Engineering Mechanics

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Module - 07
Lecture - 02
Rotational Motion – II

In the previous lecture, we have established that for a system of particles we can work with angular momentum.

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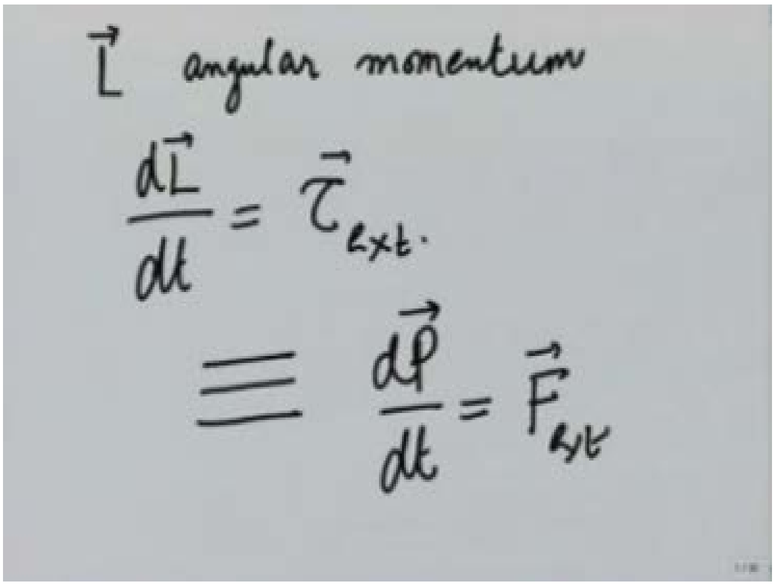

The angular momentum satisfies the condition that is rate of change is equal to torque which is applied externally on a system of particles. I showed in the case of a 1 particle system that, this was equivalent to applying the rate of change of momentum is equal to the net external force.

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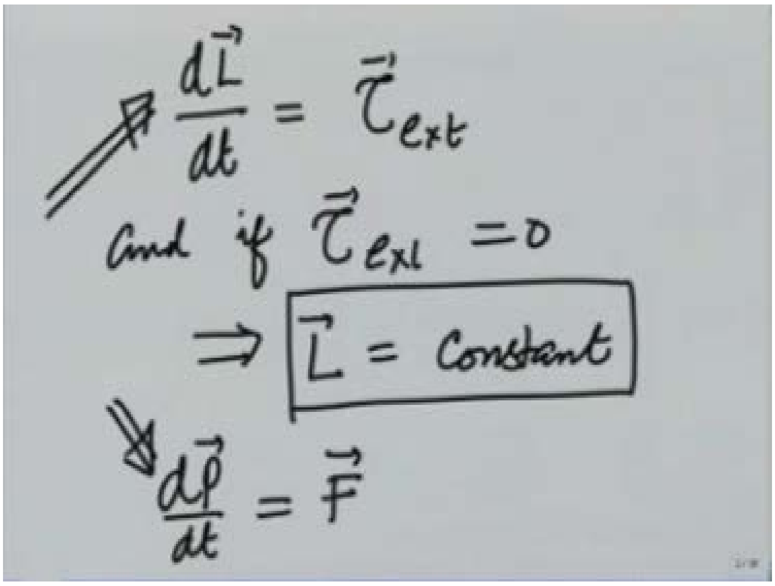

We also saw that F d L over d t is equal to tau external and if tau external is 0, this implies that L or the angular momentum is going to be constant. Let me start this lecture by taking a 2 particle example and applying this condition to get my answer. Again showing that the result that I get is completely consistent with the momentum equation d P over d t equal to F.

So, I would have establish through these 2 examples, that it is useful enough in certain conditions, a particular rigid body rotation to consider this equation rather than this equation.

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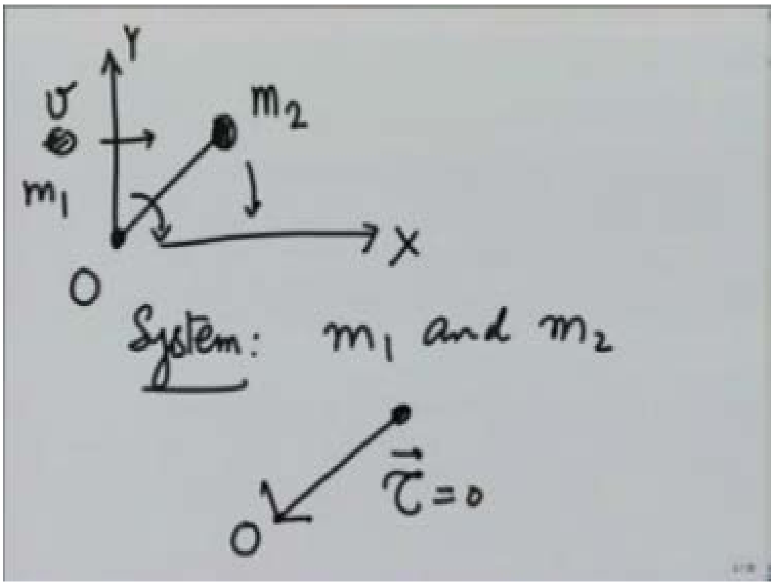

The example I take is let me take a rigid mass less rod, whose one end is fixed at point O and it is free to rotate like this about this point. At the other end of this, I have a mass m 2 let this and this be my X and Y coordinates. There is a mass m 1 that is travelling in this direction with speed v and strikes mass m 2.

What I would like to know is F mass m 1 got stuck with m 2 what will be the angular speed of rotation of the system. After all when mass m 1 hits m 2 this rod being rigid it cannot go up and down the only freedom it have it has is to move like this. Let me consider the system to be consisting of masses m 1 and m 2. Then if I look at the rod, the only force that is applied externally on the rod is at this point O.

This gives no torque because the force is not away from the origin. So, about this point O there is no torque applied. Therefore, if I take angular momentum about point O it is going to be conserved and that I can use to calculate the final omega.

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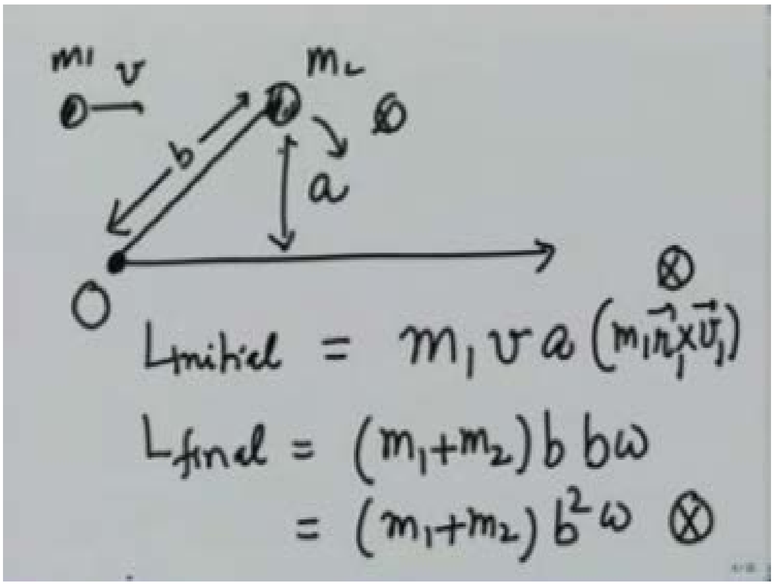

So, the system I have is this, mass m 2 mass m 1 coming in the speed v. Let this distance from the x axis be a and let the length of the rod be b. Remember it is very important that is I specify the point O because angular momentum and torque are origin dependent quantity. So, it is only about point O that the angular momentum is conserved.

So, L initial by definition of L is going to be m 2 is not moving. So, it does not contribute only m 1 v times a. You can check for yourself this is equivalent to m 1 r 1 where r 1 is a vector describing the position of particle 1 cross v 1. The direction is going to be into the pitch. After the masses get stuck and they start moving with speed angular, speed omega L final is going to be now the masses m 1 plus m 2 will be moving with speed omega. So, therefore, the velocity is going to be v times omega and times v gives me the final L final, which is m 1 plus m 2 times b square omega. Again the direction being into 3 plane of this paper. Equating the two because tau external is 0 therefore, L final and L initial must be the same.

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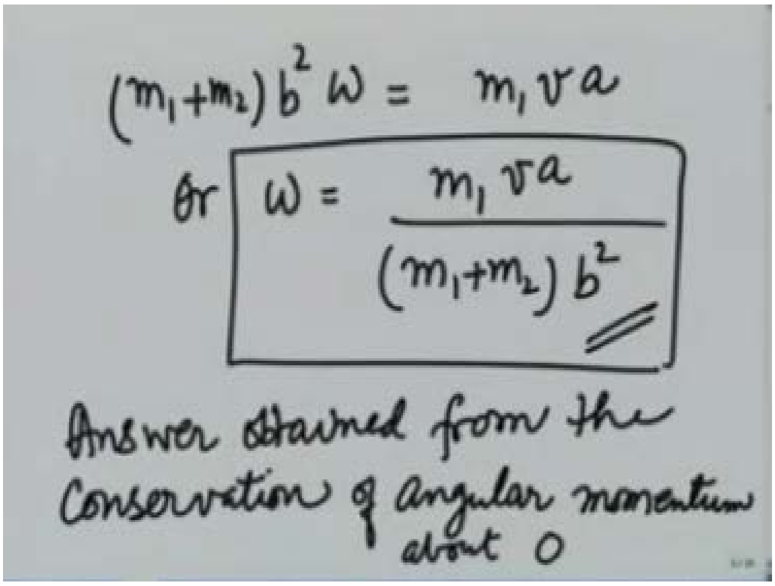

Therefore, I should have m 1 plus m 2 b square omega is equal to m 1 v a or omega equals m 1 v a divided by m 1 plus m 2 times b square. That is my answer which I obtained from the conservation of angular momentum. Let me write this answer obtained from the conservation of angular momentum. To be very specific about O. Do I get the same answer if I apply my conventional d P d t equals F let us check that to remember this answer. If we do that let us see what happens.

