Engineering Mechanics
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Module – 08
Lecture - 01
Simple Harmonic Motion - I

Having looked at, the motion of particles and rigid body motion now, we will cover a very specific kind of motion that is very very useful and has lot of rich physics in it.

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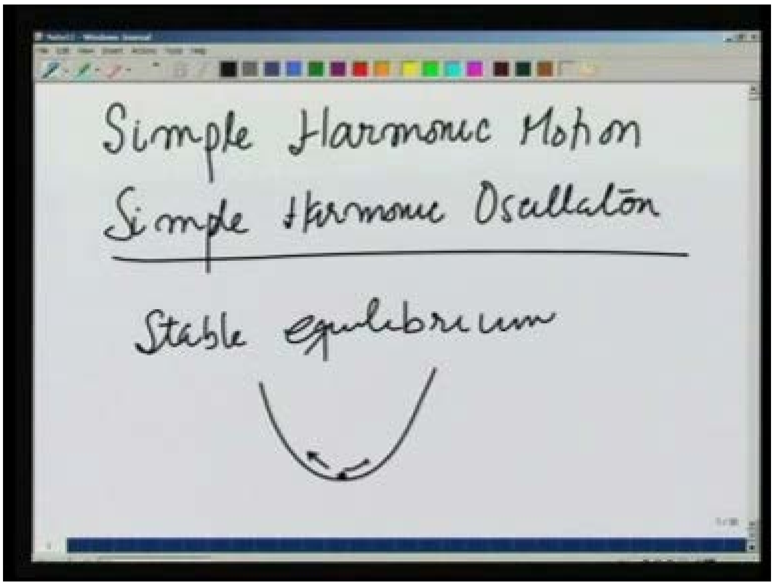

It is known as simple harmonic motion or simple harmonic oscillation and let us see how it comes about. Whenever a particle is in a stable equilibrium that is, it is at the bottom of some kind of a well. So, that when it moves it always comes back to its equilibrium position then, I can expand the potential energy about that point this could be any shape.

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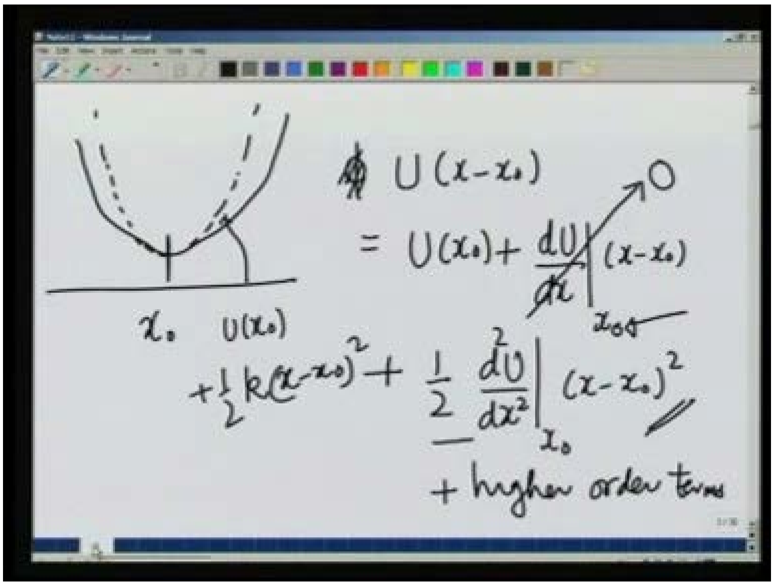

About this point, let us call this point x 0 and I am focusing in focusing on 1 dimension then, phi a let me use the notation that way be have been using potential energy U at a point x away from x 0. Can be written as potential energy at x 0 plus the derivative of potential energy at x 0 points x minus x 0 plus 1 half. The second derivative of the potential energy at x 0 this is also a x 0 x minus x 0 square. As long as x minus x 0 is small, now plus higher order terms.

If x 0 is the point which is of that of stable equilibrium then, the force which is the derivative of potential energy is going to be 0 at this point. And therefore, the first correction that comes to Ux 0 is this term. If I retain only this term and drop the higher order terms then, I would be approximating the potential about this table equilibrium point something like this; something like a parabola which is nothing, but Ux 0 plus 1 half some constant k x minus x 0 square.

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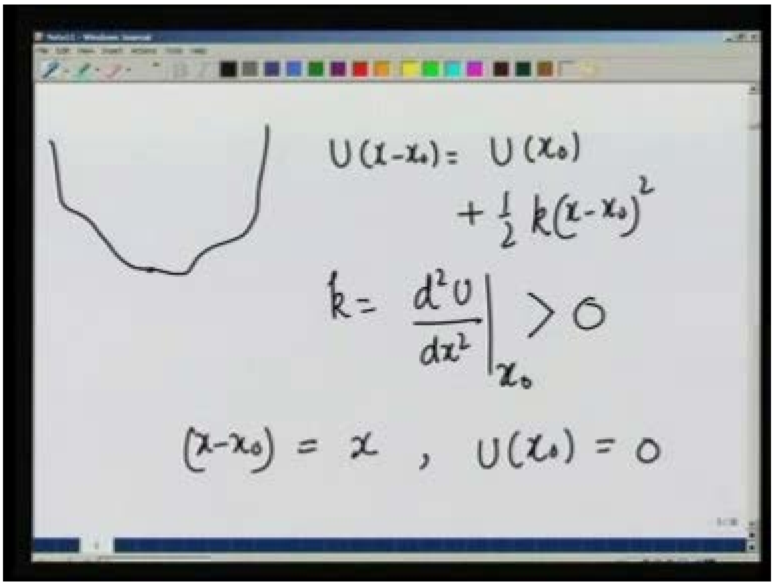

So, let us see what we are saying is if the particle is in some equilibrium position at some point. Then in general, I can write the potential energy about that point x minus x 0 is equal to the potential energy at x 0 plus 1 half k x minus x 0 square to a good approximation, where k is the second derivative of the potential energy at this point. And since, this is a stable equilibrium point or the minimum of potential energy this is greater than zero

Let me for convenience now, write x minus x 0 as only x that is in effect I am taking x 0 to be 0 and Ux 0 to be 0. Remember potential energy is always with respect to some reference point. So, I am choosing my reference so that, Ux 0 0.

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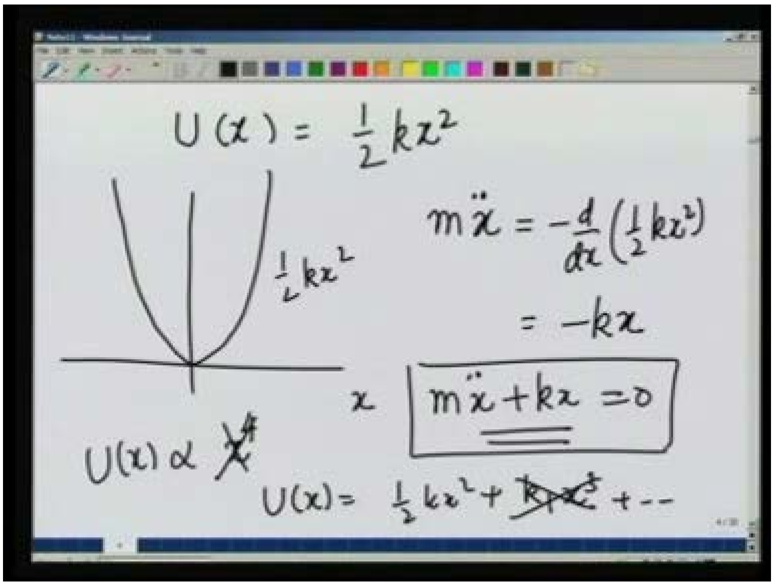

So, I can now write the potential energy as Ux is equal to 1 half kx square and that you recognize right away as the potential energy of a spring stretched by distance x. So, the potential energy with respect to x looks something like this, which is 1 half kx square Therefore, if a particle of mass m is moving in this potential is equation of motion will be given as, mx dot is equal to minus d by dx of 1 half k x square which is minus kx or mx double dot plus kx is equal to 0.

So, this is the simplest possible equation about a stable equilibrium point why I call it the simplest is because, there could be cases where even the second derivative of the potential is 0. Therefore correction would be higher order for example, I could have a potential Ux which is proportional to x raise to four or I could have higher order derivatives which are also important. For example, I could have Ux is equal to 1 half kx square plus let us say, k 1 x cube and so on.

If I neglect all these and I assume that only the second order is nonzero and this is the equation of motion and therefore, this is known as the simple harmonic motion.

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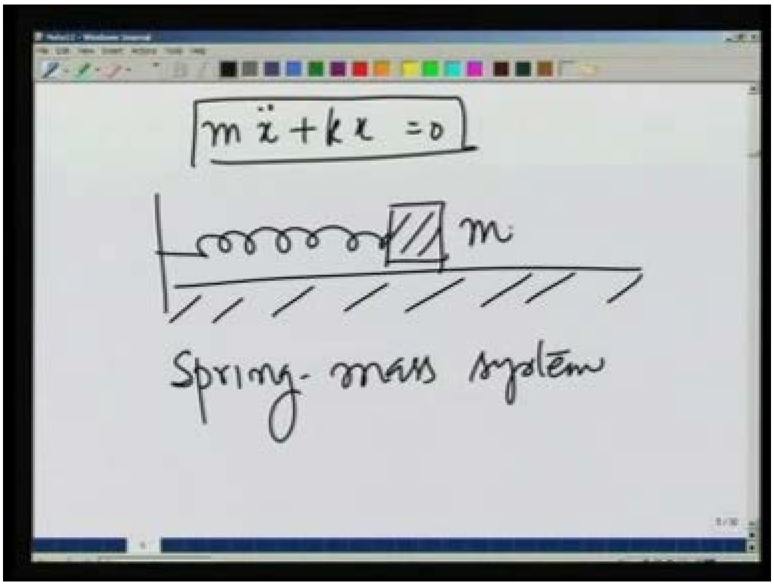

So, let us see now what we have is mx double dot plus kx is equal to 0 this is also the equation for a mass tied at the end of a spring which is oscillating on a frictionless plane. So, the prototype of simple harmonic motion is going to be a spring mass system, with this mass m and a spring constant k. And you can already see that there are many examples where I can apply this as the first form of approximation to the real motion.

For example I could have a particle at the bottom of a cup; a clamped rod which is vibrating a simple pendulum and so on. So, therefore, this is a very very good system and very useful system when we want to study a motion about an equilibrium point. Let me now give you 2 examples of how the potential energy looks like and how we approximated and what the force on the system looks like.

