Engineering Mechanics
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Module - 06
Lecture - 03
Work and Energy - II

Now, a motion can be understood or what restrictions are on the motion from these concepts, so we started those lectures from number one.

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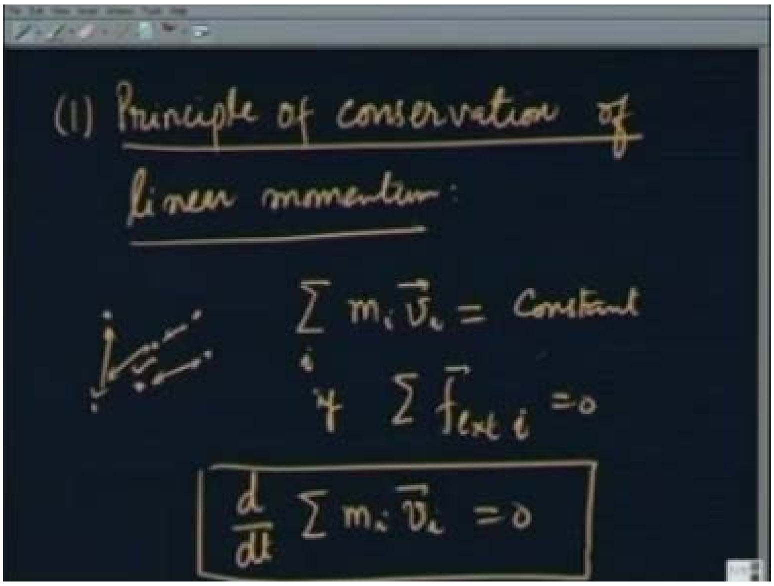

The principle of conservation of linear momentum and what it said was that if I have many particles interacting with each other as long as these inter particle forces obey Newton's third Law the net momentum. That is m I v I product summed over is a constant if summation of external forces on each of the particles is 0 that is if the net external force is 0, the momentum the total momentum of the system remains a constant.

In other words I can write d over d t of summation m I v I is equal to 0 and that is the principle of conservation of linear momentum related with this when we considered momentum of many particles was the concept of center of mass which is defined.

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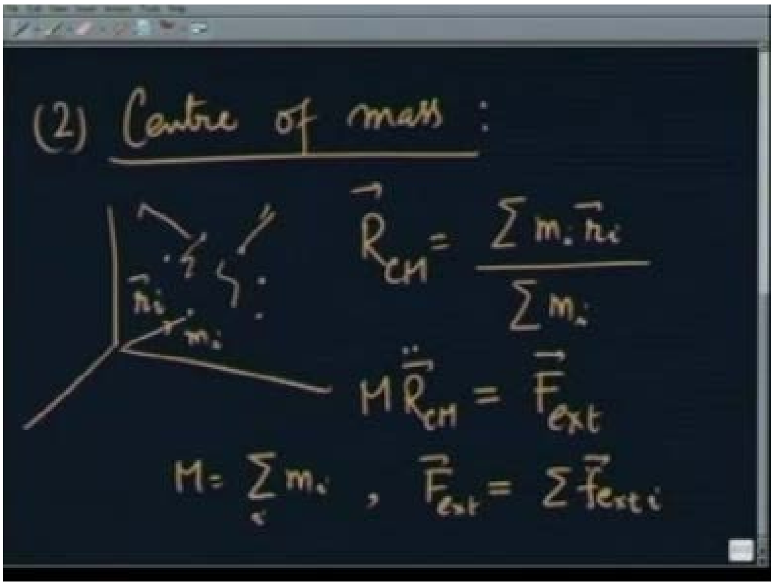

Suppose, there are many particles at different positions let me call this 1 r I, let the mass be m I, then the center of mass coordinate R equal to summation m I r I over summation m i. The beauty of center of mass was raised that if I apply an external force no matter how these particles are moving, maybe knowing anything, the body may be deforming the orientation may be changing. The center of mass always moves as if the entire mass is sitting there and the net force F external applied there, where m is the total mass of the system and F external is a net force that is summation of f external I on each individual.

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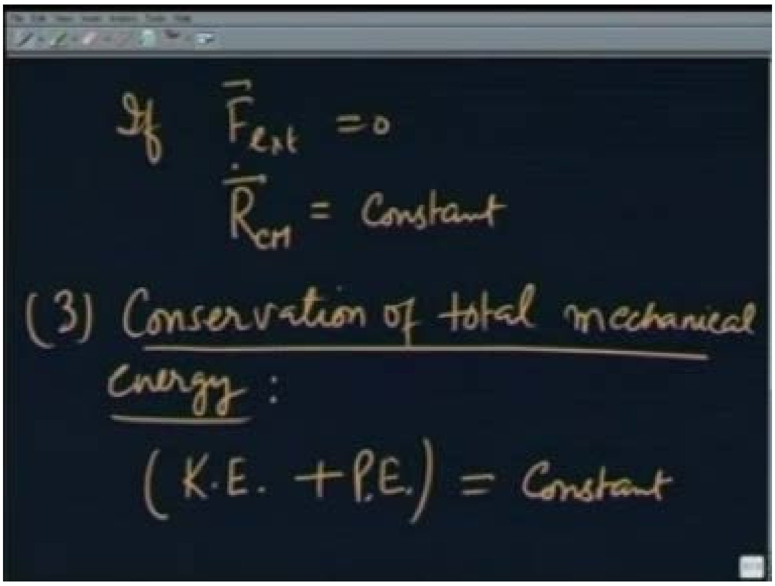

In other words, if F external the net external force is 0 I am going to have R CM dot is equal to constant that is the center of mass would keep on moving with constant velocity. We solved 1 example using this in the previous lectures third thing that we learnt was the conservation of total mechanical energy and what it said was in a conversed, where the work done is independent of the path. The kinetic energy plus the potential energy of a system of particles is constant, in this lecture we are going to look at these principles once again through examples to learn how we can use them effectively to learn about motion of these particles.

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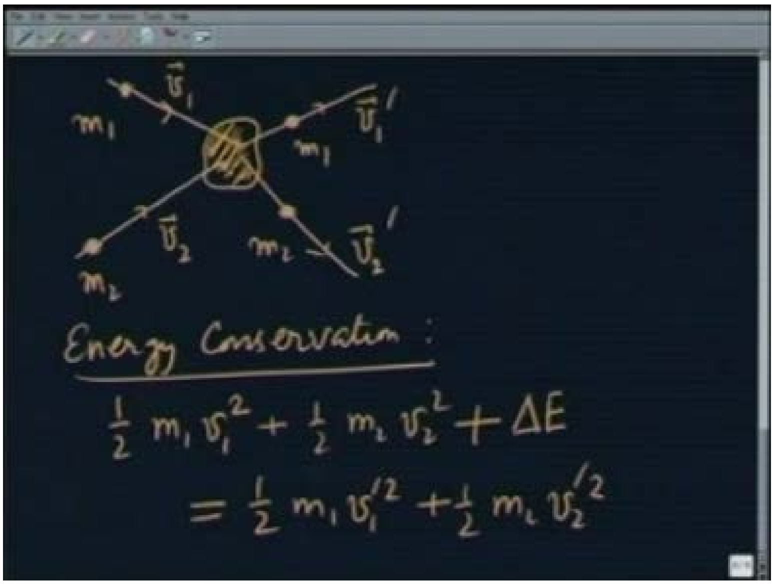

As a first example, I am going to take 2 particles colliding with each other and going out let the particles be m 1 and m 2 masses. This is coming in with velocity v 1 this is coming in with velocity v 2, this is going out with velocity v 1 prime again m 1 and m 2 goes out with velocity v 2 prime, I am going to assume that the inter particles forces act only in a very slow small region. So, before the collision and after the collision the particles are essentially free, the other thing I am going to assume is that this region is very small. Therefore, during the interaction when the particles go in interact and come out they hardly move.

So, P E the potential energy remains roughly the same and therefore I can write by energy conservation that the K E. The kinetic energy coming in which is one half m 1 v 1 square plus one half m 2 v 2 square plus I will add a term delta E to take care of energy, which may be released during their interaction or which may be lost during interaction. So, this must be equal to one half m 1 v 1 prime square plus one half m 2 v 2 prime squares that is what energy conservation gives me or let me write the equation again and then we will talk about delta E.

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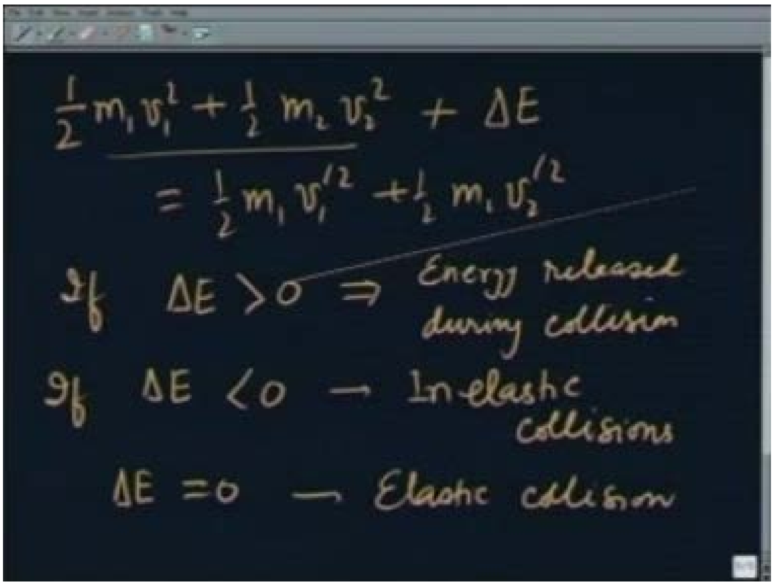

So, the energy conservation gives me one half m 1 v 1 square plus one half m 2 v 2 square plus delta E is equal to one half m 1 v 1 prime square plus one half m 2 v 2 prime square.

Now, if delta E is greater than zero; that means, the kinetic energy after interaction is going to be more than the kinetic energy that was coming in and that means, some energy has been released during the interaction. So, this implies energy released during collision this may be the case for example, 2 particles colliding and some internal energy comes out. If delta E is less than 0 energy is lost during interaction and this is usually the case which we call in elastic collisions.

In third case may be the delta E is 0 and that is an elastic collision, so we have seen what the conservation of energy is going to give us I remind you again that we have assumed the region of interaction is. So, smaller that the potential energy hardly changes and the energy between the particles is essentially that of free particle, therefore, I am writing only the kinetic energy.

