Engineering Mechanics
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Module - 06
Lecture - 04
Work and Energy - III

In the previous lectures, we have been talking about work energy work energy theorem work done by a force and have define terms like kinetic energy potential energy and so on. However, if you noticed keenly, you would have observed that I have in those lectures avoided talking about defining or talking about a conservative force or also I have restricted myself to 1 dimensional cases. In this lecture, we going to start dealing with slightly more complicated situations we are going to go to 3 dimensions and look at what conservative forces are how we can define them and how we can identify them.

Also, define potential energy in 3 dimensional cases some of the terms and language that are be using you may be familiar with particularly the mathematical terms. In this lecture, we are going to put them in the context of work energy potential energy and so on to start with, let me remind you that when we defined work.

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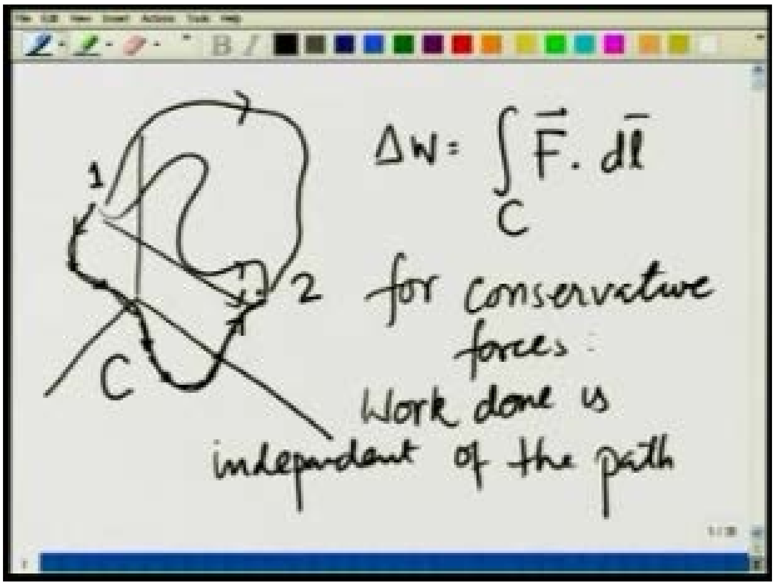

Now, I am going to generalize to 3 dimensional case if I go to form point 1 point 2 through some path the work done is defined as an integral F that is a force that I am applying dot dl with a force F is a force applied by me. If I am considering the work done by the force field, then the force is a force field later when we discuss more we will be focusing on this once more. This sometimes gets confusing whether it is a work done by the force or work done on the body work done by us and so on.

In any case, when I define this work, what it means is I am moving a particle in small steps along the path and calculating the work done along each of this segment and adding it up. So, I am going to put a sign C here to indicate, then I am moving on this curve C let me now talk about conservative forces. If you recall the conservative force is the 1 in which no matter what I do whether I chose this path or this path or go straight from 1 to 2 or go in some arbitrary way. The work done is always the same, so for conservative forces work done is independent of the path.

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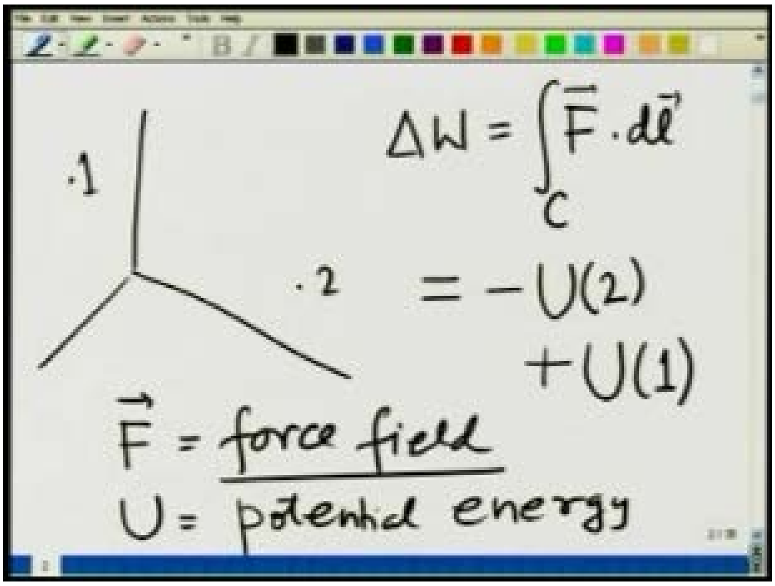

Therefore, I can safely say that I am talking about point 1 and point 2 the work done delta w which is equal to F dot dl along the path C does not really depend on path C, but only on the end points 1 and 2. Therefore, I can write this as let me just define use this to define potential energy as minus U 2 plus U 1 in this case. Now, let me express it F is the force field that is the force that is being felt by the body when it is moving I am not applying for this force this force is present in the space, where body is moving.

For example, it could be the force due to the gravitation of earth if something is moving around the earth or if 2 charges are moving it could be the force applied by the 1 charge on the other. So, now I am being very specific this force is the force given to us and this U I call the potential energy this is similar to the definition that I already used once, but now we are trying to go in to 3 dimensional cases.

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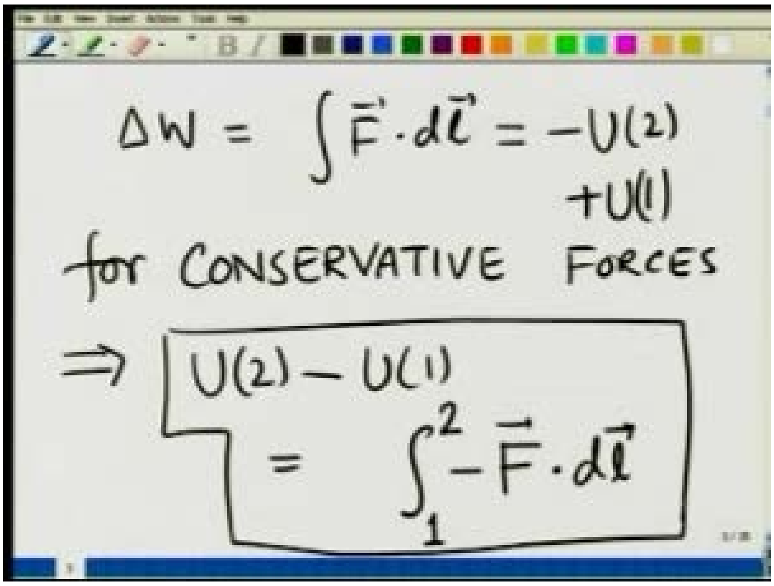

So, I define for conservative forces this equals F dot dl remind you I am not going to put any C here because independent of the path and this is going to be minus U 2 plus U 1. Let me be very specific for conservative forces and this implies that U 2 minus U 1 is equal to going from point 1 to point 2 irrespective of the path minus F dot dl. This is what gives me the interpretation of the potential energy, so through looking at the work and then identify that the work done is independent of the path I define potential energy by this definition, let me try to understand what it is.

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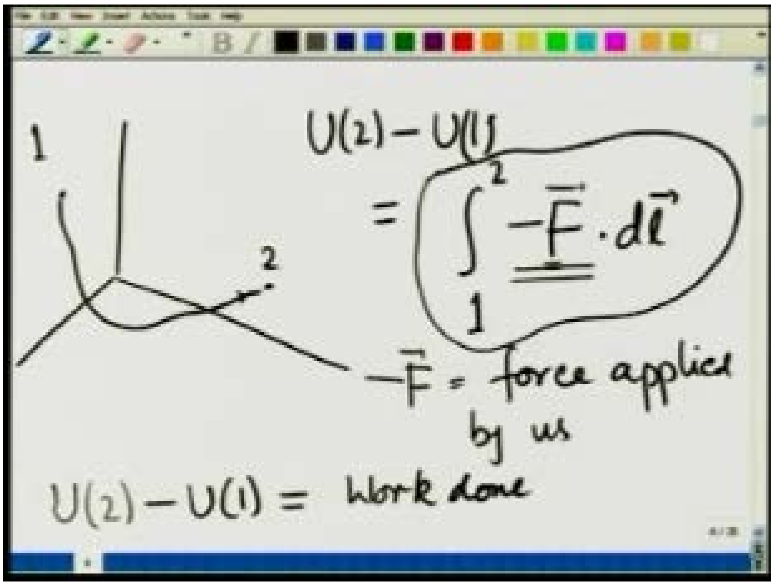

So, I am writing let me again make a figure between 0,1 and 0.2 and U 2 minus U 1 is equal to going from 0.1 to 0.2 minus F dot dl. Imagine I am bringing the body from 1 to 2 holding it in my hand such that I keep it in equilibrium or semi equilibrium because I am aligning it to be moved by the force in which it is moving. I apply sufficient force almost equal to F so that it moves all almost in equilibrium then minus F is going to be the force applied by us. So, I am bringing this body slowly applying just in a force so that it does not accelerate and let it come from point 1 to point 2 in equilibrium.

Then, this quantity minus F dot dl is going to be the work done by me because I am holding the body I am applying a force minus F. So, this is the work done by me, so by definition then U 2 minus U 1 is work done by us or by me in brining the body from 1 to 2 let me write it very clearly in the next slide.

