Engineering Mechanics
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Module - 05
Lecture - 04
Motion of Particles with Drag

In the previous lecture, we saw how two solids when they come in contact apply frictional force on each other, and how that affects their motion. In this lecture we are going to look at another kind of frictional force, when a body moves through a fluid or gas. You are well familiar from Stokes's law that.

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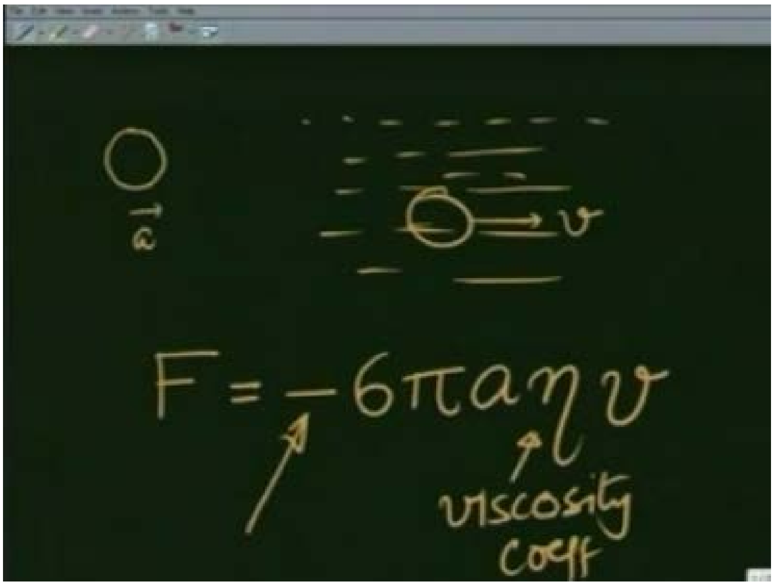

When a spherical body of radius a, moves through a gas or fluid at speed v, it is moving with speed v then, there is a force on a due to viscosity which is F equals minus six pi a eta v. This minus sign denotes that, the force is opposite to its direction of motion, eta is the viscosity coefficient. This is 1 example of the drag force that anybody moving through a fluid or gas experiences. Of course, this is the lowest order force.

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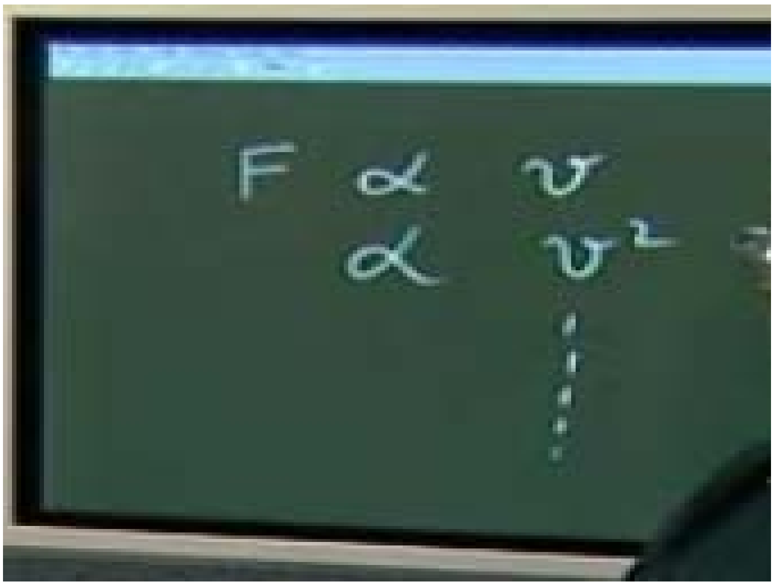

We may have forces which are proportional to v, which are proportional to v square or higher powers. These forces oppose the motion, and in this lecture we are going to look at how to deal with these forces when they are included in the motion of a particle.

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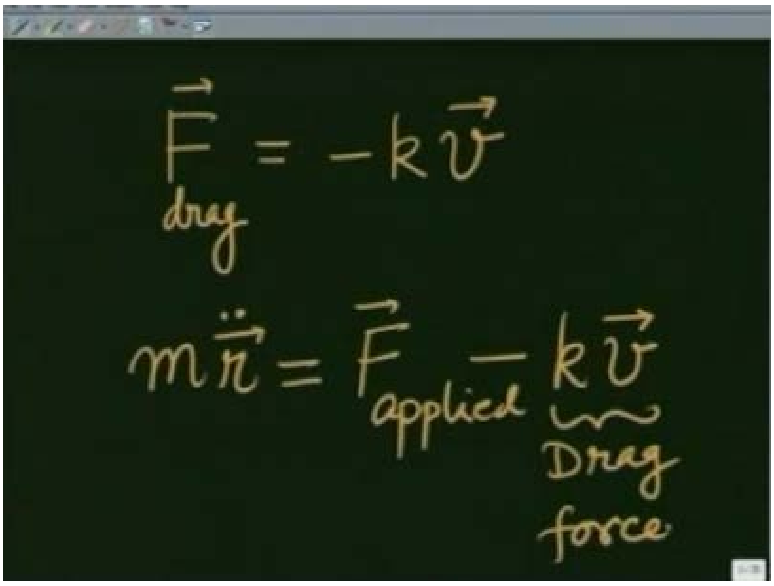

The simplest one as I said earlier is the force which is proportional to v, and I am going to write this as F equals minus kv. And I am going to call it drag force, k is the coefficient of drag, minus sign again denotes that, the force is opposite to the motion.

And therefore, the equation of motion is going to be mr double dot, mass times acceleration equals F applied that, the force that I apply from outside, minus kv, which is the drag force.

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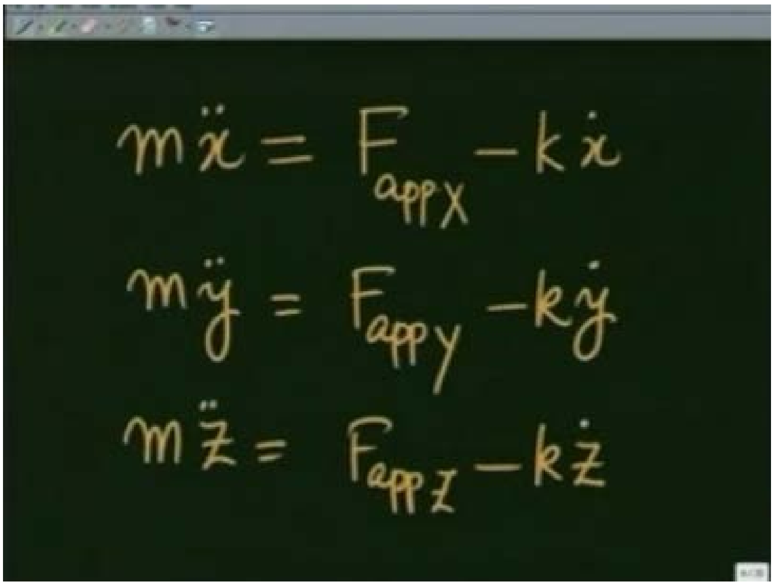

If I write these in component form I get mx double dot equals F applied x component, minus kx dot, my double dot equals F applied y component, minus ky dot and similarly, mz double dot equals F applied z, minus kz double dot. This is the simplest of drag forces and in the next few examples, we are going to see how to incorporate this in the motion.

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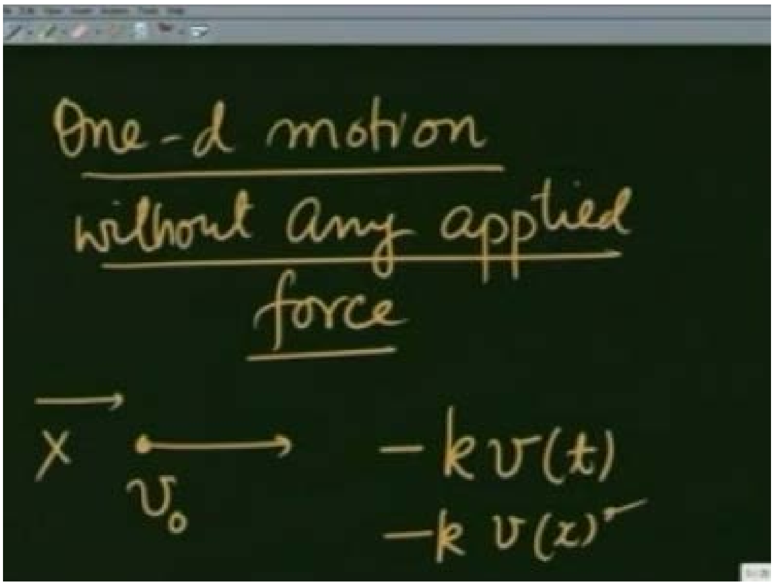

The first example I am going to take is going to be 1 dimensional motion without any applied force. There is no applied force, but drag is there. Therefore, initially I have to throw the particle with some initial speed v 0, and as it moves along it experiences a force which is minus kv, v is changing with time or I can also write v is changing with the distance. Let this direction be x direction that is I have written x here.

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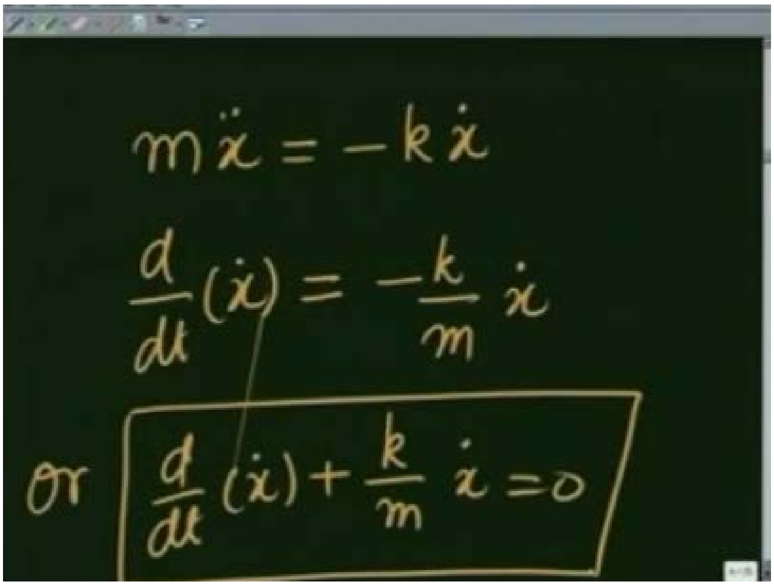

The equation of motion for this is going to be mx double dot equals, minus kx dot. In more familiar terms, it is going to be d over dt of x dot which is nothing but x double dot is equal to minus k over mx dot or d over dt of x dot, plus k over mx dot equal to 0. This is my equation of motion that, I want to solve for this particle. You can check by direct substitution that the solution is going to be.

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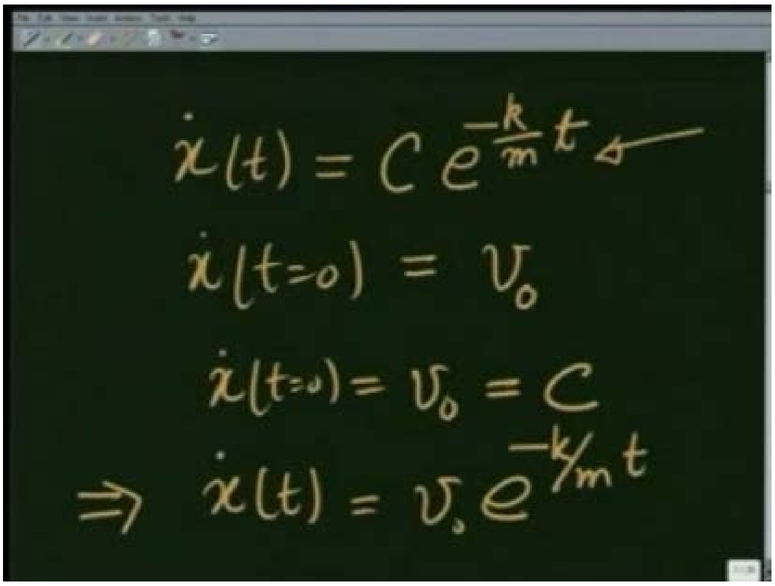

x dot as a function of time equal to some constant C e raise to minus k over m t, what is C? Let us write x dot at t is equal to 0, that is initial time, through the particle with speed v 0. And therefore, I must have x dot at t equal to 0, is equal to v 0, and when I substitute t equal to 0 here, I get this, this is equal to C. And therefore, my general solution is going to be x dot, that is the velocity of the particle equal to initial speed, initial velocity minus k over m t. Let us plot it and see, how does it look?

