Engineering Mechanics
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Module - 07
Lecture - 04
Rotational Motion – IV

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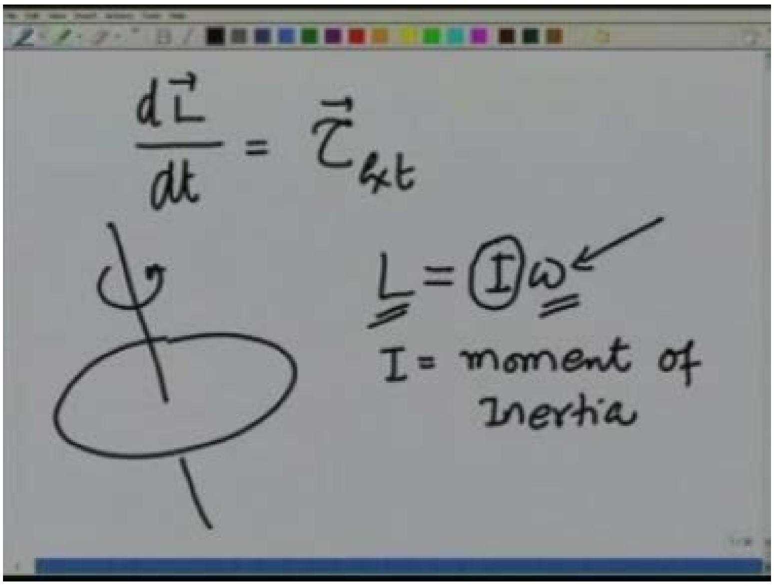

In the previous 2 or 3 lectures what we have learnt is that the dynamics of a rigid body is governed by the change in its angular momentum, by the externally applied force.

However, we dealt with simple problems where the axis of rotation was fixed in space and the body was rotating about it. In this case, the entire relationship became very simple and we showed that L equals I omega, where I is the moment of inertia and omega is the angular speed. In the fixed axis rotation, where the axis was either fixed in space or it could move parallel to itself or we could in rotations was change the magnitude of omega and therefore, change the magnitude of L.

We also applied the principle of conservation of angular momentum, where we in 1 demonstration change the value of I by pulling my hands in and out. All we did in these problems was only change the momentum, the magnitude of angular momentum and angular velocity.

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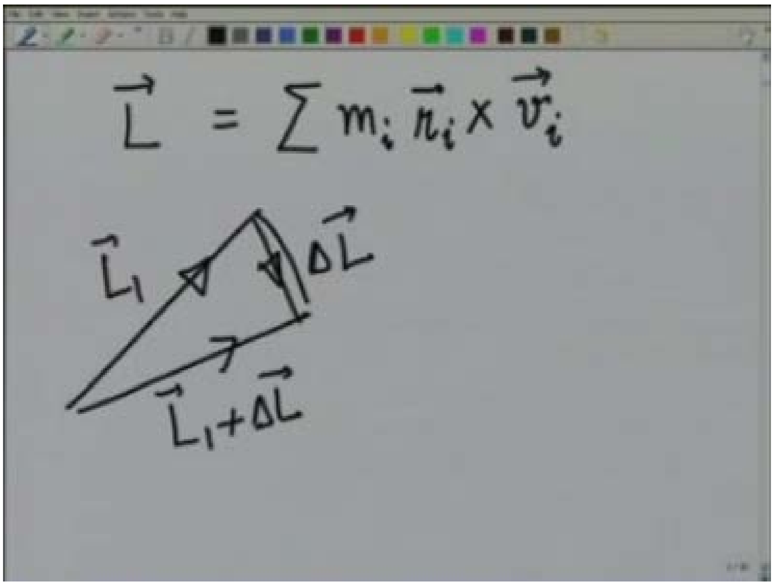

However, when we started we wrote L as a vector quantity which is equal to, for a given distribution of masses mi, ri cross vi. So, in principle if there is a vector L, I should also be able to cause a change in it not just by changing its magnitude, but changing its direction. So that, if this was L 1 and this was L 1 plus delta L, this could be the change delta L and its magnitude. In that case the axis of rotation may also change and these kind of changes in delta L give rise to a little more complicated dynamics, the body may change orientation in many different ways. This I show by a few demonstrations after this.

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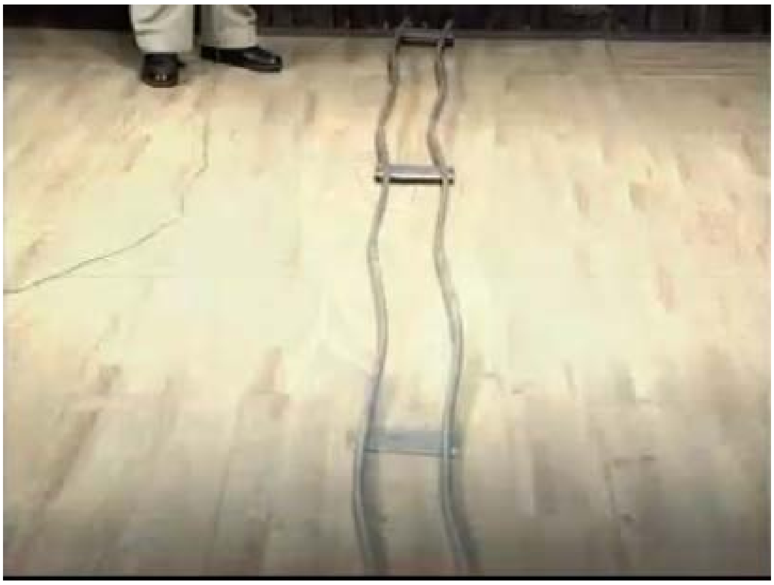

In this demonstration, I have these four cylinders let me show the shapes clearly to you.

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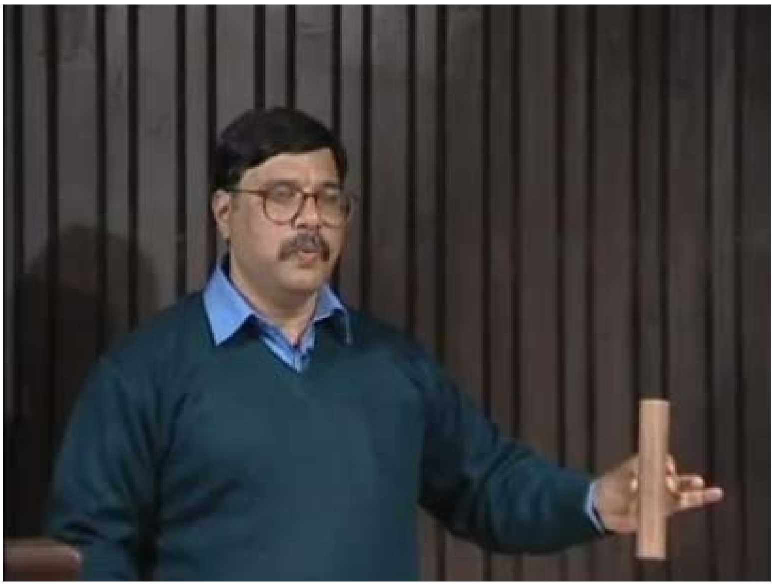

One is plane cylinder, the other 1 is cut like this.

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The third 1 is like this.

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And the fourth one is shaped curve like this.

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The question we ask now is, which of these cylinders is going to go along this curved path when rolling from this side to that side and let us see what happens. Let us first take this plane cylinder and roll it.

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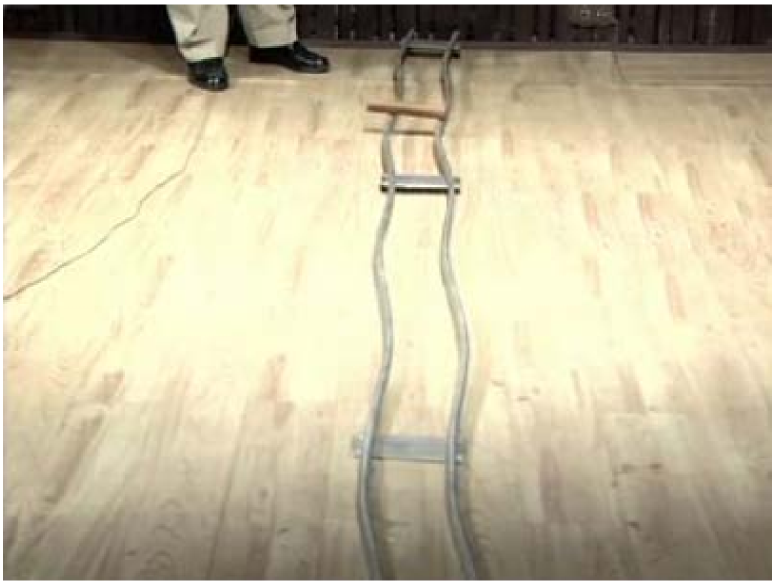

And you see it go straight, but falls out of the tracks after some time. Let us look at this cylinder and let this roll and let us see what it does.

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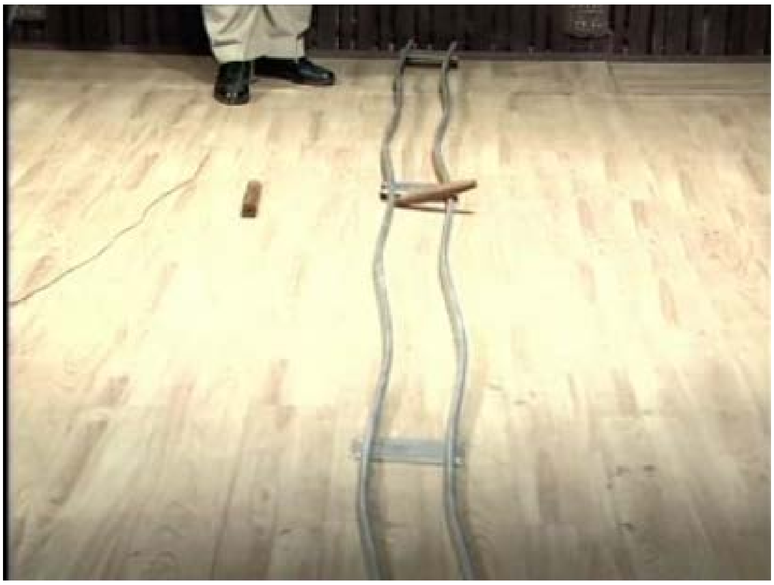

It rolls and you see it curves along the curve, the curves properly let us do it once more.

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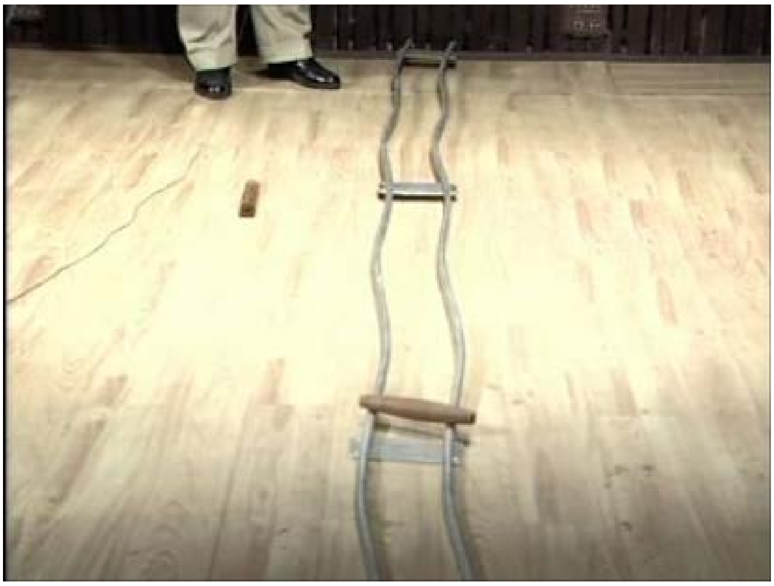

You see this one goes through clearly, how about this cylinder?

