Engineering Mechanics
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Module - 06
Lecture - 05
Work and Energy - IV

In the previous lecture on force and energy, we saw how a force is obtained from corresponding potential energy by taking its gradient. In this lecture we explode this little further and then go on to discuss non conservative forces as a real life example of calculating the force from a potential. Let us now look at the electric field and the potential or equivalently the potential energy and the force on a charge an a in an electric dipole.

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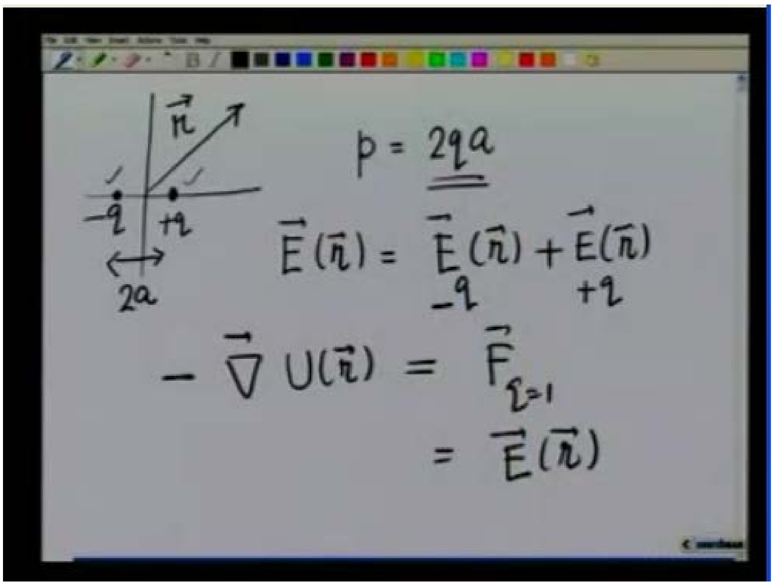

So, I am going to consider a dipole sitting at the origin with charge with minus q on negative y x axis and charge plus q on the positive side of the x axis with the distance between them being 2 a.

I want to find the electric field at a general point r form the origin, I will do, so by taking superposition of fields due to charge minus q charge plus q adding and then taking the limit a going to 0. So, that limit is taken in such a manner that p equals 2 q a remains unchanged that defines for me point dipole setting at the origin. So, I could find the field at r by superposition of field due to minus q at r plus field due to plus q at r and get my answer in an easier way. This will tell you how talking about potential helps at times could be that I find the potential energy of a unit charge at point r and take its gradient. Negative of that will give me force on that unit charge which by definition is the electric field at r, let us do this as an exercise.

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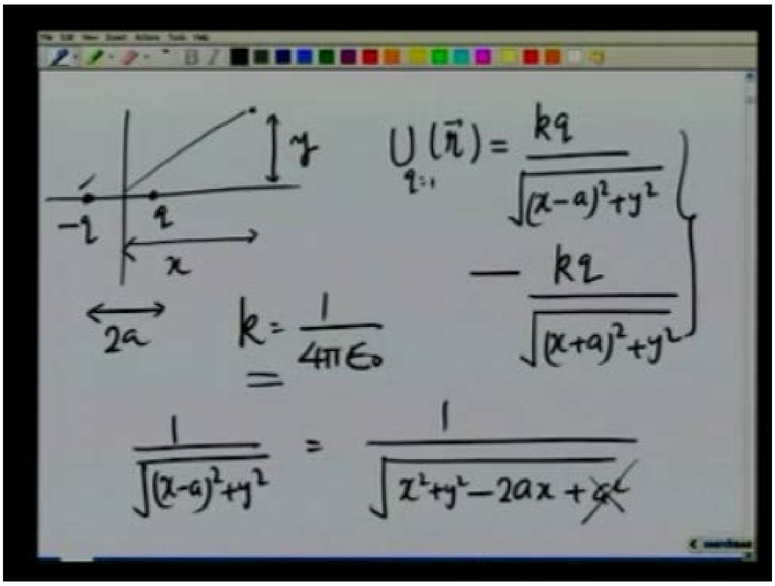

So, recap I have dipole sitting here I want to find the field at distance r which is x from the origin and y this way. Finding potential in this case is easier because I am not doing a vector sum, therefore potential energy for unit charge q equals 1 at r. The r is given in 2 dimensional spaces this x and y point is going to be potential due to the first charge plus the potential due to the second charge. It is going to be k q over square root of x minus a square plus y square minus due to the other charge this one, k q over square root of x plus a square plus y square. Here, I just want to remained to this distance between the charges is 2 a and k is one over four pi epsilon 0. I am not so much concerned about these quantities more about taking gradient of this.

Since a is very small, so far away from the dipole I can write 1 over square root of x minus a square plus y square as 1 over square root of x square plus y square minus 2 ax plus a square. I am going to neglect this terms because finally I am going to take the limit going to 0 and keep only q times a term, so I neglect that.

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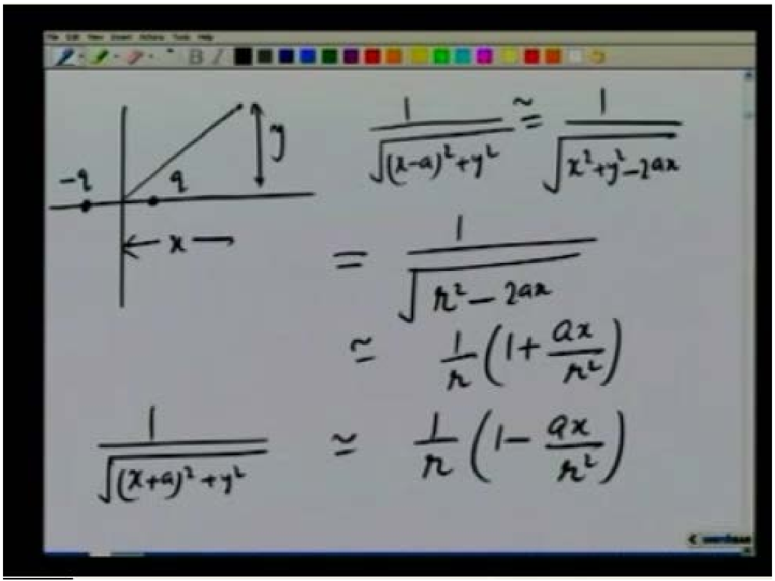

Making the figure again, I am trying to find the potential due to a dipole at a far away point which is x from the origin this way y this way this is q minus q. I made the approximation that 1 over the square root of x minus a square plus y square is equal to one over the square root of x square approximately plus y square minus 2 a x which I can write as 1 over square root of r square minus 2 ax. This is approximately equal to one over r one plus ax over r square, similarly I can write 1 over square root of x plus a square plus y square approximately as 1 over r 1 minus ax over r square.

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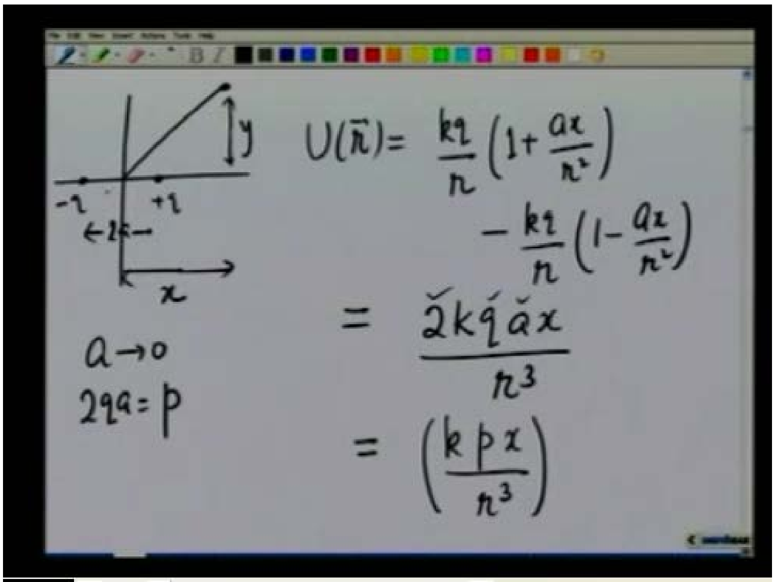

Combining the two, I get the potential for this dipole separated by 2 a minus q plus q as U at r. Here, r is this point distance y distance x from the origin as k q over r 1 plus ax over r square I am working under those approximation, where a is very small minus k q over r 1 minus ax over r square. That gives me U r to be 2 k q ax over r curbed, now when I take the limit a going to 0 such that 2 q a remains constant equal to the dipole moment that I want to have. So, then in that limit becomes 2 q a gives me the dipole moment k p x over r curbed, you can check that the dimensions are alright p is q times the distance this distance divided by this distance q. So, this is the potential, now I want to find the electric field due to this dipole at point r.

