Engineering Mechanics
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Module - 07
Lecture - 05
Rotational Motion – V

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In previous lectures on rigid body, what we have seen it is that.

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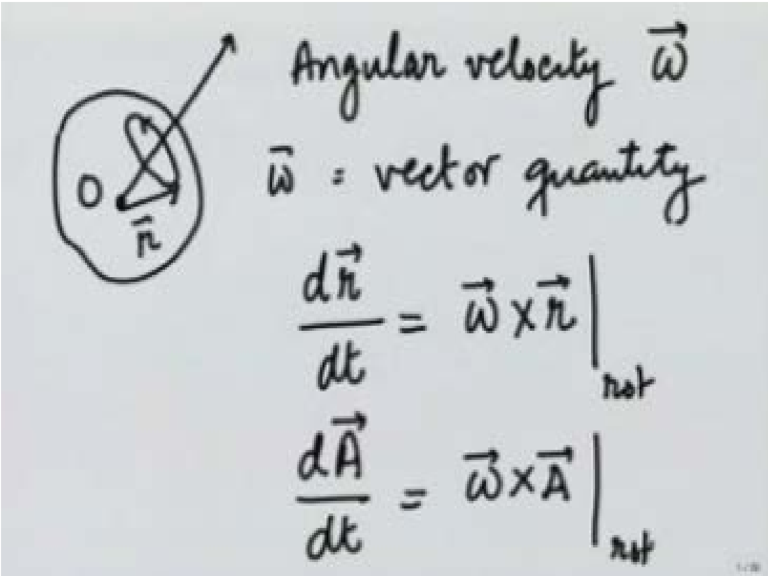

If you have a rigid body rotating about an axis, let us take our origin here then, the angular velocity omega is a vector quantity. And any vector which is attached with the body r, rotates about the axis, because this vector is rotating there is a change in the vector. Here I have taken vector r so, dr over dt because of this rotation is omega cross r. We have fixed the convention of direction of omega and this is purely by rotation, the direction of omega is such that if the body is rotating along the fingers, the thumb gives the direction of omega.

This equation is true for any vector not just r. So, in general any vector that is rotating about omega is the rate of change is given by omega cross A and I am writing rotating here to indicate that this is purely due to rotation.

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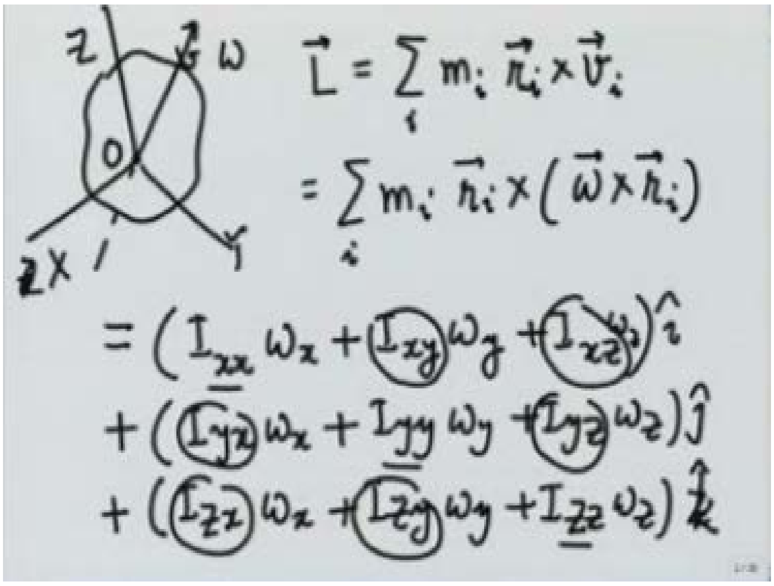

Using this therefore, when a body rotates with angular speed angular velocity omega this is the axis, this is the origin. You see I am specifying origin all the time because, we always take angular momentum or any quantity with respect to the origin. Then the angular momentum L which is defined as summation over all the point, all the points in the body which are moving mi ri cross vi is written as summation I mi ri cross vi is dri dt. And this is purely by rotation this quantity and this we worked out comes out to be Ixx omega x, plus Ixy omega y, plus Ixz omega z in I direction, plus Iyx omega x plus Iyy omega y plus Iyz omega z in j direction, y direction and Izx omega x plus Izy omega y plus Izz omega z, in the z direction.

Let me write it as k, where X Y and Z directions are given X, Y and this could be a Z direction. And therefore, in general to calculate the angular momentum I need all these quantities. The diagonal elements Ixx, Iyy and Izz are the moments of inertia about the X axis, about the Y axis and about the Z axis respectively and these quantities, the off diagonal elements are the products of inertia. So, in general it is quite a quantity to calculate.

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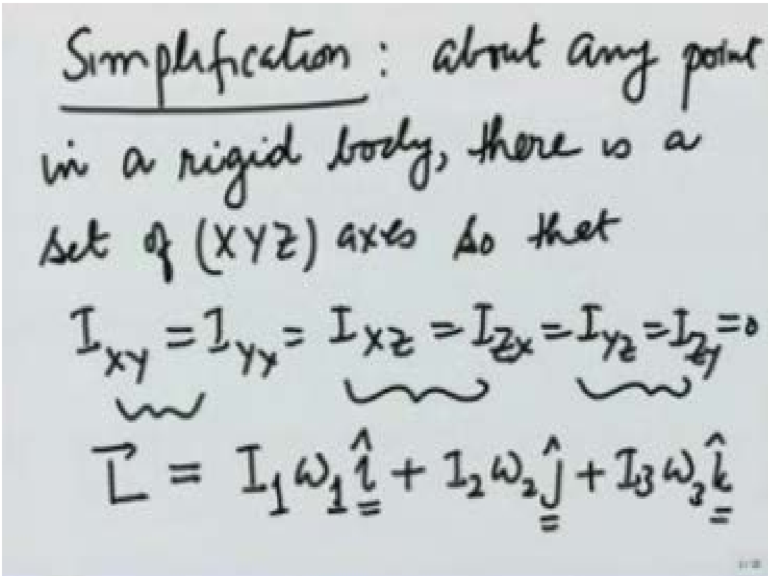

However, what we note is that there is a simplification and that is about now in any point in a rigid body there is a set of X, Y, Z axis. There is 1 set which is in 1 particular direction so that, all of diagonal elements Ixy, Iyx, Ixz, Izx and Iyz, Izy these anyway are equal, but they are all equal to 0. So that, the angular momentum is now quite simple and let me change instead of X, I am going to call it axis 1, omega 1, 1 denotes X direction. So, omega 1 is the component of omega n X direction I plus I 2 omega 2 j plus I 3 omega 3 k. So, the expression really get simplified, the only catch now is that this i j and k unit vectors are along the principle axis or principle axes 3 X Y Z of the body.

Now, principle axes are going to have a fixed orientation with respect to the body and therefore, these axes are attached with the body and the body rotates these axes also rotate. So, that is the case. So, we have to take care of that somehow the other when we develop our dynamics.

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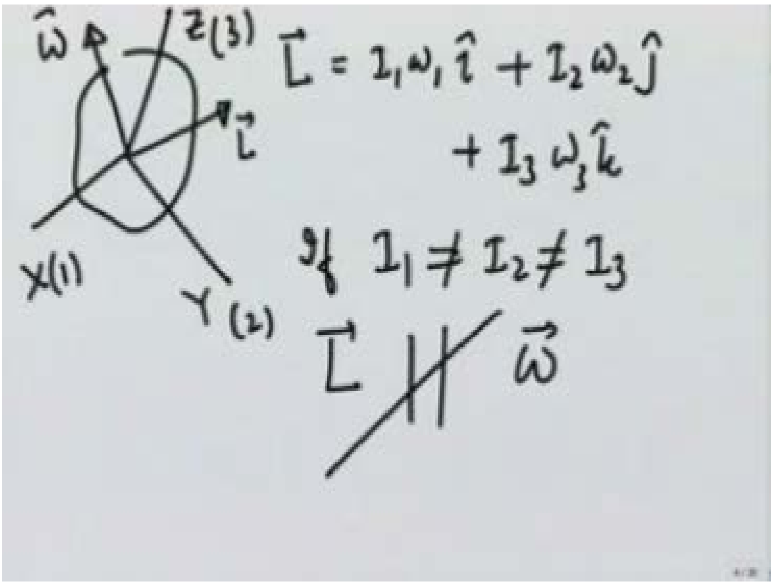

But this is quite a simplification that for a given body suppose, these are the principle axis X, Y, Z. Then the angular momentum is going to be given as I 1 omega 1 and for now, let me now change this to 1, 2 and 3. In this direction plus I 2 omega 2 in Y direction, plus I 3 omega 3 in k direction, this is diagonal. Notice however, if I 1, I 2 and I 3 the 3 moments of inertia are not equal then, L is not parallel to omega. So, if suppose this is omega so that, it has components in X, Y and Z direction 1 tend to 1 3 direction this could be L.