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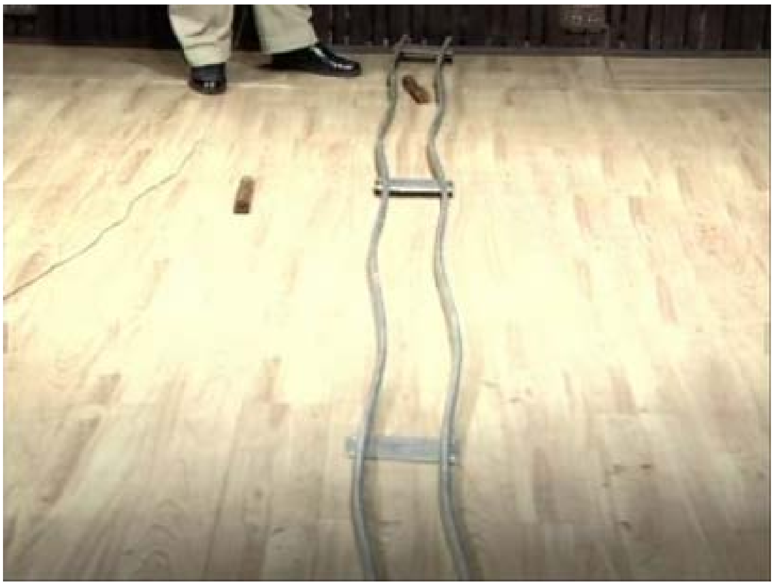

This goes over to one side and about this cylinder?

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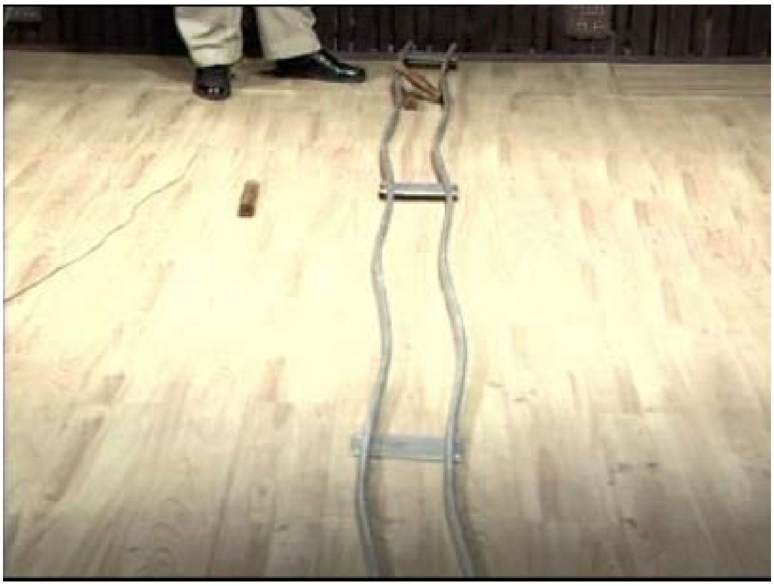

This also falls over to one side.

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In this demonstration I had this bicycle wheel, which can spin on its axis. Let us see what happens when I put this end here.

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I pivot it here it falls down.

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If I leave it, it will fall down. Let us see what happens if I give it a spin, if I give it spin and leave here and you see it rotates.

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If I give it the spin, the other way you see it will rotate the other way.

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So, what we observe is that when a rotation is given instead of falling down, this starts going around like this and this is known as precession, if I make it go faster it goes around slower.

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If I let it go slow then, it goes around very fast.

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Slower it gets faster it goes. So, these are the things that we should be able to explain using rigid body dynamics. In this demonstration, what I want to show you is if this wheel is not rotating, I can lift it up like this.

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You see when I apply a torque like this, it goes like this.

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Now, let me spin it and try to take it up, you see does not go up.

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It is going sideways, if I push it down it goes sideways this way.

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Whereas, when it is not spinning I can take up and down by applying a torque like this.

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The moment I give it a spin, I give a torque up it goes that way.

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I give a torque down it goes this way.

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This is again a manifestation of rigid body dynamics and how torques and angular momentum interplay makes the dynamics very interesting.

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Here I have a device known as the gyrocompass, in this there are 2 frames that can rotate about 2 perpendicular axis independently, this frame can rotate like this about this axis.

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This frame can rotate like this about this axis.

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And there is a spinning wheel, which can rotate about axis perpendicular to both of these like this. So, in a way X axis, Y axis and Z axis here three independent axis about which rotation can take place.

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Now, I give the inner wheel a spin and you see what happens when I rotate this. If I rotate this you see the spinning wheel aligns with this rotation, let me show to you again.

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I give the inner wheel a spin, when I rotate this, the moment I rotate the outer frame the spinning wheel the spin axis aligns with the outer frame rotation axis. See it again, I give it a spin rotate it the moment I rotate it, it aligns with this.

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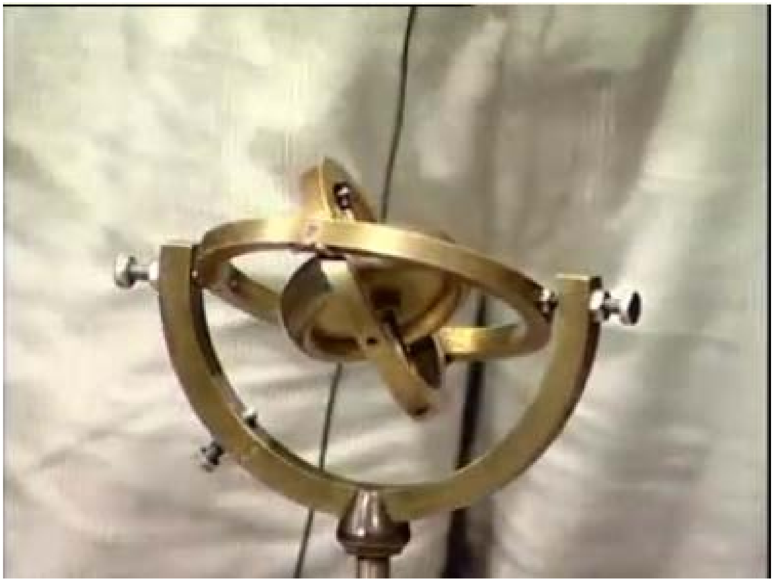

This can be used as a compass, another interesting aspect in rigid body dynamics is I have a box of sweets here.

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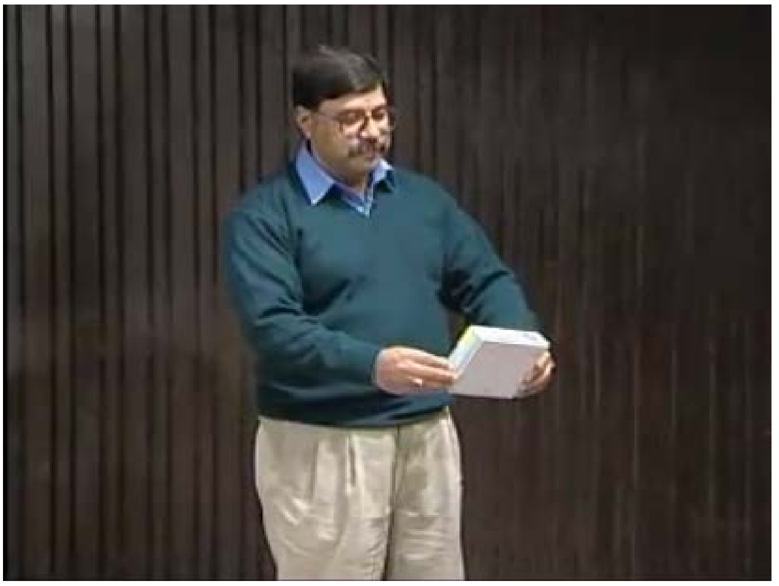

With three sides unequal, observe when I give it a rotation like this.

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And let it drop, it drops observe it carefully it drops pretty much rotating about the same axis.

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On the other hand, if I give it rotation about this.

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This, axis it again rotates in a very stable manner and drops.

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Observe now, when I do it about this axis what happens.

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By the time it comes down, it has started rotating about all axis observe it carefully.

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It has started tumbling, we should be able to explain this using rigid body dynamics equations of motions.

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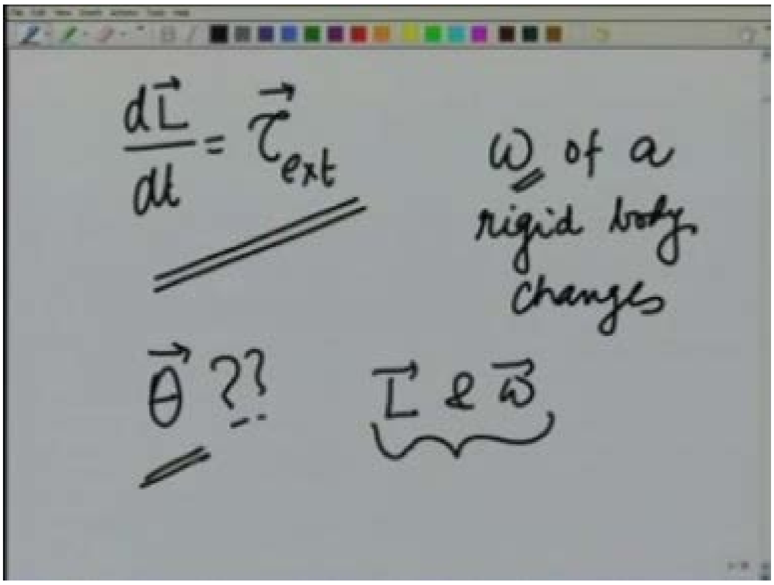

Whatever I have shown in these demonstrations, can be easily explained, by using the equation that rate of change of angular momentum is tau external. Not only that, you see I was also while doing this demonstrations showing you how angular speed or angular velocity of the body was changing. So, after having solved this equation I should also be able to tell you how omega of a rigid body changes and therefore, with time how the body changes its orientation. To develop the theory of this, we need to be very very specific about what does omega represent. Is it a vector, it is a scalar? How about the orientation being described by vector theta, can we do that and how are L and omega related?

So that, once I calculate how L is changing I could relate it to change in omega and from omega I can find how the angle of a body with respect to different axis is changing. These are questions for which the answers come when we develop relationship between L and omega. We see how when L changes how omega changes, we see how theta is related to omega and so on so, we do that now.