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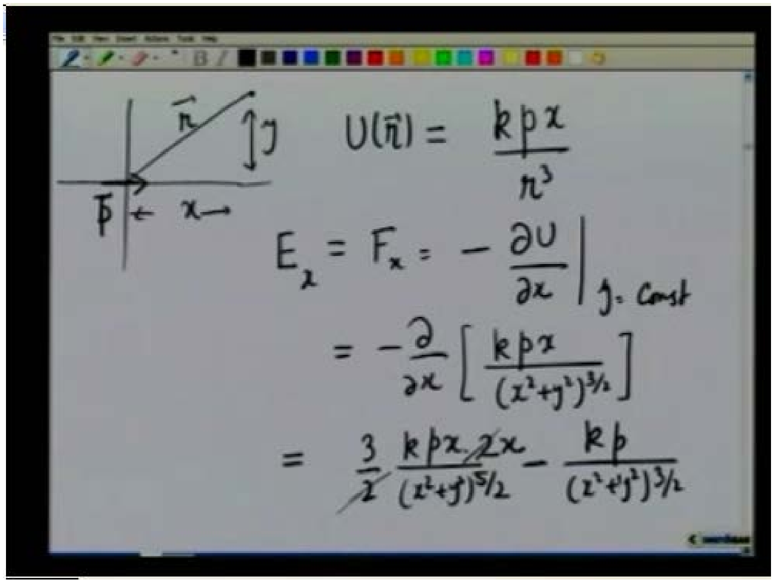

So, I am finding, so there is a dipole is here of the magnitude p pointing in x direction. So, let me now make this dipole I am making slightly big is in x direction its magnitude is p. I am trying to find the electric field at this point r distance away which is x this way and y this way, I found that U of r or a single charge. So, this is a potential is equal to k p x over r cubed, so I can find the x and y components of the force of the electric field because electric field is same as force on a unit charge is equal to F x is equal to minus d U d x which is equal to minus d over d x.

I am taking a partial derivative, so when I do that y remains a constant of k p x over x square plus y square raise to three halves. When I do that, I find I differentiate this, first I will get three over 2, this minus sign goes with that k p x over x square plus y square raise to 5 halves times 2 x minus k p I differentiate x, so I get divided by x square plus y square raise to 3 halves, this two cancels.

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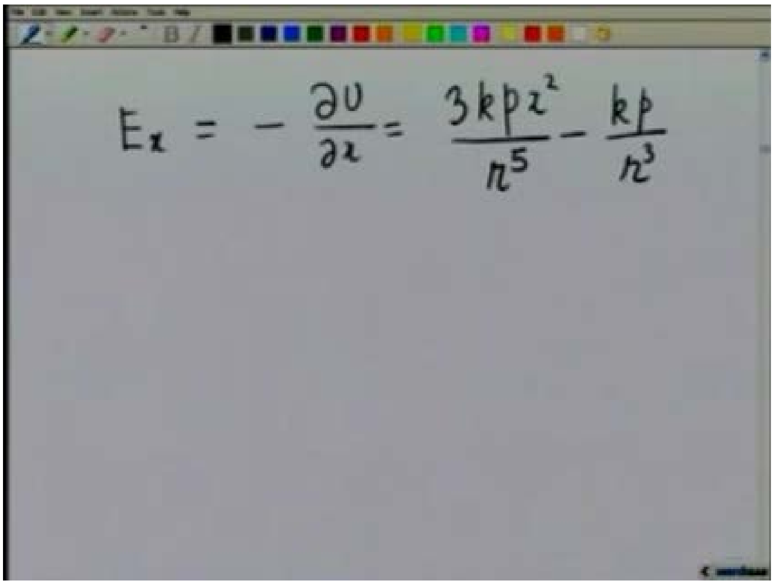

So, the expression for the electric field x component that I get is going to be x which I took to be the partial derivative of U with respect to x comes out to be 3 k p x square divided by r raise to 5 minus k p over r cubed.

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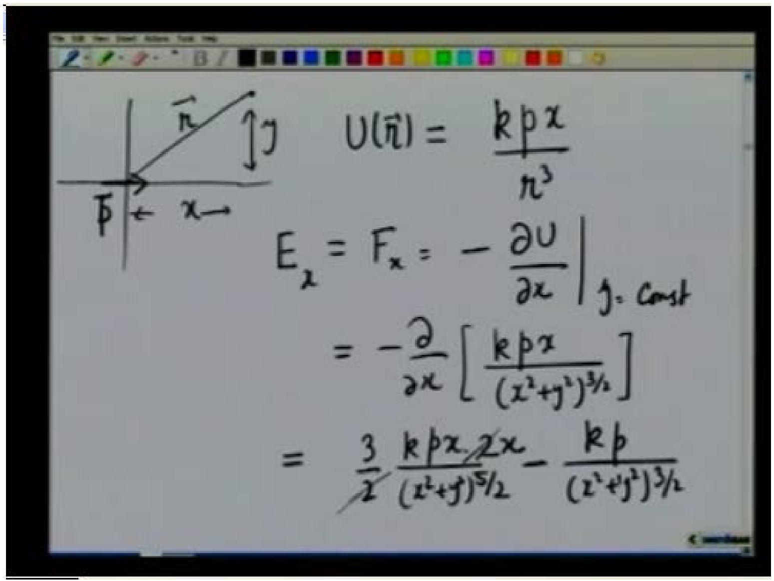

Let me see if it is correct going back to the previous slide this 3 k p x square over r raise to 5 minus k p over r cubed.

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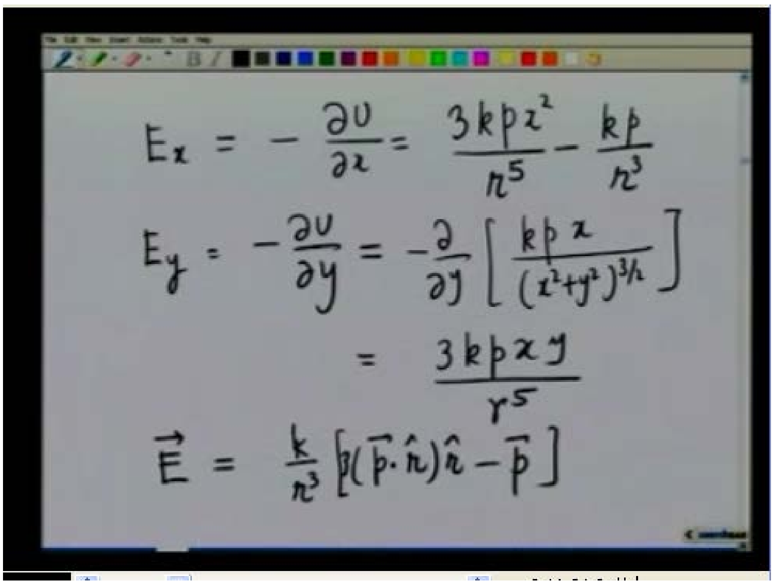

Similarly, I can find E y which is minus partial of U with respect to y and this comes out to be minus partial of U with respect of y k p x over x square plus y square raise to 3 halves and that gives you three k p x y over r raise to 5. These are the two components with electric field you can check for yourself that this is consistent with the general expression of the electric field which is given as 1 over r cubed rather k over r cubed p dot unit vector in r direction times 3 r minus p.

Getting this directly by superimposing the fields due to 2 charges would be slightly more difficult. You see that taking gradient of the potential which is slightly easier to find is an effective way of finding the field. Having talked about all these quantities, you may be now wondering what happens to the minimum maximum of the potential in three dimensional cases.

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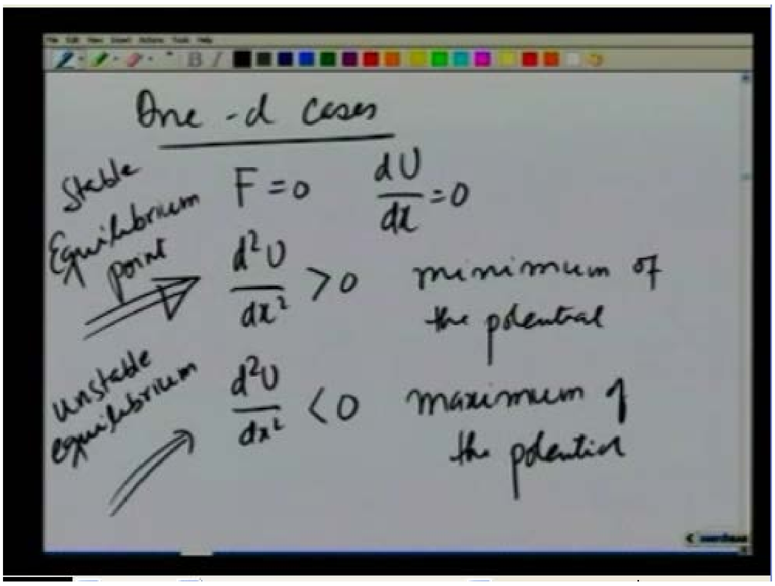

In one d cases, I knew that the force was 0 when d U d x was 0 if d 2 U dx square was greater than 0, then I had a minimum of the potential. Similarly, d 2 U dx square less than 0 gave me maximum of the potential and we have talked in the past this gave me an equilibrium point or I should say a stable equilibrium point. If I moved a body from this point it will come back and this gave me an unstable equilibrium point, the body had force 0, but if I moved it away from there, we will just take off it will not come back what about in 3 d cases?

