Engineering Mechanics
Prof. Manoj Harbola
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Module - 07
Lecture - 01
Rotational Motion - I

So, far we have been looking at the motion of a single particle or a collection of particle, but only that of it CM that is the center of mass.

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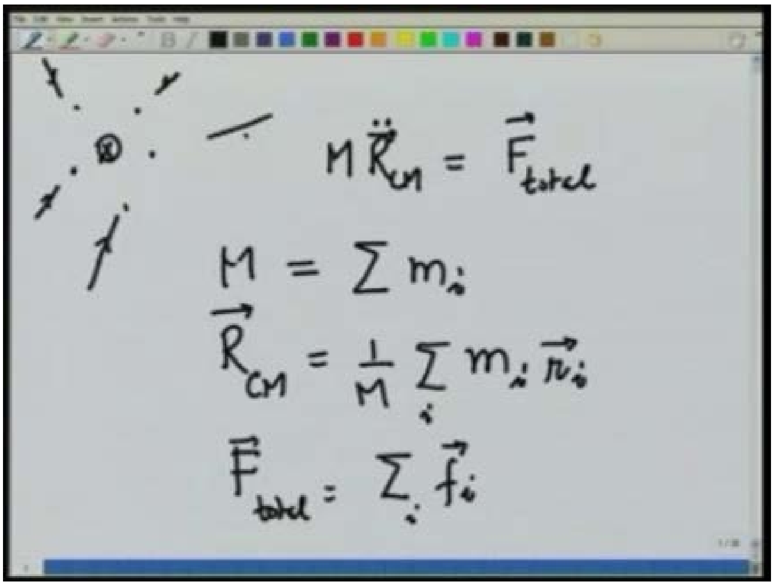

So, for example, if we have a collection of particles like this and I apply different forces on each particle. Suppose, this happens to be its center of mass what we have been looking at is how the center of mass moves. That is the equation that we have been solving in the previous lectures was the total mass M R CM double dot where double dot refers to the second derivative of the R vector is equal to total force applied.

Here, M is the sum of individual masses there is the total mass the vector R CM is defined as 1 over M summed over m i r i that is a vector for each individual particle and F total is the net force being applied on the system totally. Now, I want to look at individual particles how its motion takes place I want to go beyond the center of mass motion.

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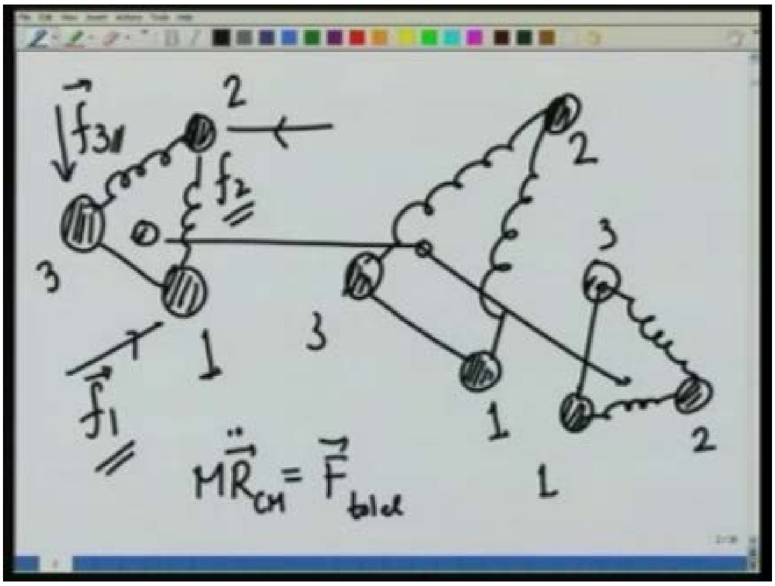

So, for example, suppose I have these particles connected by a spring for simplicity I will just take 3 of them and I apply different forces on each particle. Let me name this particle 1, particle 2, particle 3, let force on particle 1 be f 1 let force and particle 2 the f 2 let the force on particle 3 be f 3. As we apply these forces in the next instant, I could have it different situation in which the particles could be may the spring has stretched this let us says rod which is cannot change. Maybe, this is spring has stretched further I could have change its orientation that is if this is particle 1 2 and 3 other situation I could have is may be particle 1 is here particle 3 is here and this is particle 2.

In other words, under the influence of these forces f 1 f 2 and f 3 the body could deform that is the distance between the particles could changes and it could changes orientation none the less the center of mass still moves according to what the net forces. That is the center of mass would the still follow this equation, but in addition to that the translation of center of mass like a heavy point particle under the net force total force f total. There could be other things that is what I said may be the deformation may be a change in orientation. So, these are beyond the translation motion of center of mass and this is what we are going to focus on to start with let us again separate the deformation motion and the change in orientation motion.

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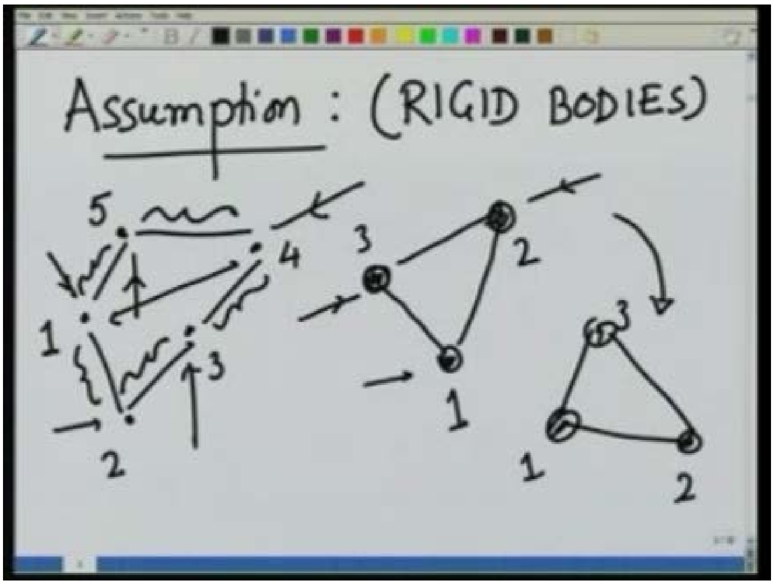

So, I am going to make an assumption and this assumption defines what are known as rigid bodies the assumption is that given a system of particles when I move them. So, let me again number then 1, 2, 3, 4, 5 when I apply different forces on each of the particles the distance between 2 particles remains unchanged. These distances remain fixed similarly, distance between this and this distance between this and this particle and so on. In other words, in the previous example where I took 1 rod 1 particle here 1 particle here and there was a spring here I replace this spring by rods that cannot change their length So, the distance between 2 particles would remain unchanged you can feel right away when I apply a force it is not going to deform.

The only thing this body can do is change its orientation, so for example 1, 2 and 3 could go to a different shape like this. Now, different orientation like this and this is known as a rigid body in rigid body the distance between any two particles of his body remains fixed. The only possible motion then is translation of center of mass plus a change in orientation; this is how we simplify the problem the deformation part we can add later. For example, deformation leads 2 say wave motion or plastic phenomena or by extension and shorting of a say spring under a force, we are leaving that right now, we are going to assume that the body is rigid.

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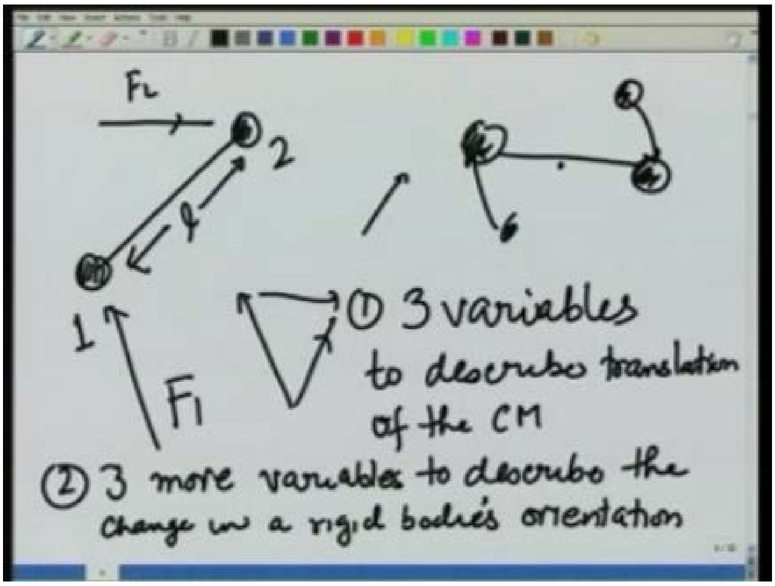

Take as an example again a body consisting of 2 masses which are attached to a rod of fixed length l. So, I would call it a rigid rod. This is a rigid body, now if I apply force here and a force here, let us call it F 1 let us F 2 what will happen is F 1 and F 2 have a net force let us say this is F 1 this is F 2 this is a net force. So, center of mass of this body would move in this direction it would have moved, so center of mass would move, but the body could itself change orientation it could be in this orientation. In general, I would required three variables to describe translation of the CM and three more variables to describe the change in a rigid bodies orientation. You can think of these three variables to describe the change in a rigid bodies orientation as three angles from three axis.

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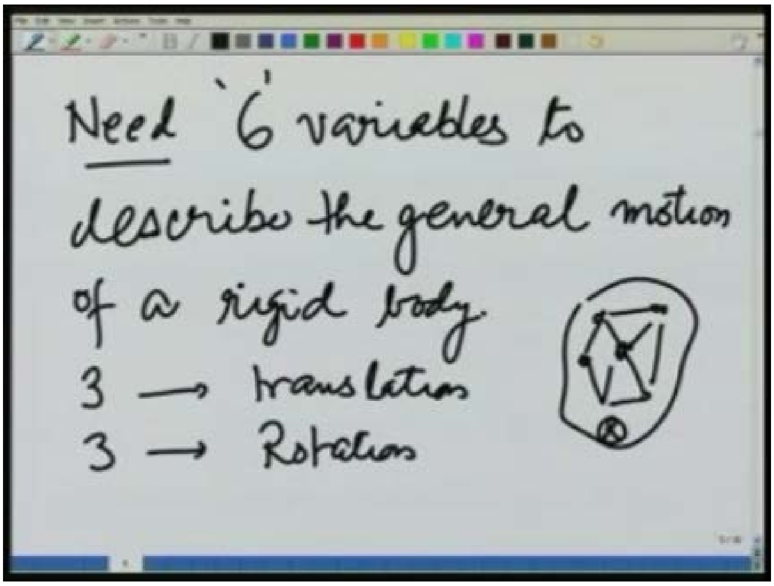

So, in general what we learn is need six variables to describe the general motion of a rigid body. You may ask why six only in a rigid body there could be ten particles, fifteen particles, ten raise to twenty-three particles why only six. That comes because there is a big constraint we are working with the constraint is no matter how many particles the distance between two particles will always remains fixed. That constrained reduces the number of variables that I need to described the motion of a rigid body to six, three of these are for translation and three for rotation. What will happen if I take a rigid body, now I can make it a general rigid body like this all the distances are fixed between these points and fix one point of this body.