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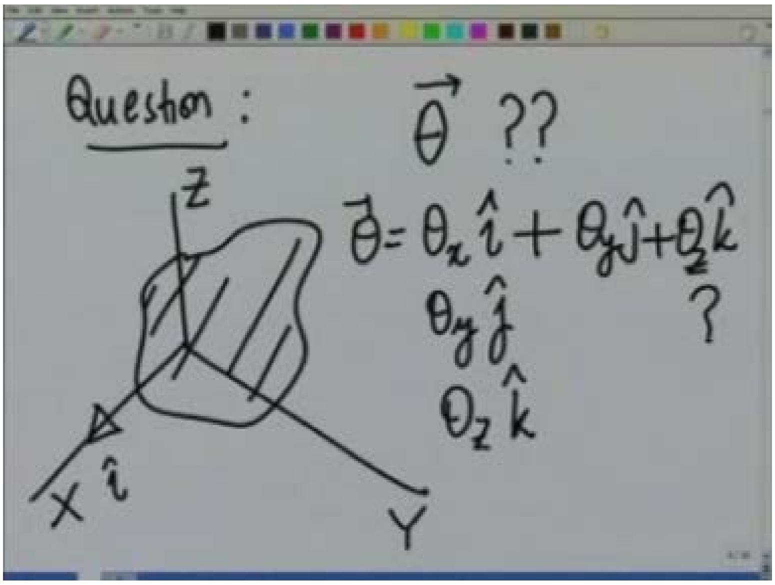

The very first question I am going to ask in this is, if I take a rigid body and rotate it with respect to axis X, axis Y and axis Z can I take its rotational angle theta as a vector. I would give the rotation about x, a direction along x axis rotation about y, a direction about y axis and rotation about z, a direction about the z axis. The question is do these quantities together, if I take this plus theta y j plus theta z k does it constitute a vector quantity and the answer to this in 1 word is no.

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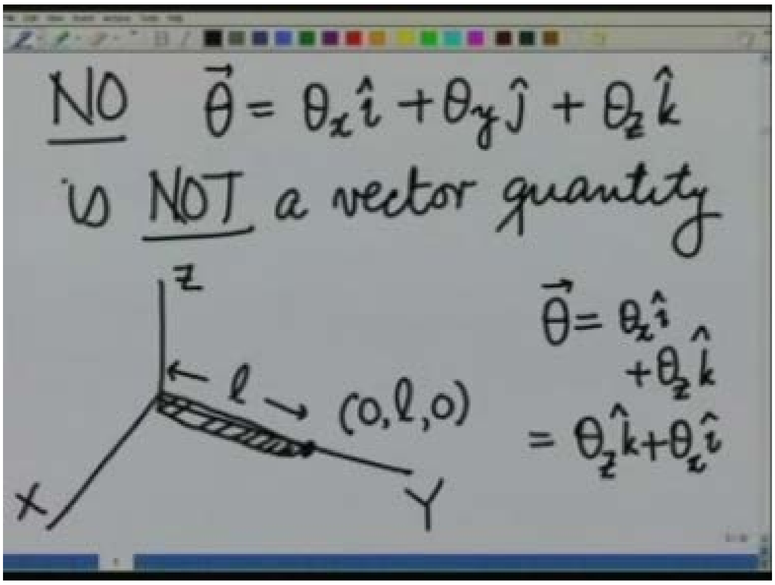

Theta, if I take this to be as theta x I plus theta y j plus theta z k is not a vector quantity and these and for this is very simple as I will now illustrate. Let us take a small rod thin of length l, along the y axis so that, it is this end has coordinates 0, l and 0. I will now do two operations on it, I will first rotate it about the X axis and then about the Z axis, in counter clockwise sense and then do it the other way, I will first rotate it about the z axis and then about x axis. If theta is a vector then, theta if this is given as theta x, I about the X axis plus theta z k, it should not really matter whether I do it this way or do rotation about the Z axis first, Z axis first and then about the X axis. Let us see if this comes out to be the same.

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So, let this be the thin rod, let this be the Y axis, let the X axis be coming towards me X Y and this is the Z axis.

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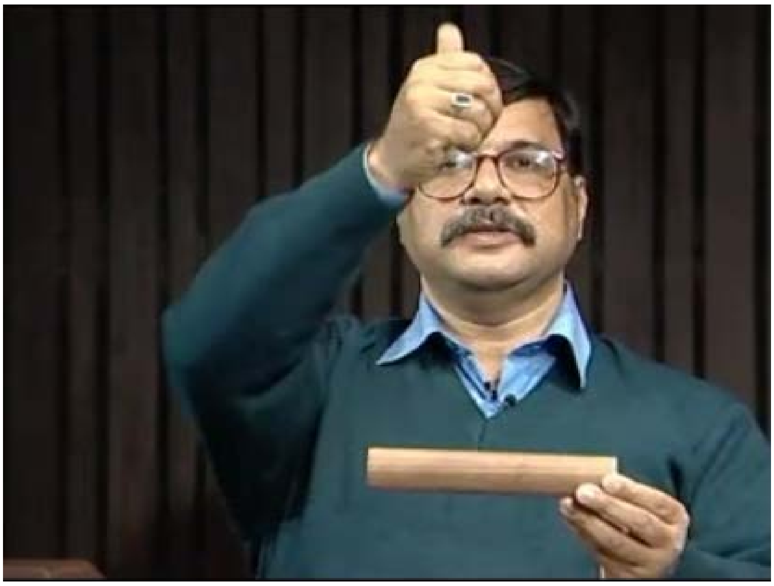

X axis, Y axis and Z axis. Let me first give it a rotation, counter clockwise sense about the X axis. So, X axis is coming this way counter clockwise sense would be like this.

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And let me now rotate it by 90 degrees about the Z axis. So, this is how it looks. Let us now do the same thing, first about the Z axis and then about the X axis. So, this is the Z axis let me rotate it by 90 degrees about the Z axis then, it rotates like this and about the X axis it if I rotate counter clockwise by 90 degree it rotates like this. So, you see in the two operations if I carry out the rotation about X axis first and then about the Z axis the rod ends up rotating In this final position.

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On the other hand if I give rotation about the X axis first, about the Z axis first and then about X axis, it ends up coming in this position.

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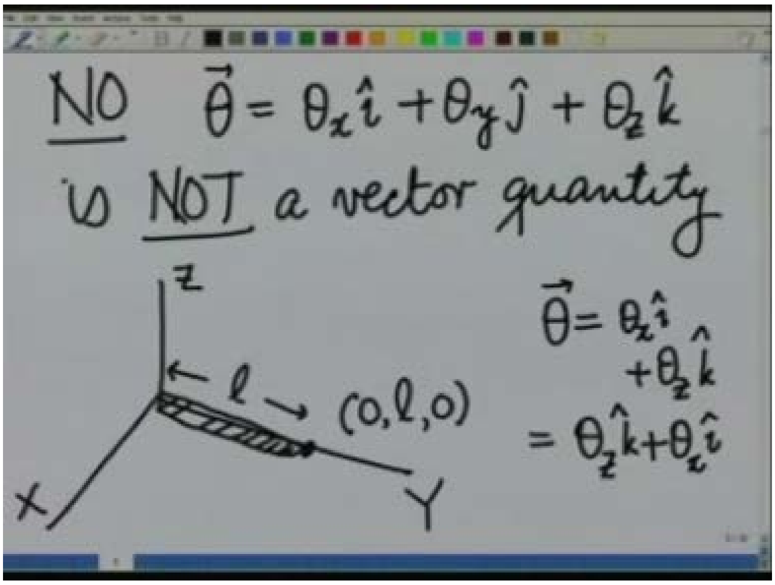

That means, the two theta x I plus theta z k is not the same as theta z k, plus theta x I. Let me show it again.

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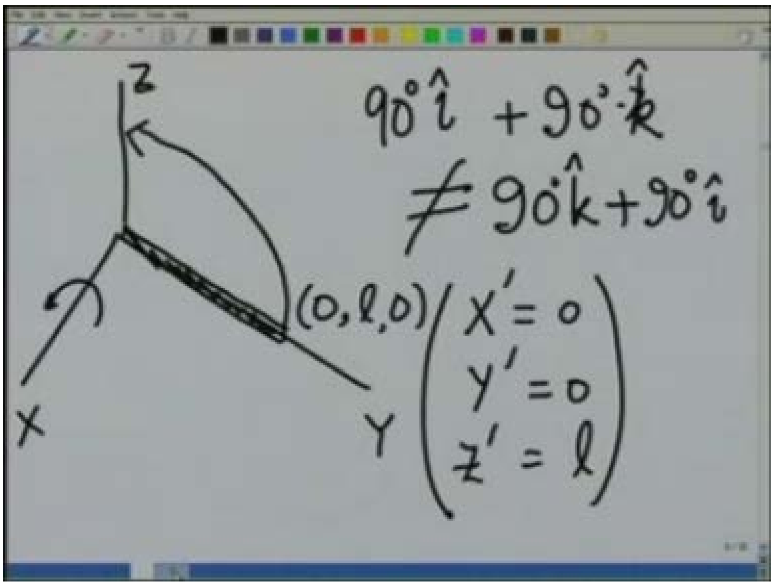

So, what I did was took the X axis, Y axis and Z axis, took a rod along the Y axis. So, that is coordinate was 0, l and 0 then, I first give it a rotation of 90 degrees about the X axis in counter clockwise sense. And then, rotated it about the Z axis again by 90 degree let me write this k. This was not the same as rotating it first about, the Z axis and then rotating it about the X axis. Let us see it mathematically, if I rotate this rod about the X axis counter clockwise, the new coordinates of this would be X prime would remain 0.

Y prime would become 0 and Z prime would become l, after the first rotation. After the second rotation about the Z axis nothing really changes, only it is, it has just rotated a bit. On the other hand let us see what happens if I rotate it about the Z axis first and then about the Y axis.

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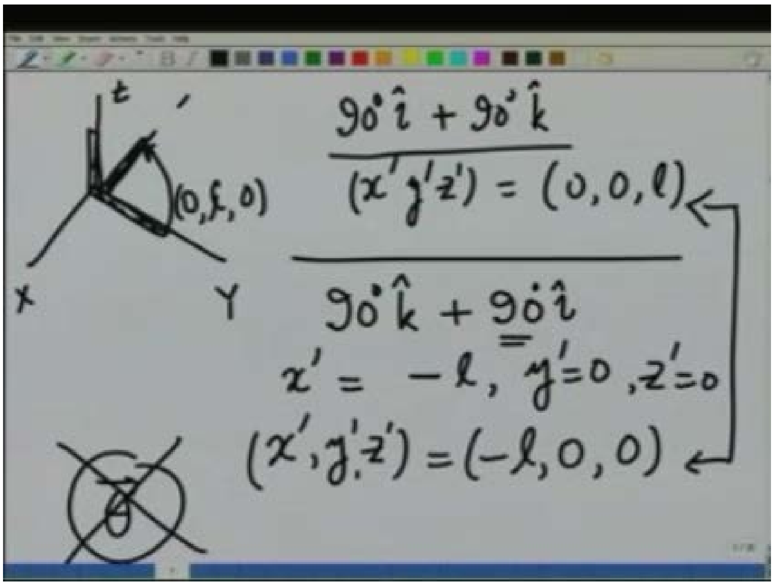

So, let us write this x y z this was the rod at coordinate 0, l and 0 if I did 90 degrees about the X axis plus 90 degrees about the Z axis, the coordinates after this rotation came out to be 0 0 and l. So, rod was in this position, let us now do first 90 degrees about the Z axis and then, 90 degrees about the X axis. When I rotate it 90 degrees about the Z axis counter clockwise, it will go this way. So that, its end would have coordinate x prime is equal to minus l, y prime is equal to 0 and z prime equal to 0 after that rotation if I rotate it by about the X axis nothing really changes.

So, the coordinates of the end are x prime, y prime z prime equals minus l, 0 and 0. You can see that the two sets of coordinates do not match at all. What that means is, that the two rotations I cannot really represent this. This as a vector it has no meaning because, it changes according to how I apply this rotations in what order I apply this rotations. So, angle theta cannot really be a vector, but there is a saving rays.