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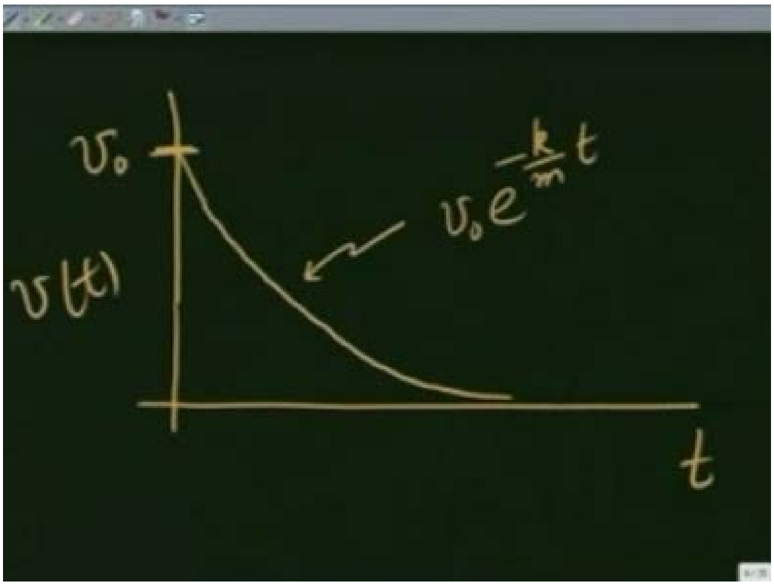

So, initially the particle started moving with some speed v 0, I am plotting vt verses t, and as time passed, velocity decreases exponentially. This curve is v 0 e raise to minus k over m t. How about the distance travelled by the particle? That is also easy to calculate once I know the velocity.

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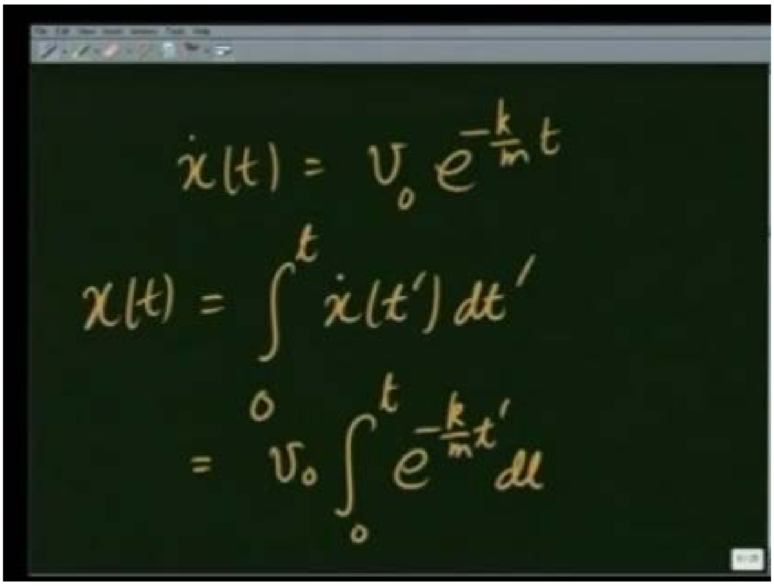

x dot t is given to be v 0 e raise to minus k over m t. So, the distance travelled in time t, is going to be integral 0 to t x dot, t prime, dt prime which is equal to v 0 integral 0 to t, e raise to minus k over m t prime, dt prime.

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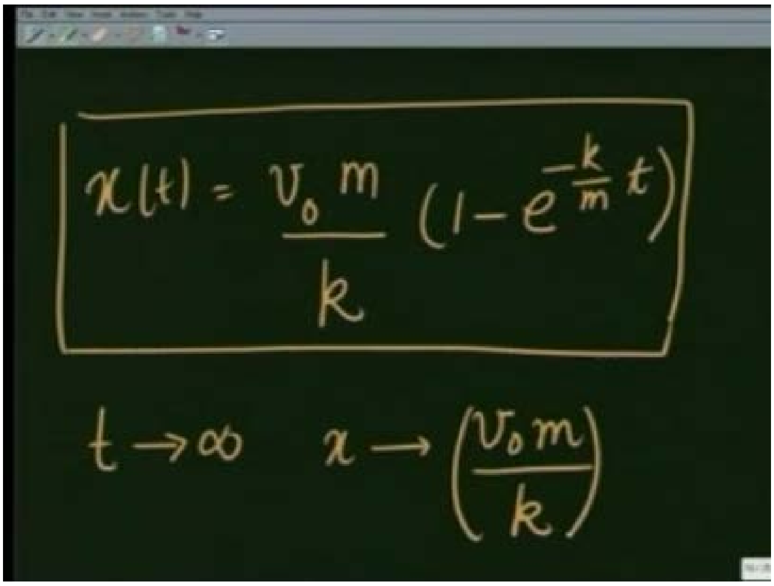

After integration I get xt is equal to v 0 m over k 1, minus e raise to minus k over m t. So, I have also calculated, how much is the distance travelled in time t. As t goes to infinity, x goes to v 0 m over k. So, that is the distance that the particle will finally, travel and stop after this. Let us see how does the plot of this look.

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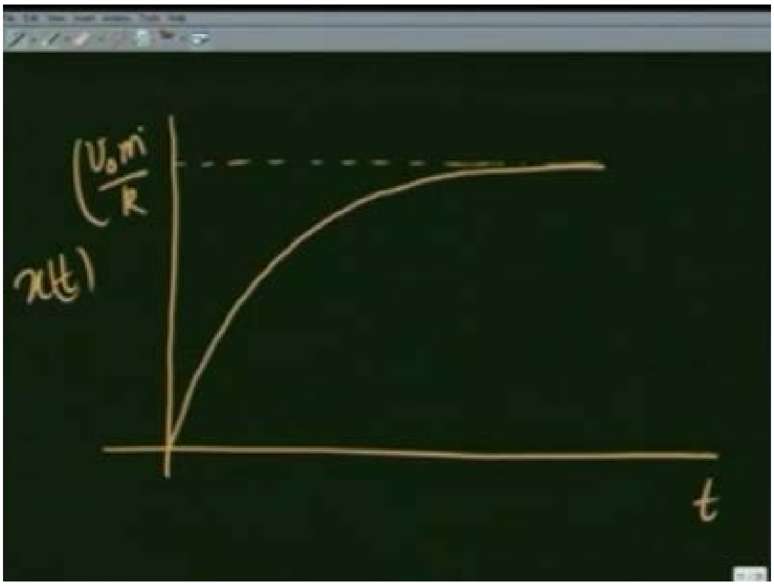

If I plot xt as a function of time. Initially obviously, distance travelled is 0, and slowly it moves up and goes like this, distance is being v 0 m over k.

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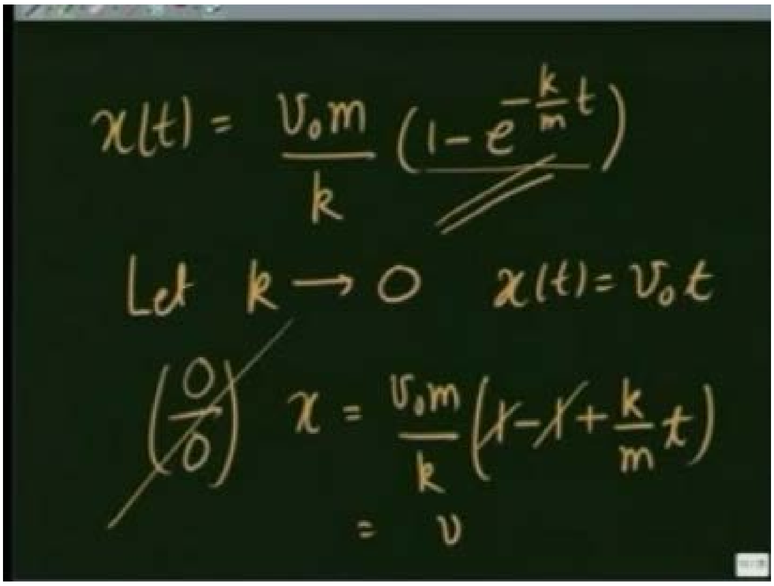

So, let us look at this expression once more. Just v 0 m over k one, minus e raise to minus k over m t. And ask if this is a correct answer, 1 way to check this is, let k go to 0, and I should get my familiar xt equals v 0 t, but if I substitute k directly here, I get an answer like 0 over 0 which is not correct. I have to be more careful and take the limit, k going to 0 keeping terms up to order k in this expression, and see if I get the correct answer. If I do that, I find x is equal to v 0 m over k 1 minus, 1 plus k over mt, this cancels and I get v 0 t, which is correct. So, this is 1 way, when you get an expression in these, when you apply these forces, you can check your answer.

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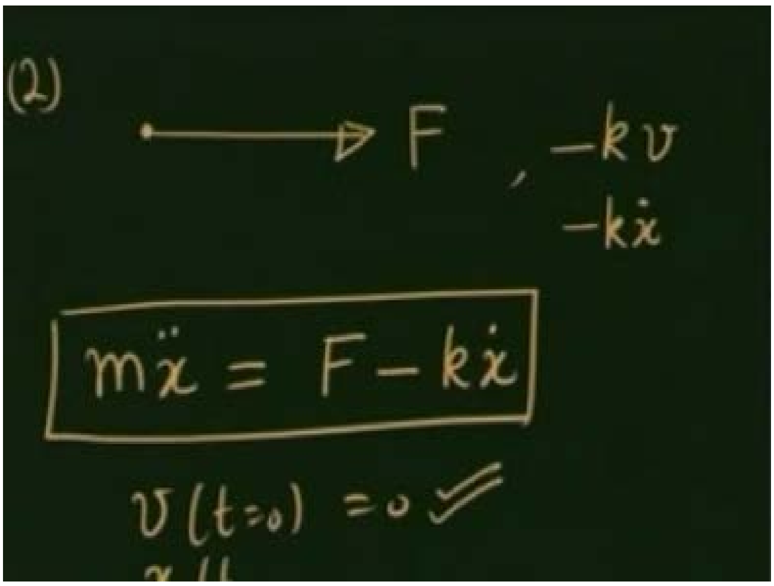

Next let us consider a particle still moving in 1 dimension, but applied on it is a constant force. So, I have a particle, I apply a constant force F again an x direction, and let there be a drag force minus kv or minus k x dot. And I want to solve how, I want see how this particle moves. The equation of motion is going to because m x double dot is equal to F minus k x dot.

In this case notice that I do not really have to start by shooting a particle with some initial speed. I can start with v t equal to 0 still 0 because I am applying a force, the particle will start moving and this is what I want to see how the particle moves. So, this is going to be 1 of my initial conditions. The other 1 is going to be, let us also say that x t equal to 0 it starts from the origin.

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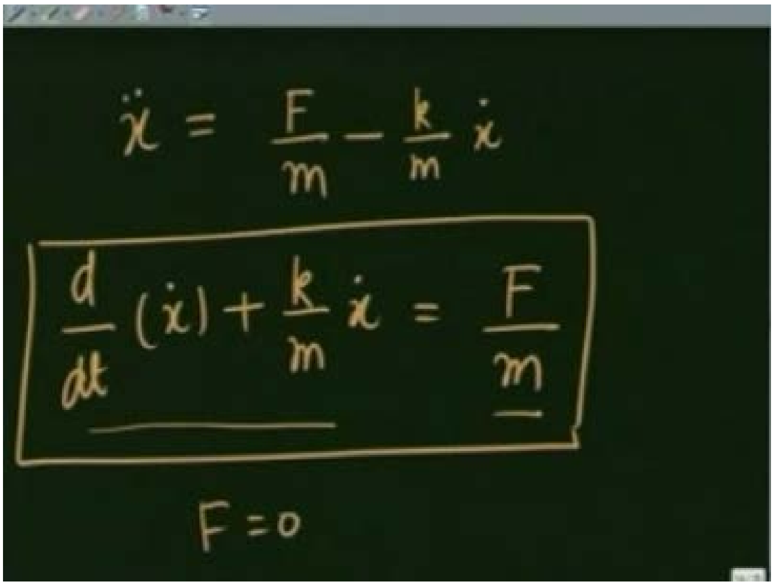

If I rewrite the equation will come out in the form of x double dot is equal to F over m, minus k over m x dot, or d over dt of x dot, plus k over m of x dot is equal to F over m. Notice, I have already solved this problem for F equals 0, that was 1 dimensional motion without any force. In such situations, where I have 1 part like this and another part like this, I can write my solution as the previously obtained solution with F equals 0, which was nothing but C e raise to, minus k over m t.

