Engineering Mechanics
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Module – 05
Lecture - 03
Motion of Particle with Friction

In the previous, lecture we have seen how to solve problems by using free body diagrams, isolating subsystems and considering forces on each subsystem. Also how the subsystems affect the motion of one another was taken into account by looking at the constraints since equations were constraints.

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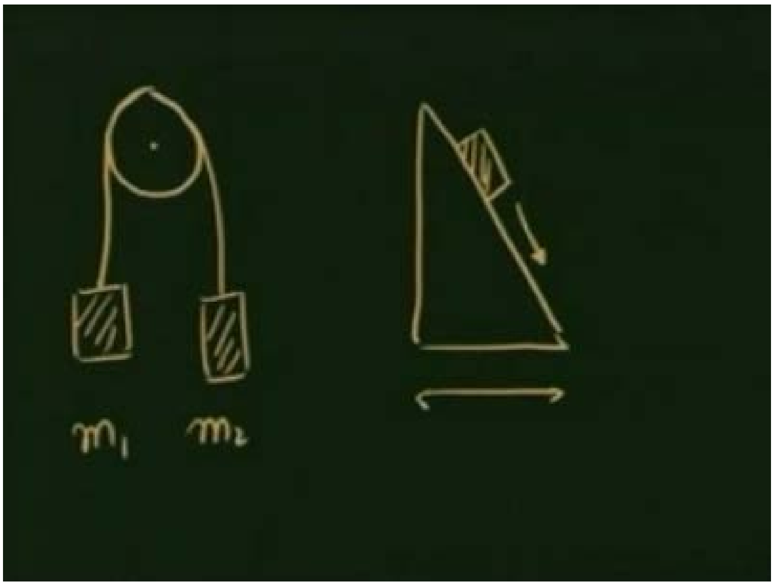

Two examples that we took were one acquits machine where I had two masses moving around a pulley m 1 and m 2. The other example that we took was a mass sliding down a wedge like this, and the wedge could also move horizontally. In these examples we ignored a ubiquitous force, that is encountered everyday in our lives, and that is the frictional force. This lecture is going to be about, how do we take into account the frictional forces, and how does it affect the motion of a particle.

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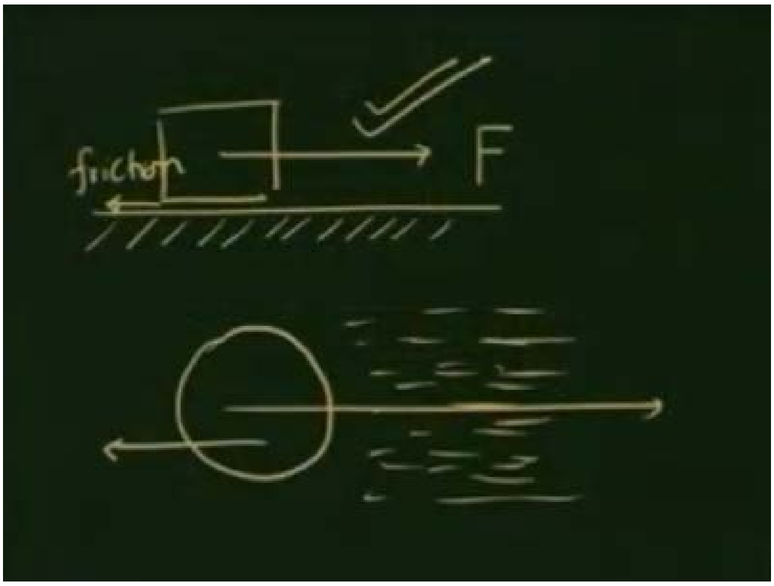

There are, when the body moves we see that its motion is usually resisted by the surface on which it is moving. So, if I apply a force F this way, there is the surface applies a force called frictional force that opposes this motion. Even if the body is not moving frictional force has a tendency to oppose a tendency to move.

Another example of frictional force is the drag force or a viscous force that we encounter when a body moves through a fluid, this force also opposes the motion. In this lecture we would be looking at both kinds of forces. So, let us start with a force of this kind which is when a solid moves on another solid.