Suppose, I pin it here then it cannot translate anymore and therefore I would need only three variables for describing its rotation. So, I hope by now I have given you a feeling of what a rigid body is and how many degrees of freedom does it have when it moves it has 6 degrees of freedom. On the other hand if I fix 1 point then it has only three degrees of freedom and that is the change in that angles required describing the change in its orientation. So, that is the rigid body and in next few lectures we are going to make this approximation that the bodies we are considering are rigid bodies and described their dynamics.

We start with simple problem of rotation about 1 axis, then we let the axis also move and in third case we also let the axis change is orientation. So, these are slowly we are going to build up these problems, but we are not going to consider the deformations in that sense all the body motion that we are considering are rigid body motions.

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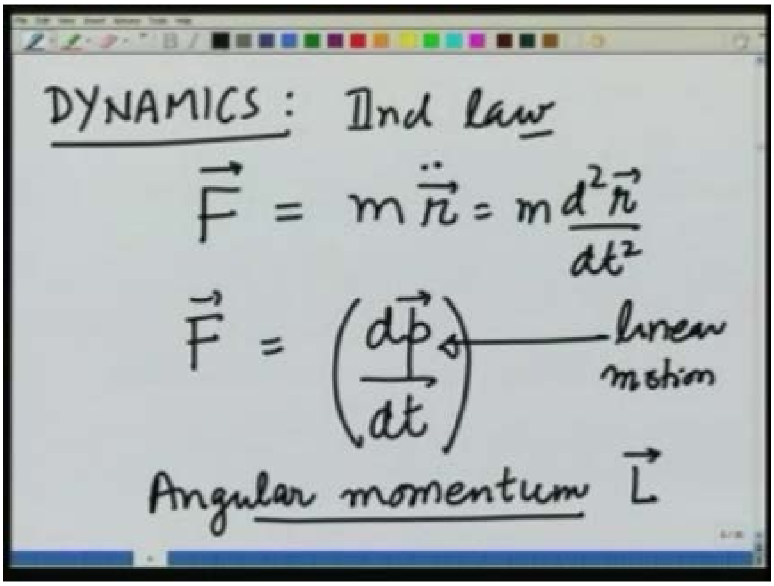

How about the dynamics of a rigid body, you recall when we considered the dynamics of a particle we started with second law of Newton that said that for a point particle force equals the mass times r double dot or mass. The second derivative with acceleration of r with respect to time, later when we learnt about momentum we said force is nothing but d p d t that is the rate of change of the momentum of the particle. The equivalent role that is played in describing the orientation of a body and rigid body dynamics is that of angular momentum that is the role that momentum placed in linear motion is played by angular momentum.

Let me denote it by a vector L in describing rigid body dynamics angular momentum is a general concept a general quantity that could be defined for any motion and by it is particularly useful in in describing rigid body dynamics. So, in the next few minutes or may be the rest of the lecture, we are going to spend time on understanding the angular momentum to develop the feeling for angular momentum. We are first going to start with definition of angular momentum and then going to describe them for a single particle.

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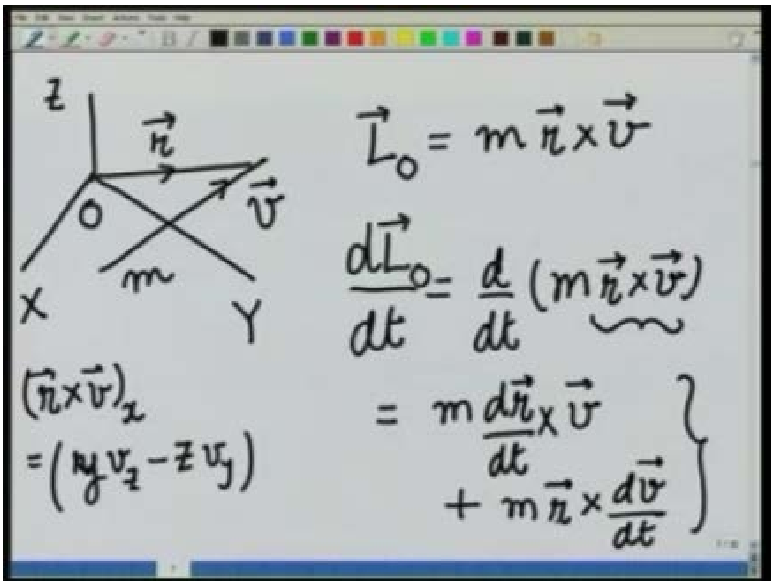

So, given a single particle moving in coordinate X Y and Z with origin at o suppose there is the particle moving like this with velocity v is mass m. Then, the angular momentum with respect to o and I emphasize with respect to o because angular moment of happens to be an origin dependent quantity is given as mass r cross v. Here, r there is the distance displacement of the particle at any given time this is the definition of angular momentum for a single particle, how about the dynamics that I was talking about the change in angular momentum with respect to time.

Analogous to what we learn, a single particle dynamics is change of the momentum with respect to time here we discussed change of angular momentum with respect to time is going to be equal to the derivative m r cross v. I am now going to use the chain rule for differentiation for those of you who feel little uncomfortable, I will leave it as an exercise that you take the components of this cross product vector and do it in terms of component.

I am going to write it directly as m d r d t that is I take the derivative of r with respect to time cross v plus the second term because I am using the chain rule m r cross d v d t. I said earlier if you are not comfortable using the vector quantities directly take the component of r cross v may be take the x component which would be equal to y v z minus z v y, take the derivative and see that finally, this emerges. So, derivative of r cross v is going to be d r d t cross v plus r cross d v d t m is common.

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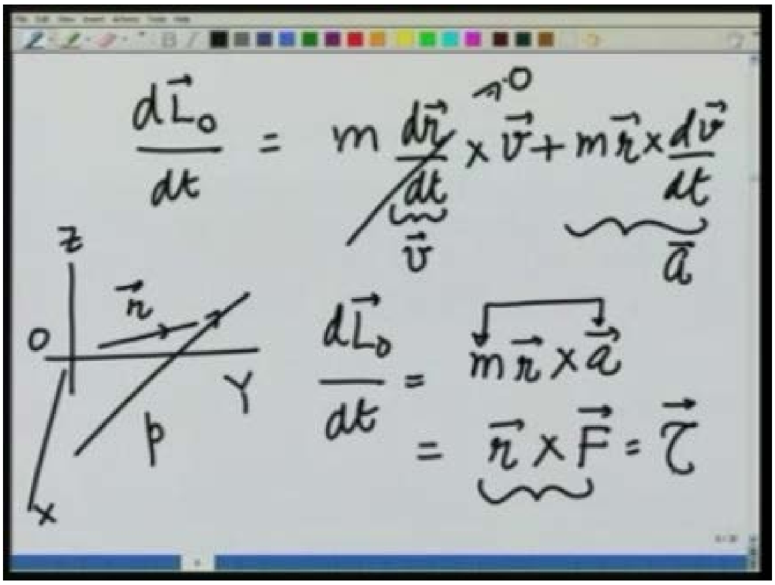

So, what we have obtained is DL with respect to o d t where let me just remind you that I have an origin o X Y Z particle is moving like this with momentum P this is r is equal to m d r d t cross v plus m r cross d v d t. We know that d r d t is nothing but the velocity itself and velocity cross velocity is going to give you 0. Therefore, what you are left with is this quantity in this d v d t the derivative of velocity with respect to time is the acceleration. So, I have d L o d t is equal to m r cross the acceleration m times the acceleration is the force. Therefore, this is equal to r cross F and we define this quantity to be the torque and I am going to denote it with this symbol call tau.

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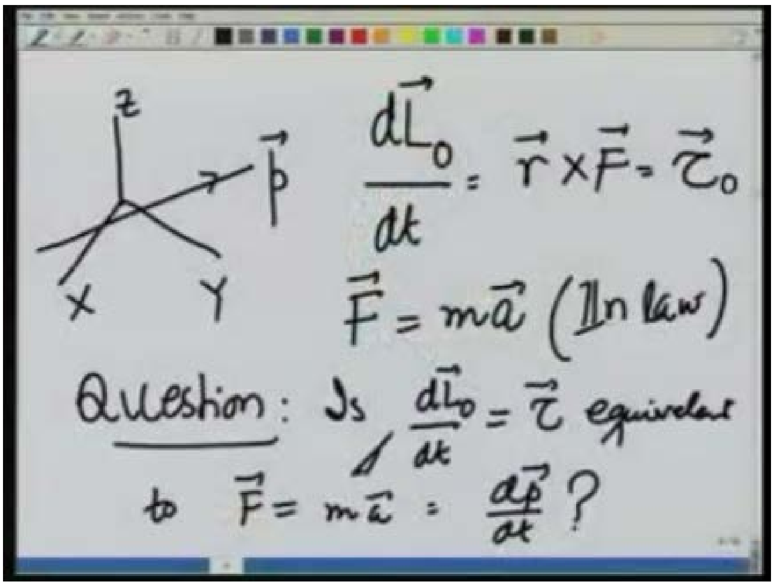

So, what we learn and I have derived for you is that given a particle moving, the change is in its angular momentum. I always denote with this o emphasizing the fact that angular momentum is a quantity that depends on the origin is equal to r cross F or the torque about o. Notice while deriving this I use the fact that F equals ma that is force equals mass times acceleration or the second law. Except that, now instead of talking about change in momentum, I am talking about change in angular momentum are the 2 equivalent.

So, question we know have is question is d L d t equals tau equivalent to F equals ma or d p d t. I am doing all these exercise to make if you familiar with angular momentum and related mathematics. Let us understand this by considering an example I will consider the example of conical pendulum. With that I will show you a couple of things first I will show you that the angular momentum indeed is origin dependent and number 2, these two are indeed equivalent.