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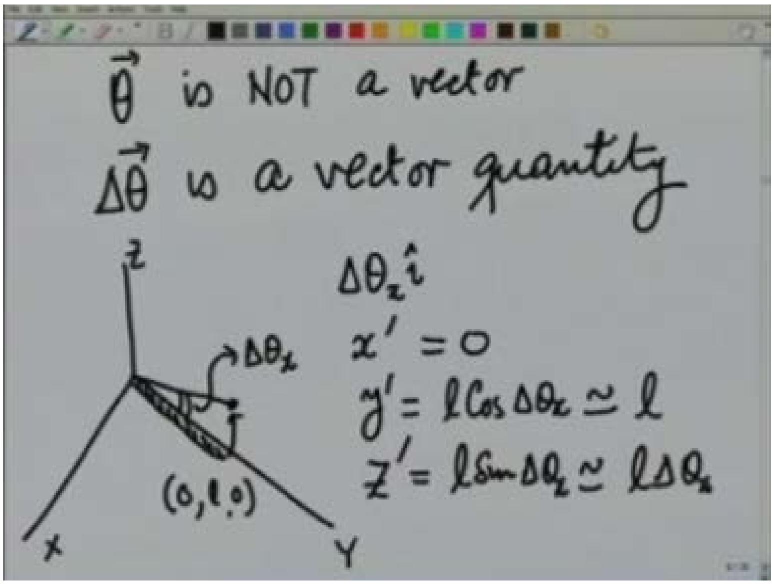

Saving rays is that although angle theta, a finite angle theta is not a vector any infinitesimal change delta theta is a vector quantity, how? Let us see that, again we go back to our rod. This is my X axis, this is Y axis this is Z axis. So, what I am going to do is apply a very small rotation delta theta x about the X axis. So a rod is rotated like this, after this rotation what about its new coordinates of this end to start with this was 0 l and 0. You will see that x prime remains 0, this angle is delta theta x you can see y prime is equal to l cosine of delta theta x, which for very small delta theta x is really l and z prime is equal to l sin of delta theta x which for very small delta theta x is l delta theta x.

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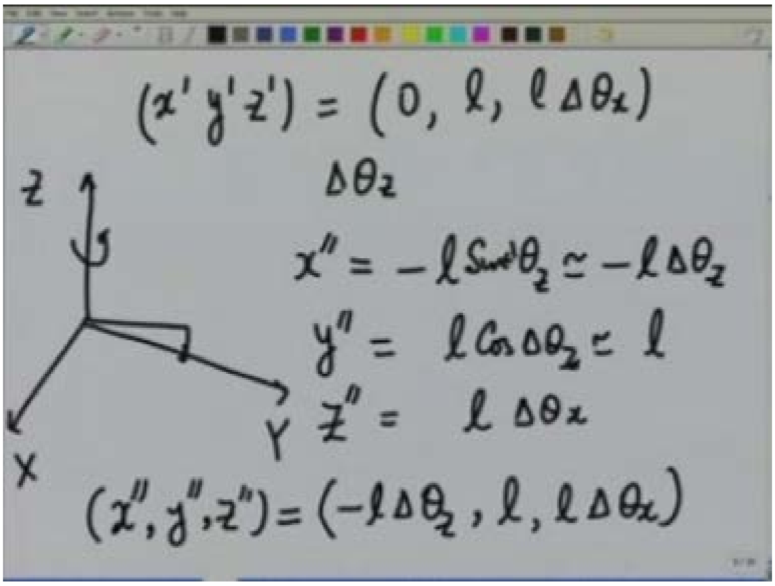

So, the new coordinates after the first rotation for the end of the rod that I get are X prime, y prime, z prime equals 0, l and l delta theta x. This is after I gave it the first rotation, about the X axis X, Y and Z. Now let me rotate this rod about the Z axis by an angle delta theta z. When that happens, you can see that x double prime which is after the second rotation is going to come from this projection going back my angle delta theta z. So, this is going to be minus l sin of delta theta z which is roughly equal to minus l delta theta z, y double prime again is not going to change.

So, this will remain l cosine of delta theta z which is roughly equal to l and z prime is also not going to change because, now its rotation about z axis. So, this is going to remain l delta theta x. So, the new coordinates that we get after the 2 rotations have been done that is a small rotation about the X axis by an amount delta theta x. And followed by a small rotation about the Z axis by delta theta z, gives you a new coordinate starting from 0, l 0 coordinate, rod starting from this position is minus l delta theta z l and l delta theta x. This is going in the order when I rotated first about the X axis and then, about the Z axis let us see if I change the order what happens.

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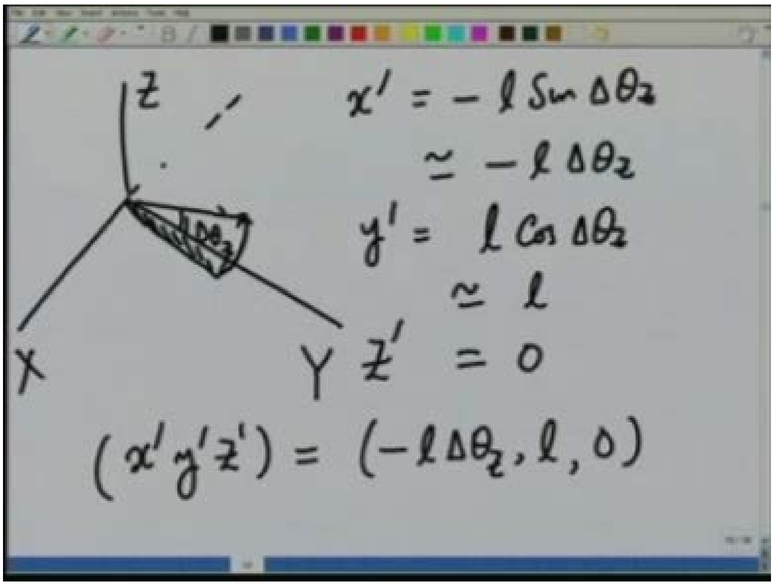

So, now what I am going to do is take the same rod X axis, Y axis, Z axis first give it a rotation about the Z axis, in the x y plane by an angle delta theta x counter clockwise. So, when it rotates like this in the x y plane, you can see that x prime this will be like this in the x y plane, is going to be this projection. This is delta theta z, is going to be minus l sin delta theta z which is roughly equal to minus l delta theta z, y prime is going to be l cosine of delta theta z which is roughly equal to l. Since this is a rotation about Z axis z prime remains unchanged. So, after the first rotation about the Z axis, I have x prime y prime and z prime is equal to minus l delta theta z l, 0. Now, let me give it a rotation about the X axis.

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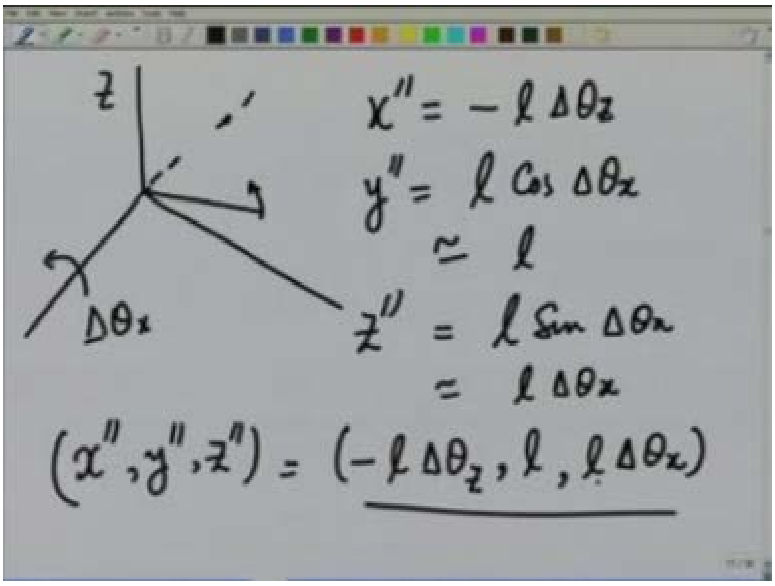

So, now I am starting with the rod in x y plane like this and give it a rotation about the X axis by amount delta theta x. Since I am giving a rotation about the X axis, the x coordinate cannot change. So, x double prime remains as it was earlier minus l delta theta z, y double prime would remain l cosine of delta theta x which is roughly equal to l and z prime. Now it is moving up, l length is moving up and therefore, is going to be equal to l sin of delta theta x because, now this will be moving up like this in the y z plane and this will be equal to roughly l delta theta x.

So, after these three rotations I end up getting x double prime, y double prime, z double prime is equal to minus l delta theta z, l, l delta theta x which is precisely the same, which I got when I had the other order first rotation about the X axis and then rotation about the Z axis.

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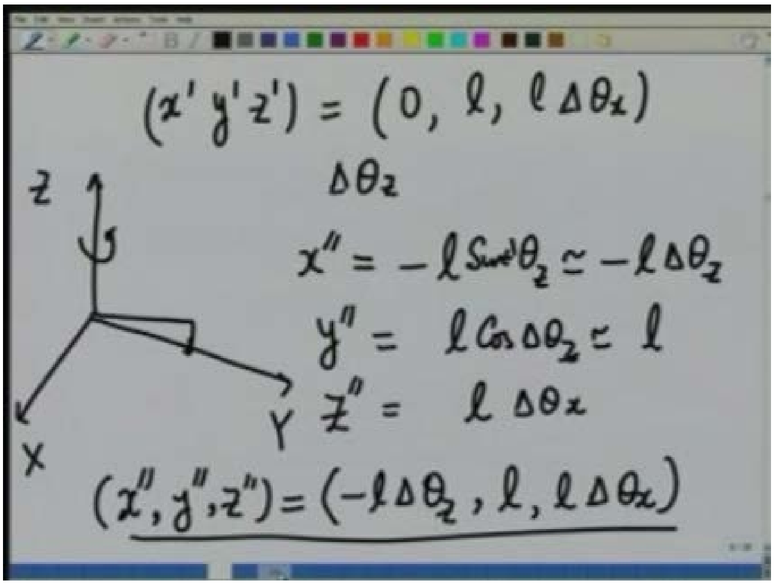

Let us see that as in the previous slide somewhere, here it is. This was earlier and here it is now, this is now. So, earlier minus l delta theta z l, l delta theta x and now, minus l delta theta z l and l delta theta x. So, when I take infinitesimal rotations when the second order product delta theta x, delta theta z vanishes what I get is that.

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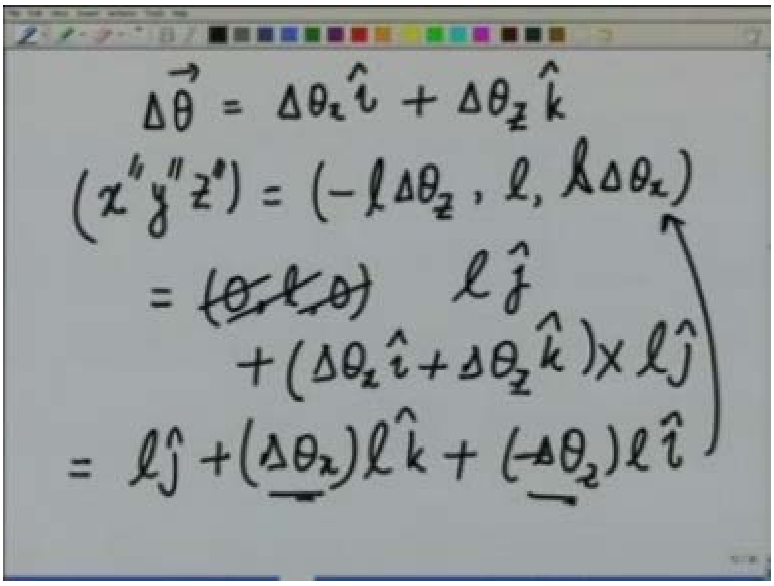

Delta theta I can write as delta theta x about the X axis, plus delta theta z about the Z axis, does not matter which all by applying, I get still the same answer I get x double prime, y double prime, z double prime as equal to minus l delta theta z, l and l delta theta

x. Which I can write as the original 0 l 0 this is initial vector. Let me write this in the vector form, l j plus delta theta x I plus delta theta z k cross the original vector, which is l j this would be by the definition l j plus I cross j is k delta theta x times l k plus k cross j is minus i.