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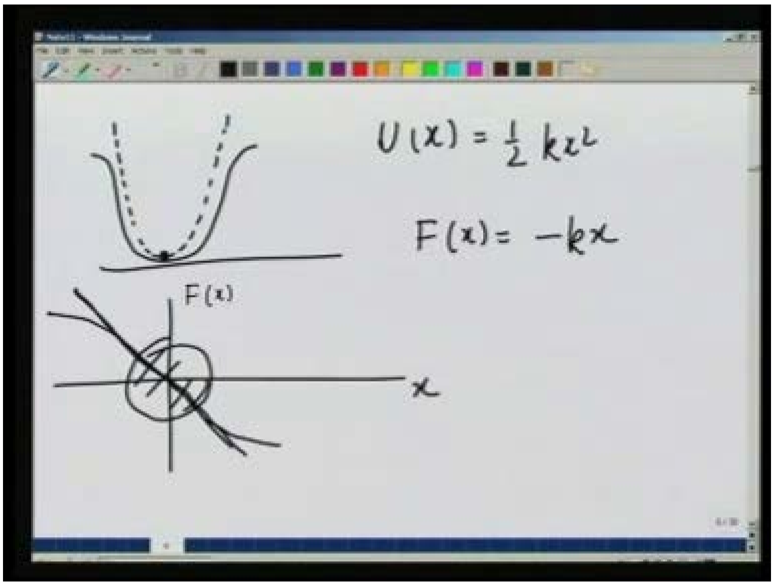

For example, I talked about particle at the bottom of a cup a cup could be of this shape and the particle could be sitting here it could have a very a functional dependence on x which is quite different. But, when I approximate this potential energy for small motion as 1 half kx square what I am doing is approximating the potential energy by this parabola. So, this is how the potential energy looks when I am approximating this motion by a a simple harmonic motion and the force could be would be Fx is equal to minus kx.

So, when x is positive I am approximate in the force like this I mean x is negative is like this line. You see here, that the in reality the force go something like this and then it becomes slightly less in magnitude; force goes something like this and then it'll becomes slightly less in magnitude. But, we approximate this as this line and therefore, as long as the motion will confine to this region, it is a good approximation.

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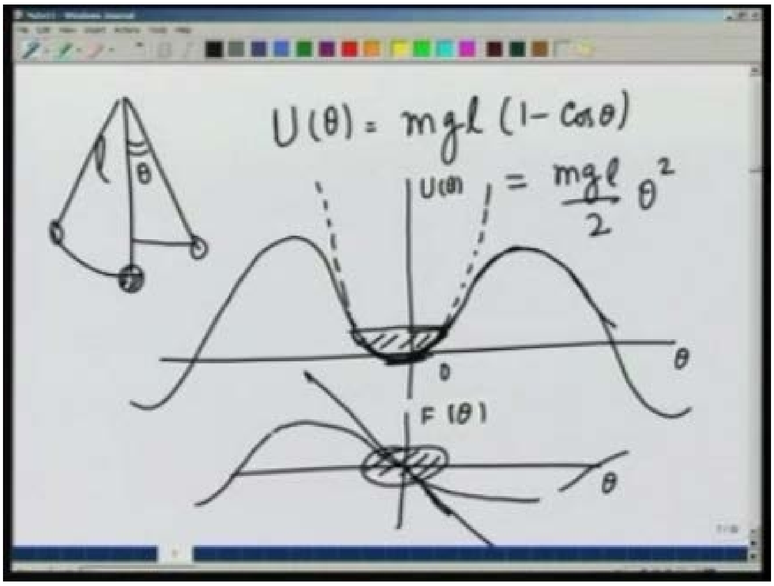

As a second example, let us take a pendulum of length l which is swinging above its equilibrium point. You can see when it is swing by an angle theta, the potential energy U as a function of theta is going to be given as mgl 1 minus cosine of theta. And therefore, the potential energy is going to looks something like this, it is minimum at theta equal to 0 then it goes like a cosine curve and so on.

However, when we approximate this as a simple harmonic motion then, the potential energy becomes mgl divided by 2 theta square and therefore, I am approximating this potential energy like this; this point is 0. So, again you can see for a good range of theta the approximation to the potential and the potential itself are quite close to each other.

How about the force? Force again we are approximate in the force to be a linear force. But as you go far away the slope goes down and therefore, the real force is something like this and there it oscillates. So, again for a good range of theta the force is linear and theta and therefore, simple harmonic motion is a good approximation for low displacements. And these lectures we are going to talk about the simple harmonic motion.

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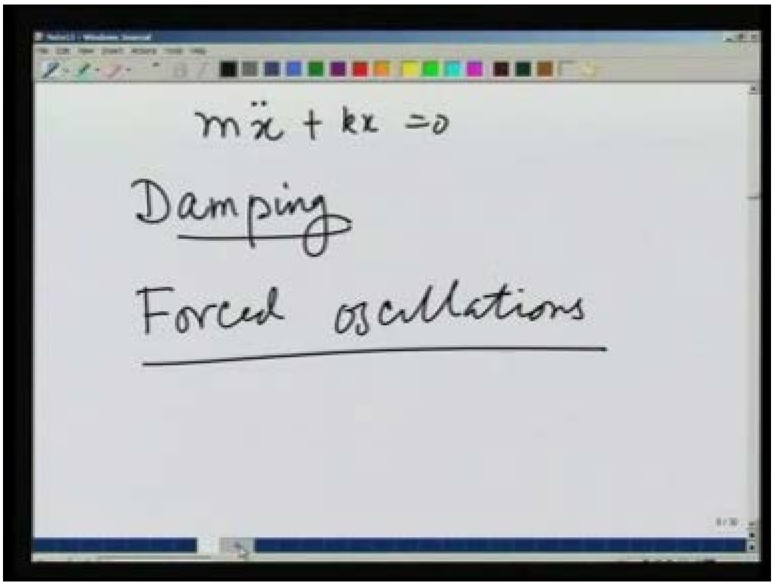

To start with we will consider the equation of motion plus kx equal to 0 and look at its solution represented by a Phasor diagram. After that, we are going to introduce damping into the system and see how it changes the motion. And after that we are going to study the forced oscillations of these systems, when the system is subjected to an external force in addition to its own force like the spring force that it experiences.

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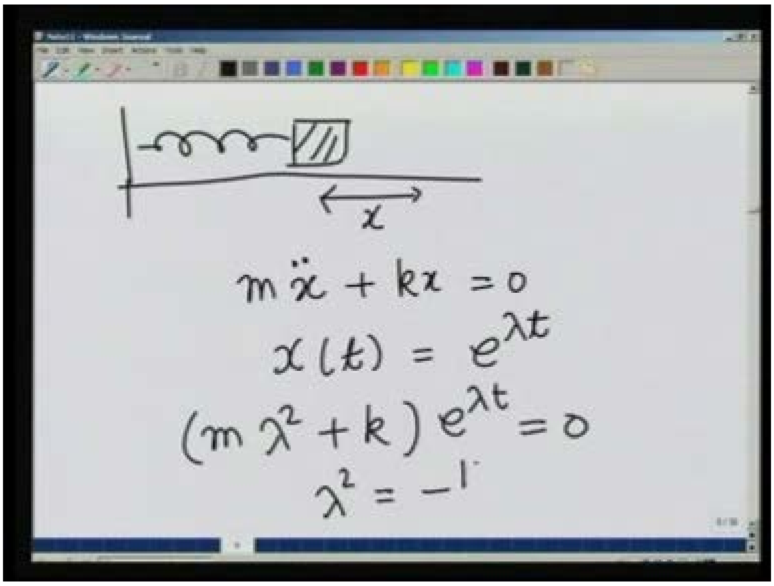

So, to start with let us take our prototype spring mass system and let us measure the displacement form the equilibrium point as x and the equation of motion as we have been saying is mx double dot plus kx is equal to 0. And we have seen in the past that, to solve such an equation we assume the solution of x to be of the form e raise to lambda t substituted in the equation then we get m lambda square plus k e raise to lambda t is equal to 0 which is true for all the times; since, it is true for all the times I must have lambda square is equal to minus k over m.

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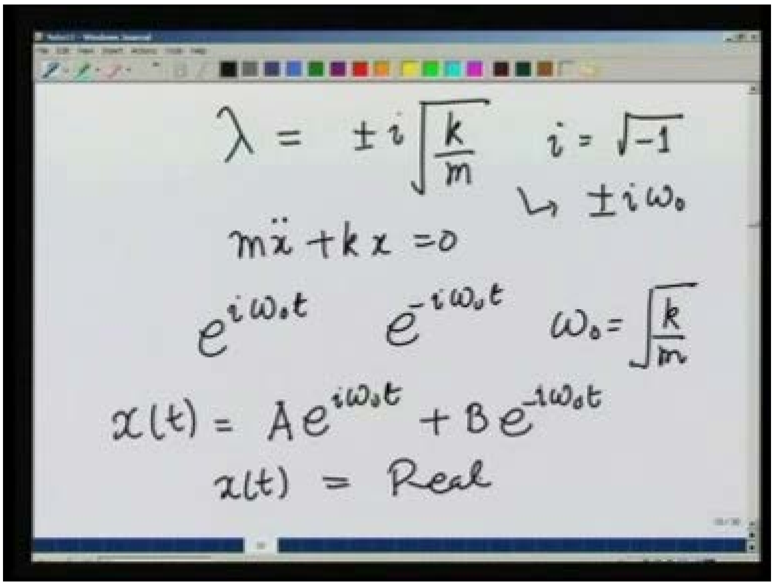

And therefore, lambda comes out to be plus minus I square root of k over m where I is equal to square root of minus 1. So, I have two solutions for the second order differential equation mx double dot plus kx equal to 0. The two solutions are: e raise to I omega 0 t and e raise to minus I omega 0 t where, I am writing omega 0 to be equal to a square root of k divided by m.

So, that lambda that I wrote earlier can be written as plus or minus I omega 0. The general solution for a second order equation therefore, is going to be xt which is a linear combination of these 2 solutions: A e raise to I omega 0 t plus B e raise to minus I omega 0 t. Since, I know that the displacement xt is real.

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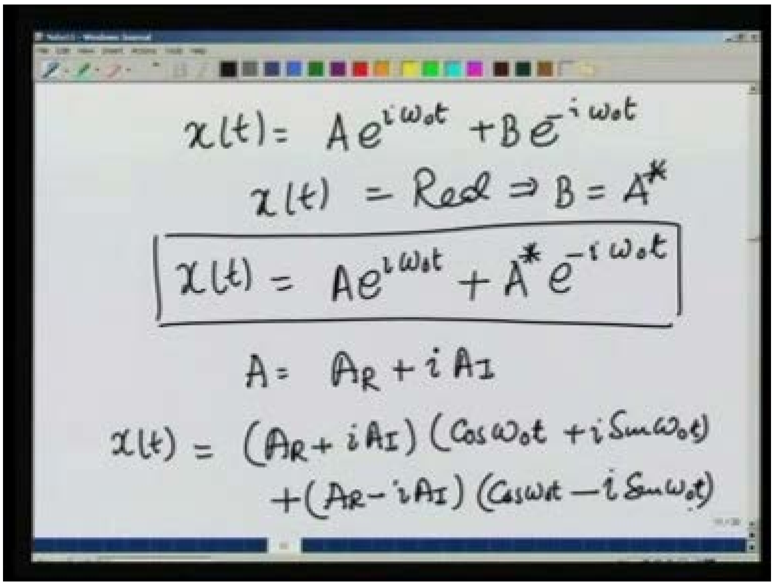

Therefore, I should have in the solution xt which, I am writing as A e raise to I omega 0 t plus B e raise to minus I omega 0 t and xt is real and therefore, B must be equal to A star. So, in general A is complex, but for xt to be real I should have the solution of the form this equal to A e raise to I omega 0 t plus A star where, A star is a complex conjugate of A e raise to minus I omega 0 t. That is the general solution since, it involves complex quantity A and A star let me write it in a slightly different form. Writing A as A real plus I times A imaginary.