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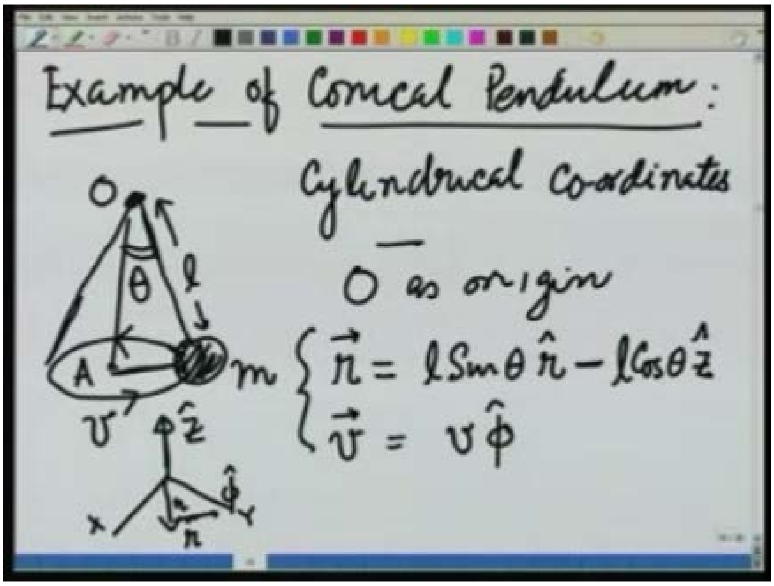

So, let us do that let us take the example of conical pendulum this is a nice example to illustrate the angular momentum concept and its derivative. A conical pendulum is nothing but a regular pendulum with a rod rigid mass less rod of length l pivoted at a the point here. Instead of moving in a plane the bob goes in a circle it is going around in a circle and this rod makes say the angle theta with the vertical.

Let me call this point pivot point o and let this point where this vertical line meets the plane in which the circle is made. Let me call this because of the symmetry of the problem I am going to use cylindrical coordinates that we learnt in our second or first lecture to describe the motion of this bob of mass m. It is moving around in a circle let this be the origin to start with o, so first case I take o as origin in that case the r vector for the ball is going to be given as l sin theta l sin theta. This is l sin theta times r remember r unit vector is the radial unit vector in cylindrical polar coordinates minus l cosine of theta z.

Let me also say that this bob is moving with velocity v in the circle v is always in phi direction. So, velocity v is equal to the magnitude v times unit vector phi, those of you who have forgotten let me remind you for a given origin the distance in the plane of X and Y the distance is given as r. Then, this v vector is r unit vector perpendicular to this is a phi vector and vector Z is a Z vector, so in X Y planar using planar polar coordinates and Z Z unit vector Z unit vector. So, these are given as r and v and now let us calculate the angular momentum for the conical pendulum.

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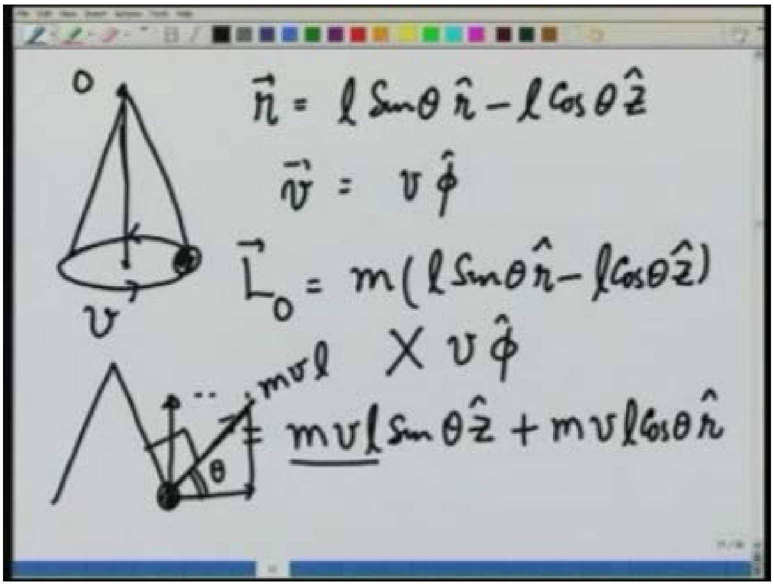

We again remained you, this is my conical pendulum the bob is moving around in a circle with velocity v is going in this direction this is origin o. We have written r is equal to l sin theta r minus l cosine of theta Z v vector is v phi this is the bob this is where A is.

Therefore, the angular momentum L about o is going to be m l sin theta r minus l cosine theta Z cross v phi R cross phi is Z. So, this comes out to be m v l sin theta Z and Z cross phi is going to be minus r you can check that. So, this is going to be plus m v l cosine theta r unit vector, therefore at any given point, let me make it again at any given point say here the angular momentum has a component in the Z direction m v l sin theta and in the r direction m v l cosine theta.

Therefore, you can see that the angular momentum is going to be something like this with this angle being theta. This magnitude is m v l and this angle is 90 degrees and you work it out you will see after all this is r cross v, so it has to be perpendicular to r and then from these component you see this is the angular momentum.

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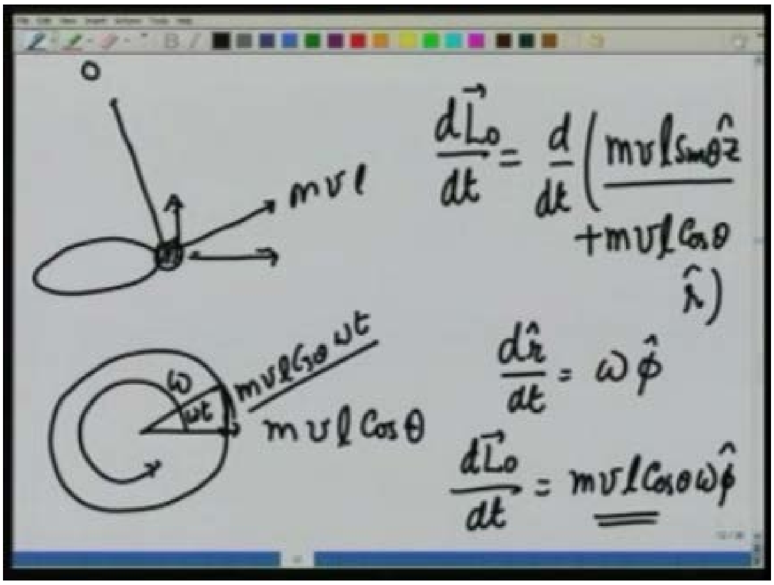

So, what we have learnt by calculating angular momentum about this point o of this bob is that it is perpendicular to this and it is magnitude is m v l, this component has in this direction and a component in this direction. As the bob moves around the r unit vector changes and therefore this component changes direction whereas, the vertical component remains constant. So, let us understand the horizontal component which is m v l cosine of theta keeps rotating and the vertical component remains constant. You can already sense what d L by d t is going to be, so d L by d t L about o is going to be d over d t of m v l sin theta Z plus m v l cosine of theta r is derivative.

In this m v l, theta Z all are constant, so this derivative is 0 in this m v l theta r constant, but r unit vector keeps on changing. If you recall from first or second lecture dr over dt is nothing, but, omega in phi direction, therefore I get d L o d t as m v l cosine theta omega and we will see what omega is in phi direction. This you could have also check because this vector is rotating in this direction with angular frequency omega therefore, the change has to be m v l cosine theta times omega. I mean this is when it moves here in omega times t time this change is roughly m v l cosine theta omega t, we divide by time and you get this answer.

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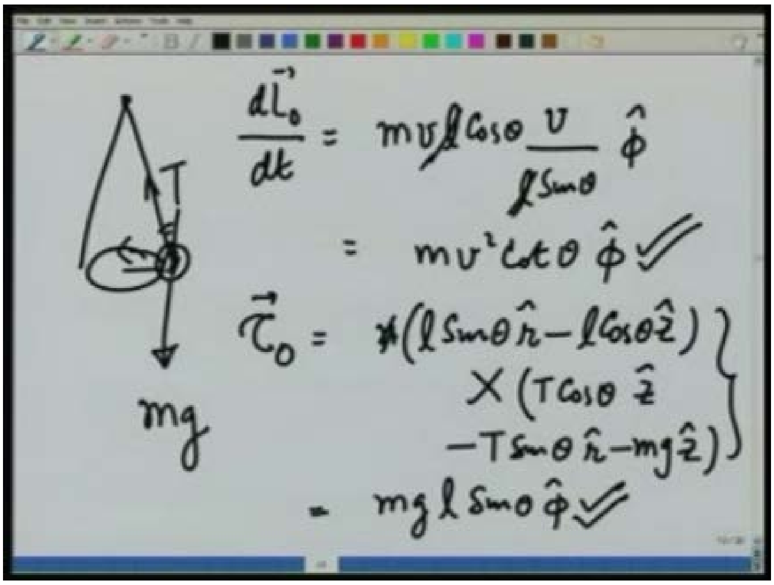

So, what we learn from this is if I take this conical pendulum which is moving like this d Lo d t is equal to m v l cosine theta omega is going to be velocity divided by this by radius. So, velocity divided by l sin theta in phi direction which is mv square cotangent theta in phi direction is that equal to tau let us do that. If I calculate the torque about this point again r is nothing but m sorry l sin theta r minus l cosine theta Z cross the force 1 force is mg and the other forces tension T. You can see is parallel to r it does not contribute to the torque, but, none the less let us write this tension T has a component. So, is this angle is theta is going to have a component T cosine of theta in Z direction minus T sin theta in r direction.

I have a minus mg in Z direction you calculate all this and you are going to get r cross Z r cross Z gives you minus phi. Therefore, you get m g l sin theta in phi direction and everything else drops out you can work this out very easily. So, what we learn now is that this quantity which is d L d t and this quantity which is torque about o must be the same.