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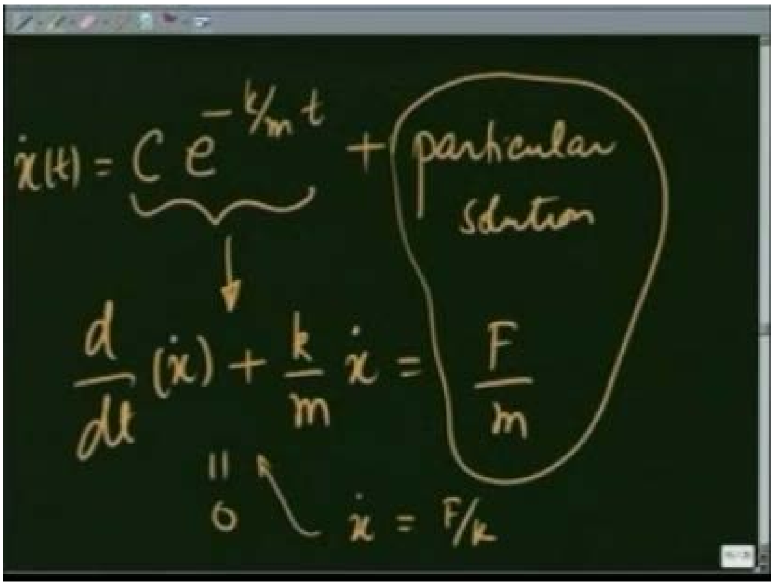

This was x dot of t plus a particular solution, that corresponds to the term on the right hand side. Remember I am solving the equation d over dt x dot, plus k over m x dot, is equal to F over m. This is a solution corresponding to this part equal to 0 the particular solution would refer to this term. And you can see, the particular solution is going to be x dot is equal to F over k. I substitute this here and I get on the right hand side F over m.

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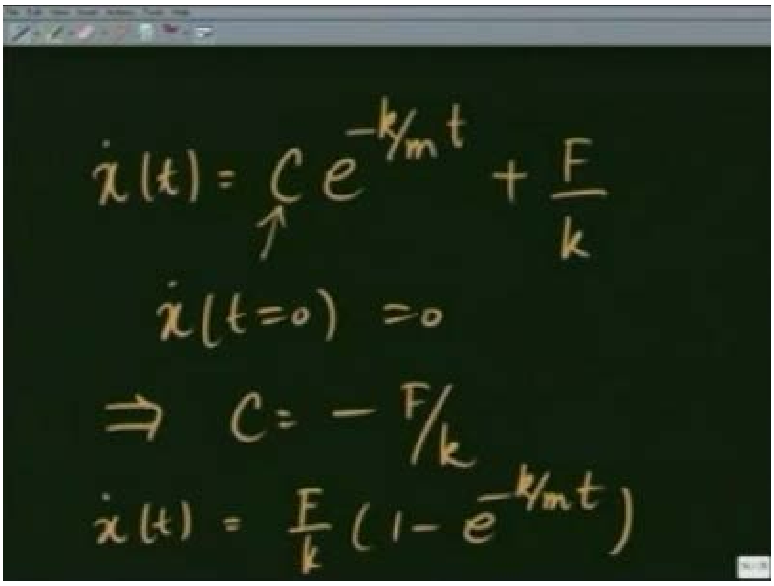

And therefore, my general solution for x dot is going to be x dot of t is equal to C e raise to minus k over m t, plus F over k. This constant C is again determined by initial condition, which is nothing but initially a particle was at rest. And therefore, I get C equals minus F over k, and the general solution therefore, comes out to be xt, other solution with this initial condition comes out to be x dot t, equals F over k 1 minus e raise to minus k over m t. Let us now look at the solution carefully.

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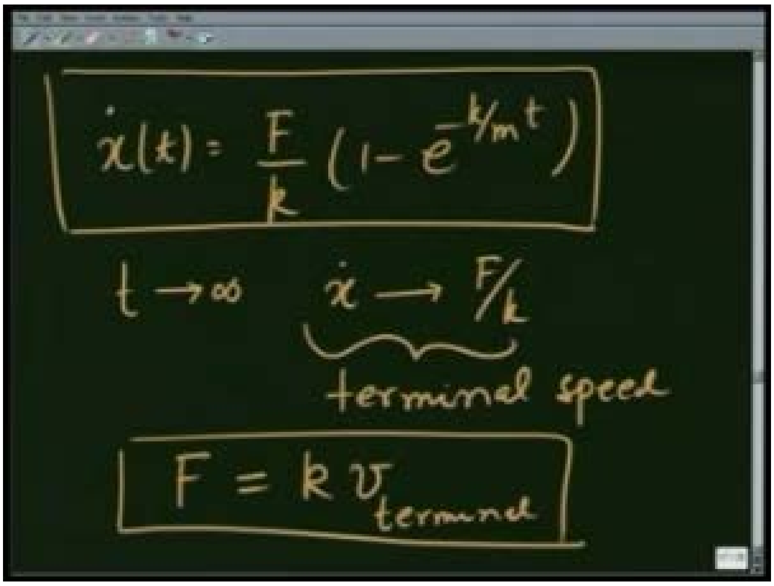

So, I have x dot t equals F over k 1, minus e raise to minus k over m t. We notice as t goes to infinity, that is after a long time I have applied the force x dot goes to F over k. This is nothing but the terminal speed, at this speed the applied force F becomes equal to k times v terminal. The two forces balance and therefore, the particle moves with a constant speed of v terminal.

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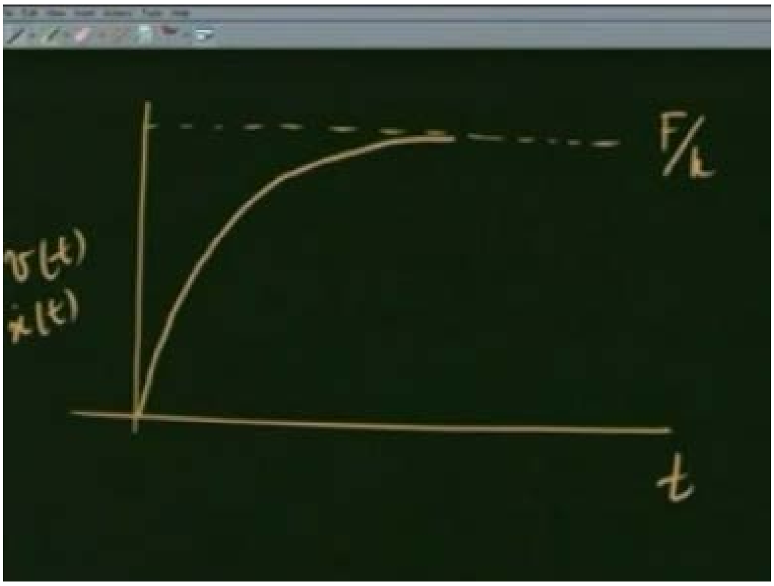

If I plot this speed I am going to get vt or x dot t same thing verses t, the final speed is F over k and the speed slowly builds up. How about the distance travelled? Well for that, I again integrate it.

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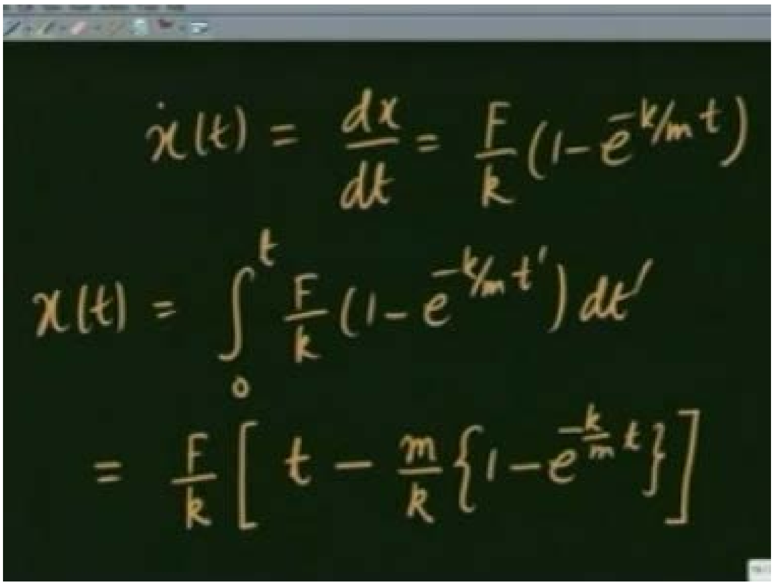

So, I have x dot t, equals dx dt is equal to F over k 1, minus e raise to minus k over m t. So, x at t is again going to be 0 to t, F over k 1, minus e raise to minus k over m t prime, dt prime integrating, I get F over k, t minus m over k, 1 minus e raise to minus k over m t. So, this is going to be the distance travelled by the particle. Now, that we have gotten the expression for the distance travelled, let us play around with it a bit, and see if it goes to correct limits.

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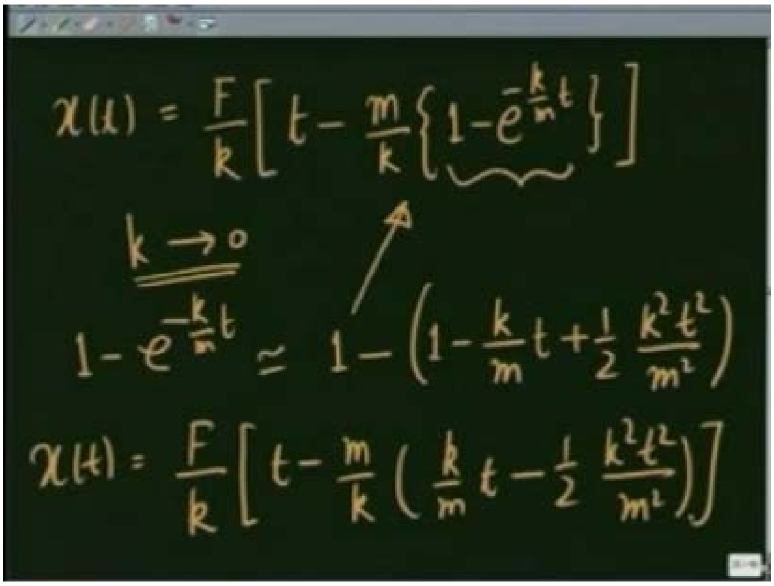

So, I have x t equals F over k, t minus m over k, 1 minus e raise to minus k over m t. So, first question I ask is, what happens if k goes to 0. Remember I said earlier you have to be careful in taking limits. So, I am going to take expand this term keep terms up to order k and see what happens. In the limit of k going to 0, I can write 1 minus e raise to o, minus k over m t, if k is really small for any t, I can write this approximately as 1 minus, 1 minus k over m, t plus half, k square t square over m square. Substituting this in here, I get x t is equal to F over k, t minus m over k, k over m, t minus 1 half, k square t square over m square.

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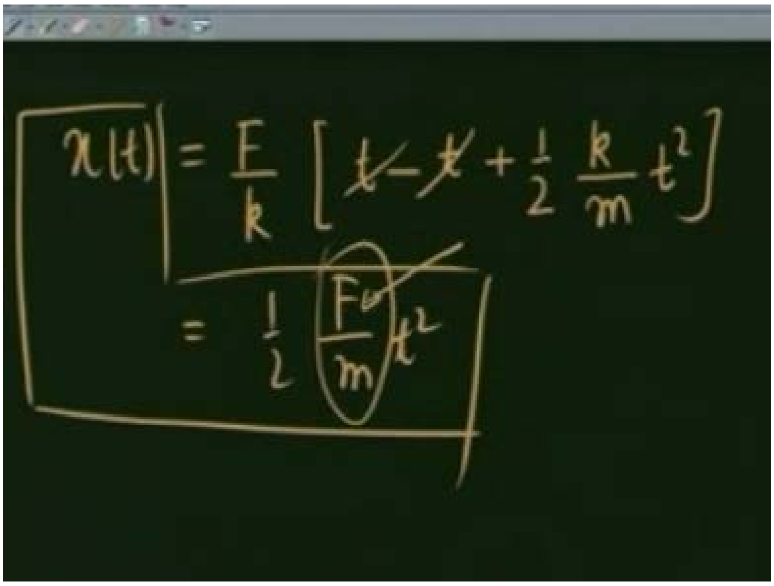

And expanding it further I get xt equals F over k, t minus t plus 1 half, k over m, t square, which is nothing but you are familiar 1 half F over m, t square which is the correct answer in the limit of k going to 0. That is I am taking particle and applying a constant force F, its acceleration is F over m, and there it moves distance 1 half F over m, t square in time t. Let us try to plot this and see how it goes.