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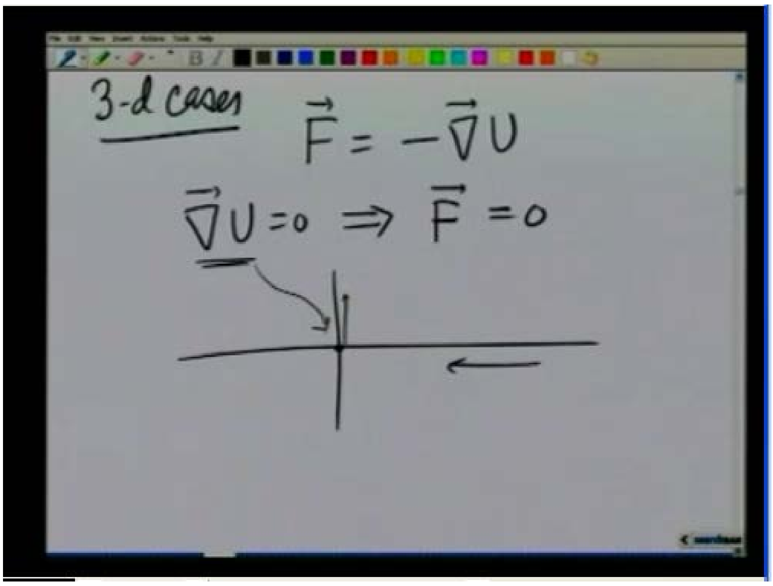

In 3 d cases, I am talking about a three dimensional force which is the gradient of U off course if gradient of U is 0 it implies that the force is also going to be 0. As far as the minimum and maximum and such things are concerned, these are slightly more complicated in 3 d cases. For example, although a point would be where the gradient is 0, it does not mean whether that it will be a maximum or a minimum for sure in three dimensions. For example, if I come along the x line suppose at the origin the gradient is 0 if I come along the x line x axis U could go through a maximum.

On the other hand, you could have a minimum along the y direction, so to find the minimum maximum in 3 d, case one really has as to see for example, for a minimum whether in all directions that point the U is minimum. For a maximum, whether coming from all directions that point is a maximum there could be more complicated cases, I just discussed coming in one direction would be a maximum and the other could be minimum the which is known as a settle point. You will learn about these things more in your electrodynamics course I thought I would just give you a gist of it.

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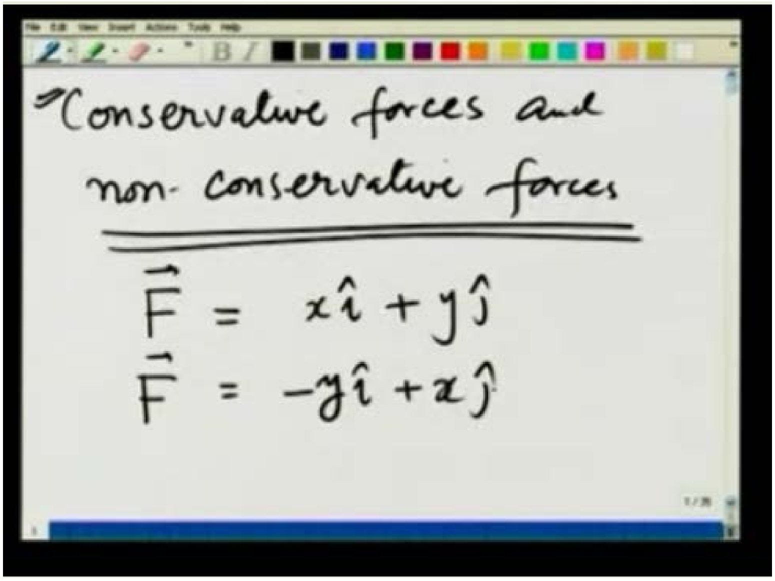

Now, let us talk about an issue that we have been avoiding. So, far conservative forces and non conservative forces how to deal with non conservative forces this you already dealt with. To work with non conservative forces, we said earlier I should not be able to define a potential energy since the work depends on the path one simplest example in one dimension is frictional force.

More you move more work you do you come back to the same point the work done is not 0 and therefore you cannot define a potential energy for a frictional force in one dimension how about in three dimensional case. Let us take to 2 examples and learn through examples, let us take 2 forces F equals x I plus y j and try to construct potential energies for these and this equals minus y I plus x j.

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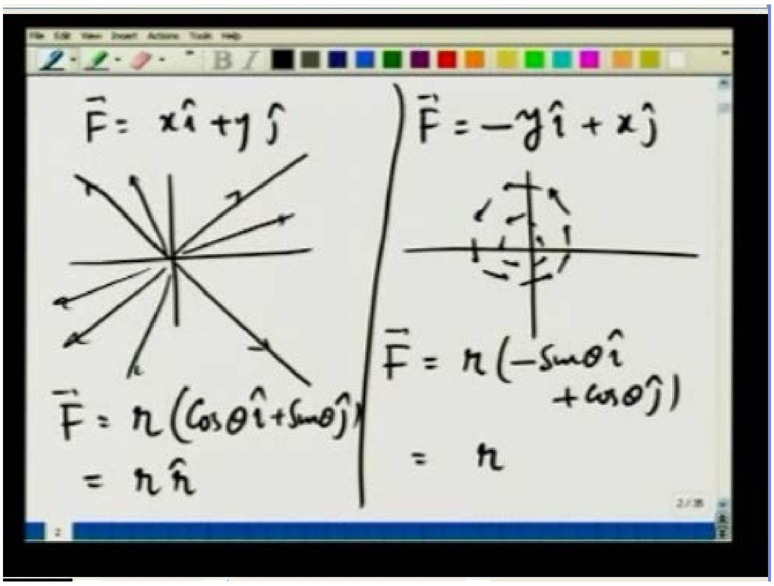

So, I am taking 2 forces let me just draw a line here F equals to x I plus y j and F equals minus y I plus x j in this case you see that the force lines are going radially outward. If we do not see directly write F as x is r cosine theta y is r sin theta, so I can write this as cosine theta I plus sin theta j and by now having learnt polar coordinate this is nothing but r in the radial direction. On the other hand, in this case the force line is going to be in theta direction at all points, so they are going to look like this.

Again, if you do not go see them clearly write this in polar coordinates and you will see that this is nothing but r minus sin theta I plus cosine of theta j which is nothing but r in theta direction. So, further out you go the force magnitude increases with respect to r and this direction is in theta direction, let us try to find the potential energy for the 2 cases.

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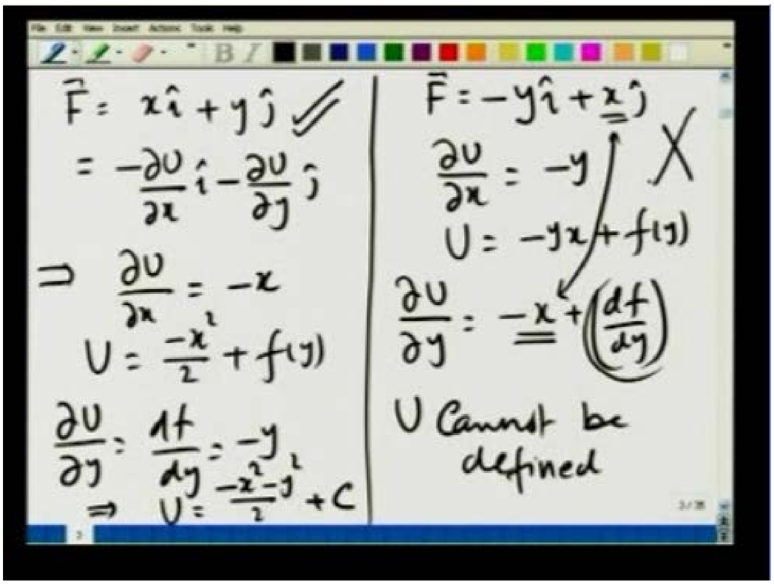

Again, let me draw a line F equals to x I plus y j which is minus partial of U with respect to x I minus partial of U with respect to y j and this implies partial of U with respect to x is equal to minus x or U could be equal to minus x square plus some function of y. Therefore, d U d y is equal to d f d y is equal to minus y and this after integration implies that U is equal to minus x square minus y square plus some constant of integration C. Let us try to do the something for F equals minus y I plus x j, so partial of U with respect to x is equal to minus y.

I can therefore write U as minus y x plus some function of y and partial of U with respect to y therefore, is going to be equal to minus x plus some d f d y, which is a strictly a function of y, but, d U d y cannot be minus x because d U d y is given to be plus x. Therefore, U cannot be defined, this is a conservative force this is a non conservative force, let me give the same take the arguments further.

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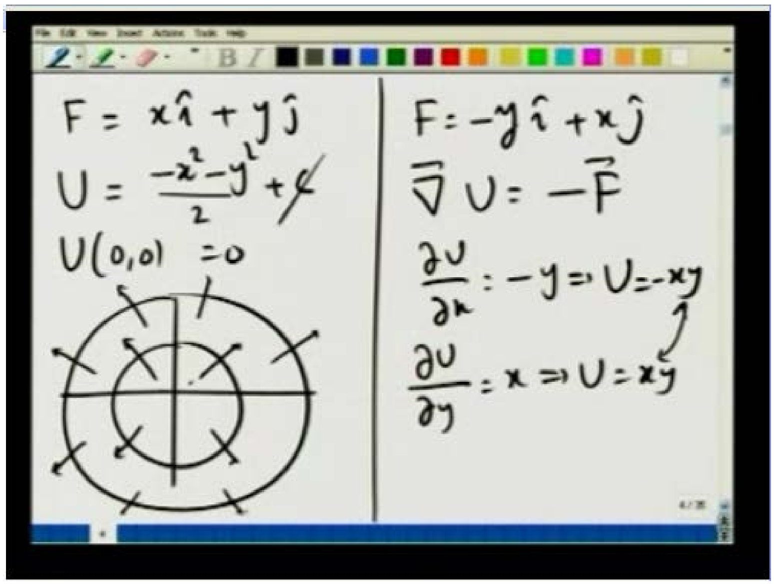

So, in the case of F equals x I plus y j I found that U is equal to minus x square minus y square plus C. I can take as a reference point U at 0, 0 to be 0, then the C becomes 0 and the constant and the surfaces are like this U becomes more and more negative as you go further and further out. Therefore, the force is in this direction is v you can already it is radially out, on the other hand when I take F to be minus y I plus x j I cannot really find a function that will give me gradient of U to be equal to minus F. Another way of looking at it is you see from d U dx equals minus y I find U should be of the form of minus x y.

