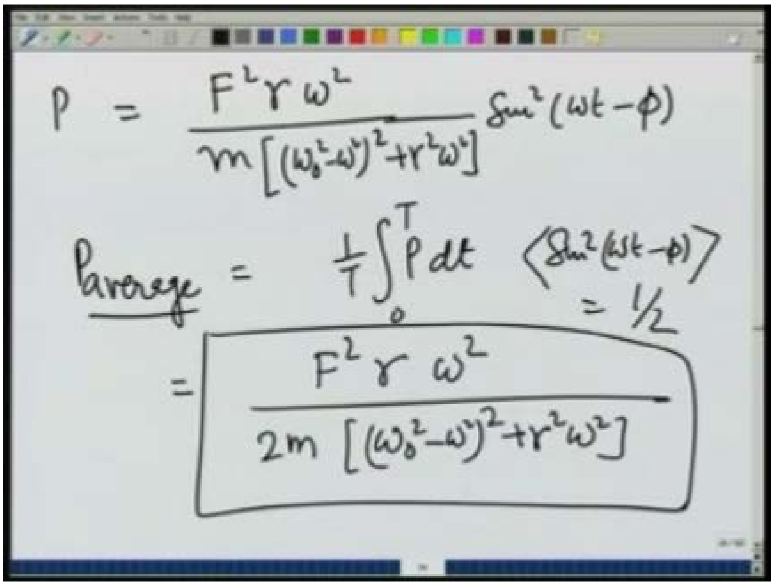

When, I open the bracket up I find that P is going to be equal to and this is power dissipated is going to be equal to let us see, if this comes out to be this is going to be F square gamma omega square over m omega 0 square minus omega square square plus gamma square omega square sin square omega t minus phi. So, power really fluctuates if go to calculate how much power is lost? Average power over one cycle that means, I integrate the power 0 to T and divide by T.

So, on an average per second how much power is being lost? Sin square omega t minus phi averaged over a cycle is always the half recall rms value from AC circuits in your twelfth. So, average power comes out to be F square gamma omega square over two m omega naught square minus omega square square plus gamma square omega square. This is the average power loss in the system. Is it equal to the power supplied?

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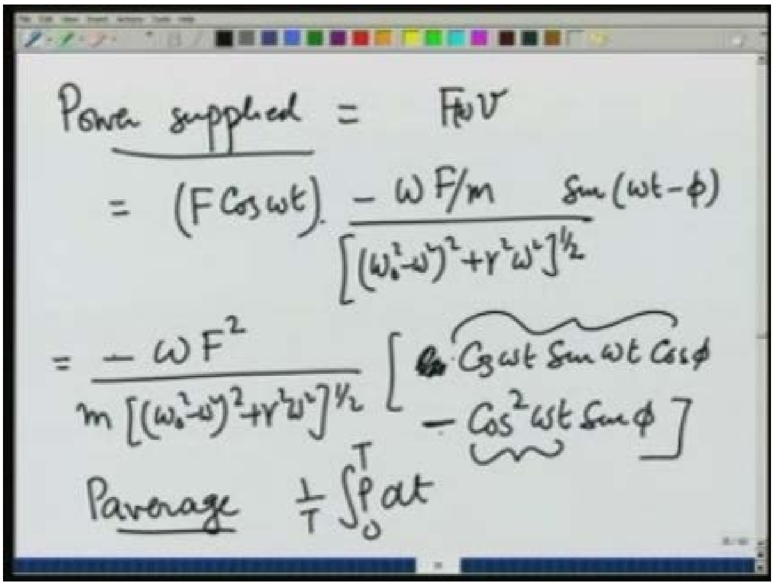

It should be let us, calculate that power supplied is going to be equal to the force supplied times of velocity and this is going to be equal to F cosine of omega t remember force is changing with time. Let me, write this Ft velocity is minus omega F over m divided by omega naught square minus omega square square plus gamma square omega square one-half times sin of omega t minus phi.

So, this can be written as minus omega F square divided by m naught square minus omega square square plus gamma square omega square raise to one-half open this up, you get cosine omega t sin omega t cosine phi minus cosine square omega t sin of phi sin omega t cosine phi minus cosine square omega t sin of phi. So, power being supplied is fluctuating and when, I calculate average power. That means, I integrate it with time t and divide by t this term is sin 2 omega t then, it is going to give me 0 cosine square omega t integrated over a cycle is going to give me one-half.