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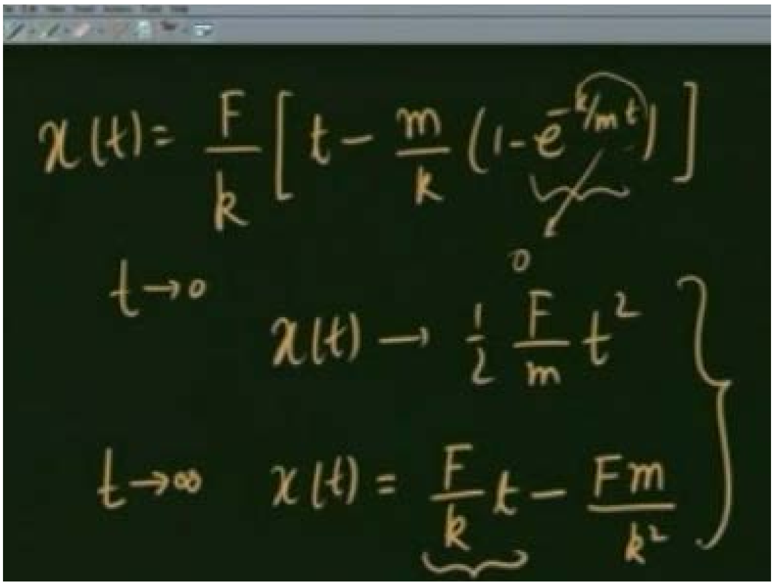

So, xt was F over k, t minus m over k, 1 minus e raise to minus k over m t, again in the limit of very small t. Basically making this term very small, we just saw that xt goes as 1 half F over m, t square.

How about large t? In the large t limit, this term would tend to 0 and therefore, I would have xt as F over k, t minus Fm over k square. This is nothing but movement with the constant speed, F over k you call from earlier is nothing but the terminal speed. So, the particle has now, after long time started moving with terminal speed F over k. So, therefore, if I combine the two, and plot xt is going to look like.

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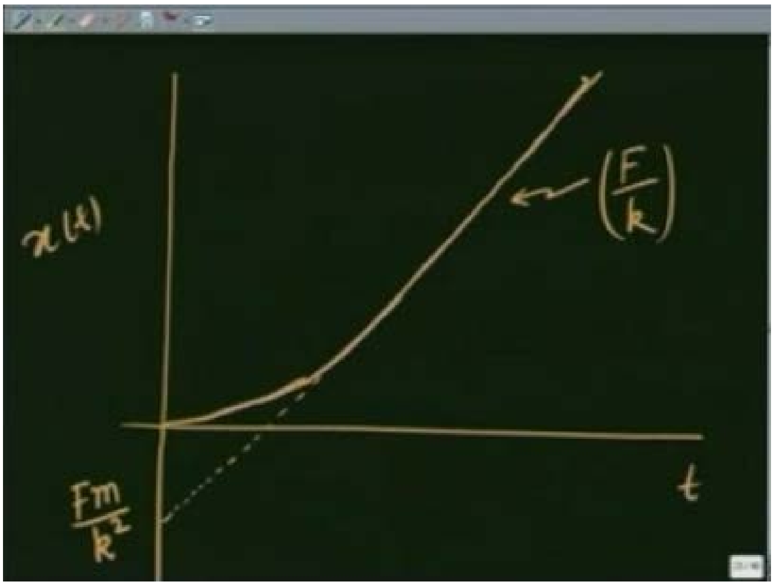

T xt initially it is going as t square and finally, it goes as a straight line. a straight line with slope F over k. which is nothing but the terminal speed and which cuts the y axis somewhere here, which is nothing but Fm over k square. So, this is how the particle is going to cover the distance with time. This is a problem where we applied a constant force. So, the most familiar problem in everyday life of this is a ball being thrown up with say some initial speed v 0.

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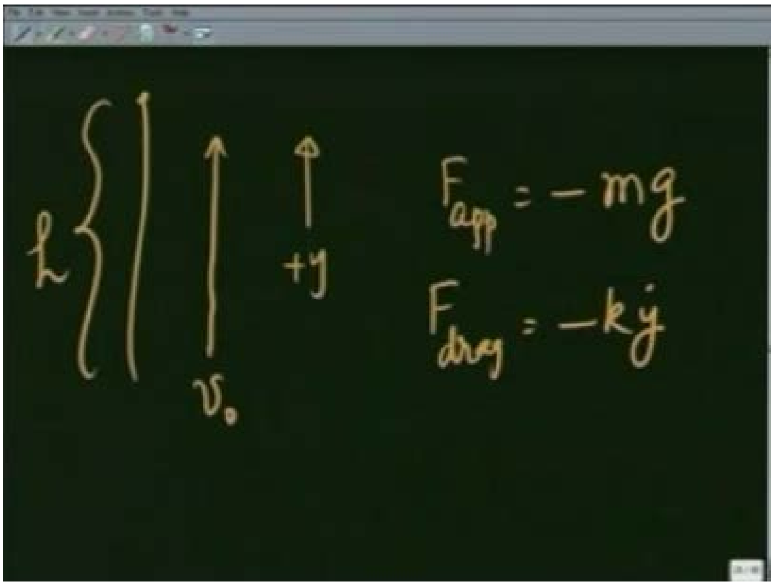

The problem that I just solved, I took initial speed to be 0, but suppose I throw a ball up then, the force and take this direction to be plus y then, the force applied is going to be minus mg. Drag force is going to be minus k y dot. and the problem becomes similar to what I solved.

I leave this as an exercise to find how long does it take for the particle to reach up. how high does it go. Because I am going to solve a slightly more complicated problem which is related to this, and that is going to be the motion in two dimension, the projectile motion in gravitational field. So, with I am done 1 dimensional motion now, let us go to the 2 dimensional motion with drag force included.

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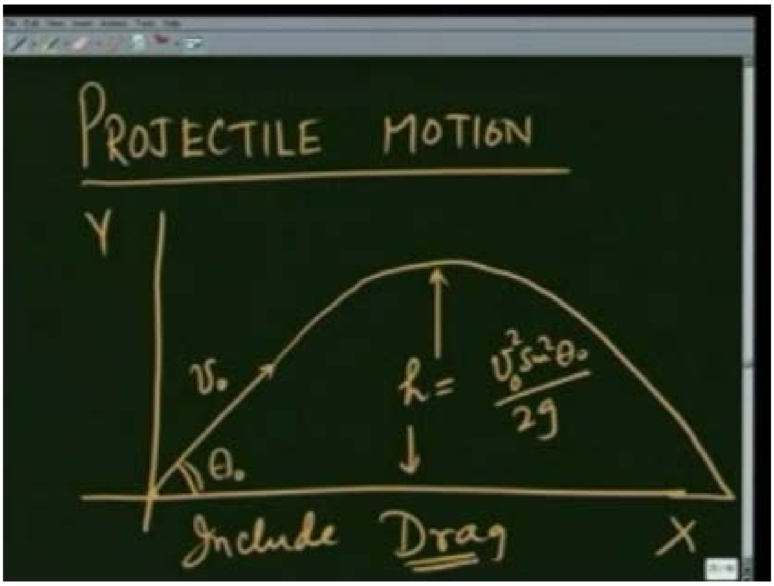

You are familiar with projectile motion, which is in the vertical plane. In this motion, I take a particle and project it, add some angle theta 0 from the horizontal, with some initial speed v 0, and without drag it follows a perfect parabola. And this problem you solved in your 12 th grade height comes out to be v 0 square, sin square theta 0 over 2 g. You can calculate the time, the range, and everything. What we are going to do now is make the problem slightly more complicated by including drag force in it, in a simplest form. And therefore, my equation of motion for the projectile is going to be.

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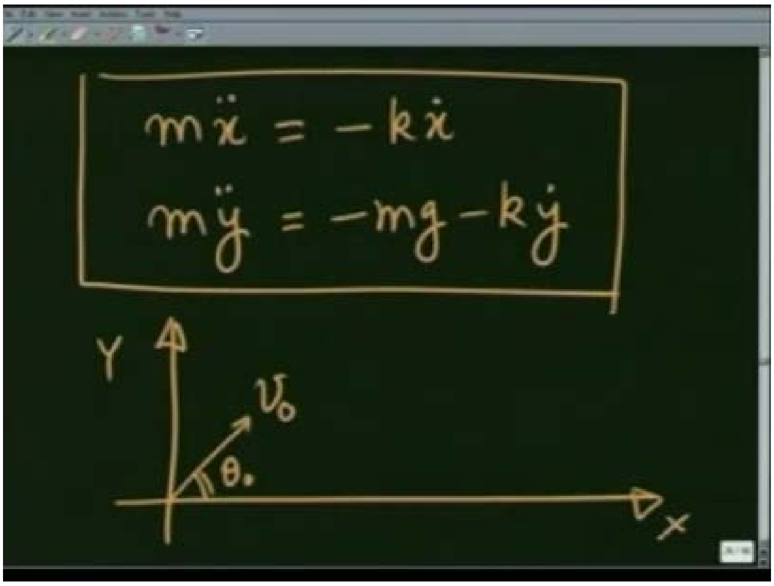

mx double dot is equal to minus k x dot because there is no horizontal applied force. And in the y direction, I am going to have m y double dot is equal to minus mg minus k y dot. These are my two equation of motion. When I am solving for a particle which is being thrown at an angle theta 0 with initial speed v 0, this I am taking to be positive y direction, this to be x direction. These two I have solved separately when I was solving the 1 d problem so, that I know the solution.

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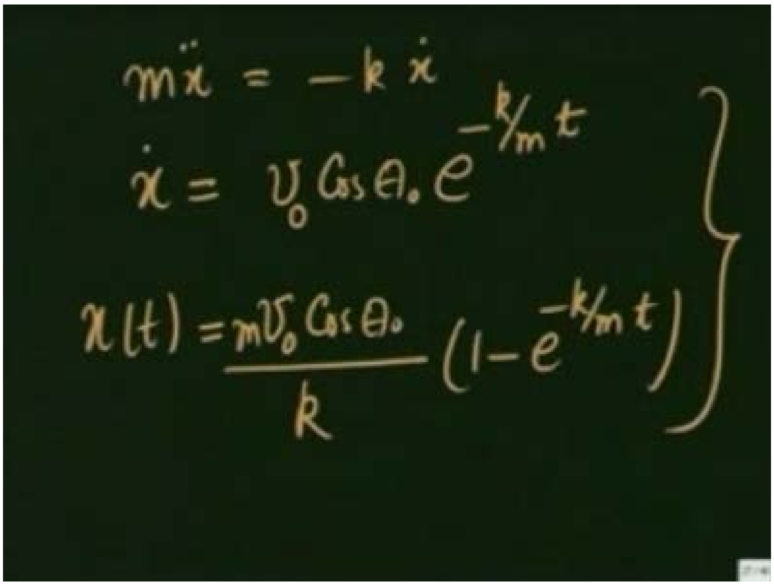

mx double dot equals minus k x dot gives me x dot, is equal to v initial, which in this case happens to be v 0, cosine of theta 0 e raise to minus k over m t. And also x as a function of time comes out to be v 0 cosine theta 0 m, over k, 1 minus e raise to minus k over m t. This is the motion in x direction, and this is related, this is going on concurrently with motion in y direction, for which I have.