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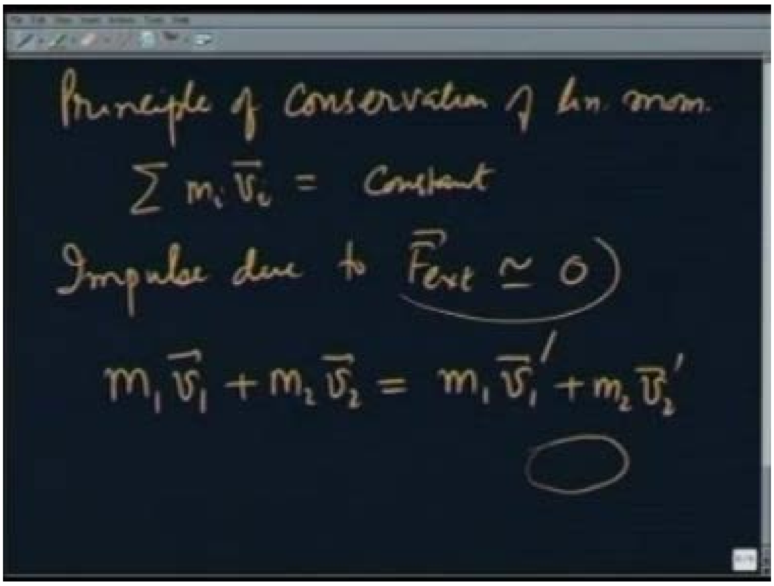

The other principle is going to be the principle of conservation of linear momentum and that tells me that summation m I v I is a constant if there is no external force. Since the interaction time is very small we neglect any external force impulse that may be acting on the particles, so we assume that impulse due to F external is essentially 0. Therefore, the only force that acts during that interaction time is that of the particles applying each other on each other and therefore I am going to have m 1 v 1 plus m 2 v 2 is equal to m 1 v 1 prime plus m 2 v 2 prime.

If there is external force I can always take care of that by adding terms here or for simplicity, we are ignoring it right now and assuming that the inter particles forces really much larger than this. So, we are taking the simplest possible scenario of 2 particles collisions and getting two conservation principles that tell us that the energy is conserved.

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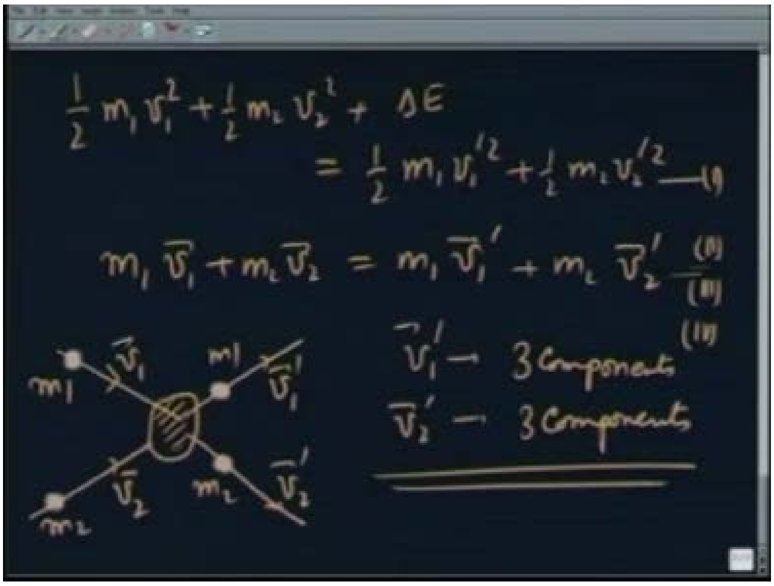

Therefore, this Equation must hold one half m 1 v 1 prime square plus one half m 2 v 2 prime square and the other equation that should hold is m 1 v 1 plus m 2 v 2 is equal to m 1 v 1 prime plus m 2 v 2 prime. In case you lost track of what is going on, so far let me remind you 2 particles are coming in interacting in a very short for short time in very small region and going out. This particle as mass m 1 this particle as mass m 2 coming with velocity v 2 coming in is velocity v 1 m 1 goes out with v 1 prime and m 2 goes out with v 2 prime.

This is one equation and for each component I can write 1 equation in this, so this is in fact, equation number 2 equation number three and equation number 4. Therefore I have four equations from which to extract information about the motion, the number of unknowns that I want to know after interaction is taken places v 1 prime three components and v 2 prime three components. Therefore total numbers of unknowns that are involved in this are 6, where as the number of equations that arise out of conversation principles are only 4.

Therefore, using conversation principles I cannot get the motion fully or I cannot get all the six unknowns fully for that I really have to solve the equation of motion. However, now I am going to show you that these principles put constraints on motion and therefore are useful to tell about what can happen what may not happen.

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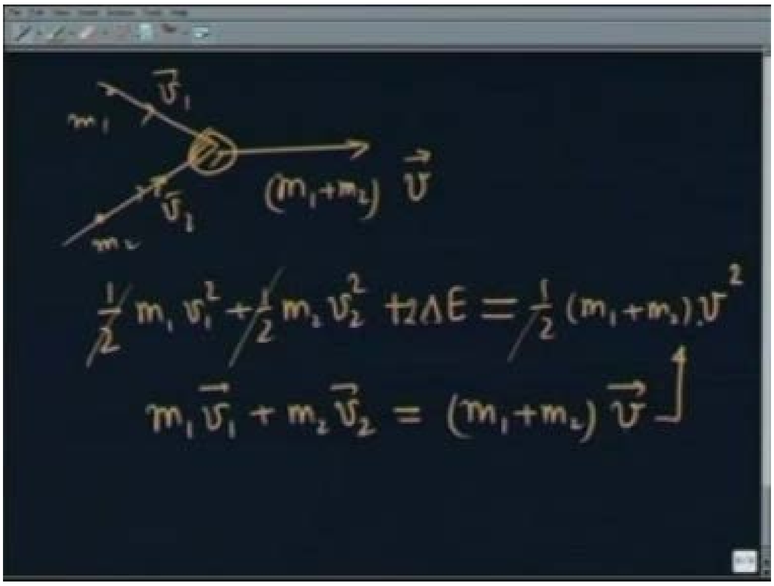

So, let us look at an interesting part case where 2 particles come in velocity v 1 m 2 with velocity v 2 get stuck together. So, that goes a particle of mass m 1 plus m 2 with some velocity v then energy conservation principle tells me that one half m 1 v 1 square plus one half m 2 v 2 square plus delta E. This is the energy released or lost during this interaction must be equal to one half m 1 plus m 2 v square and momentum conservation tells me that m 1 v 1 plus m 2 v 2 must be equal to m 1 plus m 2 v. Let us see by using this 2 conservation principles what can I learn about the motion or the restrictions on the motion from this equation. I am going to substitute here before that let me cancel this one half one half one half put a 2 here.

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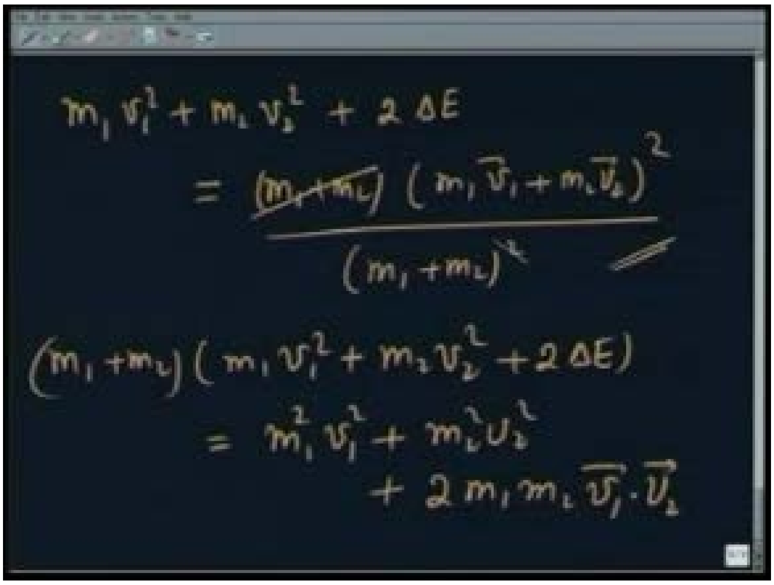

Now, substituting this v here I get M 1 v 1 square plus m 2 v 2 square plus 2 delta E is equal m 1 plus m 2 and I substitute for v which is m 1 v 1 plus m 2 v 2 that vectors here square divided by m 1 plus m 2 square, which is equal to I can take away this power. Therefore, I get m 1 plus m 2 m 1 v 1 square plus m 2 v 2 square plus 2 delta E is equal to m 1 v 1 square plus m 2 square v square plus 2 m 1 m 2 v 1 dot v 2. In case you gone to last, let me remind you what I did I wrote the energy conservation principle I wrote the linear momentum conversation principle. From the linear momentum conversation principle, I substituted for the final velocity in the energy conservation principle and I am just rearranging the terms collecting.

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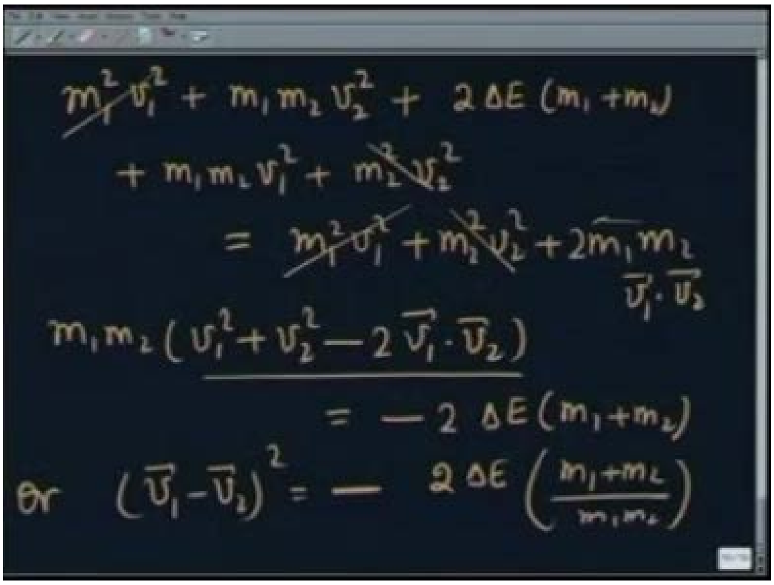

Then, I am going to get m 1 square v 1 square plus m 1 m 2 v 2 square plus 2 delta E m 1 plus m 2 plus m 1 m 2 v 1 square plus m 2 square v 2 square is equal to m 1 square v 1 square plus m 2 square v 2 square plus 2 m 1 m 2 v 1 dot v 2. This fellow cancels with this, this fellow cancels with this and I get the rearranging terms I will take it to the left hand side I will get m 1 m 2 v 1 square plus v 2 square minus 2 v 1 dot v 2 is equal to minus 2 delta E m 1 plus m 2. I can write this as v 1 minus v 2 square and therefore get v 1 minus v 2 square is equal to minus 2 delta E m 1 plus m 2 over m 1 m 2, this is a very important result and now we are going to see what all teaches us about the motion.