When I substitute this I get xt is equal to A real plus I A imaginary and I can open up e raise to I omega 0 t as cosine of omega 0 t plus I sin omega 0 t plus the second term which is A real minus I AI where AI is the imaginary part cosine of omega 0 t minus I sin of omega 0 t.

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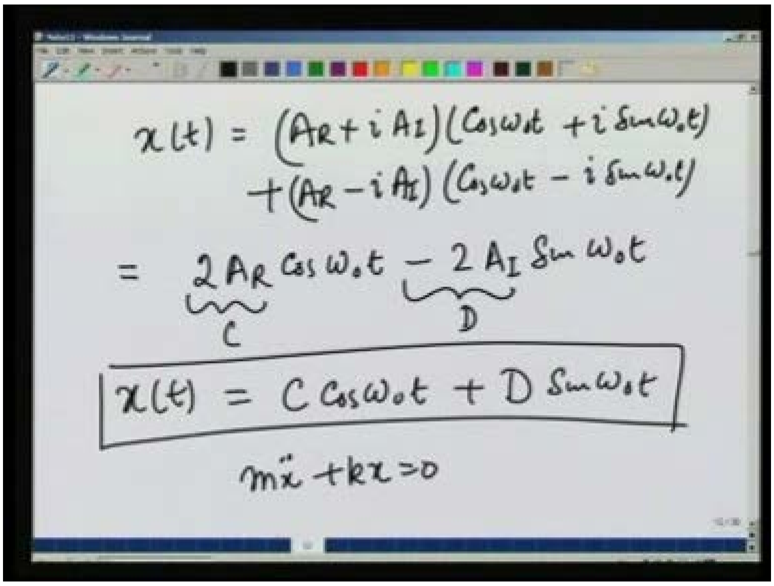

When I open this up the final solution that I get is going to be xt is equal to which is wrote as A real plus I AI cosine omega 0 t plus I sin omega 0 t plus A real minus I, A imaginary cosine omega 0 t minus I sin of omega 0 t will be equal to 2 A real cosine of omega 0 t minus 2 A imaginary sin of omega 0 t.

If I call these constants C and minus 2 AI D I can also write my solution as xt equals C cosine omega 0 t plus D sin of omega 0 t. That's another way of writing the solution equation mx double dot plus kx equal to 0, which is equivalent to the solution that I got earlier in terms of e raise to I omega 0 t and e raise to minus I omega 0 t.

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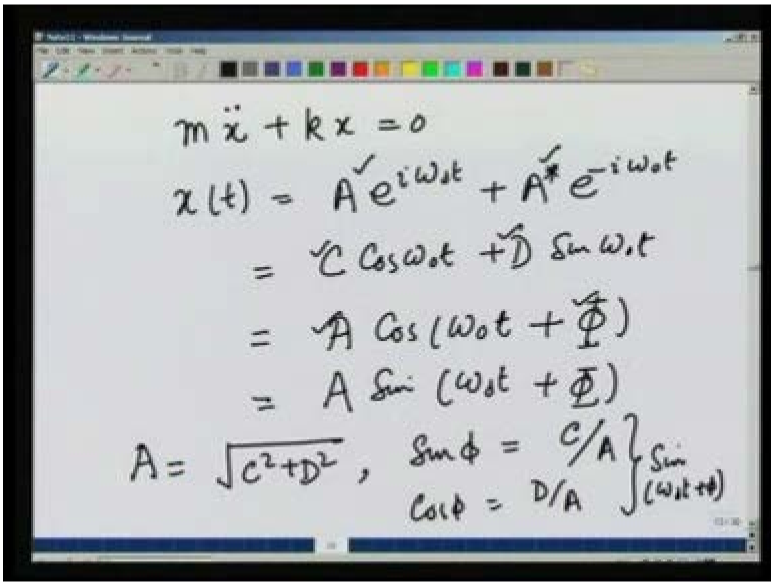

So, what we have learnt so far is, given this equation for a spring mass system or the motion of a particle about its equilibrium point. Its motion is going to be described by the displacement xt is equal to A e raise to I omega 0 t plus A star e raise to minus I omega 0 t or equivalently C cosine of omega 0 t plus D sin of omega 0 t. And third way again you can write this as some amplitude time’s cosine of omega 0 t plus a phase constant called phi. Or also, if you like sin functions I can write this as sin omega 0 t plus a phase constant called phi it is our choice how we want to represent the solution.

Where A is going to be in this case a square root of C square plus D square and depending on whether I write it as cosine omega 0 t plus phi or sin omega 0 t plus phi. The sin of phi is going to be C over A and cosine of phi equals D over A in the case, when I write this as sin of omega 0 t plus phi. So, we have in the motion 2 constants A real A imaginary or C or D or A or phi and these are determined by the initial condition of the motion as we will see.

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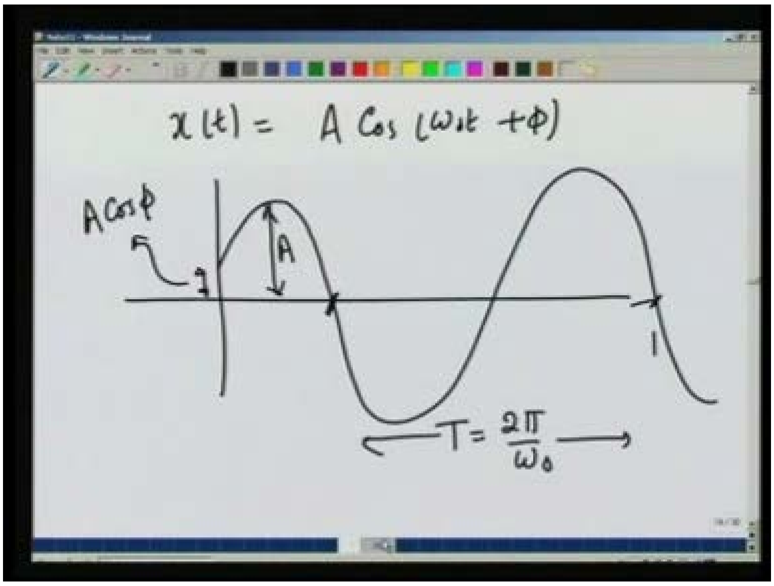

Let us try to see how this motion looks. So, let me just write xt is equal to A cosine of omega 0 t plus phi how this motion looks. In general, this would look something like this. Where this maximum distance is going to be A, the time between this and this 2 equivalent point is going to be t is equal to 2 pi over omega 0. And this initial displacement is A cosine of phi.

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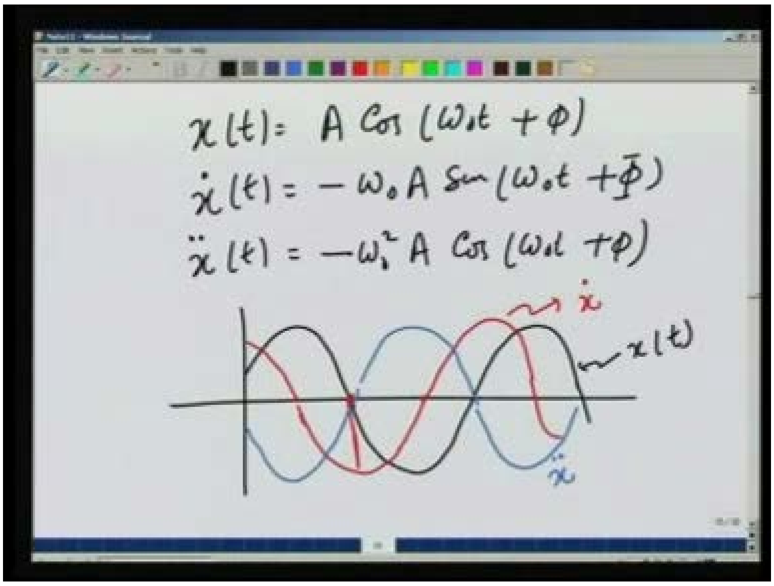

How about the velocity of the particle? That is very simple since, I am given xt is equal to A cosine of omega 0 t plus phi the velocity x dot t is going to be minus omega 0 A sin of omega 0 t plus phi. How about the acceleration? X double dot t is going to be minus omega 0 square A cosine of omega 0 t plus phi. Let us plot them and see. So, as I said xt in general would look something like this, x dot let me plot it in a different colour; is going to be slope of this and it is going to look something like this.

This is x dot you can see here the slope is positive; that means, the velocity is positive. At the maximum displacement the velocity becomes 0, as a spring mass system the mass goes to its maximum displacement it stops momentarily. Then it starts coming back with negative velocity again when it passes through the equilibrium point when the displacement is 0 its velocity is maximum it is going back with maximum speed. And then its speed starts slowing down reach a 0 again when it reaches the other extreme and so on.

The acceleration let me show that in a slightly different colour let us say blue is going to be negative of the displacement times omega square. So, that is going to look something like this, I am not shown the magnitudes I have just shown how in general the velocity and the acceleration are going to look black one; obviously, is the displacement.

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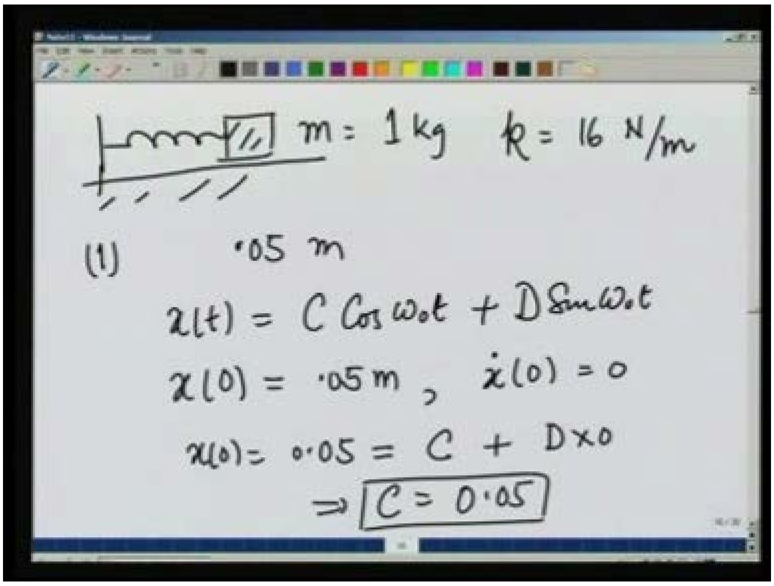

As a first example of solving this equation, let us go back to our a spring mass system and let us take mass of the block to be 1 kilogram. Let us take k to be 16 Newton’s per mete and let us first displace the mass by 0.05 meters; let us say to the right and leave it. Let us see what happens. So, in general the solution xt is C cosine of omega 0 t plus D sin of omega 0 t and as I said earlier C and D are determined by the initial conditions. Initial conditions given are let x at time t equal to 0 is 0.05 meters.