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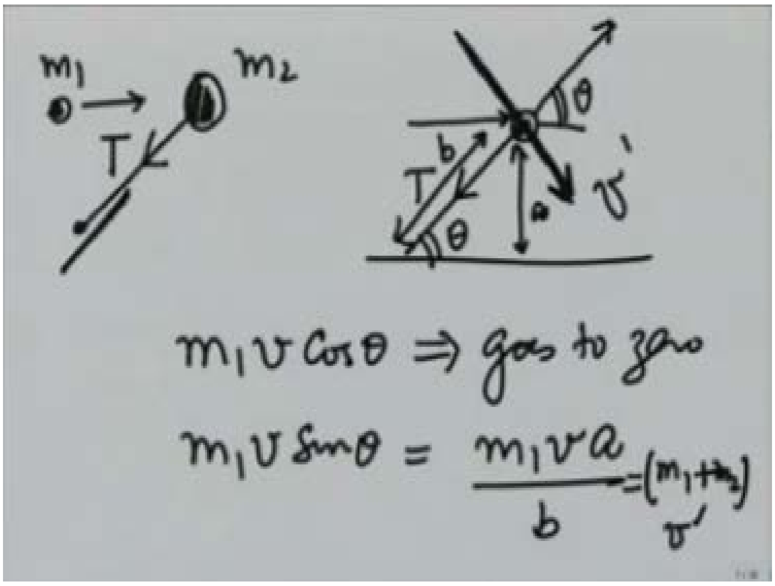

There is a rigid rod and m 1 strikes it. The only direction this rod can apply a force on m 2 is in this direction. Let us say this force extension T. Since, the rod is rigid it can withstand any momentum change, in this direction, in this direction and apply an equal and opposite force to make that momentum 0. So, when this particle comes in with momentum P, it is coming in like this. It strikes mass m 2, the component of momentum in this direction and this direction will go to 0 because this T would be sufficient to make it 0, it is a rigid rod after all.

The only component of the momentum that will survive will be this. If I call this angel theta when this angle is also theta. So, the initial momentum m 1 v cosine theta will go to 0 because of this force T applied by the rigid rod goes to 0. The only component that will survive because there is no force in this direction will be m 1 v sin theta, which will be equal to m 1 v a divided by b. Where a is this distance as I said earlier and b is the length of the rod.

This momentum remains the same, but now after the mass m 1 get stuck with mass m 2 the mass changes and therefore, the velocity v prime is going to change and this is therefore, is going to be m 1 plus m 2 times v prime where v prime is a new velocity in this direction.

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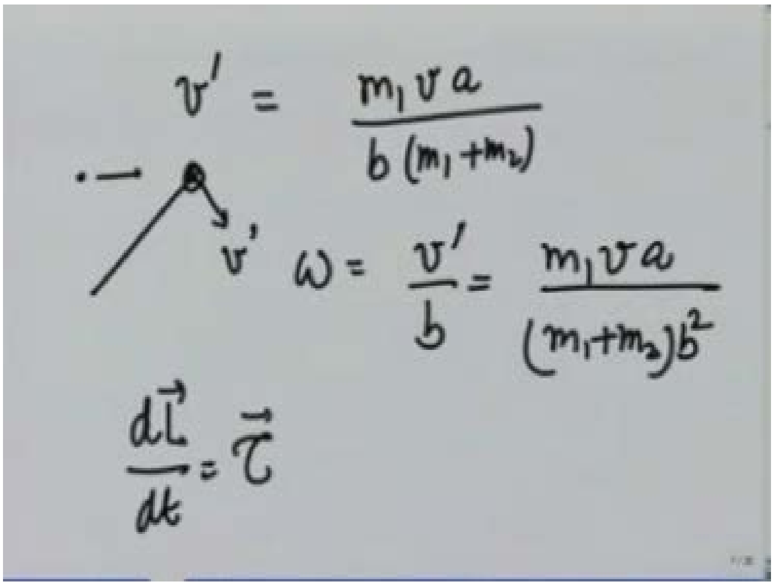

This gives me v prime is equal to m 1 v a over b m 1 plus m 2. So, to recall again after masses get stuck this fellow move immediately after that the velocity v prime. Therefore, omega is going to be v prime over b which is equal to m 1 v a over m 1 plus m 2 b square which is the same answer as obtained earlier by applying the conservation of angular momentum.

So, the point I am trying to make through these examples is that applying the formula d L over d t equals tau or is consequence the tau 0 means L is a constant is equivalent to apply Newton's second law. Therefore, depending on the situation and particularly rigid bodies we are going to use this which is much more convenient to use having developed all the machinery for describing rigid body motion that is the angular momentum. Change in angular momentum its relationship with the torque applied and the conservation of angular momentum. Now, we are ready to apply this machinery to rigid body rotation. We start with simplest of the problems in rigid body rotation.

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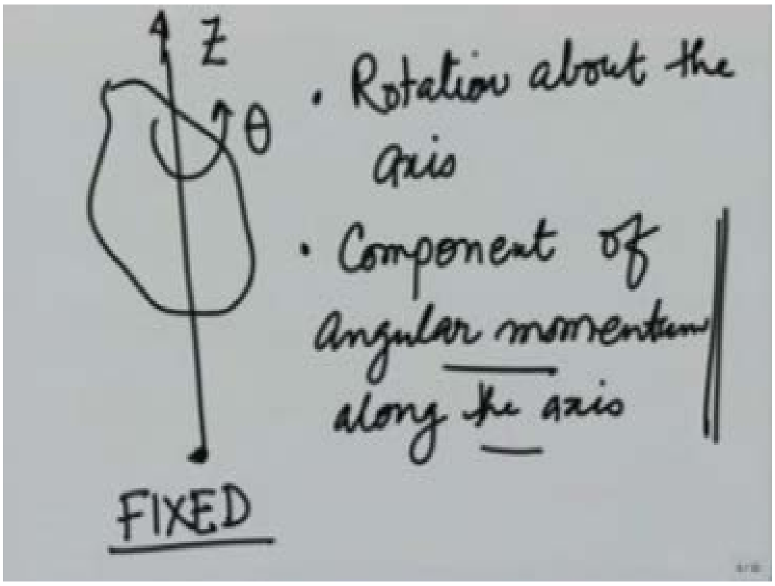

That is the motion of a rigid body about a fixed axis in space and for convenience I call it the Z axis. The only relevant parameter in this is going to be how much does the body rotate about this axis because the only thing I am considering is rotation of this body about this axis. So, all the relevant quantities are going to be rotation about the axis and component of angular momentum along the axis.

Any other component of angular momentum, even if it changes is sort of compensated for or balanced by the torques applied on the axis to keep it fix in space. So, right now what we are considering is really the axis is fixed in space and body is rotating about it. I am not even allowing for the axis to translate, that we will consider the next step. So, we are considering a fixed axis rotation with no translation.

As I said other components of angular momentum in this case are compensated for by the torques that I applied to hold the axis in place. The only component is the component of angular momentum along the axis. So, let us first calculate that.

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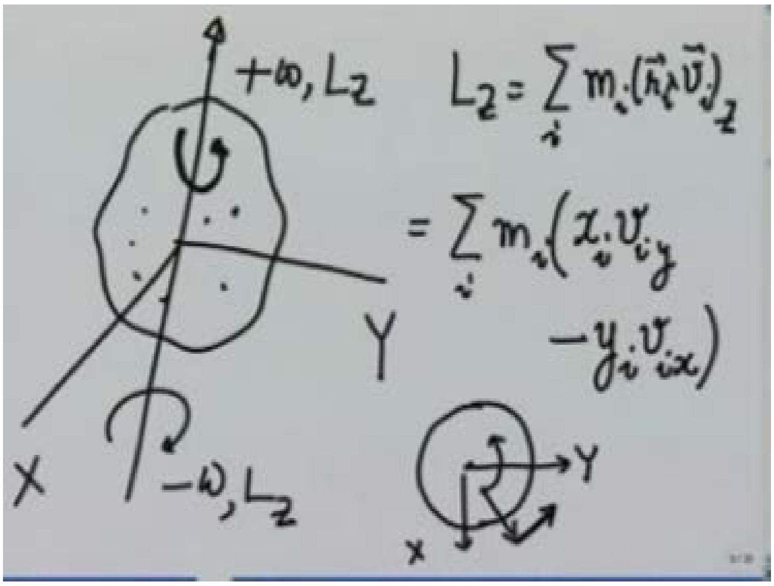

So, let us take this as the Z axis, this as the X axis, this as the Y axis and the body is rotating like this. Since, L is a vector quantity I also need a convention for its sign I will take the right hand rule. That is if I point my thumb in the direction of the axis the direction of fingers curling give me the positive direction omega and L and the other way is going to be negative.

So, this way is going to be positive omega and the other way is going to be negative omega, same thing with L z same thing with LZ. So, let us calculate L by definition L z is going to be summation I m i r i cross v i z component, which is going to be equal to summation I where I really the first 2 different points different masses on this body, m i x i v i y th component minus y i v i x component. By definition of the angular momentum.

Now, let us look at any point I th point moving it is really moving in a circle about this axis this. As I said earlier let it this be the X axis let this be the Y axis. So, velocity v at any given point is in this direction phi direction, we called your plane are polar coordinate and r is always be in R direction. The magnitude of the velocity let me take the figure again,

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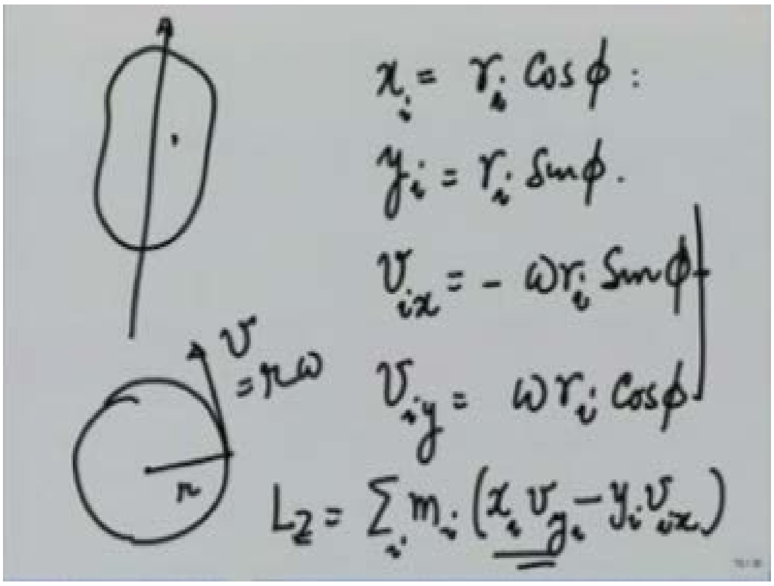

This is the rigid body rotating, this is point I. Point I is really moving in a circle like this and this magnitudinal velocity being r where r is this distance omega. The x component of i th particle is going to be r i cosine of phi y component is going to be r i sin of phi velocity is in phi direction. Therefore, i th particle x component is going to be equal to omega r i sin phi with a minus sign and v i y is doing to be omega r i cosine of phi.

Recall L z is nothing but summation I m i x i v y i for i th particle e minus y i vx for the i th particle. Substitute all these quantities in this is going to give you r i square cosine square phi. This term is going to give you r i square sin square phi sin square phi is a minus sign so minus, minus will compensate.