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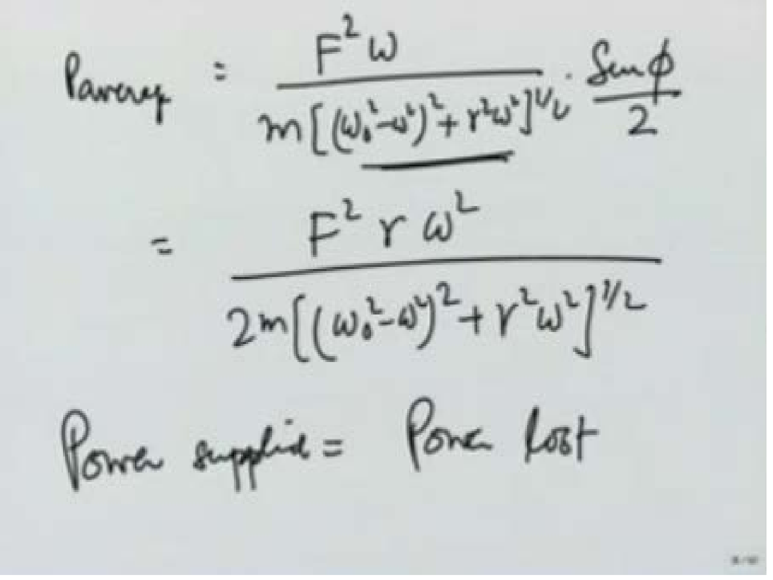

So, therefore, and this minus sign takes care of this minus sign and therefore, average power supplied is going to come out to be F let us, see that is on the previous page F square omega over m F square omega over m omega naught square minus omega square square plus gamma square omega square raise is to one-half sin of phi. Sin of phi you recall is, gamma omega divided by this quantity.

So therefore, this comes out to be F square gamma omega square n one-half. Because, the cosine square omega t being average divided by 2m omega naught square minus omega square square plus gamma square omega square raise to one half which is exactly the same as we obtained earlier by calculating the power being lost. So, power supplied is equal to power lost as must be the case for the steady state motion to sustain itself.

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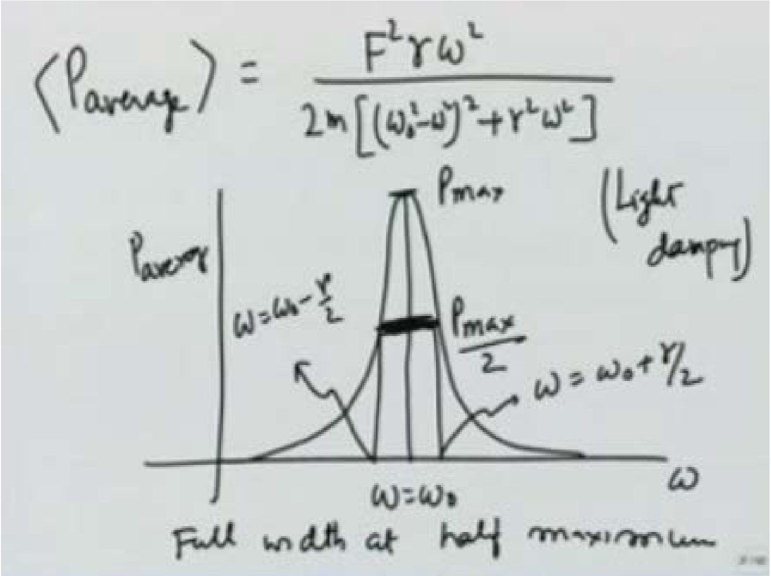

So, we find that the P average supplied or lost is equal to F square gamma omega square over 2m omega naught square minus omega square square plus gamma square omega square. Let us, see how this power changes as I change the applied frequency? I am again considering light damping you will see that this looks something like this, where the maximum is obtained at omega equal to omega 0.

So, you see when there is resonance that means the amplitude is maximum the power that I need to supply to the sustained motion or the heat that is generated in the system due to friction is also maximum. And at omega equals omega naught plus gamma over 2 and omega equals omega naught minus gamma over 2 the power becomes half of that the P power.

So, suppose this is P maximum at these 2 frequency this becomes P maximum divided by 2. So, this width the frequency width by is gamma this is known as width full width at half maximum. Let me, recap when I plot for light damping P average verses frequency P max is at omega equal to omega 0. And the power observed or power supplied falls by half at frequencies omega naught minus gamma over 2 and omega naught plus gamma over 2.