And since, we have just realized it x dot at t equal to 0 0, when we substitute this condition I get x at 0 is equal to 0.05 should be equal to C plus D time 0 or I get C equals 0.0 5. We have determined 1 of the coefficients.

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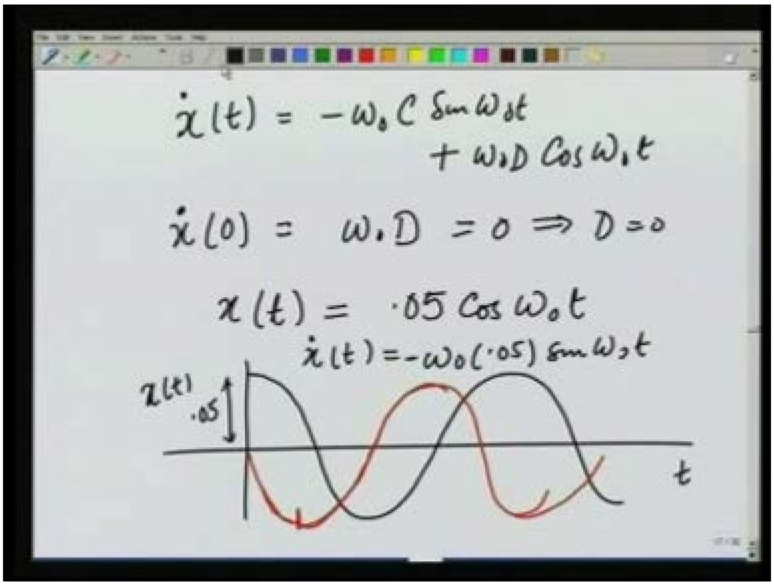

For the second coefficient I take derivative of xt which is going to be equal to minus omega 0 C sin of omega 0 t plus omega 0 D cosine of omega 0 t and at t equal to 0, this quantity is omega 0 D and this is given to be 0 this implies D is 0. So, in this situation when we took the particle and took it to a maximum distance and left it the solution looks like xt is equal to 0.05 cosine of omega 0 t.

If, I plot it on xt plot it will look something like this where this distance is point 0.05, if I were to plot the velocity of the particle. I will take the derivative and that would come out to be x dot t is equal to omega 0 times 0.5 sin omega 0 t with a minus sign here. And that would look like this, initially the particle is moving towards the left it has negative velocity the velocity reaches maximum and then it slows down and then goes to the other side.

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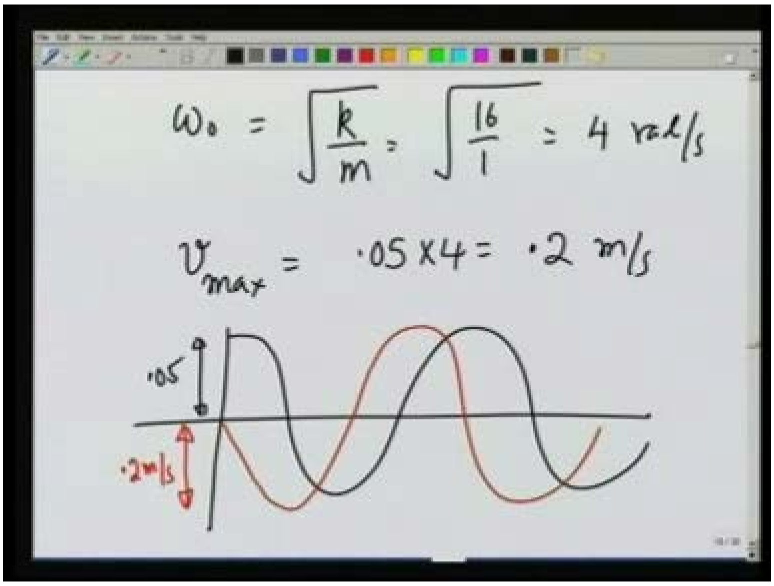

Here omega 0 is equal to square root of k over m which is 16 over 1 m equals 4 radians per second so that, the maximum speed which is 0.05 times 4 is point 2 meters per second. So, to see it again this is how xt looks when plotted and this is how the velocity looks where this maximum; I have not drawn it to the scale is point 2 meters per second and this maximum is 0.05. Let us take a different set of initial conditions and see what answer do we get.

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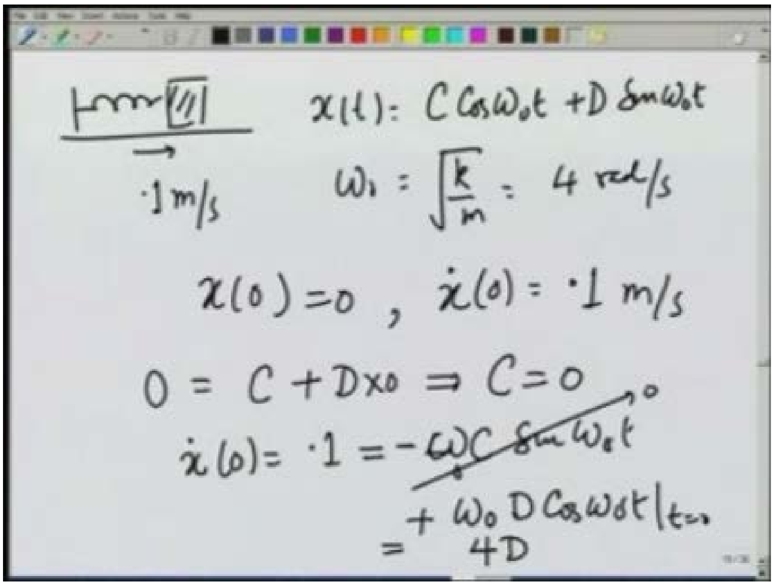

Now, let me take the same system whose solution we have already determined is xt is equal to C cosine of omega 0 t plus D sin of omega 0 t. Where omega 0 is equal to square root of k over m is equal to 4 radians per second. Let me take, a something with which I can hit the block and let me give it initial speed of point 1 meters per second to the right from the equilibrium so that, I have x 0 is equal to 0 and x dot 0 is equal to point 1 meters per second. With this I will get 0 is equal to C plus D time 0 which gives me C is equal to 0.

So, this time C is 0 on the other hand x dot at 0 is given to be point 1 meters per second and this would come out to be omega C sin omega 0 t and this is already 0 because, we have seen C is 0 plus omega 0 this is with the minus sign D cosine of omega 0 t. And this tells me that point 1 is equal to omega not we have determined to be four D at t equal to zero.

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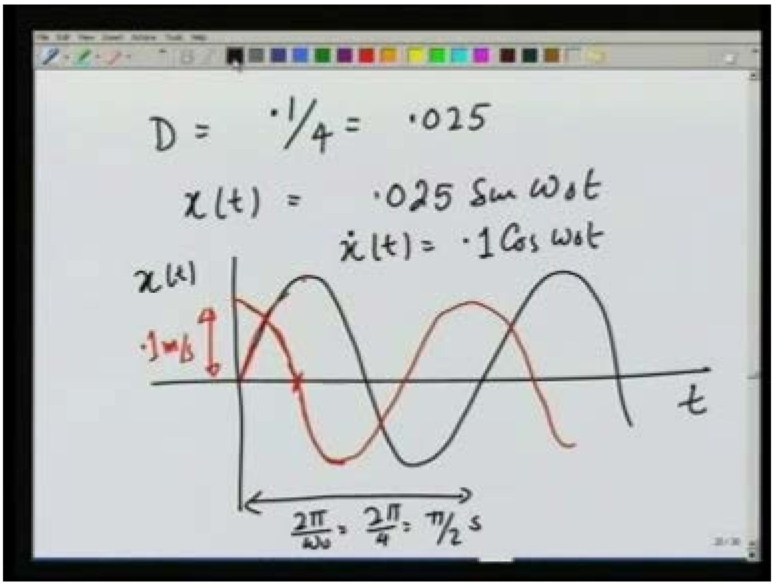

And therefore, D is equal to point 1 over four which is 0.025 in this case therefore, when I take the spring mass system and give it a initial velocity at the equilibrium point of point 1 meters per second. The solution comes out to be point 0 25 sin of omega 0 t. When plotted against t the solution would look like this, where the time period that is this is t this is xt the time from here to here is 2 pi over omega 0 which is 2 pi over 4 or pi divided by 2 seconds.

The velocity in this case x dot t is going to be point 1 cosine of omega 0 t as you can see easily by differentiation and therefore, when I plot it will look something like this, with the maximum being point 1 meters per second at t equal to 0, which we solve with which we really hit the mass in the beginning. Again you can see initially the slope is positive and therefore, velocity is positive the slope is decreasing slowly. So, velocity keeps going down crosses 0 at the maximum displacement and then goes negative.

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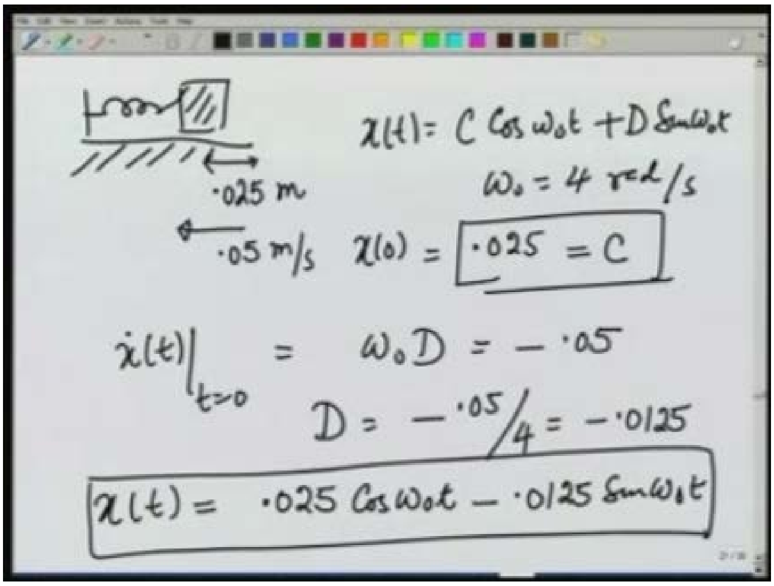

To get more familiar with such things, let me take a third example where I take the same system and displace it by some distance. Let us say 0.025 meters and give it a velocity initially in this direction negative direction of let us say 0.05 meters per second. Again, looking at the system, I have xt is equal to C cosine of omega 0 t plus D sin of omega 0 t. Where omega 0 is given to be 4 radians per second I have x at 0 is equal to 0.025 and this is equal to C.