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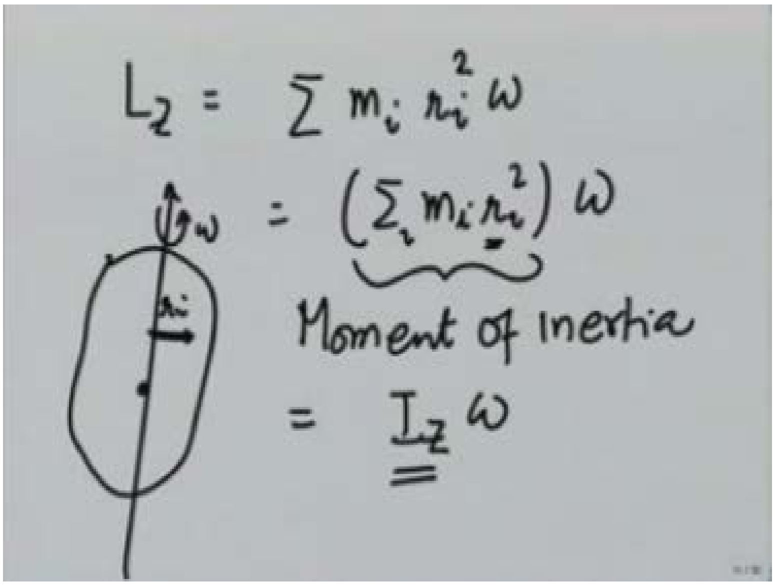

Therefore, I get L z for this body which is rotating about the Z axis with omega as summation m i r i square omega which I can write as summation part inside m i r i square omega and this we call the moment of inertia about the Z axis. So, let me call this I z omega. This you have seen in your twelfth grade, but now I have given you a rigorous derivation is starting from the basic definition.

I z is nothing, but mass times the perpendicular distance from the axis you. Remember r i is this. It is not the distance from the origin, but the perpendicular distance from the axis multiply m i times r i square add it up that I z times omega is L z.

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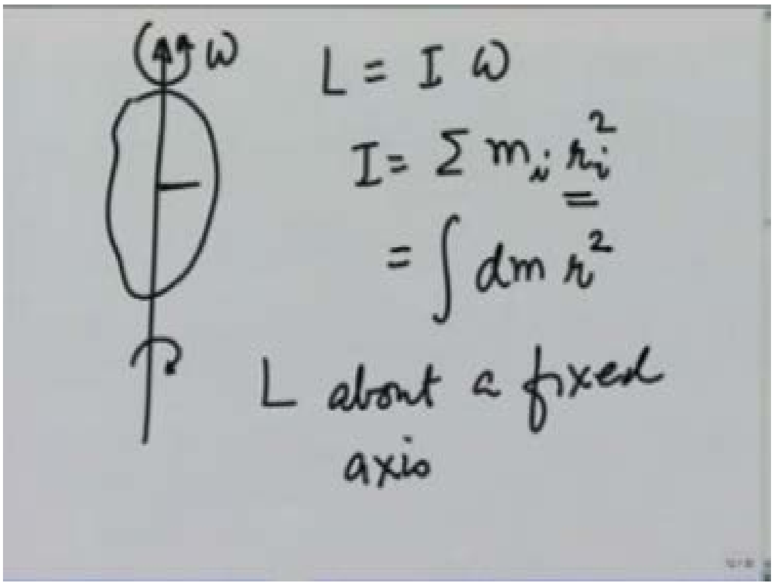

So, what we have found is that given a body which is rotating about an axis its angular momentum L is going to be given as I omega. I have dropped the axis Z where I is nothing but summation m i r i square where r i is the perpendicular distance from the axis or if the mass distribution is continuous I can write as d m r square.

Again the sign convention is going to be, if I take my thumb in the direction of the axis of rotation, then this curl finger gives me the positive omega. When thumb is pointing in the positive z direction or positive direction whichever I choose and the other way gives me negative.

Remember L is a vector quantity. So, this is equivalent when I talk about 1 dimensional motion momentum which is a vector quantity who have negative or positive values depending on which way the particle is moving. So, now with this I am now going to illustrate a few things regarding angular momentum. Considering this definition of L about a fixed axis. Having discussed angular momentum. Now, I am going to illustrate how angular momentum really affects the motion or rotational motion in different situations.

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As the first example I take this platform which can be freely rotating. So, if I stand on it start rotating I will have an angular momentum about the axis. I will show you first the effect of how changing the moment of inertia changes my angular speed in order, that the angular momentum remains constant.

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So, let me stand on this platform push myself slightly. So, that I start rotating and

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I will take my hands out you see I have slowed down.

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As soon as I take my hands in I will start moving again. So, let me show it to you again I rotate myself as soon I take my hands out I slow down I bring them in I start moving faster. May be the effect is not visible to you because my hands going in an out may not change the moment of inertia so much.

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So, I have taken 2 weights to increase my moment of inertia if I move my hands in and out. I will repeat the experiment at this time the speed change should be visible to you. I stand here I am not yet rotating, but I want to show you if I take my hands out. Since these masses move out m i r i square for the masses would increase the angular. The moment of inertia would increase and therefore, the speed should go down if the angular momentum has to remain constant. So, let me rotate myself and I move out you see I have slowed down as soon as I take them in I move fast. I take them out I slow down I take them in I move very fast.

So, this is an effect of conservation of angular momentum I will in a minute explain it again on the screen, but now let me show you the effect of changing the masses.

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Now, I will take instead of light masses 2 heavier masses and you will see that the effect will be much more dramatic because as I take the masses out or in because they are heavier the change in the moment of inertia would be much larger. So, let me do it again for you I stand here take my hand out I am moving slowly. As I come towards you I will take my hands in and you will see I am moving very fast. Again I will take them out and I start moving slowly. As I put them in I move very fast you see the effect of conservation of angular momentum and the change of moment of inertia.

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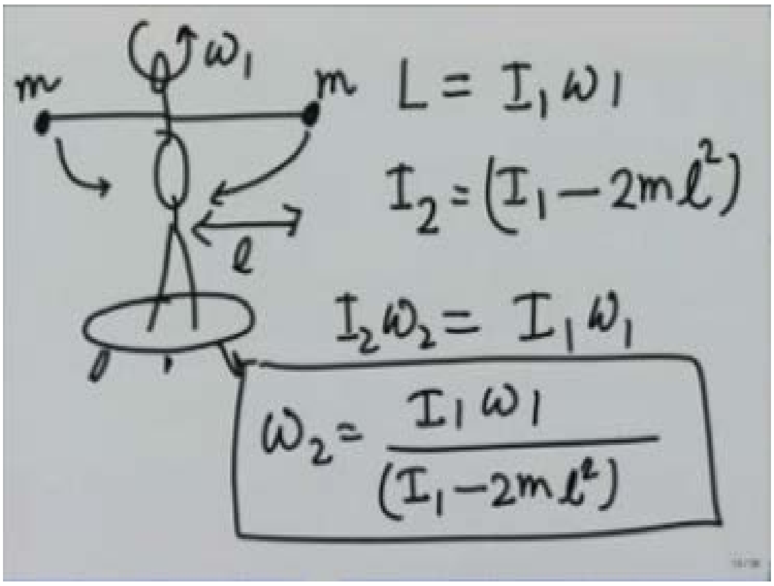

Let me explain this on the screen what we did was outstanding on this platform which could rotate. Let me stand like this and I held my hands out with heavy masses here. I was rotating with some angular speed omega, let me call this omega one. So, my angular momentum L as we just derived was I 1 and omega one. As soon as I brought these masses N. I changed to I 2 which is I 1 minus let us say roughly 2 m l square where m is the mass of each of these and l may be the length of my arm, but however, there is no external torque.

This platform is nearly friction less and therefore, angular momentum must be conserved. So, mu omega that is I 2 omega 2 should be such that the angular momentum remain unchanged. After all when I am moving my arms in or out, the other forces that I are being applied by my joints are internal to the system and therefore, omega 2 becomes I 1 omega 1 over I 1 minus 2 ml square.

As soon as I stress then out it again slows down. So, this shows you the effect of change of mass on moment of inertia, change of distance of moment of inertia and a consequence of the conservation of angular momentum. Now, I take another demonstration in which I will take this bicycle wheel.

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Which I can rotate by my hand. If I give it this rotation it acquires an angular momentum in the positive Z direction, if I take this to the Z direction. Let us see what happens if I stand on this platform and give it a rotation, I start moving. As soon as I stop it I also stop let me give it the rotation the other way I will start moving the other way.

Let me stop it and I will stop let me give it to rotation this way stop let me give the rotation the other way I move this way. Let us try to understand this as to what is happening.

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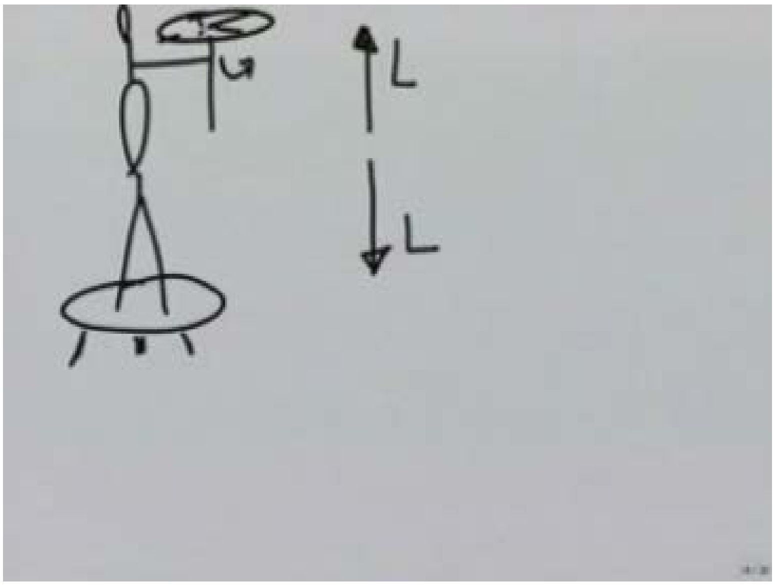

What is happening here is that I am standing on this platform, holding this wheel. As soon as I give it a rotation like this, it starts it has gained an angular momentum L in this direction. However, when I apply the force considering myself and the wheel as a system there is no external torque on the system. After all this platform is friction less. So, I and the wheel with me should start rotating in such a way, as to develop an angular momentum in the opposite direction of exactly the same magnitude. So, that the net angular momentum remains 0. Therefore, my body starts rotating the other way let me show it to you again.

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I took this wheel I stood on the platform, 0 angular momentum as soon as I give it a rotation this direction I start moving the other way. As soon I stop it I also stop I will rotate it the other way and then I start rotating the other way. This is a manifestation of the conservation of angular momentum. As a third example of angular momentum conservation and through this I also wish to, show the vector nature of angular momentum.