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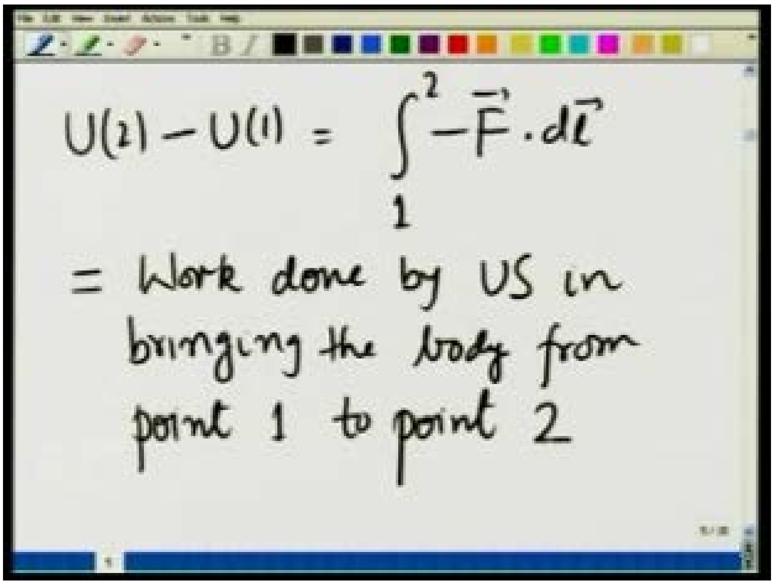

So, U 2 minus U 1 is equal to 1 to 2 minus F dot dl again I remind you I am not writing any C because this is I am talking about conservative force and therefore, the work done is independent of the path and it depends only on points 1 and 2. This then is equal to work done by us in brining the body or mass m from point 1 to point 2 and again this is true for conservative forces. So, this gives me the definition of potential energy U it is the work done by us if I bring a body form point 1 to point 2 keeping it in equilibrium with the force feel in which the body tends to move, let us take 2 examples.

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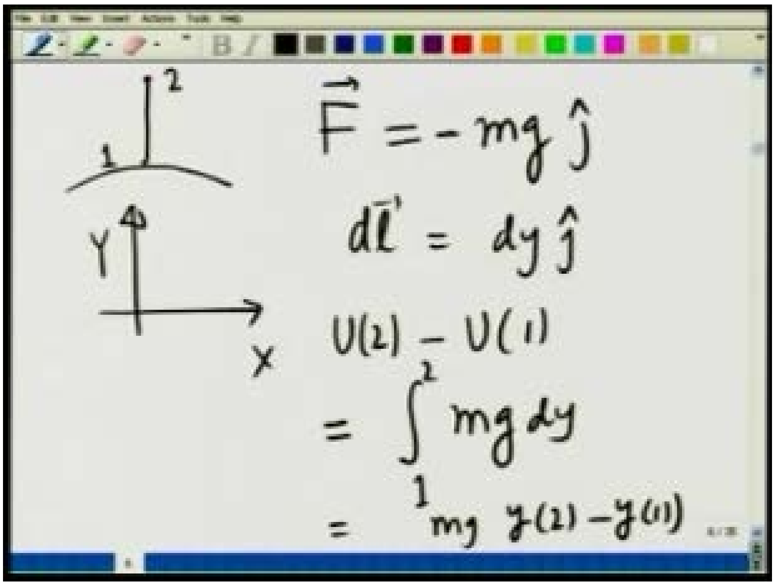

Let us take a familiar example of a body moving say from point 1 on the surface of the earth to a point 2, which is not very high above the earth so that I am going to make the approximation that the force remains the free the cone same. So, the force applied or the force field s the body is moving in a force applied by the earth is nothing but m g j with a minus sign because I am going to take this as my y axis this is my x axis only y axis matters in this case.

When I move the body up dl is going to be d y j and therefore U 2 minus U 1 is going to be minus F dot dl. Therefore, mg d y going from 1 to 2 and this is equal to m g y 2 minus y 1 which is your familiar expression for the potential energy when we work in terms of vectors sometimes we get in to confusion.

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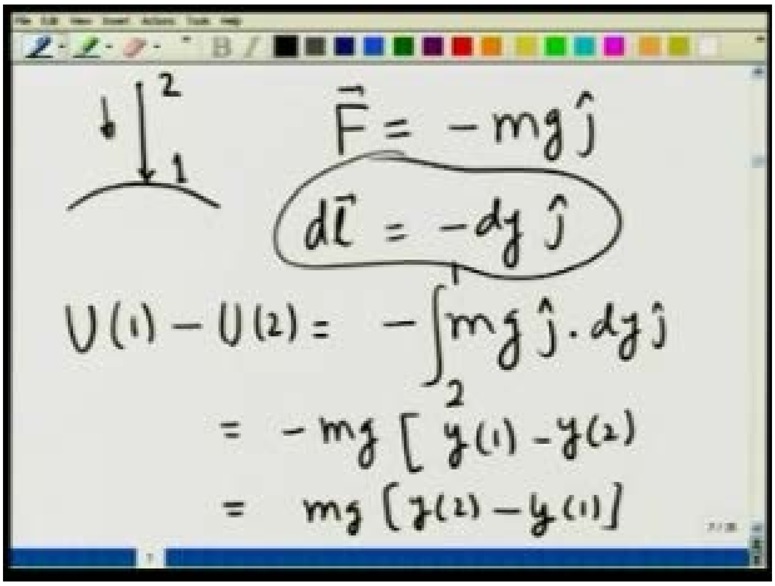

Let me just take that as an example let take this as an example to show that suppose instead of moving up I decided to come this way from point 2 to point 1 in that case again force F is equal to minus m g j. You are going to now say since the body is moving this way dl is going to be minus dy j and therefore U, since I am moving from point 1 2, this is going to be U 1 minus U 2 is equal to minus F dot dl and thus going to be minus m g j dot dy j integral.

I end up getting minus m g this is moving from 2 to 1 and therefore I am going to get this y 1 minus y 2 which is equal to m g y 2 minus y 1. Now, you notice in the previous slide I had obtain U 2 minus U 1 as m g y 2 minus y 1 and now I am getting U 1 minus U 2 as m g y 2 minus y 1. Where a mistake and this is is where lot of confusion at times arises the mistake is here you see when I am taking dl to be minus d y and then again putting limits from 2 to 1 I am actually double counting the minus sign.

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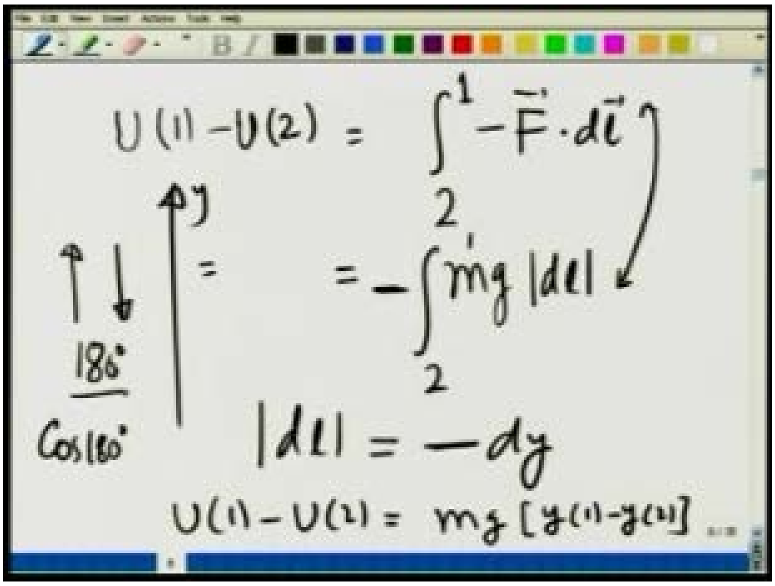

The proper way to do it is would be to write again U 1 minus U 2 is equal to integral going from 2 to 1 minus F dot dl and this is going to be equal to now you see dl is this way I am moving down minus F is this way. Therefore, minus F dot dl the angle between them is 180 degrees the magnitude of minus F is m g magnitude of dl is dl with a minus sign in front that minus sign comes because of cosine of 180 degrees. I get this integral going from 2 to 1 and now I know magnitude of dl is nothing but minus dy because y I am measuring going up.

Now, you put in everything correctly you get U 1 minus U 2 to be equal to m g y 1 minus y 2 which is negative of U 2 minus U 1 correct answer. So, you see you have to be careful in doing this always make sure that you are not double counting going from one point to the other the general displacement is d x I plus d y j plus d z k and thus going to be to your dl the sign is properly taken care of by these limits. If you take it explicitly, then you have to be careful as to what is the relationship between dl the direction of it and a general displacement vector d r which is equal to d x I plus d y j plus d z k in this case the relationship happens to with a minus sign in front.

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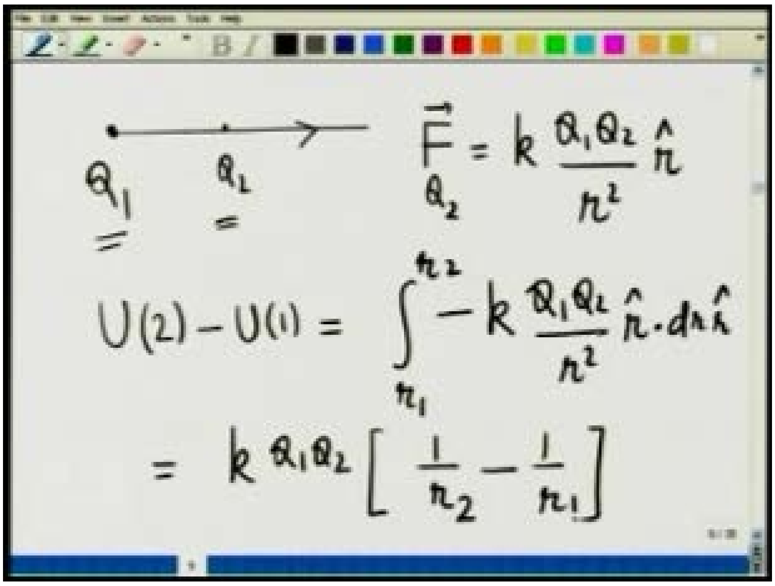

As another familiar example let me take 1 charge Q 1 and let us assume is fixed somewhere, let us take the other charge Q 2 and move it let both the charges be of the same sign. So, the force is moving out going out wards since you know polar coordinates by now I can write this force on Q 2 as k q 1 Q 2 over r square along the direction r. I want to calculate the potential energy of Q 2 in the field of Q 1. So, by definition if I move from point 2 to point 1 this is going to be equal to say from r 1 to r 2 let me move radically minus F. So, that is going to be minus k Q 1 Q 2 over r square r dot d r and that gives me k Q 1 Q 2 1 over r 2 minus 1 over r 1.