So, C is already determined x dot t at t equal to 0 is going to be omega naught D and this is given to be minus 0.05. And therefore, D is equal to minus 0.05 over 4 which is minus 0.0125. And therefore, my solution xt is going to be of the form 0.025 cosine of omega 0 t minus 0.0125 sin of omega 0 t where omega 0 is 4.

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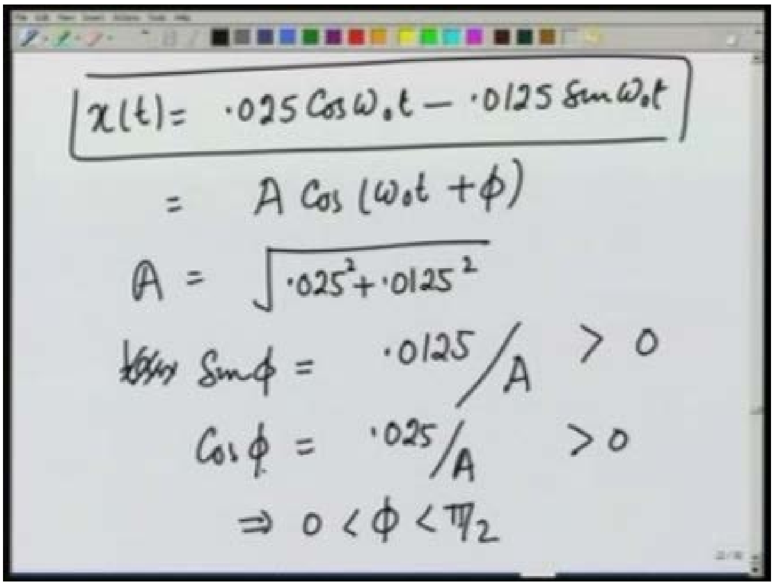

Let us look at the solution. So, xt is equal to 0.025 cosine of omega 0 t minus 0.0125 sin of omega 0 t. If I write it in the form, some amplitude cosine of omega 0 t plus phi you can see that A is going to be a square root of 0.025 plus 0.0125 square is square. And tangent of 5 or better to write sin of phi is going to be equal to 0.0125 divided by the amplitude. And cosine of phi is going to be equal to 0.025 divided by the amplitude; both are greater than 0 and this tells me whatever the value, the phi is between 0 and pi by 2.

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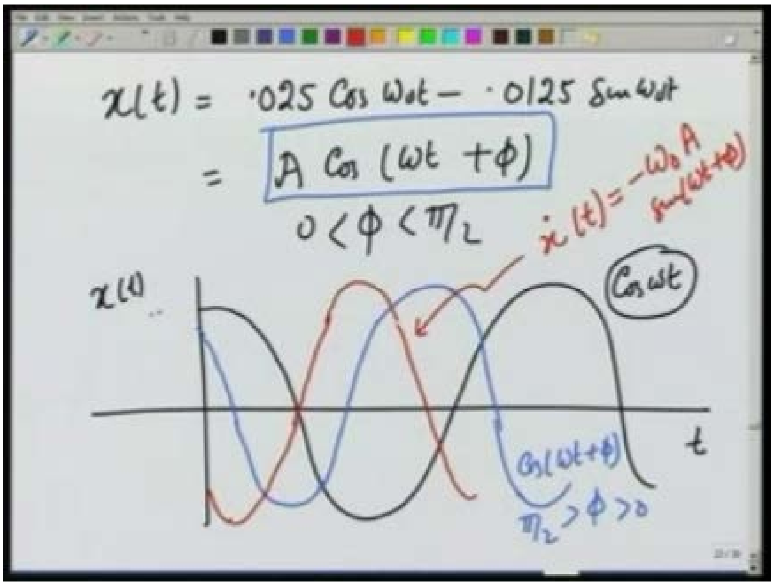

And therefore, if I plot this function xt is equal to 0.025 cosine of omega 0 t minus 0.0125 sin of omega 0 t which we wrote as, some amplitude cosine of omega t plus phi with phi between 0 and pi by 2, it will look like I am plotting xt verses t. At t equal to 0 the function has already achieved what it would have achieved little later if the function was pure cosine omega t. So, cosine omega t looks like this, this is cosine omega t. The function that I have now is this it'll start somewhere from the back side and go like this.

This is cosine omega t plus phi with phi greater than 0, but less than pi by 2. You can see the velocity is going to be negative because the slope is negative here it starts from here and becomes maximum at the equilibrium point and starts going down goes to 0 becomes maximum at the equilibrium point and so, on This is x dot t which in this case is going to be minus omega 0 A sin of omega t plus phi.

So, this is how the motion is described if instead of pushing it in at t equal to 0 from same distance I push it out the phase would be slightly different and I leave that as an exercise for you.

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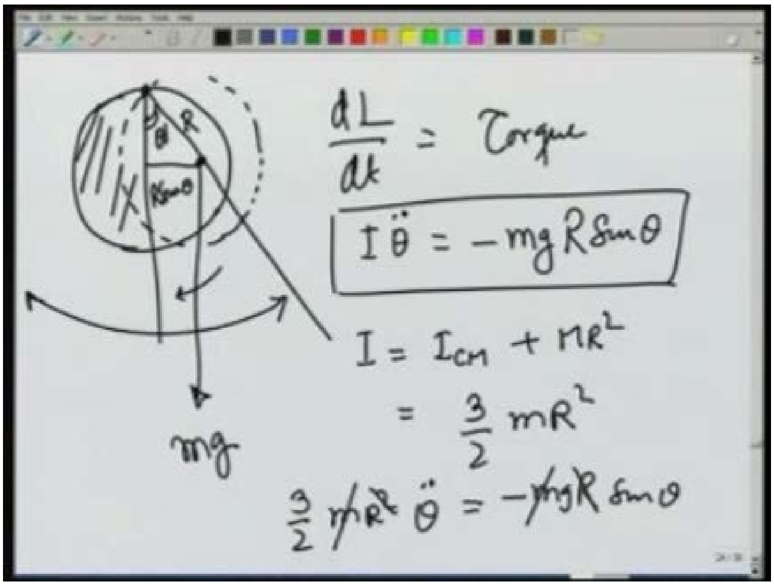

As a second example of simple harmonic motion let me now, take a disc and pivot it at a point here so that, it can oscillate about that point like this. Let this be 1 of the diameters. So, its diameter goes back and forth and I want to know what the frequency of oscillation is. As we have already learnt in rigid body dynamics that, whenever the forces and masses are distributed we use the angular momentum. So, in this case also since the disc is has distributed mass the force at each point we use angular momentum concept and therefore, the dynamics is described as L dL dt is equal to the torque.

In this case, a torque is going to be due to its weight mg which will be at distance if the radius is R; R sin theta if this angle is theta. And this is going to be opposite to the displacement and therefore, I can write I theta double dot is equal to minus mg R sin theta that is the equation of motion governing the motion of this disc when displaced from its equilibrium position. Now, I about the pivot is going to be I about cm plus MR square and which is going to be equal to three by 2 mR square.

Therefore, the equation is of motion is going to be three over 2 mR square theta double dot is equal to minus mg R sin theta m cancels 1 of the R cancels.

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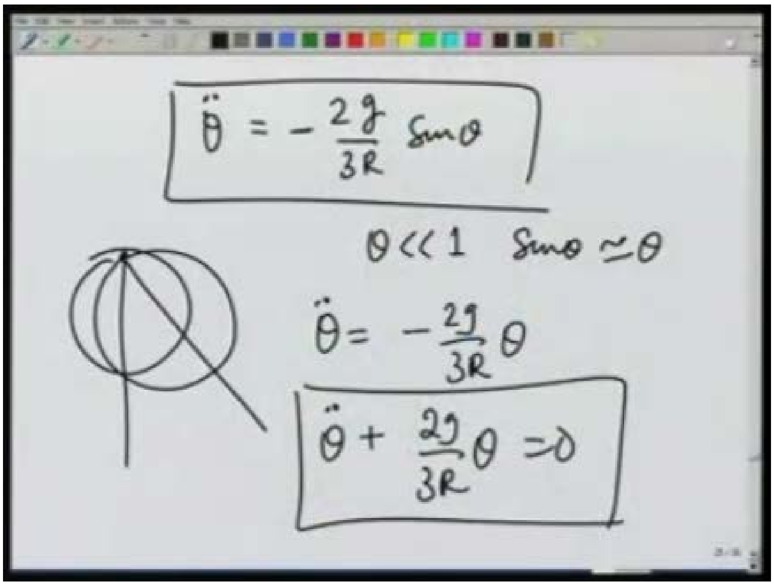

Therefore, the equation of motion that I have is theta double dot is equal to 2 g upon 3 R sin of theta. We are considering this disc which is being displaced from its equilibrium position like this. For theta less than one so that, I can approximate sin theta roughly as theta the equation of motion becomes theta double dot equals there is a minus sin here minus 2 g over 3 R theta or theta double dot plus 2 g over 3 R theta is equal to 0. This is in the same form as the simple harmonic motion equation.

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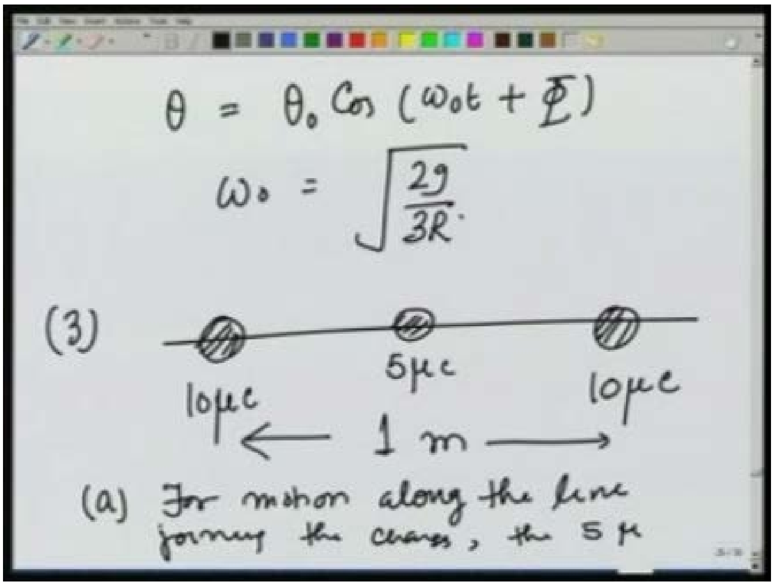

Therefore, theta is going to oscillate like theta is equal to some amplitude theta 0 cosine of omega 0 t plus phi where phi and theta 0 depend on the initial conditions; where omega 0 is going to be square root of 2 g over 3 R. So, this is another example that tells you how we see simple harmonic motion in everyday life.