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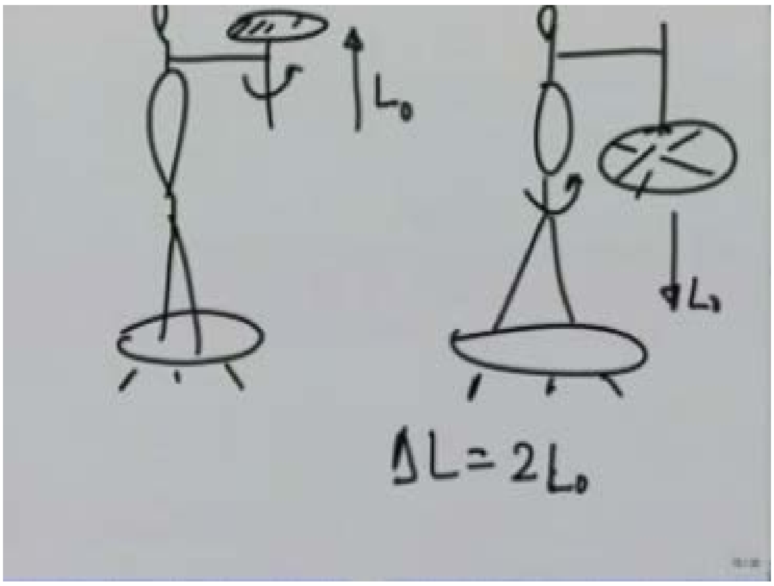

I take this wheel stand on the platform give it a rotation in one direction. I stand on the wheel and suddenly I will turn the wheel the other way. That means, initially the angular momentum that the wheel has is in this direction. All of a sudden what I will do is, I will hold this point the downwards. So, that the L could now change direction. If L is a vector quantity and it change the direction like this. The net change in L could be 2, the initial let me call this L 0, 2 L 0.

However, there can be no change in the angular momentum because there is no external torque and therefore, I should start rotating in such a manner, that the net angular momentum change is 0. That means I should rotate may be the other way to keep the angular momentum change 0. Let us see if that really happens. So, I will take this wheel which is rotating.

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Stand on the platform right now I am stationary I will turn this and you see I will start rotating I turn this back I stop. I will turn this again and I will start rotating let me show it to you more clearly by rotating it faster. I stand here I turn this I start rotating I turn this back I stop. So, this is again a manifestation of the conservation of angular momentum.

Effect I would like to show you in this is if I hold this further away from the body and then tilt it or keep it near my body and tilt it there would be a difference. Hopefully it will be visible to you on the camera. That I would like you to think about I will hold it near my body and tilt it you see the speed. I will now hold it further away from my body and tilt it when is it. That should be expecting to move faster and when is it when I should be expecting to be moving slower.

See it again, hold it near tilt it. Then again give it the speed hold it far and tilt it. It is really that, I am moving the center of mass of the wheel further and bringing it nearer by holding it further nearer and that makes a difference in my rotation speed when I tilt it. So, let us try to analyze what happened when I was holding this wheel.

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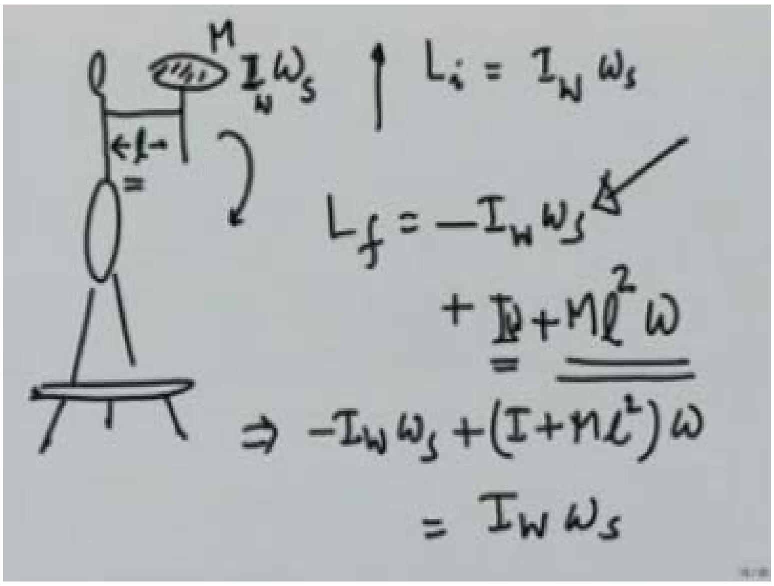

Let it is moment of inertia be I wheel let it move with a speed omega s let this mass be M and let the length of my arms be l and I was holding it on the platform. When I tilted it the initial momentum L initial was I wheel times omega s, L final is going to be minus I wheel omega s because I have tilted it like this. Therefore, I have to start moving let my own moment of inertia be I plus there will be a moment of inertia of the wheel M l square.

Or you can say let the moment of the angular momentum of the wheel is angular momentum of a center of mass plus angular momentum of it about the center of mass plus my own angular momentum. Therefore, that gives me minus I wheel omega s plus I plus M l square omega equals I wheel omega s.

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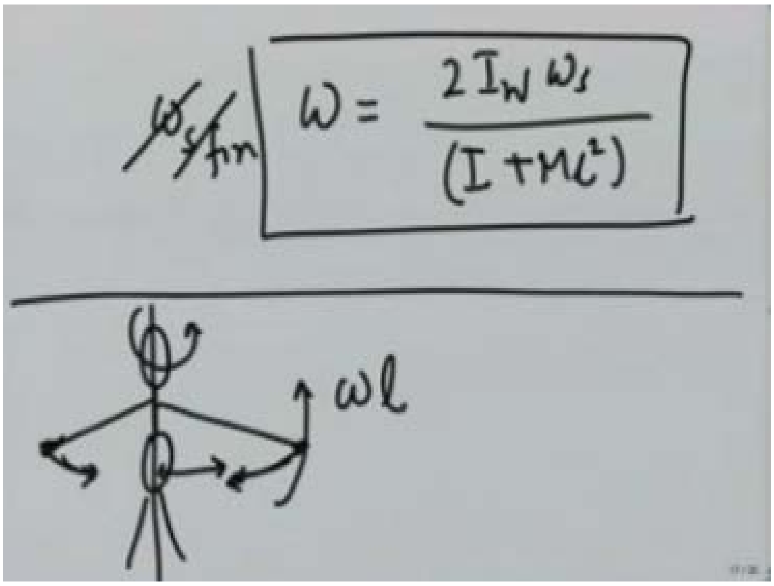

Or omega s final sorry this will be omega with which I move is equal to 2 I wheel omega s over my own moment of inertia plus M l square. From the conservation of angular momentum. Let me also try to understand when I took this demonstration of having 2 masses far out and pull them in, why did I start rotating faster. Can I understand this in terms of Newton's laws.

Certainly yes you see when the hands are stretched out they are moving with a speed let us say omega l. As I bring my arms in the speed goes down because the length or the distance from the axis of rotation has gone down. I have to apply a force on the masses to slow them down. In the process masses applied force on me to speed me up.

The balance gives me the final angular momentum, but you see doing it this way is slightly move involved. Therefore, the conservation of angular momentum comes in handy to solve this problem. Hopefully by now you convinced that in rigid body rotation it is the angular momentum that is the key quantity of interest and that determines the dynamics. Let us now see some other quantities or equivalent quantities and rigid body dynamics which are equivalent to one particle dynamics that will be useful.

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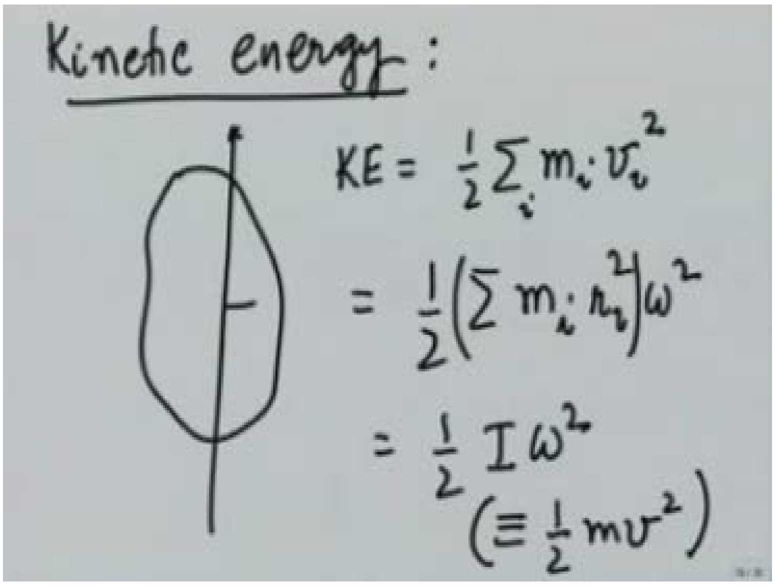

So, let us look at kinetic energy and again I am going to consider this simple case of particle moving about 1 axis. Kinetic energy is equal to 1 half summation i m i v i square and therefore, I can write this as 1 half summation m i r i square where r i is a perpendicular distance from the axis times omega square which is nothing but I put the bracket here 1 half I omega square. Which is equivalent to 1 half mv square of 1 part of the dynamics.

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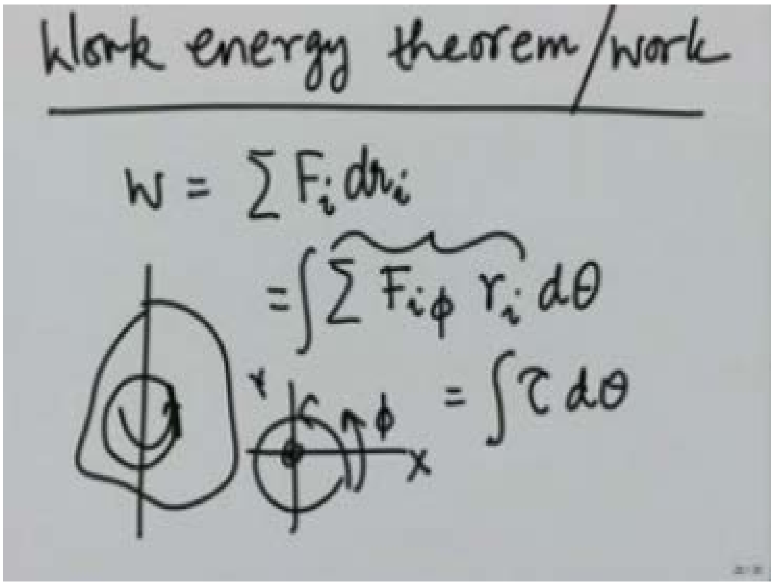

What about work energy theorem or work done itself. Again we will use the basic definition that work done is nothing but summation F on i F particle distance moved by it in the direction of the force d r i summed over. So, if a body is rotating like this and I apply a force on this, the only component since it can move only in phi direction. Only component of force that will work is that in the phi direction. So, I can write in this case is summation F i phi direction we have by phi I mean the in the sense of cylindrical coordinates, this is phi X Y and this is Z axis about which this is rotating.

F i phi distance is going to be r i d theta which is nothing but this quantity is nothing but the torque. So, torque times d theta. This is going to be partial work done for full work I put the integration.