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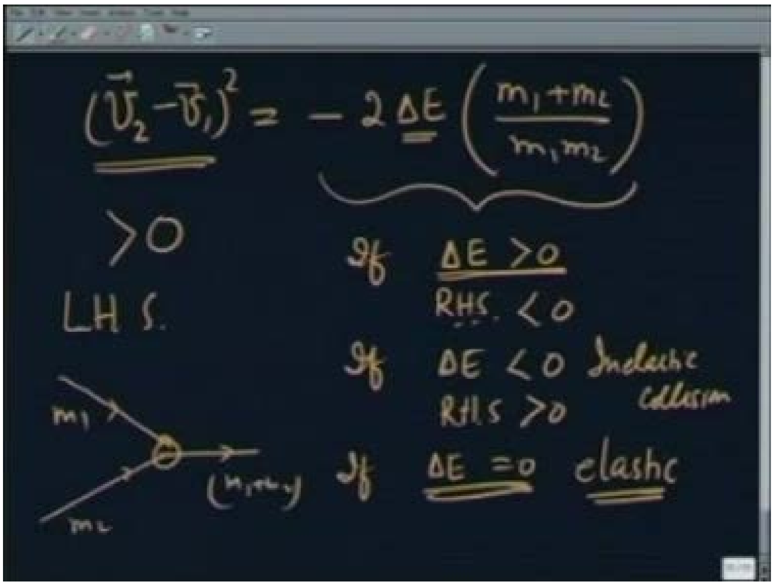

Let me write this again on the next slide what I have obtained by combining the 2 conservation principles v 2 minus v 1 or v 1 minus 2 square is equal to minus 2 delta E m 1 plus m 2 over m 1 m 2. Notice that the left hand side is a perfect square, so it is always greater than 0 on the other hand right hand side, so, this is a left hand side the right hand side could be greater than 0 zero or less than 0 depending on the sign of delta E. If delta E is greater than 0, then the right hand side less than 0 and therefore the left hand side and the right hand side can never be equal if this conditions is satisfied.

What does this mean recall again what are we doing two particles coming in interacting and going out as 1 particle m 2 m 1 m 1 plus m 2 if during the interaction sum energy is released. That means delta E is greater than 0 I can never satisfy this equation and this interaction will never take place. So, if 2 particles during interaction release energy they will never get stuck together and go out as 1 particle. For example if 2 atoms collide and make a molecule, they will nerve do it in free space during their forming of formation of that molecule some energy is released.

On the other hand, if delta E is less than 0, then right hand side is greater than the 0; that means, during the interaction sum energy is released sum energy is lost. That is an inelastic collision and this does take place as you have seen your eleventh and twelfth grade also. Third case delta E is equal to 0 that interaction is also not possible because the right hand side is 0 and the left hand side is greater than 0.

So, in this is an elastic collision in elastic collision 2 particles will never really get stuck together through the example that we just solved. We saw how the combination of 2 conservation principles tells me whether a particular reaction or a particular collision is allowed or whether it takes place or not.

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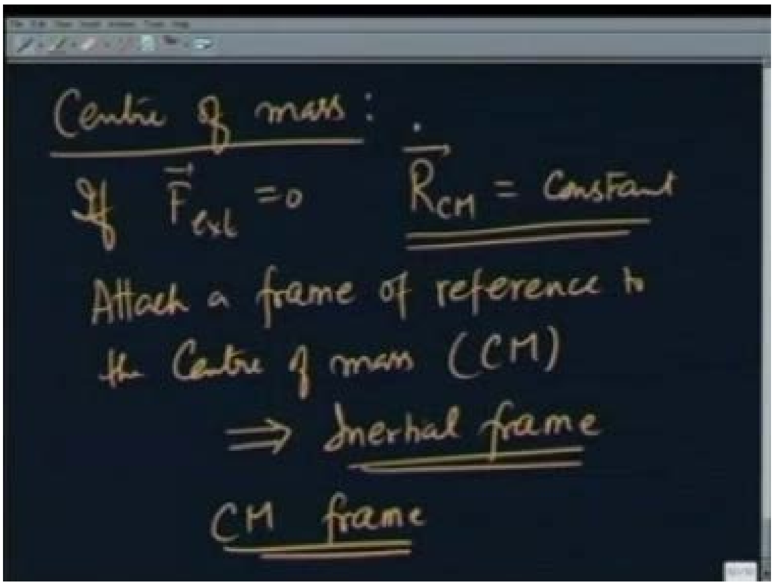

The other concept that we have introduced in the past two lectures is that of center of mass and in the coming few minutes, I want to spend some time on this to show how important a concept it is and how it makes solving problems easier times. As I told you earlier, if there is no external force, then the center of mass moves with a constant velocity. If it is moving with a constant velocity, then I attach a frame of reference to the center of mass.

So, let us attach a frame of reference to the center of mass or in short CM that frame is also going to move with a constant velocity and therefore that is going to be an inertial frame. If it is an inertial frame, Newton’s second law is valid there also and I can equally will describe the motion of particles in this inertial frame attached to the center of mass which I am going to now call the CM or the center of mass frame, why would I want to do that?

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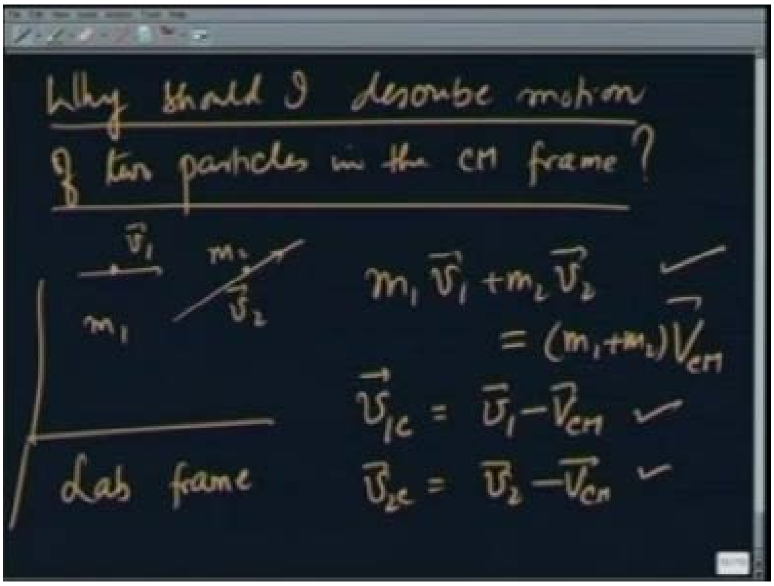

So, why should I describe motion and in this case since I am taking example of 2 particle system. So, let me write motion of 2 particles in the CM frame, why should I do that recall that by definition CM frame is such that the total momentum of particles in the frame is 0. Again, going back to 2 particles m 1 moving with v 1 m 2 moving with v 2 in the frame attached to the ground which I am going to call lab frame lab because usually collision experiments are done in laboratory.

Therefore, I am going to call this lab frame I have m 1 v 1 plus m 2 v 2 is equal to m 1 plus m 2 VCM by definition. On the other hand, if I go to center of mass frame the velocity in the center of mass frame, which I am going to refer to as v 1 C is going to be v 1 minus VCM and v 2 c is going to be v 2 minus VCM combining this equation this equation.

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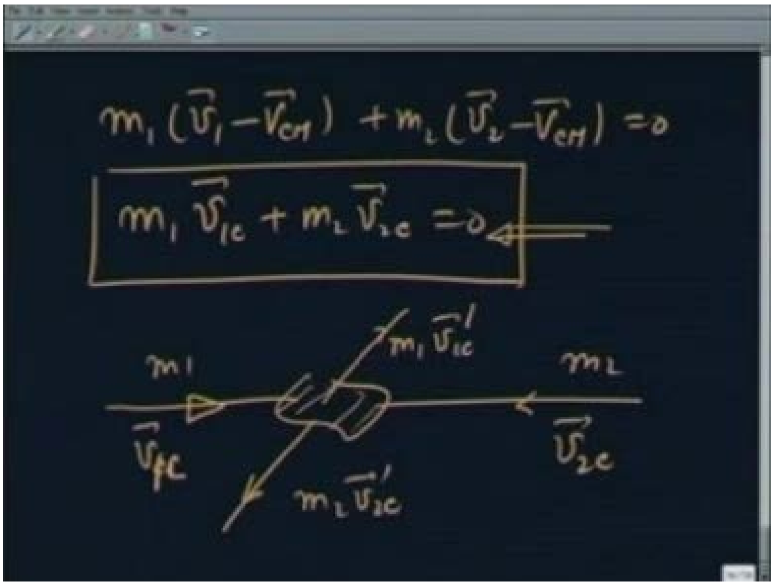

This equation you can see right away that in the center of mass I am going to have m 1 v 1 minus VCM plus m 2 v 2 minus VCM is equal to 0 or m 1 v 1 c plus m 2 v 2 c is equal to 0, the net momentum in the center of mass frame is always 0. That is understandable by definition if I am sitting on the center of mass frame the velocity is as such that the net momentum has to be 0. Therefore, if the particle is colliding if one particle in the center of mass frame is moving like this the other 1 is going to move like this after collision also since the net momentum is 0.