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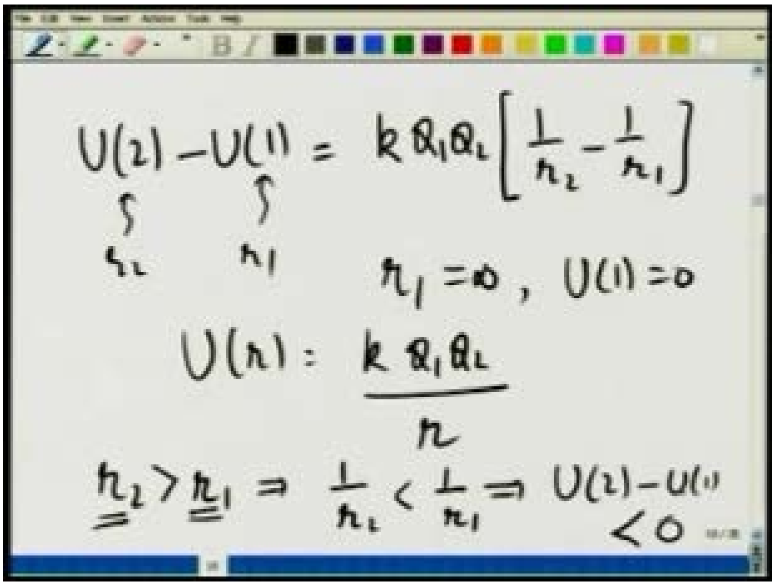

So, in the force field of charge Q 1 when Q 2 moves U 2 minus U 1 is going to be equal to k Q 1 Q 2 1 over r 2 minus 1 over r 1 where this refers to r 1 this refers to r 2. Usually, what we do is define a point for example, in this case r 1 to be infinity where I take U 1 to be 0 as a reference point and then your U at r becomes k Q 1 Q 2 over r. Notice again when I calculated this I put minus k Q 1 Q 2 over r square r as the r unit vector as the force and that is the force by applied by us the external agency to keep the charge Q 2 in equilibrium while moving it.

Of course if r 2 is greater than r one; that means, 1 over r 2 is less than 1 over r 1 I am moving the body from inside to outside this implies U 2 minus U 1 is negative. The work is being done on the external body on the body that is keeping the charge in equilibrium when the particle moves or the charge particle Q 2 moves from a point r 1 to r 2 which is when r 2 is greater than r 1 you are extracting work. The work is U 2 minus U 1 is negative work done by me is negative that means the work is being done on me the force field is doing work on us.

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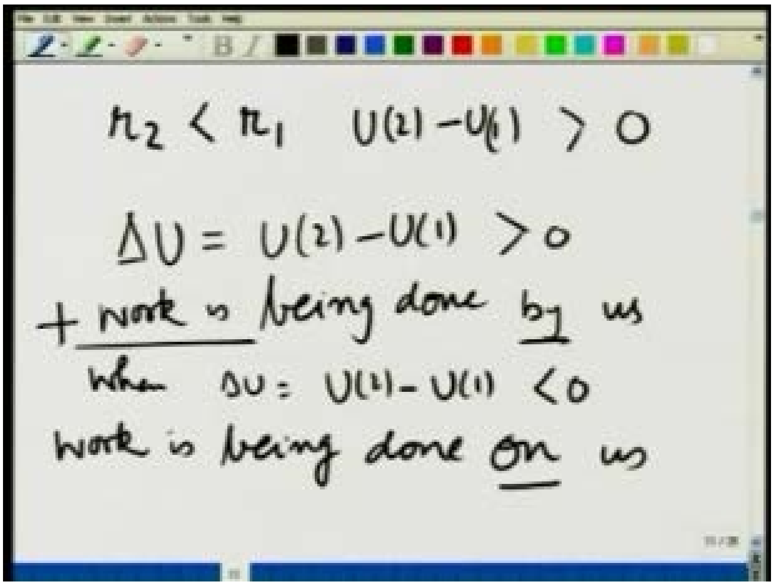

Similarly, if r 2 Is less than r 1, in that case U 2 minus U 1 would come out to be greater than 0 and therefore, the work done by us is going to be positive we are pumping in energy we are doing work. Work is not being done by on us these are points I am emphasizing because lot of people get confused on such things as whether the work is being done by us work is being done on us and so on.

So, when I am defining potential energy if delta U that is U 2 minus U 1 is greater than 0, then work is being done, let me call it positive work is being done by us. When delta U which is U 2 minus U 1 is less than 0, work is being done on us keep this in mind having looked at these examples let us now still keeping in still focusing on conservative forces ask the inverse question.

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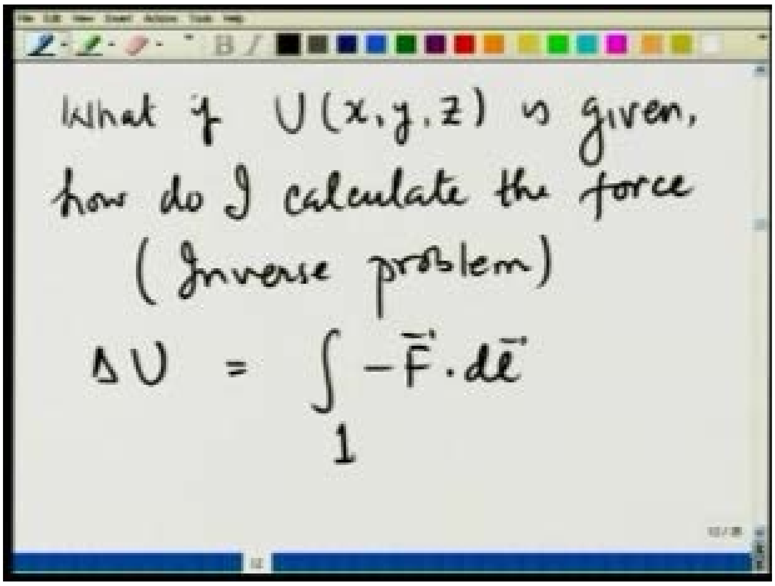

What if U the potential energy is given as a function of position of the particle that is U x y z as given how is given how do I calculate the force. If you like, I call this the inverse problem because what I done, so far is defined delta U is as work done by us in moving a particle from 0.1 to 0.2. I am now asking given this how do I calculate this let me remind you what we did in 1 dimensional case in the 1 dimensional case.

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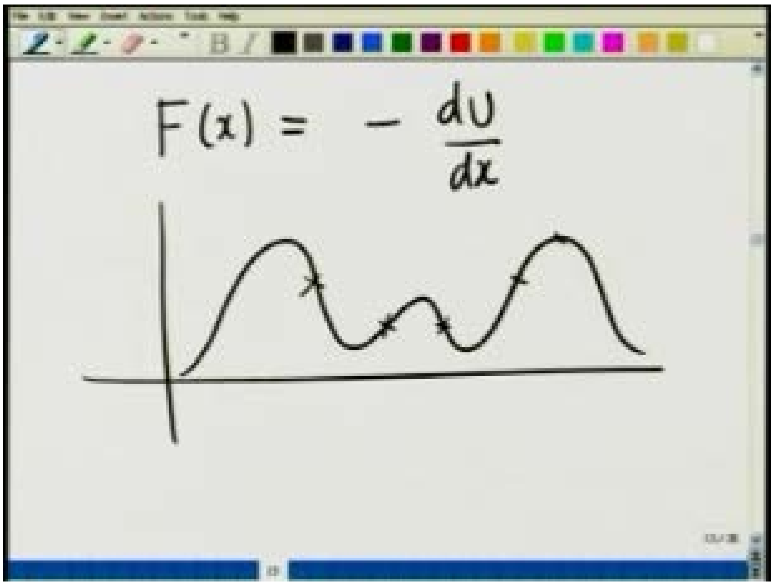

We said that force at any point was minus d U d x where x is the displacement direction if you recall from the previous lecture I had. In fact, made a potential hill something like this and I had pointed out how forces are and what are their directions at different points.

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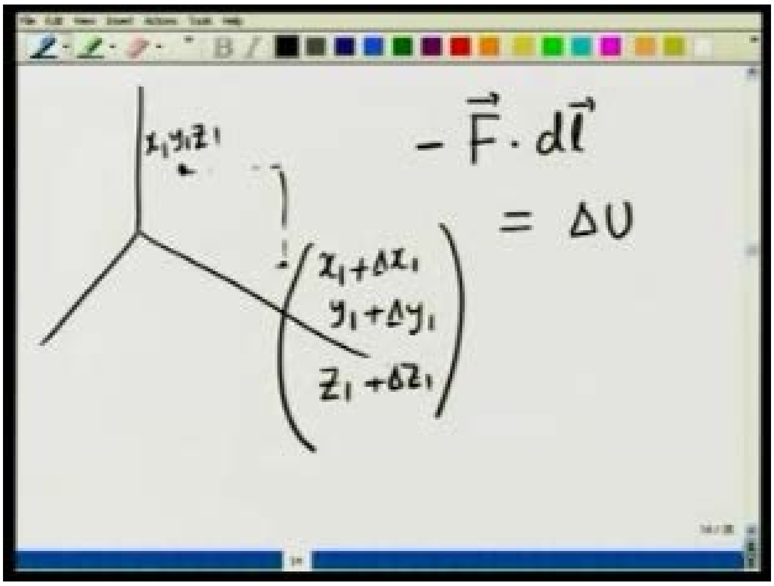

Now, what do we do in 3 dimensional case we again go back to our basic definition and see. If I move from 1 point x 1 y 1 z 1 when nearby point let me call it x 1 plus delta x 1 y 1 plus delta y 1 and z 1 plus delta z 1 then the work done since this is a conservative force for you. F is going to be F dot dl work done by us is going to be minus F dot dl and this is going to be delta U that is the change of U between point this point and this point.