On the other hand, if I look at d U d y to be equal to x I find U to be of the form of x y both cannot be true, so this is a non conservative force I cannot really get any potential for this. Now, the question we asked do I always have to try to find the potential energy and if I cannot find it, then I say it is non conservative or is there any easier way to look at it. If I all the time I have to integrate the force it becomes a difficult task at it, so happens that they happens to be a differential way of doing the same thing and now I like to talk about that it also gives me the definition of a quantity called curve.

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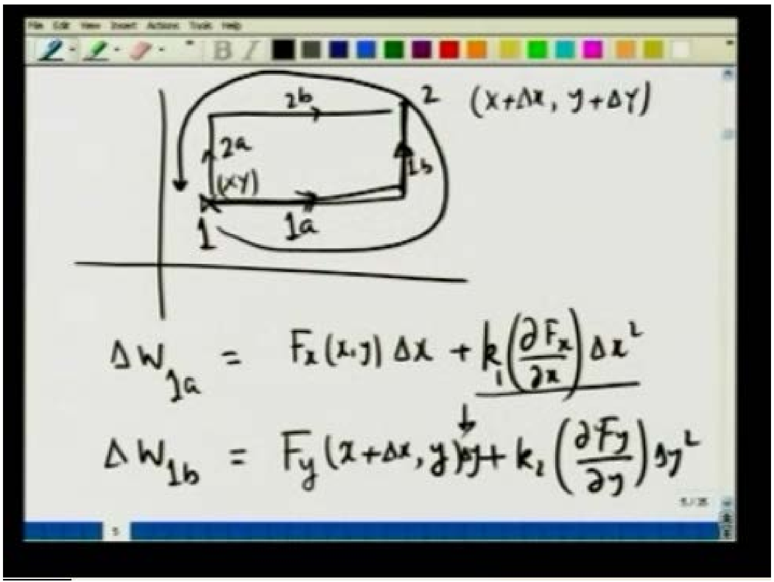

So, go back to the definition restricting myself to 2 dimensions and generalization into three dimension is quite easy I go from point one to point 2 and if I find that the work done is independent of which two paths I choose is going to be a conservative force. If it is not independent or it depends on the path, which I choose, then it is going to be non conservative force also put another way. If I go from point one, go around this path and come back to the same point if the work done is 0, then it is a conservative force if it is non 0, then it is not a conservative force.

So, let us try to do that let me go from point x y to point 2 which is x plus delta x y plus delta y and without any loss of generality, let me take this path to b 1 a 1 be let me take this path to be 2 a 2 b. So, work along path 1 a since I am moving along the x axis, this is path one a is going to be F x at x y times delta x plus some correction because F x itself might change I am I am going to go all the way to second order. So, some correction let us call it delta F as F x changes along the x axis delta x square and some constant, this is going to cancel. So, I am just putting some constant k k could to be one half or something.

As I move along path one b that is from here to here only thing that is changes y, so this is going to be F of y at x plus delta x plus y has not change plus some correction, let us call it k one k 2 as I move along y F y may change delta y square. I am going to do this, there should be a delta y here because I am let has a distance, so let me expand this.

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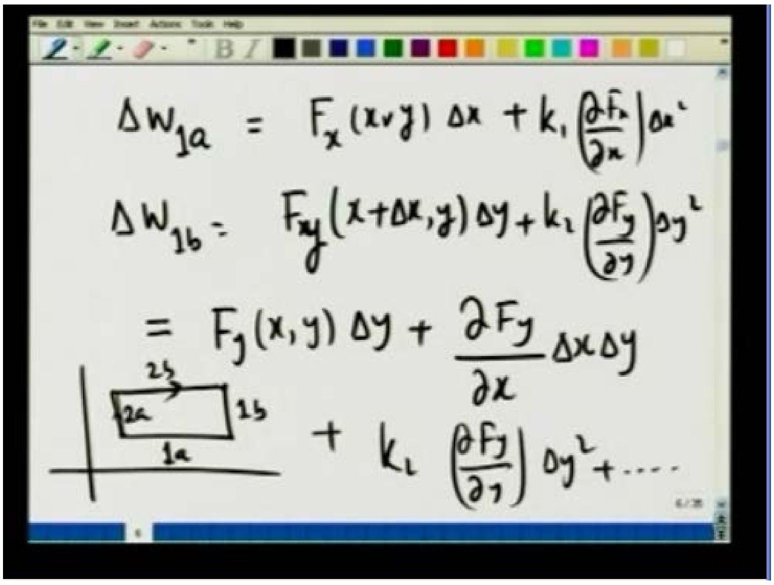

So, I get delta w along one a is equal to F x x x y delta x plus some constant k 1 partial of x delta x square is going to be F x at oh sorry F y at x plus delta x y delta y plus some k 2 d F y d y delta y square. If I expand this further I get F y at x and y delta y plus delta F partial of y component of F with respect to x changes. Since I moved by delta x delta y by Taylor series expansion plus a second order term k 2 y square, so I am keeping all the terms to second order and there could be some higher term. Let me remind you again I am calculating this work in going from point 1 to point 2 along path 1 a and 1 b. I could do similar excise while going from path 2 a in 2 b liking go this you can take similar steps and I leave this as an exercise.

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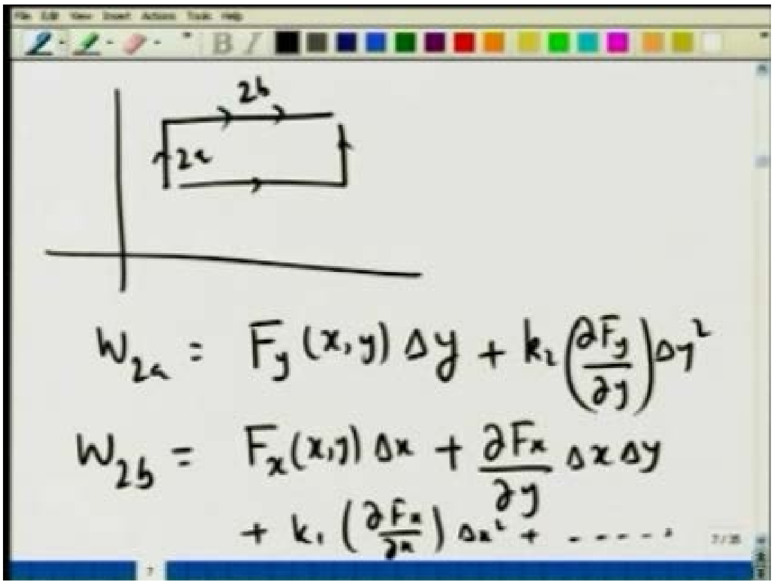

You would find that the work done when I go from along this path comes out be along 2 a comes out be F y at x y delta y plus you can write k 2 delta F y del y del y square and along 2 b this is 2 a this is 2 b. It comes out be F x at x y delta x plus partial of F x with respect of y delta x delta y plus k 1 partial of x with respect to x delta x square plus higher order terms. So, let me flash the previous expressions again this is delta W 1 a one b expanded it looks like this. This flash back and you see when I did the work along path one I got these contributions and I worked along path 2 I got these contributions comparing you will see that this term appears in this case.

Similarly, this term and this term, so I will remove this I will focus only on this this term and this term appears in this case also. This term appears and this term appears only terms that I am left with are this and this, so if I were to take the difference moving along path one a and one and path 2.

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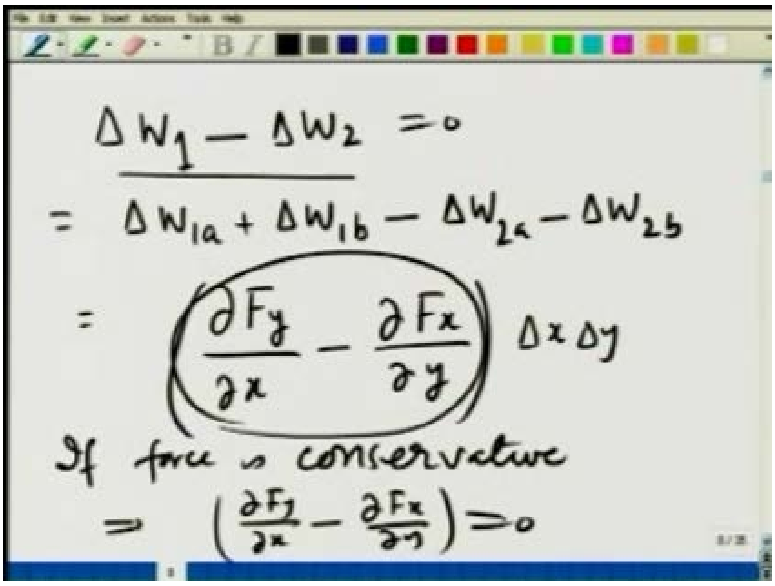

I will find that work done along path 1 minus work done along path 2, which is delta W 1 a plus delta W 1 b minus delta W 2 a minus delta W 2 b and let me flash those things back again. This was delta W 1 a this delta W one b these terms canceled, similarly in this case this is delta W 2 a delta W 2 b the only terms which were left were these this also canceled. Therefore, I get this to be equal to partial of F y with respect to x minus partial of F x with respect to y delta x delta y. If this were to be 0 that is if force is conservative, then this term must be 0 this implies partial of F y with respect to x minus partial of F x with respect to y must be 0 otherwise not.