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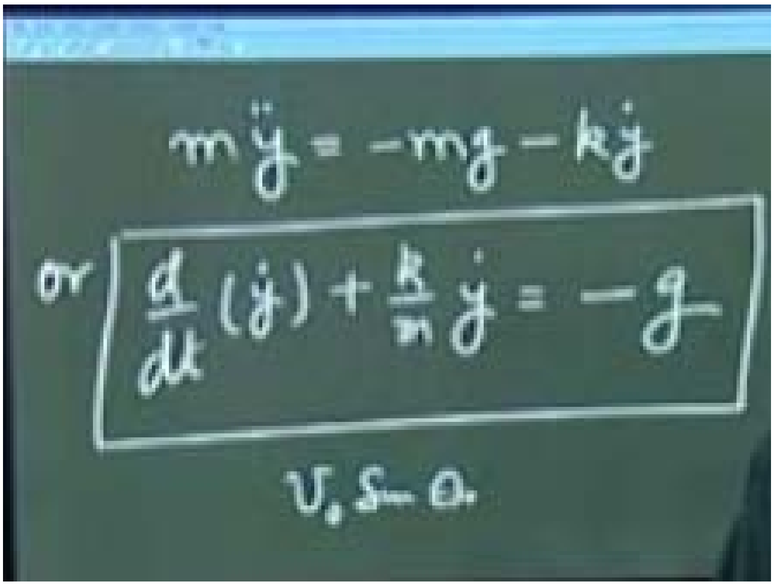

My double dot, is equal to minus mg, minus k y dot or d over dt of y dot, plus k over m, y dot is equal to minus g. In our 1 dimension example, we have already solved this. The slightly different parameters instead of this minus g, I had an F there, the initial velocity was 0, here initial velocity is v 0 sin theta 0. So, let us solve this for that. As I said earlier in the case of 1 dimension, the general solution is going to be summation of the particular solution, and the solution of the homogenous part.

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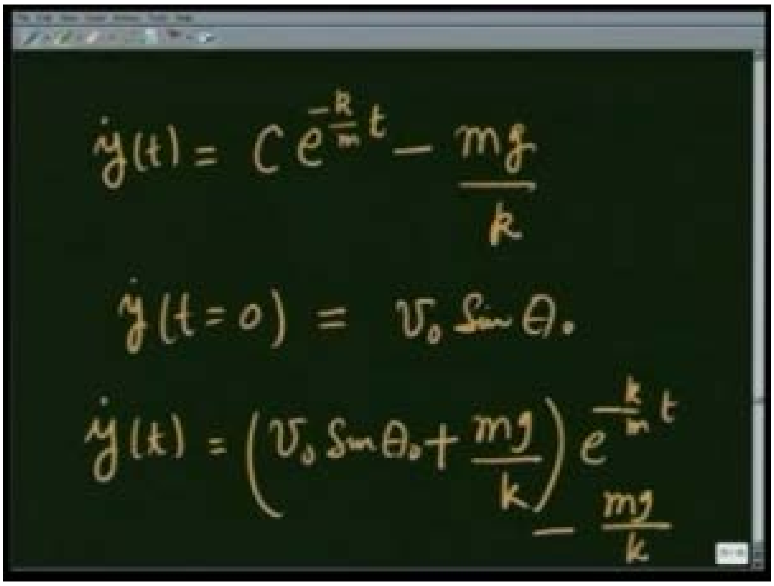

And therefore, y dot as a function of t is going to look like C e raise to minus k over m t, minus mg over k. And C is determined by the initial condition which is that y dot at t is equal to 0, is equal to v 0 sin of theta 0. And therefore, I have y dot of t is equal to v 0 sin of theta 0, plus mg over k, e raise to minus k over m, t minus mg over k.

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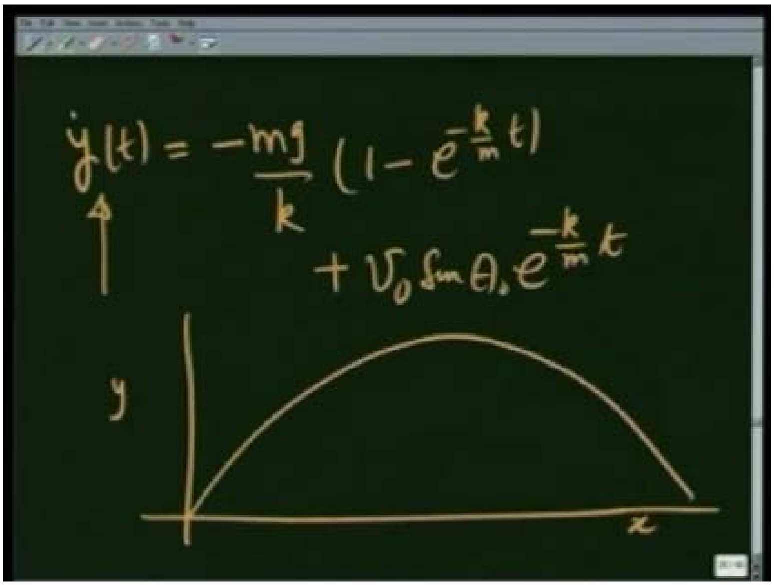

Or I can also write this as y dot of t is equal to minus mg over k, 1 minus e raise to minus k over m t, plus v 0 sin theta 0 e raise to minus k over m t. This is how the part the velocity a particle in y direction varies with time. If I want to find the trajectory of the particle that is, I want to find, how does it go in the space then, I have to solve for x and y and plot them 1 y verses x, one versus the other. However, before we find the trajectory, let us play around with the expression for the velocities that we have already obtained.

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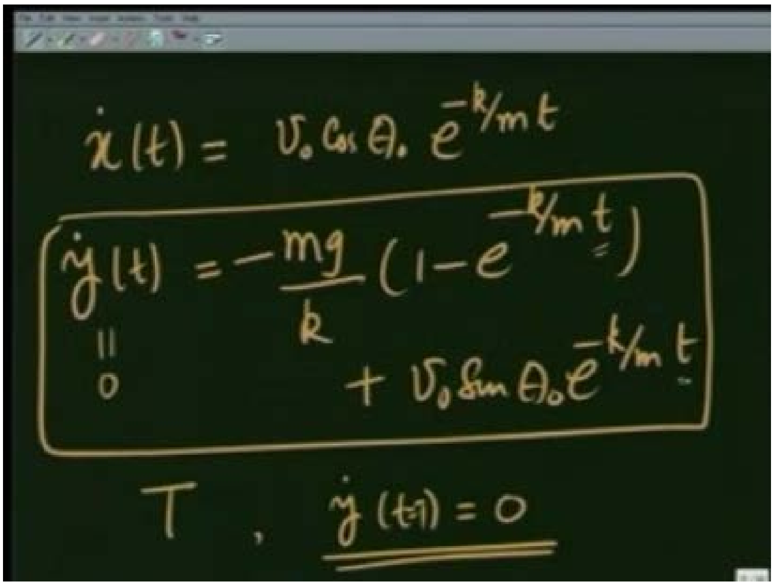

x dot t is v 0 cosine of theta 0, e raise to minus k over m t, and we have just found that y dot t is equal to minus mg over k, 1 minus e raise to minus k over m, t, plus v 0 sin theta 0 e raise to minus k over m t. At this point I may ask, how long does it take time T before y dot becomes 0. That is after I throw the particle, after I project the particle when does it come to stop or when is when does it reach its maximum height? So, for that I substitute this is equal to 0, and corresponding time would get me the answer. Let us do that, if I do that I get.

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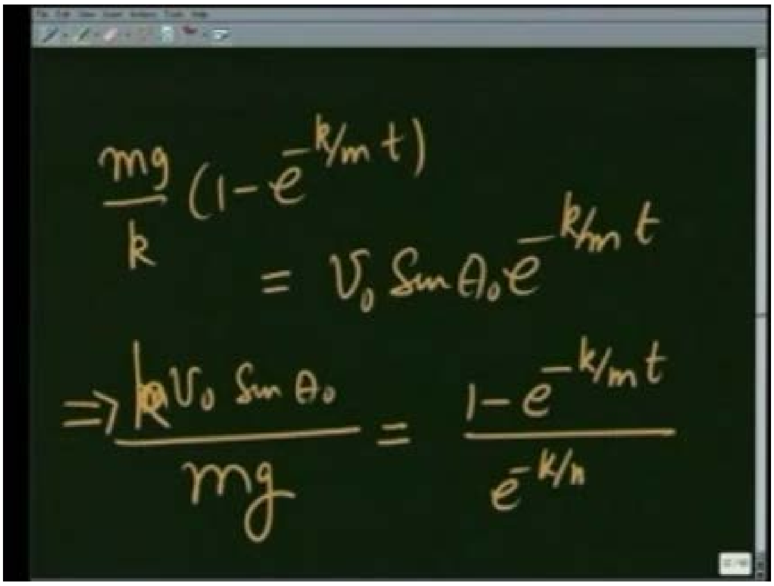

mg over k, 1 minus e raise to minus k over m t, is equal to v 0 sin theta 0 e raise to minus k over m t. And this gives me v 0 sin theta 0, m v 0 sin theta 0 k over mg is equal to, 1 minus e raise to minus k over m t over e raise to minus k over m t.

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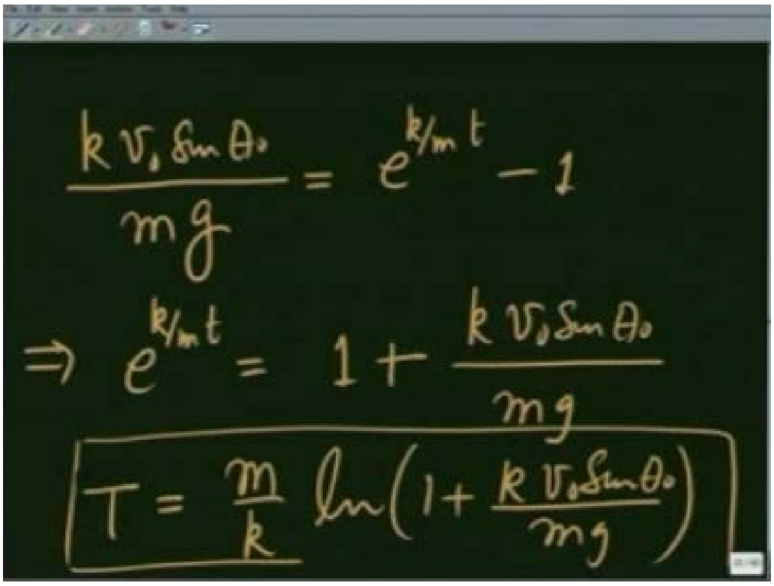

Or k v 0 sin theta 0 over mg is equal to e raise to k over m t minus 1, which implies e raise to k over m t is equal to 1 plus k v 0 sin theta 0 over mg or time T, when its velocity becomes 0 or reaches highest point is equal to m over k log of 1 plus k v 0 sin theta 0 over mg. Let me again ask and check my answer. Is this correct in the limit of k going to 0? Sure it is.

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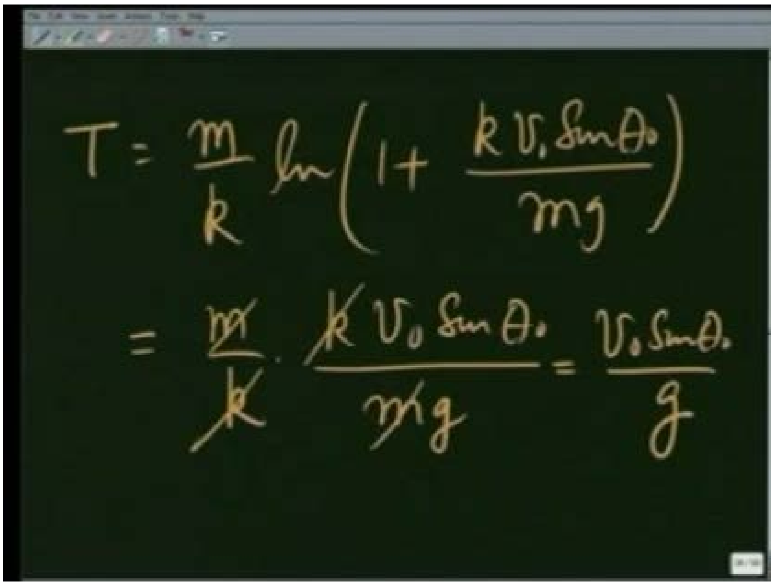

Because T is equal to m over k log of, 1 plus k v 0 sin theta 0 over mg. In the limit of very small k can be written as m over k expanding log, I get k v 0 sin of theta 0 over mg this cancels, this cancels and I get my familiar answer, v 0 sin theta 0 over g.

After doing this exercise, let us go on to find the trajectory. For trajectory, what I will do is, find x and y separately and plot them. Recall that without any drag force, one could eliminate T and find x and y, a direct relationship between x and y. This is not possible in this case so, going back to our expressions.