So, minus delta theta z l I which is precisely the same as this. So, what we understand is that when I apply very very small rotations, the rotations commute and I can represent them as vectors. Although I took here the example of delta theta x and delta theta z, which are rotations about X and Z axis, I could have chosen any other combinations. And shown that even if I apply a third, small rotation about the Y axis they will all commute.

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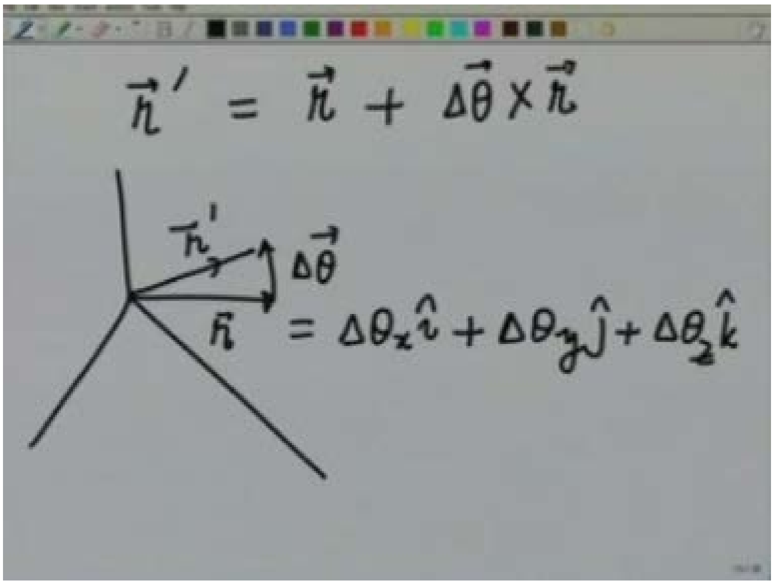

So, what we get from this is that I can write the new vector r which I obtain if I had a vector somewhere, let me generalize the result and I rotated it by a small angle delta theta. Which, I will write as a vector which could be delta theta x, about the X axis delta theta y about the Y axis plus delta theta z about Z axis. Then I could write r prime as equal to the original vector r, this is the original vector and this is r prime plus the change in r which will be written as delta theta cross r, having shown this let me now show it in a slightly different way.

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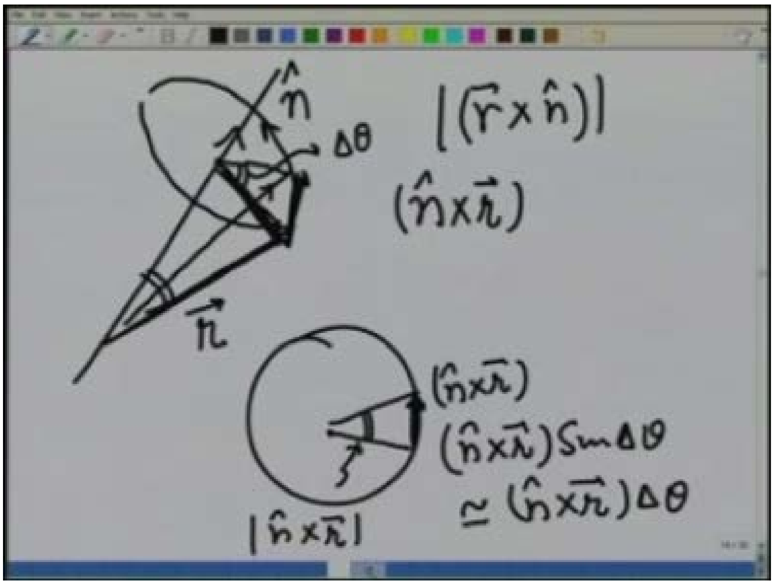

Let us take an axis of rotation, in direction n unit vector at direction n let there be a vector r although I am writing it for r, r could any general vector and let this vector r rotate about this axis like this. So, that after infinitesimal rotation about this axis, its new position is here. It has rotated by an angle delta theta about the axis n, how much is this change. For very small angles this change can be written as this length is going to be r cross n magnitude because, this is going to be this length is nothing but this sin of this angle which is nothing but r cross n magnitude. This direction is perpendicular to this and you can see that this is actually n cross r direction.

So, what we see if I make the circle here this magnitude is n cross r magnitude and it has change by this amount in the direction of n cross r by an amount n cross r sin of delta theta. Which is roughly equal to n cross r delta theta and this is your delta r because, this is how much it has change, let me make the figure again.

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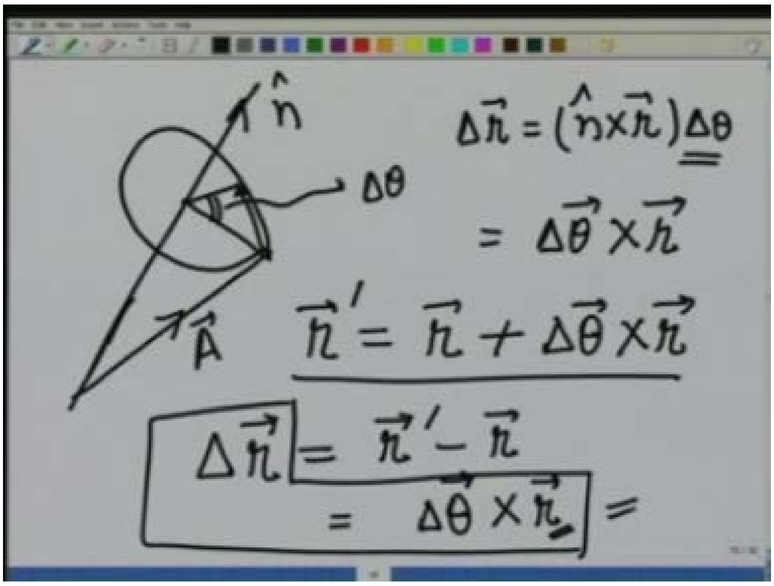

I have taken an axis in the direction of unit vector n, taken a vector r which is rotated by a small angle delta theta and I want to know this change. This direction is automatically n cross r and this change delta r is n cross r right direction, right magnitude of this radius times delta theta. And therefore, taking the direction of delta theta along the axis of rotation I can write this as delta theta cross r and therefore, the new r r prime is really the original vector plus delta theta cross r what I have shown you is in the previous derivation.

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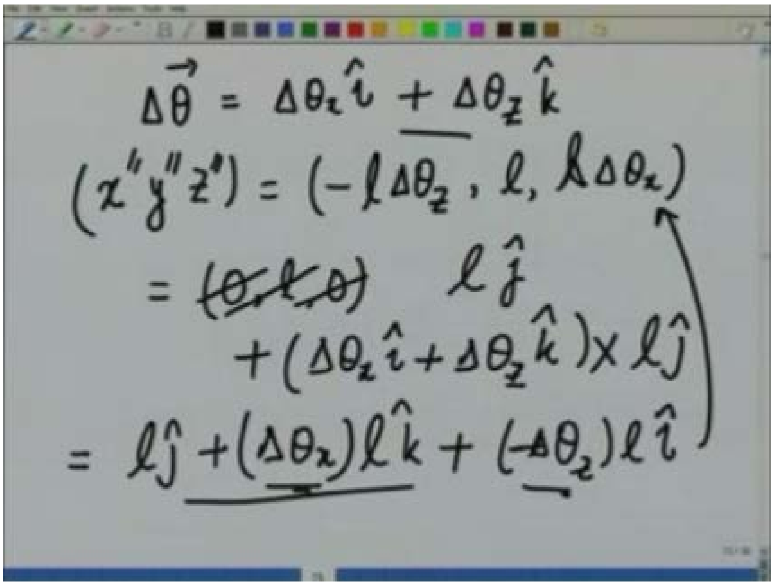

Where I wrote delta theta is this and then derived delta r and after that wrote r prime equals r plus delta theta r. Now, I have taken a general axis of rotation n and shown again that r prime can be written like this. Therefore, this is the general result that if a vector rotates about an axis in unit vector direction n by a very small angle delta theta, I can write the change as this cross product. This is an absolutely general result, although I have written there here taking r as a vector this could in general be any vector A. The change would still be this.

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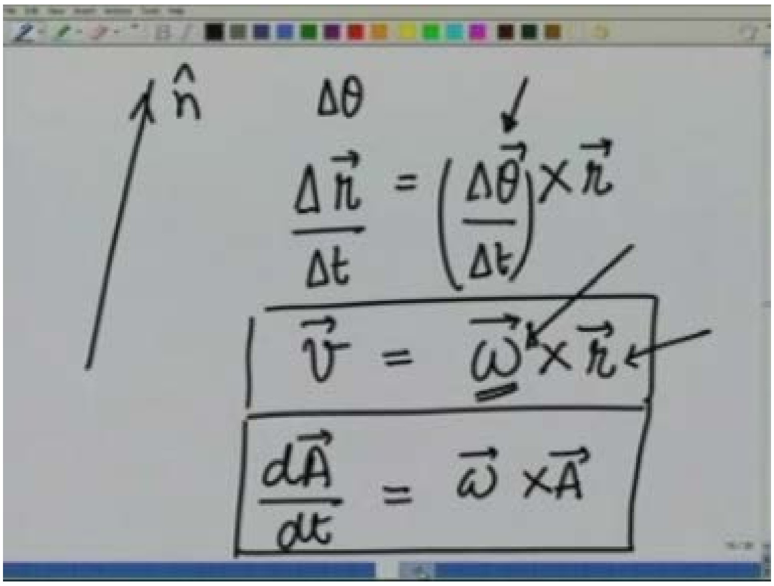

So, this is a first general result we get that for a rotation about an axis pointing in direction n, a small angle delta theta rotation gives rise to change delta r is equal to delta theta cross r. If I divide by delta t, I get velocity is equal to this, I write as the angular velocity omega cross r, this is being very very specific to the vector displacement vector r. In general, the rate of change of a vector rotating about an axis pointing in direction n is going to be equal to omega cross A, another general result.

These are very very useful result for use in rigid body dynamics and we will be making use of them quite frequently. Just like delta, theta can be represented as a vector delta theta over delta t can also be represented as a vector, which I now call angular velocity.