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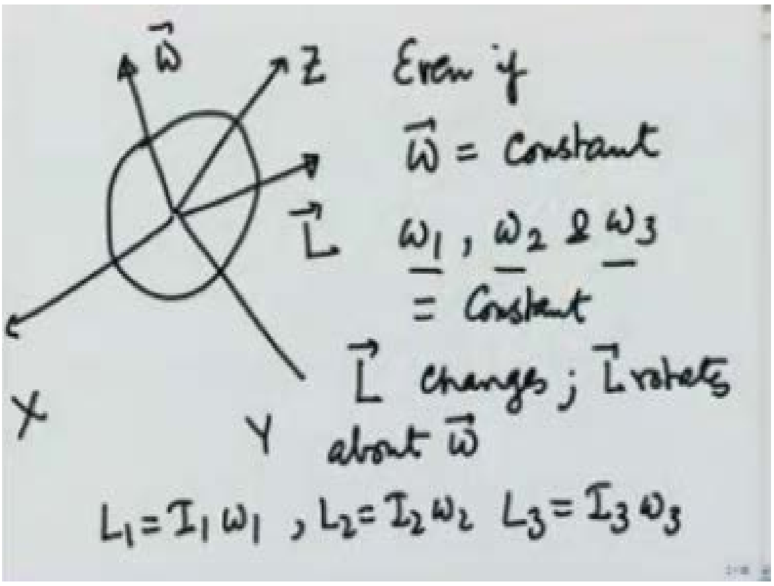

Even if we take omega to be a constant and even if omega 1, omega 2 and omega 3 that is, the components of omega along the principle axis omega 1, omega 2 and omega 3 are constant. L is going to change because, L rotates about omega, why does it do so? Because, L 1 equals I 1 omega 1, L 2 equals I 2 omega 2 and L 3 equals I 3 omega 3. All 3 are constant because, of omega 1, omega 2 and omega 3 remaining constant, but the principle axis are themselves rotating. So, these components although they are fixed, they are rotating and therefore, L itself rotates about omega.

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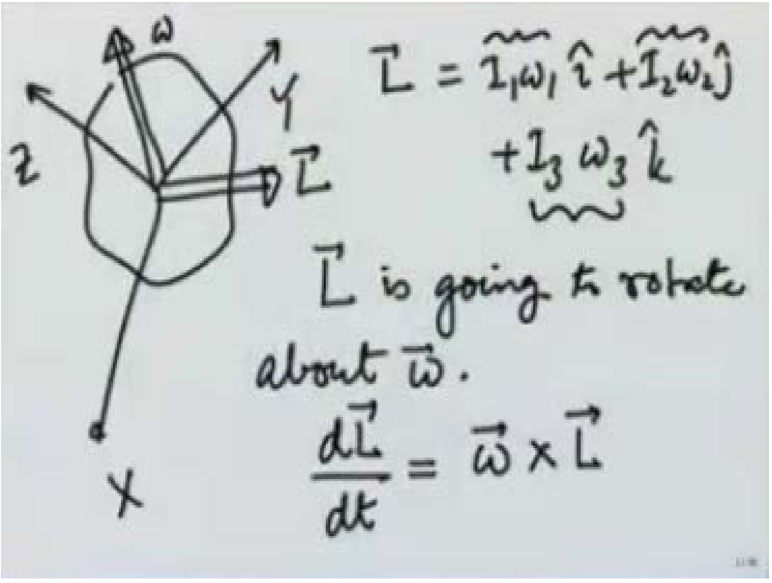

So, what we see is in the case of X, Y, Z principle axis omega and L are not being parallel to omega L could be I 1 omega 1 I, plus I 2 omega 2 j, plus I 3 omega 3 k. All these numbers may remain unchanged, but L is going to rotate about omega. Because, the principle set of axis being attached to the body are rotating about omega and the rate of change, because of this rotation is going to be omega cross L. We will see a few examples of this kind later in the lecture.

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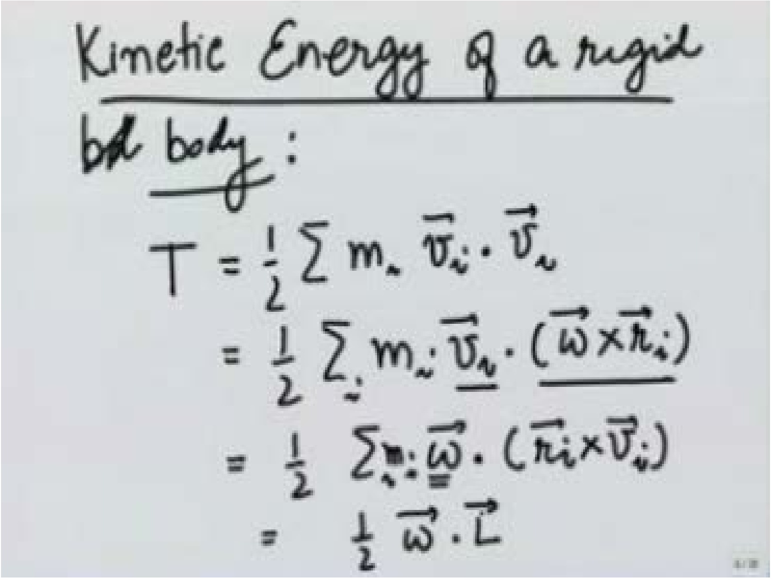

But before that let me also for completeness show you, what the kinetic energy of a rigid body is when it is rotating. So, let us say the kinetic energy by definition is equal to summation mi vi dot vi, which I can write as 1 half summation over I mi vi dot omega cross ri. This is a dot product of a vector with a cross product. So, I can use the permutation formula and write this as 1 half summation I omega cross ri cross vi and mi is always there. Since this omega is constant, summation mi ri cross vi is L, this comes out to be 1 half omega dot L.

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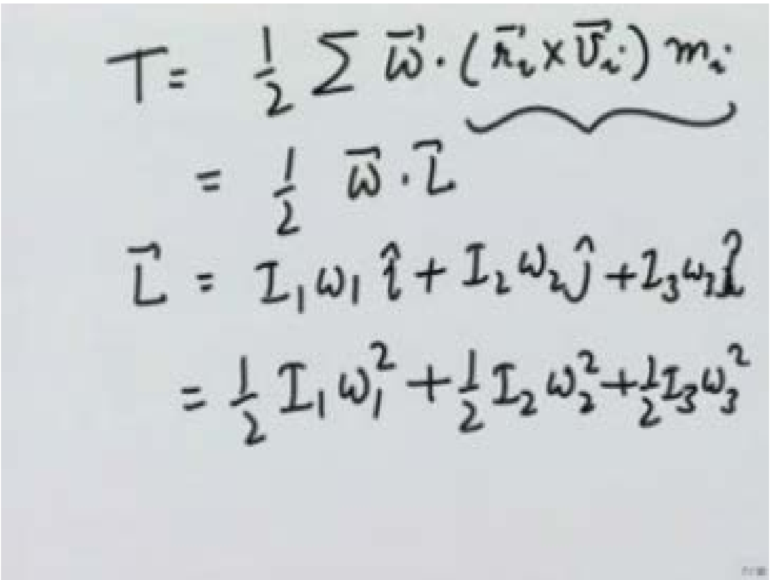

So, the kinetic energy of a rotating body is equal to one-half summation omega dot ri cross vi, I am deliberately writing mi on this side. And this summed over is L, which is one-half omega dot L and using L equal to in the principle axis notation omega 1 I, I 1 omega 1 I, plus I 2 omega 2 j, plus I 3 omega 3 k. This comes out to be 1 half I 1 omega 1 square, plus 1 half I 2 omega 2 square, plus 1 half I 3 omega 3 square. So, you see if I work with the principle axis of the rotating body, my expressions it becomes simple added complication is that, now the principle axis itself is rotating with the body. So, we have to now look at how to handle this.