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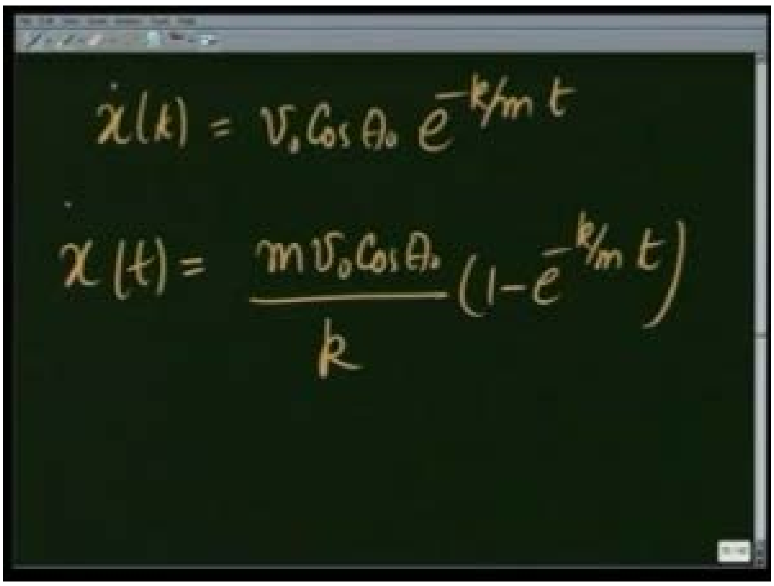

We had x dot t is equal to v naught cosine of theta 0 e raise to minus k over m t, which gives me xt is equal to m v 0 cosine of theta 0 over k, 1 minus e raise to minus k over m t. This is my expression for x, you are also familiar with this from the earlier part of the lecture.

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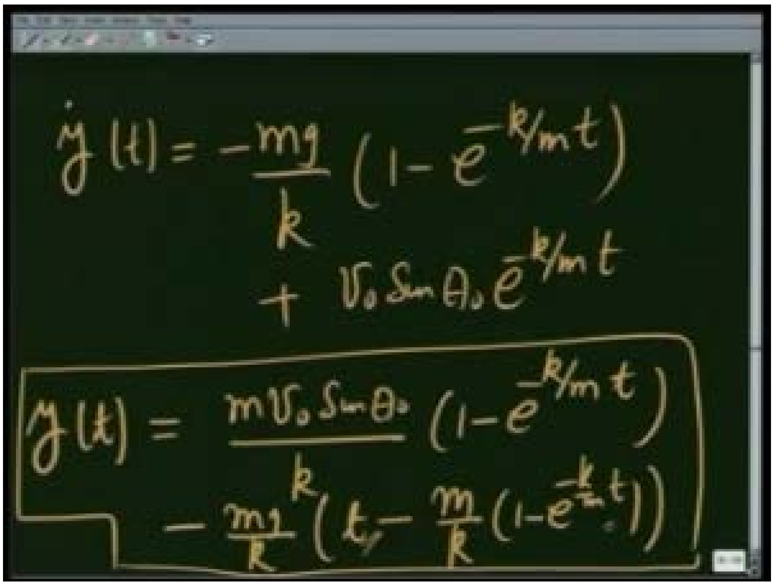

Similarly, I have y dot t which is nothing but minus mg over k 1 minus e raise to minus k over m t, plus v 0 sin theta 0 e raise to minus k over m t. Integrating this I get an expression for y as a function of time and it comes out to be m v 0 sin theta 0 over k, 1 minus e raise to minus k over m t, minus mg over k t minus m over k, 1 minus e raise to minus k over m t. You see because t appears in linear form as well as this form, that is why I cannot eliminate it to get a direct expression between x and y. However, for given time I can calculate x and y and plot the trajectory.

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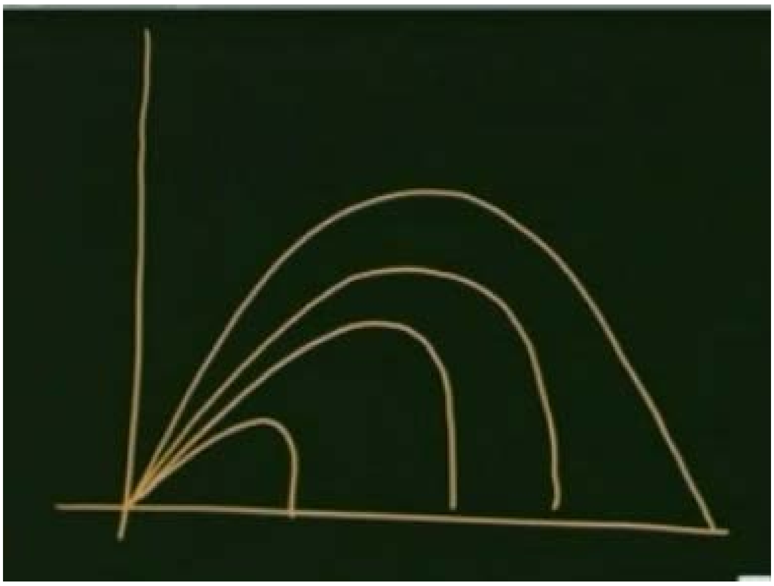

If I do that for different case, and a fix v naught and sin theta naught, I find that for k equal to 0 it is a perfect parabola. However, as k is introduced that means, drag force comes into play, its height and range both start changing and all have got some numbers here, which I will show you how for different parameters it changes. As an example of how the trajectories are affected for different values of that coefficient or by this drag.

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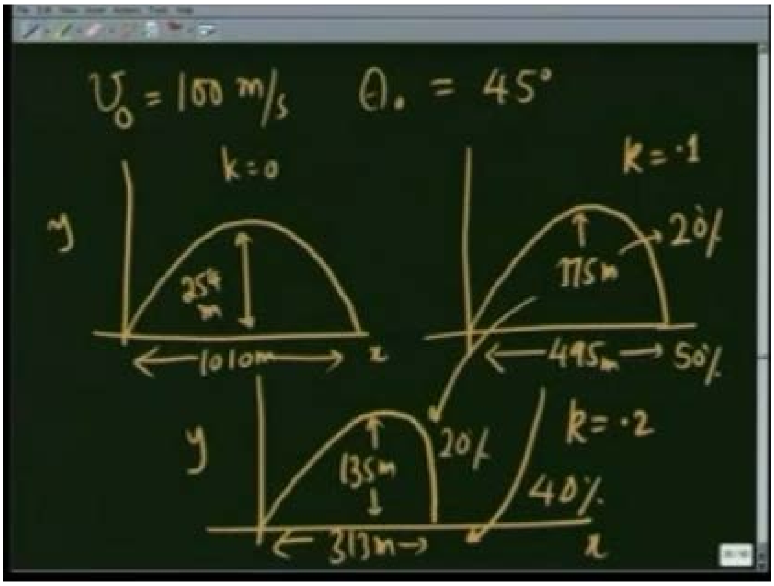

I have taken a sample problem of a particle being thrown in the speed of 100 meters per second at an angle theta 0 equal to 45 degrees, and I will plot for three different case, the trajectories which I have calculated. I take k is equal to 0, which is the case of no drag or no viscosity, in that case you will familiar as this comes out as a parabola. Its height is about 254 meters, and the range comes to about 1 kilometer 1010 meters. Now, I take k to be point 1, the trajectory changes slightly. It goes and comes down, but not as a parabola, but something like this, its height now reduces to 175 meters and range comes out to be 495 meters.

Notice, that the reduction in height is about 20 percent whereas, range as gone down by about 50 percent. If I go further and make k equals 0.2, the trajectory becomes even more like this with height now becoming 135 meters, and the range becoming 313 meters. Again notice, that the reduction in range is much more which is about 40 percent whereas, the height goes down by about 20 percent.

So, you see how introducing drag or this frictional force due to fluid or the gas affects the motion of a trajectory. This is a very important problem when we consider a motion of particles some motion of say bombs or motion of artillery pieces being thrown, being projected in a battle field. An interesting question you may ask at this point is, you all know from your 12th grade that.

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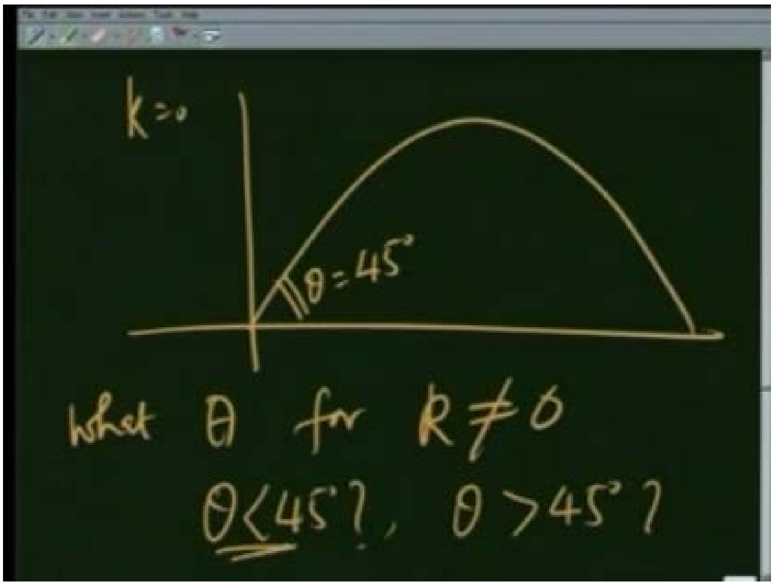

If k is equal to 0, the maximum range which is attained is when theta is equal to 45 degrees. What theta for k not equal to 0? Is theta less than 45 degrees? Is theta more than

45 degrees? The answer to this question is the theta should be slightly less than 45 degrees.

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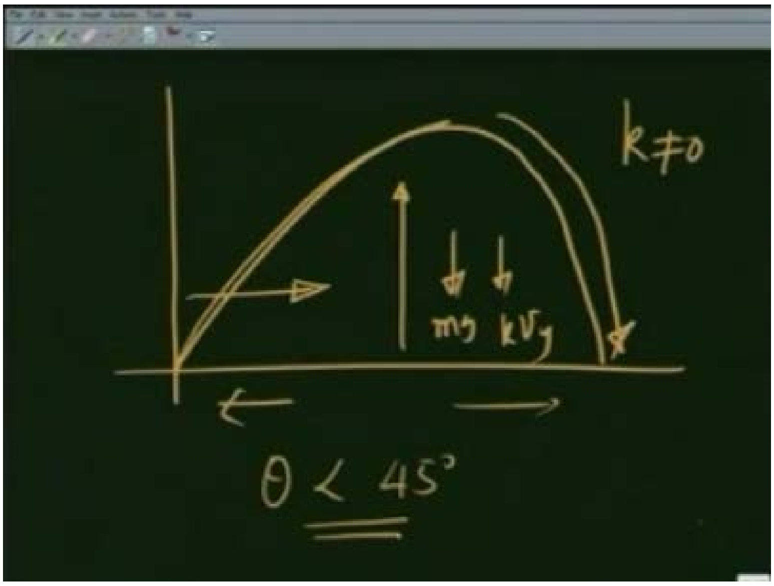

Because you saw that the height is not affected so much by introducing of k as the range. So, what I want to do is give slightly more of x component to the velocity, if I want to fire the projectile further and further. The other way to look at the same problem is that, when the particle is moving is up, it spends more time because it is being pull down by mg, it is also being pull down by kV y spends more time in going the same height then, when it is coming down.

So, in that time, you want to cover as much distances as possible because x is, x component of the velocity is also going down. So, to attain maximum range, I should fire at theta less than 45 degrees. So, far I have considered the drag force which is proportional to the velocity, and in the opposite direction, and shown you through some examples, how it affects the motion. This is the simplest possible drag force.