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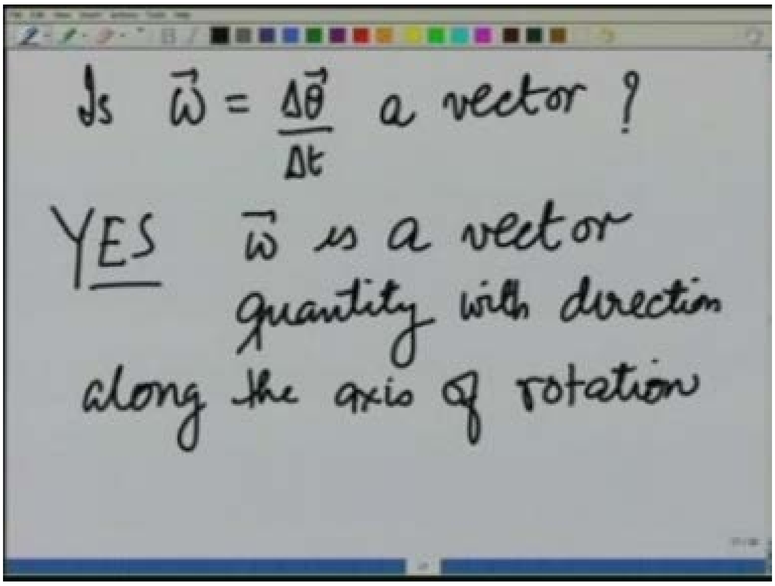

So, second thing is angular velocity which is delta theta over delta t, a vector the answer is yes. Omega is a vector quantity with direction along the axis of rotation and the sense is the again right hand rule, if the something is rotating like this the thumb gives me the direction of omega. You may ask so what, I have identified omega as a vector quantity, how does that help me. See in physics when I take a quantity it has to be either a scalar or a vector then, I know exactly what kind of algebra I can perform on this.

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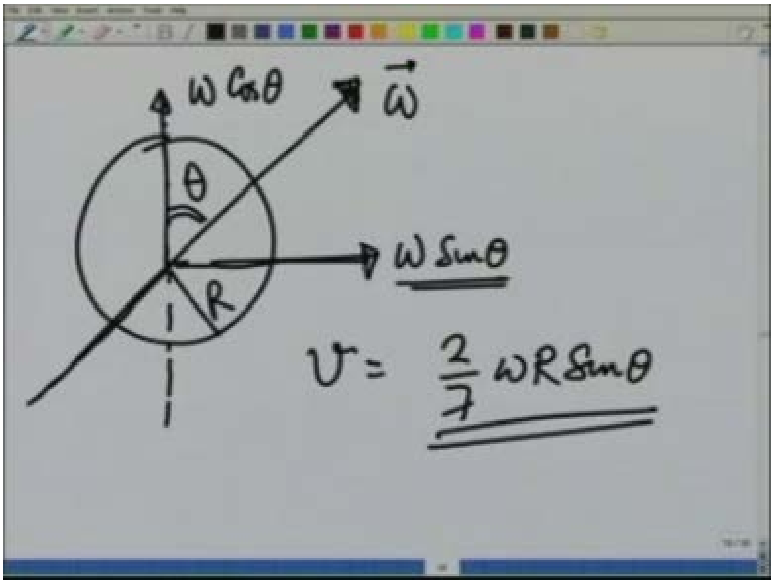

So, identifying the angular velocity as a vector quantity does give me a handle of handle on things. For example, let me just illustrate this with a very simple example if I take a ball, a solid ball give it a spin omega about an axis and leave it on the floor which has friction at an angle theta from the vertical. The axis of rotation makes an angle theta with the vertical and now I ask what would be the eventual rolling speed of this ball.

I know if omega points exactly up then the ball cannot roll, eventually it will stop I know if it is pointing this way it will roll with certain speed, that we can calculate. How about when makes an angle. Now, I make use of the vector nature of angular velocity, take its component in this direction omega sin theta and take its component in this direction omega cosine of theta. And these are 2 independent vector quantities, omega sin theta omega cosine theta, omega cosine theta cannot contribute to rolling at all. So, eventually due to friction it will stop rolling, it will stop rotating like this.

On the other hand omega sin theta is responsible for rolling. So, the question I may ask now what if I take a ball and rotate it with a speed omega sin theta and leave it on the floor with the axis of rotation horizontally. What will be its rolling speed? You can calculate that using the techniques that we used in the first three lectures, its final speed comes out to be omega equals two-seventh omega R sin theta if R is the radius of the ball. So, you see we could do this right away because we identified omega as a vector quantity and could take its component and realize that only the horizontal component contributes to rolling.

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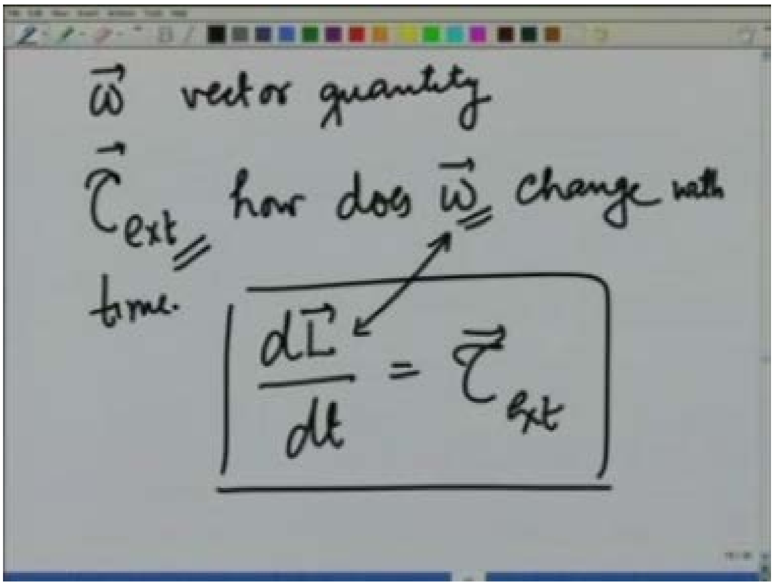

Now that we have identified the angular velocity omega, as a vector quantity. The next question I ask is if to a rotating body I apply an external torque tau, how does omega change with time, the equation I know is that d L over d t is equal to tau external. So, to relate the change of omega with tau external, I need to relate L and omega the angular momentum and omega and this is what we do next.

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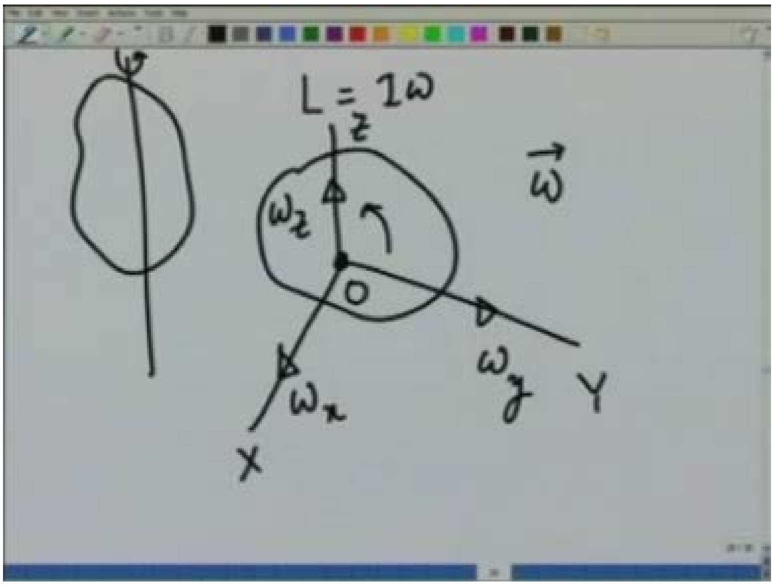

We have already done it in simple cases where we wrote about, about when a body was rotating about a fixed axis, that L was I omega, but now we do it in general. Suppose we take a body, which is fixed at 1 point and it is rotating with some angular velocity omega. So that, if I take X, Y and Z axis there is a omega x there is a component of omega in this direction omega y and there is a component of omega in direction omega z. What is this angular momentum about this point?

So, right now what we taking is a fixed point rotation, I know if the body is also translating, we can always go to the center of mass and calculate the angle of moment of center of mass and angular moment about the center of mass and we can do a motion, a general motion also. So, as long as I can do a fixed point rotation about the center of mass or any other point the general motion has no problem in it. So, let is now evaluate given a point o, what would be the angular momentum, given angular velocity omega.

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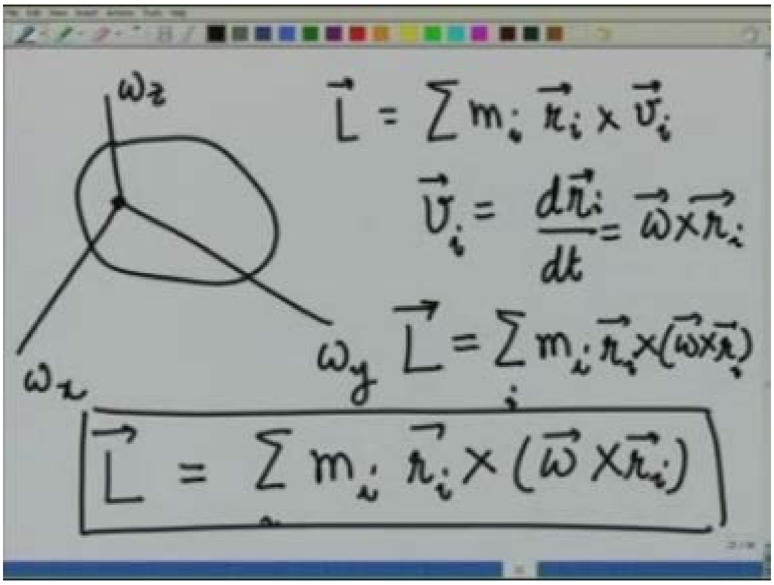

We again go back to the definition this is a body it is rotating about this point, with component omega y, omega x and omega z. The angular moment of vector L is given as summation mi ri for ith particle cross vi and we just saw that, I can write vi which is nothing but dr over dt as omega ri cross ri. And therefore, the angular moment of vector L is equal to summation I, mi, ri cross omega cross ri. So, this is the general vector L summation I, mi, ri cross omega cross ri and this is what I want to evaluate now remember now, omega has three components.

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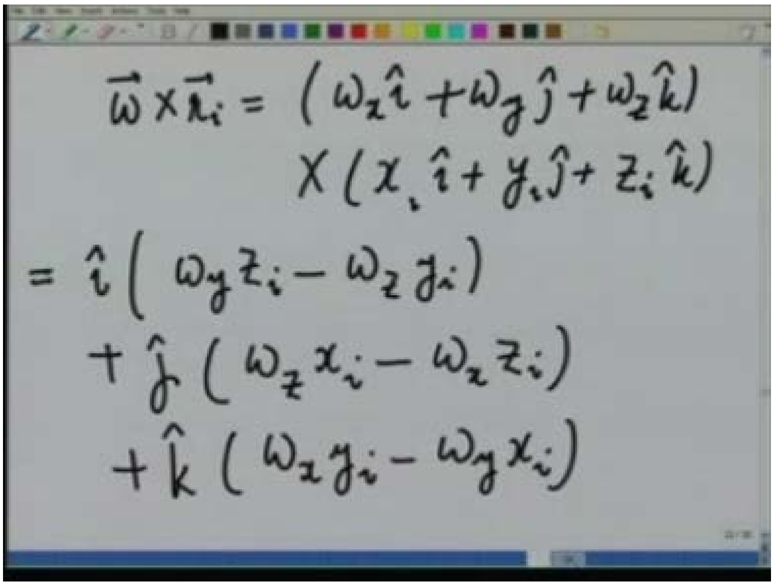

So, let us write omega cross ri first which is nothing but omega x in x direction omega y plus omega z cross x I, plus y I, plus z I k and this comes out to be in I direction. It will be omega y zi minus omega z yi plus j omega z xi minus omega x zi plus k omega x yi minus omega y xi.