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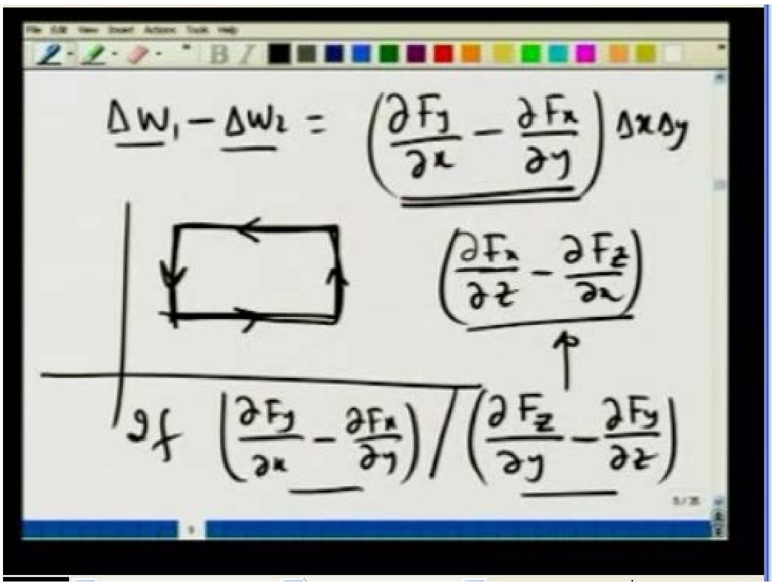

So, I get delta W 1 minus delta W 2 is equal to partial of Fy with respect to x minus partial of Fx with respect to y delta x delta y. This delta W 1 minus delta W 2 is nothing but this is delta W 1 is the work done in moving from this point to this point and minus delta W 1 is the work done and moving back along path 2. So, this is a work done in moving the entire closed path and if this is non zero, then the force is non conservative. I can give similar arguments for other dimensions movement along the plane of y z or movement along the plane of x z.

I find in general that if delta F y over delta x minus delta F x over delta y or delta F z over delta y partial y minus partial y over partial z. Let me write it here partial F x with respect to z minus partial of F z with respect to x if they are any one of these is non zero you have a non conservative force. If these are all 0, you have a conservative force is there a compact way of writing this the answer is yes.

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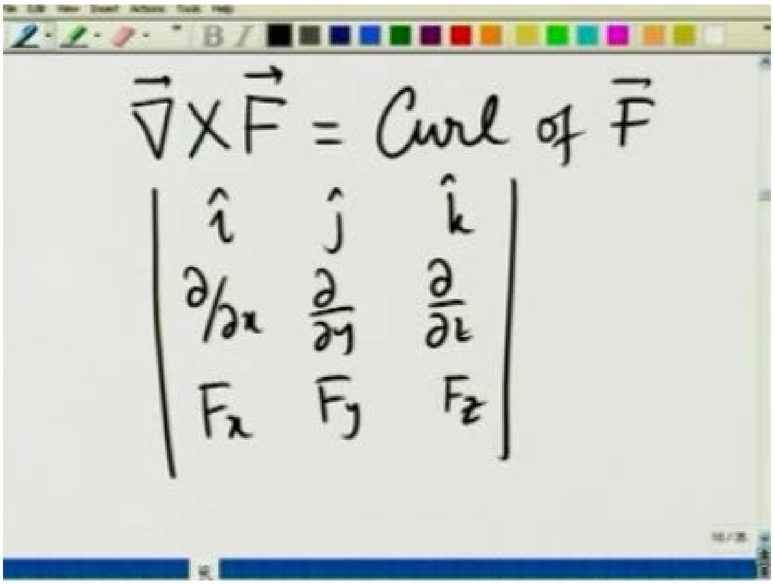

These are all components of a vector form by this which is known as curl of F and you can see this I j k this is by definition F x F y F z determinant, this gives me all three components that I wrote in the previous slide.

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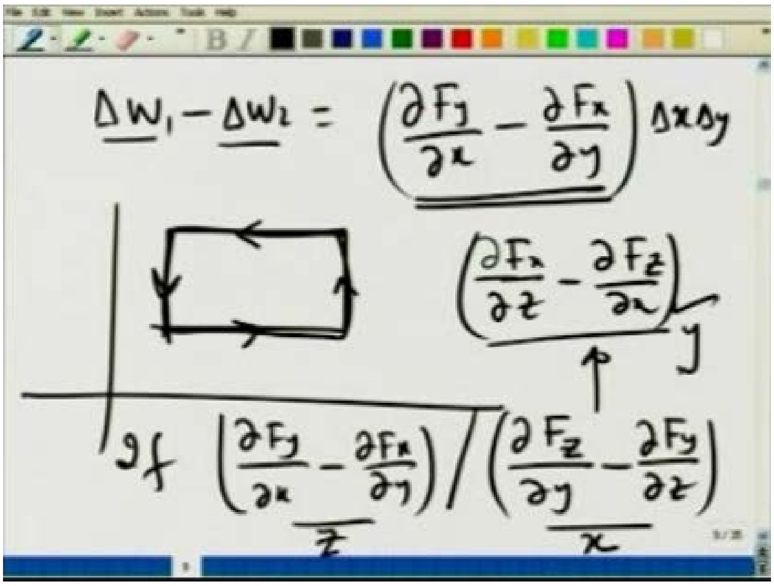

This is a z component this is the x component this is the y component of that curl that I wrote here.

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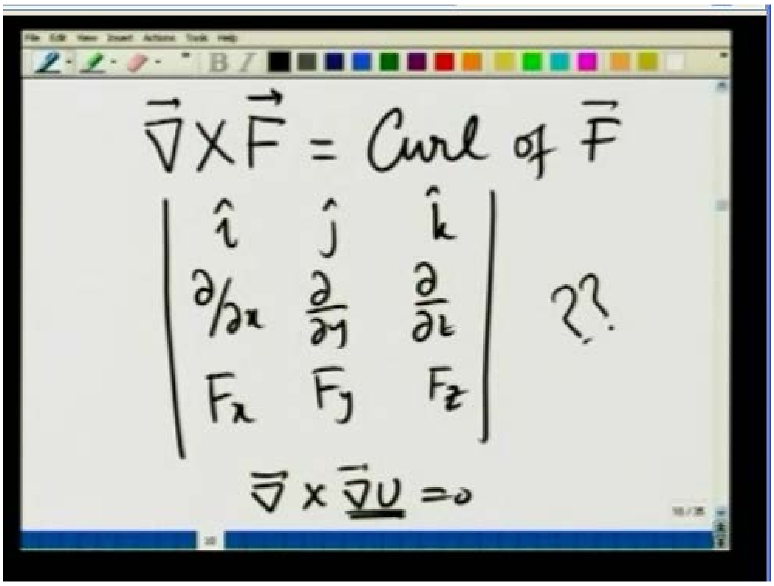

If any of these components is non zero, force is non conservative, if they are all 0 the force is conservative. So, I found given a force I do not really have to calculate the potential energy I can straight away take its curl if it is non zero non conservative force if it is 0 conservative force. This also tells you that by the way that curl of a gradient must be 0 because if I define a gradient; that means, I am taking about a conservative force.

What does this curl mean, can I get more feeling for it, the answer is yes, as a name suggest curl is something that describes a curly sort of thing.

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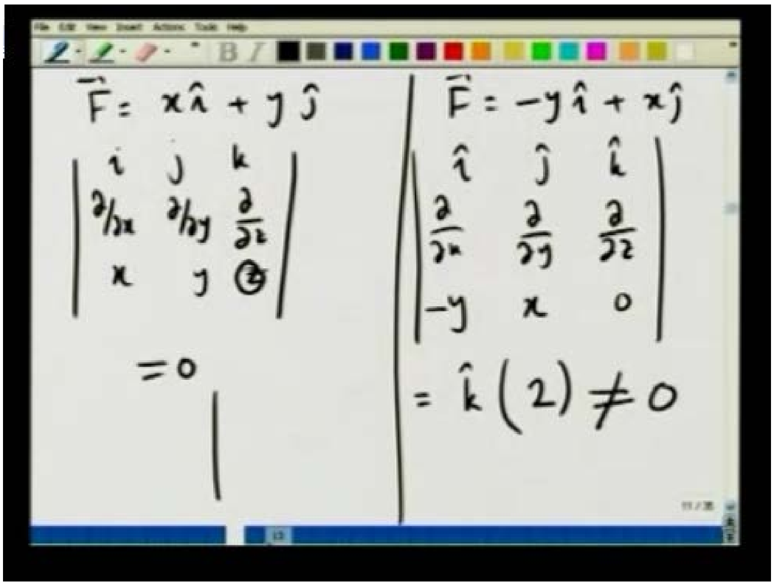

Let us again go back to our two examples F equals x I plus y j and F equals minus y I plus x j and take the curl, curl is going to be I j k partial with respect to x partial with respect to y partial with respect to z of x y and z and you can see this 0.This is 0 here, on the other hand if I look at the curl of the other example j k partial of x partial of y partial of z this is minus y x 0. You will see that it gives k component or z component which is equal to 2, so this is non zero, let me again make how these force will looking.