No matter what they do, whether energy is lost energy is gain whatever they will again either go in opposite direction or move towards each other. So, this was m 1 CM frame it was coming in with velocity v 1 c m 2 coning in velocity v 2 c. So, the net momentum is 0 out goes this with m 1 v 1 c prime m 2 with v 2 c prime again the net momentum is 0. Let us see it clearly that what happens in the lab frame and what happens in the center of mass frame.

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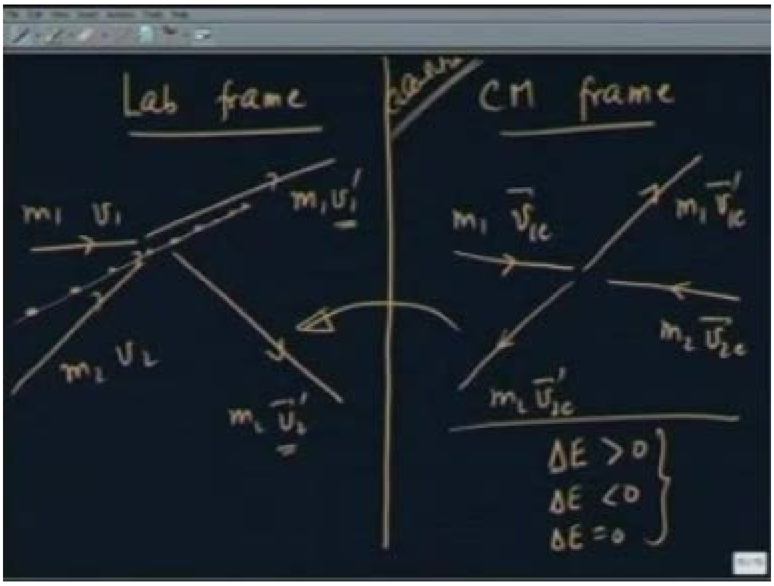

So, let me draw a line here In the lab frame that is sitting on the ground or sitting in my laboratory, where I am doing the experiment one particle comes in. The other particles comes in this is m 1 v 1 m 2 v 2 m 1 goes out m 2 goes out with different velocities v 1 prime and v 2 prime. In the center of mass, frame m 1 comes with v 1 c m 2 has to come with opposite velocity m 2 v 2 c so that the net momentum is 0 and m 1 goes out with v 1 c prime m 2 has to go the other way m tow v 2 c prime. The center of mass off course it was moving like this will keep on moving as it is, so let us say this was the center of mass this during entire interaction will keep on moving like this.

So, no matter what is happening delta E as we introduced earlier may be greater than 0, delta E may be less than 0 or delta E may be equal to 0. This picture will always remain the particles will always move in opposite directions and that simplifies the description of motion. Of course finally, to compare with the result of experiments, I have to transform back form center of mass to my lab frame. So, this is good for calculations and this is a comparison the lab frame is a comparison with experiments, so I should also be going back and forth between the 2 systems I should known how to do that.

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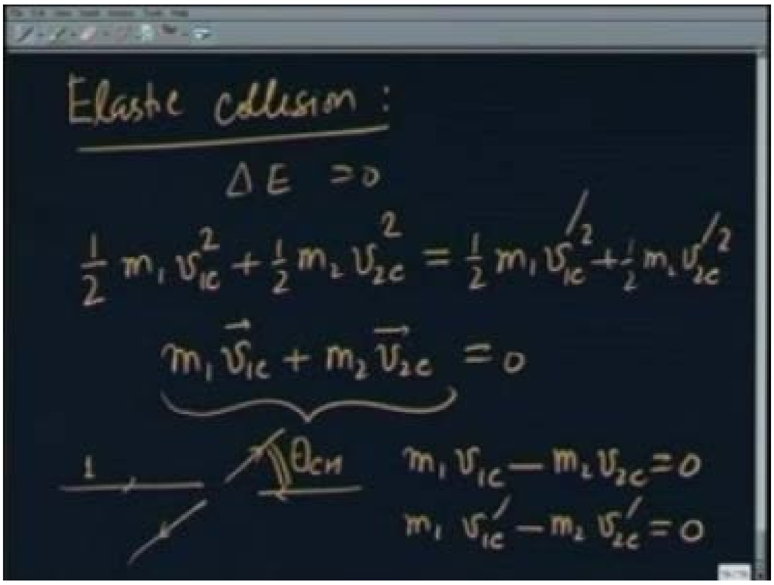

To start with, I will take a simple example of elastic collision Again between two particles and describe it in the center of mass and also see how to go from center of mass to the lab frame. We call that in an elastic collision delta E is equal to 0 and therefore, in the center of mass frame I am going to have half m 1 v 1 c square plus one half m 2 v 2 c square is equal to one half m 1 v 1 c prime square plus one half m 2 v 2 c prime square. Similarly, I am going to have m 1 v 1 c plus m 2 v 2 c is equal to 0 I recall in center of mass frame see this is 0 the velocities have to opposite to each other. So, I can write this equation in a simpler form as m 1 v 1 c minus m 2 v 2 c is equal to 0.

After the collision the particles go out, again directions opposite to each other I am going to have m 1 v 1 c prime minus m 2 v 2 c prime is equal to 0. You may feel there are more variables in this than the equations or they there are more equations here than the number of variables where that is not true because in this frame I do I am not telling you what this angle of the scattering is. For example, the particle one comes in like this, it is getting scattered by this angle which I will call theta CM we are not specifying that I cannot really solve for that unless I know the equation of motion.

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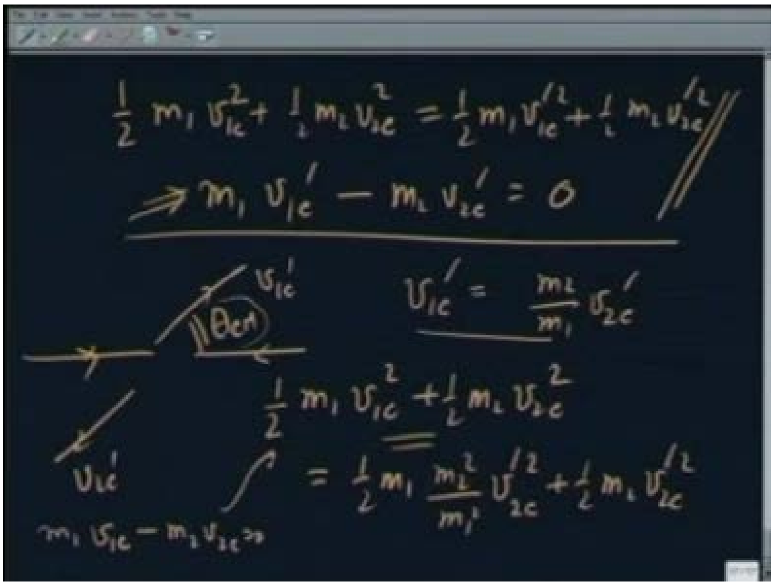

Let us see what these three equations tell me, so I have one half m 1 v 1 square c square plus one half m 2 v 2 c square is equal to one half m 1 v 1 c prime square plus one half m 2 v 2 c prime square. Other equation is m 1 v 1 c prime minus m 2 v 2 c prime is equal to 0 as I told you I have v 1 c prime v 2 c prime. The only thing I can get from these equations then is v 1 c prime and v 2 c prime I can not specify this scattering angle. This is constant with what we learnt earlier that conservation laws can give you only some information about the motion, but not all if I want to get this theta CM, I really solve the equation of motion.

I must know in detail what the interaction is, but going back to the equations of conservation applying these two together, I get v 1 c prime is equal to m 2 upon m 1 v 2 c prime. Let me substitute this in the energy conservation equation to get one half m 1 v 1 c square plus one half m 2 v 2 c square is equal to one half m 1 v 1 c prime, I will substitute this I get m 2 square over m 1 square v 2 c prime square plus one half m 2 v 2 c prime square. How about the left hand side, I can use the same trick on the left hand side to get v 1 c in terms of v 2 c recall that like this I also had like this equation. The momentum conservation equation I also had m 1 v 1 c I am writing on the left bottom corner minus m 2 v 2 c is equal to 0.

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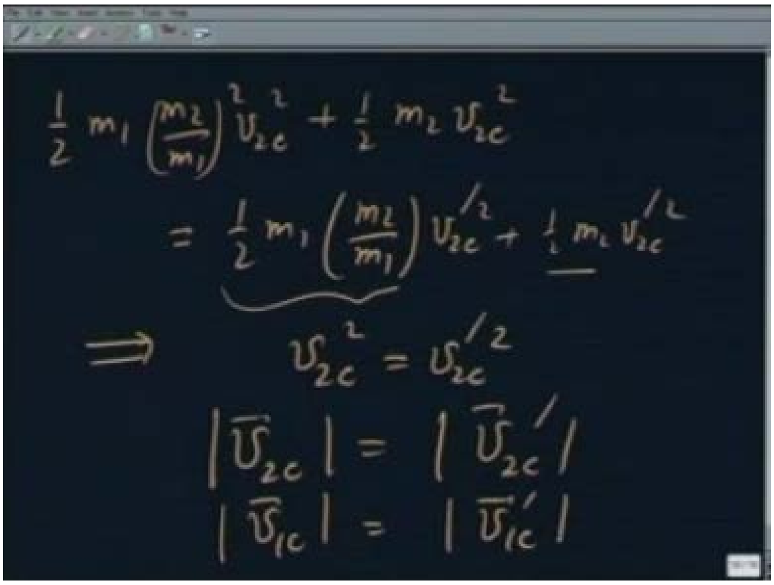

So, I can make this substitution here to finally get one half m 1 m 2 over m 1 square v 2 c square plus one half m 2 v 2 c square is equal to one half m 1 m 2 over m 1 v 2 c prime square plus one half m 2 v 2 c prime square. Now, you see the common factor this and this together are the same and this implies v 2 c is equal to v 2 c prime square or the magnitude of v 2 c remains the same during the collision. If this is the case, then obviously, v 1 c also remains the same as v the particles collided go out.