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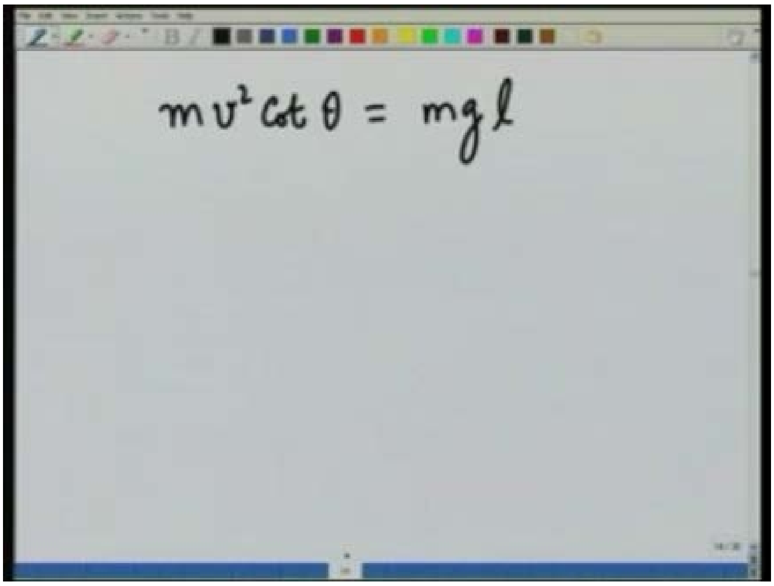

Therefore, we get m v square cotangent theta is equal to m g l.

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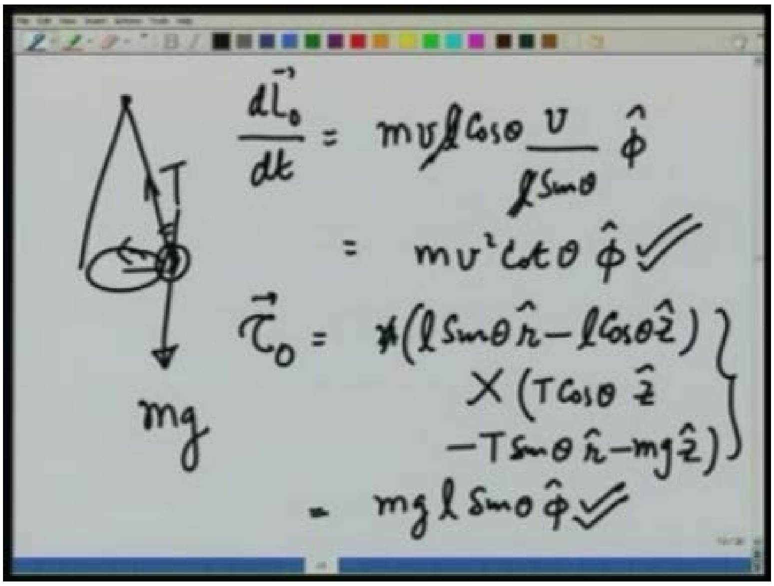

Let me see it was m g l sin theta.

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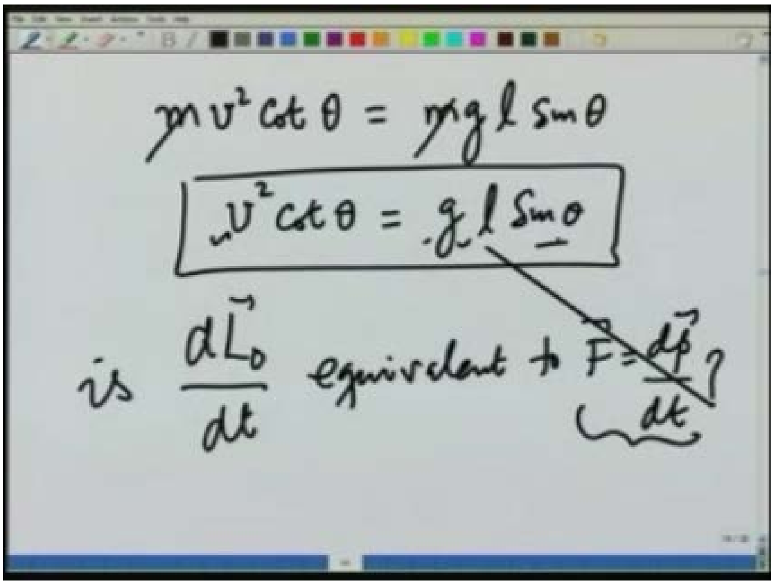

Here, m g l sin of theta both were in phi direction m drops out and therefore we get v square cotangent theta equals g l sin of theta. This is what we get maximum I can get from this is theta as the function of v and g l, so again I raise the question is d L o d t equivalent to F equals d p d t. The answer seems to be know because only thing I am getting from here is the angle theta, whereas F equals d p d t also gives me the tension, let us see that and then we will understand it better.

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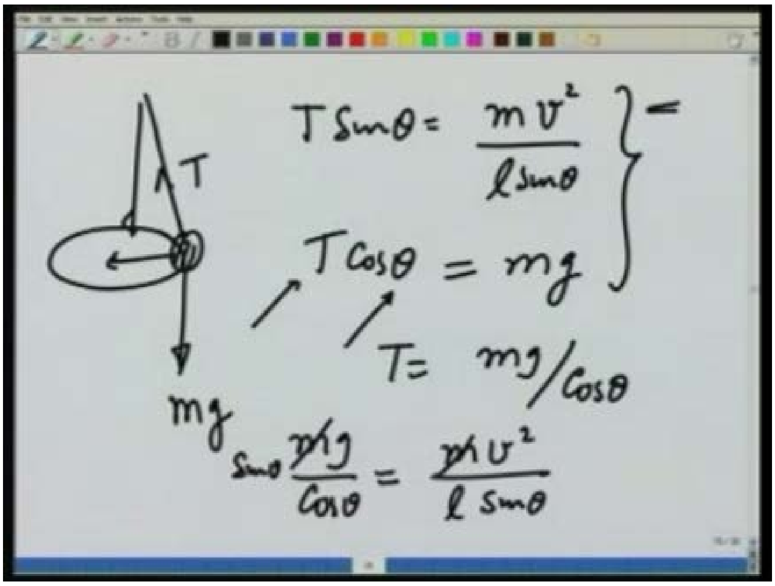

So, when this bob is moving in a circle like this is the tension T in the rod this is mg this component of T provide the centripetal acceleration. Therefore, T sin theta must be equal to mv square over the radius of the circle which is l sin theta and T cosine of theta must be equal to m g. We get 2 equations here and therefore, we can solve them to get 2 quantities that is theta and tension both, whereas when are applying d L by d t equals tau we are writing only 1 equation.

The reason is that when you take cross products some components drop out, so what is the way out, so let us first solve this. So, from here I get T equals mg over cosine of theta and when we substitute in this equation we get mg over cosine of theta equals mv square over l sin theta and there was a sin theta here also, m drops out.

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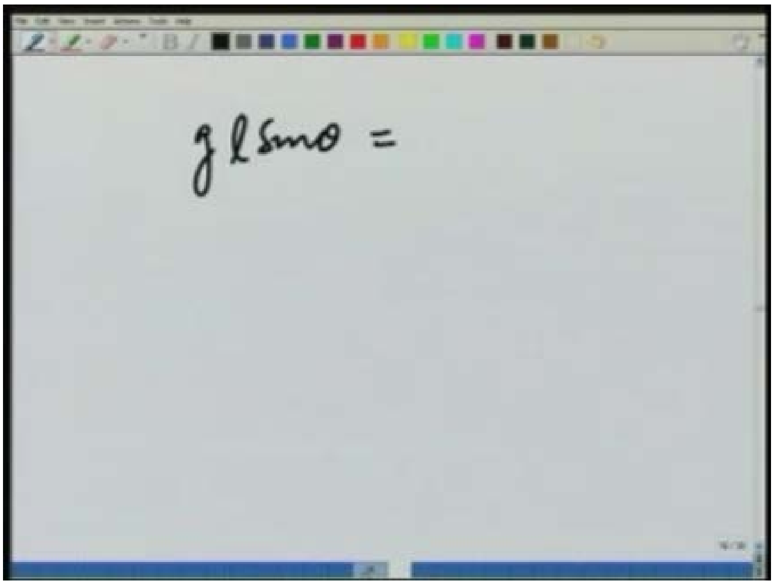

Therefore, we get g l sin theta equals let us see what it was.

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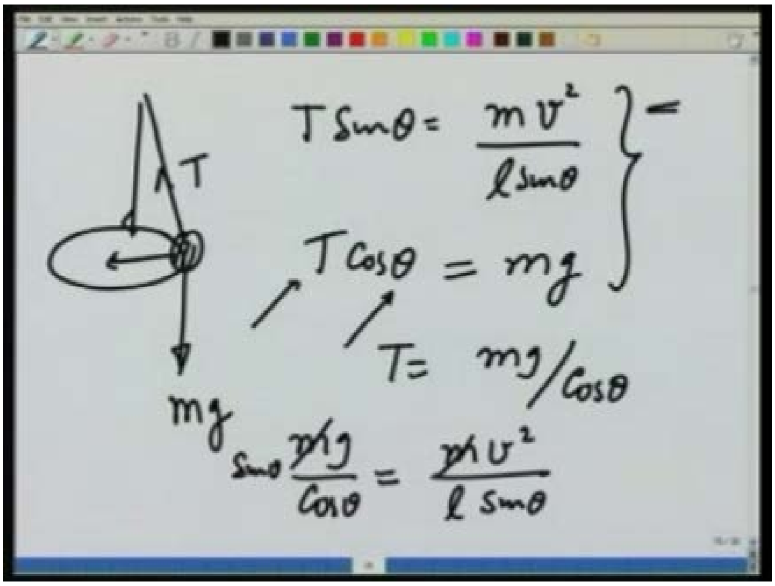

G l sin theta equals v square cotangent theta.

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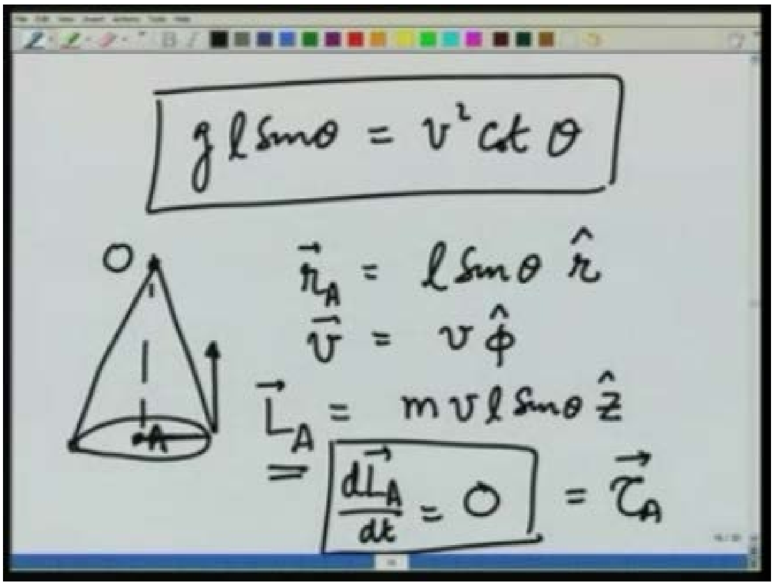

This is an equation that we had obtained earlier, however these equations also give me T to find T using the torque equation what I have to do I have to find another equation. That equation comes if I take torque about this point, earlier we took torque about this point angular momentum about that point now we going to do it about point A. So, let us again see if I take A as origin then r with respect to A is going to be equal to l sin theta that is this distance l sin theta in the r direction.