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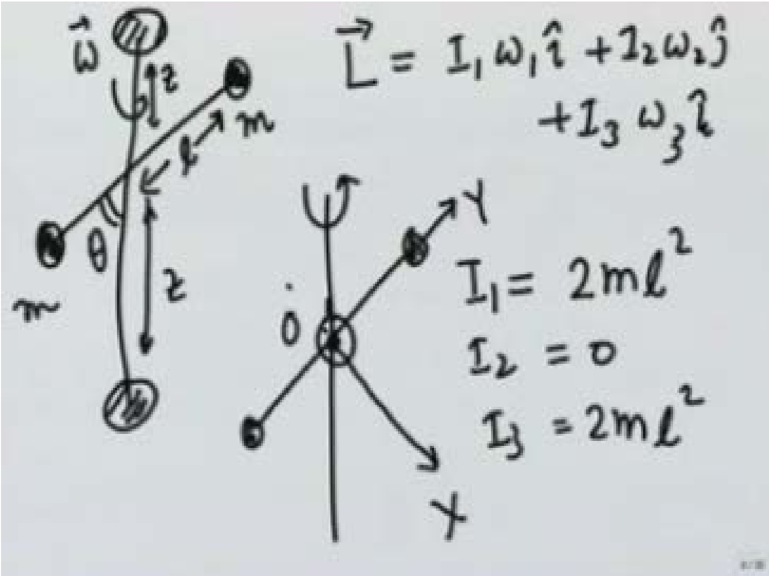

As an example let me go back to our old friend, a light rod with masses m on both sides rotating about an axis and let us fix this axis. On some ball bearings here, let this height be z, let his also be z, let this angle be theta, let the length of the rod be 2 l so that, half of it is l. And I want to first calculate the L, the angular momentum of the body, remember we calculated this last time, but I want to do it again. And this is rotating with angular speed, angular velocity omega like this. L and the principle axis rotation is going to be I 1 omega 1 I, plus I 2 omega 2 j, plus I 3 omega 3 k where, these I j k refer to the principle axis of the body.

Now when the body is like this, this is the rigid body. Let me choose this is the origin O, the X axis like this, Y axis like this and Z axis is coming out. You can see these are the principle axis and you will calculate, this we did last time I 1 to be equal to 2 ml square that is the moment of inertia about the X axis. I 2 moment of inertia about the Y axis is coming out to be 0 and I 3 moment of inertia about the Z axis will come out to be again 2 ml square. This is omega so, omega has 2 components.

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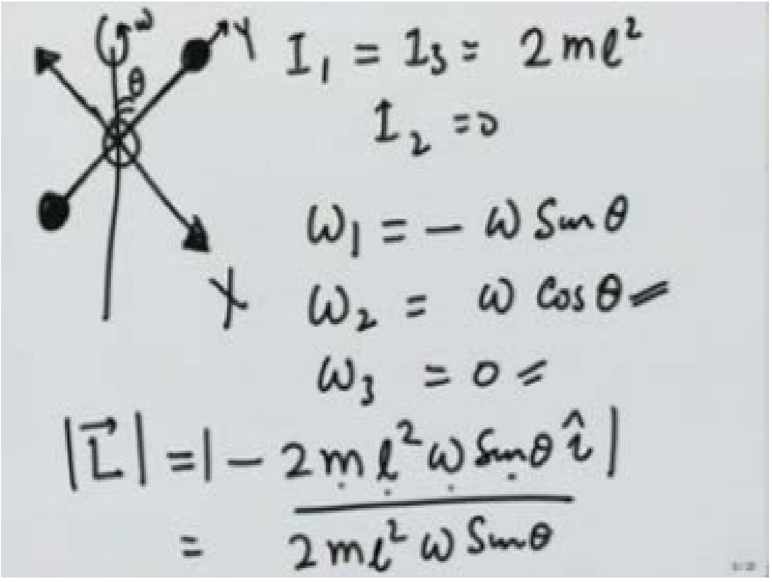

Let me go to the next thing and show you, this is a body, this is omega, this is theta I 1 is equal to I 3 is equal to 2 ml square I 2 is 0. Let me remind you this is my X axis, Y axis and Z axis coming out and as I said earlier as the body rotates these axis are fixed in the body, these axis will also move. So the components really refer to, the components at the time when the, this, this rod is in the plane of this screen. So, omega 1 you can see is going to be in this direction is equal to minus omega sin theta, omega 2 is along the Y axis is going to be at this point omega cosine of theta and omega 3 is 0.

And therefore, I get L is equal to I 1 omega 1 gives me minus 2 ml square omega sin theta I and rest of the other 2 are 0 because, I 2 0 it gives you 0 with this and omega 3 is 0. So, the angular momentum is in this direction at this point, at this time it is in this direction as the body rotates, it will also come out of the plane. It will come out and going again like this. Now you see as far as the mass length omega and sin theta these are concerned they are constant. So, the magnitude of angular momentum is going to be 2 ml square omega sin theta and magnitude remains constant. So, the only change in omega occurs because it is rotating about an axis.

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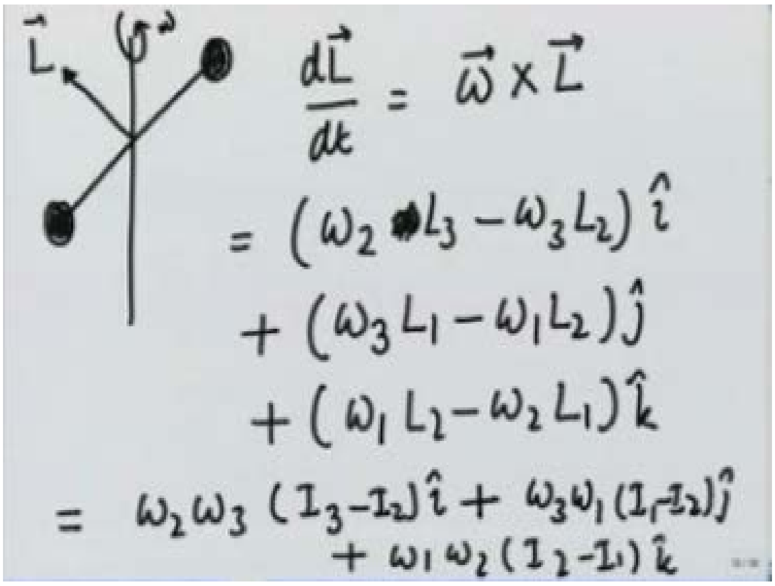

L so, change in L is going to be equal to omega cross L and in general I can write this as omega 2, omega L 3 minus omega 3 L 2 i, plus omega 3 L 1 minus omega 1 L 2 j, plus omega 1 L 2 minus omega 2 L 1 k, which in this case can be calculated to be omega. Let me write it further omega 2, omega 3 I 3 minus I 2 i, plus omega 3 omega 1 I 1 minus I 2 j, plus omega 1 omega 2 I 2 minus I 1 k.