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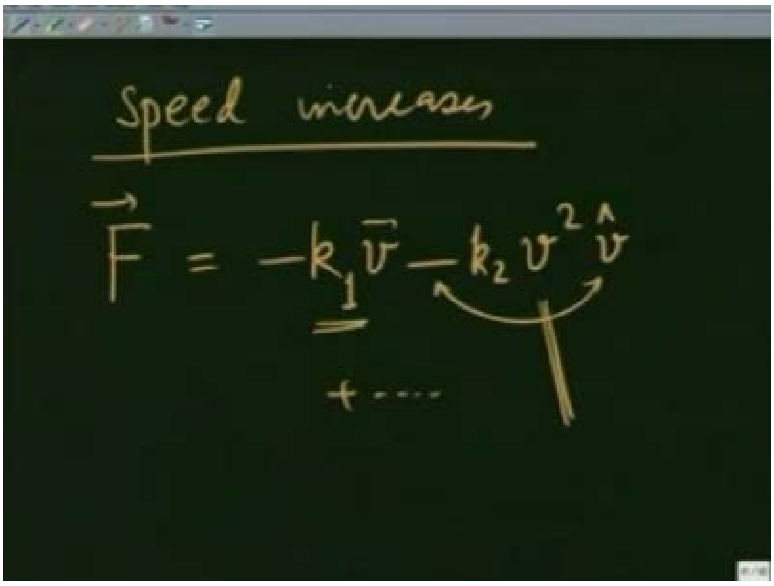

As the speed increases more dependence on velocity may come in the form of F may be equal to minus k v, minus, let me write this k 1, k 2 v square, left side a unit vector v so that, it tells you this is opposite to the velocity vector, and higher order terms, this being the simplest. When these terms come into play, problem becomes non-linear.

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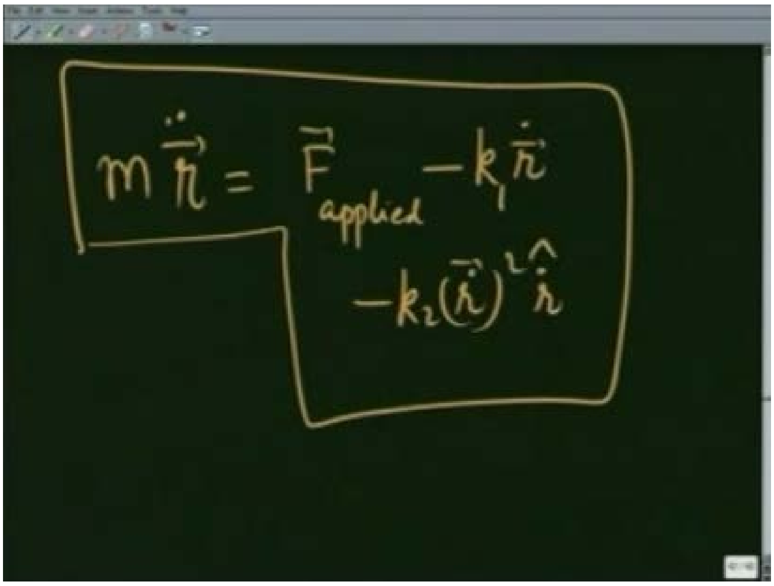

Because now you have m r double dot is equal to F applied, minus k r dot k 1 r dot, minus k two r dot square r dot vector. And these are not easy to solve. In most of the cases, you have to apply numerical methods to get the final solution. However, there are some cases where you can apply some techniques and get solutions. I will solve one example to illustrate this, and that would complete our lecture on drag forces.

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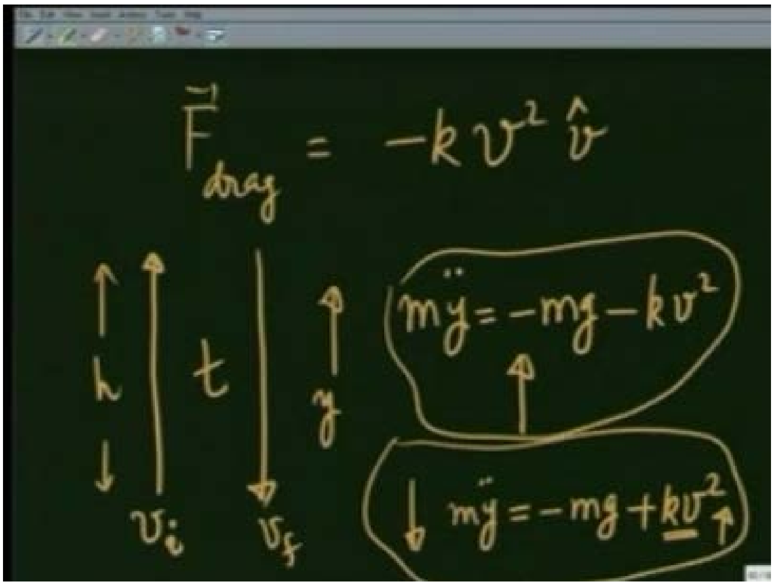

The example I take is that of F drag being proportional to v square. And obviously, it is in direction opposite to its velocity. And I want to find, if I throw a particle up with some initial speed, v initial, how long does it take to reach up? What is the height it can go up to? And what is the speed when it comes down? v final.

Let us see what all answers can we get. If I write the equation of motion since, this is a 1 dimensional case, let me take y to be going up, I can write m y double dot is equal to minus mg minus k v square, this is when the particle is going up. Notice in this case, I have to be careful when considering the motion of particle coming down. When the particle is coming down, I would have m y double dot is equal to minus mg plus kv square because in that case the force, drag force would be acting in upward direction. That is why I took this particular example. So, let us see how do we go about solving this?

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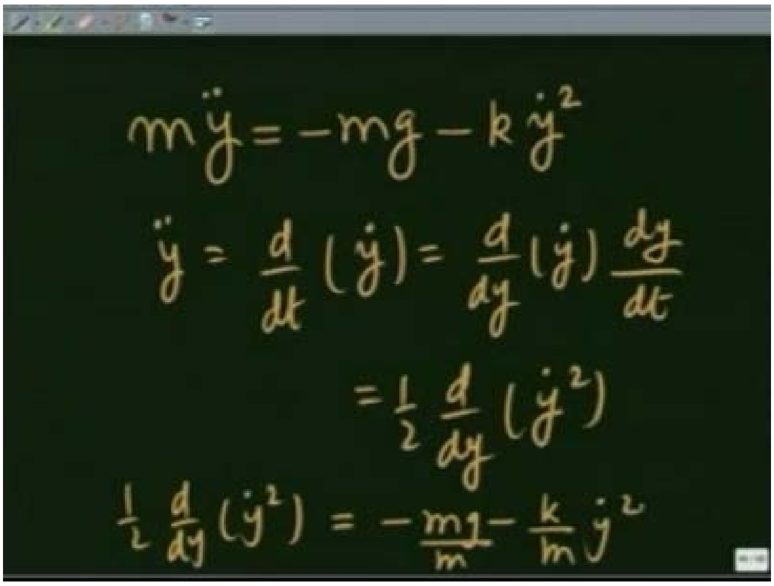

While the particle is moving up I write m y double dot equals minus mg, minus k y dot square, and I go back to my technique of writing y double dot as d over dt, y dot which is same as d over dy, y dot, dy dt which is same as d over dy of, y dot square 1 half. And therefore, I can write my equation as 1 half d over dy, y dot square is equal to minus mg over m, minus k over m y dot square.

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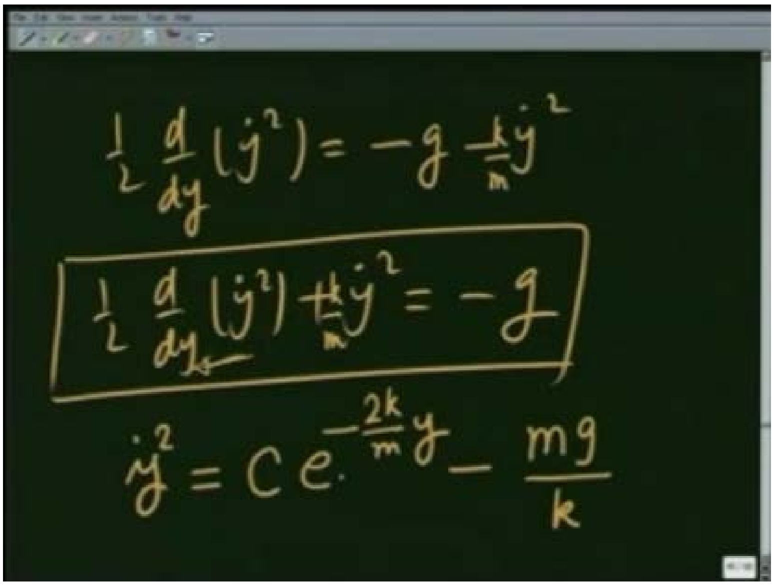

And I have 1 half d over dy, y dot square is equal to minus g, minus y dot square, 1 half d over dy y dot square, plus y dot square equals minus g. I can solve for y dot square directly in terms of y. You see I have eliminated time. So, in this problem it will not be possible for me to calculate how long does it take before the particle comes to stop, but I can certainly calculate, how high the particle goes before it comes to stop.

You can directly see that, the solution I have forgot k over m here. So, there is going to be a k over m here, y dot square is going to be of the form C e raise to minus 2 k over m y, minus mg over k. You can directly substitute this and see if I substitute y dot square with mg over k, it gives me minus g and this satisfies the homogenous part. Now, two initial condition.

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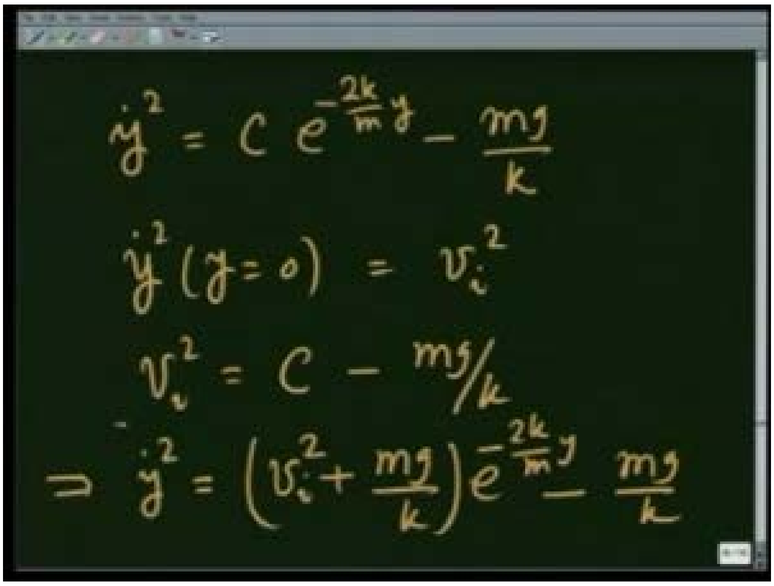

So, I have y dot square equals some constant e raise to minus 2 k over m y, minus mg over k, and I know the initial velocity y dot square, when y is 0 is equal to v initial square. And therefore, v initial square is going to be equal to C, minus mg over k and that gives me y dot square is equal to vi square, plus mg over k, e raise to minus 2 k upon m y, minus mg over k.

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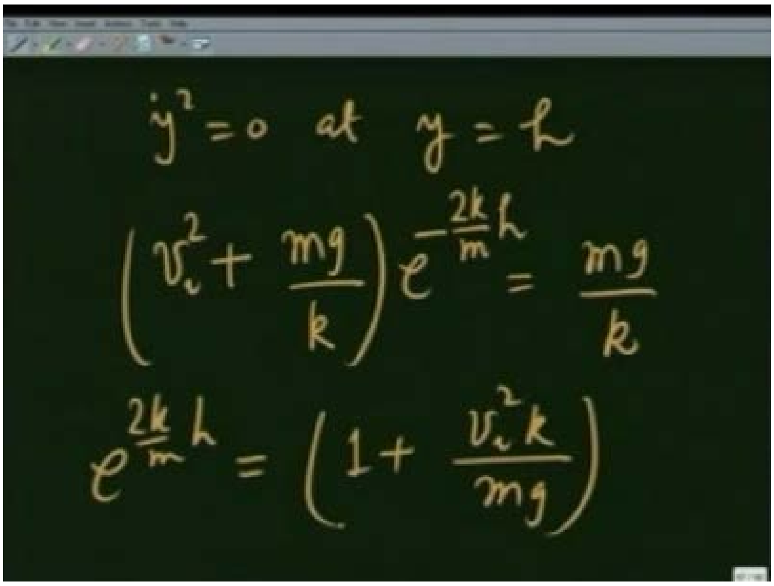

To find how high the particle went, I take y dot square to be 0 at y equal to h and therefore, I get vi square, plus mg over k, e raise to minus 2 k over m h, equals mg over k. And that tells me that, e raise to 2 k over m h is equal to 1, plus vi square k upon mg.