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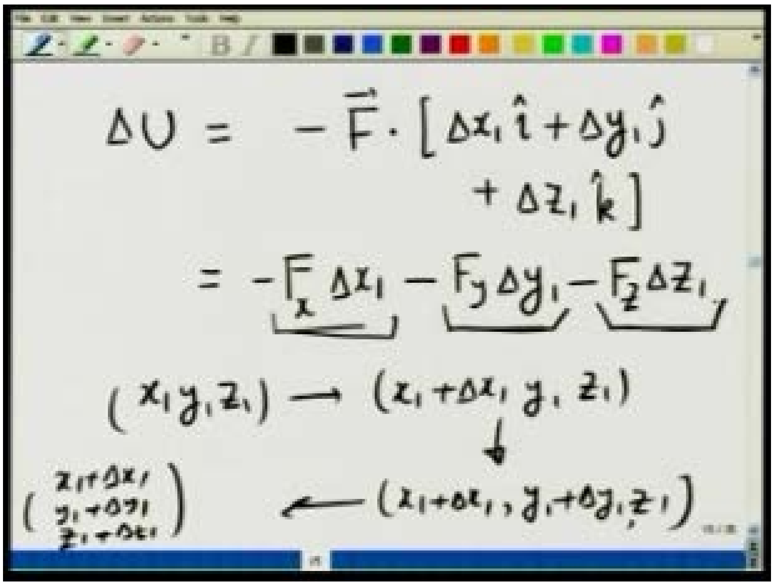

So, between 2 points I am defining delta U which is equal to minus F dot delta x 1 I plus delta y 1 j plus delta z 1 k. Therefore, I could write this as minus F x delta x 1 minus F y delta y 1 minus F z delta z 1 notice this comes out very neatly it does not matter which order I go form point 1 to point 2 because I am talking about conservative force filed. I could go from x 1 y 1 z 1 to x 1 plus delta x 1 y 1 z 1 go from here to x 1 plus delta x 1 I am moving particle along the x axis first then along the y axis.

Then, from there I could go to x 1 plus delta x 1 y 1 plus delta y 1 and z 1 plus delta z 1 plus delta z 1. Along each path, when I move along the x axis I do this much work when I move along the y axis I do this much work and when I move along this z axis I do this much work.

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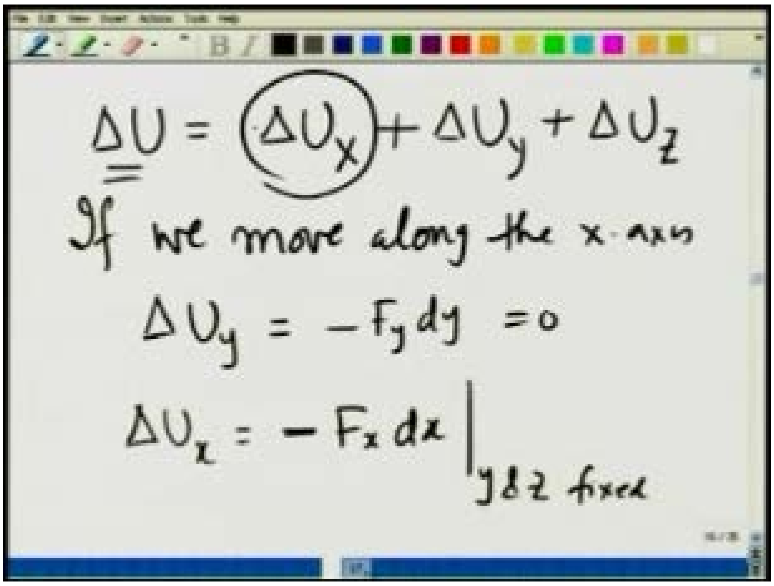

If you like I can call this delta U is equal to change in U when I move along the x axis change in U when I move along the y axis plus change in U when I move along the z axis. When I move along the y axis there is no work done being done against F x I am moving perpendicular to F x. So, if we move along the x axis then delta U y which is going to be minus F y d y is equal to 0. All the change in U comes only due to this, so if I keep y and z fixed I have delta U x which is equal to F x d x keeping y and z fixed. No work is done against forces in y or z direction when I move along the x axis, let me put a minus sign here because this is a force applied by us.

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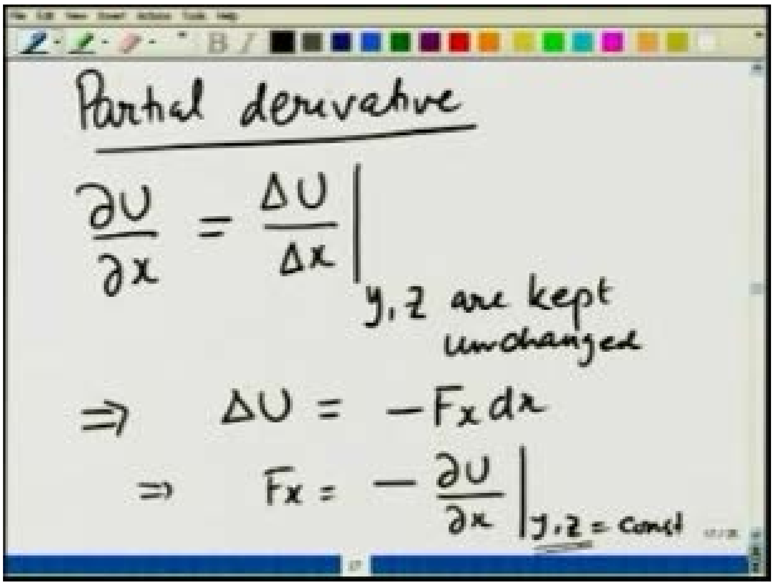

Therefore, let me out define something call the partial derivative, which I will say is delta U over delta x partial derivative with respect to x and y and z are kept unchanged and let me call this d U d x. You see then therefore from the definition that delta U if I move along the x axis is going to be minus F x d x gives me F x as partial derivative with respect to U and it is understood y z are constant. So, this is pretty much like 1 dimension that I am calculating the potential energy; however, since I am talking about the 3 dimensional case I have to be slightly careful. When I am moving along the x axis, I do not change y and z because that will unnecessarily mix up thing.

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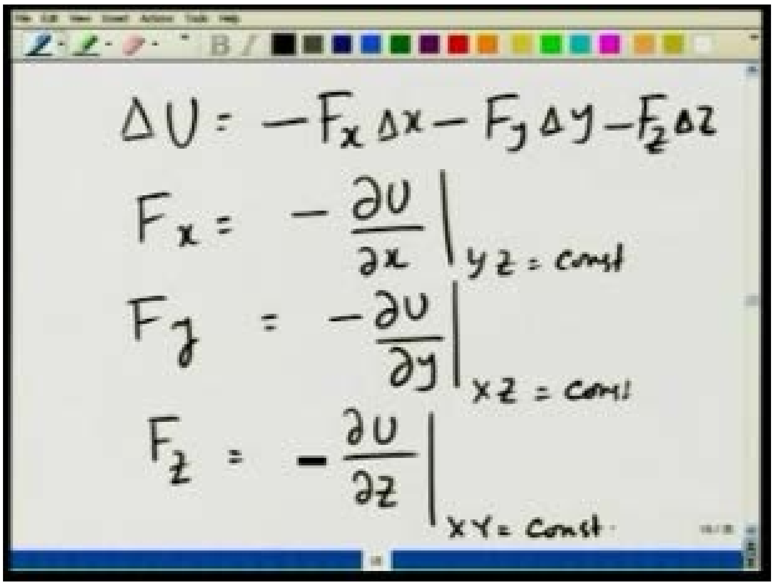

Similarly, I can now write that since delta U Is equal to minus F x delta x minus F y delta y minus F z delta z. So, if I am moving along the x axis change is in U is only due to the force component along the x direction. Therefore, F x is equal to minus partial derivative of U with respect to x, similarly F y the component to the force along the y direction is minus d U by d y. Force along the z direction is going to be minus or the component to the force along the z direction is d by d z by writing this partial derivative.

I mean in this case x and y remain constant I do not change them I move along the z axis in this case I keep x and z constant and this case I keep y and z constant. So, I have introduced a new term call the partial derivative physically what it means is since I am talking about a conservative field.

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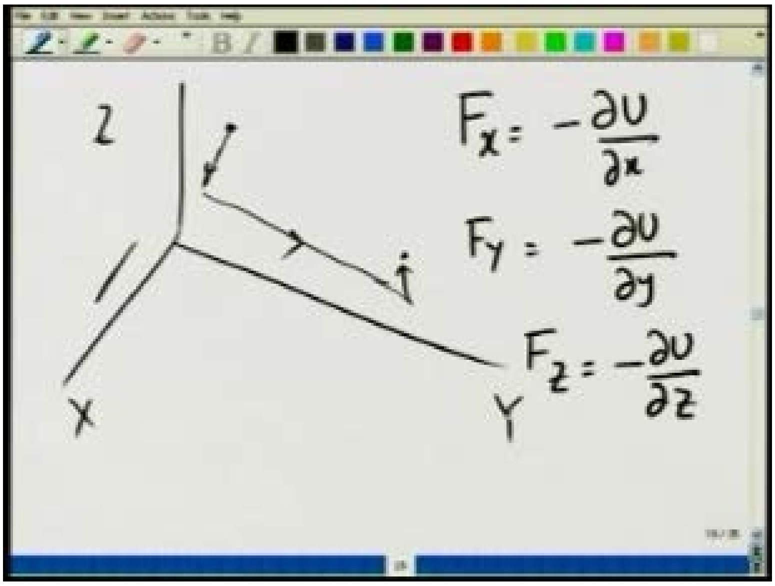

When I am moving from point one, this point to this point I am first moving along, let us call this x axis the y axis the z axis I move along the x axis find out how much is the change in potential energy. That is going to be only due to the component of the force along the x direction then I move along the y axis and then I move along the z axis and I calculate in each case partially how much is the change in potential energy.