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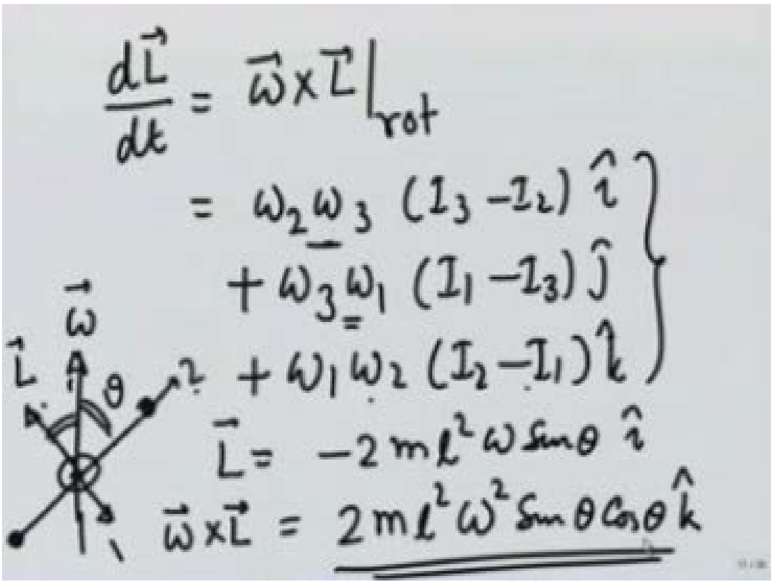

So, what we notice is that in general purely by because of the rotation, only because of the rotation, the quantity because, the L is rotating about omega. This comes out to be omega 2 omega 3, I 3 minus I 2 along principle axis X plus omega 3, omega 1, I 1 minus I 3 j plus omega 1, omega 2 I 2 minus I 1 k at any given instant. And in the case of this rod that we took, L is in this direction. So, at any time L is equal to minus 2 m l square omega sin theta i omega is in this direction, omega cross L you can see will come out to be coming out of the paper in direction 3 this was 1, this was 2 and 3 was coming out.

So, it will be omega cross L, omega is this way would come out in direction 3 and its magnitude is going to be magnitude of omega, magnitude of L times sin of this angle, this is theta, this is 90 minus theta or the sin of this angle is going to be cosine theta. You will see this will come out to be 2 ml square omega square sin theta cosine of theta in the direction k. Do we get the same answer from this? Of course, you can see omega 3 is 0. So, these 2 terms the first term and the second term are going to give me 0, the third term gives me omega 1 times omega 2 times I 2 minus I 1 I 2 is 0.

So, it comes out to be I 1 which is 2 ml square with a minus sign, times omega 1, omega 1 Omega 1 is minus omega cosine theta. So, that comes out to be omega cosine theta omega 2 is omega sin theta another omega sin theta in the answer. So, let me write its expressive.

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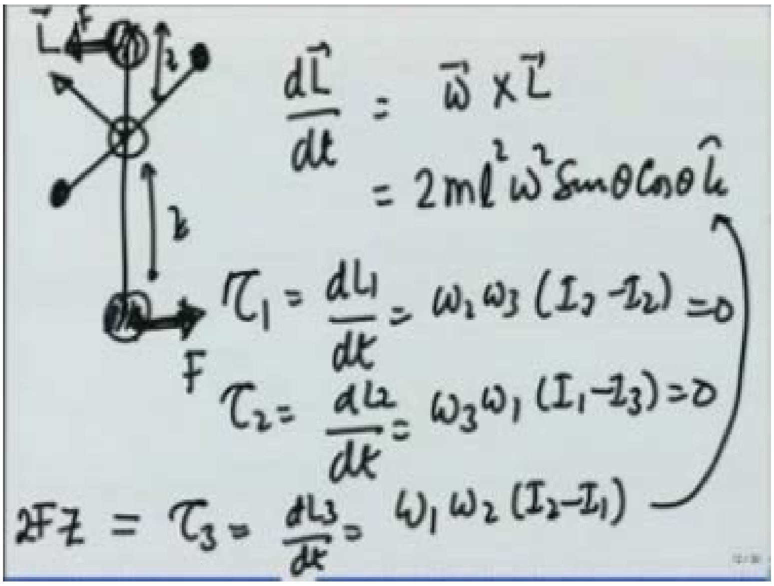

So, what we got in this rod if it is rotating like this, this is L and therefore, dL dt which was omega cross L came out to be 2 ml square omega square sin theta cosine theta k. On the other hand if I calculated using tau 1, which is dL 1 dt purely due to rotation it is going to be omega 2 omega 3 I 3 minus I 2 comes out to be 0. Tau 2 which is dL 2 dt which is equal to omega 3 omega 1 I 1 minus I 3 that also come out to be 0. Tau 3 which is dL 3 dt this is equal to omega 1, omega 2 I 2 minus I 1 will come out to be this.

So, the torque is in this direction and therefore, to keep it rotating I have to keep applying a torque. As the body rotates, the torque direction also changes and the torque is provided by these bearings here. If this height is z and it is symmetrically placed in z like this then, at this point when the body is in the plane of the screen the force here would be like this and the force here applied by the bearing on the axis would be like this. So that, there is a torque coming out of the paper the magnitude of the torque would be equal to 2 FZ where, F is this force. And therefore, I can also calculate how much is the force applied on the axis by these ball bearings.

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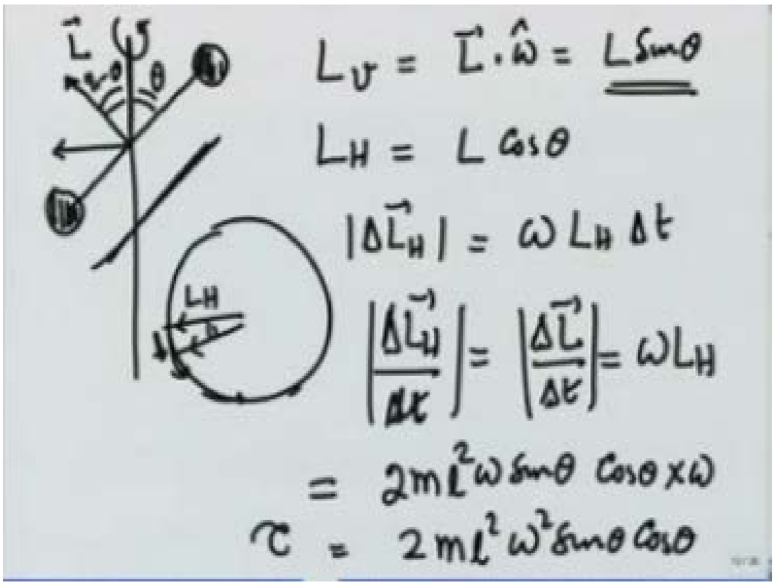

There is another way of looking at it, the same body when it is in the plane L we had seen is like this and the body is rotating like this. L has 2 components, let me call it L vertical which is along omega and L horizontal. This is going to be, if this angle is theta L sin theta this is going to be L cosine theta. As the body rotates L is also rotating about it, its magnitude is constant. You will see that this component along omega, this component remains unchanged. The only component that rotates is this horizontal component. Let us see how it rotates, this is the horizontal component LH and it is rotating like this.

So, at any given time this small change in the vector is going to be this way and this is going to be equal to the rate of rotation omega, that is the component perpendicular to omega. So, it is rotating with omega LH delta t and therefore, delta LH over delta t magnitude is going to be same as delta L over delta t magnitude. Because, it is only the horizontal component that is changing and this is going to be omega times L horizontal and this you can see is going to be L verse 2 ml square omega sin theta this is 90 minus theta.

So, L horizontal is going to be sin of 90 minus theta of this, which is cosine of theta times omega, which comes out to be 2 ml square omega square sin theta cosine theta. And at this point when the body is in the plane of this screen, horizontal component is going to be coming out of the screen. So, that is the direction 3 direction of dL dt and this is going to be the torque required.

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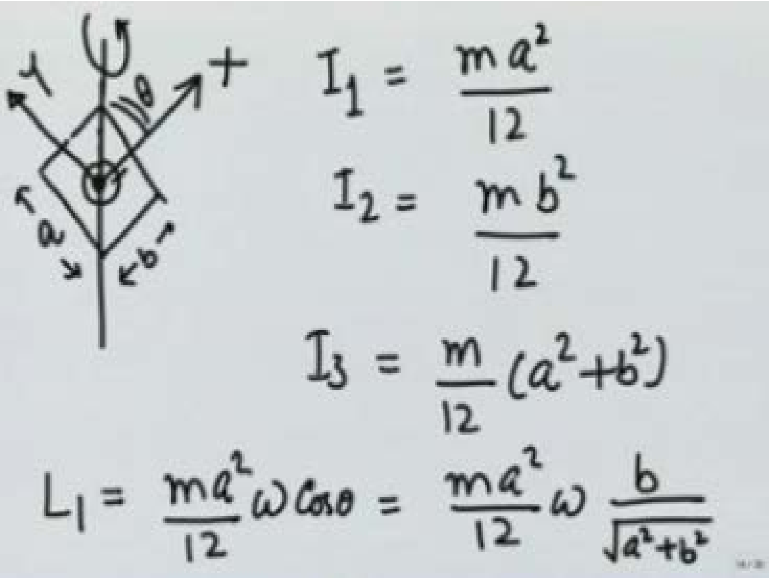

Let us now take another example that we did last time that of a rectangle moving about, rotating about its diagonal, the length of the rectangle is a and the width is b. Let us take our origin to be its center then, it is easy to see by symmetry that these are the principle axis. Let us call this X axis, let us call this Y axis and Z axis is coming out of the plane. If the mass of the rectangle is m then, I 1 that is Ixx is going to be ma square over 12, I 2 is going to be mb square over 12 and I 3 is going to be m over 12 a square plus b square.