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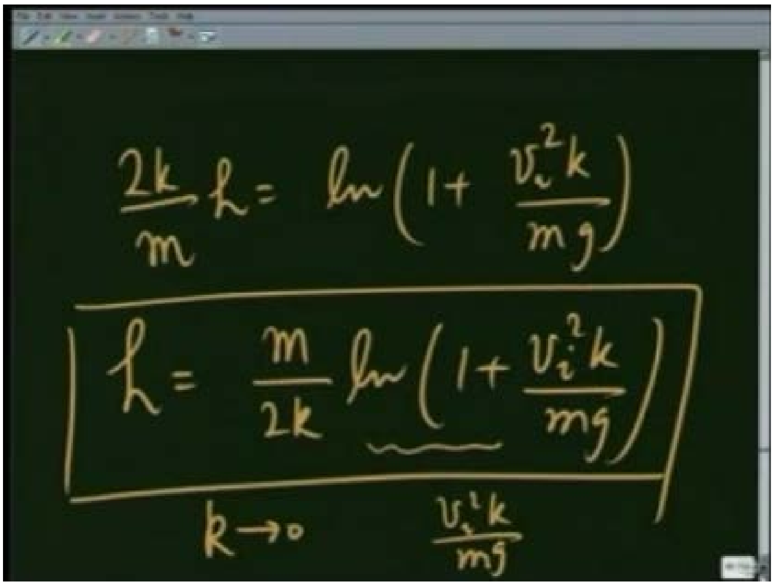

Or 2 k over m h ,equals log of 1, plus vi square k upon mg, or the height it went up to is equal to m over 2 k log of 1, plus vi square k upon mg, this is the height at which it stops. Again, I will go back to limit of k going to 0 and see if my answer is correct. When the limit k equal to 0 is taken, I can expand the log term and write this as vi square k over mg.

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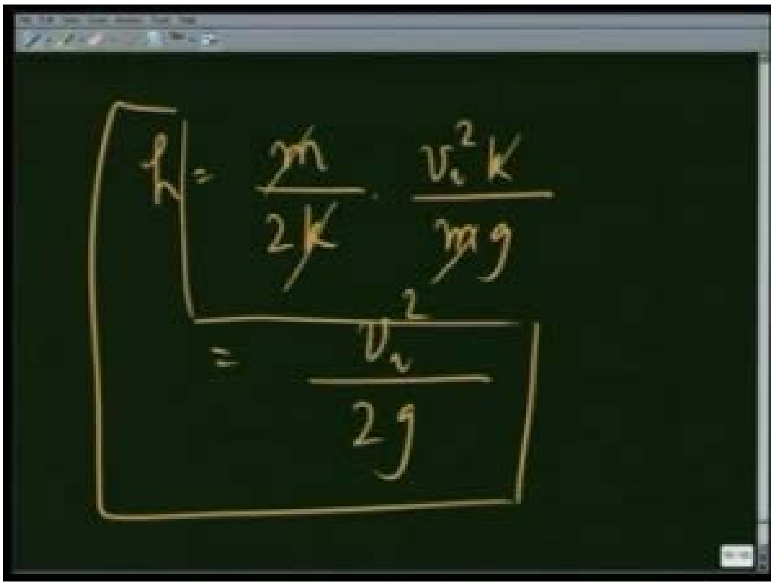

And therefore, the height that I get is h equals m over 2 k, vi square k over mg, which is our familiar result from your previous classes. So, I have calculated the height up to which the part will go, will go with the drag included. I leave it for you as an exercise that when it comes back.

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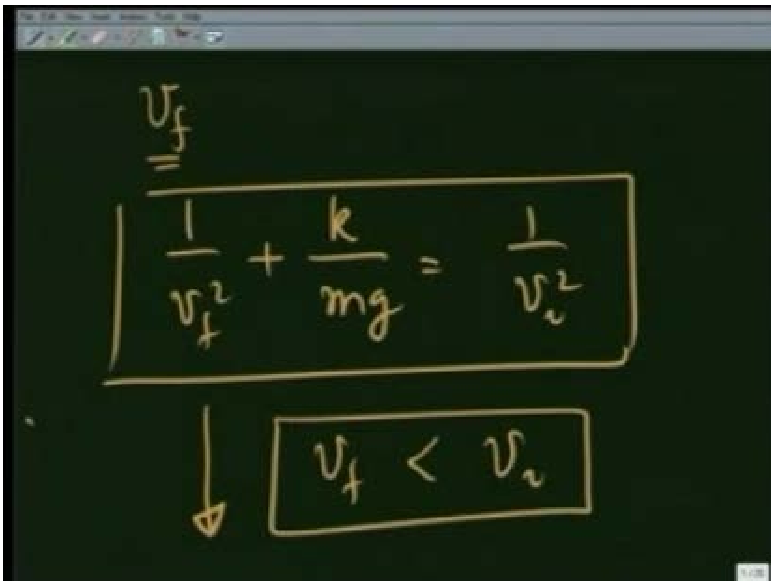

The velocity v final then, the relationship between v final and v initial square is going to be 1 over v final square, plus k over mg is equal to 1 over v initial square. The word of caution though, when considering the motion of part of the coming down you should take proper account of the force of friction.

The force of friction changes direction, and I was showed in one of the previous slides, that you have to change the sign of the force. V final of courses is going to be smaller than v initial because in this friction, the particle has loss energy. So, what we have done in this lecture is, shown through some examples, how the drag force affects the motion of a particle.

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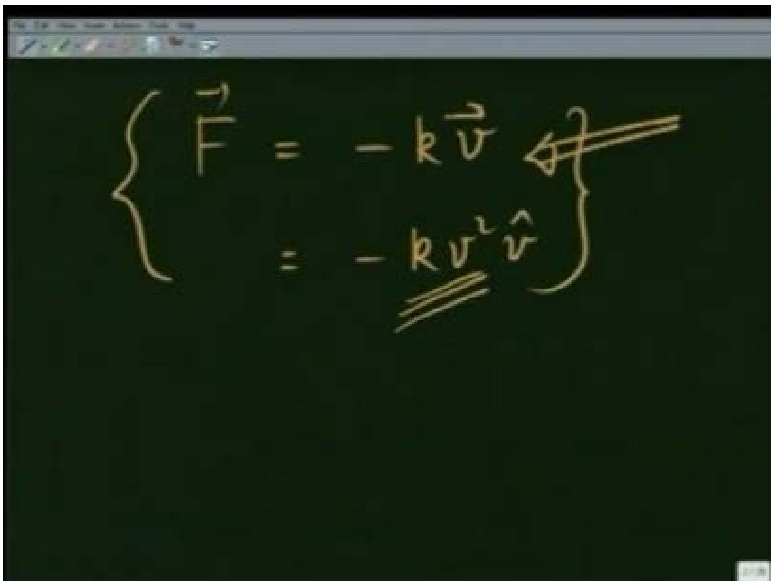

I have considered the simplest possible form where F is proportional to the velocity itself, and another form where F is proportional to the velocity square. This is encountered when velocities are small and suffices in most of the cases.

Another mathematical advantage or analytical advantage with this is, that most of the cases I can solve it analytically. If the speeds become higher then, higher orders of v's power come in to play and then, one has to solve the problem numerically. With these we finish the first part of these lectures on engineering mechanics. In this part I made you familiar with different co-ordinate systems that we use besides.

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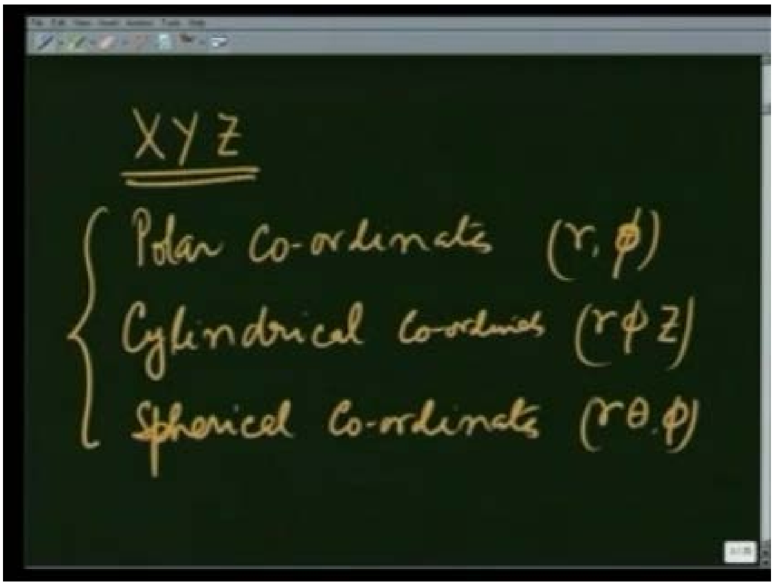

The xyz co-ordinate system, cartesian co-ordinate systems. We considered the planar polar co-ordinates r and phi. We considered cylindrical co-ordinates r, phi and z. We considered spherical co-ordinates r, theta, and phi. And we used some of them while solving problems.

Not only these co-ordinates systems are useful in solving mechanics problems, you will find them very useful in solving problems in electrodynamics and wherever there are symmetries. After this introduction to co-ordinate systems, we considered the motion of particles single particles with constraints and learnt.

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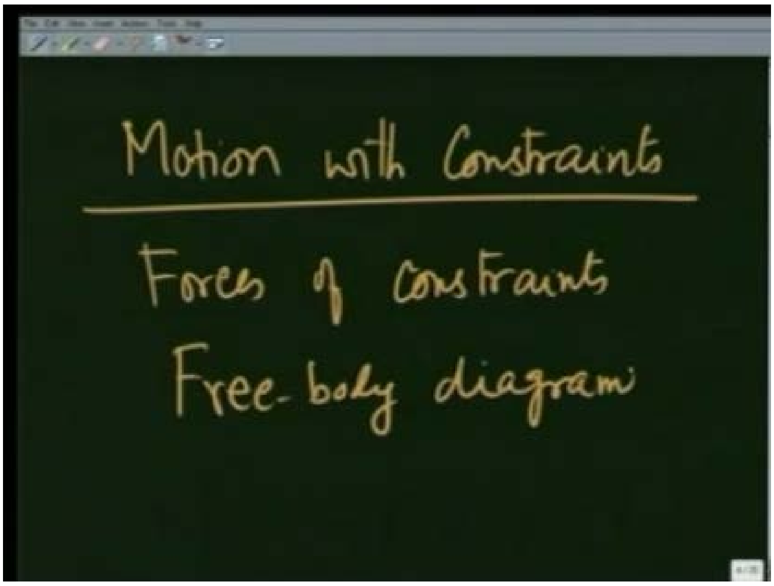

What are forces of constraints? What is free body diagram? How to take constraints into account and how to solve problems when constraints are there? Following that, we considered a very special class of forces.

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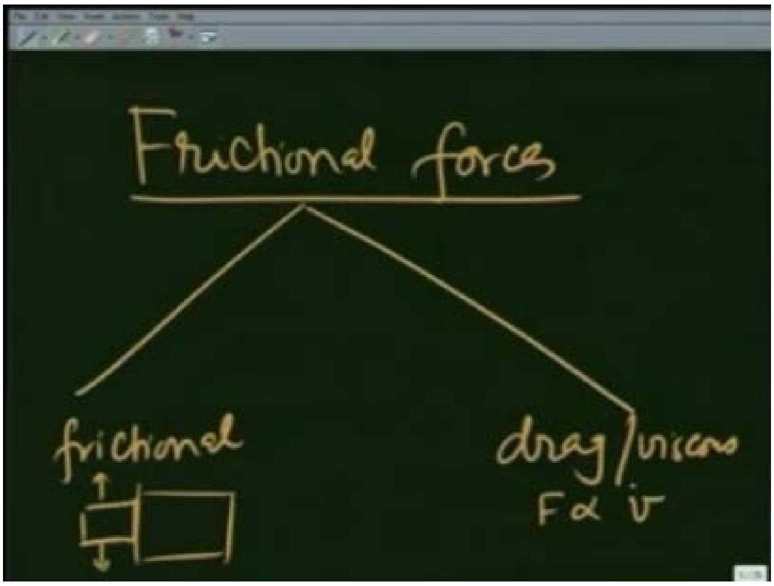

The frictional forces and took them into account when solving dynamical problems. In these we considered regular frictional force that you are familiar with, when one body slides over the other body, solid body. And we also considered drag or viscous force. In their simplest possible forms, one where F is proportional to the velocity and also proportional to the v square. In the coming few lectures, we are now going to get more sophisticated, considered the work done by forces, do work energy theorem and then, go to many particle systems where we will consider rigid body motion and things like those.