Engineering Mechanics
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Module - 07
Lecture - 06
Rotational Motion - VI

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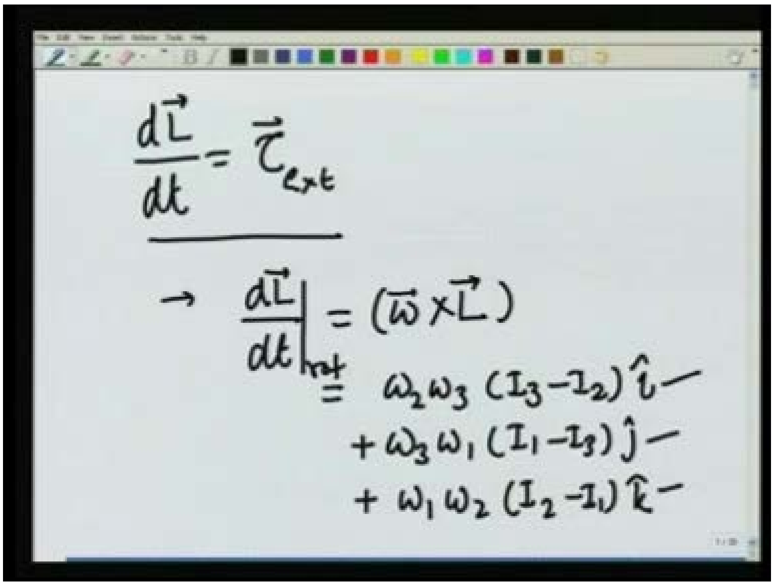

In the previous lecture, I have shown you that the Rigid body dynamics is described by this equation and we apply it to certain situations in which omega was a constant not only was it a constant its components along the principle axis were also constant in that case we saw that, dL dt was really nothing but, omega cross L only and this we wrote as omega2 omega3 I3 minus I2 i plus omega3 omega one Ione minus I3 j plus omega one omega2 I2 minus Ione k, where i j and k are the unit vectors along the principle axis of the body the principle axis; obviously, rotate with the body.

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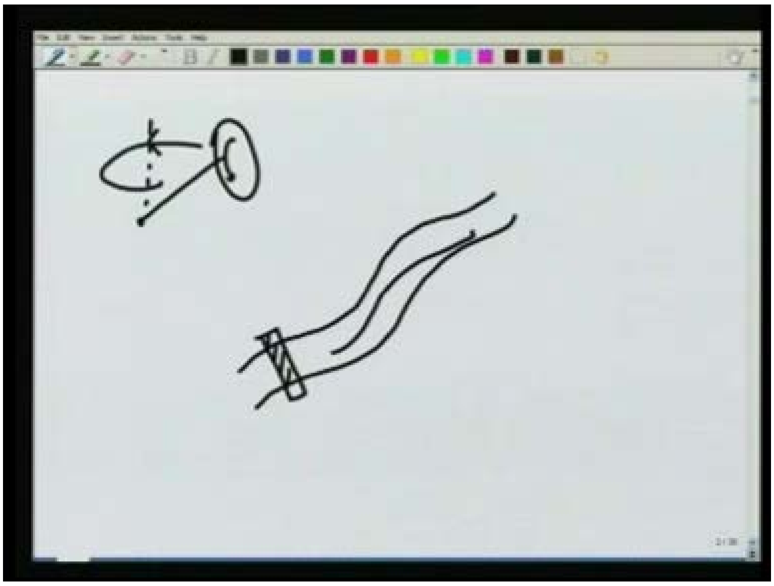

This was purely due to the rotation of the body in addition I, also explained 2 demonstrations to you one was the precession of a rotating wheel when you put this point on a stand, the wheel started also precessing about this axis I also, explained to you the gyroscope how the gyroscope spinning will aligns with the outside frame omega, before we make this lecture more quantitative.

Let me explain to you, another demonstration if, you recall we had demonstration where I had a track like this on which I let 4 cylinders row and we found that only one of the cylinders could really negotiate all the tracks and go all the way through. Let me show that demonstration to you again and then we will explain it.

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I have these 4 cylinders here of different shapes

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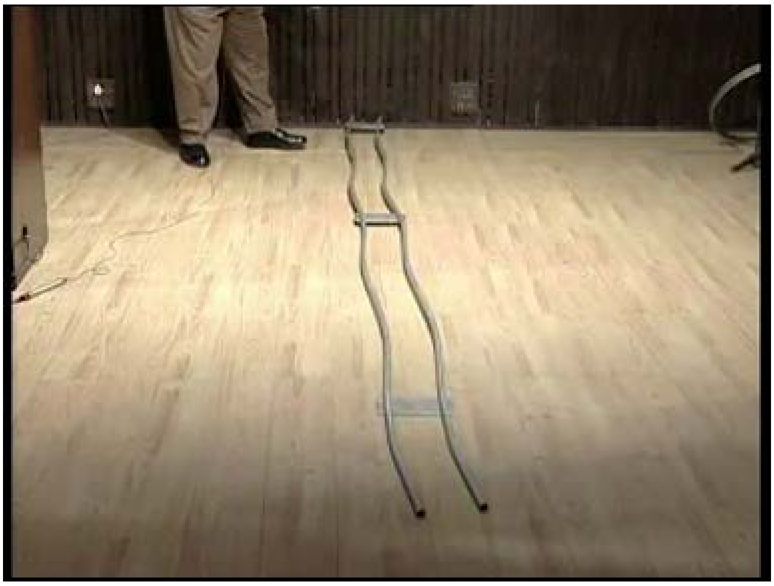

And what we see is that, if I role the straight cylinder down it does not really go through and falls of the track, it cannot really turn with the turning of the track.

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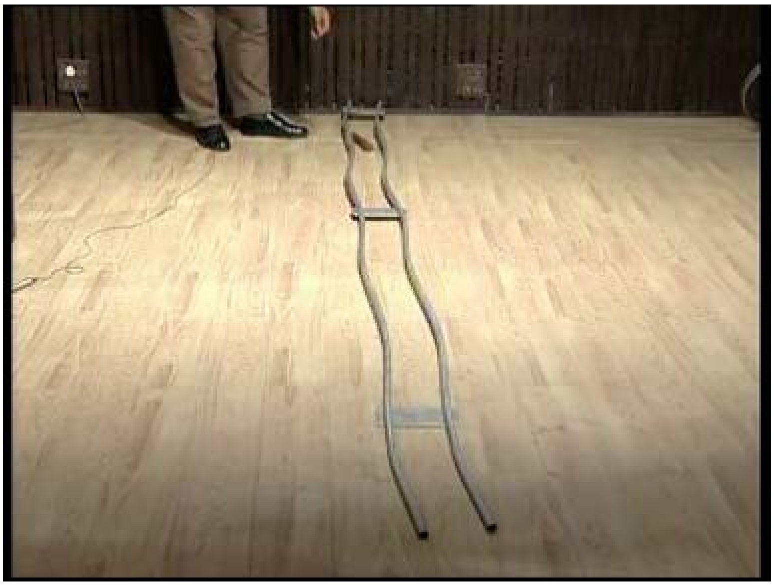

Similarly, if I take the cylinder which is shaped as you can see like this, if it goes it turns only in one direction.

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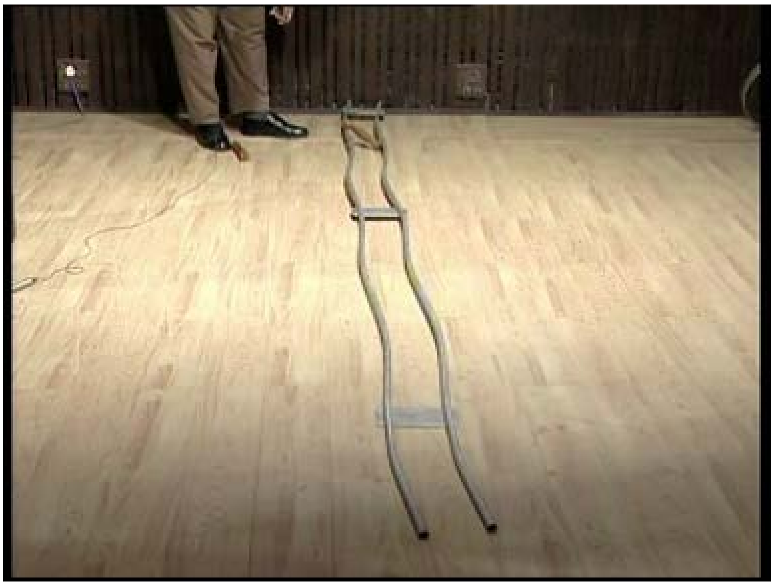

This one is also similar expect that we have given bulge in the middle and this also turns only to one direction and falls off the only cylinder that goes and negotiates curves in both directions is this one and let us see this.

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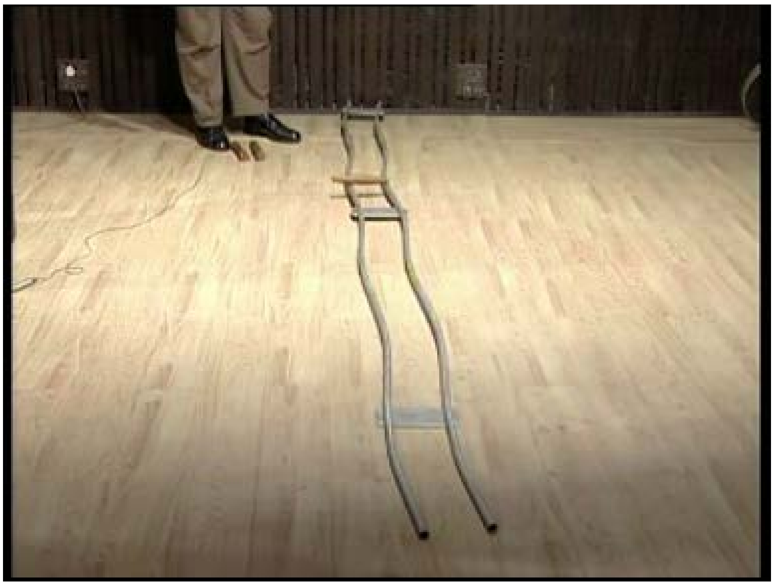

Let us, now try to understand why only one particular shape goes on the track all the way negotiating all turns both turning left and turning right and others cannot.

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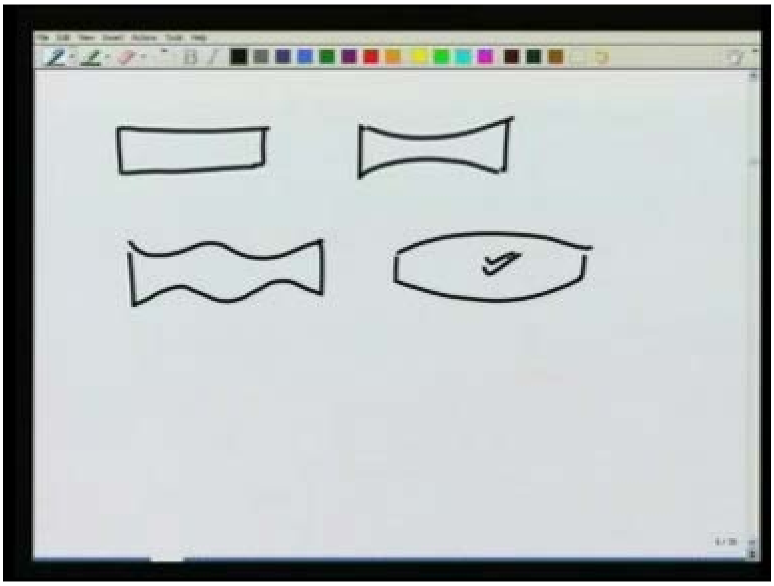

A 4 cylinders that I had, were one the plain one another one which was like this third one which was like this and the fourth one which was like this it was this cylinder that went all the way. Let us understand, why it happened? When the, cylinder is rolling on the track.