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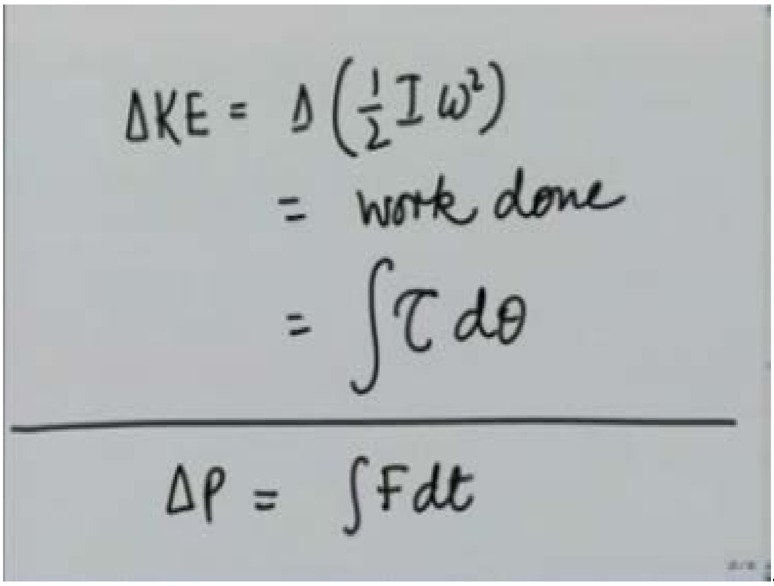

Once we determine the work, the work energy theorem is that change in the kinetic energy that is delta of 1 half I omega square is going to be the equal to work done, which is nothing but tau d theta. Similarly, if I apply an impulse to a single body, what happens to this momentum, this momentum changes by the amount of impulse F d t.

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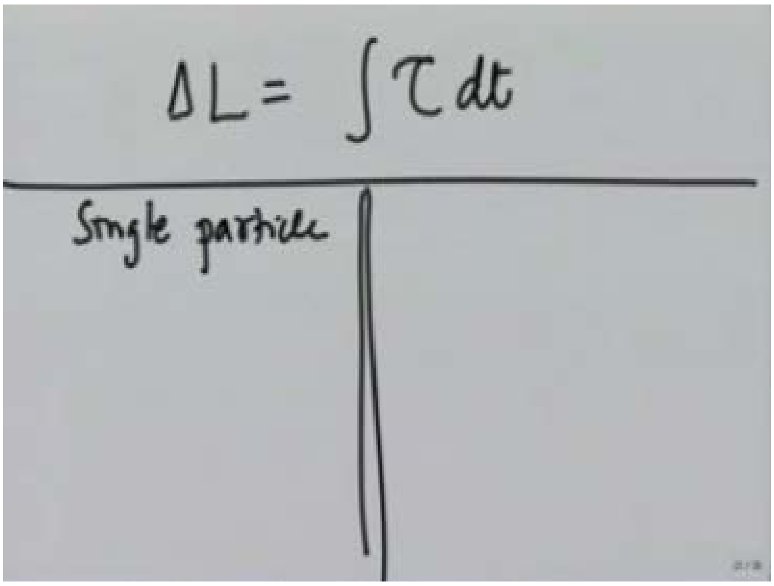

The similar manner if I apply an impulse and that provides a torque is angular momentum is going to change by the torque impulse. So, let us see in the case of rotation about a fix axis what are the equivalent quantities of single particle. So, let us see what are the equivalent quantities in single particle and rigid body dynamics.

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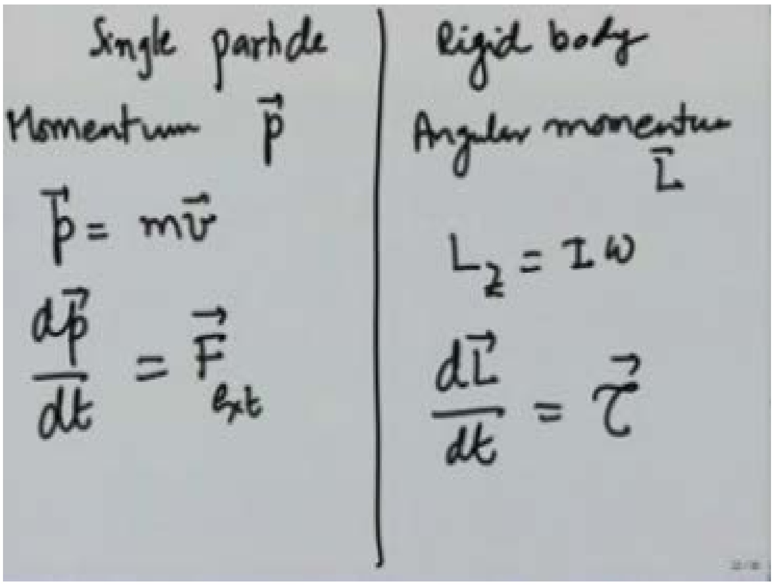

So, let we write single particle here and a rigid body. Here I have momentum p here I have angular momentum L. Momentum p equals mv and the case that we have discussed so far L along a particular axis equals I omega. Momentum satisfies the equation d p d t equals the force. Angular momentum satisfies the equation d L d t equals the torque.

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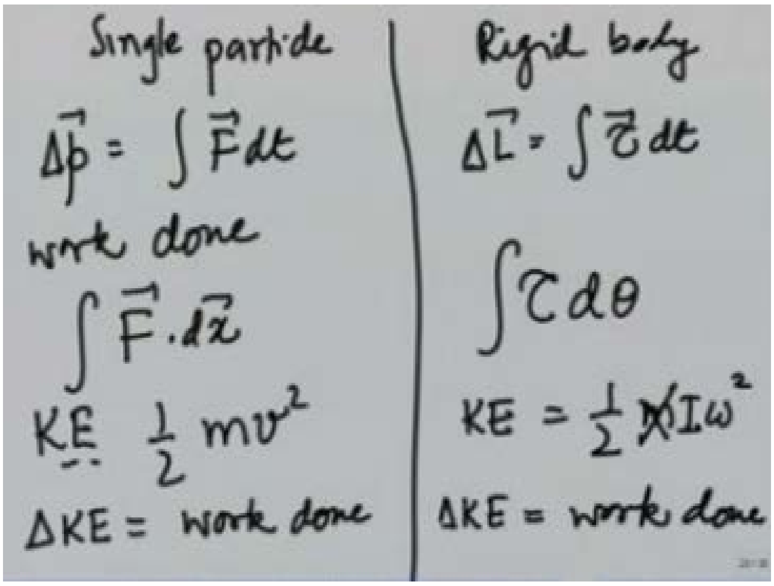

If there is an impulse, in that case the change in momentum is equal to the impulse and the phase of rigid body if there is an impulse change in angular momentum is equal to torque impulse. The work done in this case is F dot d x integral, in this case we considered again the case of single axis is tau d theta integral. Kinetic energy is 1 half m v square. Kinetic energy in this case we derived was 1 half m I omega square.

Work energy theorem tells you that delta K E is equal to work done, same thing here. Delta KE is equal to work done. Having done this simple case we will make the materials slightly more complicated in the next lecture by considering a combination of translation of a body and at a rotation about a fix axis. We still call it fix axis rotation as long as the direction of the axis of rotation does not change it only translate.

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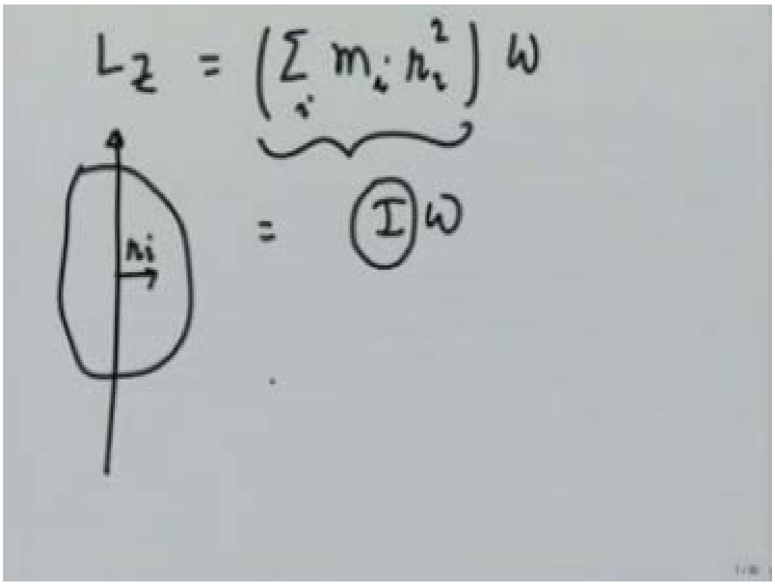

Having obtained the expression for L z as summation I m i r i square omega, where ri is the perpendicular distance from the axis of rotation and we call this quantity the moment of inertia I omega. Let us calculate this quantity I for certain simple and slightly more involved elements.

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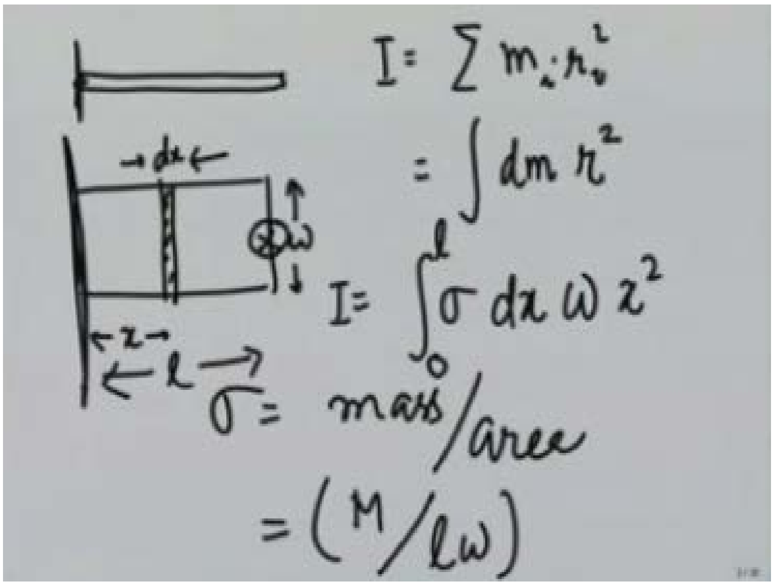

For example if I have a rod or even a rectangle rotating about the axis here or this rotating about the axis here. What is the moment of inertia, this is rotating right now let us say going into the plane or coming out of a plane. So, we can generalize this definition which we wrote as m i r i square as equal to integral some mass d m, which is at a distance r square and integrated over the entire body.

For example in this case, if I take a small stick here at distance x, the width of this strip is dx then I for this would be given as integral sigma. I will define what sigma is in a moment, dx times the width of the rectangle times x square. Where x square is the square of the perpendicular distance from the axis of rotation of this mass d m, sigma is mass per unit area. Which in this case would be a total mass divided by l times w where l is the total length of the rectangle. The integration is going to be from x equal to 0 to x equal to l.