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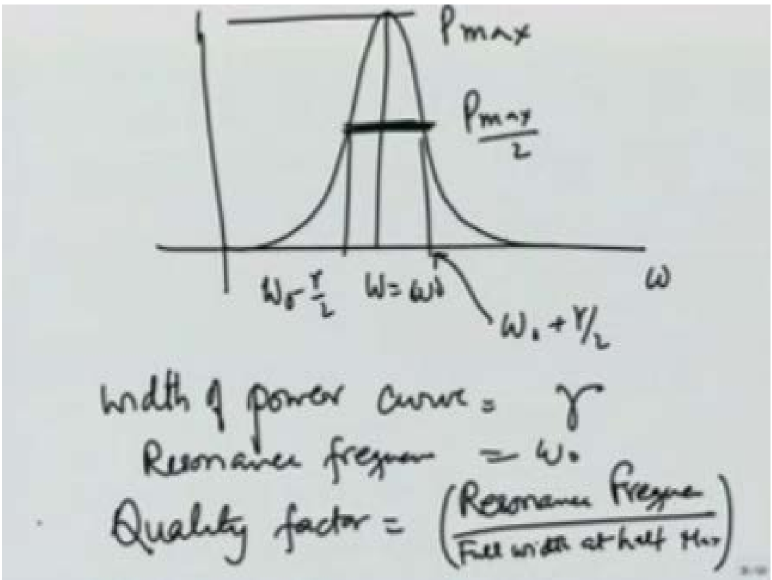

This is known as the width of power absorption curve. So, width of power curve is gamma resonance frequency is omega0. You see, right away that the quality factor is also related to the resonance frequency and the full width at half maximum. This is another way of looking quality factor q that we are introduced in the previous lecture. So, far we have looked at steady state solution.

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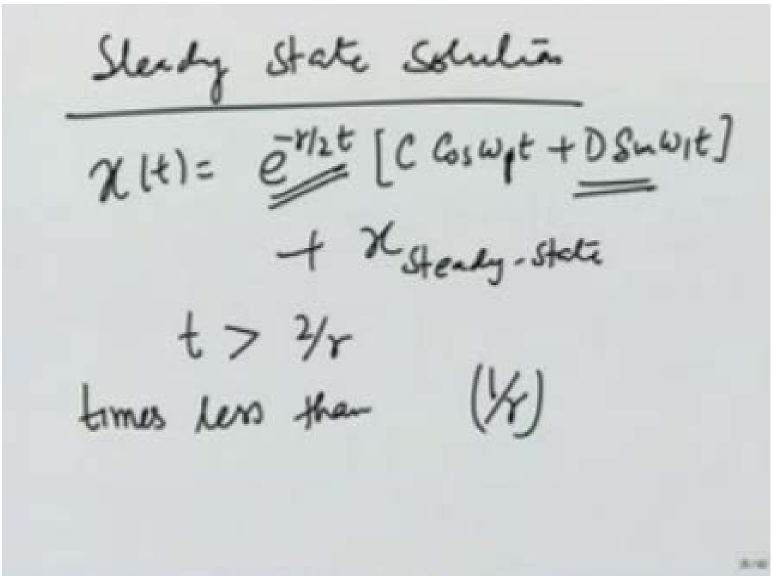

But we call that the general solution xt is e raise to minus gamma over 2 t C cosine of omega 0t plus D omega 1t. Because, damped oscillator sin of omega1t plus x the steady state. So, this factor really makes the solution homogeneous solution go down for times greater than 2 over gamma. For times less than say 1 over gamma we are going to have significant contribution from this solution also. So, in general for such times we are going to have 2 parts of the solution: 1 part which is the decaying part the other part which is the steady state part and the total solution is the sum of the 2.Let me, show it here The total solution may look something like this and it finally, it goes to the steady state solution these are known as transients.

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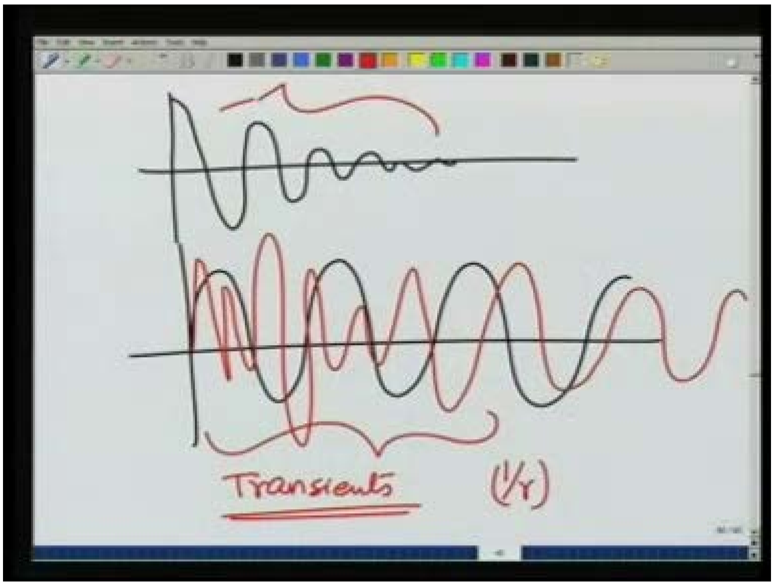