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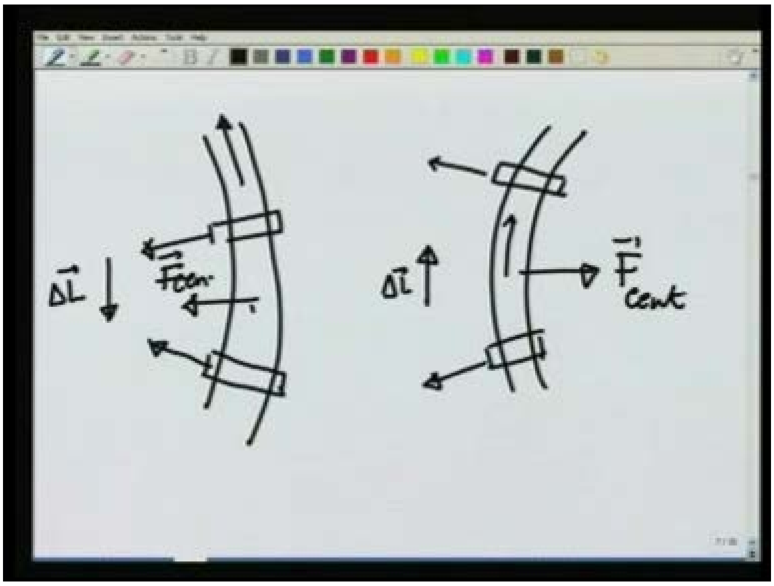

Let us take, the rolling direction to be this then when the cylinder is here its angular momentum L is in this direction as it goes around the track its angular momentum becomes this and therefore, there is a change in the angular momentum in the direction opposite to its motion on the other hand when it turns the other way if, the cylinder is here again I am taking cylinder moving this way then the angular momentum is in this direction and when it reaches here the angular momentum is in this direction and therefore, the change in the angular momentum is in the direction of the motion this is delta L delta L, on the other hand when it is turning on this track it requires a centripetal force in the direction to the left and on this track it requires a centripetal force the direction to the right.

So, that a center of mass moves properly on the track where do these forces come from? These forces are provided by the normal reaction on the cylinders due to the track and let us understand, how the interplay between these normal reactions and the torques they provide really help only one particular cylinder to negotiate all the curves.

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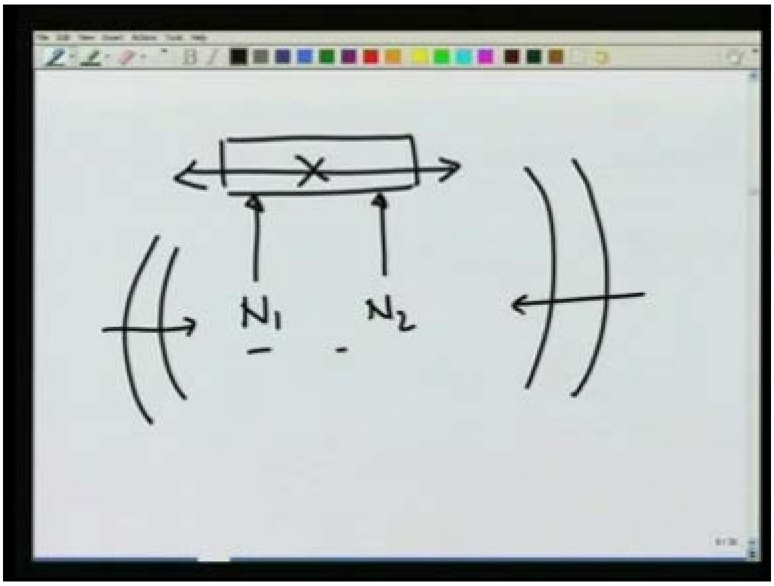

Let us first take, the example of a plane roller where the normal reaction is going to be like this perpendicular to the surface of the cylinder and therefore, whether the track is going this way or this way there cannot be any component in this direction there is no component of these 2 forces None and N2 and therefore, the normal reactions cannot provide a proper centripetal force and the plane cylinder does not turn at all it just goes straight.

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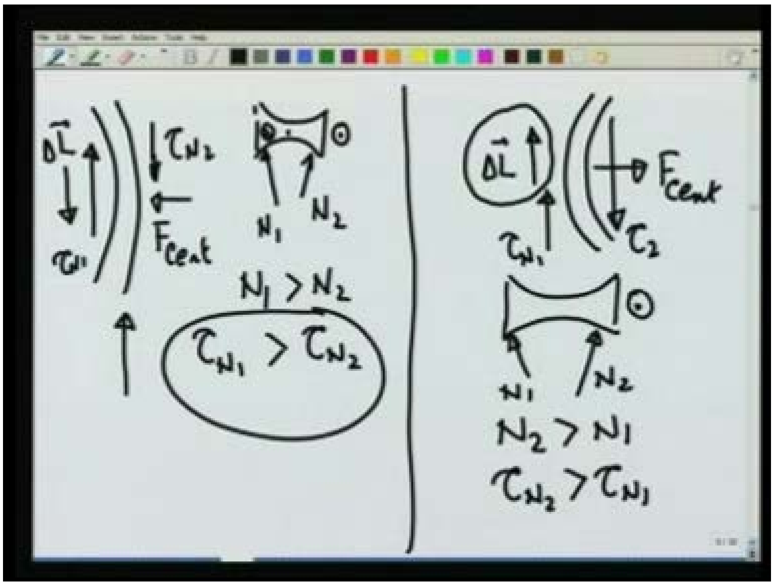

How about the cylinder which is shaped like this? The 2 are tracks to turns we are looking for are like this, in this case remember delta L is in the direction opposite to the direction of motion and this is F centripetal in this case, the change in the angular momentum delta L is along the direction of the motion and this is F centripetal. Let us look, at this cylinder from behind then the normal reaction on this side will would be None and on this side would be in this direction N two.

If, the centripetal force has to be to the left you can see in this case None has to be greater than N2 and therefore, the center of mass can turn to the left. on the other hand if, None is greater than N2 the torque due to None is also going to be greater than torque due to N2 assuming they are equidistance from the center of mass. However, torque due to None here goes into the cylinder and on this side it comes out into the cylinder here means I am looking from behind is along the track in the direction of motion this is tau None. And this is tau N2 tau N2 is smaller than tau.

Therefore, the net torque would be in the direction of motion which is opposite to the required direction for delta L to change in the direction opposite to motion. And therefore, this cannot make the roller turn this way and therefore, it is cannot negotiate the curve. Let us see, it in the other case if, it has to turn to the right then this is N2 this is None in this case since the centripetal force is to the right N2 has to greater than None. And therefore, torque due to N2 would be greater than torque due to None torque due to N2 as we saw earlier is coming out of this plane.

Therefore, torque due to N2 is in this direction. And this is larger than torque due to None. Again you see that tau2 minus tauN2 minus tau. None is in direction opposite to that required to cause this change in delta L. And therefore, it cannot negotiate this term turn going to the right also therefore, this cylinder does not go over the track properly.

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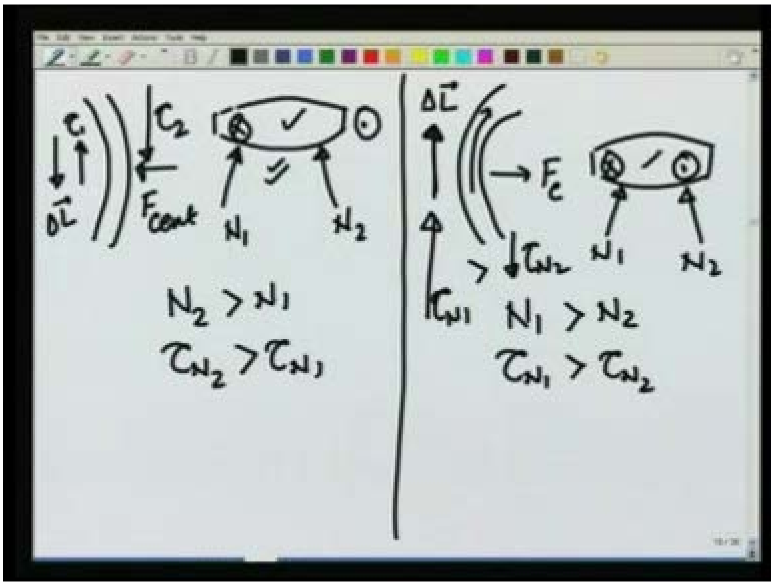

Now, we come the cylinder which went all the way to the track and which is shaped like this again remember delta L direction is this therefore, torque has to be in the direction opposite to the motion. And this is F centripetal and on the track turning to the right this is F centripetal and delta L direction therefore, the torque direction has to be along the direction of the motion. Now, in this case None and N2 the normal reactions are going to be like this and since the centripetal force is to the left N2 should be greater than None.

Therefore, torque due to N2 is also going to be greater than torque due to None, but torque due N2 is coming out torque due to None is going in looking from the backside torque due to None is in this direction torque due to N2 is in this direction. And this is greater than torque due to None. So, the difference is in the direction which is proper for to cause this change in delta L. And therefore, this cylinder can easily turn to the left.

Let us see, what happens? When it is negotiating a curve going to the right. In this case again making the normal reactions None and N2 we see that F centripetal is to the right and therefore, None should be greater than None; that means, torque None it is also greater than torque due to N2 torque None is going N2 the cylinder that is in the direction of motion. And torque N2 is coming out and therefore, this torque is greater than torque in the opposite direction and you can see again that the difference is giving the right direction for delta L to be along the direction of motion. And therefore, this cylinder can negotiate both the turns going to the left or going to the right and as you saw in the demonstration it went all the way through.

So, this demonstration is essentially to show you, an interplay between forces change in the angular momentum and the required centripetal force that can make particular roller go all the way along a track and some other rollers not along the track. Let me now go to the general case and ask what happens? If the angular velocity of a rigid body rotating also changes.