The velocity is nothing but v phi and therefore angular momentum about A is going to be m v l sin theta r cross phi is Z. You see when I take angular momentum about point A only the vertical component remains the horizontal component has vanished. Therefore, when I take d L A over d t, everything on the right hand side is constant and therefore this comes out to be 0, if that is the case, torque about point A must also be 0 and let us figure that out now.

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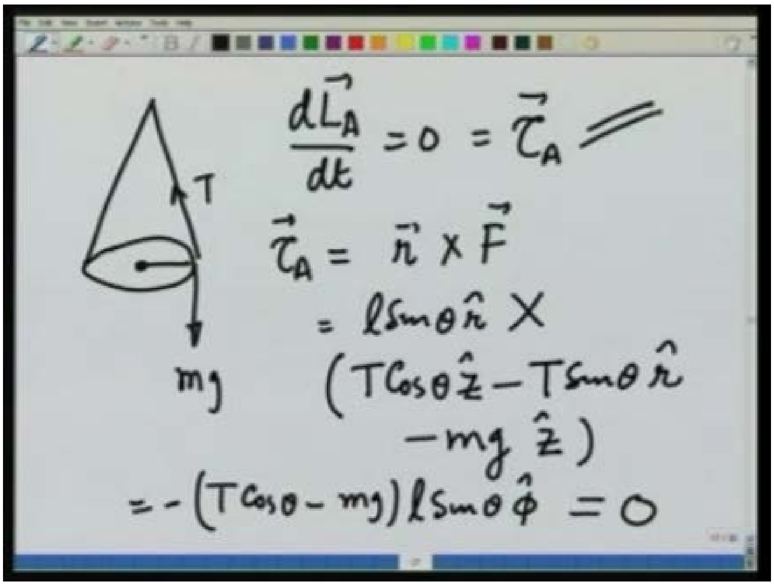

So, what we have learnt if I take l about point A, I get d L A d t is equal to 0 and this also must be equal to torque about point A, this is the tension. Let us calculate torque about point A and I am using strictly mathematical definition. Here, r cross F r is nothing but l sin theta in r direction cross F is nothing but T cosine of theta in Z direction minus T sin theta in r direction minus mg in Z direction, r cross r r cross r is 0. Therefore, when I take torque about A I get T r cross Z r cross Z is minus phi T cosine of theta minus mg times l sin theta in minus phi direction and this must be 0 because d L A d t is 0.

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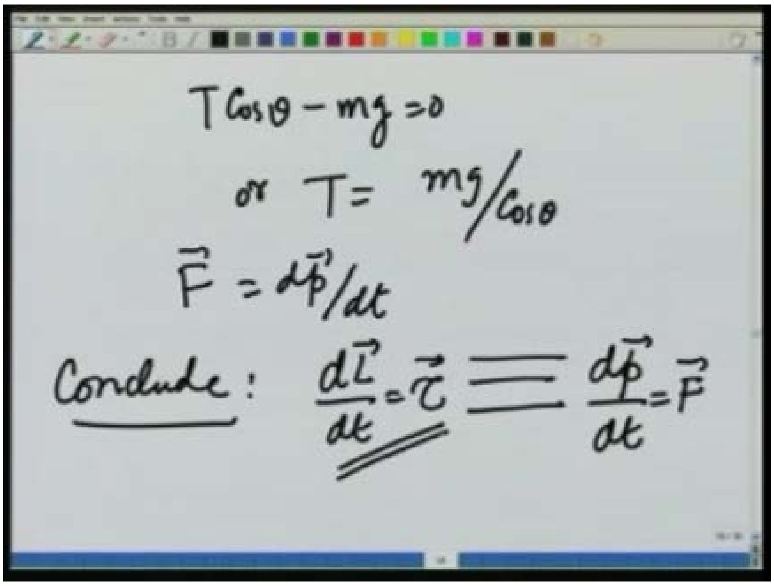

This gives me T cosine theta minus mg is equal to 0 or T equals mg over cosine of theta, which is the same result as I obtained from the Newton's second law direct application F equals d p d t. So, what we conclude is conclude is that d L d t is equal to tau is absolutely equivalent to d p d t equals F, the only certainty is that in applying this I may have to take torque and angular momentum about two different points.

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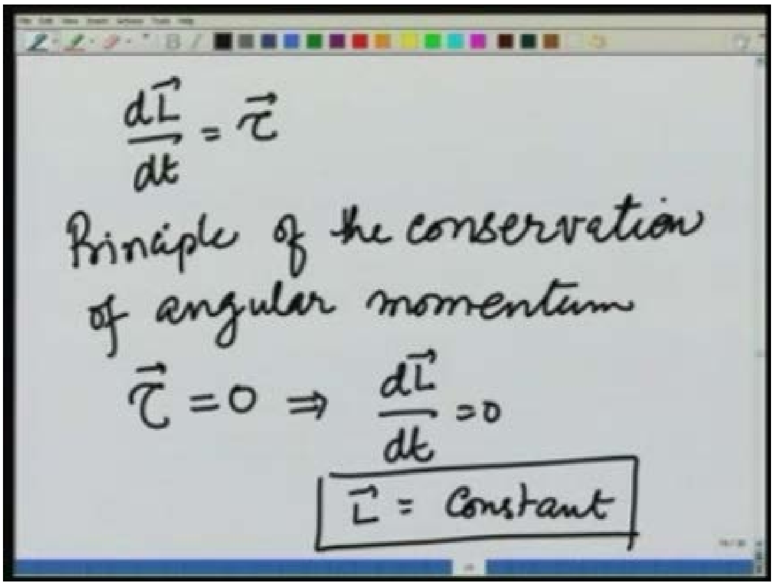

Having learnt that d L d t although for a single particle is equal to tau, now we enunciate the principle of the conservation of angular momentum and that else if tau is 0 this means d L d t is 0 or L is a constant. You are already familiar with the use this condition in a previous in may be your lectures in twelfth grade, let me just show it to you.

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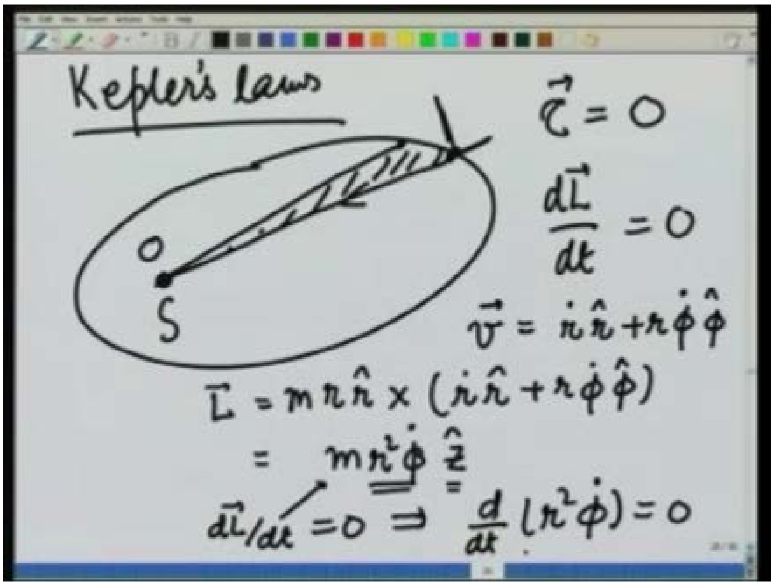

You are all familiar with Kepler's laws and one of them is when the earth is moving around the sun or any planet is moving around the sun in equal time they cover equal areas. This is nothing but a manifestation of the conservation of angular momentum let me show you how. If I take this sun as the origin the force is always directed towards the origin and therefore tau the torque r cross F because r and F are parallel is 0 and therefore the angular momentum must be conserved. Now, you see in these case the velocity at any given point, recall your planar polar coordinates could be in r dot direction and perpendicular to this.

So, velocity in general is r dot r plus r phi dot in phi direction and therefore angular momentum is r r r cross v v is r dot r plus r phi dot phi times m. This is equal to r cross r 0 mr square phi dot in Z direction, m is the given mass for a given particle r and phi dot keep changing Z is a fix vector. So, therefore, d L d t is equal to 0 implies that d over d t is of r square phi dot is equal to 0.

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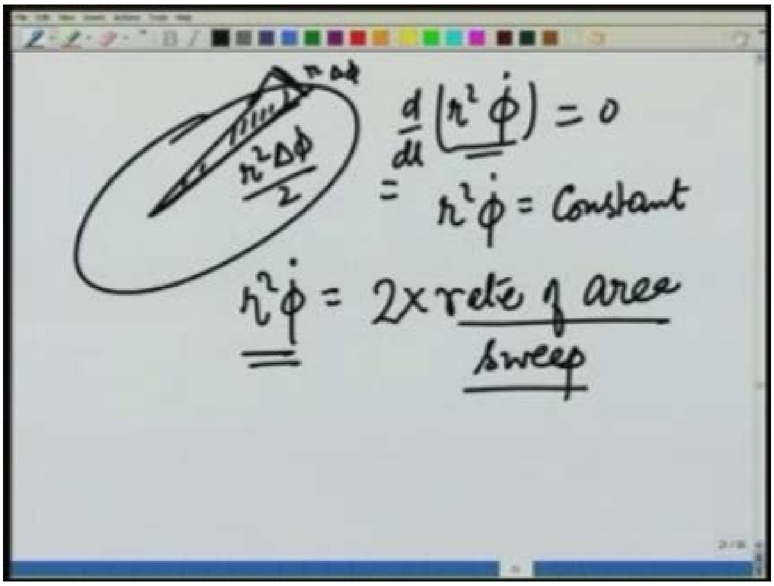

You can see that this r square phi dot is nothing but r square phi dot is equal to 0 r square phi dot therefore is a constant and r square phi dot is nothing but rate 2 times rate of area sweep. That is given an r this area you see is roughly r square delta phi in time T divided by 2, this is r this is r delta phi. So, the change of area in time delta T is r square delta phi divided by 2 divided by delta t give it r square phi dot over 2 as rate of change of area and constancy of this implies constancy of rate of area sweep.