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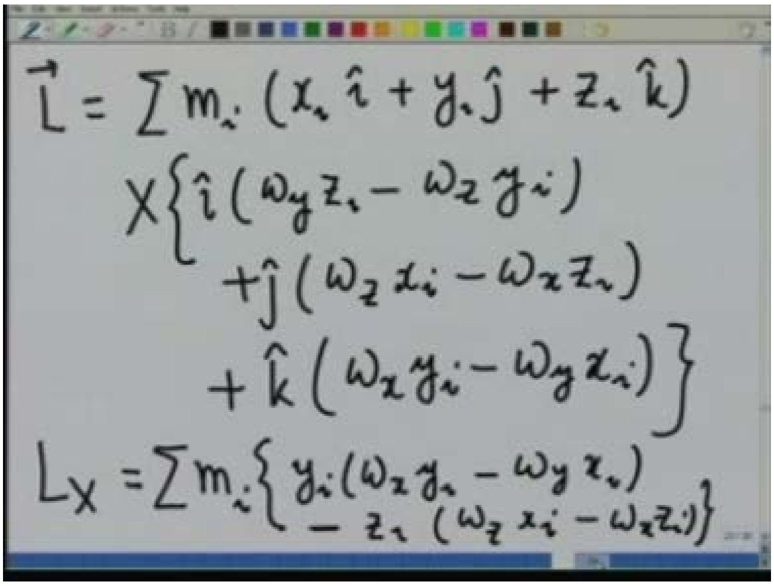

And therefore, the angular momentum L is going to be equal to summation mi xi i plus yi j plus zi k cross i omega y zi minus omega z yi plus j omega z xi minus omega x zi plus k omega x yi minus omega y xi. For simplicity let me first take only the x component of L, this will come to be summation mi for x component let us start multiplying j cross k would give me 1 component. So, that will be yi omega x yi minus omega y xi plus k cross j would give me a component, that will be minus zi omega z xi minus omega x zi.

When I collect like terms you will get omega x yi square plus omega x zi square minus omega y xi yi minus omega z xi x z zi.

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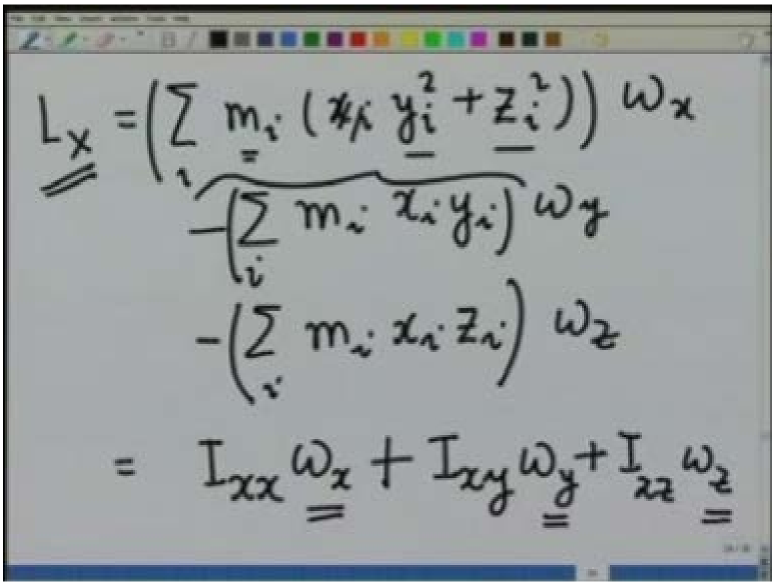

So, let us collect these terms and write Lx as I would go back and collect terms summation I mi xi no, this is yi square yi square plus zi square. So, it will be yi square plus zi square minus times omega x minus I, mi we will see again it will be xi yi omega y. So, we will write xi yi omega y and similarly, minus summation I mi xi zi omega z let me put these terms in brackets and identify them you recall this is the perpendicular distance yi square plus zi square of ith particle from the x axis.

So, this is nothing, but moment of inertia about the X axis let me write this as Ixx omega x plus let me identify this term minus summation I mi xi yi. Let me define this as product of inertia Ixy and multiply by omega y plus, last term I identify as product of inertia as Ixz omega z. So, you see relationship between the x component of the angular momentum and different components of angular velocity is slightly more involved. You can similarly do calculations for other components and I leave it for you to work out.

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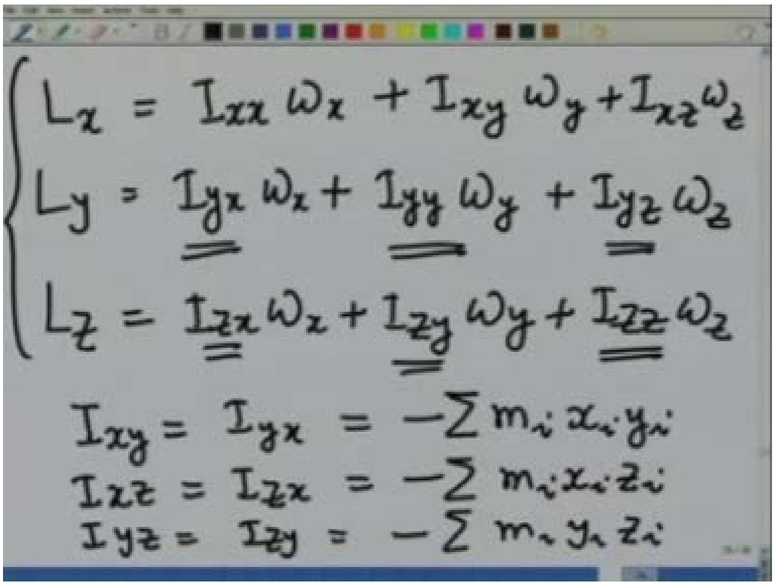

So, that you will find that Lx which you just evaluated is Ixx omega x plus Ixy omega y plus Ixz omega z. Similarly, you will find Ly would come out to be Iyx omega x plus Iyy omega y plus Iyz omega z where, Iyx is again a product of inertia involving mi yi and xi. This is moment of inertia about the y axis and this is product of inertia involving y and z and Lz similarly would be, Izx omega x plus Izy omega y plus Izz omega z. Where, Izz is the moment of inertia about the z axis Izy and Izx are products of inertia.

Let me define them Ixy is going to be equal to be Iyx which is equal to minus summation mi xi yi Ixz is going to be equal to Izx, which is equal to minus summation mi xi zi and Iyz is going to be equal to Izy, which is equal to minus summation mi yi zi.

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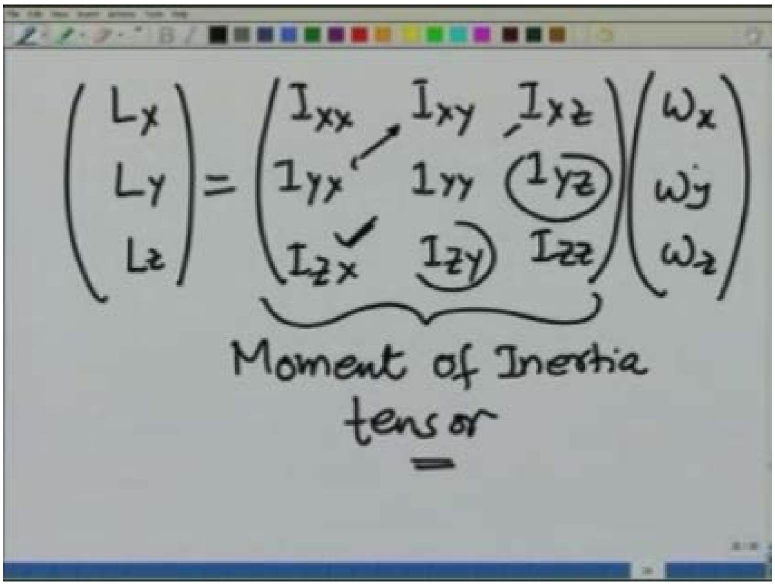

In general this relationship can be written as a matrix multiplication as vector Lx, Ly, Lz three components are equal to Ixx, Ixy, Ixz, Iyx, Iyy, Iyz and Izx, Izy, Izz a matrix times omega x, omega y, omega z and this quantity, this matrix is known as moment of inertia tensor. It is a tensor because, this has nine components which transform in a particular way. So, the situation is quite complicated Lx to calculate Lx, Ly and Lz we need these nine components well really six components because, these components are equal zx is equal to this and zy is equal to yz. Six components multiply by them omega x, omega y, omega z and calculate the angular momentum.

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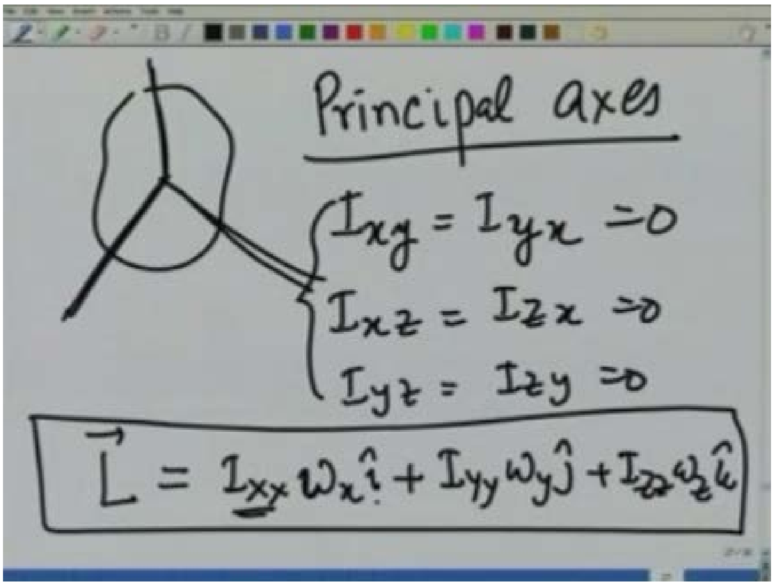

Fortunately for us there is a simplicity involved for any rigid body, about any point there exist a set of axes known as the principle set of axes. So, that the half diagonal elements Ixy, Iyx that is 0 Ixz, Izx 0 and Iyz, Izy is equal to 0 and therefore, L vector simply becomes equal to Ixx omega x I plus Iyy omega y j plus Izz omega z k.

For any rigid body any point I can find a set of axes, which are obviously oriented in a certain way about the body. So, they are very specific to the body and attached to the body. So that, the half diagonal elements of the moment of inertia tensor are 0 and L becomes assumes a very simple form like this.

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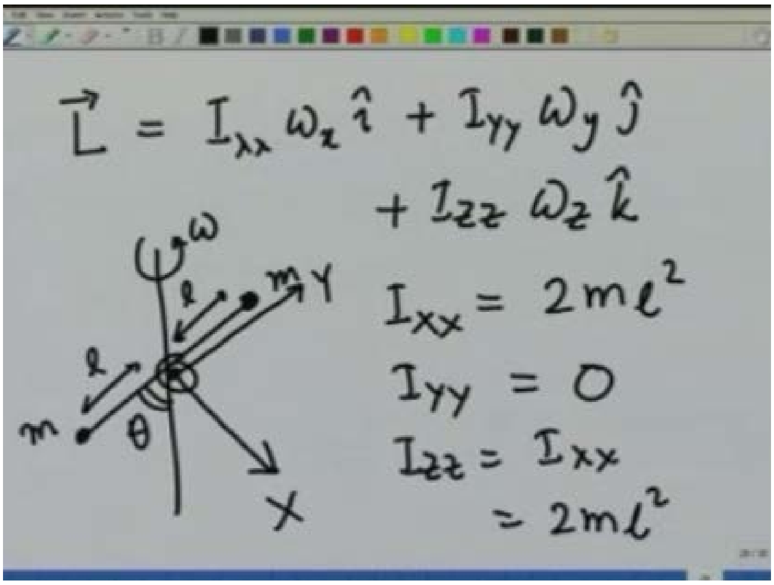

So therefore, I can write by taking components along the principle set of axes as Ixx omega x in I direction, Iyy omega y in j direction, plus Izz omega z in k direction. Let us take a few examples suppose, I take the old familiar example that we did in the very first lecture when we started rigid body dynamics. A rod having 2 masses m at distance l from the centre rotating with angular velocity omega about an axis, which is at an angle theta from it.