When I move along the x axis, change in potential energy comes only due to the x component of the force and therefore it is related to the partial derivative of U with respect to x. When I move along the x axis, change in potential energy comes only due to the component to the force along the y axis and therefore that is related to the force the change in U with respect to y.

This is precisely why I get F x equal minus d U d x F y equals minus partial of U with respect to partial y. Similarly, when I move along the z axis the only component of the force, that effects that changes the U is the z component and therefore, this is partial of U with respect to partial of z. So, I have got now all 3 components, but remember I am talking about vector quantities is there any compact way of writing it.

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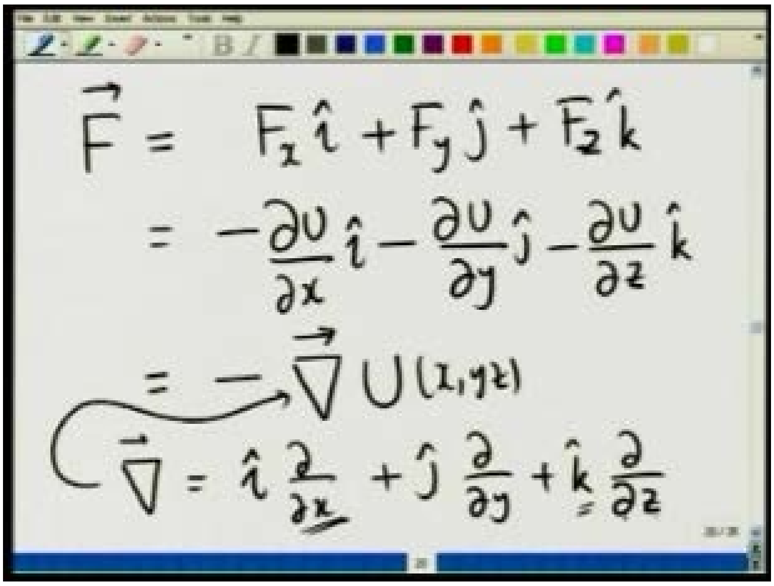

So, I write the force the net force, which is equal to F x I plus F y j plus F z k. This comes out to be minus partial of U partial change derivative of U with respect to x I minus partial derivative of U is with respect y j minus partial derivative of U is with respect to z k. The compact way of writing it is by introducing a new operator call the gradient operator, I am going to write this as minus a vector differential operator acting on U. Here, I will write this as I partial with respect to x plus j partial with respect to y plus k partial with respect to z.

When, I write this in front of a potential energy U x y z or any function of x y z, what it means is take partial derivative with respect to x and put. I multiply by this, I take partial derivative with respect to y multiply by j unit vector take partial derivative with respect to z multiplied by k unit vector add all three. So, this gives me a vector after taking the partial derivative, this is known as the gradient operator.

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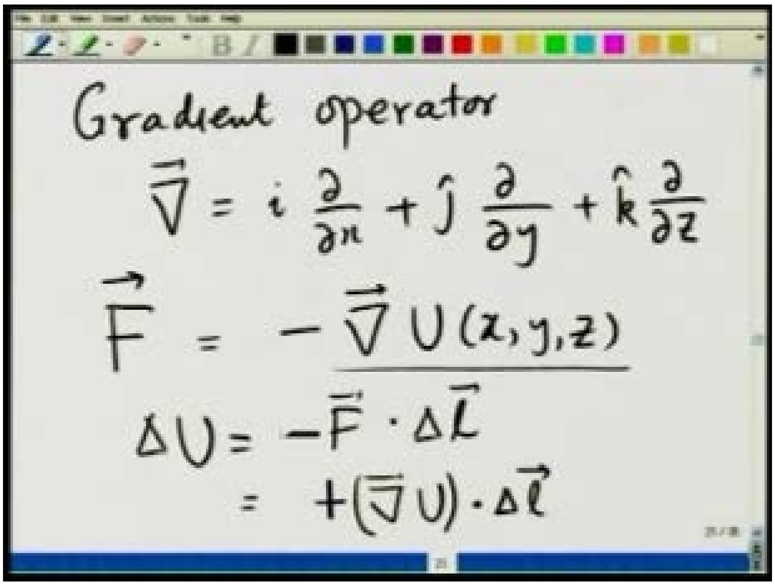

So, I have introduced a gradient operator this is like the 3 dimensional analog of the 1 dimensional differential operators, which I will write as this d by dx plus j I should write partial say partial of with respect to y plus k partial derivative with respect to z. Since I am talking about 3 dimensional case, this gives me a vector quantity and the force of which the potential is given is given as minus the partial the gradient of the potential energy U x y z. Again, if you are feeling uncomfortable will go back to 1 dimensional case and you will see there, I get plus or minus signs in the derivative. That means, either the function is increasing to the right or decreasing to the right depending on the sign.

Similarly, this in 3 dimensional case gives me in a particular direction if I go how the function U is changing remember again delta U is nothing, but, F dot delta l or dl and this is nothing, but minus partial of U dot this is a vector quantity delta l. So, it really again describes the change in the in a function U, but in a 3 dimensional case depending on which direction I am moving slightly more than the 1 dimensional case will now try to get some more insights into this.

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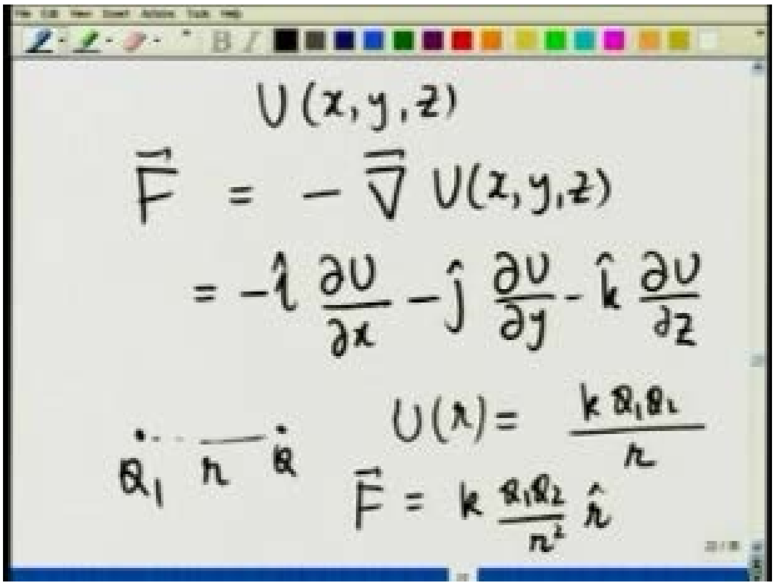

As far as the definitions are concerned, it is very clear that if I want to get given a function U x Y z if I want to get the force F this is going to be equal to minus gradient of U x y z. It is in explicit form is going to be minus I partial of U with respect to x minus j partial of U with respect to y minus k partial of U with respect to z. As a simple example let us take again a charge Q 1 and charge Q 2 at a distance r from it and we have already seen and you also know that U r is given as k Q 1 Q 2 over r. If I want to get the x y and z components you take this derivative and you will see that the force F would come out to be correctly as k Q 1 Q 2 over r square r, I leave this as an exercise.

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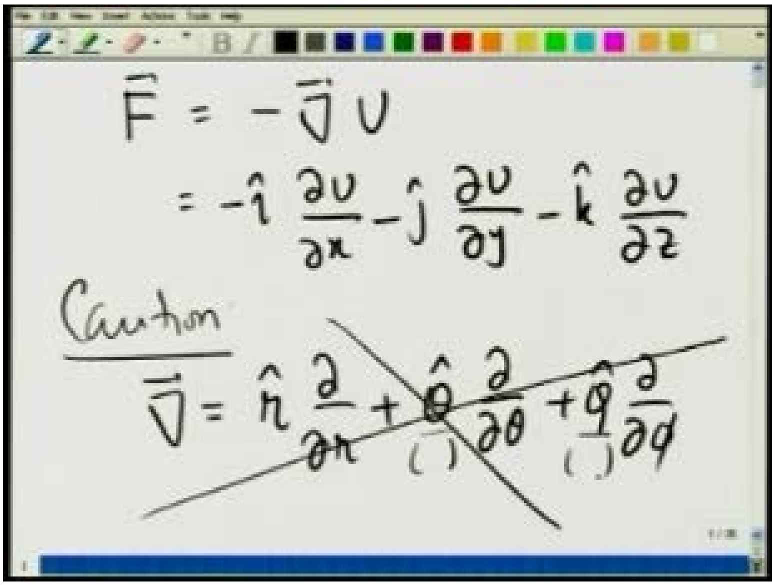

Having defined the force, having obtained the force on the definition of gradient which I wrote as minus I partial U partial x partial of U is with respect to y minus partial of U with respect to z. I would like to give a word of caution since now you are familiar with different coordinate systems do not directly write gradient in other coordinates. For example, if I take spherical polo coordinates as this is not correct is not even dimensionally correct because there should have in the denominator dimension of length does not even have that. To obtain these things in other coordinates systems, you have to make a proper transformation, for example in this case in these spherical polar coordinates.