And therefore, the various angular momentum components are going to be L 1 equal to suppose, this angle is theta then, L 1 is going to be ma square over 12 omega cosine of theta and you can see cosine of theta is going to be this, divided by this. So, this is going to be ma square divided by 12 omega, cosine theta is going to be b divided by a square root of a square plus b square. Let me read all the whole thing.

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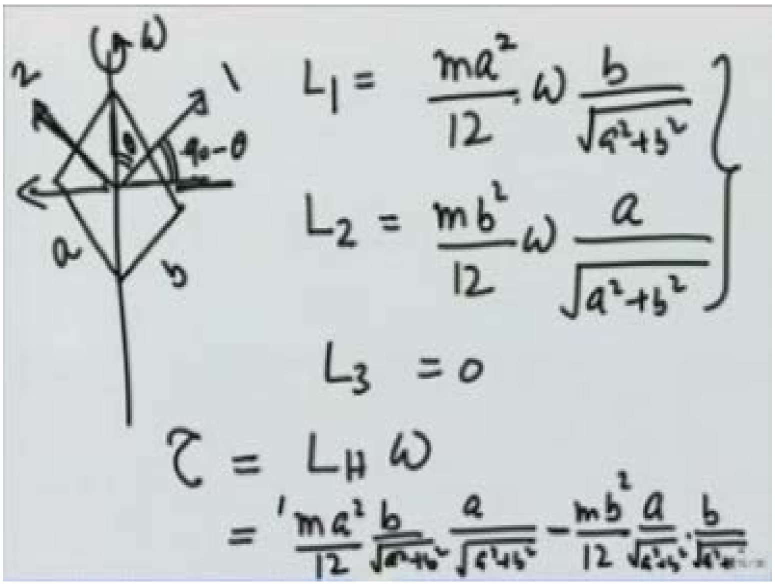

This is the diagonal, this is half the plane is axis 1, axis 2 this is theta and this is rotating with omega and therefore, L 1 comes out to be m a square divided by 12 omega cosine theta, which is b over square root of a square plus b square. Similarly, L 2 this is axis 2 is going to be coming out to be m b square over 12 omega sin theta, which is a divided by this, a over a square root of a square plus b square and L 3 is 0. So, you got an L 1, L 2 and L 3 you can see right away that L is not parallel to omega.

How about the torque, to maintain this rotation the torque would be equal to we can use different arguments as we used earlier. Right now let me just use L horizontal times omega, that is if I calculate the horizontal component and multiplied by omega it is only the horizontal component of L that is rotating and vertical component that along omega remains unchanged. So, L horizontal is going to be L 1 times cosine of this angle, which is really this is 90 minus theta so, sin theta.

So, ma square over 12 b over square root of a square plus b square times sin of theta, sin of theta is a divided by square root of a square plus b square, that is in this direction. The

2 component would have a horizontal component going in this direction. So, it will be minus m b square over 12 a over a square root of a square plus b square times b over square root of a square plus b square times omega.

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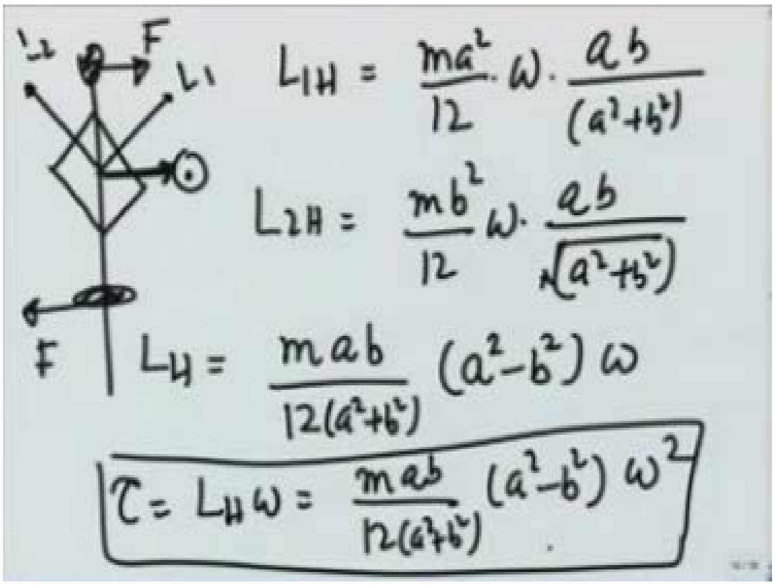

Let me rewrite it, if I have this body, this is L 1, L 2, L 1 horizontal component is m a square over 12 omega sin theta cosine theta, which I can now write as is a b over a square plus b square. L 2 horizontal is mb square over 12 omega sin theta cosine theta which I can write as a b over square root of times divided by a square plus b square there is no square root. This you see from the last screen let us call the product a b divided by a square plus b square, a b divided by a square plus b square times omega.

And therefore, L horizontal is going to the right that is equal to m a b over 12 a square plus b square, a square minus b square. And therefore, the torque which is L horizontal times omega is going to be 4 times omega, m a b over twelve a square plus b square a square minus b square times omega square. How about the direction? This is rotating like this so, this will be going into the plane and that will be the direction of the torque. Suppose, I fix bearings on the top and the bottom to get torque going into a plane I would need a force which is like this here and like this here. The net force is 0, but it gives a torque which is going into the plane of the screen. So, this is the torque required to keep the rectangle rotating with a constant speed omega.

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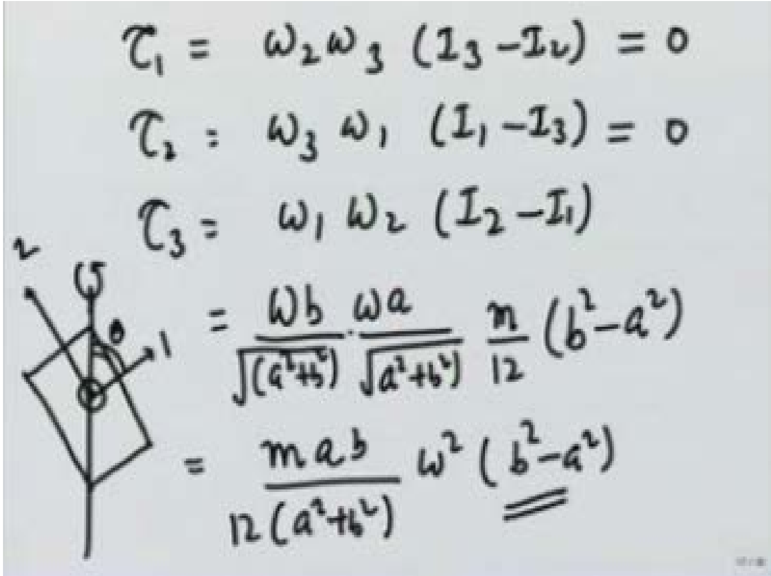

Let us now see how I get the same answer instead of using this horizontal component from the general expression, which told me that Tau 1 is equal to only by pure rotation omega 2 omega 3 I 3 minus I, 2 tau 2 is omega 3 omega 1 I 1 minus I 3 and tau 3 is equal to omega 1 omega 2 I 2 minus I 1. Again recall this is what we are talking about, this is the way the rectangle is since omega is this way, this is 1 at this point, this is 2, 3 is this way, omega 3 is 0, this comes out to be 0, this comes out to be 0. How about the tau 3? It comes out to be omega 1, which is omega cosine of this angle.