As a third example of showing that around in equilibrium point the motion is really simple harmonic I take 2 charges of ten micro Coulomb each separated by a distance of 1 meter. And I take a third charge of five micro Coulomb put right in the middle right in the middle electric field is 0 and therefore, it is an equilibrium point. I want you show number 1 for motion along the line joining the charges.

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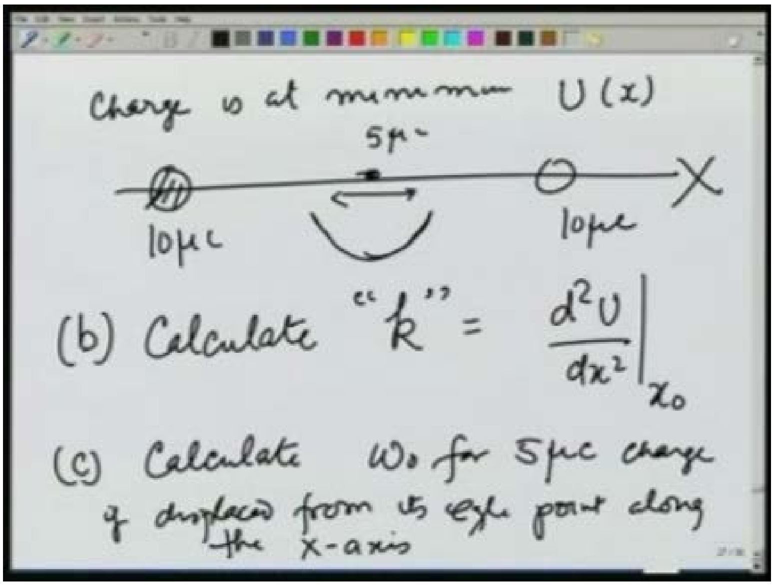

The 5 micro Coulomb charge is at minimum Ux. So, what I am asking to do is take these 2 charges 10 micro Coulombs each put a 5 micro Coulomb charge in the middle keep them fixed these are fixed. And consider the motion of this 5 micro Coulomb charge about this equilibrium point I want you show as I move about this the energy goes up potential energy goes up.

As long as I confine the motion along this line, number 2 calculate the spring constant K which we have already decided is d 2 U over dx square at the equilibrium point. And therefore, see calculate the oscillation frequency omega naught for 5 micro Coulomb charge if displaced, from its equilibrium point along let us call this the X axis the X axis.

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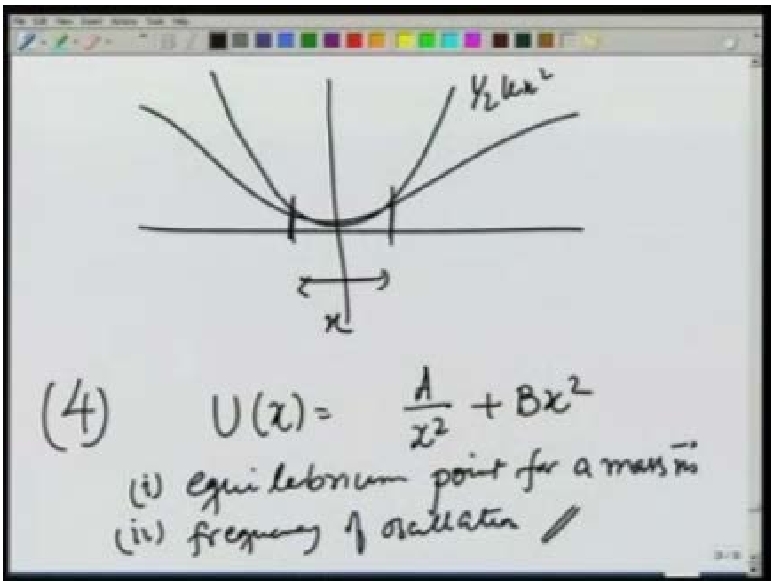

What you can also do is, calculate this potential exactly It may look something like this and see how the approximate potential that you are writing as 1 half kx square looks like in comparison to the real potential. Also up to what range of x is this approximation good and up to what range you can describe the motion about the equilibrium point as the simple harmonic motion.

As the forth example, I would like you to consider a potential of the form A over x square plus Bx square and calculate equilibrium point for a mass m and b. Calculate the frequency of oscillation when you displace this mass from the equilibrium point You can of course, go ahead and see how good the approximation of kx square is in approximate in this potential and for what range of displacement is this approximation good.

So, these are the exercise I would like you to do before we proceed and of further.

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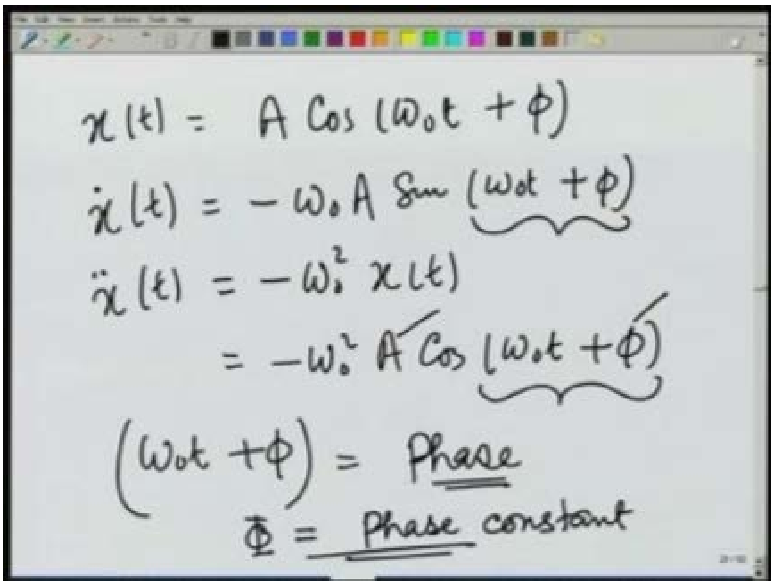

So, we have looked at how the motion of a particle moving in a potential of half kx square form looks like. It looks like something like cosine omega 0 t plus phi the corresponding velocity is minus omega 0 A sin omega 0 t plus phi. And the acceleration is exactly in the opposite direction of the displacement minus omega naught square xt which is minus omega naught square A cosine of omega 0 t plus phi.

Let me now, focus a bit on this quantity omega naught plus phi which is known as the phase of the oscillation. You can see that the motion finally, or the constants of motion A and phi depend on the initial conditions. So, phi is known as phase constant and depending on the phase constant the motion is different with respect to t.

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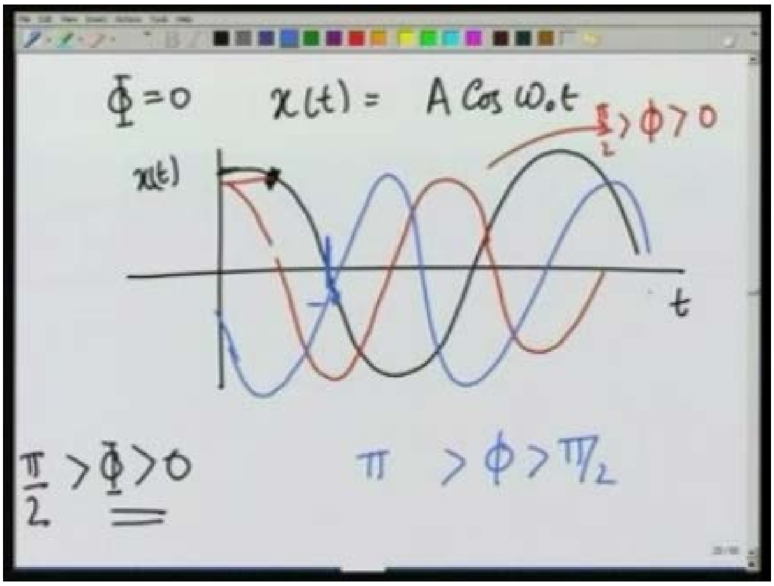

For example, let us take the case of phi is equal to 0 in that case xt would look like A cosine of omega 0 t. And if plotted against t the motion looks like this xt goes back and forth starts from the maximum goes down to equilibrium point and oscillates back and forth. What happens in the case when phi is greater than 0 a phi is greater than 0 and let us first take the case of it being less than pi by 2.

In that case, the motion starts at t equal to 0 such that it would have already achieved a phase that it had at little later time in the case of phi being 0. And in that case therefore, the motion starts something like from this point like this; this is a case of phi being greater than zero, but less than pi by 2. What about the case now when phi is greater than 0 just write it with blue or greater than pi by 2 and less than pi. In that case also, the motion would start at t equal to 0 as if with the displacement that it would have achieved at a later time.

But, for phase greater than pi by 2 this is phase greater than pi by 2 so, motion starts somewhere here. So, in that case the motion would look something like this you can see in the initial phase how the motion depends.

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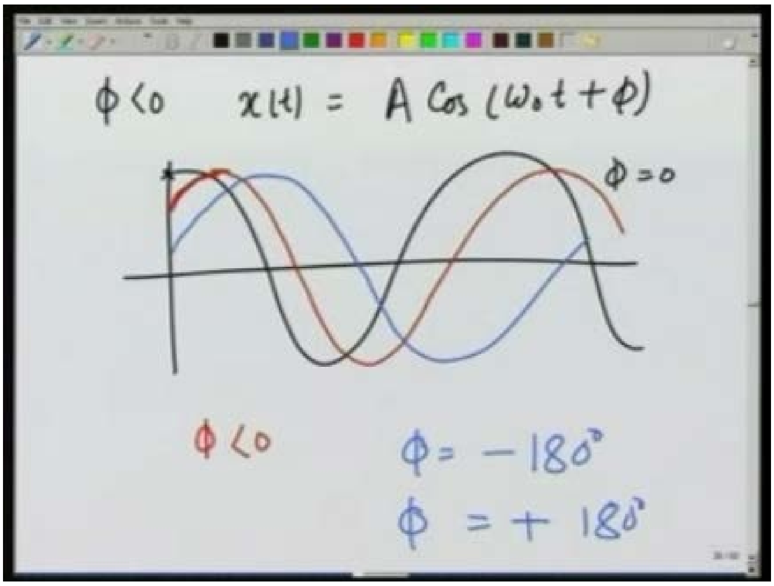

Let us look at the case 1 pi is less than zero. So, the xt which is equal to A cosine of omega 0 t plus phi would be such that if I were to take the case of phi is equal to 0. The motion would catch up it will achieve the phase which it had at 0 at a slightly later time. So, let us plot the case of phi less than 0 it would look something like this. So, the motion is such that, it is achieving the phase which it had at 0 time in this case phi equal to 0 case slightly later.