The Kepler's second law with says that the radius vector in a planets motion sweeps equal areas and equal time is nothing but a manifestation of the conservation of angular momentum. This becomes a powerful tool to solve problems involving angular momentum as we will see in the examples as we go along. Having done all this for a single particle are now going to move to many particles because after all when we deal with a rigid body, it consist of many different point particles.

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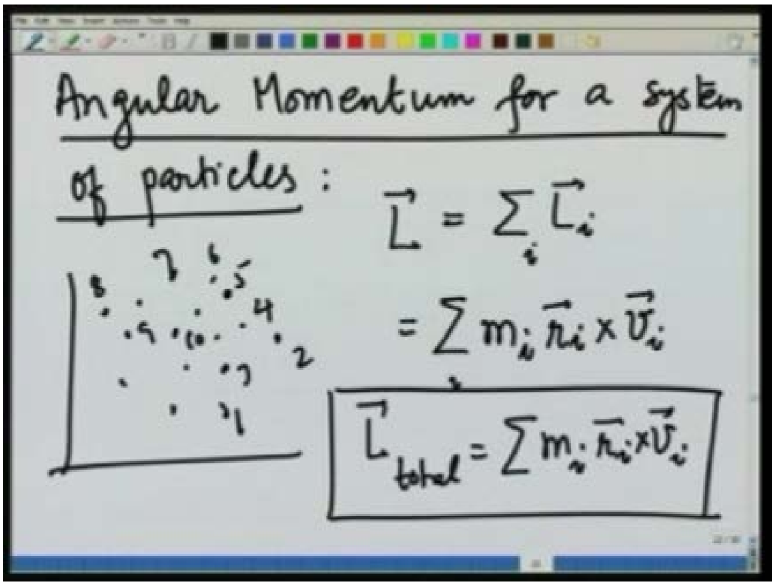

So, now we are going to go to make our self familiar with angular momentum for a system when I say system that means, many particles of particles. So, what we have going to consider is there many different particles given and by definition angular momentum is going to be the sum of angular momentum for each 1 of these particles where I runs from 1 through m 4, 5, 6. They may be m particles 7, 8, 9, 10 and so on, so this is nothing but summation m i that is mass of the i th particle m 1, m 2, m 3, m 4 r i cross v i.

That is it, L total is nothing but summation over m i r i cross v i, we can use this definition now to relate the total angular momentum to the angular momentum of the center of mass and plus angular moment of about the center of mass. Recall from out previous lectures where I was talking about center of mass and I kept on a emphasizing the center of mass is a very important quantity. I will use it again and again and you will see again a rigid body dynamics, it becomes a very central concept.

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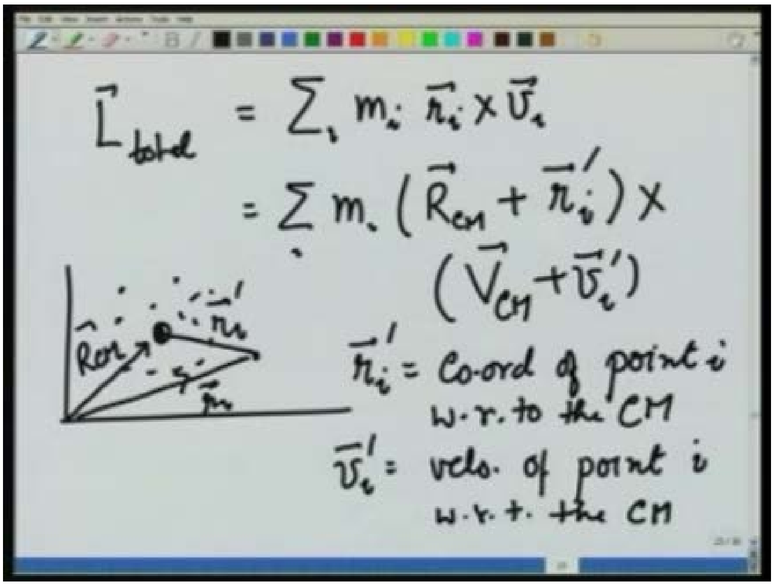

So, let us now write L total which is equal to summation I m i r i cross vi as summation I m i, let me write r i as suppose I just make a 2 dimensional plot this is the center of mass of distribution of particles. I will call center of mass coordinate R CM and the coordinate of a point particle i th point particle with respect to CM is r i time. So, this is R CM and with respect to this the point is given at r i prime and this is r i. Similarly the velocity would become V of CM plus vi prime where let me write explicitly r i prime is coordinate of point I with respect to the center of mass v i prime is the velocity of point I with respect to the CM.

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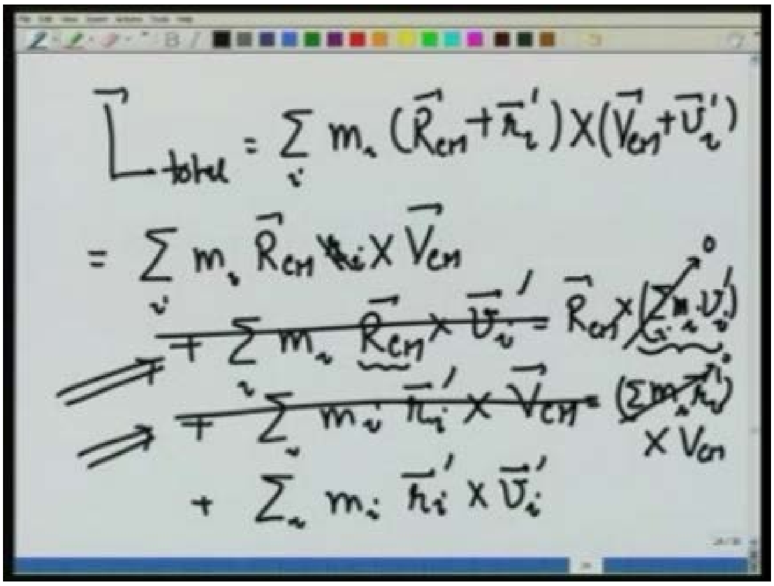

So, what we get is given a distribution of particles L total is equal to summation I mi R CM plus r i prime cross V CM plus v i prime. This I can open up and write as summation I m i R CM sorry cross V CM plus summation I m i R CM cross v i prime plus summation I m i r i prime cross V CM plus summation I m i r i prime cross v i prime, look at this term second term R CM is common to all.

So, this can be written as R CM cross summation I mi v i prime, similarly look at the third term V CM is common. So, this can be written as summation m i r i prime cross V CM a minutes thought will tell you this is the total momentum with respect to the center of mass and that is always 0. Similarly, this is coordinate of the center of mass with respect to the center of mass that is also 0, so these two terms drop out what I am left with?

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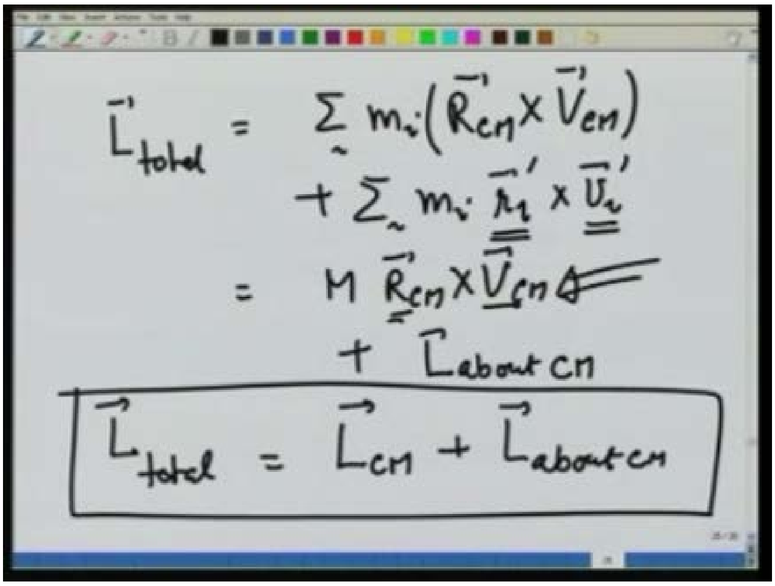

Therefore, that L total is equal to summation I m i R CM cross V CM plus summation I m i r i prime cross v i prime again this is the common quantity. So, this is only summation m i which is the total mass R CM cross V CM plus this is nothing but coordinate with respect to center of mass velocity with respect to center of mass therefore this becomes nothing, but, L about CM. This quantity is nothing but as if there is a massive particle of mass m moving with velocity V CM at distance R CM. So, this is nothing but the angular moment of center of mass and therefore I can write L total as equal to L of CM plus L about CM.

This is general result irrespective of whether a body is rigid or not, you see we have not used the rigid body condition anywhere. The total angular momentum of the body is going to be a sum of the angular moment of a center of mass plus angular moment about the center of mass.

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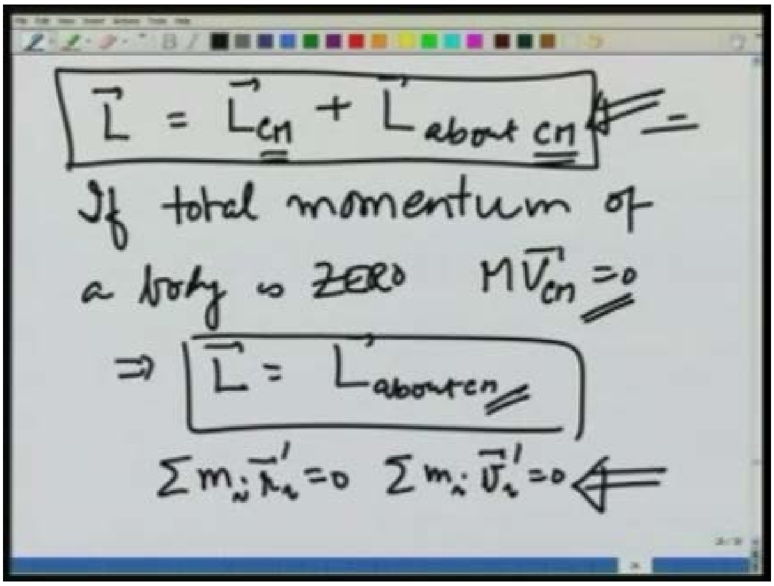

This equals L CM plus L about CM and you can see if total momentum of a body is 0 that is MV CM is 0 this implies L equals L about CM. Therefore, in that case the angular momentum of the body no matter where I take it from is independent of the origin because this is always come out to be L about CM. This is only because the total angular momentum total momentum of the center mass is 0. In general, this is the result you can see, this is also a result which depends on the property of the center of mass that is summation m i r i prime with respect to center of mass is 0 and similarly summation mi v i prime is equal to 0.