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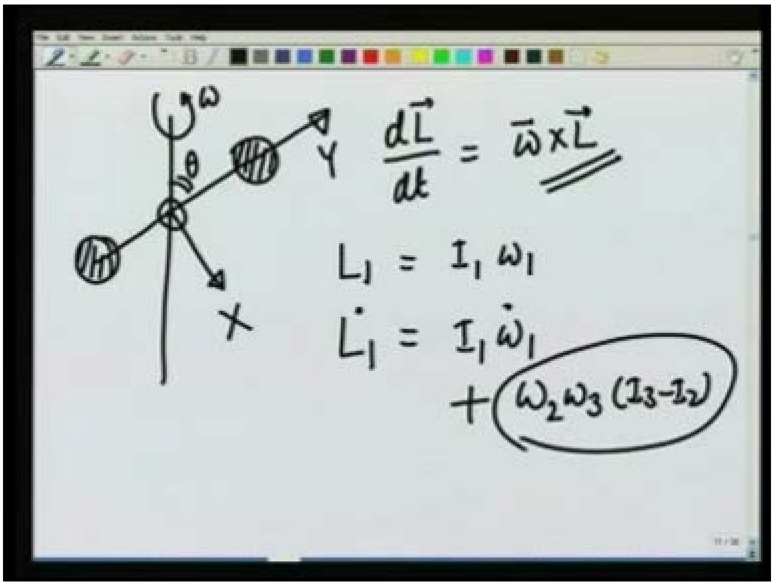

For this, we again go back to our old friend a rod with 2 masses on its side rotating about an axis at angle theta. Recall earlier that, we had solved this problem with dL over dt equals omega cross L, because omega was not changing with time in addition omega had constant components along the principle axis of the body which I took to be x in this direction y in this direction and z therefore, coming out. Let me now see what happens? If, I also change this omega with time in that case recall that, the component along the x direction is Ione omega one

So, if I change the omega the angular velocity omega one is going to change and therefore, Lone dot is equal to Ione omega one dot. In addition there is a change in the component of angular momentum in direction one, because of this rotation which we earlier wrote as omega2 omega3 I3 minus I2. This you recall, we had been using. So, there are 2 sources of change in the component of angular momentum along direction one, one because angular velocity itself is changing and 2 because L is not in the direction of omega and therefore, it rotates about the angular velocity

So, notice Lone itself is only Ione omega one, but when we take its change it is caused both by the change in omega and by the rotation of L vector around omega vector. And therefore, in general I am going to have Lone dot which is same as dL one dt is equal to Ione omega one dot plus omega2 omega3 I3 minus I2.

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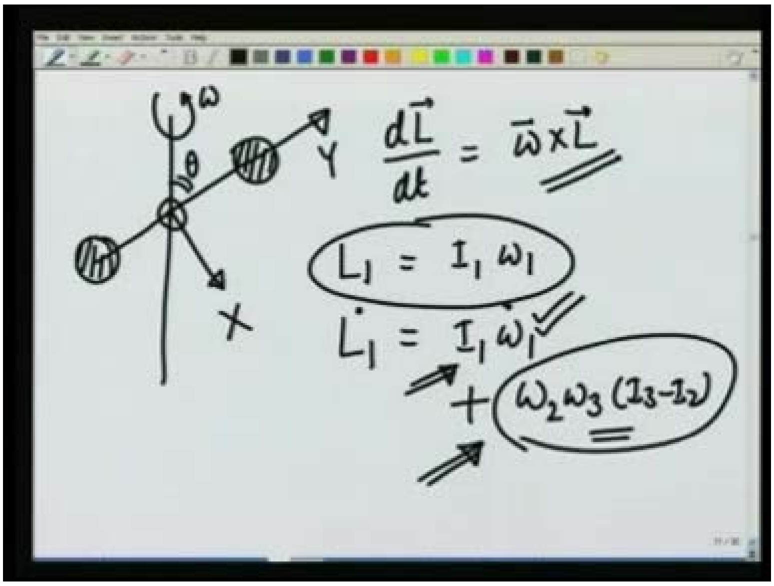

Similarly, L2 dot the change in direction 2 would be caused by change in the component of angular velocity along direction 2 plus because L is rotating that change anyways caused. So, omega3 omega one Ione minus I3 same thing for L3 is going to be I3 omega3 dot plus omega one omega2 I2 minus Ione although Lone is equal to Ione omega one L2 is I2 omega2 and L3 I3 omega3, because as the body rotates L also rotates somewhat omega that, causes these changes.

In addition the change of omega itself causes these changes the net is the sum of these 2 and therefore, this is how the components of angular momentum along the 3 axis change. This obviously, then must be equal to the component of torque along principle axis one this must be component of torque along principle axis 2 and this must be component of torque along principle axis 3.

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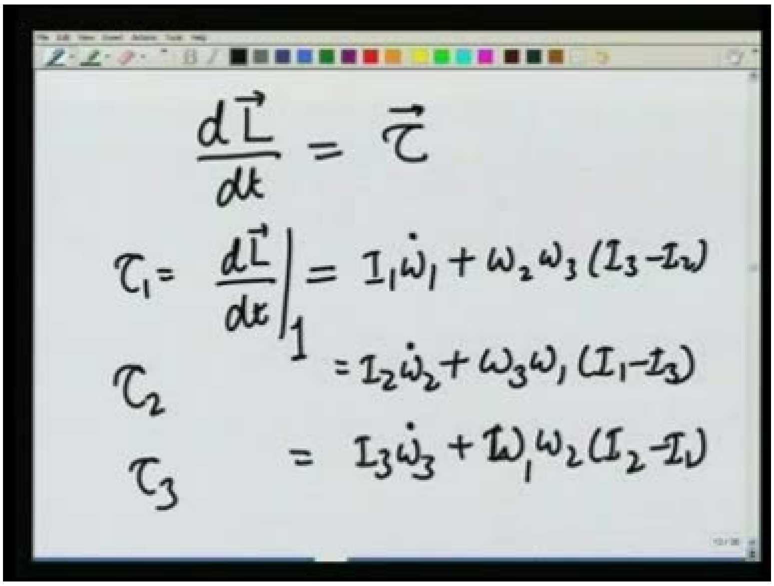

These equations are nothing, but writing dL over dt is equal to tau by taking components along the principle axis principle axis of the body. So, tau one should be dL one dL over dt along principle axis one. And that comes out to be Ione omega one dot plus omega2 omega3 I3 minus I2 similarly tau 2 comes out to be I2 omega2 dot plus omega3 omega one Ione minus I3. And tau 3 comes out to be equal to I3 omega3 dot plus I omega one omega2 I2 minus I one.

I remind you again that, although Lone is Ione omega one L2 is I2 omega2 and L3 is I3 omega3 the change in these components involves both the change in omega component itself, as well as a change due to the rotation of L vector around the omega vector and therefore, the net change happens to be the sum of these 2. These are known as Euler equations it is because of this turn that; the solution becomes very interesting and this is what we are going to see. As you had already seen, in some demonstrations because of the distribution of velocities, because of the angular momentum the motion of rigid body is can sometimes a very counter intuitive.

These equations involve all that, in this if you solve them and interpret them properly we get all the answers. I, also want to remind you that you had been solving the Euler equations or we had been solving the Euler equations in the previous lectures without formally calling them so.

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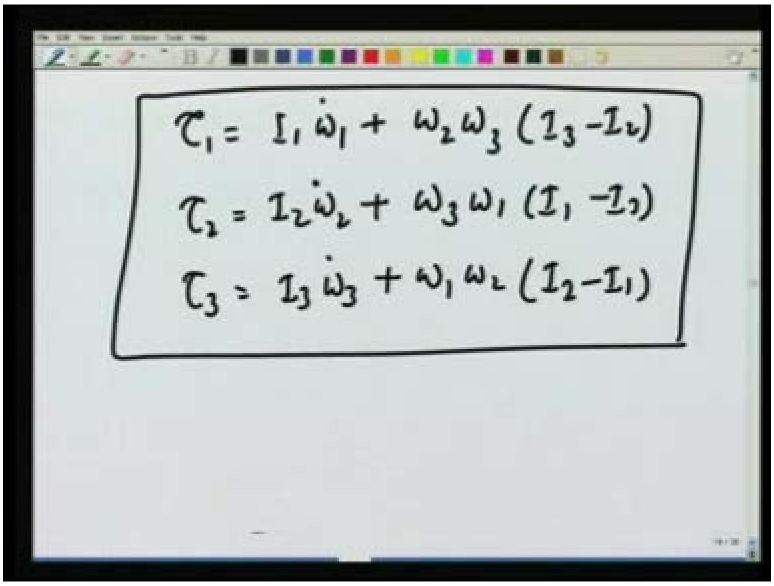

So, let me write equations again Tau one equals I one omega one dot plus omega 2 omega 3 I3 minus I2 tau 2 equals I2 omega 2 dot plus omega 3 omega one Ione minus I3 and tau 3 equals I3 omega3 dot plus omega one omega2 I2 minus I one where the directions 1, 2 and 3 refer to the principle axis of the rigid body. And omega one omega 2 omega 3 are the components of the angle of the angular velocity along those axis and omega one dot omega 2 dot omega 3 dot are the components of change in omega along this axis. We will, now apply these equations to some other examples and see what we get.

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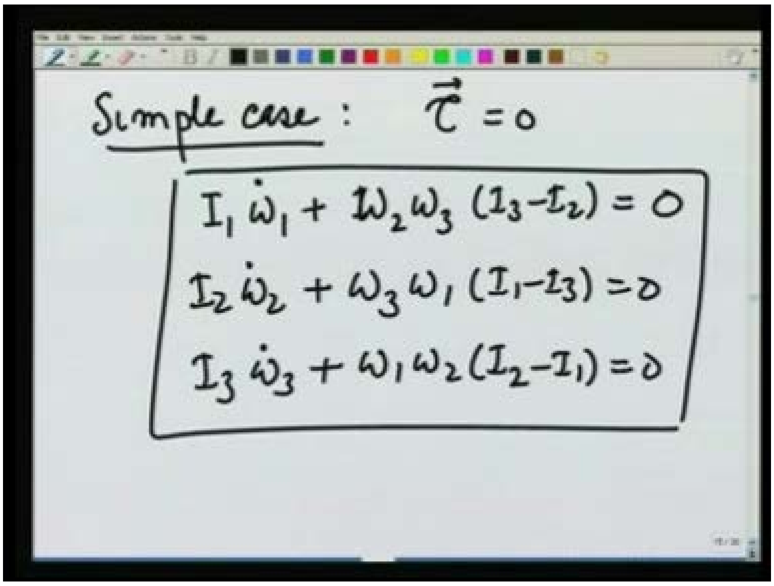

The first simple case that, I take when the external torque is 0 if, the external torque is 0 then the equations that, we should be solving are Ione omega one dot plus omega2 omega3 I3 minus I2 is equal to 0. I2 omega2 dot plus omega3 omega one I one minus I3 is equal to 0 and I3 omega3 dot plus omega one omega2 I2 minus Ione is equal to 0. This would be the case for example.

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If, I take this Box and drop it down after giving it some rotation about the center of mass there would be no torque because the gravitational force is acting on the center of mass. So, it will be a torque free rotation. Let us see, what do the equations predict for us in the case of this Box we can see there are 3 principle axis, one axis is going like this.

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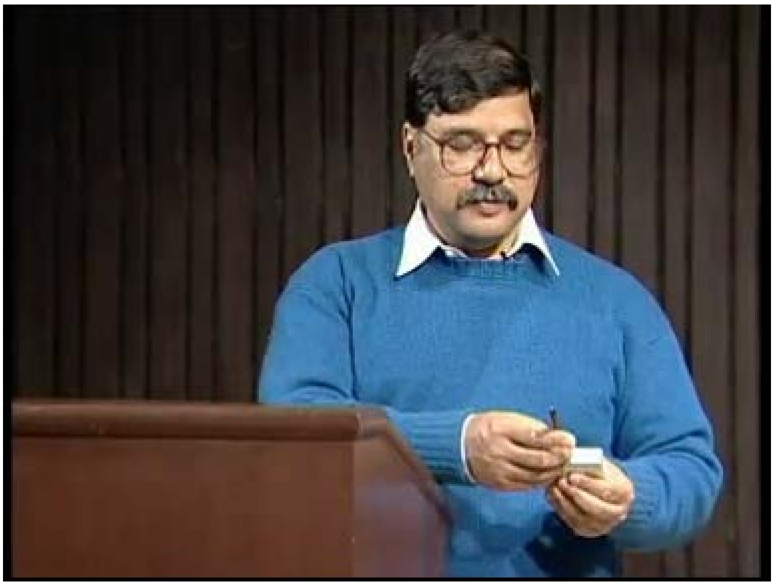

And one is like this.