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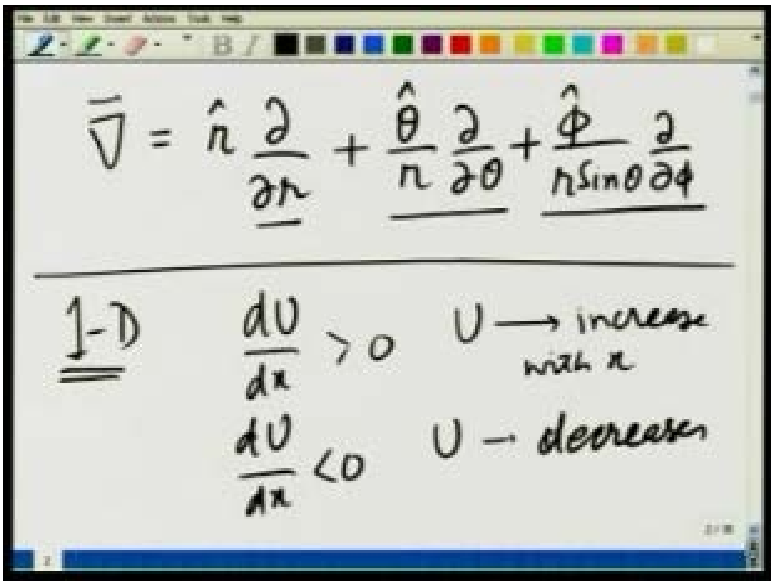

For example, the partial derivative is given as in direction of r it is indeed partial with respect to r. On other hand in direction of theta it is like this and the direction of phi it is divided by r sin theta d by d phi. That makes perfect sense because this is distance covered in phi direction this is the distance covered in theta direction this is the distance covered in r direction. So, this a word of caution that when you go from 1 coordinates system to the other take proper transformations to define gradient operator.

Let us now try to get some feeling of this because in 1 dimension is quite easy you been thinking in terms of 1 dimension and you can see that if I go if. So, let us just first say 1 D case the familiar case is if d U d x is greater than 0 U increases with x if d U d x is less than 0 U decreases with x. What can we talk about in 3 dimension does it increases in all the directions how does the gradient give you in which direction is the function changing and so on.

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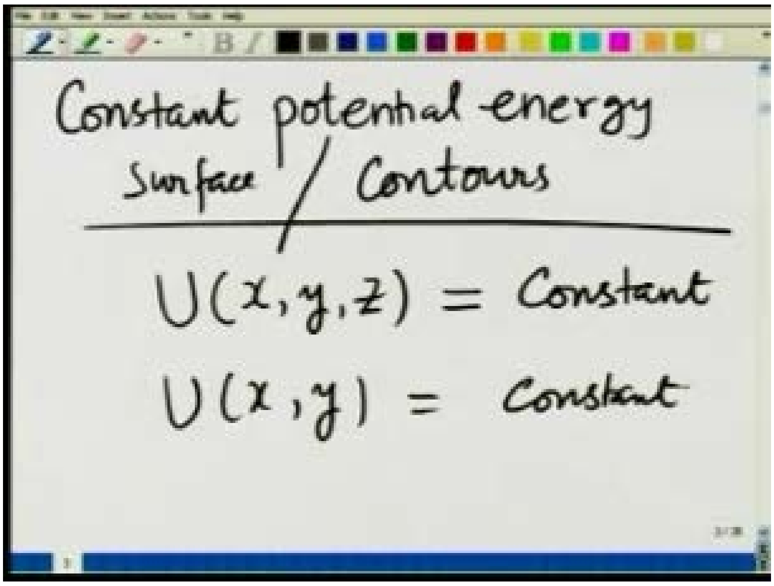

So, for that let me first in introduce a concept of constant potential energy surface or in 2 dimensions what are called contours and probably when you are doing your geography or something in your eighth or ninth grade, you have come across such term as contours. Now, for a constant potential energy surface I define that surface in 3 dimensions as U x y z that surface over which this is a constant or the contour in 2 dimensions is U x y is a constant.

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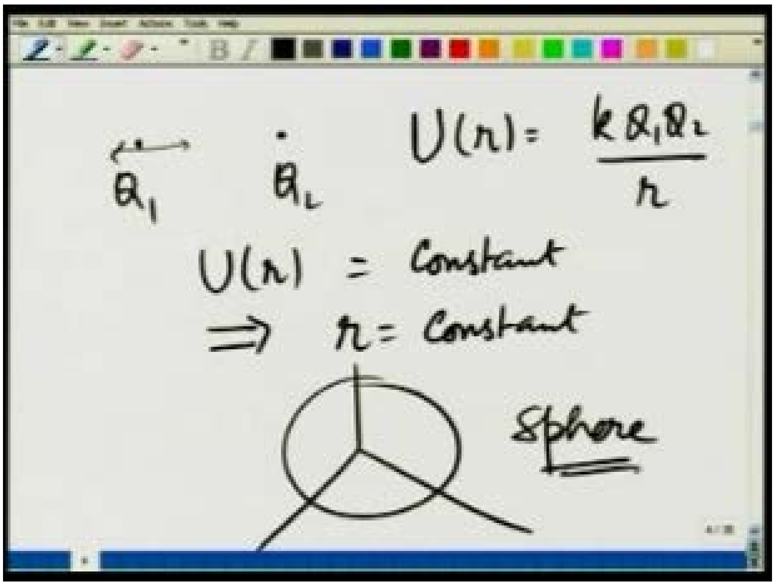

For example, if I take the case of One charge Q 2 moving in the force field of this Q 1 then I know that potential energy U at r is given as k Q 1 Q 2 over r and U r is equal to constant. This would imply that r is a constant and therefore in this case the constant potential energy surface is going to be a sphere.

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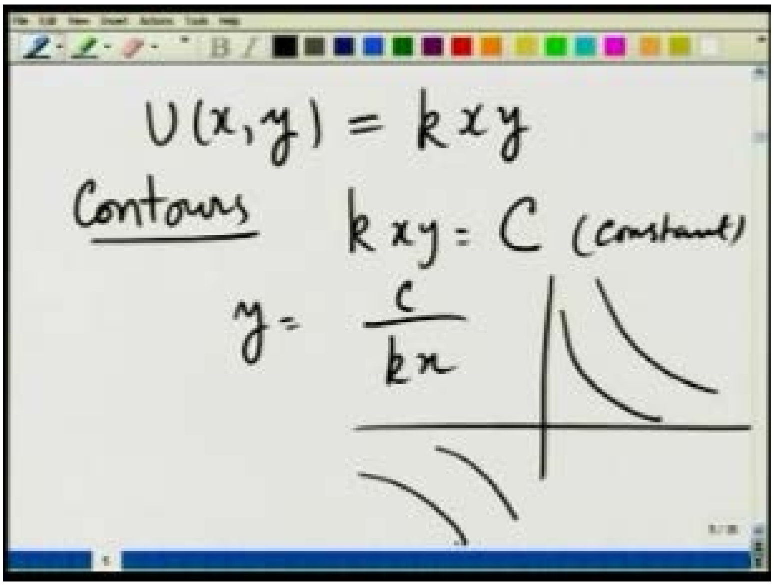

As another example just cooked up example let me take U x y to be equal to some constant k times x y and if I am looking at contours curve over which U is a constant I should have k x y is equal to C a constant. Therefore, y equals C over k x which are nothing, but, different hyperbola depending on the value of C. So, these are the contours for this particular function, now why do I need this I need this to get a feel for what gradient operator is?

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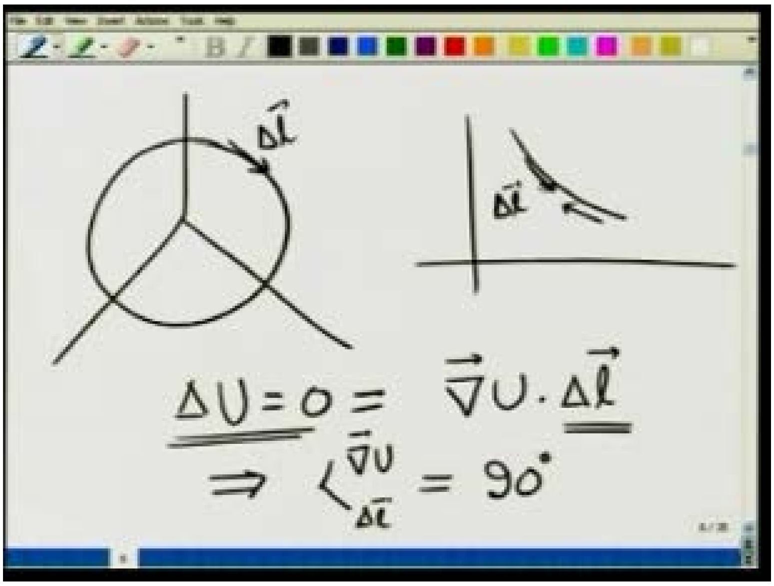

So, let us take a 3 dimensional case or 2 dimensional case where I have a contour or a constant potential energy surface and let me move along this along the surface. For example, I can move in the case of Coolum potential, if I move along this sphere then I am keeping U constant. Similarly, in the case of U being equal to k x y if I move along this hyperbola U remains a constant and therefore, if I move along the contours delta U 0.

By definition delta U is also equal to gradient of U times delta l where delta l is a distance move this could be delta l in this case on this sphere in any direction this would be delta l in this case this way or this does not matter. None the less, what no matter what I take this to be delta U comes to be 0 always if I move along a constant energy surface or a contour. This implies that the angle between gradient of U and delta l along a constant energy surface of the contour is equal to 90 degrees.