So, this will come out to be omega cosine of this angel is b over a square plus b square, square root times omega 2 which is omega sin theta, which is omega a over square root of a square plus b square times I 2 minus I 1 which is m over 12 b square minus a square. So, this gives me the same answer m a b over a square plus b square 12 omega square b square minus a square. This is remember, third component this is negative means is going into this plane of the screen.

So, this gives me the right direction as well as the right magnitude, as we calculated earlier which was this. This was a square minus b square, but it was going into the plane. Similarly, here since axis 3 is coming out of the plane I get a negative tau 3 so that means, torque again is going into the plane.

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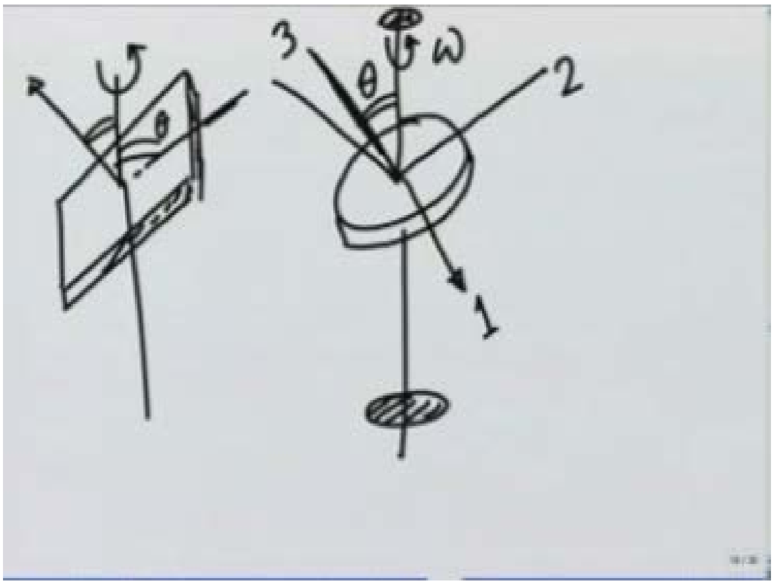

As a variation on this problem, let me ask you to do a problem where this plate is kept at, the rectangle is kept with its plane making an angle with the vertical and it is being rotated like this. I want you to calculate so, that u there is normal to the plane in that an angle with the rotation axis. I want you to calculate how much torque would be required to keep it rotating. I leave it for you to solve, although I am going to solve a related problem that of a disc tilted at an angle from the vertical and rotating like this so that, it is perpendicular to the disc makes an angle theta with the axis of rotation.

I want to calculate how much torque would be required to keep it rotating at a constant omega. Again you see, now I will take this middle point center of the disc as the origin and see that this, 1 of the diameters is a principle axis. So, as the other diameter and third axis going to be coming out of perpendicular. So, it will be in this direction itself, this is direction 3 and I want to calculate what its angular momentum is and what is the torque required to keep it moving. So, let us make this picture again.

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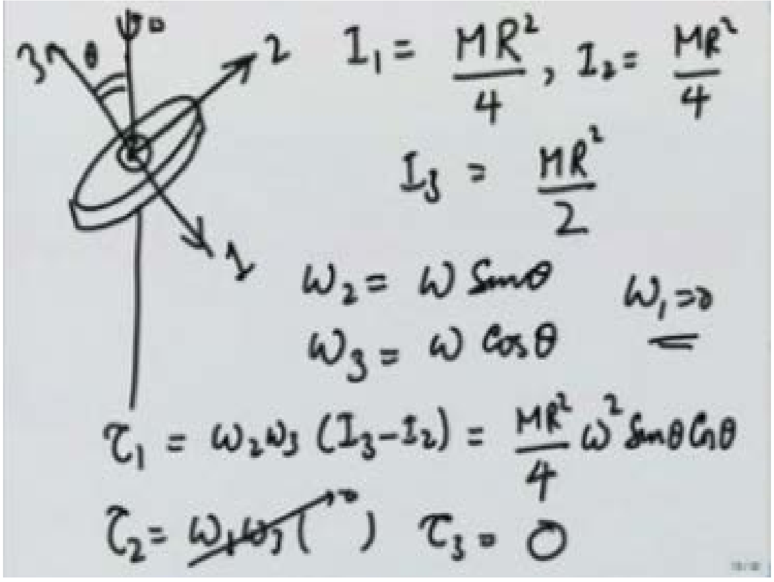

This is a disc rotating, the disc perpendicular at an angle theta with omega, this is axis 2 this is axis 1 and this is perpendicular is axis 3. We can see I 1, we had calculated it earlier the moment of inertia about one of the diameters is M if the radius is R square over 4, I 2 is also going to be M R square over 4 and I 3 is going to be M R square over 2. Omega has components at this point in direction 2 and direction 3, omega 2 is equal to omega sin theta. Omega 3 is omega cosine theta, if I want to calculate torque tau 1 and omega 1 is obviously 0, omega 2 omega 3 I 3 minus I 2 gives me I 2 minus I 3 is M R square over 4 omega square sin theta cosine theta.

Tau 2 is going to involve omega 1 omega 3 whatever this gives the 0 because, omega 1 is 0 and so, is tau 3, which is also going to involve omega 1 and that is also coming out to be zero So, in this case the torque is going to be in the direction along 1. At the, this given situation the direction 1 is going to be coming out of the plane of the paper. So, let me make it again.

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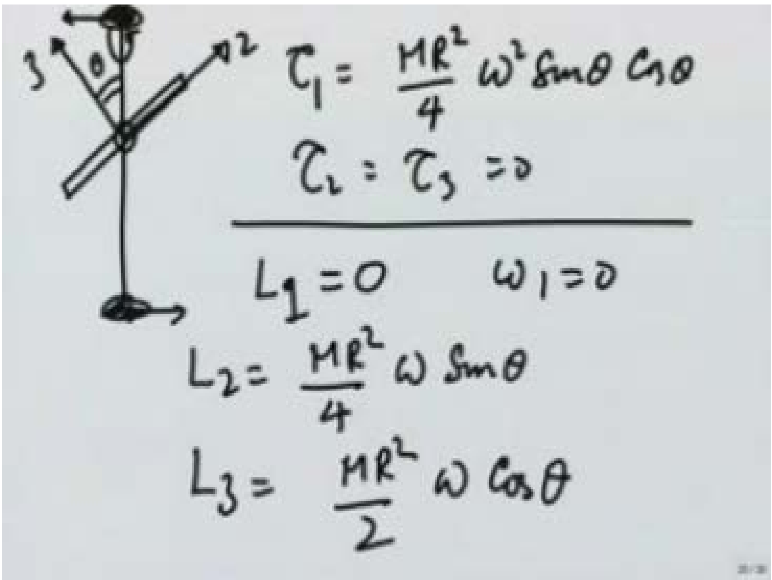

Just look at this disc, in the position when it is like this 1 is coming out of the plane, 2 is like this 3 is perpendicular to the disc and it is rotating like this. This angle is theta and we see in this case that the torque is coming out to be tau 1, which is M R square over 4 omega square sin theta cosine theta as we saw in the previous slide. Tau 2 and tau 3 are 0. So, torque direction is this coming out of the plane of the paper. So, if I have bearings here holding this, the force is going to be this way and this way. Let us look at the same thing again from the, by taking the horizontal component of the angular momentum.

The angular momentum is going to have no component in direction 1 because, omega 1 is 0. L 2 is going to be M R square over 4 times omega sin theta and L 3 is going to be M R square by 2 omega cosine of theta and therefore, let me make it again.