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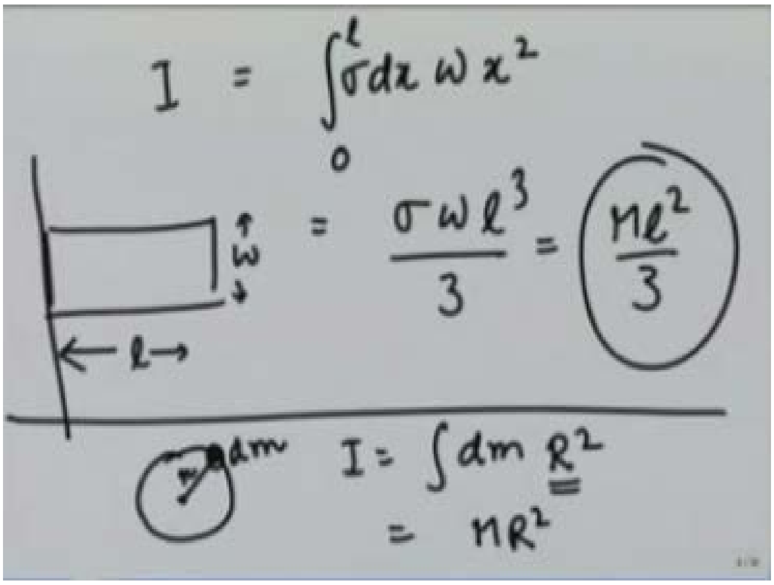

So, let us do that and when we do that I get I is equal to integral d x sigma, omega, x square 0 to l where I am talking about this rectangle rotating about 1 of a side length l width w. So, this comes out to be sigma w l cube over three. Sigma w l is nothing but the mass. So, M l square over 3 a result that you are well familiar with.

As the second example let us take a ring of mass m and calculate moment of inertia about a center. Then if I take any mass d m here or over the periphery the mass is at the same distance from the axis r. Therefore, I is going to be dm radius square integral R is a constant. So, it comes out to be MR square another familiar result.

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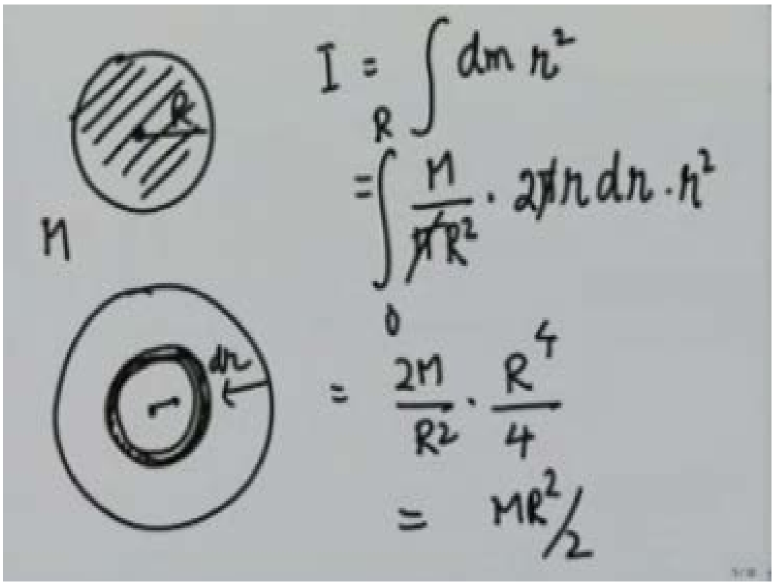

As a third example let us take a disc, a solid disc of mass M and radius R and I want to calculate this moment of inertia about an axis passing through it center. What I can do for this is divide the disc into a small small rings of width of radius d r. Then the moment of inertia for the disc I know if it is mass is d m is going to be d m r square and if I integrate this it gives me the moment of inertia for the disc

D m is going to be mass per unit are of the disc which is M divided by pi R square times the area of this small section that I have taken and this is going to be 2 pi r d r times r square integrated from 0 to R. What I get is this pi cancels, 2 M over R square times R raise to 4 divided by 4 and the result comes out to be MR square by 2. That is the moment of inertia of a solid disc about and an axis passing through the center. The result could easily be generalized to a cylinder also, which has which has this disc elongated along the Z axis.

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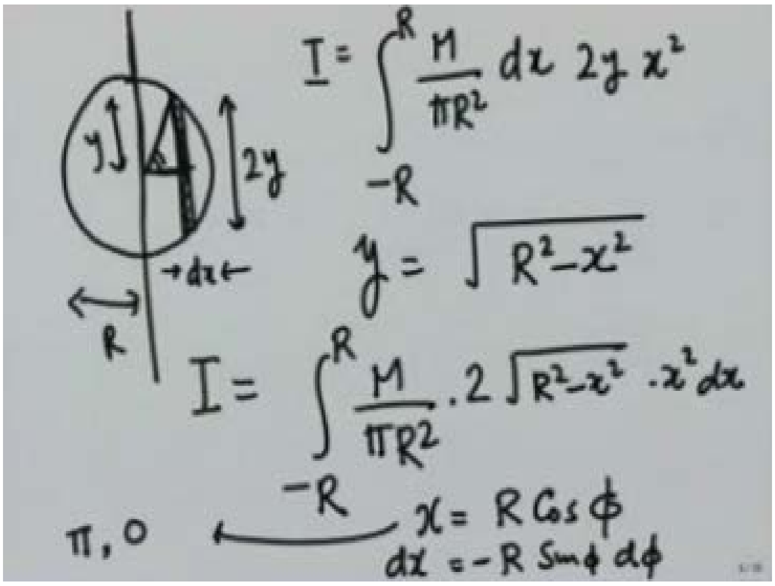

Next example I take the same solid disc, however, now I take the axis as its diameter. So, this diameter is the axis. For calculating a moment of inertia about this I will take a strip on the disc like this of width d x. So, that the mass is going to be M over pi R square dx times this length which I will call 2 times y 2 times y where y is this length. Multiply this by x square and integrate from minus R to R because this distance is R.

What is y, y is you can see from this right angle triangle is going to be equal to square root of R square minus x square. Therefore, the moment of inertia about 1 of the diameters is going to be integral from minus R to R M over pi R square times 2 y the square root of R square minus x square times x square d x.

Let me take x to be equal to R cosine of pi that gives you dx is equal to R sin of phi d phi with a minus sign in front. The limits for this are going to be for minus R is going to be pi or x equal to R is going to be 0 with this minus sign the limits change. In any case I can write this as let me write it again.

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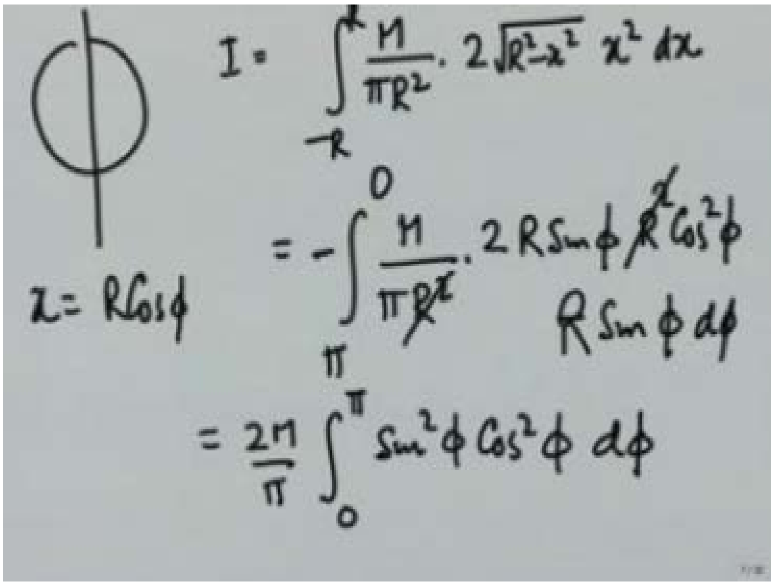

I am talking about the moment of inertia about the diameter I is equal to M over pi R square times 2 square root of R square minus x square x square d x. Integral is from minus R to R, I have taken x to be equal to R cosine of phi and with that this integral changes to pi to 0 with a minus sign in front. M over pi R square 2 R square minus x square this will become R sin phi x square is going to be R square, cosine square phi d x is going to be R sin phi d phi.

I have already taken care of minus sign outside, which I can write as this as this R square cancel 2 M over pi, 0 to pi sin square phi cosine square phi d phi which is equal to 2 M over pi 0 to pi sin square phi cosine square phi d phi which is equal to M over 2 pi integral 0 to pi sin square 2 phi d phi, change to variable alpha equals to phi.

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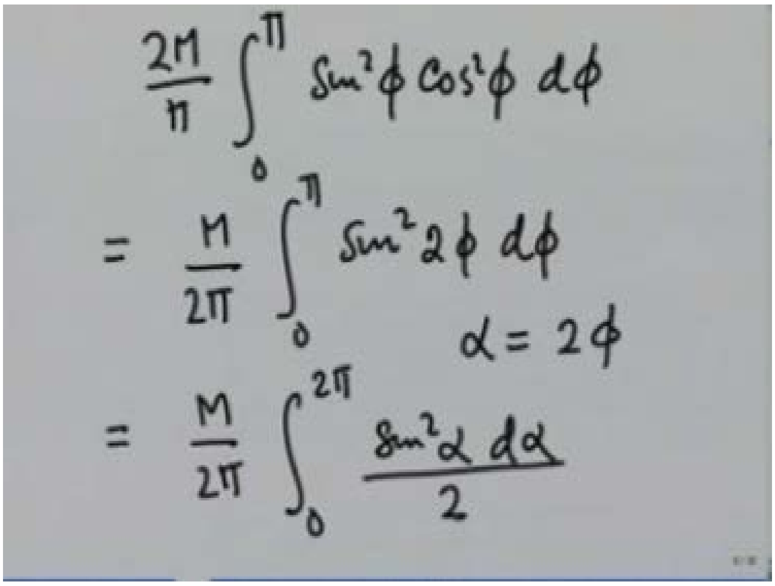

This becomes equal to M over 2 pi 0 to 2 pi sin square alpha d alpha divided by 2.

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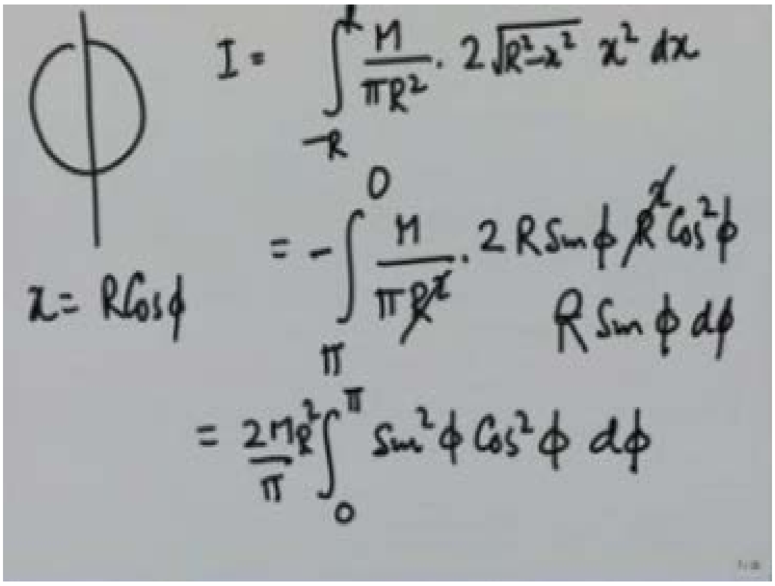

I have forgotten R square somewhere. There was an R square here. So, that will remain.