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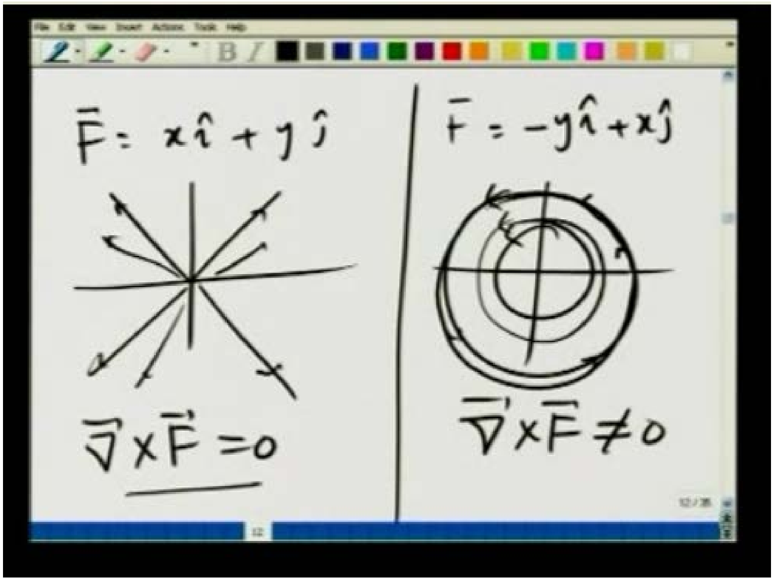

So, F equals x I plus y j the force was looking something like this and in this case F equals minus y I plus x j the force field was looking something like this in this case curl of F is equal to 0. In this curl of F is not equal to 0, so this quantity curl seems to give you a non zero value when things seem to be moving around in circles or they are curling they turning around. On the other hand, if they do not they go in straight line curl is 0 you can see in this case very easily if I take a part and then go around in a circle work done is going to be non zero. So, it is a non conservative force, this is a conservative force to get more feeling for this let us look at some more examples.

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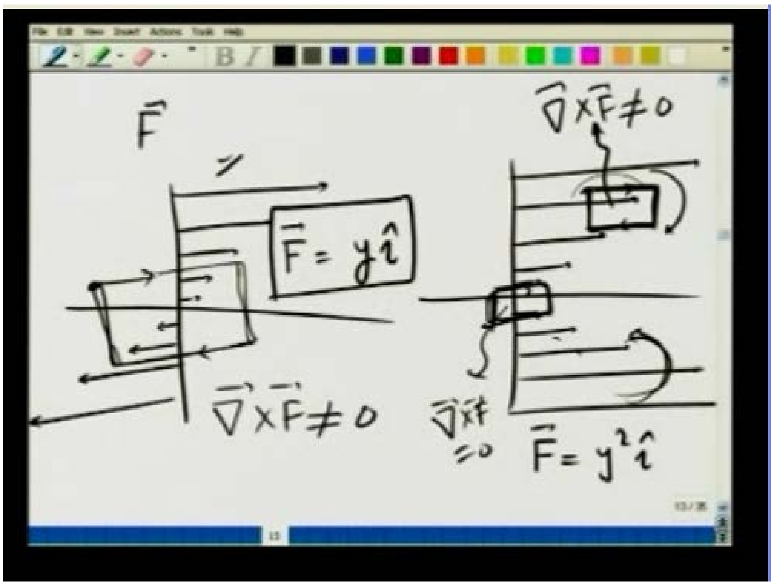

Let us take a force field F I just take make pictures which is something like this as you go out it increases as you go on the negative side it again increases. Let us take another one as you go up it increases as you go down it increases when the same direction. You can see if I take a particle around here like this I do positive work when I go this way, no work same positive work and come back here I have gathered some work. Work done in going around in a closed path is non 0 that means, curl of F is not equal to 0.

Similarly, here if I go around the origin in symmetric way you will see I pick up some positive work 0 work negative work equal to this positive work and 0 again. So, around the origin the work done seems to be 0, but if I take a closed path here, I do some work here 0 work less work because of force is become less and come back here. So, I picked up some work in going around a circle, so on this point curl of F is not equal to 0 around the origin near the origin curl of F is equal to 0. Again, you see just looking at the pictures the curl seems to give you a feeling for this moving feeling for a rotation on this point this seems to be the lines are becoming smaller and smaller.

So, this sort of curling around they are curling around here, they are not curling around they are changing the same way on both sides and this is really the meaning of curl just to quantify this. Let us say this could be something like F is equal to as you increase y F magnitude increases in x direction. So, this could be a function that is described by this sort of geometric shape, in this case as you no matter which way you go on the y direction F could be y square i, so let us see if really our feeling of about curl is correct or not.

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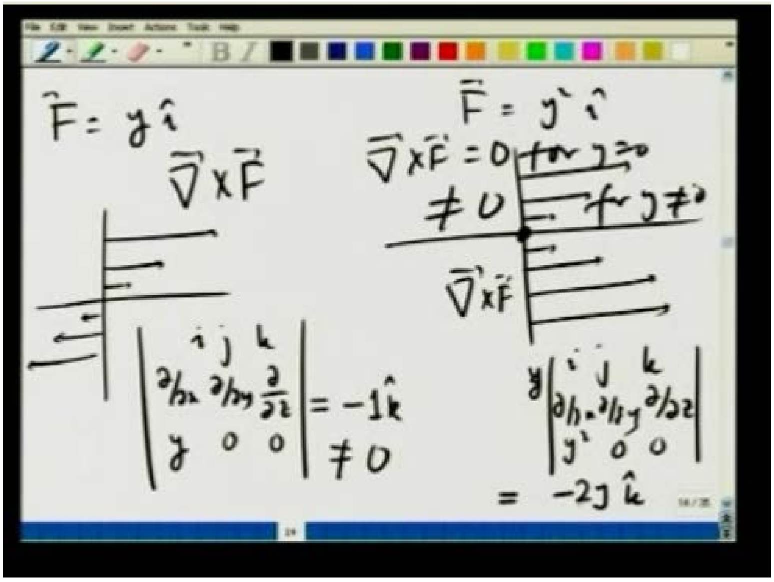

If I make it again in this case I had force field like this and I wrote F equals to y I, then I also had F equals y square I and the force filed looks something like. If I calculate the curl is I j k partial with respect to x partial with respect to y partial with respect to z this is y this is 0, 0 and you will see that it has a k component which is non 0, this gives you minus 1 k, this is 1. So, this is non 0 all the time, on the other hand in this case if I take y square, so I j k partial with respect to x partial y partial z y square 0 0 it gives me a curl which is equal to 2 y k.

I am calculating curl of F I am calculating curl of F and this is nonzero for nonzero y, but, 0 at y equals 0. So, our feeling that at the origin curl is 0, so this gives you curl of F is equal to 0 for y equal to 0, but non 0 at any other y for y not equal to 0 curl of F is not equal to 0.

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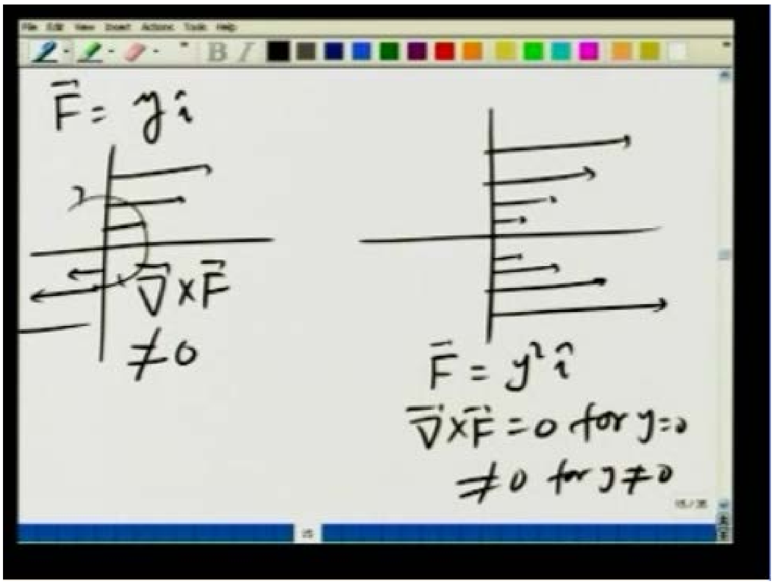

Let me make it clean once more F equals y I curl of F as we can calculated previously is not equal to 0 F equal to y square I curl of F is equal to 0 for y equal to 0 not 0 for y not equal to 0. So, you can see that the curl seems to give you a feeling here again the some sort of rotation involved here, there is no rotation involved it gives you a feeling for a rotational kind of thing, these quantities curl and so on. What introduced initially in the studies of fluids that is where the origin lies and you can get a feeling again a physical sort of feeling if you think in those terms think of any given force field.