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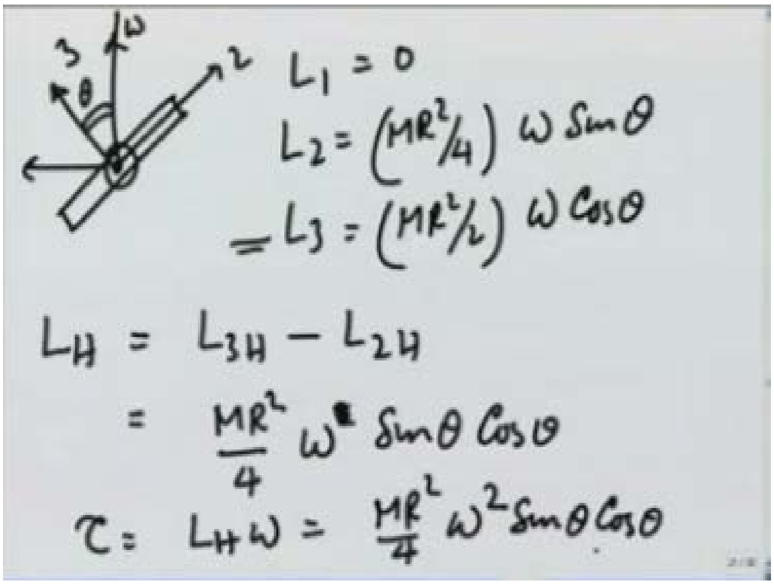

Two and at this point 1 is coming out, 3 is like this L 1 we saw is 0, L 2 is this is omega. This is theta L 2 is M R square over 4 omega sin theta L 3 is M R square by 2 omega cosine of theta. So, L horizontal is going to be L 3 horizontal minus L 2 horizontal in this direction. You can see L 3 is greater than L 2 and this comes out to be M R square over 4 omega square sin theta cosine of theta. This L H will not have omega square, tau that is a change in L is going to be L H times omega which is going to be M R square over 4 omega square sin theta cosine theta, which is the same answer that we obtained earlier.

Tau 1 equals M R square over 4 omega square sin theta cosine theta in direction 1. And here also we see that, since this horizontal component is pointing towards the left it will be coming out of the paper of the screen as the disc rotates and therefore, again it is in direction one.

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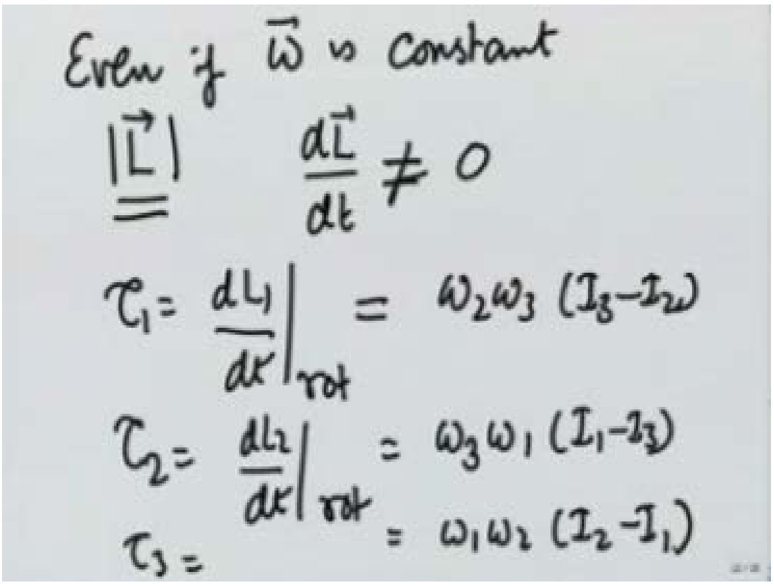

So, through these 3 examples what I have shown you is even if omega is constant L magnitude could also be a constant, but d L over dt is not equal to 0. Because, L is not in the direction of omega, in all these cases where we had the components omega along 1 2 and 2 and 3 in such a manner that L magnitude was a constant. Although there was a torque applied, but dL dt was not equal to 0 and therefore, you know the torque required. We could calculate the torque either by using general equation which is tau 1 which is dL 1 by dt.

Purely due to the effect of rotation which was equal to omega 2, omega 3 I 3 minus I 2, tau 2 which was dL 2 over dt, purely because of rotation which was equal to omega 3, omega 1 I 1 minus I 3 and tau 3 which was dL 3 by dt purely because of rotation equals omega 1 omega 2 I 2 minus I 1 or by taking the horizontal component of L and multiplying by omega C. And see how it is sweeping around, when the body is rotating. Having done these examples now I am in a position to also explain what we saw in the demonstrations.

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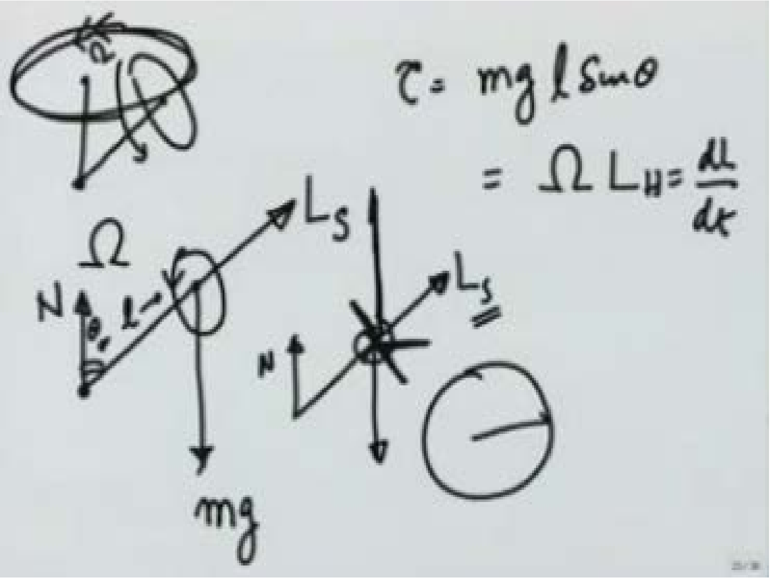

Recall the demonstration where I had a bicycle wheel which was rotating, let us say it was rotating like this and when this rotating wheel was pivoted at this point, it also is started going around like this, what is known as recession. Let us have a look at that.

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What I showed you was here was a bicycle wheel which I rotated like this.

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And when I put it here, it started going around like this, when I take the rotation the other way, it rotated the other way. Let us try to understand this. Our observation is that it is rotating. When I have this bicycle wheel and it is turning like this, we can see that it has an angular momentum in this direction. Let me call it angular momentum due to the spin. Since it is pivoted here and it is being pulled down like this, it is not moving up and down although very slight movement is there, that we will explain later. We can for the time being assume that there is a normal reaction N, which is equal to m g so that, center of mass does not move up and down.

What this N and mg together they do is, they provide a torque or a couple which is equal to. Suppose, this length is l and this angle is theta from the vertical then the torque provided by them is m g l sin theta and what about the direction? The way I have shown it right now, let me make it slightly better, the bicycle wheel is like this, this is m g, this is N. So, the way I have shown it the torque is going into the plane of the screen. So, this L s must change and how does it change? It has to change direction, it has to change into, the delta L has to be into the screen.

For the time being we will assume that, the spin is so large that this rotation recession that I am talking about does not really affect its spin angular momentum much. Although, in in principle you should be taking components of this rotation also to describe angular momentum, but right now we will ignore it. Then you see that Ls has to change going into the plane.

(Refer Slide Time: 46:25)

If I go back to this demonstration, this is the direction of angular momentum and when there is a torque applied like this, the angular momentum has to go like this. If the angular momentum has to go like this, the body rotates and this is precisely what we see. Let us do that now, let us give it an angular momentum in this direction. Therefore, the wheel has to rotate like this and when I leave it, it rotates like this.

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If I give it the rotation the other way, if I rotate it like this then the angular momentum would be in this direction. The torque N cross g is this way so, the angular momentum has to rotate in this direction and therefore, it should rotate the other way let us do that.

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And you see it goes the other way, we can also calculate the rate of this rotation. So, this torque must be equal to the magnitude of rate of rotation. Suppose, this rotation is at rate omega that means, the angular frequency about this vertical axis is this way and this should be equal to omega times L H which is dL dt, why? Because, the vertical component of L does not change, the horizontal component sweeps as we saw earlier. Let me make a nice picture again.