Let me take line perpendicular to it as the X axis, line along the rod as Y axis and obviously, then line coming out of the screen is going to be Z axis. By symmetry you can see that these are principle set of axes because along Y the x component is 0, you will find Ixy becomes 0.

Similarly, you can find other components are going to be 0, except the Ixx is going to be ml square times 2 2 ml square Iyy is going to be 0. Because, from the Y axis there is no distance of a masses and Izz by symmetry is going to be same as Ixx which is 2 ml square. So, question is what is its angular momentum?

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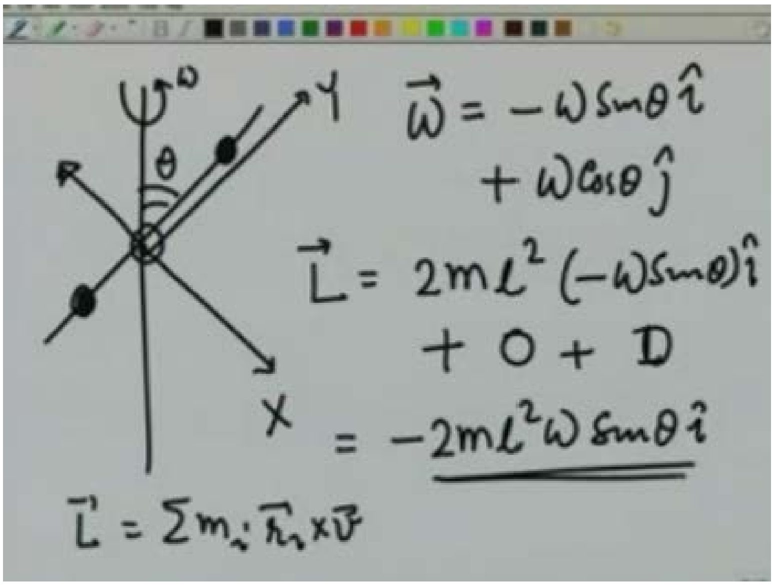

This is a rod rotating about an axis making an angle theta here with omega this is the principle axis X, principle axis Y, principle axis Z obviously, the principle set of axes rotates with the body. Now, at this given position you will see that omega has 2 components one along the negative X axis of the, for the body. So, which is be minus omega sin theta I, where I represents the direction the principle axis X plus omega cosine of theta j and there is no z component.

Therefore, L vector is going to be 2 ml square Ixx times omega x which is minus omega sin theta plus Iyy which is 0 plus Izz which is non-zero, but omega z is 0. So, this is 0. So, L comes out to be minus 2 m l square omega sin theta in I direction that is in this direction. Obviously as the as the rod rotates L will also rotate because, these set of axes are also rotating with the body, they are very specifically oriented about the body, but this is L.

Recall from the first lecture this really comes out to be the value of L because, if I calculate L by summation mi ri cross v, v for this mass is going into the screen with the speed omega L sin theta and r is L. And therefore, you get an answer for this the angular moment comes out be ml square omega sin theta for this also it comes out be ml square omega sin theta both pointing in negative x direction. So, this really is the answer.

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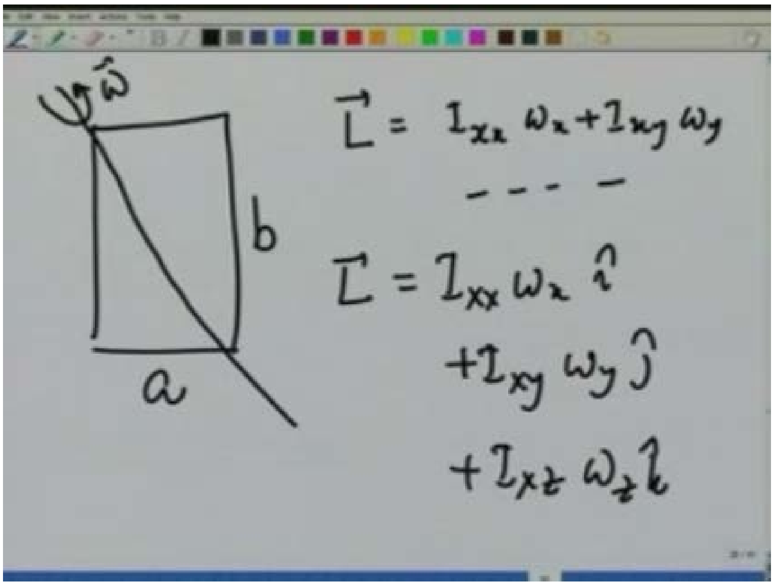

As the next example let us take a rectangle of width a, length b and let it rotate about one of its diagonals with speed omega. I want to find what is its angular momentum. Now, either I should be doing L equals the whole thing Ixx omega x plus Ixy omega y and so on or I can go to the principle set of axes calculate component omega and then, straight away write L equals Ixx omega x I plus Ixy omega y j plus Ixz omega z k.

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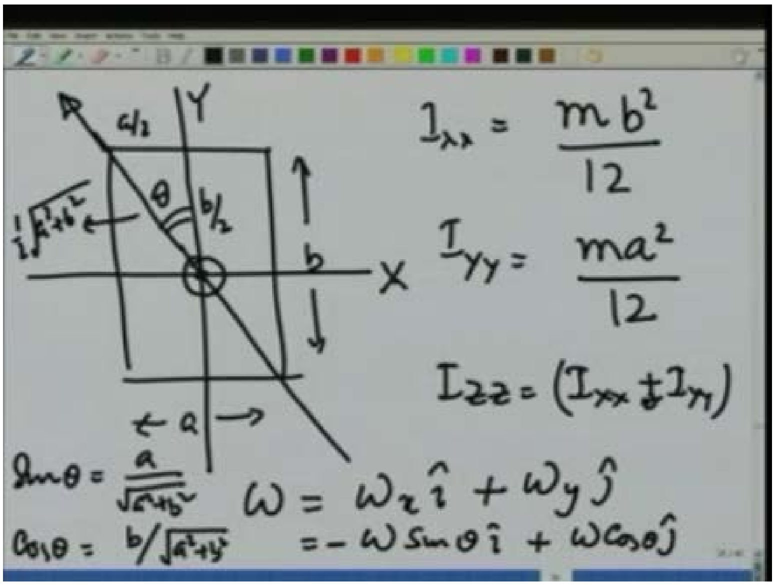

Now, again I leave it as an exercise for you to show that for a rectangle, the principle set of axes are this. Let us call this the X axis, Y axis passing, if I take the centre as the origin passing through the origin as this X axis is this Y axis is this way and Z axis obviously coming out. This length is b this length is a, so Ixx is going to be if the mass is m b square over 12 Iyy is going to be m a square over 12 and Izz is going to be by perpendicular axis theorem axis plus Iyy.

This is omega, let me call this angle theta then omega, as an omega x I plus omega y j where omega x is omega sin theta in the negative x direction plus omega cosine theta in positive y direction. Sin theta and cosine theta are easy to calculate because, these distances are known this is b by 2, this is a by 2 and this is going to be 1 half square root of a square plus b square. And therefore, sin theta is equal to a over square root of a square plus b square cosine theta is equal to b over square root of a square plus b square.

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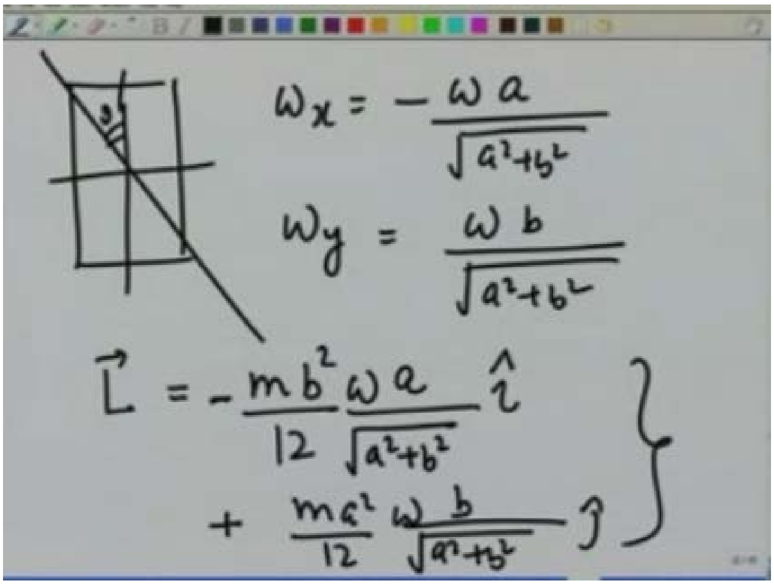

And therefore, I have Omega x as equal to minus omega this is theta a over square root of a square plus b square omega y is equal to omega b over square root of a square plus b square and therefore, L vector is going to be m b square over 12. Omega a divided by square root of a square plus b square in I direction with the minus sign plus m a square over 12 omega b over a square root of a square plus b square j. So, this is not parallel to omega, it could be in some other direction.

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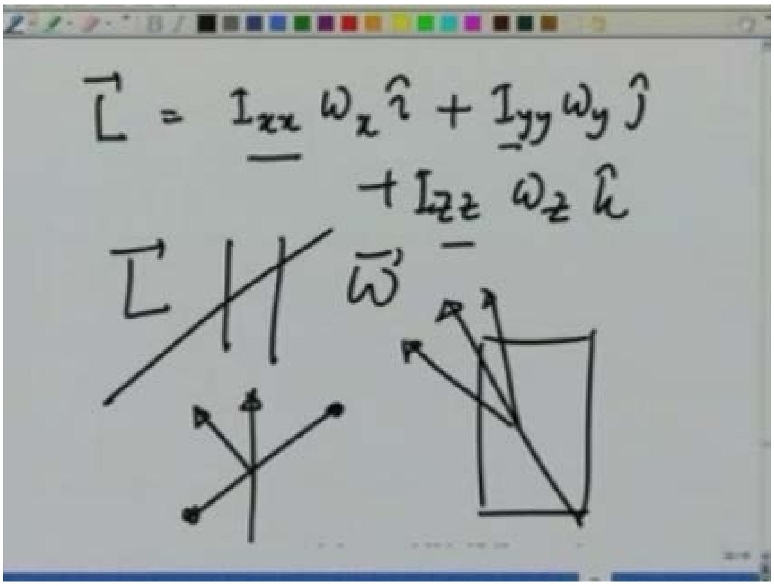

What you notice in these 2 examples is that, if L is written as Ixx omega x I plus Iyy omega y j plus Izz omega z k, L is not parallel to omega unless all these I's happen to be equal. So, in general for example, in this rod example omega was this way, L was this way. In the rectangle example omega was this way, L could be at some other angle either this way or this way depending on a and b. So, L and omega are not parallel and L rotates about omega and this gives rise to very interesting dynamics, as we will see in the coming few lectures.