Because, they exists for a short time of the order of 1 over gamma. However, what you notice the transients could give you a very haphazard motion Take an oscillator force it with the certain frequency and you will see initially, it is back and forth no regular pattern and those are the transients. After some time these transients or the homogenous part dies down. And what you see is the steady state solution? This is 1 thing that I thought I should mention.

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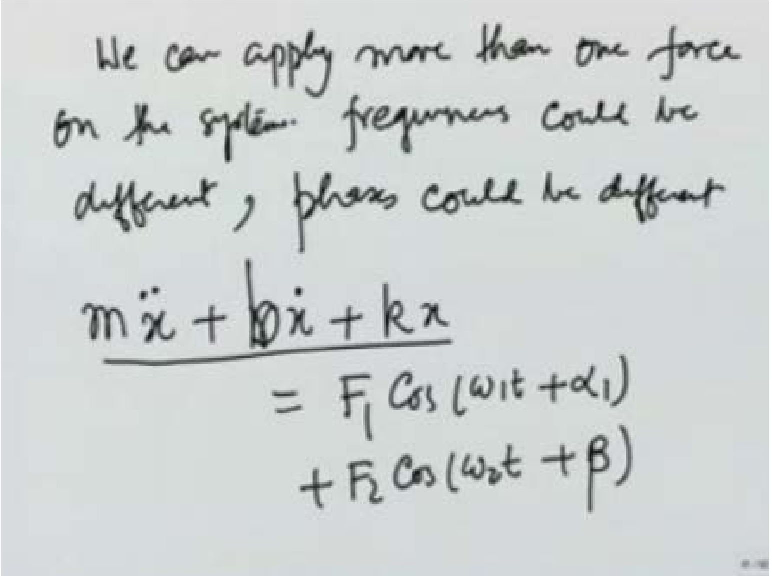

Finally, I would like to state that I could also apply more than 1 force on the system not only that their frequencies could be different phases could be different. When, I say phases could be different I mean when one force is maximum other force need not be maximum at that time. So, what would be the motion in general? As I said in the beginning of the lecture a general periodic force can be a sum of many different frequencies.

So, let us say if I have applied 2 forces this should be b kx 2 forces: F1 cosine of omega1t plus alpha1, where alpha is a phase plus F2 cosine of omega2t plus beta. So, there are 2 different phases: alpha and beta. 2 different forces what would be the general motion? Recall that, this is a linear equation and therefore, principle of super position holds. If I were to look at only steady state solution x a steady state.

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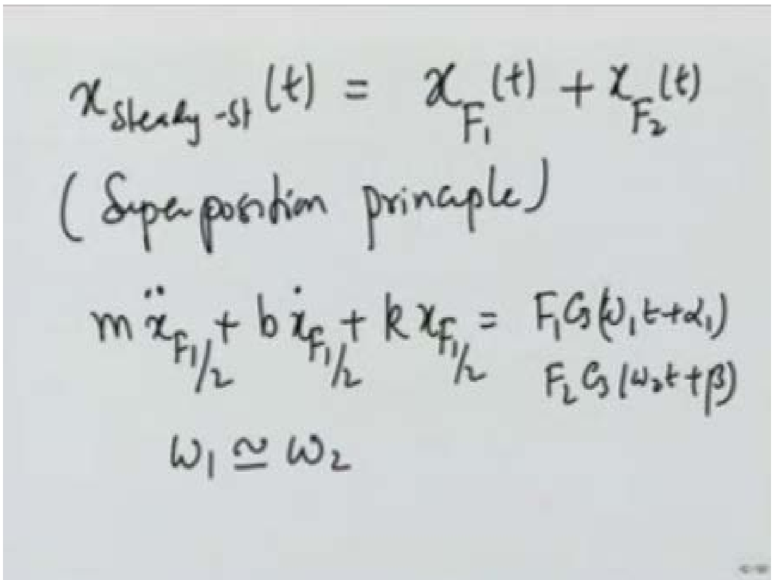

So, it would be solution corresponding to F1 plus solution corresponding to F2 this is because, of the super position principle that tells me, that any linear equation for any linear equation the sum of the 2 solutions is also a solution. You can substitute it back and see because, the equation when you substitute mx F1 double dot plus bx F1 dot plus kx F1 is equal to F1 cosine omega 1t plus alpha1 is satisfied. And for 2 F2 cosine omega2t plus beta is satisfied.