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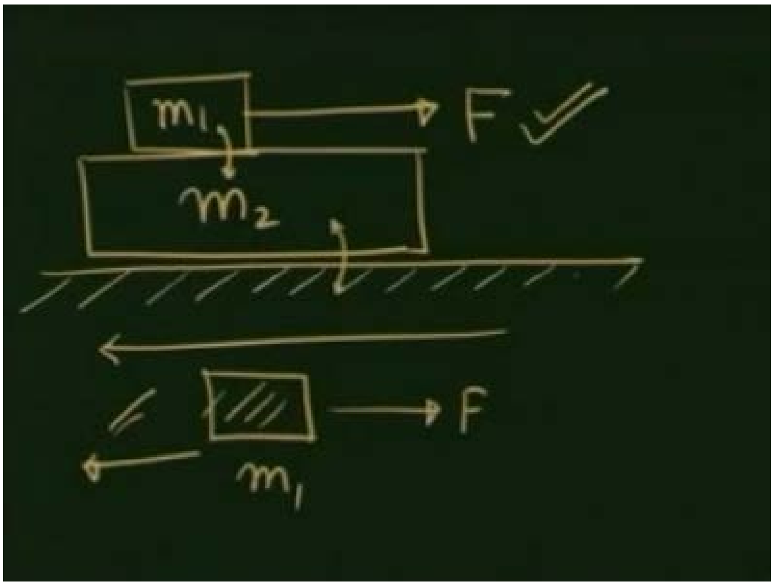

Let us take a mass m 1 and another mass m 2 and let me pull this mass with some force F. If you recall your everyday experience, you would see that the top mass or the bottom mass does not move until the force has reached certain value. And that is because their tendency to move is being opposed by the frictional force either between these two masses or between this mass and the surface on which this mass is resting.

So, first question is, is the frictional force of constant amount or does it vary as I vary this force? Let us give an argument, if suppose the frictional force were of constant amount and that means, this mass m 1 which is on the top, experienced a constant force in this direction. Then, even if I did not apply any of this force F because of this forces, the mass would tend to move this way and I know that does not happen. And that means, a frictional force adjusts itself as I apply the force, how does it adjust itself? It is just sufficient to oppose the motion or the tendency to move.

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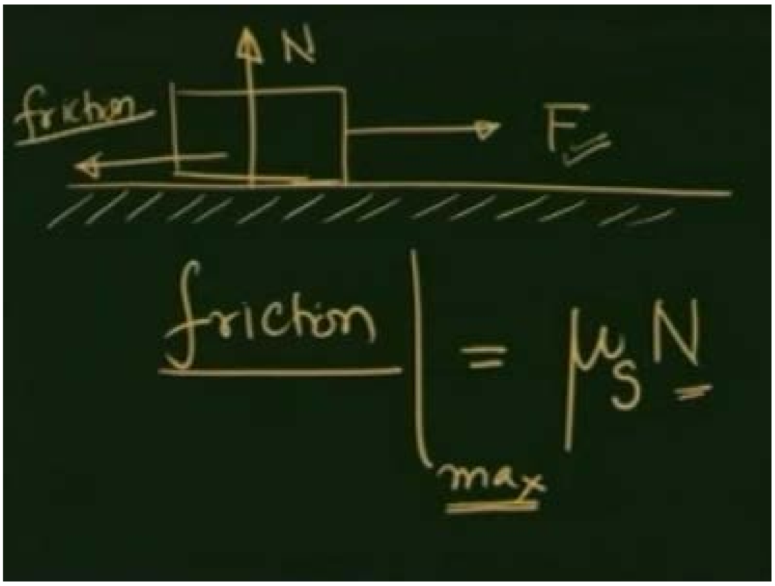

So, for example if I take a mass sitting on a surface and I try to pull it by a force F, it will, its tendency to move would be opposed by this frictional force. It is observed experimentally that the maximum frictional force that a surface can apply on a given mass is given by mu, where mu is a constant times N and I will put a subscript s here, to indicate that this is static friction. Friction is slightly different when the body starts moving, and we will be discussing that.

Notice that, I have written a maximum here and that is to indicate that, that is a maximum possible frictional force that is available. Of course, the friction force, the frictional force adjusts itself as I vary this force. So, the maximum possible is this otherwise, it is always less than just sufficient so, that this body does not move. What is N? N is the normal reaction of the surface on the body. So, N is a force that is perpendicular to the surface on which the body is moving.

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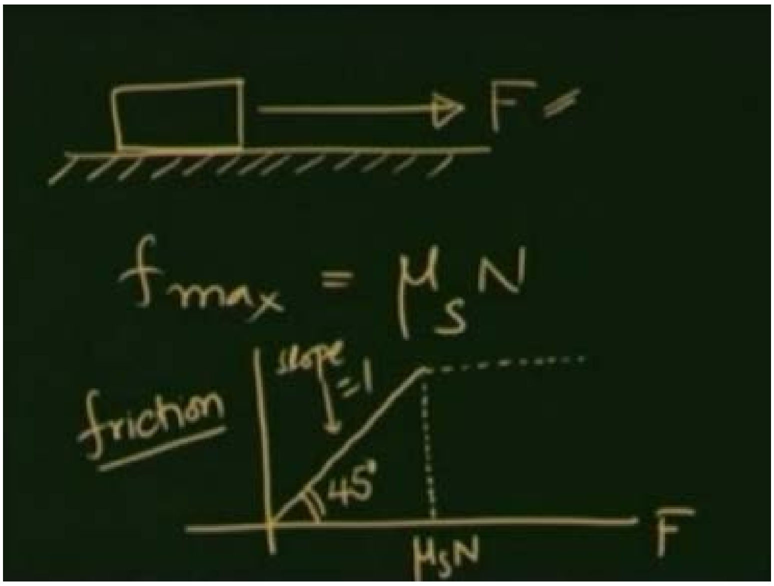