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And one is perpendicular to this.

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The Ione, I2 and I3 all 3 are different in this case. So, let us see, what do these equations predict?

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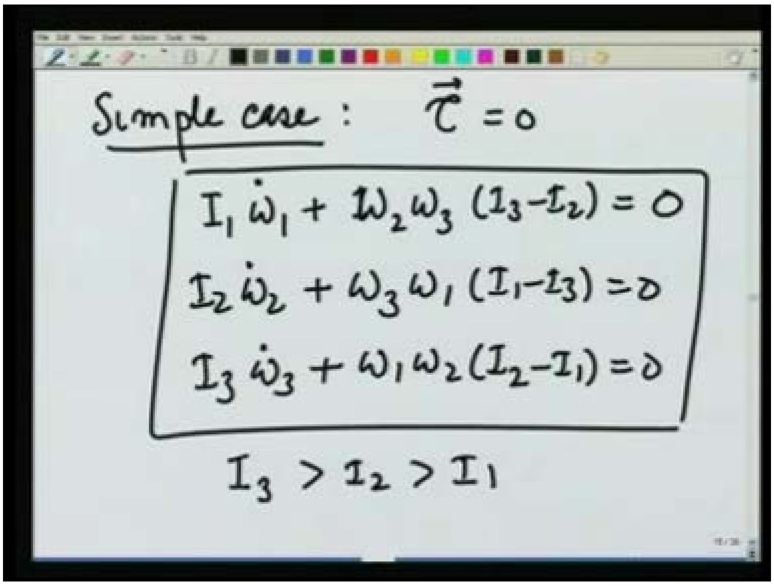

For simplicity, I will take I3 to be greater than I2 to be greater than Ione it really does not matter because I, can just name the axis about which the moment of inertia is a largest as axis number 3.The one which is intermediate as axis number 2 and one which is smallest moment of inertia as axis number one.

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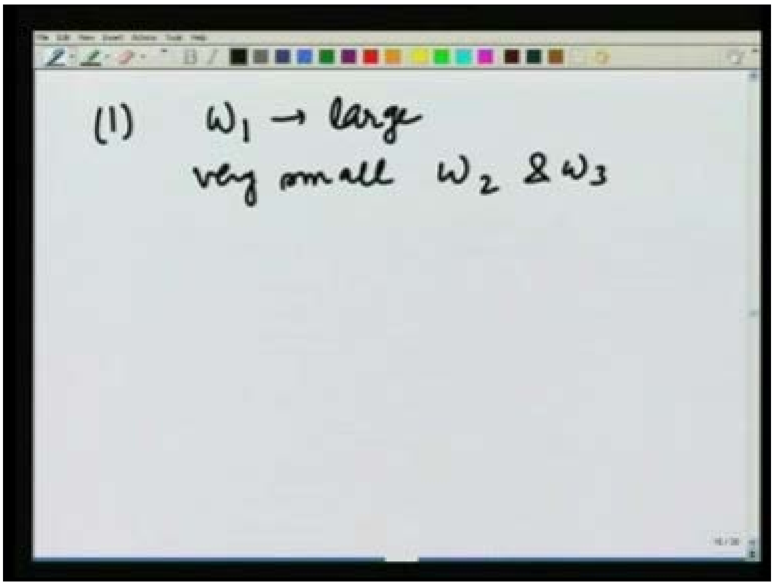

So, let us first take the case if, I give it a spin omega one large and very small omega2 and omega3. That is, I give it a spin about the axis about which the moment of inertia is smallest and that probably would be this axis. And I give it a rotation.

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Along the way I can give some rotation because, I am spinning it about axis 2 and axis 3 also and let it go and see what happens? In this case, the equations would be Ione omega one dot plus omega2 omega3 I3 minus I2 is equal to 0. Since, omega2 and omega3 are very small I can in the first approximation neglect this remember I am just trying to give you qualitative picture of what is happening? And therefore, I can write Ione omega one dot is equal to 0 or omega one is a constant.

Therefore, if I give it a large spin about the axis around which the moment of inertia is smallest it will go with constant angular speed about that point it'll keep on rotating constantly. What happens to omega2 and omega3 which are very small to start with?

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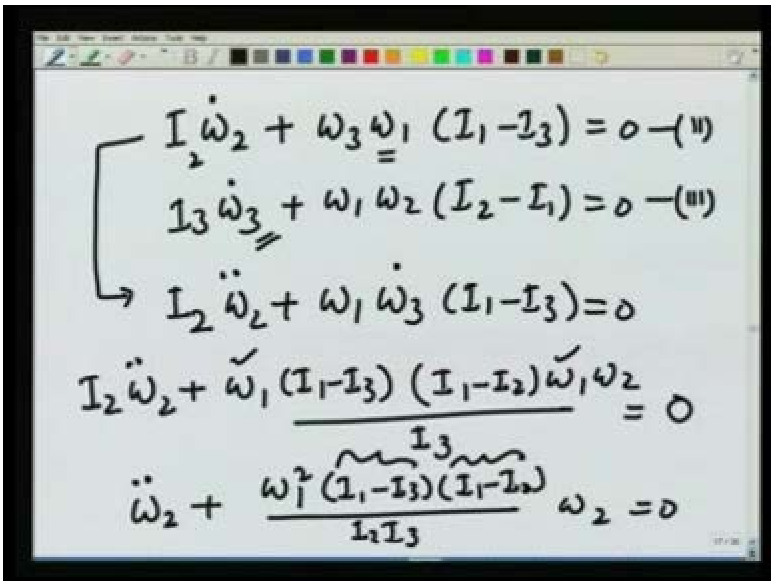

Let us see that so, the equation for omega2 and omega3 is I2 omega2 plus omega3 omega one Ione minus I3 is equal to 0 and I3 omega3 dot plus omega one omega2 I2 minus Ione is equal to 0. Let us differentiate this once more. So, that we write this in entire equation in terms of omega2 alone I would have I2 omega2 double dot plus omega one is already shown to be a constant omega one omega3 dot Ione minus I3 is equal to 0 this is equation number 2 this is equation number 3.

Let us substitute for omega3 dot from equation number 3 to get I2 omega2 dot plus omega one Ione minus I3 and I substitute for omega3 dot it will be minus sign. So, I will write this as Ione minus I2 omega one omega2 divided by I3 is equal to 0. So, the equation I get is omega2 double dot plus omega one square I multiplied this and this Ione minus I3 times Ione minus I2 divided by I3 omega2 that is equal to 0. This is the equation that, determines time evolution of omega2 notice.

Since, Ione is the smallest moment of inertia Ione minus I3 is negative and. So, is Ione minus I2 2 negative quantities multiplied would give me positive number. And therefore, I should also write I2 here because I have divided by I2.

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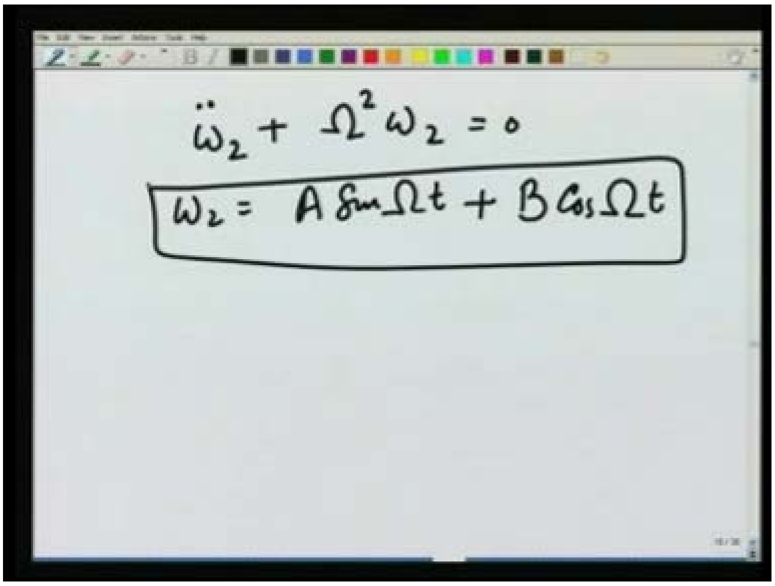

Therefore, I get the equation for omega2 as omega2 plus some positive number. Let us call it omega square times omega2 is equal to 0 and the solution for this is I know is omega2 equals some constant A sin omega t plus B cosine of omega t. So, as the body would fall omega one would be constant omega2 would be just changing periodically how about omega3 I leave it as an exercise for you show that, omega 3 would also change in the similar manner sin omega t plus cosine omega t with some constants in the front. Again using the fact that Ione is the smallest of the moment of inertias and therefore, if I give it a spin about axis one omega2 and omega3 would change slightly they will change periodically.

So, it will shake a bit, but it will move with constant angular speed about axis one. This also the second part I will, leave as an exercise for you if initial spin is given about I3. That is; largest moment of inertia axis you can again show that omega3 would remain a constant and omega one and omega2 would be changing in this manner.

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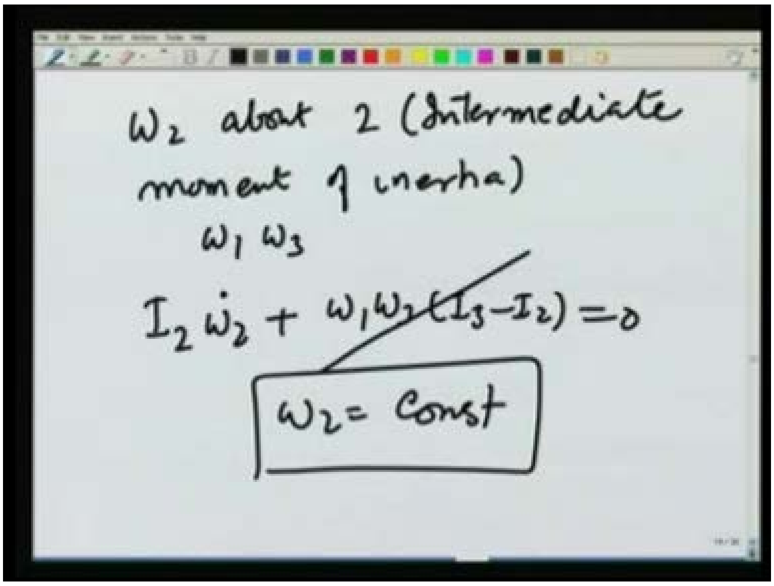

Let us see, now the case when I give omega2 about axis 2 which is intermediate moment of inertia axis and the some small omega one and omega3. So, again we will see that I2 omega2 dot plus omega one omega3 I3 minus I2 is equal to 0 this I can neglect in the first approximation.