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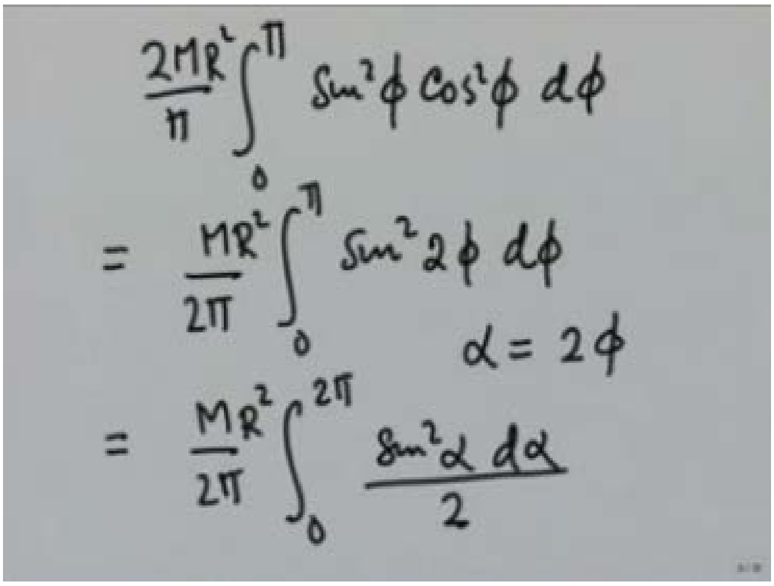

There is an R square here there is an R square here there is an R square here. So, what we get for.

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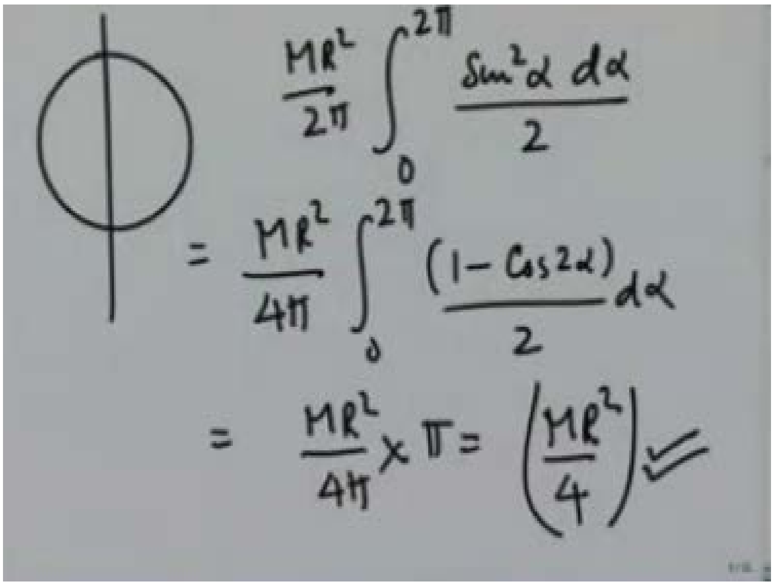

The moment of inertia about diameter is M R square over 2 pi, integral 0 to 2 pi sin square alpha d alpha over 2 which is equal to M R square over 4 pi integral 0 to 2 pi, sin square alpha can be written as 1 minus cosine 2 alpha over 2 d alpha. This you can see right away is nothing but MR square over 4 pi times pi which is M R square by 4. A result again you are well familiar with. So, for a disc about its diameter the moment of inertia is M R square by 4.

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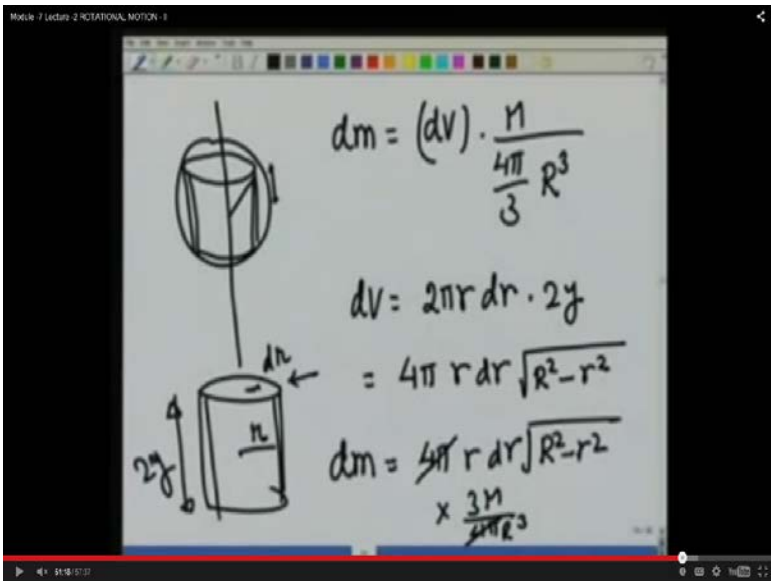

As last example let us do a sphere about one of the diameters. For this what I am going to do is take a cylindrical shell around the diameter. So, that the mass of this cylindrical shell is going to its volume, let us call it dv times the density which is going to be M over 4 pi by 3 R cubed. The volume of this cylindrical shell, if I take this to be of radius r and thickness d r is going to be dv equals 2 pi r d r. This whole length let us call it 2 y times 2 y where y is this height.

Which I can write as 4 pi r d r y as we calculated in the previous example is going to be R square minus r square. Therefore, the mass is 4 pi r d r square root of R square minus r square times M, 3 M over 4 pi R cubed. This 4 pi cancels and therefore, I get as the mass of the sphere as d m equals r d r square root of R square minus r square M over R cubed.

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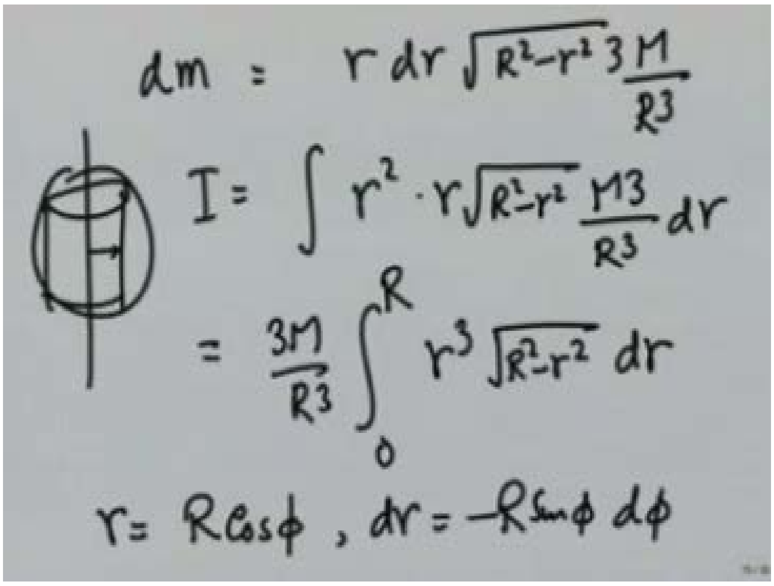

This is a cylinder and therefore, the moment of inertia of this sphere is going to be the perpendicular distance of this. This is like a ring is r square times r square root of R square minus r square M over R cube or there is a 3 here times d r. So, this is equal to 3 M over R cube R varies from 0 to R, r cube the square root of R square minus r squared r. To evaluate this integral again I use r is equal to R cosine of phi. So, that d r is equal to minus R sin of phi d phi and limits are going to be from 0 to pi by 2.

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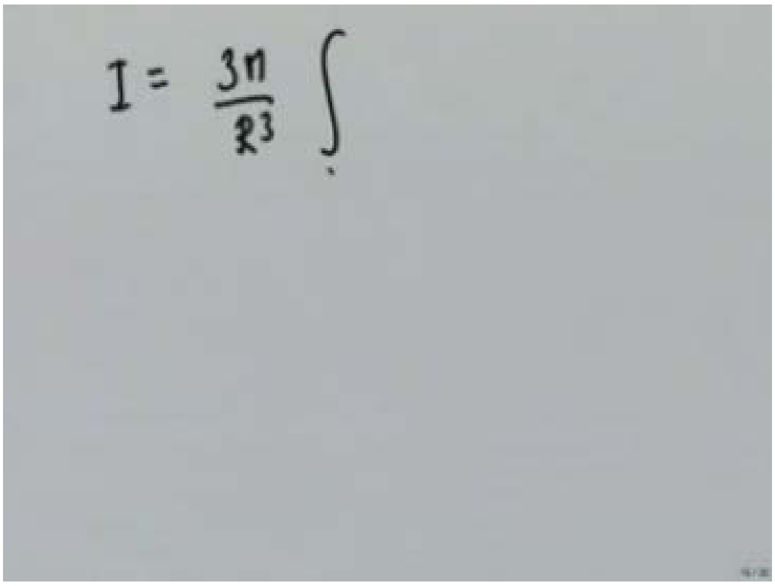

So, substituting I am going to get I equals 3 M over R cube integral and see this.

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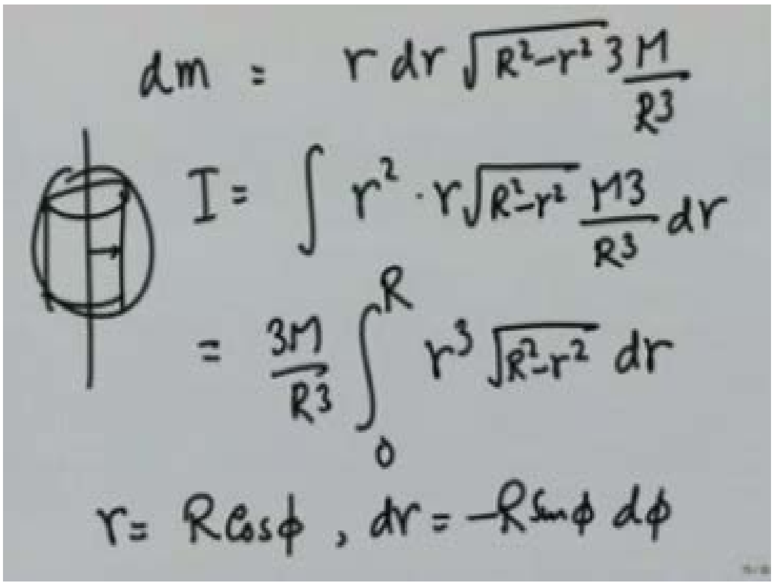

This is 0.