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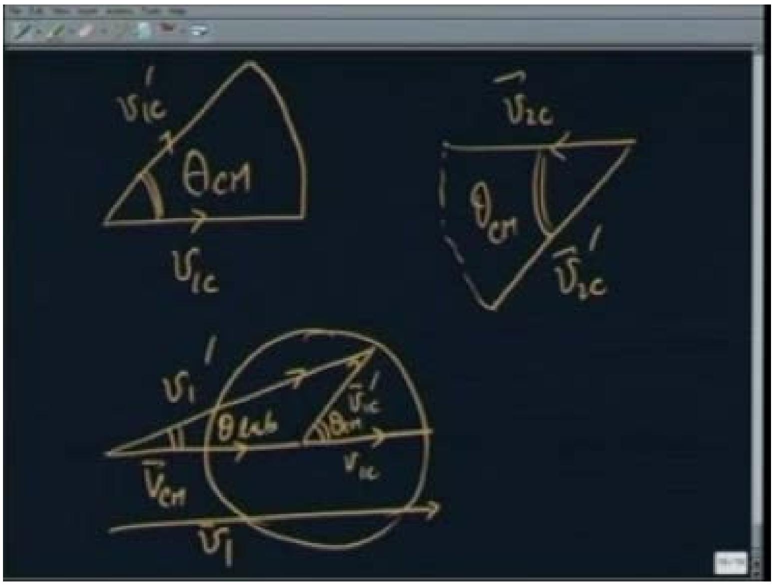

Graphically, what is happening is this particle is coming in let us say with v 1 c all that happens during the collision is this vector this length remains the same just rotates by angle theta CM. I said earlier I cannot really not calculate theta CM using conservation principles similarly, v 2 c this is v 1 c prime v 2 c comes in and all that happens during the collision is this also rotates by an angle theta CM. So, this is v 2 c v 2 c prime as simple as that off course if I want to compare the results that I get with the lab results I have to transform back to the lab that will see in a minute.

This is so simple, so if I make a circle this was v 1 c and this is what happen to it after the collision suppose this a centre of mass velocity was. Then, from here to here was v 1 in the lab frame and after the collision VCM plus v 1 c prime is going to be the velocity in the lab frame and therefore this is going to be v 1 prime this was theta CM and this is theta lab.

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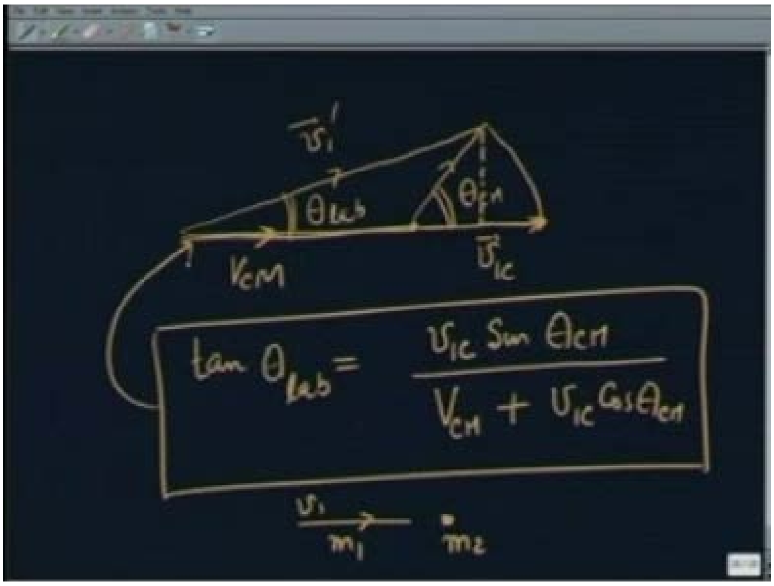

So, looking at the motion and center of mass frame I find that if this was V CM and in the center of mass frame this was v 1 c all that happens during the collision with particle 2 is that this particle moves this direction this direction of motion rotates by theta CM. So, this final velocity is v 1 prime in this direction and obviously, this particle was moving like this, so it has scattered by this much theta lab, the relationship between the two is very clear. Now, I have tangent of theta lab is equal to if I drop a perpendicular here it will be v 1 c magnitude sin of theta CM divided by VCM plus v 1 c cosine of theta CM.

Notice in this case, I have been making VCM and v 1 c in the same direction that is the assumption this need not be the case always this. However, is the case if I take mass 2 to be stationary and mass 1 to be coming in with some velocity v 1, so this formula is correct in this case when VCM and v 1 c are in the same direction.

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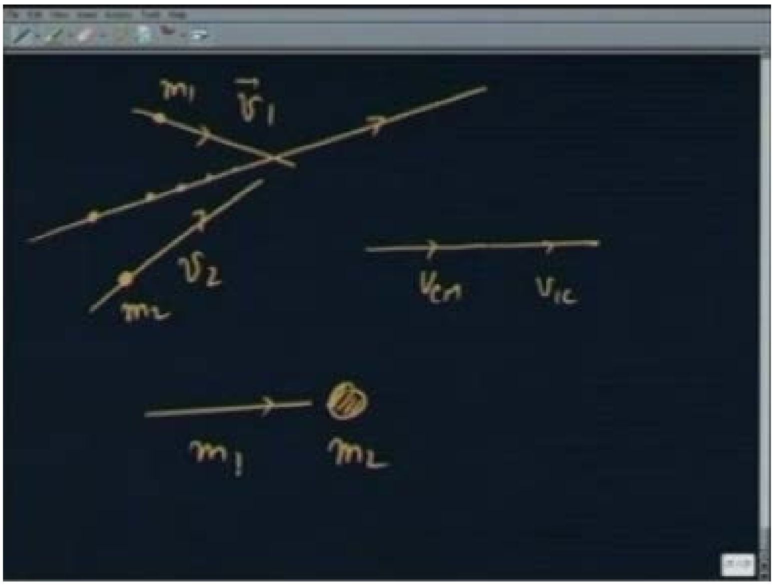

On the other hand, if I have one mass coming in like this in the lab frame, the other mass going like this in the lab frames this is m 2 this is m 1. Then, V CM would be moving would be something like this mass center of mass would be moving in this direction, they will not be co directional. The relationship between theta lab and theta CM would be slightly more complicated. So, the simplest case that we have considered is when VCM and v 1 c are in the same direction. Therefore, this I said is case where mass m 2 is stationary and mass m 1 is moving in, let us see what we learnt from this relationship.

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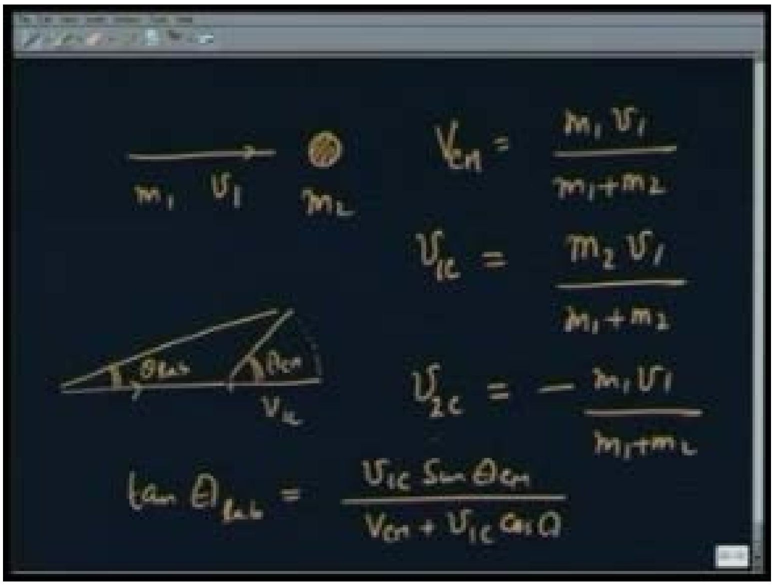

So, we specialize through a case where mass m 2 is sitting mass m 1 is coming with the same velocity v 1 then VCM by definition would be m 1 v 1 over m 1 plus m 2 v 1 c would be equal to v 1 minus VCM. Therefore, m 2 v 1 over m 1 plus m 2 and v 2 c would be equal to the since mass 2 is stationary to start with minus m 1 v 1 over m 1 plus m 2 you can see that m 1 v 1 c plus m 2 v 2 c is equal to 0. So, this is VCM v 1 c and this just rotates in elastic collision by theta CM, therefore this is theta lab I have tangent of theta lab equal to v 1 c sin theta CM divided by VCM plus v 1 c cosine of theta CM.

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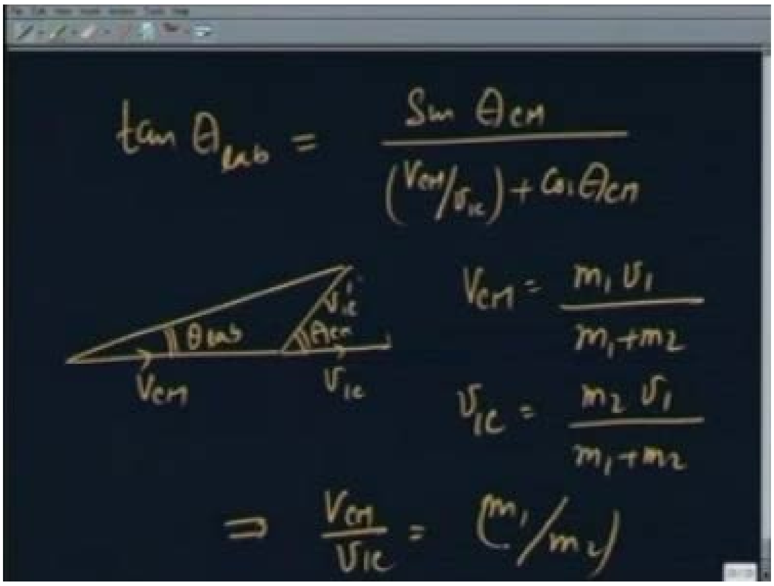

Then, tangent of theta lab is equal to sin of theta CM divided by VCM over v 1 c plus cosine of theta CM. Let us see picture again this was VCM v 1 c v 1 c prime theta CM and this is theta lab. Now, we said when v 2 or the mass 2 was stationary VCM was equal to m 1 v 1 over m 1 plus m 2 and v 1 c is equal to m 2 v 1 over m 1 plus m 2 and therefore, VCM over v 1 c is equal to m 1 over m 2.