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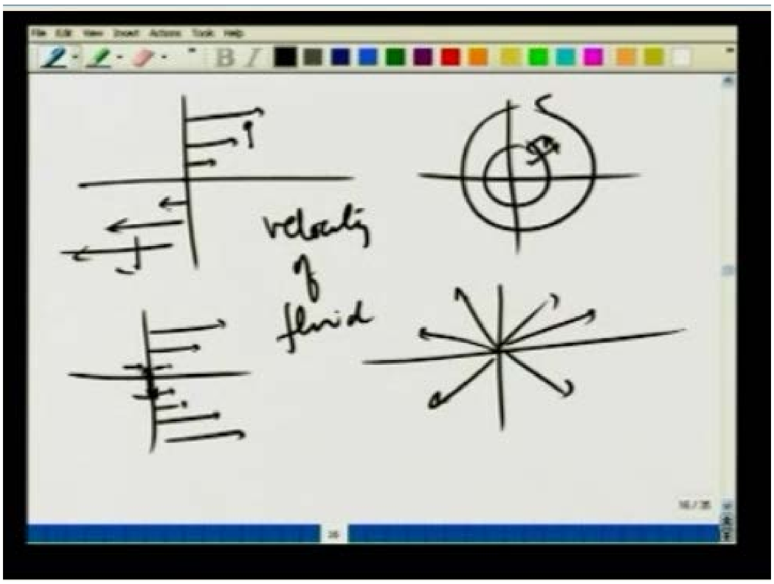

For example, the one that we took this or this or this or the radial one and imagine these lines to be representing velocity of a fluid flowing and think take a small match stick put it somewhere. So, I take a match stick put it somewhere here if you feel that the match stick rotate then curl is not 0 if you feel match stick would not rotate curl is 0. So, for example, in this case no matter where you put the match stick it will tend to rotate if this is the velocity of the fluid, curl is non 0 anywhere in this case. No matter where I put the velocity is becoming larger and larger as you go outside the match stick would tend to rotate curl is non 0.

In this case, if I put the match stick at the centre it is being pushed by the same force on the same velocity on both the sides it will not rotate curl is 0, but if I put it somewhere here, it will tend to rotate like this curl is non 0 in this case. No matter where I put it will not rotate curl is non 0, so this sort of physical way of looking at the quantity curl having learnt about curl and getting some feel for it. Let us talk about little more, so that you get a better feeling for it, I will first discuss a fantastic theorem about curl that really arises from the way we derived it.

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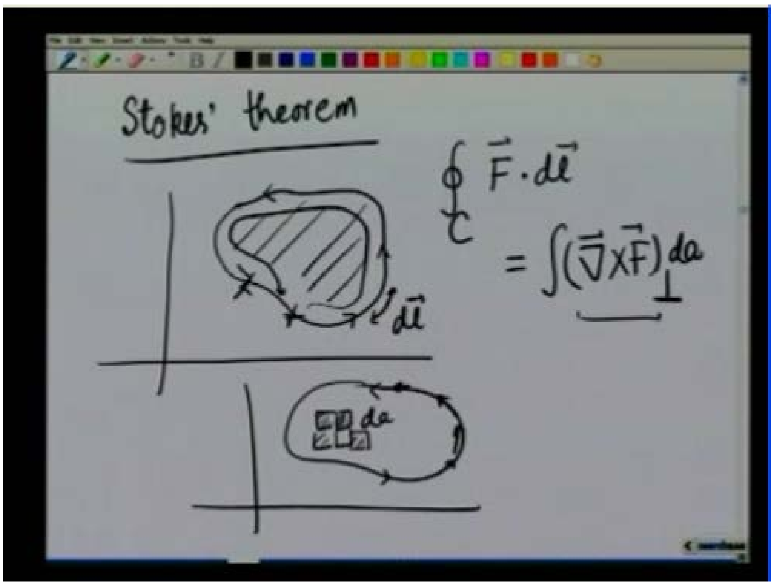

So, this is known as Stokes theorem and what it states is that if I take a closed path and given a force or a vector function F which I multiply over dl where dl is a line element path and integrate over this closed path.

For example, I could be calculating the force in sorry this should be the other way force in going around this close path from this point and coming back to the same point. So, I am going this way and coming back to this point and I want to calculate the work done of course is the force is conservative that means, a curl is 0 the work done is 0. If it is not conservative curl is non 0 and this integral is given as the curl of the force field is perpendicular component to this the this surface on which the curve is being made times the area d a.

Let me elaborate a little bit I am taking a close path, so I am calculating F dot dl along this path this is related to the curl. So, I can think of a small area da on the surface find the curl there multiply with a component of curl perpendicular to x surface with that delta area small area and added up. It is a very useful theorem and you can see right away how curl becomes very important quantity for us. The proof is also very nice and I would like to go where it, so that you sort of get a nice feeling about curl.

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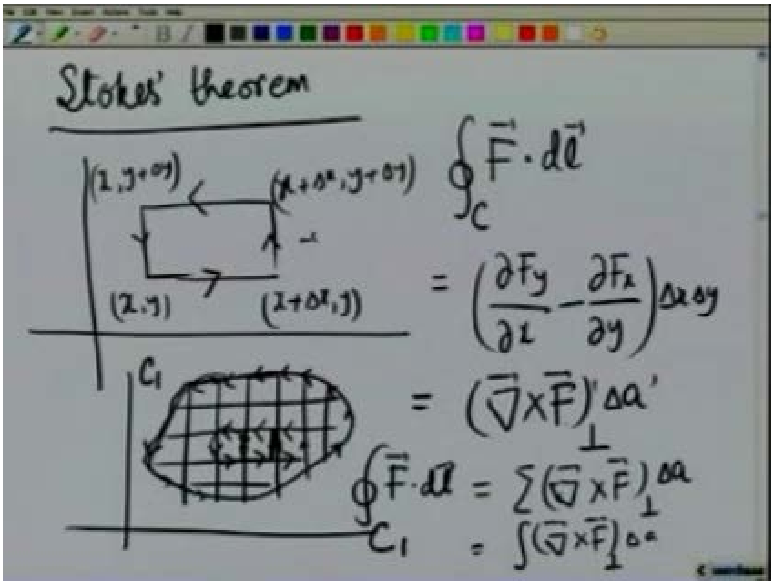

For proving, I will go back to the way we obtain the quantity curl where we obtained was it was by taking a small rectangular path going around it starting from x y going to x plus delta x y going up to x plus delta x y plus delta y coming back to x y plus delta y. Then, getting back to where we restarted from, in that case we found when I go counter clockwise then the work done F dot dl along this path close path was equal to delta F y delta x minus delta F a partial F x partial y times delta x delta y.

This is nothing but the perpendicular component of F on the x y surface times the small area delta a sort of Stokes's theorem and a miniature and infinitesimal version. Now, would I can do is take a big path and divide it into this small infinitesimal areas and for each area I can go around like this for this area I will go like this. For area I go this I will go like this and in doing, so finally, when I add F dot dl around each area you will see that opposite going paths cancel each other. Finally, the only place where they would not cancel is this outside line, so finally, if I add all these F dot dl's, I am going to get F dot dl around this path, let me call this C one to distinguish it from this one.

For each path, I know it is delta F y over delta x minus l partial F x over partial y or perpendicular to of the curl times delta. So, this is going to be a sum over curl of F perpendicular times delta x or delta x delta y or delta a, which is nothing but I can write this in integral forms it is sum over small infinitesimal areas F delta x. You have to be careful in these because we are talking about directional quantities in that you follow the convention when I go counter clockwise and then the thumb. If I curl my fingers around the counter clockwise direction, the thumb gives me the direction along which I should take the component of the curl of F.

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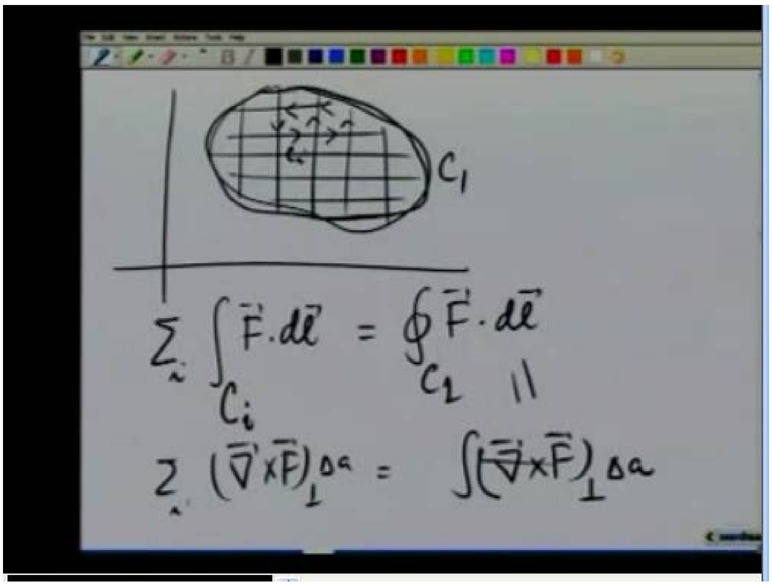

So, let me rewrite this what I did was I took a close path divided this into small grids or infinitesimal areas I took this F dot dl around each path and added it all up when I added it all up all these counter going paths canceled the contribution of F dot dl. Finally, I was left with the only this, but the area integral that is, so for each small one I had F dot dl is small one let me say it C i when I summed it over it gave me F dot dl around this path C 1. This is C i and when I summed over perpendicular component of F with the area it gave me integral curl of F perpendicular delta a.

These two are equal and that is a Stokes, here again I remind you be careful about the direction we are talking about directional quantities. I am going to follow right hand rule when a fingers curl right hand figures curl thumb gives me the direction along which I should take the component of the curl.