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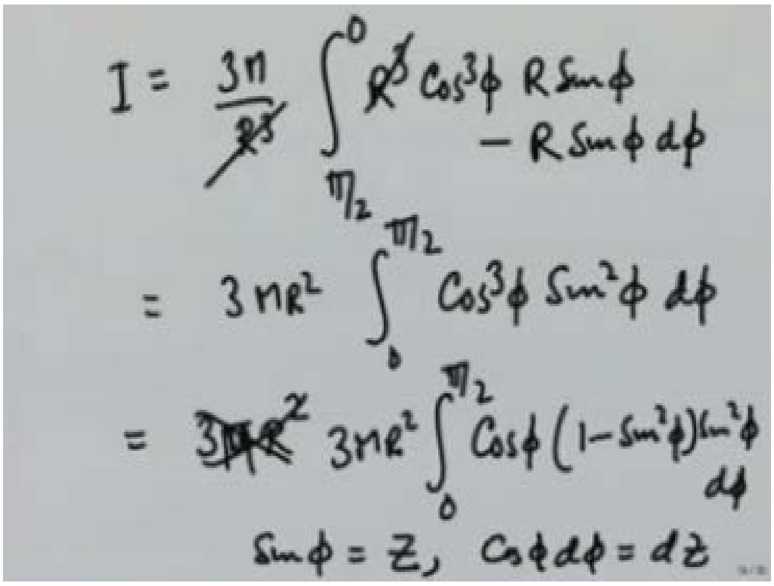

So, at 0 it is pi by 2 to 0 for r cubed I get R cube cosine cube phi square root R square minus r square comes out to be R sin phi times minus R sin phi d phi. This R cubed cancels and I get 3 MR square 0 to pi by 2 cosine cube phi sin square phi d phi.

To evaluate this integral let me write this in a particular way 3 M R square 3 M R square 0 to pi by 2 cosine phi cosine square phi will be left, which I will write as 1 minus sin square phi sin square phi d phi. Let sin phi be equal to Z. So, that cosine phi d phi is d Z and limits are going to be from 0 to 1.

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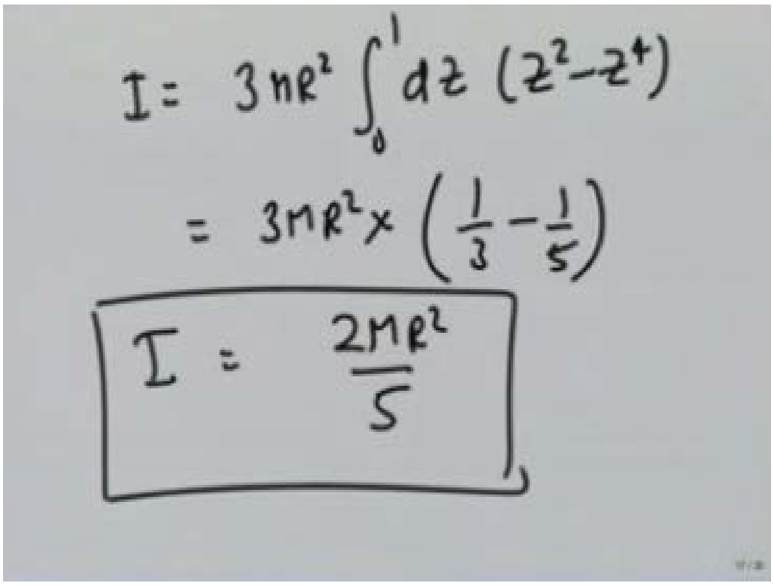

Therefore, the moment of inertia I is going to be 3 M R square 0 to 1 d Z. Z square minus Z raise to 4 which is nothing but 3 MR square times 1 third minus 1 fifth which is 2 M R square by 5. That is the moment of inertia of a sphere about 1 of it is axis as another example of momentum angular momentum conservation.

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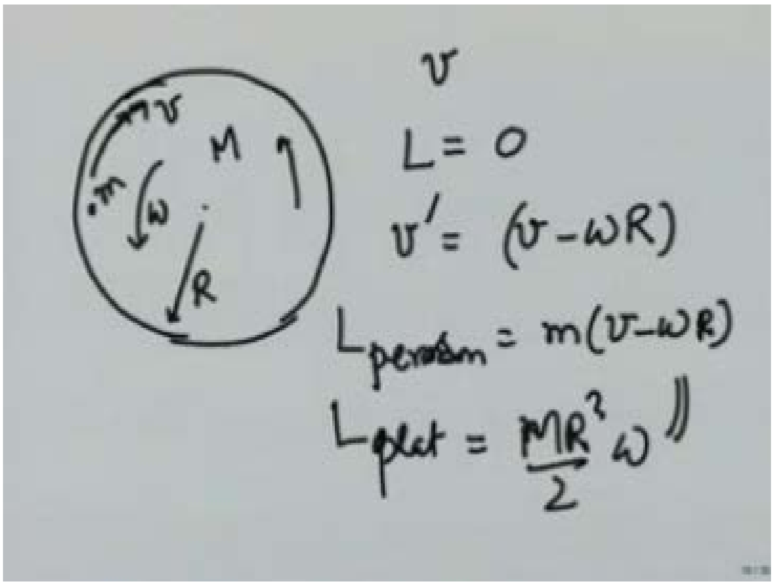

Let us consider a huge platform which can rotate without friction in one particular direction or any direction. Let us take a person of mass m moving on his periphery with a speed v with respect to a platform. Let the radius of the platform be R and let it is mass be M, we want to know with what speed would this platform be moving, with what angular speed would this platform be moving.

So, again initially if there is no angular momentum, then L should be equal to 0. If the person moves with respect to the platform in this direction with v, the platform starts rotating the other way with omega. So, that the net velocity of the person with respect to ground is going to be v minus omega R. Therefore, the angular momentum of the person is going to be mv minus omega R.

On the other hand, the angular momentum of the disc in the opposite direction of the platform is going to be M capital M R square by 2 times omega because this platform is like a disc. The 2 should be equal in magnitude because they are opposite in direction. So, that the net angular momentum is 0.

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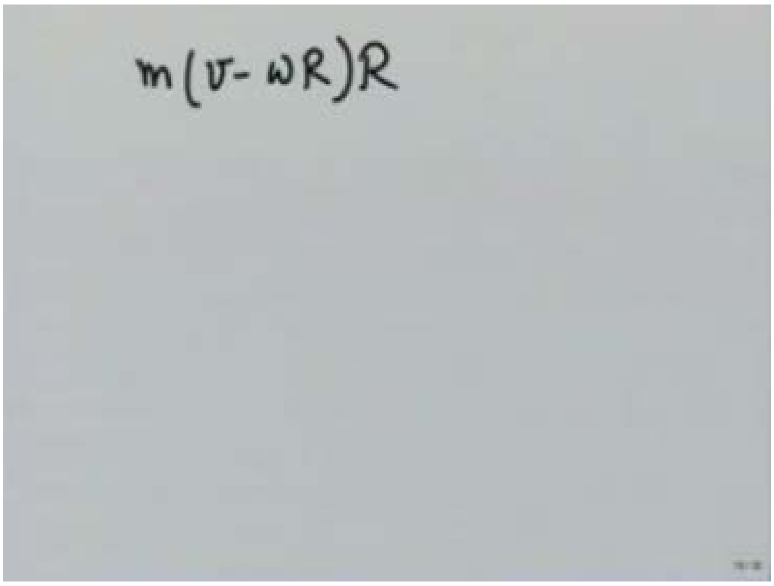

Therefore, what we get is m v minus omega R times R is the angular momentum.

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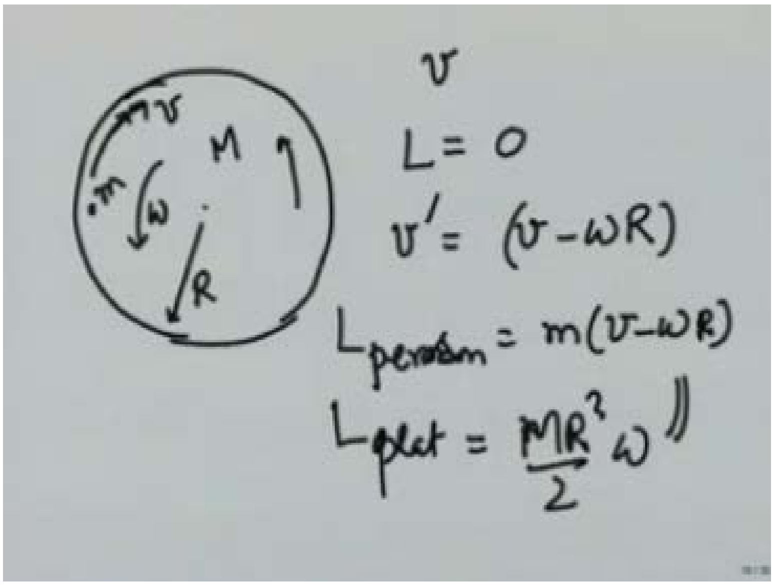

It is connected in the previous slide. It should be times R here for the angular momentum.

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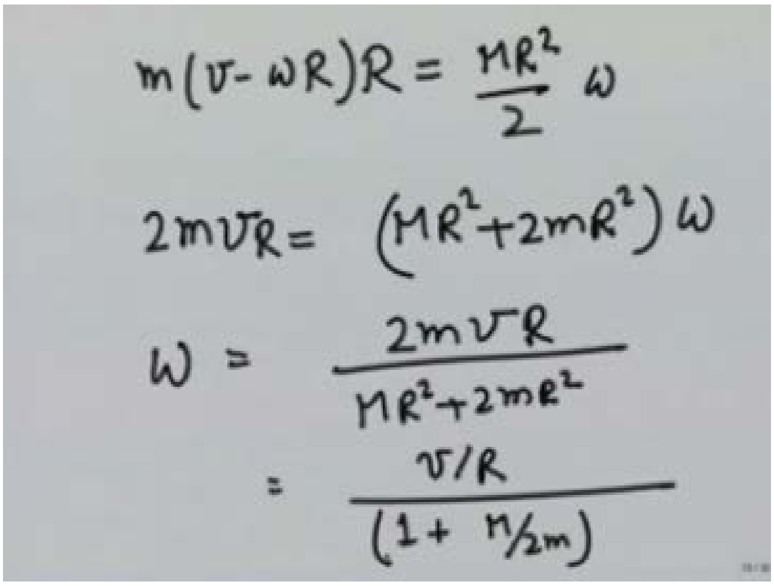

Should be equal to M R square by 2 omega which gives me 2 m v is equal to M R square plus 2 m R square. Omega or omega equals 2 m v over M R square plus 2 m R square v. There is an R here and v R which is nothing, but v over R divided by 1 plus M over 2 m.