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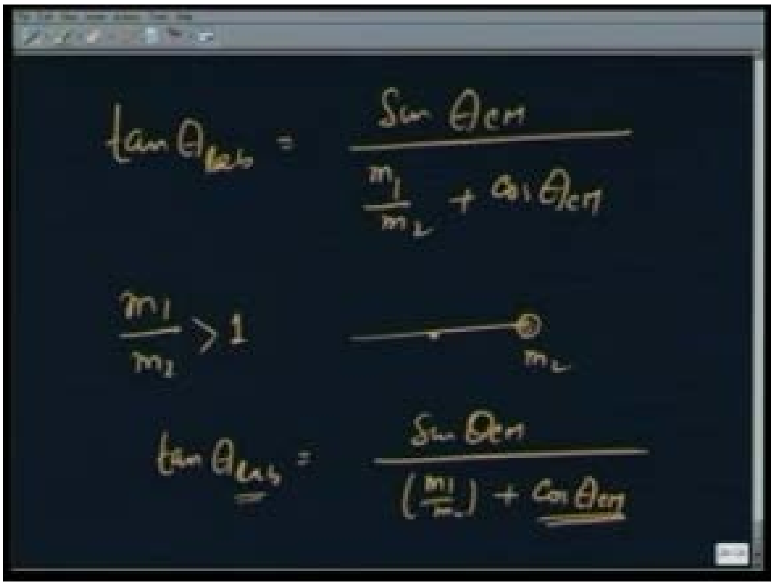

This implies that tangent of theta lab is equal to sin of theta CM over m 1 over m 2 plus cosine of theta CM. So, we applied the conservation principles in center of mass frame in this the in an elastic collision the magnitudes of velocities in the center of mass frame remain the same this exchange direction total energy is conserved. Now, let us see the case when m 1 over m 2 is greater than 1; that means, the particles which is coming in is heavier than the particle on which it is sitting. In that case, you will see that tangent of theta lab is equal to sin theta CM over sum number which is greater than 1 plus cosine of theta CM the minimum value of cosine theta CM is minus 1 and therefore theta lab is going to actually show a maximum.

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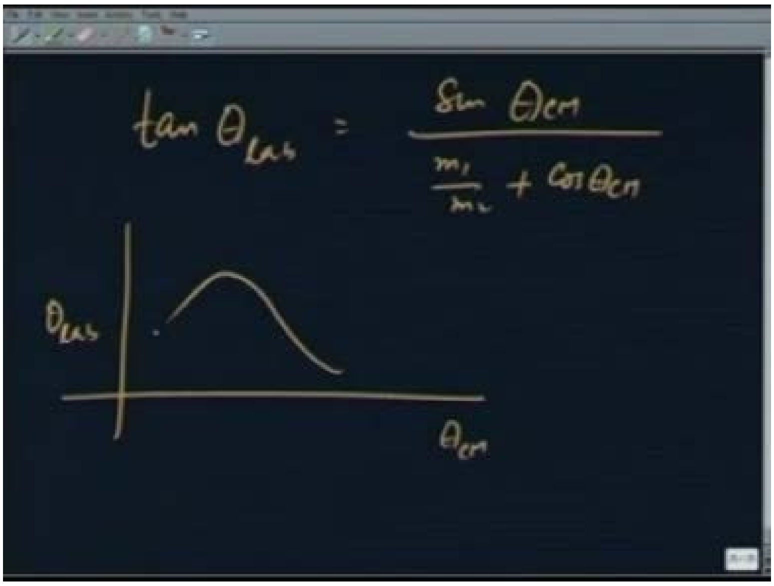

Tangent theta lab is equal to sin theta CM over m 1 over m 2 plus cosine of theta CM if I plot theta lab verses theta CM will show some sort of a maximum. Let us try to understand that graphically, but before that you try to recall from your everyday experience if you hit a very heavy ball on say marble, a kancha, you see does not really reflect back it always goes forward. There is some maximum angle through which it can get scattered on the other hand do the other way hit a kancha on a very heavy say cricket ball it can go in any direction. So, when m 1 is greater than m 2 that is I am hitting a cricket ball throwing a cricket ball on a stationary kancha, it always deflects forward there is a maximum angle.

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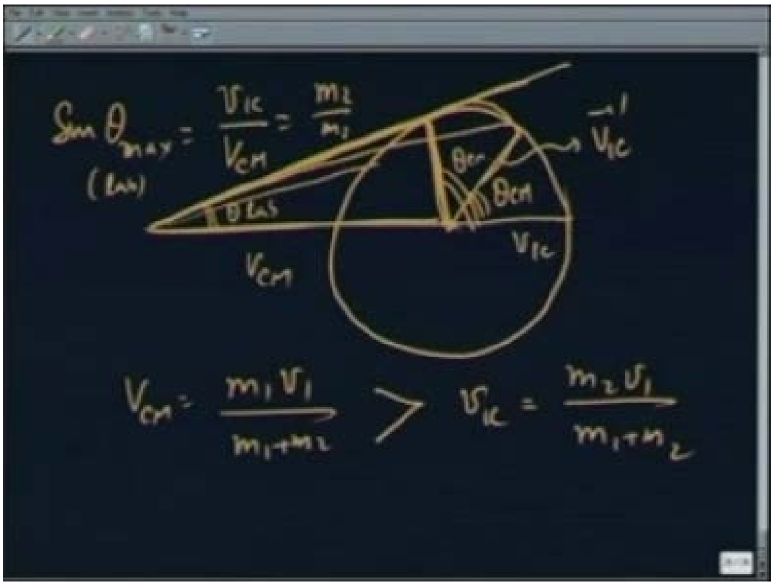

To understand this graphically, let us make V CM, we call V CM is equal to m 1 v 1 over m 1 plus m 2 and it is greater than v 1 c which is equal to m 2 v 1 over m 1 plus m 2. So, this vector is smaller and in magnitude than this an v 1 c can go in any angle this is theta CM and this is theta lab. You will see that as this vector v 1 c prime vector rotates theta lab is going to become a maximum when this touches it like this when this is theta CM, then again going to become smaller. So, we see graphically also how the angle theta lab is actually has a maximum it has a maximum when v 1 prime is as its tangent and you can see from here that for sin theta max in lab is going to be this divided by this or v 1 c over VCM which is m 2 over m 1, let me show it more clearly again in the slide.

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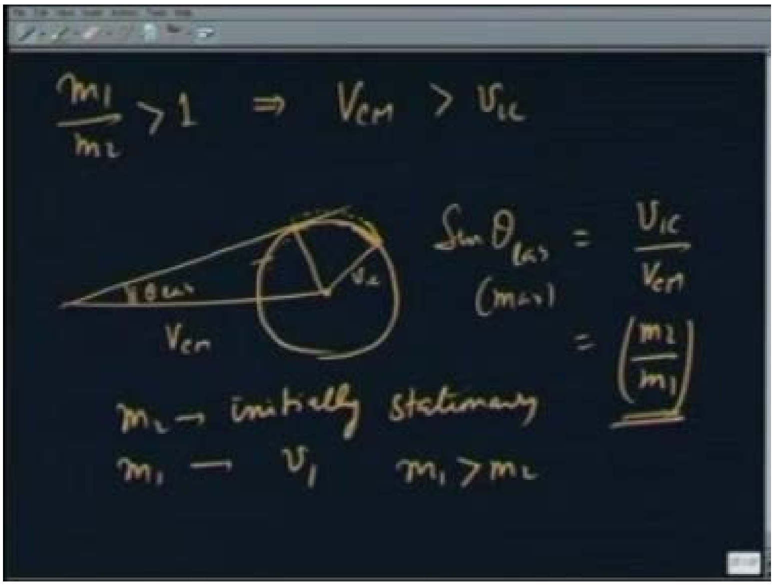

When m 1 over m 2 is greater than 1, this implies VCM is greater than v 1 c in magnitude and therefore this is V CM I draw a circle from here of radius v 1 c. That shows how much v 1 c can rotate lab angle would keep on increasing if v 1 c is here, here, here, here, here, here, here and at this point where this becomes a tangent theta lab would be a maximum and as v 1 c rotates.

Further, theta lab would become smaller as you can see, so sin of theta lab maximum is equal to v 1 c over VCM which is equal to m 2 over m 1 recall again this is a case when m 2 is initially stationary and m 1 is coming in with velocity v 1 and m 1 is greater than m 2. In that case, there is a limit of scattering angle in the lab, on the other hand, so this is the information that we obtain by applying looking at the motion in center of mass frame and then transforming to the lab frame.

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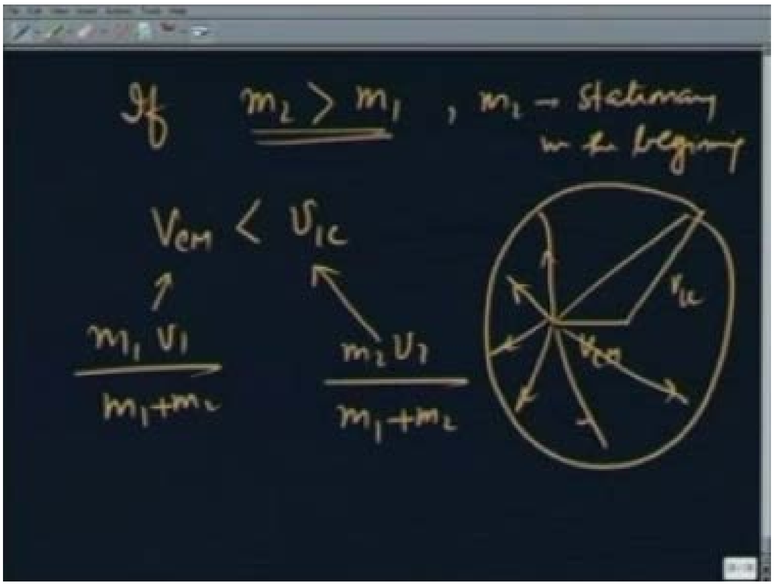

On the other hand, if m 2 is greater than m 1 and still stationary and m 2 stationary in the beginning then VCM is going to be less than v 1 c because VCM is m 1 v 1 over m 1 plus m 2 v 1 c is m 2 v 1 over m 1 plus m 2. Then, m 2 we are taking to be greater than m 1 and thus that case VCM would be this, but, v 1 c would be larger in magnitude and this when it rotates around the lab velocity can make any angle.

So, we learn that in these collision processes where m 2 is stationary m 1 comes and hits it if m 1 greater than m 2 slightly cricket ball hitting a marble. There is a maximum angle through which the cricket ball will get reflected on the other hand if it is marble hitting a stationary cricket ball marble would go at any angle as you can see from this picture, so we see that in is an elastic collision.