So, this is decomposition cannot be done about any arbitrary point that is I cannot say L equals L some point o prime plus L about that point o prime it is only with the center of mass. The center of mass satisfies these two equalities that I can decomposed the angular momentum in this manner this must be kept in mind.

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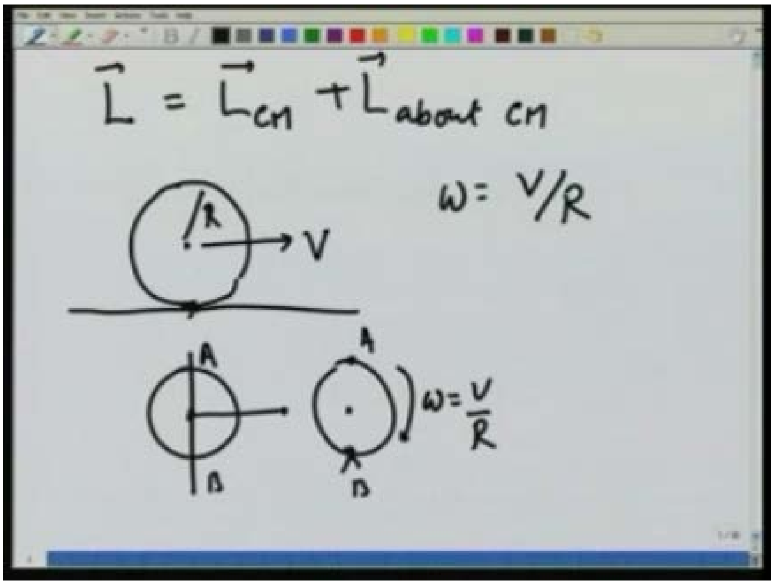

So, what we see is that L is equal to LCM plus L about CM let us take a simple example of it. Let us take a bicycle wheel rolling on the ground if it is rolling this point is not moving instantaneously and therefore, if it is rolling with the speed V and it is radius is R I have omega equals V over R. So, I can consider this motion as if the center of mass is translating with velocity V and top of it the. So, this body let me just see this is point A point B this whole thing is translating with velocity V and on top of it with respect to center of mass, this is moving with omega equals V over R, so that instantaneously this point is at rest.

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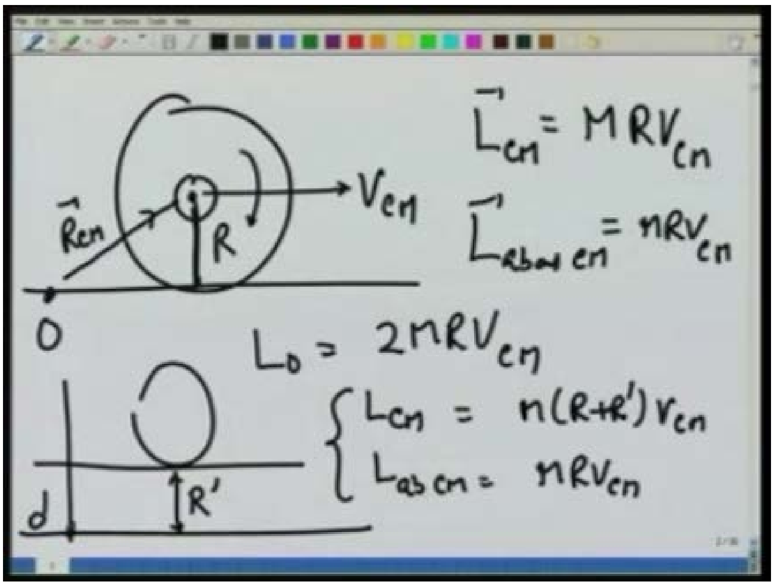

So, the angular momentum for this if I take the origin here this is R is going to be this is R CM this is V CM. So, you can see right away that L of center of mass is going to be M R V CM where R is the radius R CM cross V will give you only the projection. Then, L about CM this is going to be in the direction perpendicular R cross V is going to be perpendicular going into the page I am not worrying about the direction. Now, L about CM since this is rotating like this is also going to be M R V CM you can calculated that and therefore with respect to o the angular momentum is going to be 2 M R V CM.

On the other hand, let this be the ground if I shift my origin here let us call it o prime say by distance R prime. Then, L CM is going to be equal to M R plus R prime V CM. However, L about CM is going to remain the same and then if you sum the two, you get L about o prime.

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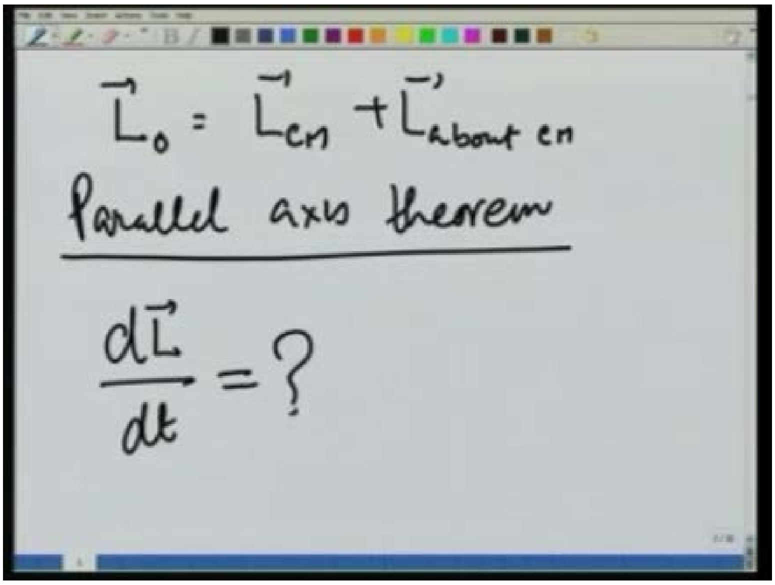

This fact that L About any point is equal to L CM plus L about CM also gives raise to something known as the parallel axis theorem in relation with the moment of inertia. I am sure you heard of this theorem, but we will come back to it later when we do rigid body dynamics. Now, we have trying to make our self familiar with angular moment, how about the dynamics, now how about the d L I am dropping these suffix t, now d L d t for a many body system let us work that out.

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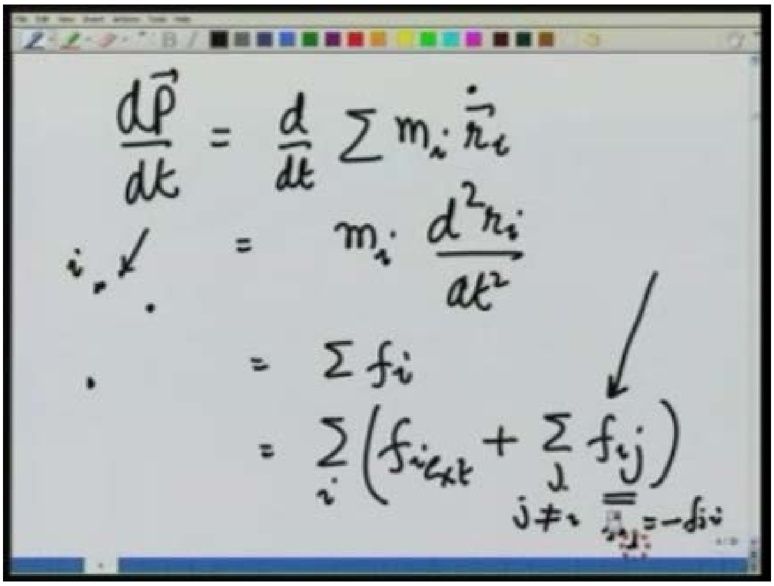

Recall when we are doing the momentum change for a Many body system we had this as d over d t summation m i r i dot which had given us mi d 2 r i d t square. This was nothing but summation over fi where fi that is on if particle the force was nothing but the external force that is something I apply externally plus the force j not equal to I on i th particle due to other particles. We had shown them and that is how we came to center of mass motion that this summation over I and j gives me 0 if f ij the forces between the articles satisfy Newton's third law. That is f ij is equal to minus f ji, the force on i th particle due to j is opposite and equal to force on j f particle due to i.

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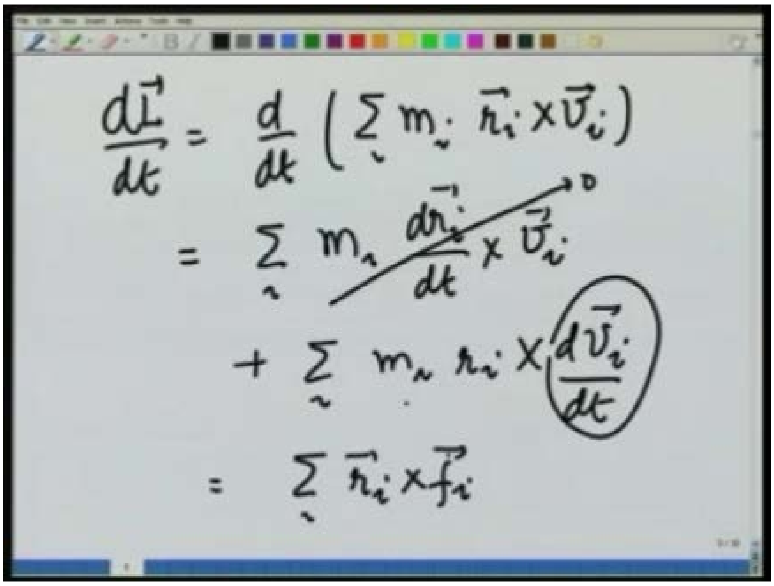

A similar derivation now we are going to do for the angular momentum and let us write this is d over d t of summation I m i r i cross vi which gives me summation over I m i d r I over d t cross v i. This being the velocity this term cross product is 0 plus summation I m i r i cross d v i d t m i times this acceleration d v i d t is nothing but the force on the i th particle. So, this gives me summation I ri cross fi, where fi is the total force on the i th particle that is the force including the external force plus the force due to other particles.