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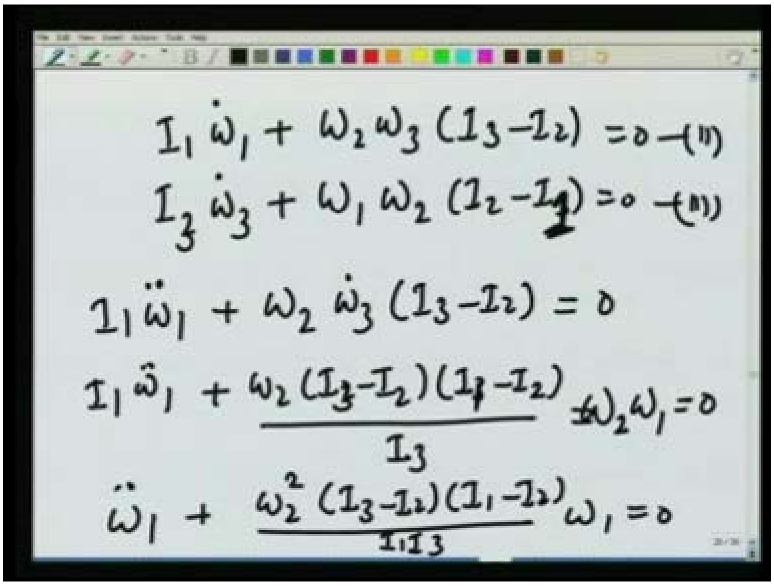

So, omega2 is a constant how about omega one and omega3 I would have Ione omega one dot plus omega2 omega3 I3 minus I2 is equal to 0 and I3 omega3 dot plus omega one omega2 I2 minus I3 is equal to 0. Again differencing this equation once more and substituting for omega3 from here we get Ione omega one double dot plus omega2 we have already shown is a constant omega2 omega3 dot I3 minus I2 is equal to 0 substitute for omega3 dot to get Ione omega one double dot plus omega2 I2 minus oh sorry I3 minus I2 times.

This I can change sign when I go to the right side I can write this as I3 minus I2 sorry this should be I2 minus Ione. So, this should be Ione minus I2 divided by I3 is equal a times omega2 omega one is equal to 0 or omega one double dot plus omega2 square I3 minus I2 times Ione minus I2 divided by Ione I3 omega one is equal to 0. So, what we did is really substituted for omega one double dot omega one dot we what we did really is we differentiated equation to once more and substituted for omega3 double dot from equation number 3.

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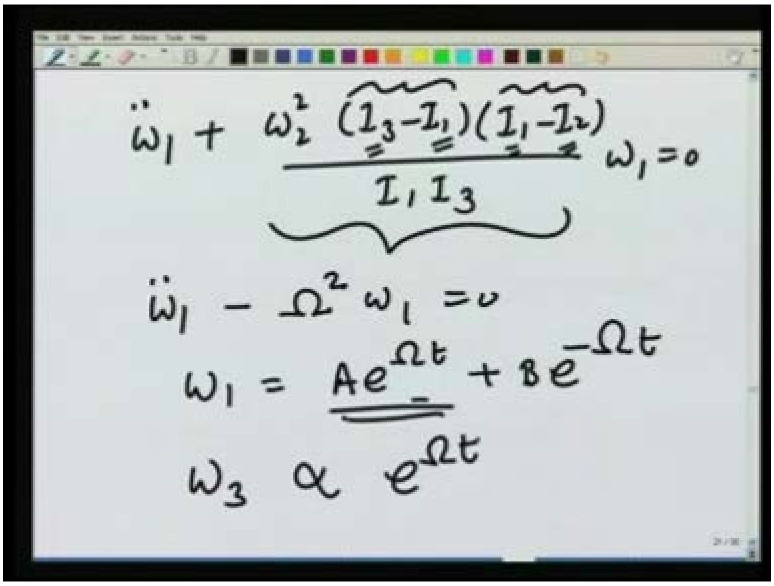

And the final equation we get let me, rewrite it is omega one double dot plus omega2 square which is a constant I3 minus Ione times Ione minus I2 divided by Ione I3 omega one is equal to 0. Since Ione is the smallest and I3 is the largest this number is positive and Ione is the smallest and I2 is intermediate this number is negative. And therefore, this entire quantity is negative. Let me write this as omega one double dot minus omega square omega one is equal to 0. And the solution for this is of the form A e raise to omega t plus B e raise to minus omega t.

So, whereas, earlier when we gave a spin about the axis which has the, smallest moment of inertia or the largest moment of inertia the other 2 angular moment angular velocity components were just changing periodically slightly. Now, omega one grows exponentially you can similarly show that omega3 would also be proportional to e raise to omega t, I leave this as an exercise. And therefore, if I give it a spin about axis number 2 which is the intermediate moment of inertia axis omega one and omega2 would go and you will see that the body after sometime does not really rotate about only one axis or it will rotate about all 3 axis.

Let us see this, I have shown this demonstration to you, in a lecture 2 or 3 lectures earlier, I will show it you again now.

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So, what we predicted is if, I give it a rotation about axis about which the moment of inertia is smallest it will go without really developing any angular velocity about the other 2 axis.

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Let me show that; in slow motion, see it goes down straight.

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Similarly, if I take the axis about which the moment of inertia is the largest which should be axis like this perpendicular this flat plane. In that case also we saw that, the angular speeds do not really develop much and you see that it comes down flat.

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On the other hand, if I take the intermediate axis.

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As it rotates the other 2 angular speeds also gain and you will see as it falls down it will start rotating.

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In a manner which will give it rotation about all 3 axis.

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You see develops angular speed about all 3.

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Now, let me take another example of torque free rotation which you observe almost every day, it is a spinning plate moving or a spinning coin what you notice when it moves or a frisbee which is given a spin that it wobbles as it moves and let us understand that.

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Let us see the wobbling of the frisbee first.

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Let us, now see this in slow motion you see the motion when it moves is like this.

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And we will try to analyze this further. So, I am going take.

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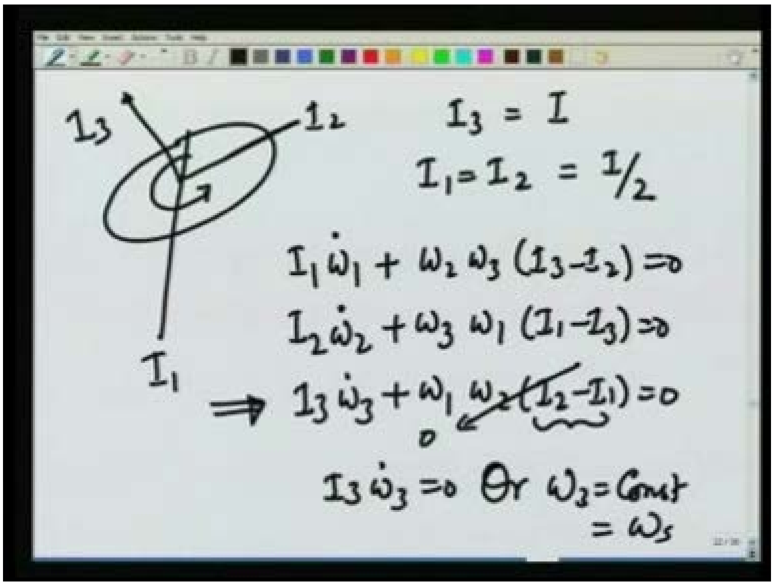

A disc; obviously, its principle axis are the diameters and the perpendicular. Let me call this Ione I2 and I3 the 3 axis I3 is equal to let me call this, I and Ione and I2 are equal and the case of a disc they are I over 2. Since this is again torque free motion about the center of mass the equations would be Ione omega one dot plus omega2 omega3 I3 minus I2 is equal to 0 I2 omega2 dot plus omega3 omega one Ione minus I3 is equal to 0 and I3 omega3 dot plus omega one omega2 I2 minus Ione is equal to 0.

Let me give, the disc a spin about axis 3 and may be little bit of spin about one and 2 and see what happens? From this equation we see that since I2 and Ione are equal this term is 0 and therefore, I3 omega3 dot is equal to 0 or omega3 is equal to constant. Let me call the angular spin angular speed. So, it is spinning with a constant angular speed what about omega one and omega 2?

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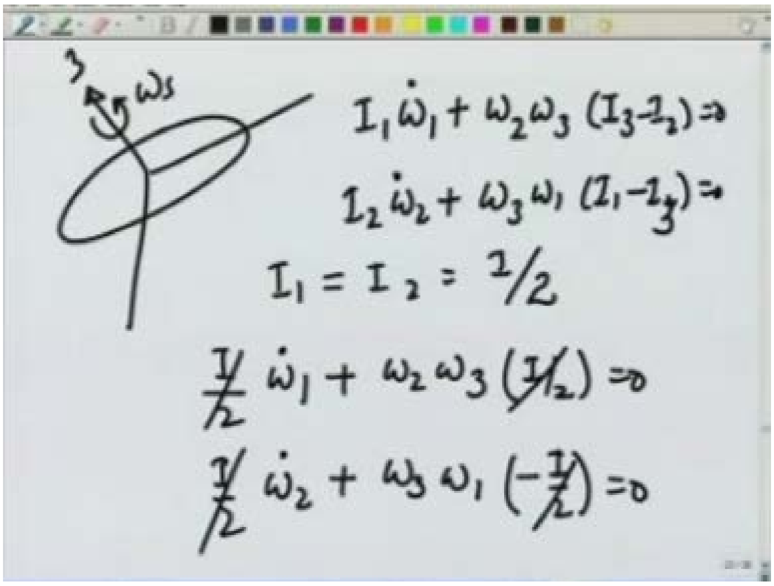

Let us see that. So, remind you again I am considering a disc this is axis 3 axis one axis 2 it is now given a spin about axis 3 and it rotates with constant spin what happens to omega one and omega2 that is what we are trying to find out. So, I 1 omega one dot plus omega2 omega3 I3 minus I2 is 0 and. So, is I2 omega2 dot plus omega3 omega one Ione minus I 2. Since Ione is equal to I2 is equal to I3 which I am calling I divide by 2 I can write these equations as I divided by 2 omega one dot plus omega2 omega3 I3 minus I2 would be I divide by 2 again is equal to 0.

And the other equation as I over 2 omega2 dot plus omega3 omega one Ione minus I2 or this should be Ione minus I3 sorry Ione minus I3 would be minus I divided by 2 is equal to 0. This is common.

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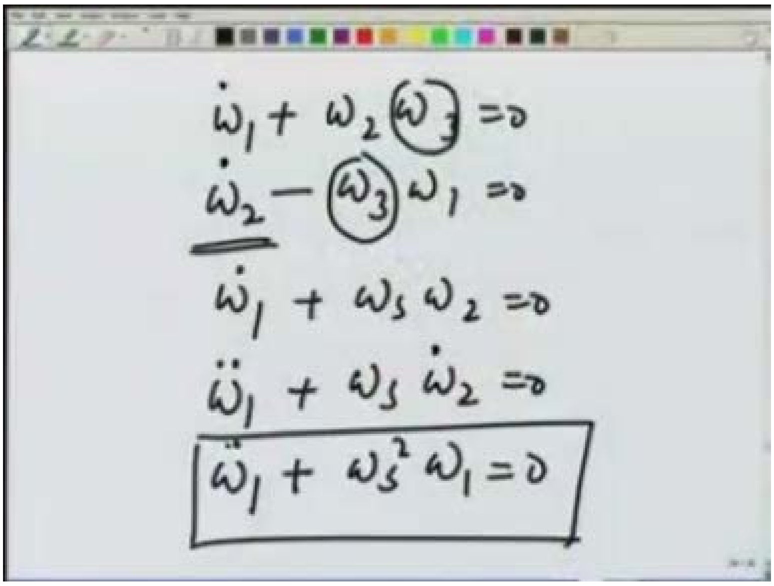

Therefore, the equations I have are omega one dot plus omega2 omega3 is equal to 0 and omega2 dot minus omega3 omega one is equal to 0 omega3 we are calling omega spin which is a constant And therefore, I have omega one dot plus omega spin omega2 is equal to 0 which I differentiate once more to get omega one double dot plus omega spin omega2 dot is equal to 0. And omega2 dot from this equation is can be substituted as omega spin times omega one and therefore, I have omega one double dot plus omega spin square omega one is equal to 0. This is the equation that, determines omega one and you see this is the harmonic oscillator equation.