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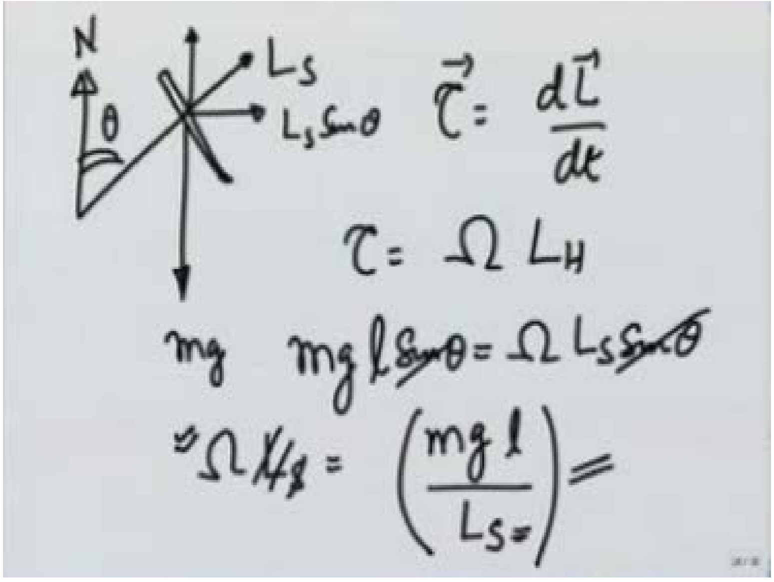

This is the wheel, this is L spin, this is vertical component which does not change, the horizontal component if this angle is theta, is going to be Ls sin theta, there is an m g acting downwards there is a N acting upward. So, tau should be equal to dL dt we have already made sure of the directions by demonstration as well as by working out here. Tau magnitude is going to be equal to capital omega, which is the rate of perception times L horizontal and this we calculated to be m g l sin theta and this should be equal to omega L spin sin theta. Sin theta, sin theta cancels and therefore, you get L spin is equal to m g l divided by omega L spin equals this and therefore, omega equals m g l divided by L spin and that is your answer.

No matter what angle it is at, the precession frequency is the same. If very high spin rate is there, omega is going to be very small and therefore, I had note any effects due to capital omega which is generally known as gyroscopic approximation.

(Refer Slide Time: 49:27)

The second demonstration that I am going to explain now is going to be this gyroscope and what we had noticed in this gyroscope was that there are 3 independently rotating objects. One is this frame which can rotate about this horizontal axis, one is this frame which can rotate about the vertical axis, which is mounted on this frame, outer frame and there is this v. And what we notice was, suppose I give it a spin and then rotate the outer frame, the spin tends to align with the spin direction of the outer frame. Let us see it again, I will give it a rotation and as I rotate this it tends to align with this. Let us try to understand this in terms of angular momentum.

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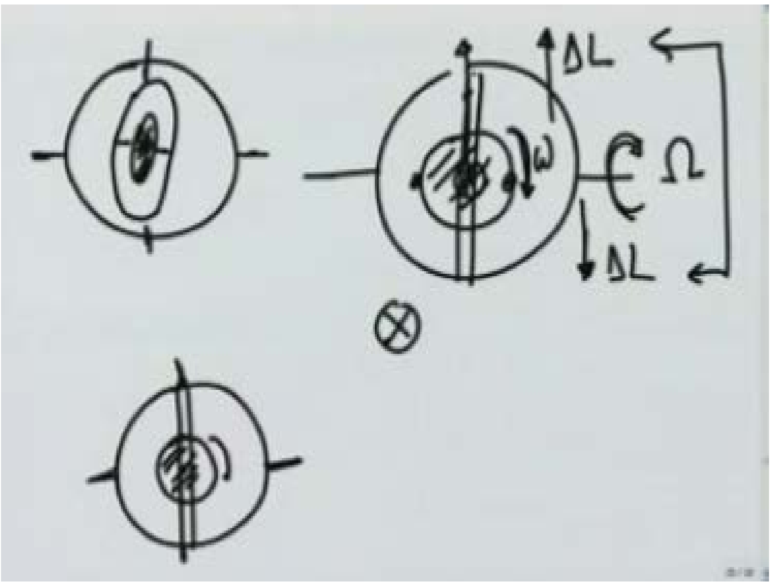

What I have is, I will try to make a picture. There is an outer plane like this which can rotate about the horizontal axis, in this outer frame there is an inner frame that you make it like this, which can rotate about this vertical axis and in this inner frame, I have a wheel which is like this. Let me look at it from a different view, the outer frame like this and let this frame, inner frame be in the position like this. You see this way and then the wheel is mounted like this.

Suppose, the wheel is given a rotation a spin by omega and we rotate after that, this is omega. What we see is this whole assembly, inner assembly rotates in such a manner so that, the spin wheel aligns with this spin.

(Refer Slide Time: 51:38)

You see that again, let us understand why that happens. When it is rotating like this, you will see that this has an angular momentum going into the plane. As soon as I give the outer frame rotation, this angular momentum would tend to go down. So, as soon as I rotate it like this, this angular momentum develops a delta L going down. However, assuming everything is friction less there is no torque on the system taking it, which can change angular momentum going down. If there is no torque there should really be no change in L, as far as the vertical direction is concerned.

So what does the, this frame do? It rotates in such a manner so that, it develops a corresponding delta L in this direction. In order that net change in the angular momentum is 0 and that means, it will tend to rotate like this, it will this frame would come out here and go in here so that, it develops an angular momentum in this direction. And that is why this whole thing aligns with omega, having qualitatively understood this that is if we have these frames, this can rotate freely about this, this can rotate freely about this and this can rotate like this.

Why this spin aligns with the outward spin? Let us try to make it quantitative to make it quantitative.

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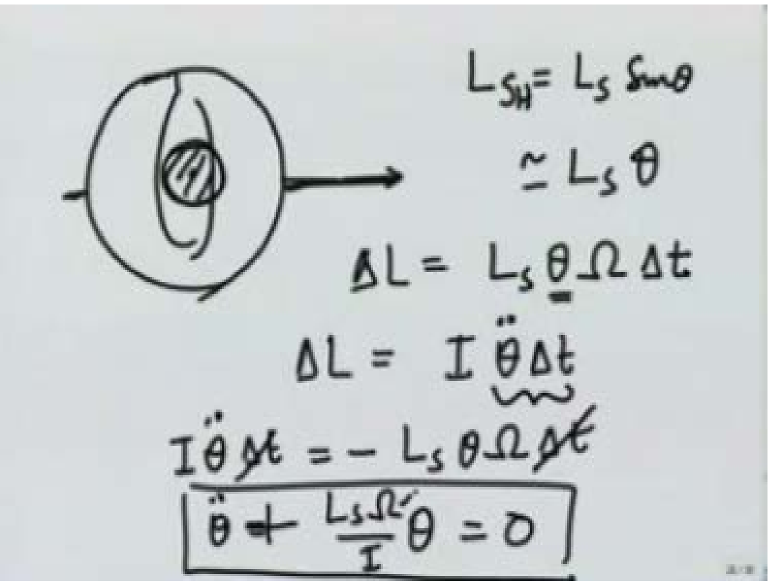

To make it quantitative, let me make this outer frame. let me make the inner frame and let this wheel, inner wheel is such that its axis is making an angle theta with this, this direction making, making an angle theta coming out of the plane of the screen. If this angular momentum is L s, that is the angular momentum of the inner wheel, this wheel spin is L s, its horizontal component coming out of the screen is going to be L s sin theta, L s horizontal I am assuming it is making an angle with this.

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So, what I am assuming is here is the axis and this fellow is making an angle like this, with this axis so, this angle. So, therefore, the component in this direction is going to be L s sin theta and along the axis is going to be L m cosine theta. L s sin theta if this theta is very very small is going to be roughly equal to L s theta.

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Now, when I give the outer frame a rotation, you see this component changes, this components develops a delta L in this direction and how much is that delta L? You can see this is horizontal it is going to go down, it is going to be outer omega times delta t times this component itself. So, delta L is going to be equal to Ls theta times omega delta t this way. However, there is no torque in this direction and therefore, delta L in this direction should be 0, what would happen? This frame would rotate in this manner so that, it develops a delta L in this direction.

So, it will gain an acceleration or velocity in this direction so that, this delta L gets 0 and that gain should be delta L is equal to I, whatever I is about this axis theta double dot because, it is going to go that way, it will change theta times delta t, this is I omega. In the direction opposite to theta and therefore, I should have I theta double dot delta t is equal to minus L spin theta omega delta t.

This cancels and I get an equation for theta, theta double dot is equal to is plus L s omega over I theta is equal to 0. This is like a harmonic oscillatory equation and therefore, if it is a spinning and I make an omega, it should oscillate about that omega axis.

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Let us see that, this is a small angle let me give it a spin like this and let us rotate the outer frame and you see this is oscillating.