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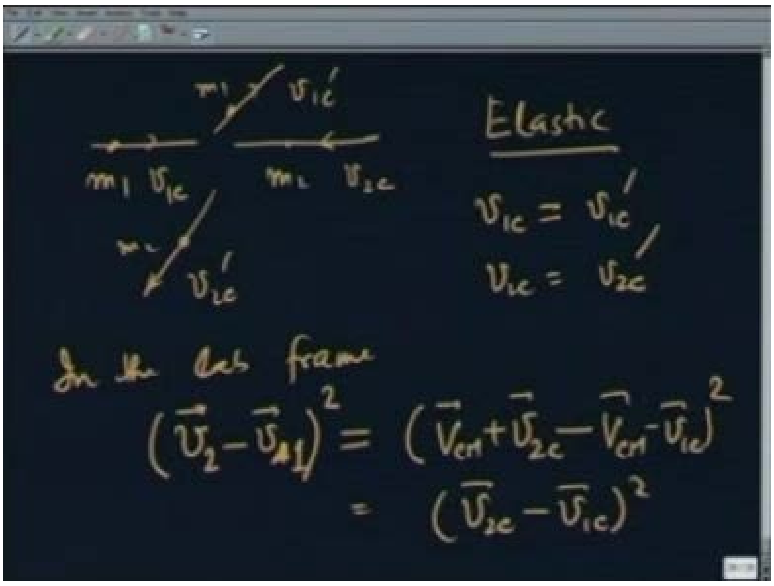

When 2 particles collide again, I am taking a general case where 1 particle is not necessarily stationary M 1 comes in with velocity v 1 c in the center of mass frame m 2 comes in with velocity v 2 c in the center of mass frame. They go out v 2 c prime v 1 c prime m 1 m 2, if the collision is elastic then v 1 c was equal to v 1 c prime and v 2 c is equal to v 2 c prime. This is a general result which we in the previous case applied to the case when 1 particle was stationary to start with a other particle hit it.

Now, let us see what the consequence of this in the lab frame the relative velocity of particle 2 with respect to particle 1 is this. Let us see what happens in an elastic collision to the square of this, I can write this as v 2 is recall VCM plus v 2 c minus v 1 is VCM plus v 1 c, so this I can write as VCM minus v 1 c which is equal to v 2 c minus v 1 c square.

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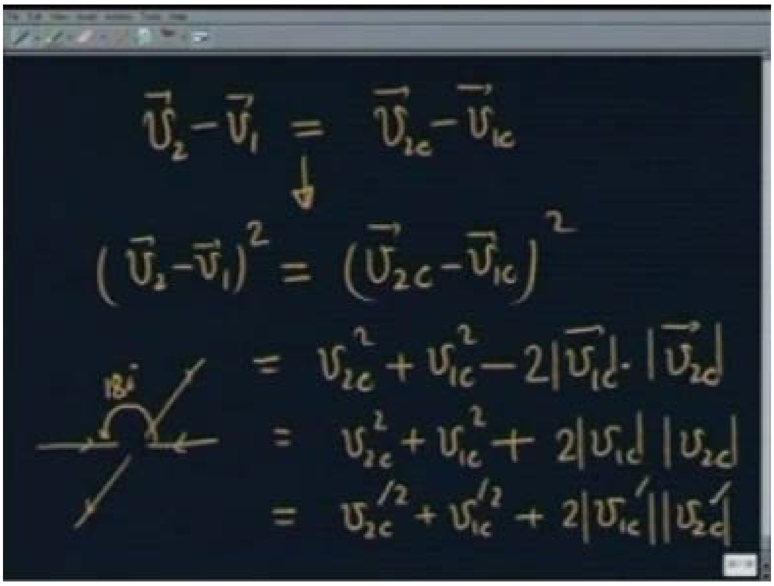

All it says is that the difference of velocities v 2 minus v 1 in which ever frame you sees it the relative velocity is the same v 2 c minus v 1 c and I am making use of that in just looking at the square v 2 minus v 1 square is equal to v 2 c minus v 1 c square. I can write as v 2 c square plus v 1 c square minus 2 v 1 c dot v 2 c, recall that in center mass frame the particles are moving opposite to each other. So, the angle between them there velocities is 180 degrees and therefore, I can write this as v 2 c square plus v 1 c square plus 2 v 1 c v 2 c.

Also, recall that when the collision is elastic the velocity is magnitudes or their squares remain unchanged and therefore, I can write this expression further as v 2 c prime square plus v 1 c prime square plus 2 v 1 c prime v 2 c prime.

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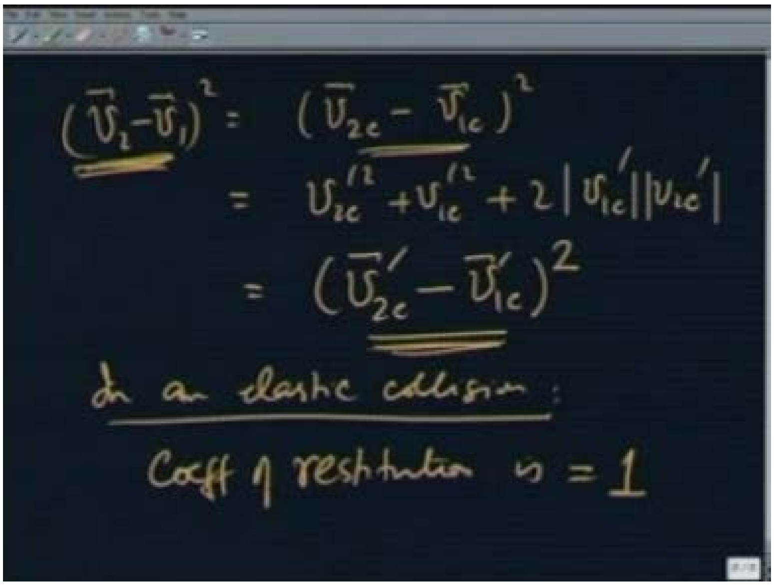

I can therefore, skip some steps and write that v 2 minus v 1 square is equal to v 2 c minus v 1 c square. This is then equal to v 2 c prime square plus v 1 c prime square plus 2 v 1 c prime v 2 c prime, which would be equal to v 2 c prime minus v 1 c prime square change this in to a dot product again using the fact that v 1 c prime and v 2 c prime are also at 180 degrees. So, what you see that in an elastic collision the relative speed square the relative the speed of approach and speed of separation remain the same which you have learned in your elementary courses.

In an elastic collision, the speed of separation is the same as speed of a approach and that is precisely what I have derived here. Again, using the center mass frame or the coefficient of restitution is equal to 1, so this is another example of how we can quickly derive these results going to center of mass frame and how we can use these principles.

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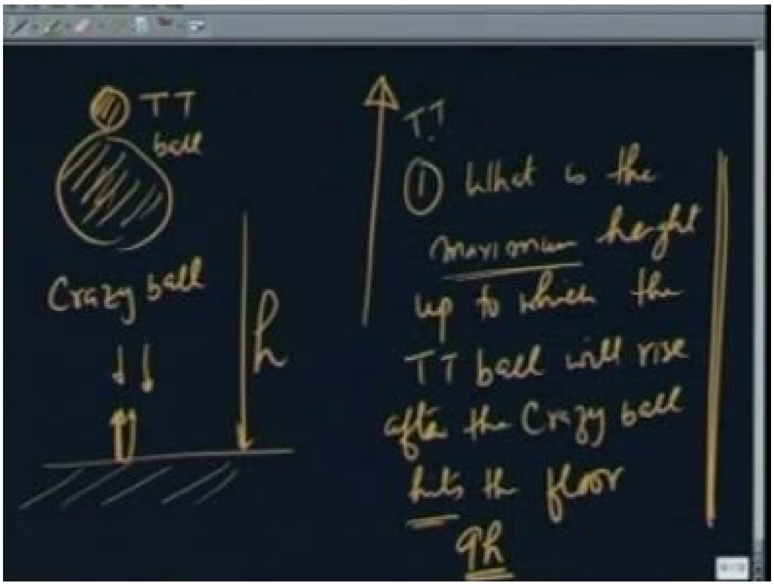

If you want to see a really effect of 2 particle collision I will give you a 1 dimensional example take a T T ball and put it on top of what they call in the market crazy ball, this ball really jumps very highs coefficient of restitution is almost 1 balance it and drop it on the ground from say a height h and observe what happens after it hits a ground. You will see that the T T ball really takes off calculate what is the maximum height up to which the T T ball will rise after the crazy ball hits the floor, perform this experiment and try to justify it using whatever you learnt.

So far, assume all the collisions to be elastic and that will give the maximum possible height also when while solving the problem assume that T T ball and crazy ball hit when they are going against each other. By the time the lower crazy ball hits the floor and goes up the T T ball is still moving down, I will give you the answer the answer comes out to be nine times height form which you drop it.

So, if you drop it from a 1 meter, will really hit the ceiling do this experiment and see and try to see if you get this result theoretically from whatever you have learnt. So, you should be able to do so, having done the examples of elastic collisions, let us now ask what happens when inelastic collision between 2 particles are going to take place.

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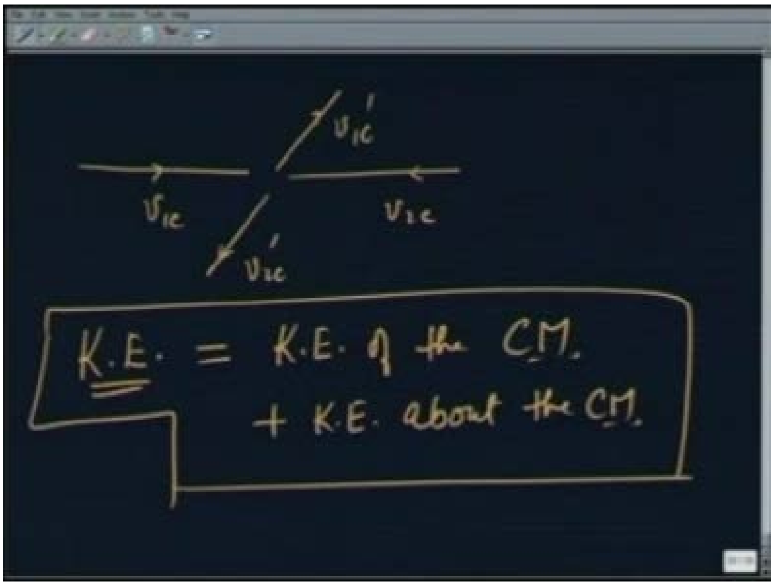

For that, what will happen is 2 particles which are coming in the center of mass frame with v 1 c v 2 c since the momentum is still must remains 0, when they go out after collision v 1 c prime and v 2 c prime would still be in opposite direction. Their magnitudes may become smaller how much energy loss takes place how much is the maximum energy that can be lost. Those are questions that are raised in an inelastic collision, so let us try to answer that for that I am going to use a result for the kinetic energy of these particles.