If I keep increasing the phase let us take it with blue this would be something like this and so on. What about if I keep decreasing the phase and go to the limit of phi is equal to minus 180 degrees how would the motion look and how does that compare with the motion phi equal to plus 180 degrees.

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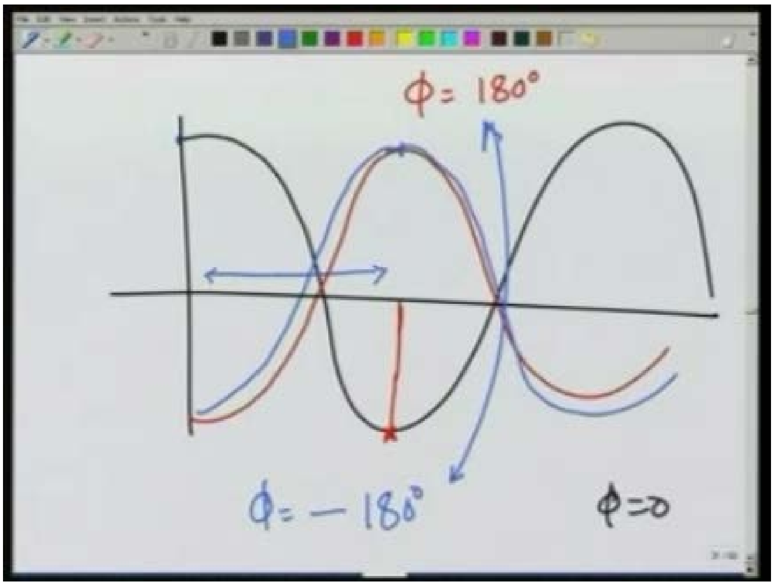

Let us see these 2 extreme cases. This is phi equal to 0 let us take the case of phi equals 180 degrees. Phi equals 180 degrees would be such that it would achieve at 0 what the 0 phase had at 180 degree. So, the motion would look something like this. What about the case of phi minus 180 degrees? It would catch up with the phase at 0 with 180 degree; a catch up with the phase at 0 at a time which is given by this distance. So, therefore, this would shift here. So, minus 180 degrees also would look something like this. So, you see the phase is 180 degrees and minus 180 degrees mean 1 and the same thing.

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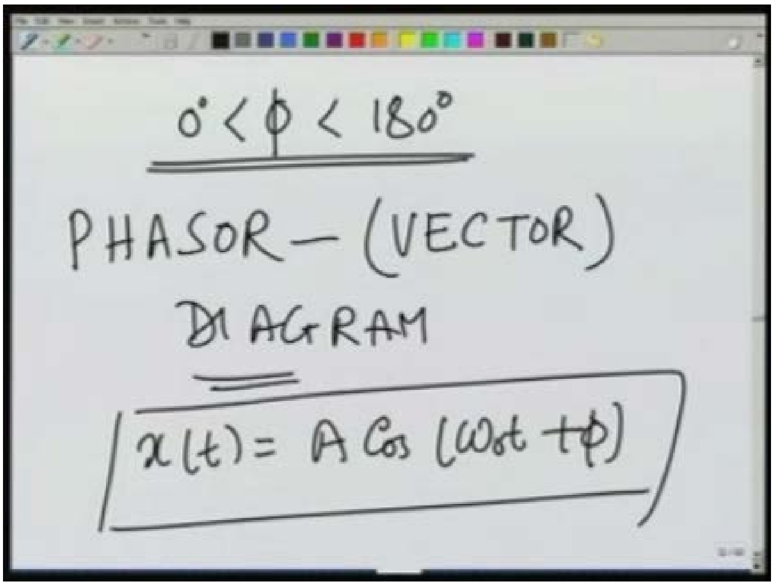

Therefore, in describing the motion it is best to keep phi limited between 0 degrees and 180 degrees. Now, let us look at a nice geometric way of looking at the oscillatory motion and this is based on the fact that when I write xt is equal to A cosine of omega 0 t plus phi.

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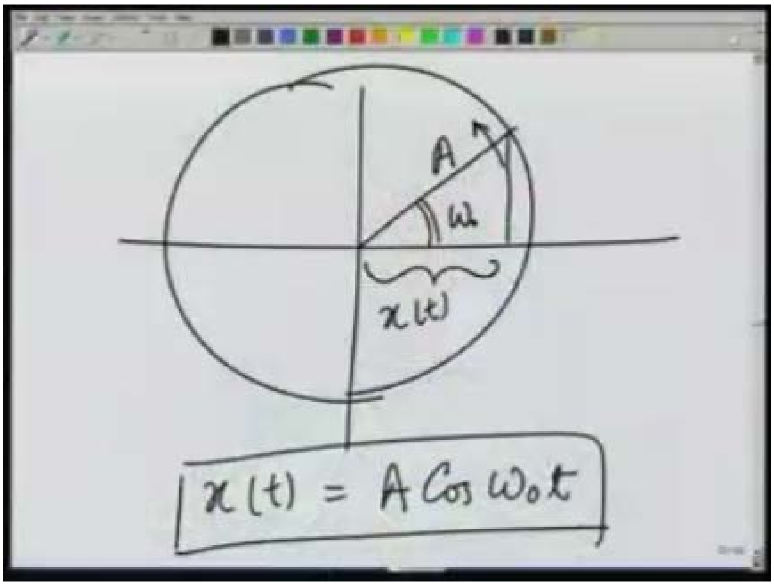

What I am really doing is, representing the x component of a radius equal to A which is rotating in this direction at speed omega 0. I am taking its x component and this x component is really describing the motion xt is equal to A cosine of omega 0 t.

So, Phasor diagram is a way of looking at this motion and it becomes particularly useful when 2 or three motions are superimposed on each other. But right now let us look at the Phasor diagram for 1 motion only.

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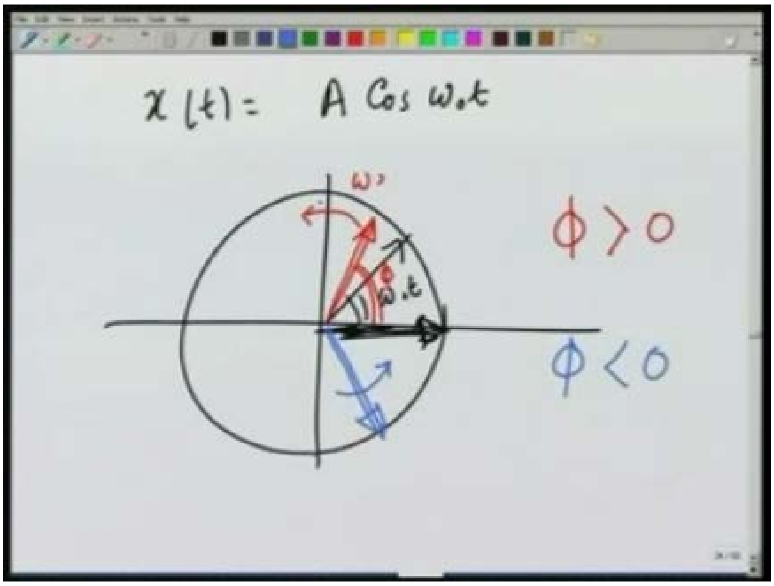

So, if I have xt is equal to A cosine of omega 0 t, the initial position of the vector would be given like this and it would be rotating counter clockwise with a speed omega 0 t; omega zero so there in time t covers a distance of omega 0 t. On the other hand if I had phi greater than 0 then it would start with an initial position which is given by phi and then would be rotating with speed omega 0

Third possibility is phi less than 0 and then it would be starting from here and would be rotating like this, its projection on the x axis would give me the displacement as a function of time. As I said earlier I can choose to be choose the phase to be either between 0 and 1 eighty degrees or between minus 90 and 90 its 1 and the same thing.

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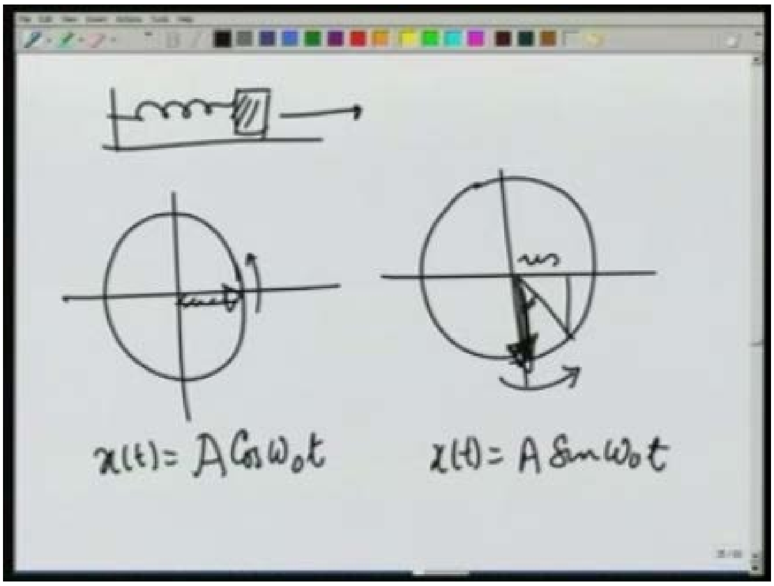

Let us then look at the earlier example of the spring mass system where we had four initial situations which we have different one was that we stretch the spring to a maximum and let go. In that case, the motion would be represented on the Phasor diagram with Phasor starting from here and going around like this.

The second situation was when we had given an initial positive velocity to the mass to the right; so that, in that case Phasor would be here and it would be rotating like this. You can see in this case xt would look like the amplitude which is a radius of the Phasor cosine of omega 0 t. In this case you can see xt would like because after time t it'll be here. So, this projection would be sin omega t A sin of omega 0 t.