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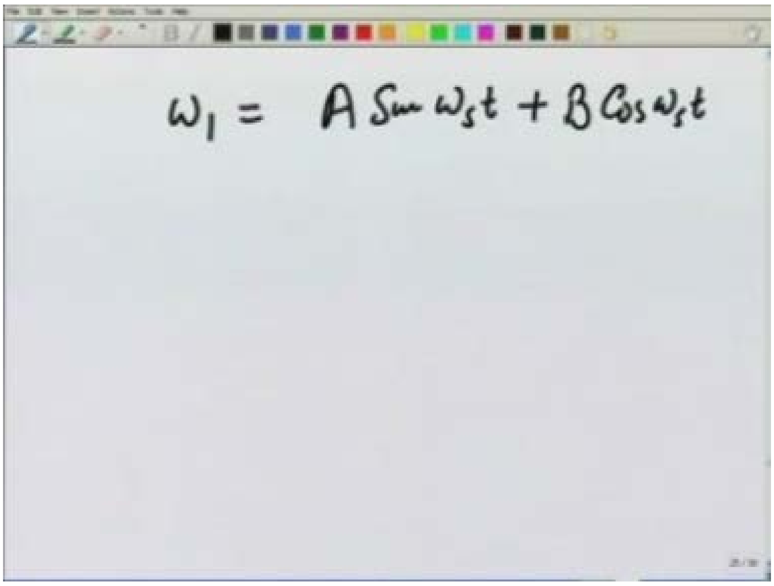

Therefore, the solutions are that omega one should be changing as some constant sin of omega s t plus some other constant cosine of omega st.

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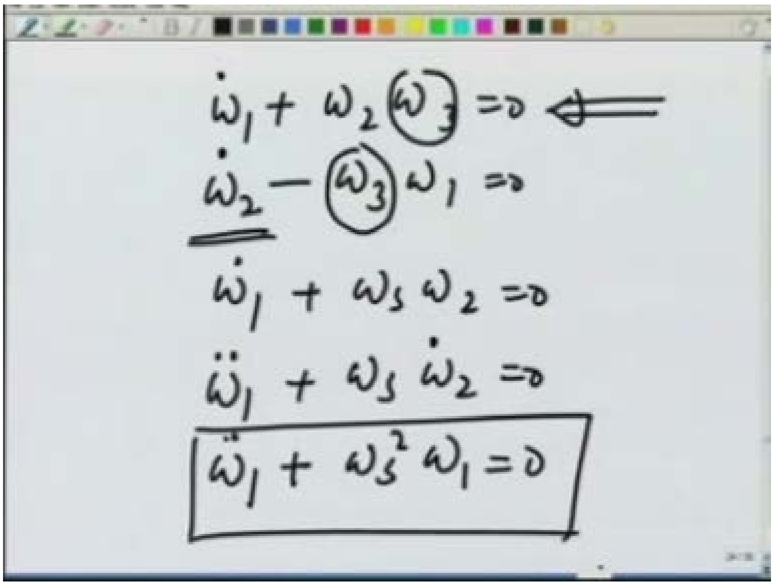

Similarly, I can determine omega2 either by differentiating again and solving the same equation or from this equation omega2 is nothing but…

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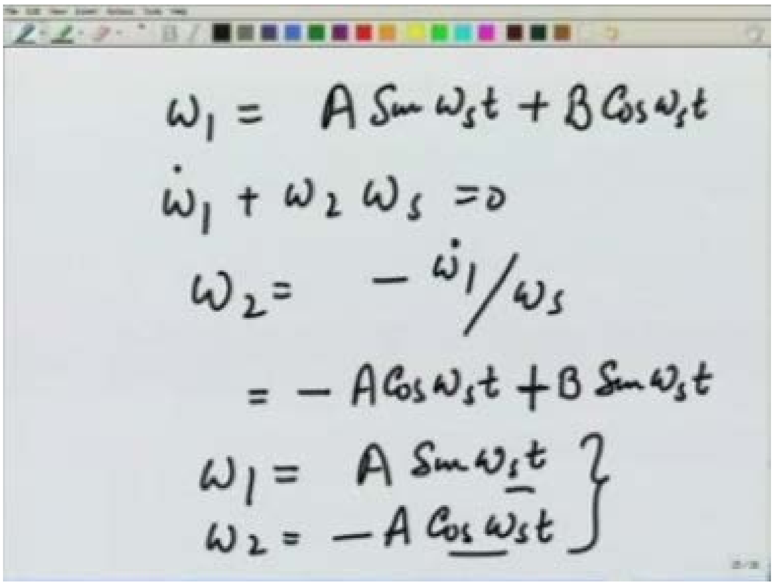

Since, omega one dot plus omega2 omega spin is equal to 0 omega2 is nothing but, minus omega one dot divided by omega spin and this therefore, becomes minus A cosine of omega st minus B sin of omega structure. And minus, minus becomes plus I can always choose my time in such a manner that; omega one is like A sin omega s t that is at time t equal to 0 omega one is 0. And omega2 is equal to minus A cosine of omega s t. So, omega one and omega2 are actually changing periodically.

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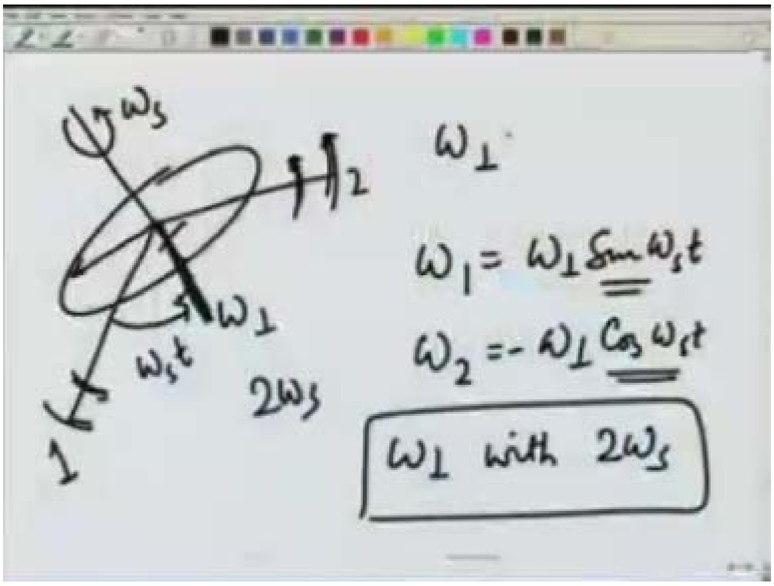

Let us see what does it mean? Here is axis 3 which is rotating at omega s here is axis one axis 2. So, with respect to these axis this component of omega in the plane one, 2 is changing with time with omega s t with respect to these axis which are also rotating so; that means, this must be rotating little faster how much faster? These axes are rotating already with omega s so; that means, the way omega perpendicular rotates. Let us call this omega perpendicular component perpendicular to the third axis rotates is rotating really at 2 omega s.

Let us try to understand that again, I have omega one is equal to some constant omega perpendicular sin omega st and omega2 is equal to omega perpendicular some constant cosine of omega st this is with the minus sign. So, this actually could have been on that side as time passes this develops a component of sin omega st and cosine omega st along one and 2. And these are already rotating with omega s; that means, in outer space in from the outside frame omega perpendicular must be rotating with 2 omega s

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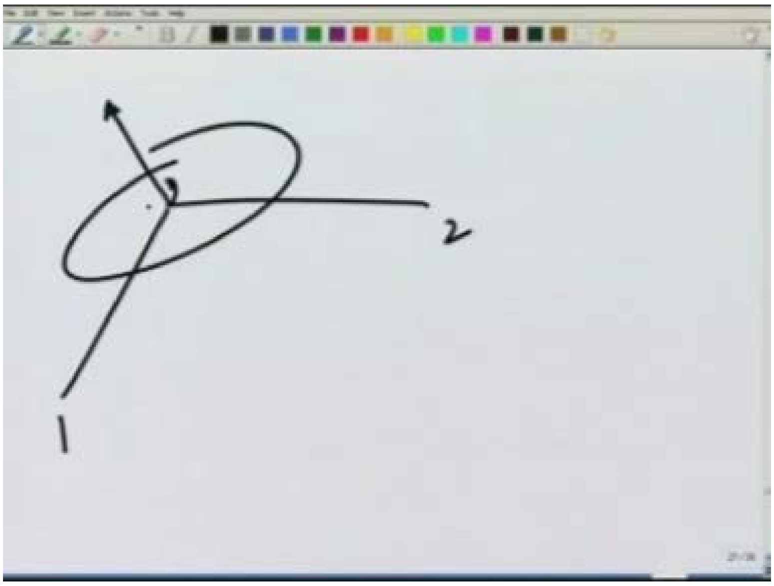

That means; omega the angular speed in this case this is one this is 2 is really about the third axis like this; third axis omega is fixed it is spinning.

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The plate is spinning like this and it is also rotating about axis one and 2 and this axis rotates you can see it sort of wobbles.

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As it move moves around, having solve the equations mathematically and indicated to you that, wobbling takes place. Let us, analyze the situation more carefully.

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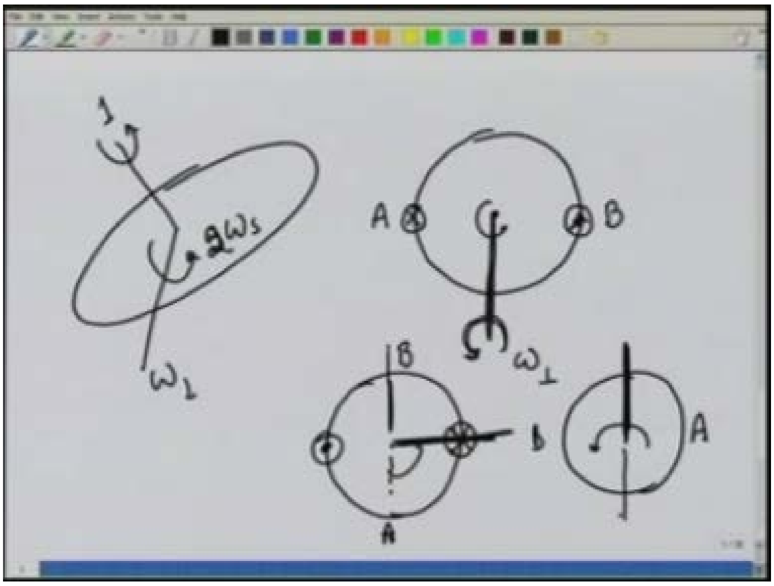

what I have shown you is if, there is this disc then if, it is spinning about this axis which we have been calling one omega perpendicular rotates about one with rate 2 omega s and because it is precessing about one at twice the spinning rate that is what results in wobbling. Let us see, how that takes place?