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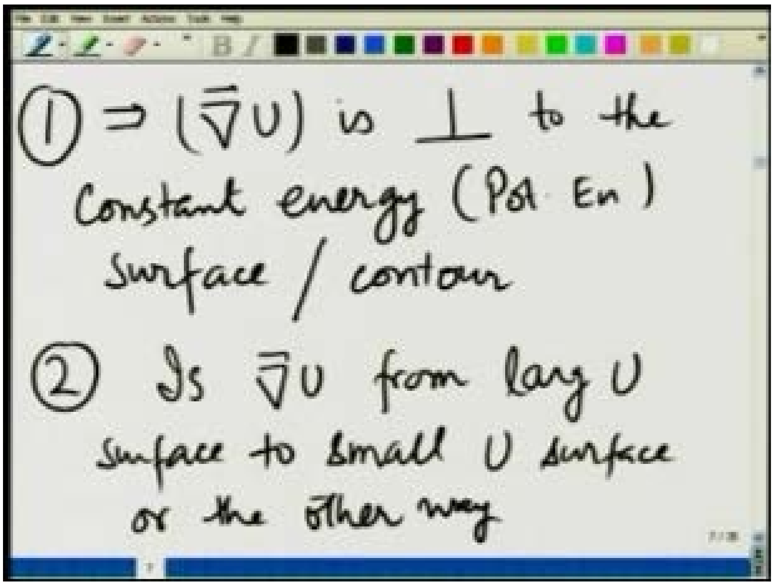

This implies that gradient of U Is perpendicular to the constant energy when I say energy I mean potential energy let me be expressing it potential energy surface or contour in 2 dimensions. So, first thing we conclude is that the gradient of a function U is going to be perpendicular to the x constant surface or the contour I still I have not fix the direction is that going to be towards increasing U decreasing U. Recall form the 1 dimensional case that I just mentioned if d U d x is positive; that means, if I go in positive x direction U would be increasing, can I say similar thing about this.

Let us look at that, then I will take an example and shows again first conclusion is that gradient of U is perpendicular to the constant energy or potential energy surface or contour, number 2. The question I ask is gradient of U from large U surface to small U surface or the other way.

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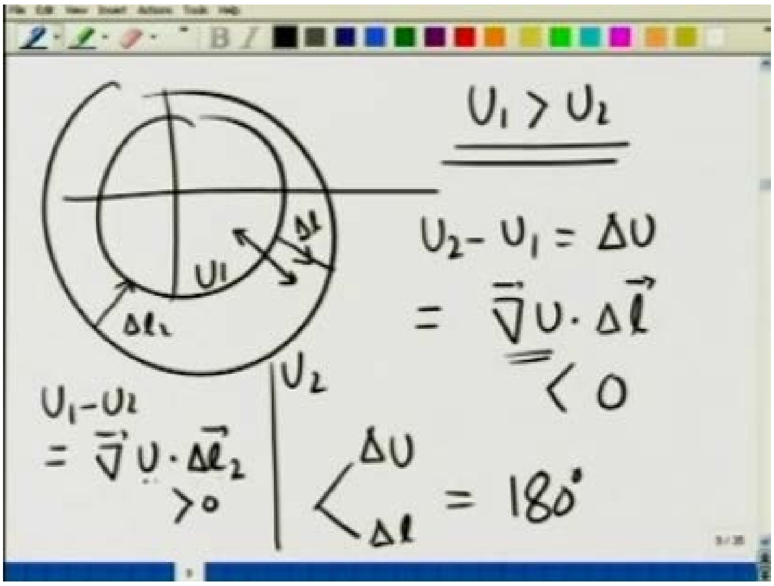

So, let me just picturize this, suppose I have let me take 2 dimensional case since it is easy, a contour let this be U 1 U 2 and let U 1 be greater than U 2. So, I know gradient of U is either in this direction or in this direction, let me come from surface 1 to surface 2 by delta l then I know that you 2 minus U 1 which is change in U is going to be equal to gradient of U dot delta l. This should be less than 0 because we I have already taken U 1 is greater than U 2 this can be less than 0 only if delta U and delta l make an angle of 180 degrees.

So, I conclude that delta U should be in the direction opposite to delta l in this case, so it is pointing away from the direction in which U is decreasing or it is pointing to the direction in which U is increasing. Let us see that explicitly if I on the other hand go from U 2 U 1 thus write it here then U 1 minus U 2 is again going to be gradient of U dot delta l. Let me call it delta l 2 and this is going to be greater than 0 and that would be the case if delta l 2 and delta U are in the same direction. So, it points perpendicular to the constant energy surface or the contours and in the direction of increasing U.

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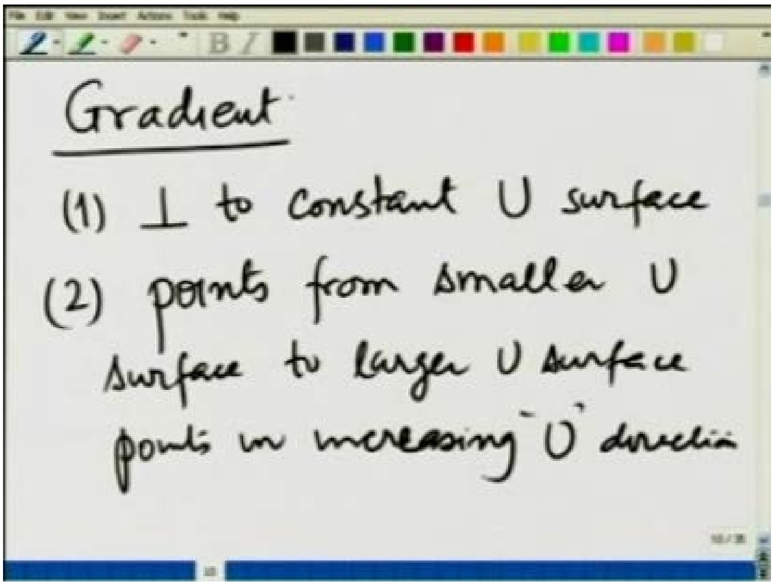

So, two things we learn about the gradient Are number 1 perpendicular to constant U surface number 2, points from smaller U surface to larger U surface or in short points in increasing U direction that is a kind of feeling. Now, either I develop and imagine a constant energy surface and then U gradient of U is going to be perpendicular to that and it is going to point in the increasing direction that is quite understandable if I go back to the 1 dimension case also.

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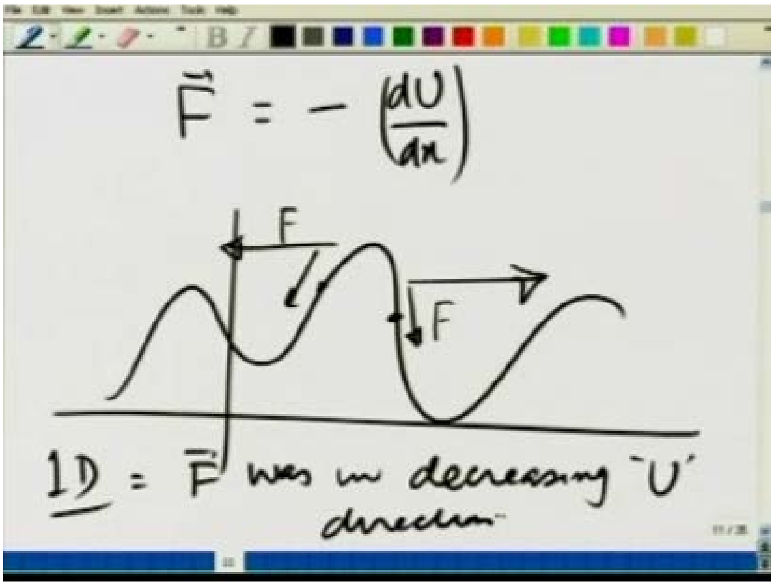

Recall, there that F was equal to minus d U dx so that the force direction if you recall that example was towards decreasing U. It was this way here this way here this is the force direction rather than writing this way the force was in this direction here this is 1 dimensional case in this case it was in this direction because d U d x is positive force in negative direction. So, it was in the direction of decreasing U, so in 1 D case, F was in decreasing U direction how about 3 D case.

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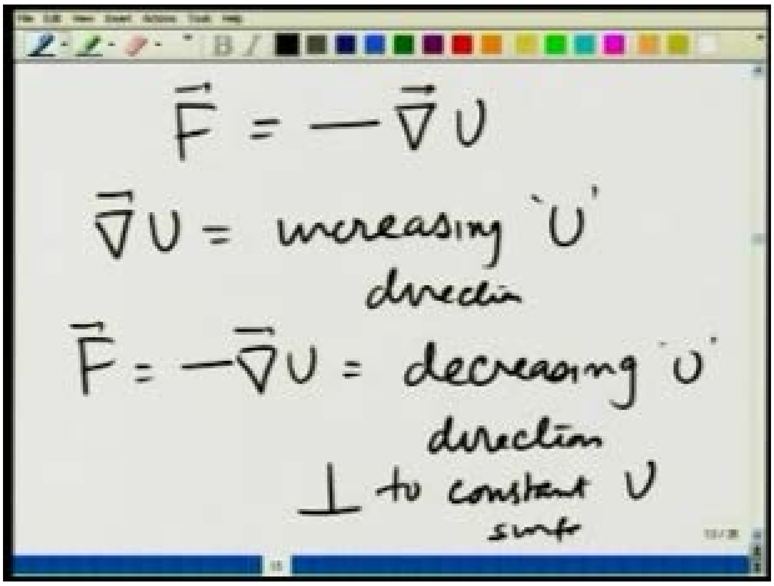

In this case, I have force which is equal to minus the gradient of U and gradient of U is in increasing U direction. So, force which is minus gradient of U is in decreasing U direction and it is perpendicular to constant U surface. So, this sort of gives you a feel for this and this is absolutely consistent with what we learned in 1 dimensional case, let us take an example.