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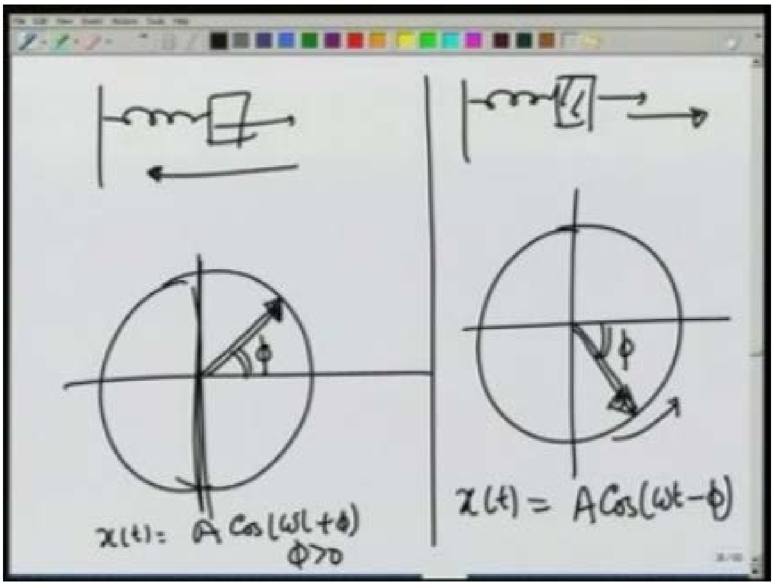

What about the case, in which we pulled it to a distance and then gave it a velocity going this way. So, the particle was displaced positively it if the Phasor type has to be on the right hand side of this line and it was moving in the negative direction. So, the Phasor would be somewhere here with this angle being phi. You can see, that xt is going to be A cosine of omega t plus phi with phi being greater than 0.

On the other hand, if we took this is spring mass system stretch it a bit and then gave it a velocity in the right, a particle is moving to the right and it is right displaced in the right direction it'll be somewhere here and would be moving like this. Xt would be A cosine of omega t minus phi with this angle being phi. So, this is the way of representing the displacement in Phasor or vector way.

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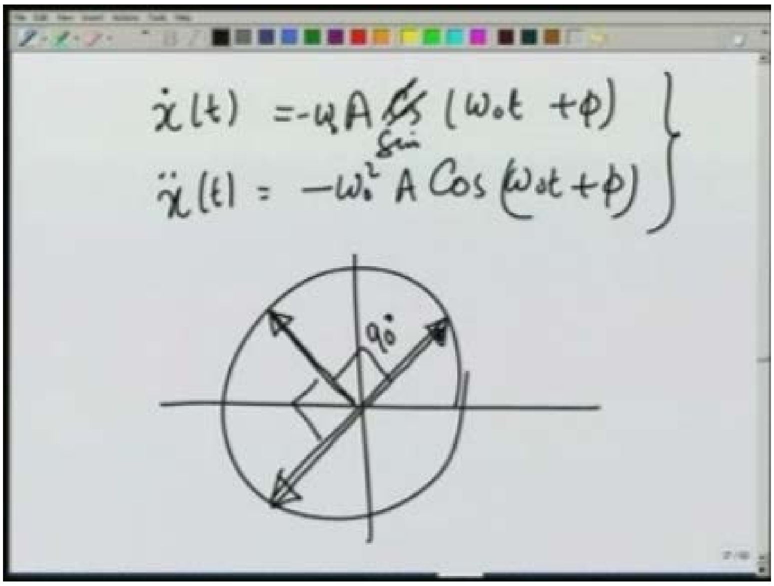

Since x dot t is given as minus omega naught cosine of omega not t plus phi and x double dot t is given as minus omega 0 square A cosine of omega 0 t plus phi. This should be sin if I represent the displacement velocity and the acceleration on the Phasor diagram. If the displacement at any given time is like this, we can easily see that the velocity Phasor would be pointing in this direction at 90 degrees to the displacement.

And similarly, acceleration be pointing in the direction opposite to the displacement 90 degrees this way. You can work it out yourself and see its satisfies all the conditions or all the directions and magnitudes everything correctly with this representation.

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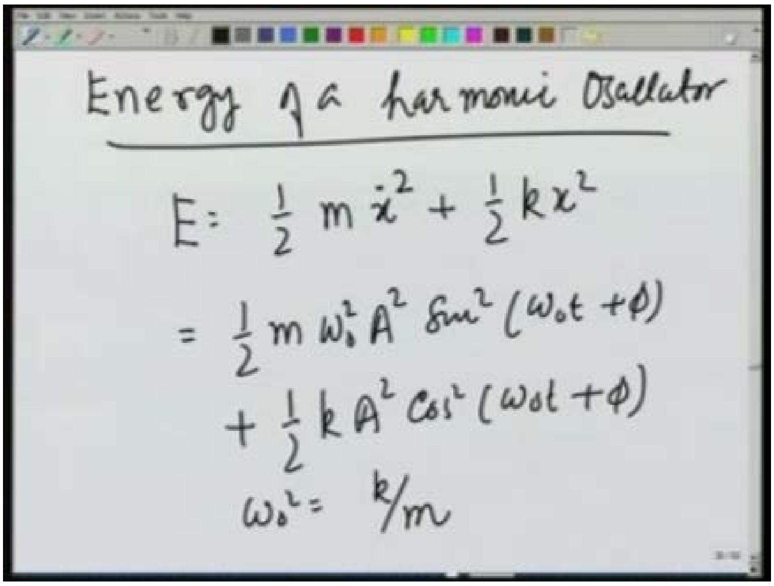

Finally to sum up this introduction to harmonic oscillator let me talk about energy in a harmonic oscillator. Energy for any system is given as 1 half mx dot square which is the kinetic energy plus 1 half kx square which is the potential energy which in this case would become 1 half m omega 0 square A square sin square omega 0 t plus phi plus 1 half k A square cosine square omega 0 t plus phi. However, omega 0 square is equal to k over m.

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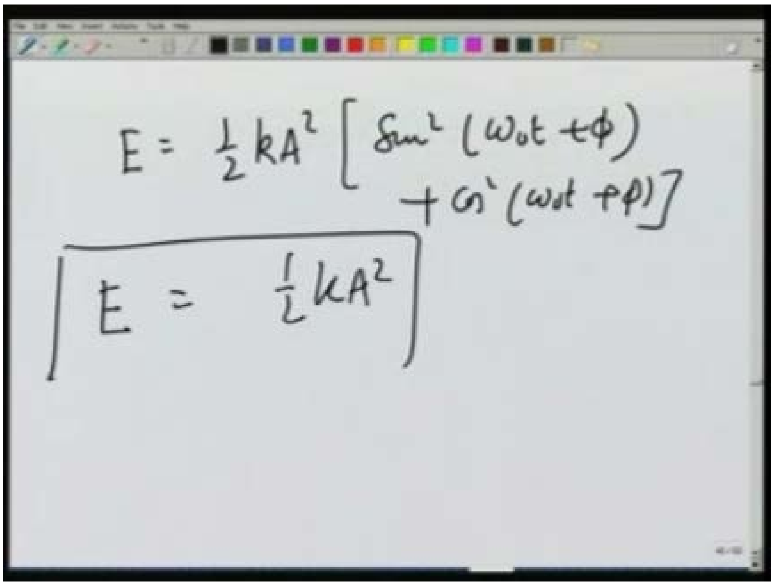

Therefore, when I added up and substitute and added up I get e is equal to 1 half k A square sin square omega 0 t plus phi plus cosine square omega 0 t plus phi which is, 1 half k A square. So, the total energy of the system at any given time is going to be equal to 1 half k A square, which is sensible. If I displace the mass to its maximum displacement, then the velocity is 0 and therefore all the energies; potential energy which is equal to 1 half k A square k A square.

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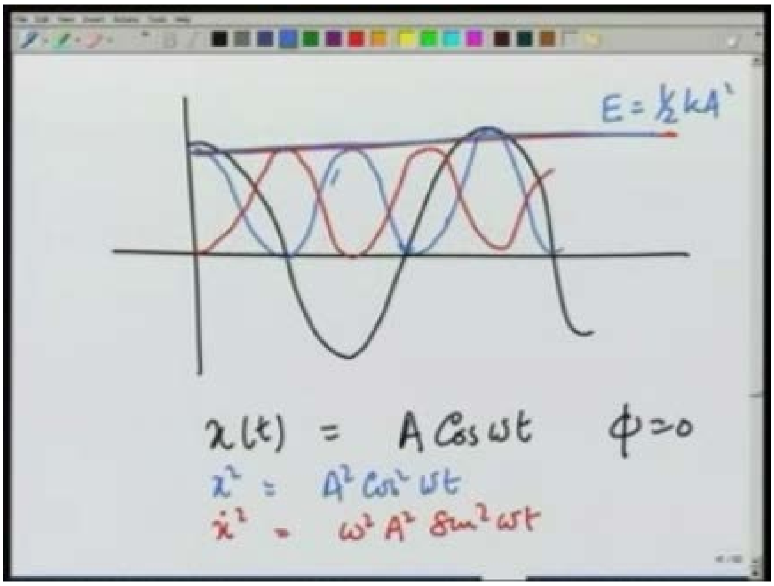

Let us see it graphically. If I were to plot xt, let us take the case of a simple motion where phi is taken to be 0 then xt would look like this. If I square it let me show the square with blue x square would be A square cosine square omega t which is all positive it'll start from here go down go like this and so on. Notice, that here it is less than the displacement curve the square of 1 is a square of less than quantity less than 1 is less than 1.

Similarly, x dot let me show this with red square would look like omega square A square sin square omega t. So, that curve would look something like this and when I add the 2 up the sum would be a constant. Let me show it with red and blue together and that is a total energy; half k A square. So, let me now sum up this introductory lecture on harmonic oscillator.

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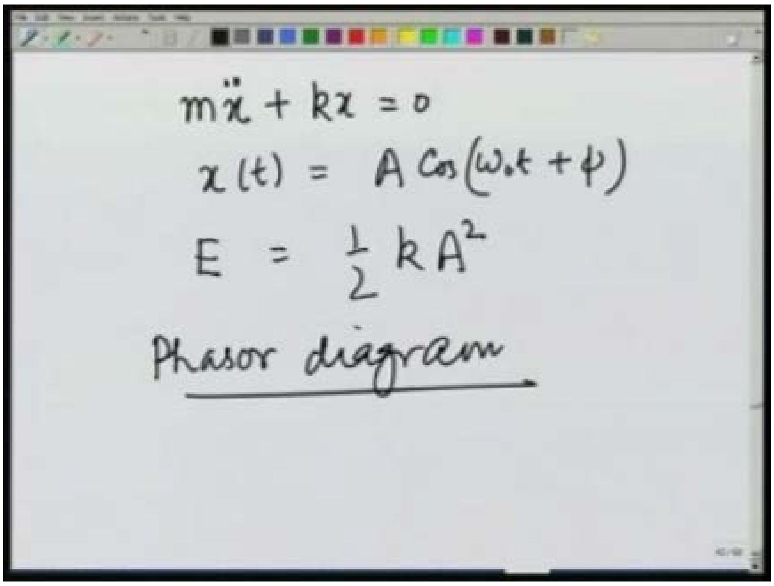

In general, about stable equilibrium point I would have for small displacement about that point an equation of motion like this. So, the motion is simple harmonic which is given like cosine omega naught t plus phi the particle goes back and forth. Or the system which has this energy this kind of motion goes back and forth about an equilibrium point. The energy of the system is 1 half k A square and motion can be described by a Phasor diagram which we will be using later in our analysis.