Let us, look at this disc from the top this is a spinning axis and let us say at this time it is rotating about this axis with omega perpendicular and it is; obviously, spinning like this. Let us call this point A and this point B at this time point A would be going into the plane and point B would be coming out of the plane. Because of this rotation in situation 2 by the time due to spin the this particular axis reaches here that is rotates by pi by 2 the rotation is taking place about this axis at this point, this is point A and this is point B.

So, that it is this point which is going down and this point which is coming up. Let us see the third situation by the time this axis reaches here. So, that this is A and this is B the disc is again rotating about this point. So, there is a mismatch between the spinning and the perpendicular motion and that is what gives rise to wobbling. Let us see that, through this frisbee again.

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Let us, take this line about which the disc initially rotating with omega perpendicular and it is spinning like this.

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What I just now showed you is by the time the spinning, this initially spinning axis reaches here…

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So, initially the disc is let us say, it is rotating like this.

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And it is spinning like this.

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By the time this reaches here the disc is rotating about this point. So, it is going down like this.

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By the time it reaches here the disc is rotating about this point and it is going like this again.

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And therefore, there is a motion which takes place like this.

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And that is the, wobbling motion. Let us now, look at the motion in slightly different wave as to how the disc rotates in space.

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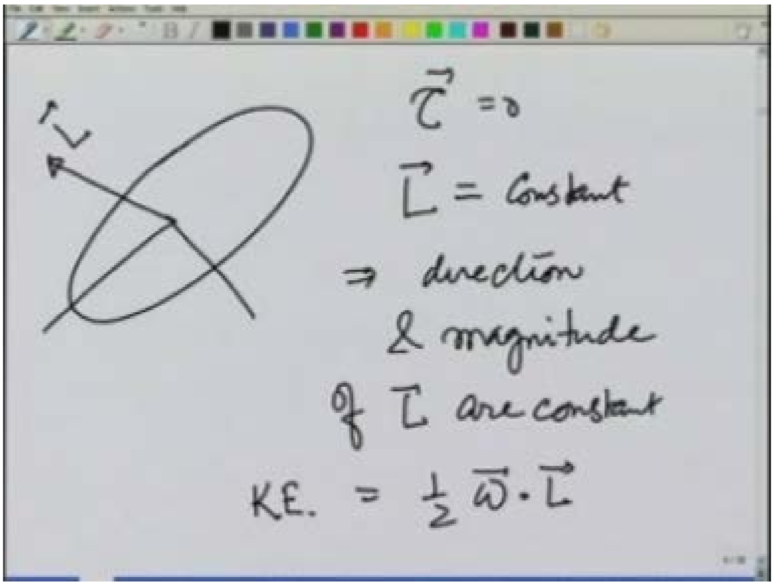

What I, have shown you is that this disc is given a spin and it can also rotate about the axis in the plane parallel to the disc. Since, it is a torque free motion the angular momentum L is a constant and this means; the direction and magnitude of L are constant. So, if I give an angular momentum initially L this is going to remain fixed in space. The other quantity which is a constant is the kinetic energy which is one half omega dot L and these two are enough to tell us how the motion of the disc should be.

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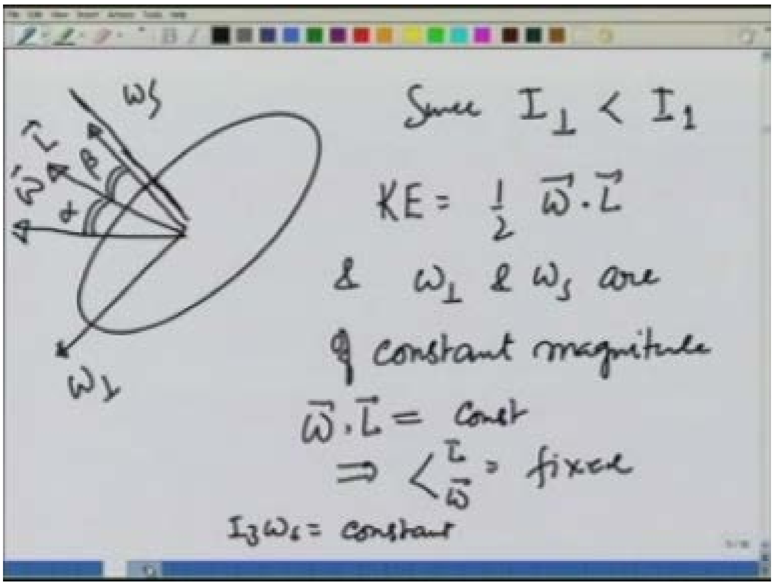

Let us now see, the relationship between omega L and the spin axis. If, this is a disc at some instant let this be omega perpendicular let this be, omega spin. So, that the net omega is something like this. Since, I perpendicular is less than Ione the spin axis. So, this I is much this I is larger I omega s would be slightly larger and therefore, you can see that L would be between vector omega and the spin axis.

Further, since the kinetic energy is one half omega dot L and we have already seen from Euler's equations that omega perpendicular and omega spin are of constant magnitude therefore, omega dot L is equal to constant implies that, angle between omega and L is fixed. Therefore this angle let me call this alpha is fixed further I3 omega s is also a constant therefore, projection of L on the spin axis is always a constant. And therefore, this angle beta is also fixed. So, let us see what we have learnt, what we have learnt is for this given disc.

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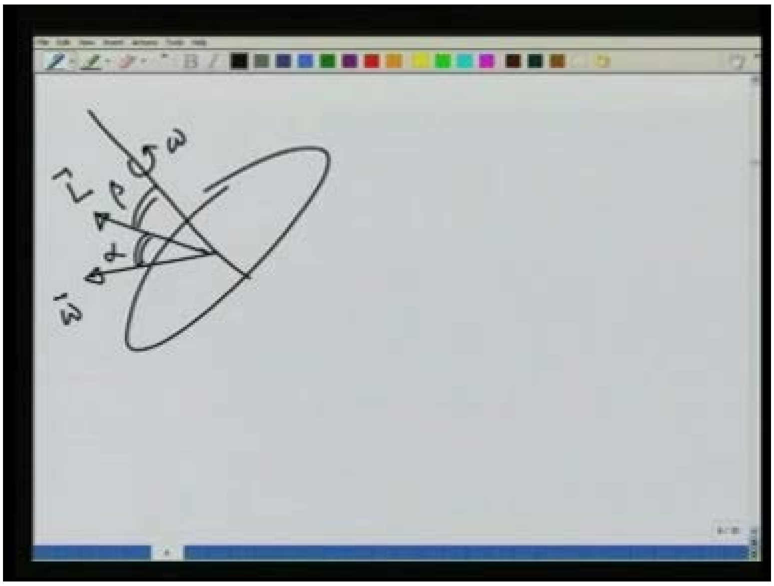

If, this is omega this is the spin axis L is going to be between the spin axis and omega with this angle being alpha and this angle being beta and this is spinning about it. So, what is happening is these 3 vectors L, L omega spin and omega they are like this.

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Let me make these 3 omega spin L and omega and this angle is fixed this angle is fixed and the disc is spinning. So, only where this can happen if, these angles are fixed these magnitudes are fixed that this and L is fixed in a space that these 2 vectors are rotating about L like this.

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And we want to know what this rotation rate is?

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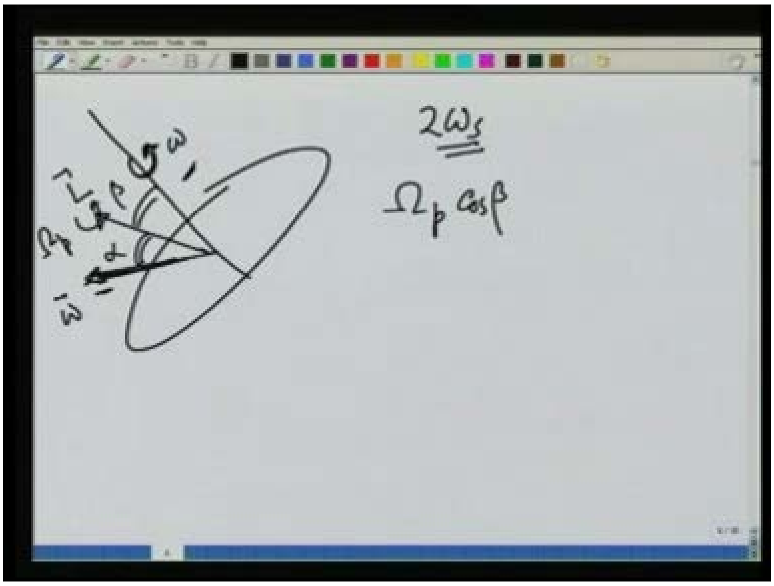

So, let this rotation rate be omega p where omega p stands for omega precession I am going to relate it to 2 omega s which I know is the rate at which omega this vector rotates above this spin axis. This we have already calculated the compose omega p along the spin axis and that gives you a component omega p cosine beta and that is really the rate at which omega vector is rotating about spin axis. Because this is rotating like this and this component of this omega precession along this gives me the rate at which omega rotates about this.

Therefore, this should be equal to 2 omega s and this gives me omega precession equals 2 omega s over cosine of beta that tells me that about this fix vector L in space the spin axis is going like this; see if, I take the frisbee again this is spin axis and suppose this is vector L.

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And this spin axis is going like this.

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In addition, the disc is also rotating about its diameter. So, the entire motion is like this.

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And that is wobbling.

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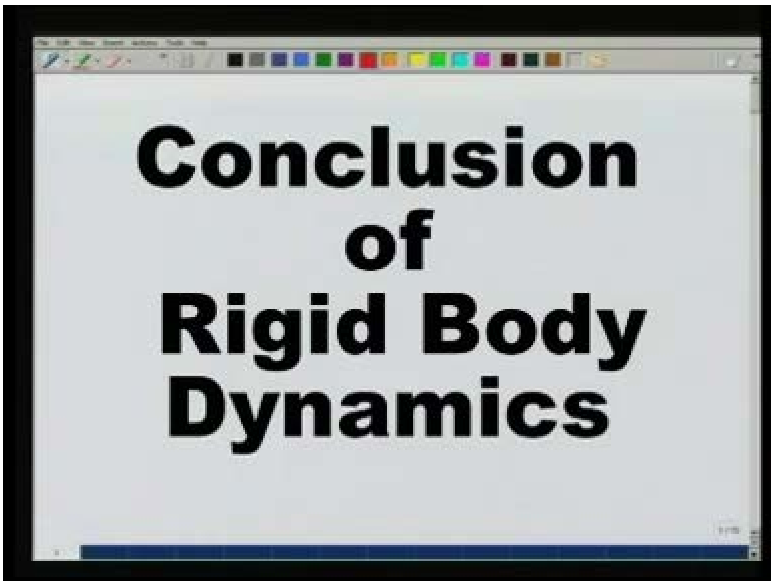

Let me now; conclude the lectures on rigid body dynamics.