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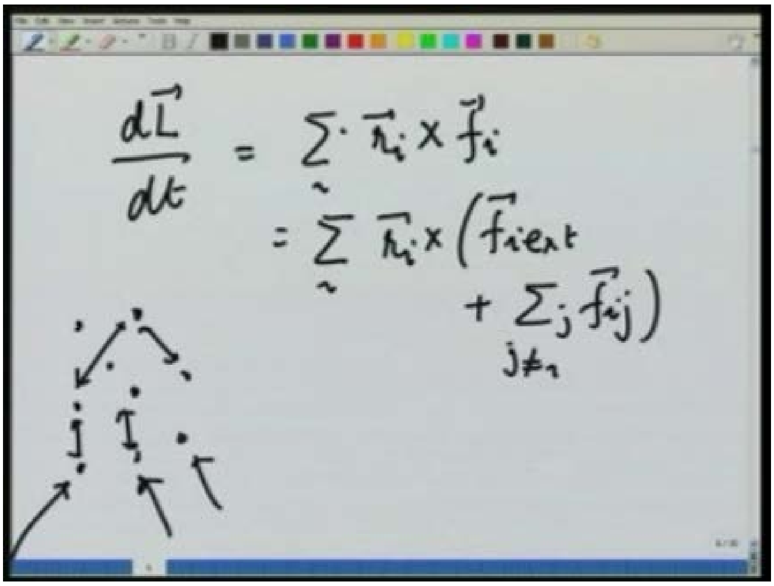

So, d L d t for a many particle system that is they are many particles which are interacting with each other they may be interacting with the each other. There are external forces applied only each 1 of them is equal to summation I m r i cross f i which explicitly I will write as I r i cross fi external plus summation over j that is all the other particles applying a force on the i th have particle j not equal to i.

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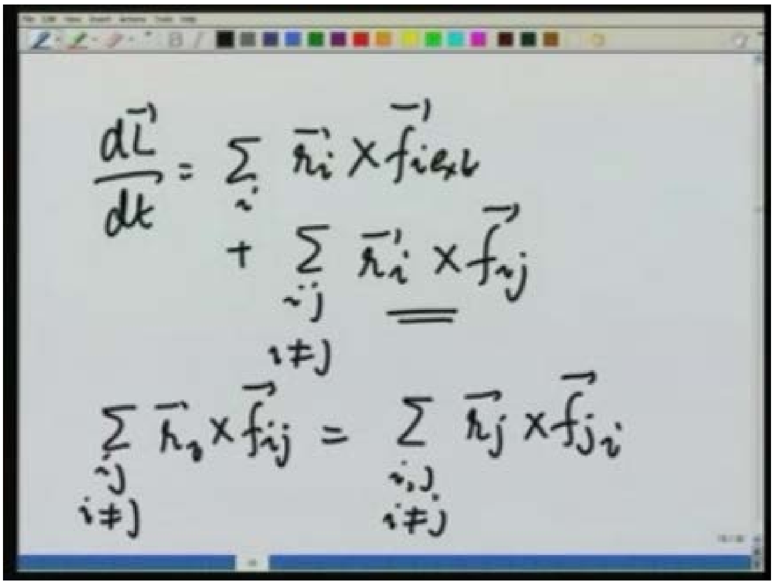

This I can now write as d L d t equals summation I r i cross fi external plus summation ij I not equal to j ri cross f ij recall this term. Now, I want to show to 0 in order to prove that dL dt equals ri fi external only. So, I can write summation ij I not equal to j ri cross f ij since I and j are being summed over I can interchange them and write this as summation rj cross f ji I will jut interchange j and I ij I not equal to j.

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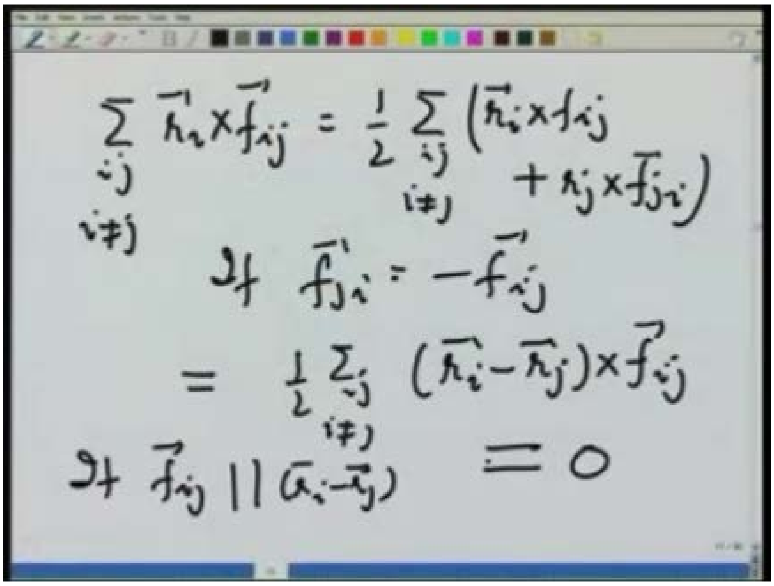

Therefore, I can write this term Summation ij I not equal to j ri cross f ij is equal to 1 half of summation ij I not equal to j ri cross f ij plus rj cross f ji and f f ji is equal to minus f ij.

This term becomes equal to 1 half summation ij I not equal to j ri minus rj cross f ij and f f ij is parallel to ri minus rj parallel or anti parallel then this term is 0. Fortunately, in mechanical problems, most of the forces are such at they are along the line joining the 2 particles and therefore, this term always 0.

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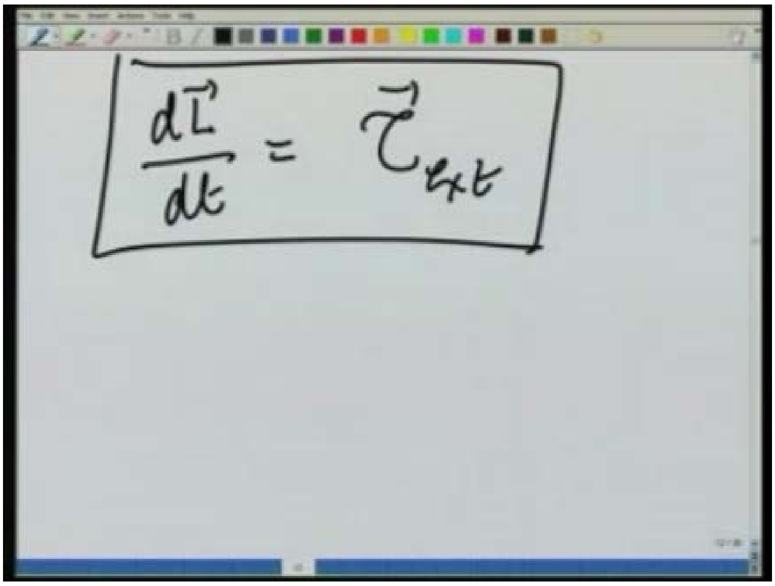

Therefore, we get d L d t for a many particles system also is equal to tau external, this is going to be our dynamics equation as I showed in the example of 1 particle solving this gives me the resultant motion of the particles.

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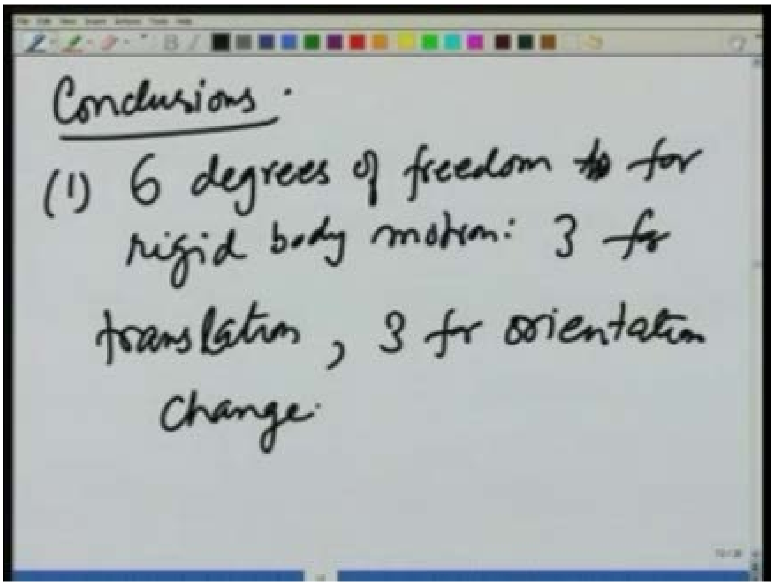

Let me conclude this lecture by saying by getting our main results number one, 6 degrees of freedom to describe to the freedom for rigid body motion. Here, 3 for translation and 3 for orientation change the tremendous reduction in the degrees of freedom comes because there is a constraint of 2 particles being at the same distance no matter what the motion is.

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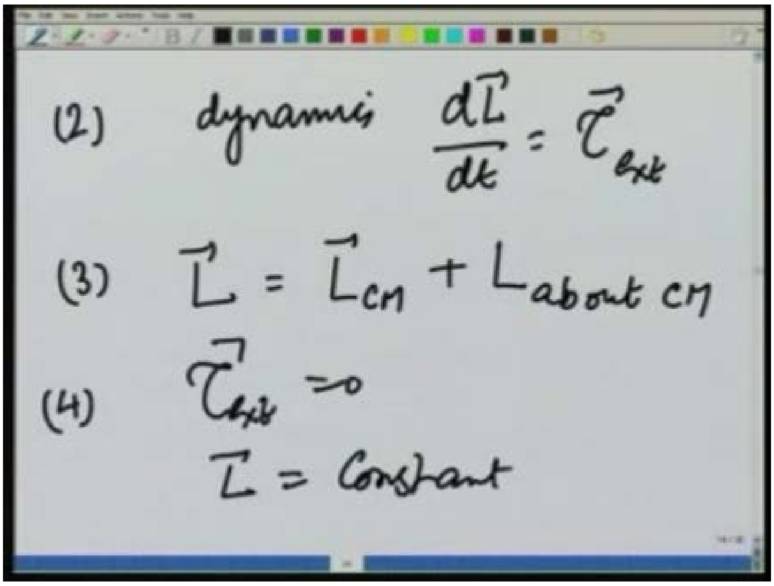

Number 2 the dynamics is going to be described by d L d t by d t equals tau external under certain condition that is the forces are equal and opposite and along the line joint 2 particles. This gives me the dynamics and L for is system is equal to L CM plus L about CM and both if tau external therefore is 0. Then, L is the constant that is the conservation of angular momentum, having developed these ideas on angular momentum, next lecture onwards, we are going to apply them to describe the motion of a rigid body.