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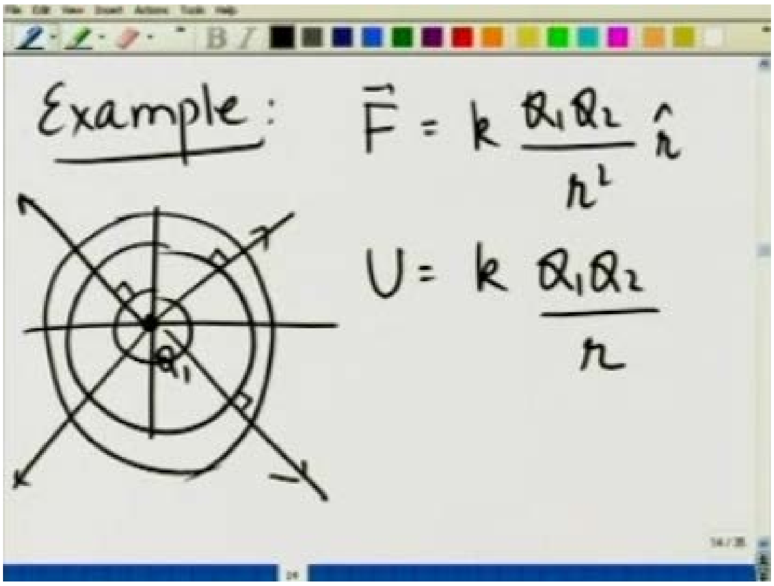

The simplest again being force between two charges as I said earlier the constant energy surface is this sphere U is k Q 1 Q 2 over r if charge Q 1 is here then force I known Q 2 is going to be radially out. You also know that potential energy goes down as I go further and further away and also since this is radially out it is always perpendicular to the constant energy surface, you should work out more examples like this to get a feel for the gradient.

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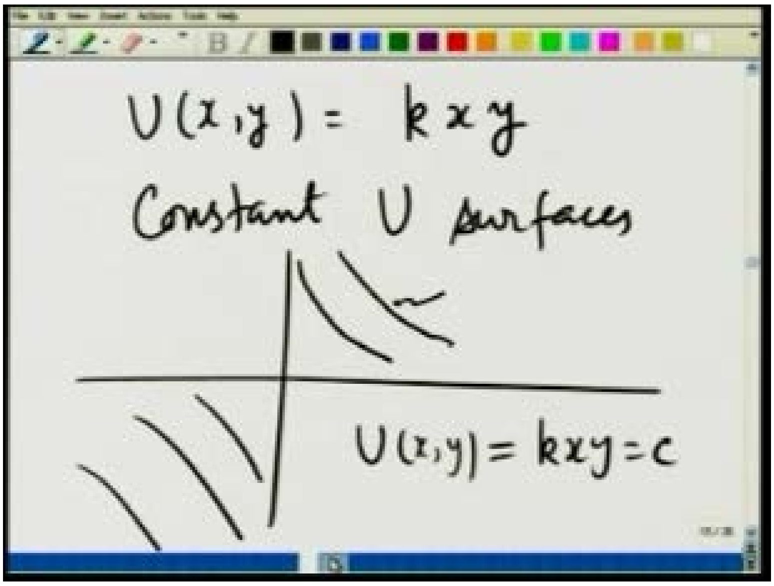

Let us take another example, let me just hook up U x y to be K x y as I said earlier as an example I took earlier and let me now plot the constant U surfaces. These are said earlier are going to be hyperbolas depending on what C is and so on, now since U x y over these surfaces is a constant, this implies that this equals k x y equals C.

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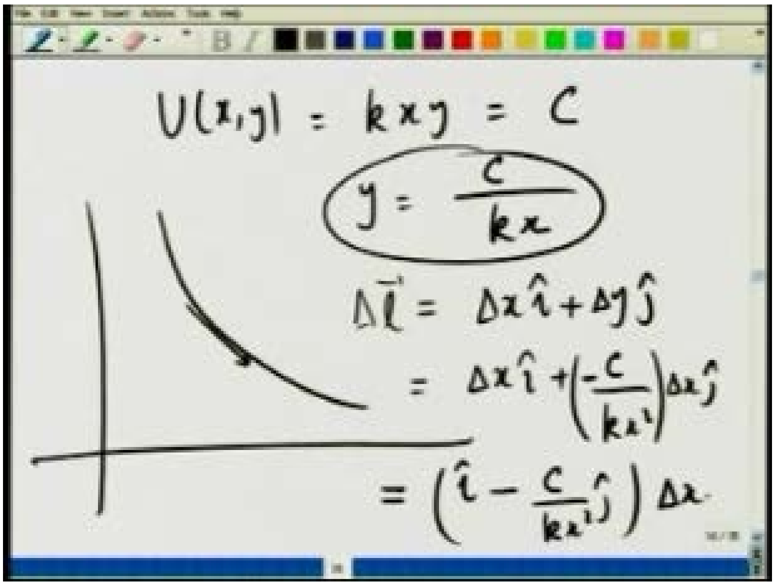

Then, U x y equals k x y equal C and; that means, along the constant energy surface, let me take 1 particular 1 of them y is equal to C over k x. If I make a displacement along the constant contour surface or along the constant energy surface of the contour, I am going to have delta l, which is equal to delta x along I plus delta y along j. This relationship delta y is going to be delta x I remain delta y is going to be C over k x square with a minus sign delta x j. So, if I move along the contour delta l is going to be I minus C over k x square j delta x at a given point x.

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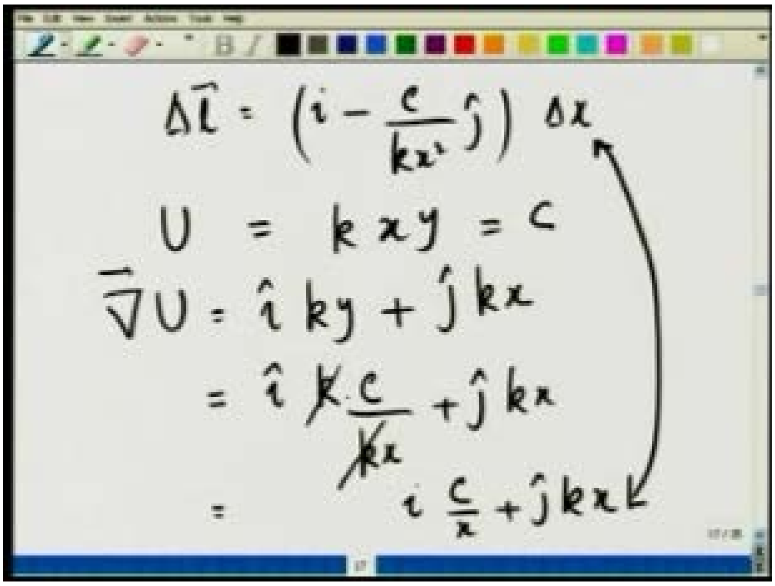

So, remember this this is I minus C over k x square j delta x delta l is I minus C over k x square j delta x U is equal to k x y. Therefore, gradient of U is going to be k y along I direction when I take partial derivative of this I keep y constant and differentiate with respect to x plus j k x. Since this is a constant, I again can write on that particular curve I k y is nothing but C over k x plus j k x. So, add a given point on that curve or the contour gradient of U is going to be I C over x plus j k x. You can see if I take the dot product of delta l and gradient of U is going to be 0 I get C over x minus C over x which is 0 at all the points and that shows you that delta U is perpendicular to delta l let me show it much more explicitly.

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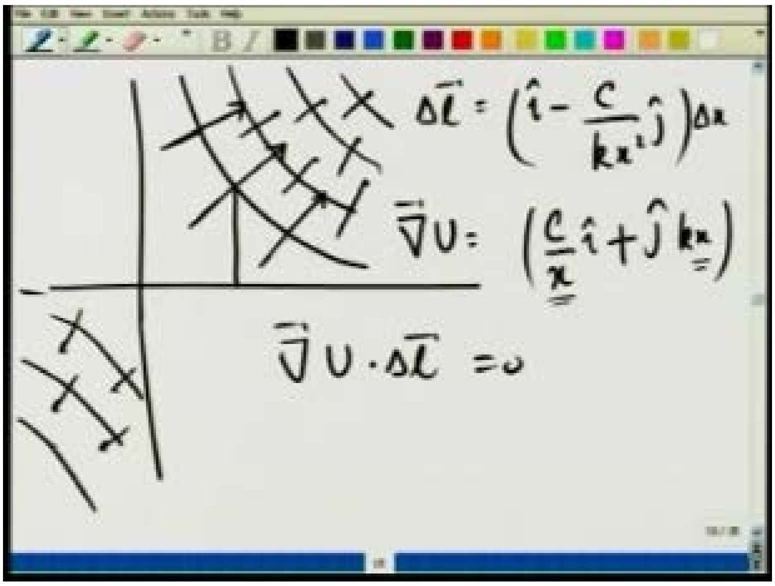

So, I got delta l which was equal to I minus C over k x square j delta x at a given point and gradient of U which was equal to C over x I plus let me see what it was plus k x j k x. So, that gradient of U dot delta l is 0 and therefore gradient of U at any point is going to be perpendicular to that surface. The direction is going to be in the increasing direction you can see form here for positive x and for x on the positive side is going to be like this you can also confirm for yourself as C increases the contours becomes like this.

So, U the gradient of U is really pointing in this direction that I hope by now gives you some feeling for the gradient operator you could also tried on the negative side on that side the gradient of U is going to be perpendicular for pointing in this direction. Again, increasing C means I am going further and further away in this contours, so far we talked about conservative forces.

How in a conservative force, the force and the potential energy are related I have shown you that the potential energy. When I take its gradient gives me the force the force is in the direction perpendicular to the constant potential energy surfaces and in the direction of decreasing potential energy. In the next lecture I will take one more example of calculating a force from the potential energy using gradient and also talk about non conservative forces.