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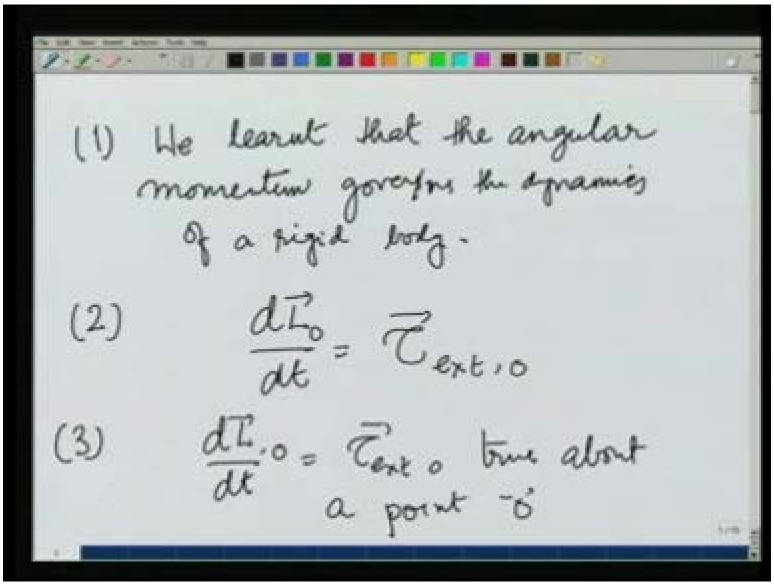

We learnt that; the angular momentum governs the dynamics of a rigid body this plays a central role angular momentum changes according to dL over dt equals tau. External depends on the point about which we are taking the torque and the angular momentum. 3 this equation; dL over dt tau external o is true about a point o.

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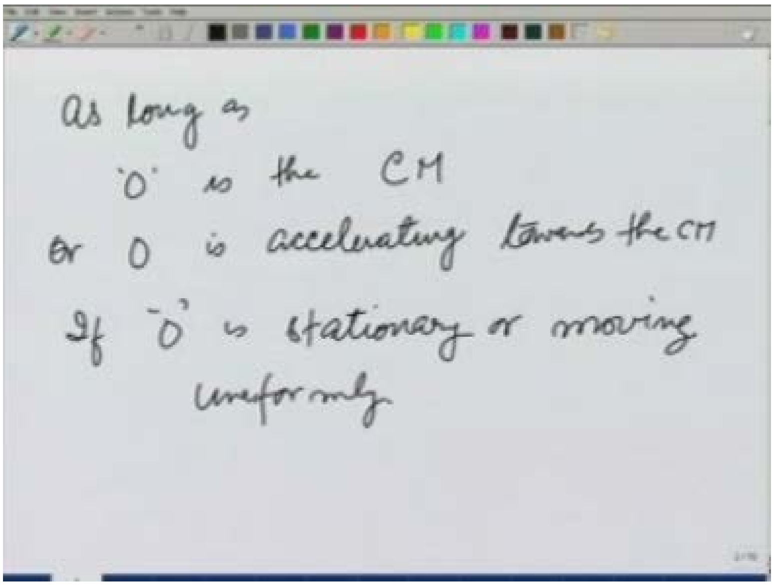

As long as; o is the center of mass or o is accelerating towards the center of mass of course, it is true, if o is stationary or moving uniformly, then what we learnt?

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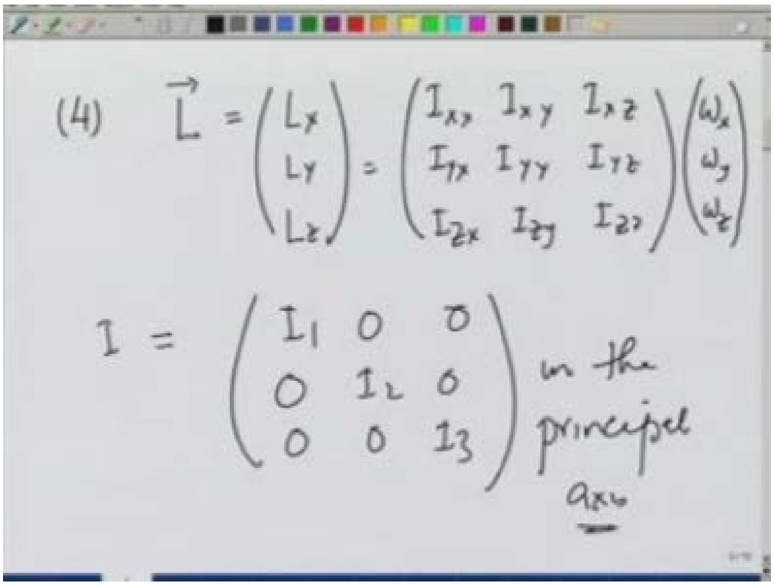

Is that; the angular momentum in general if I write its 3 component is Lx Ly and Lz is really a product of the moment of inertia tensor. And the angular velocity; however, if I use the principle axis notation, principle axis representation in that case the moment of inertia tensor I becomes diagonal Ione 0 0 0 I2 0 0 0 I3 in the principle axis and in that case.

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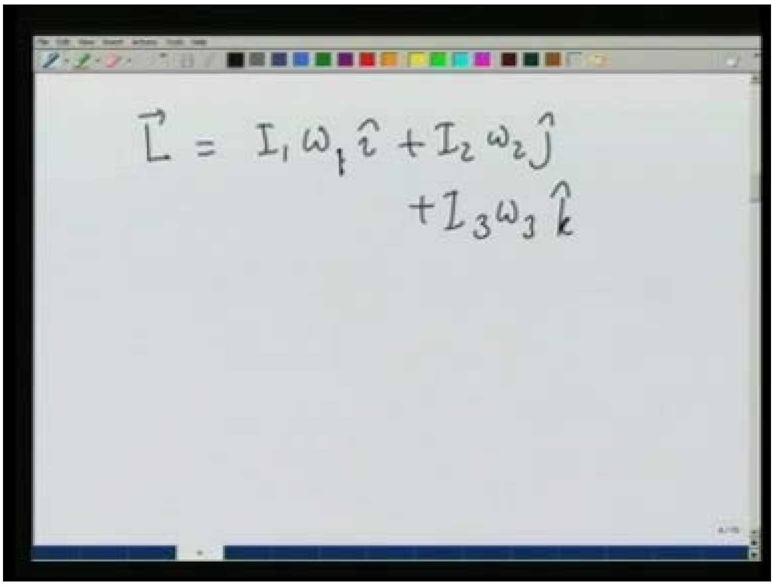

The components of L become very simple Ione omega one along direction one I2 omega2 along direction 2 and I3 omega3 along direction 3. However, these principle axes are also rotating in space.

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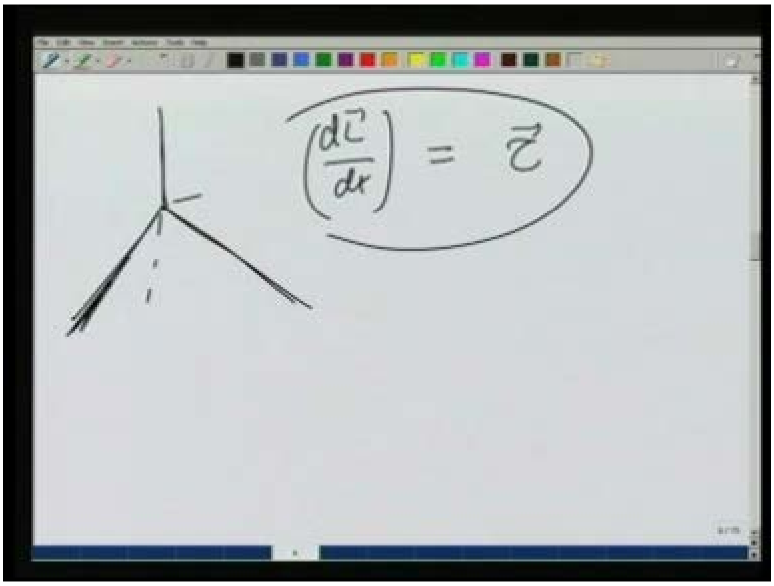

So, in describing the motion N principle axis what we do is at any given instant we take components of dL dt along the principle axis directions at that instant of course, with time the principle axis are changing and then this is equal to tau, all we are doing is taking the components of this equation along the principle axis.

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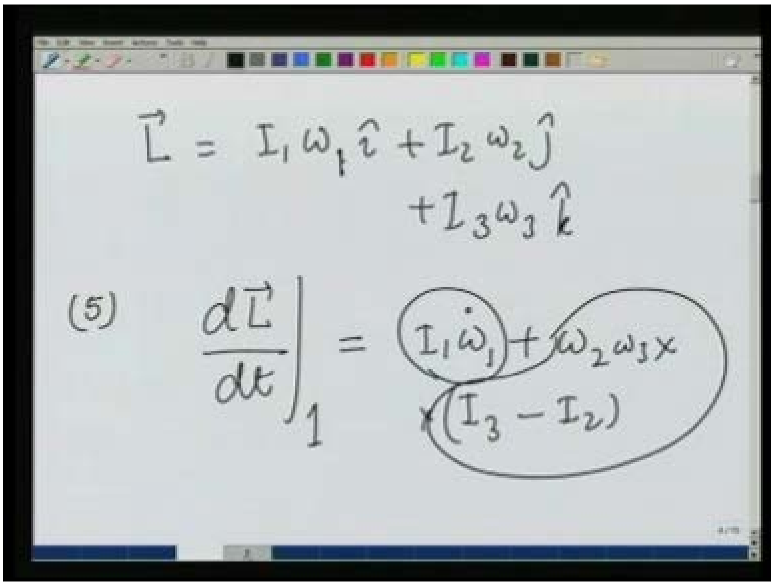

And therefore, when I take dL over dt is component along one, where one denotes the principle axis one or x principle axis it becomes Ione omega one dot plus omega2 omega3 times I3 minus I2 where this part is because the component of omega along axis one is changing and this part comes due to the rotation of L.

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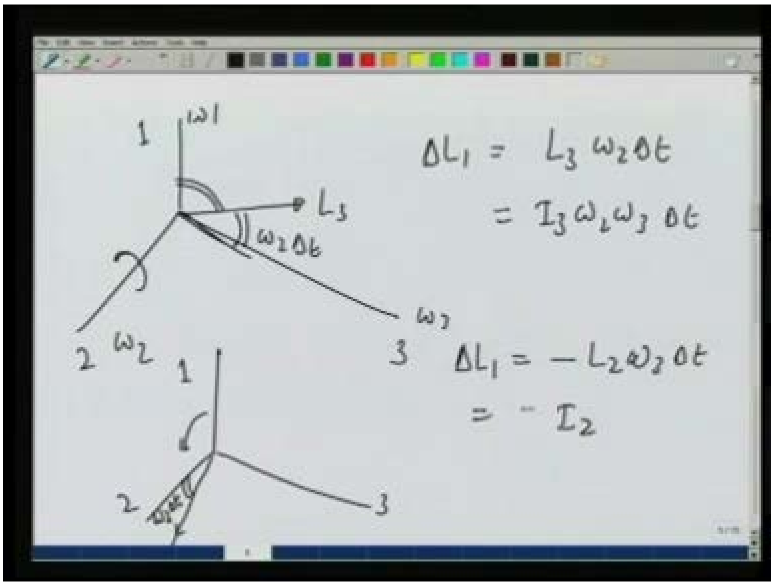

Let us see that, suppose; the principle axes are given like this one 2 3 I have omega one omega2 and omega3 going like this at any instant suppose; the body is rotating about axis 2 then L3 would have rotated in this direction. And you see this develops a component along one if, this angle is omega2 delta t then delta Lone would be L3 time's omega2 delta t which comes out to be I3 omega2 omega3 delta t.

Similarly, if the body is rotating about 3 one towards 2 then L2 would have gone down by omega3 delta t. And therefore, L2 would develop a component along one which will be minus L2 omega3 delta t which is minus I 2.