So therefore, their sum satisfies on the right hand side this being added. I would like you to solve this problem particularly. See the case when, omega1 is nearly equal to omega2. See what the sum of these 2 solution look like? What you will observe? Is that in this case when omega1 is nearly equal to omega2. And let us, take alpha to be to be equal to beta to be 0.

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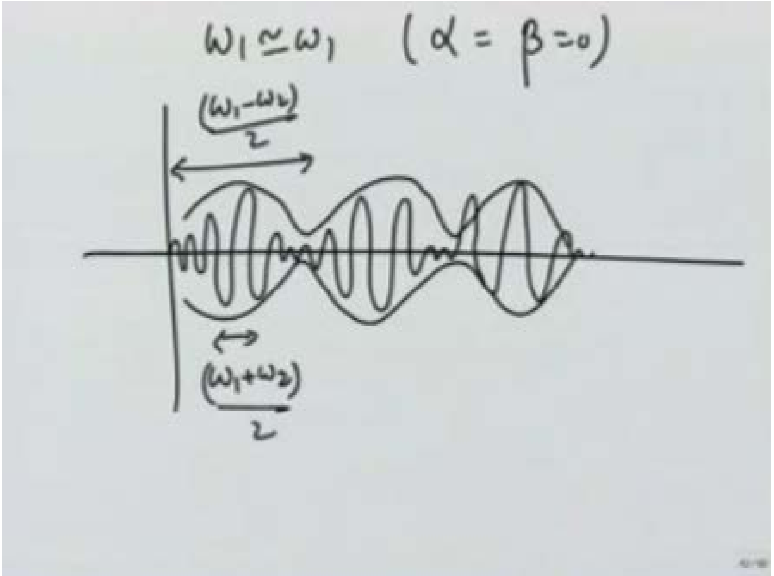

You will see that the solution would be of this form. So, the amplitude would be varying with time and the frequency of oscillation will be roughly equal to omega1 plus omega2 over 2 and this frequency of change of amplitude would be omega1 minus omega2 over 2. I would like you to do this exercise yourself by using by adding the 2 solutions. So, this is like the phenomena of beats.

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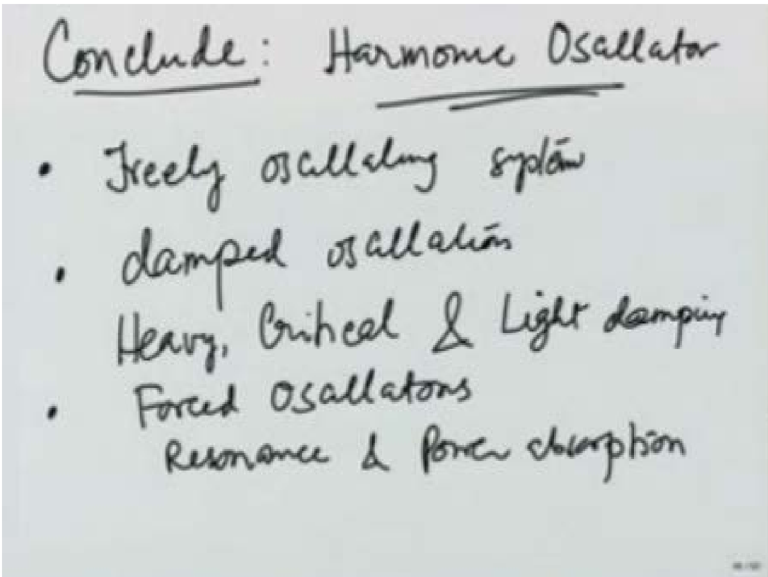

So let us, conclude our lectures on Harmonic Oscillator. These are some basic ideas about Harmonic Oscillator you can obviously, this is such a robust model that you can keep doing more and more. We have looked at free freely oscillating system we have also, looked at Damped Oscillations in the regime of heavy, critical and light damping.

We have also looked at Forced Oscillations, the resonance and power absorption of a Damped Forced Oscillation. I have also indicated to you How to get solutions for more than 1 force with different frequencies? What I have not covered is, how to do this using complex algebra and many other terminologies like, response functions and things like those, but given these basics I am sure you be able to cover them on your own.