I am going to show that kinetic energy of a system of particles is equal to kinetic energy of the center of mass plus kinetic energy about the center of mass. We are going to use this result here to see how much is the maximum energy loss that can take place during an inelastic collision, but this result by itself is of great importance and we will use it quite lot later when we describe rotational motion of extended bodies.

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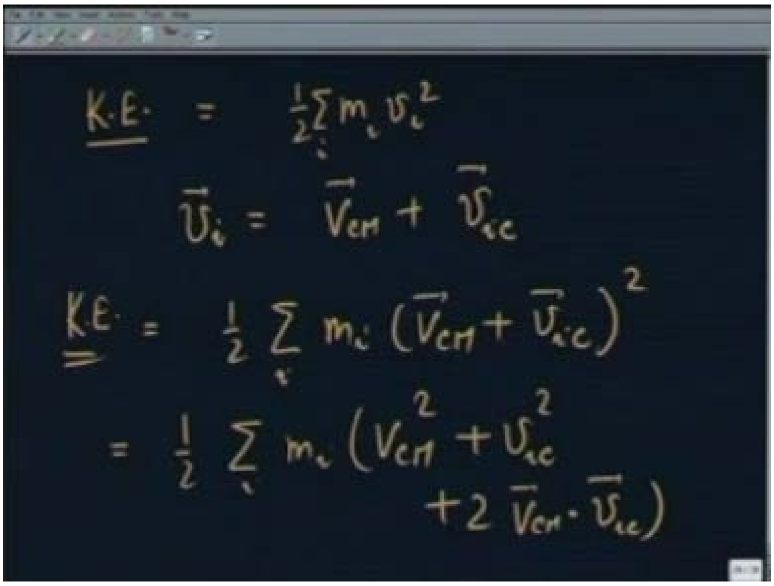

To see this, let us write kinetic energy of a system particles is nothing but one half m i v I squares summed over i. Let me now use a fact that v i is nothing, but, v of the center of mass plus v i c that is velocity in the center of mass frame. Therefore, the kinetic energy can be written as one half summation I m I VCM plus v I c whole square and this comes to be one half summation I m I VCM square plus v I c square plus 2 VCM dot v I c.

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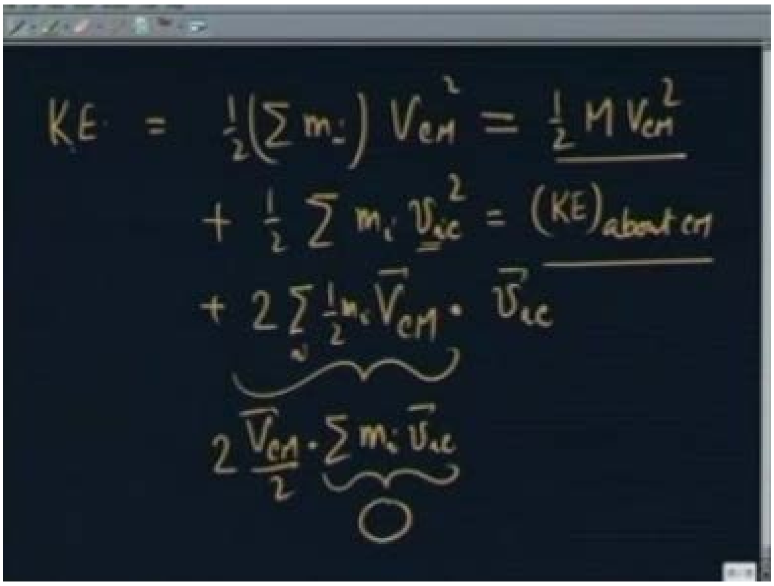

Let us write these three terms separately to get kinetic energy is equal to one half summation m i V CM square plus summation over one half summation m I v I c square plus one half V CM dot v I c and there is an m I here and summed over i. This term I can write as one half V CM is a constant that has no index i, so summation m i's total mass VCM square.

This term I can write as K E about CM because v I c measures the velocity about the center of mass and for this term VCM I can again take out write this as VCM over 2 summation m I v I c dotted here. However, this is the momentum in the center of mass frame and therefore, 0 and therefore the only two terms which are left are this.

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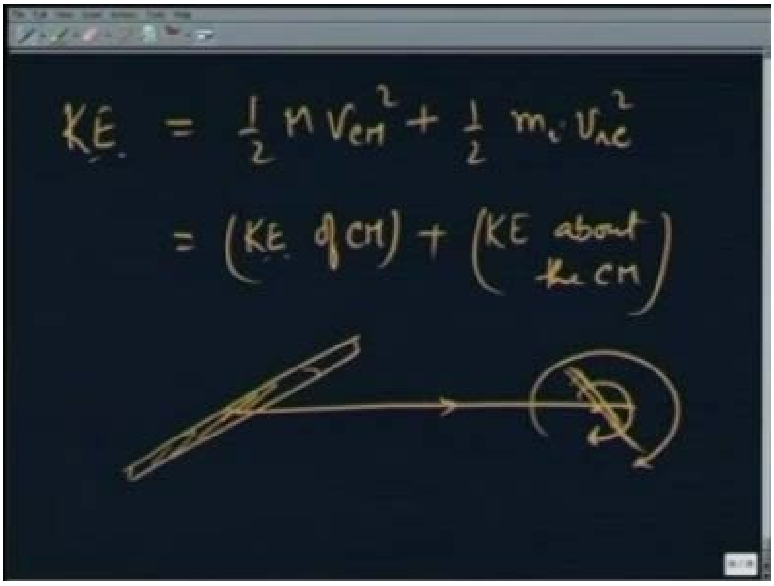

I can write the kinetic energy of the system as one half m V CM square plus one half m I v I c square the third term gave you 0 which is nothing but K E of center of mass plus KE about the center of mass. This is the beauty of center of mass later when we do rotational dimension, you will see that I can do a similar composition for the angular momentum also.

To see an example of this let us take a rotating rod, suppose I throw this, so the center of mass is moving in 1 direction if it is force free motion, the rod is rotating the next time the orientation could be this. Then, I could write its kinetic energy as the kinetic energy of a center of mass plus the rotational energy about rotational kinetic energy about the center of mass, we will be making use this of this again in discussing rotational dynamics.

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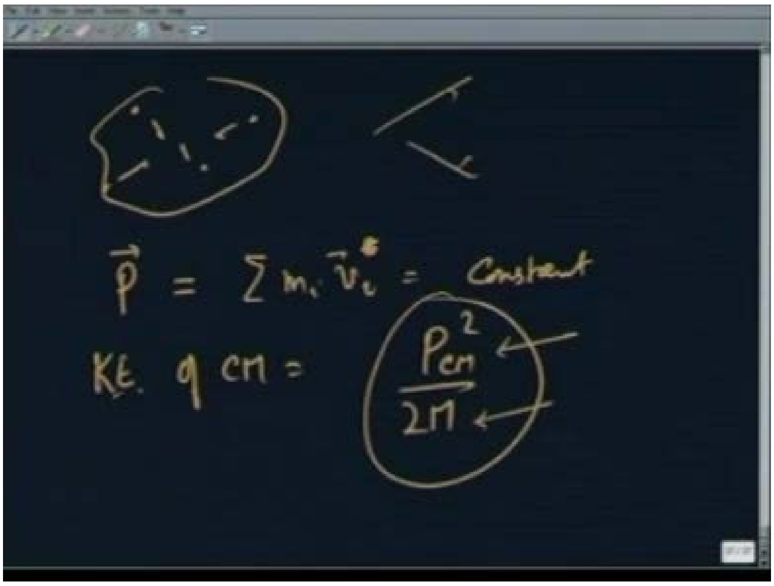

What is here is that now if two particles or more are interacting, colliding and then going out I should have the net momentum p always conserved. So, that is going to be summation m I v I which is a constant the K E of center of mass, I can write as PCM square over two total mass since PCM is constant mass is a constant. This term would always remain there no matter what whether energy is lost or whatever the kinetic energy center of mass is going to be P CM square over 2 m.

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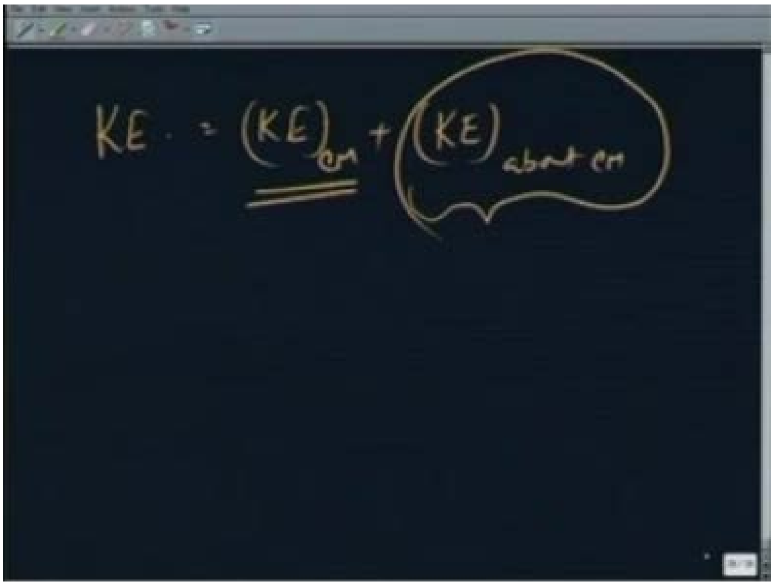

There is a total kinetic energy is K E CM plus K E about CM this must always remain there because the momentum conservation. So, the maximum energy that I can loose in an interaction is K E about c this is another result of importance shows us how going to center of mass sometimes helps. In fact, lot of time it helps and we will be making use of it again and again to conclude in this lecture I have taken some examples to show you the importance of application of conservation principles.

How they can give us some information about the motion when particles interact although the full information has to be obtained only through the solving of the equation of motion lot of important restrictions can be derived based on conservation principles.