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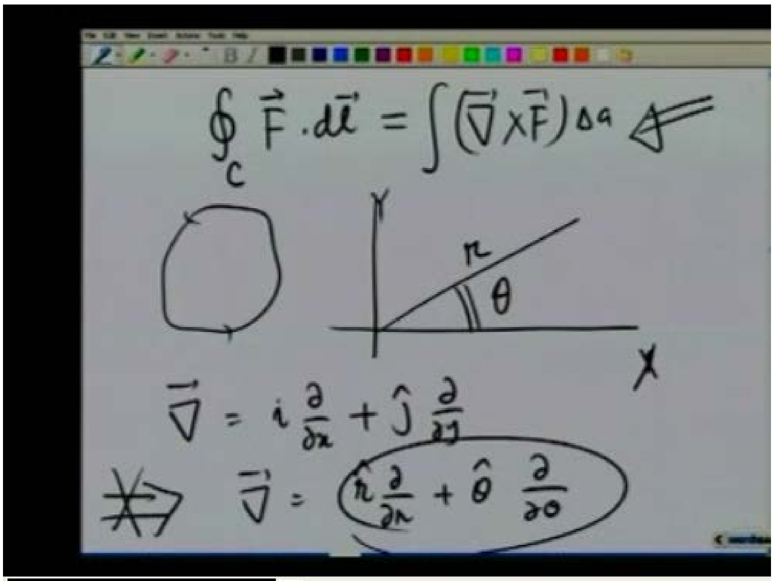

As I told you earlier, this Stokes's theorem F dot dl is curl of F times delta a integral over that surface really comes from the way we defined initially the curl quantity. So, it is a very useful quantity when I want very useful theorem when I want to see curl in different situations or curl in different coordinate systems. As an example, let me take now what will be the expression for curl in my polar coordinates, remember in the beginning of the previous lecture. When I was talking about gradients I had warned you that when I write gradient as I d by d x, let me just confined myself to two dimensions.

It does not mean, it does not imply that in polar coordinates it is going to be r d partial with respect to r plus theta partial with respect to theta. In fact, this is not even dimensionally correct, same thing is true for all other quantities and as an example I will just derive curl for you using this definition or this theorem stokes theorem which really initially introduced in infinitesimal way the quantity curl.

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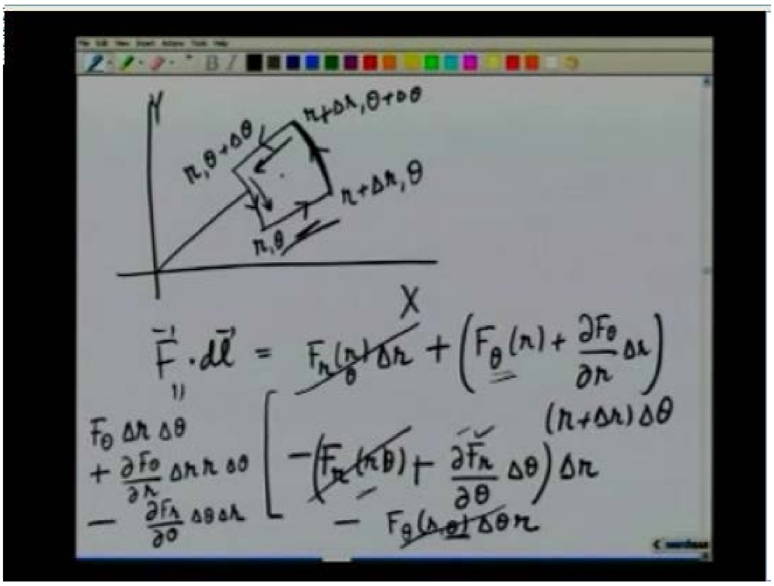

So, if I am talking about polar coordinates I am going to choose my path in polar coordinates as this following right hand rule I go counter clockwise. So, for the curl the component will be coming out of this screen, this point is r theta this point is r plus delta r theta this point is r plus delta r theta plus delta theta and this point is r theta plus delta theta. I will go quickly over this if I want to calculate F dot dl over this path is going to be Fr at r delta r that will be the contribution from this plus F theta at r.

However, as I go out theta component changes it changes by partial of F theta by partial of r delta r and the distance I cover is this one and this distance is r plus delta r times delta theta, now I am coming back. So, this becomes minus F r, however, this is Fr at theta plus delta theta, so this is going to be minus Fr at r theta. I should also have theta here minus how the radial component of F changes with respect to theta delta theta and the distance I cover. Let me make it plus put the brackets here delta r and finally, when I come along this path is going to be minus F theta at r theta delta theta times r.

You can check for yourself these are the only linear terms the other terms, if I really look at the other variations of these quantities they will give the second order terms. So, those higher order terms third order terms in delta theta and delta r, so I am not worrying about those. Now, you can see this will cancel with this and similarly, F theta r delta theta will cancel with this one part of this the other part I will get F theta at r theta delta r delta theta. So, let me just write it here then I will take it over to the next screen.

I am going to get this equal to F theta delta r delta theta the other component F theta r delta theta cancels with this. Then, I am going to get plus partial of F theta with respect to r delta r, r delta theta minus this term will be left which is partial of F r with respect to theta delta theta delta r.

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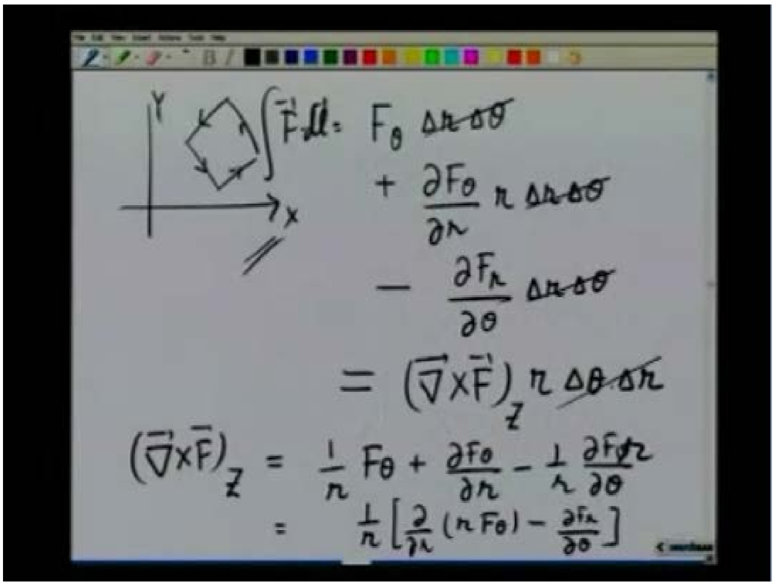

So, let me rewrite it what I have determined is F dl and this comes out to be I went along this path, let me copy this comes out to be F theta delta r delta theta plus del partial theta partial r r delta r delta theta minus partial Fr with respect to theta delta r delta theta. This is F dot dl by definition or by Stokes theorem this must be equal to curl of F coming out of the screen this is x this is y. So, it will be z component times the area which is going to are delta theta times delta r and that gives me the z component of curl of F and z direction in terms of polar coordinates.

This comes out to be one over r F theta plus partial F theta over partial r minus 1 over r partial F theta with respect to a partial Fr with respect to theta, which in short I can write as one over r partial r r F theta minus partial Fr over partial theta. So, you see I have been able to derive using the basic definition the curl in different coordinate system and use the basic theorem of this.

This is just to give you a feeling for these quantities you will obviously, be using the advanced version of these or using it more and more in your coming courses in electrodynamics advance mechanics and so on. So, you should practice a lot of problems on this play around try to get may be the other components of curl in cylindrical coordinates or in spherical coordinates. To conclude, I will give you potential energy in three dimensional how potential energy and force are related.

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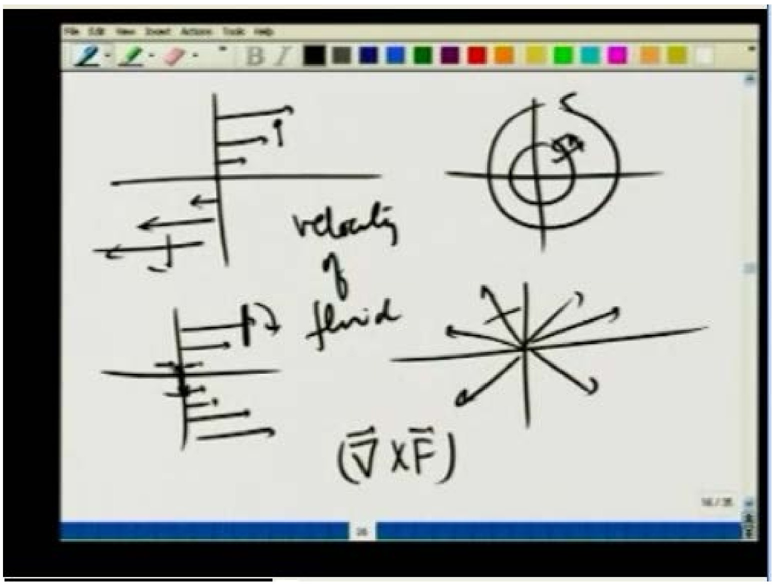

How we can think of a conservative and non conservative force and how we can test them using differential form of differential way of taking curl and this sort of some of our introduction and some details of work energy potential energy and so on.