So, the maximum frictional force on a block on a surface is given by f max, which is mu s times N. If I apply a force here, the frictional force adjust itself, so, that the body does not move. If I plot the frictional force against the applied force F which is given here, you would see that frictional force goes exactly as F, this angle being 45 degrees so, that the slope of this line is equal to 1. Right, when the applied force exceeds, the maximum possible frictional force the body would start to move, and the frictional force may become a constant. I am drawing this line because this force drops slightly as I will explain in coming few minutes.

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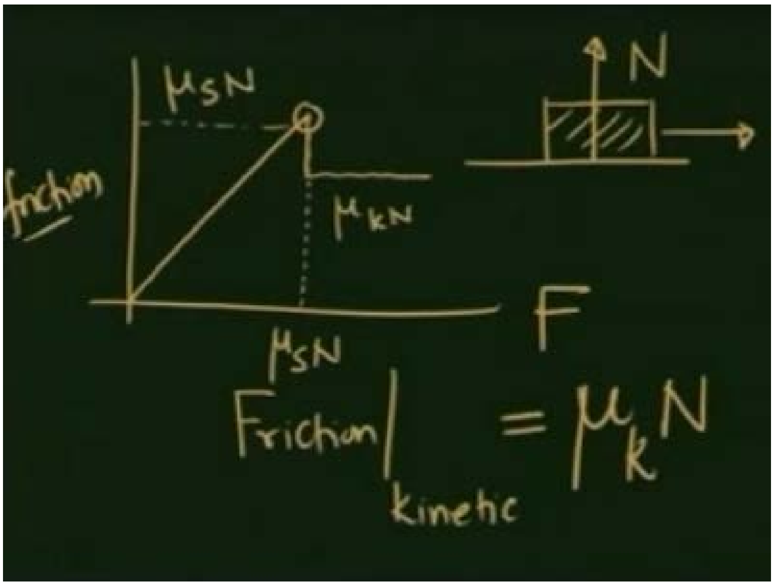

Of course, once the body starts moving the frictional forces slightly less. So, that I write as friction kinetic, and it is given by mu k, the normal reaction.

So, that if I again plot the applied force verses friction, it will go up at 45 degrees until the maximum value of the frictional force mu s N has been reached, N as I said earlier is the normal reaction of the surface. And as the body starts to move, the force drops a bit, but then is a constant, this is equal to mu k N, this point obviously is mu s N.

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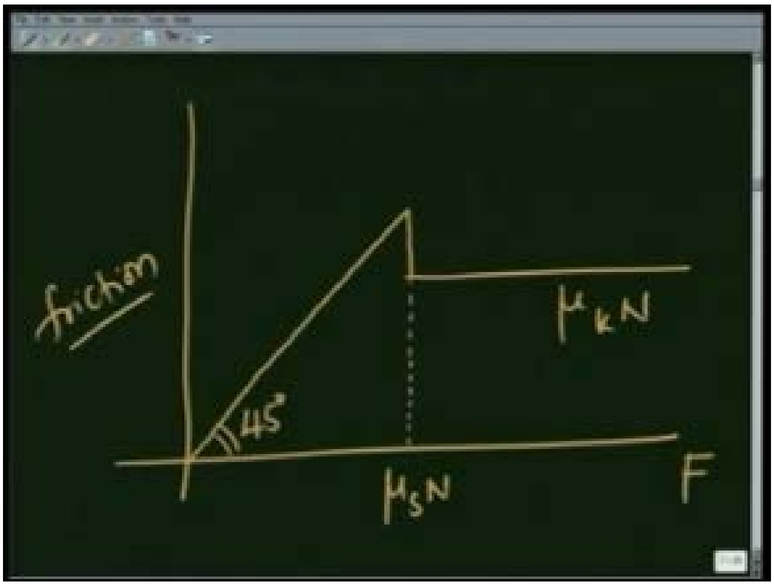

Let me redraw it for clarity so as I take a body, apply a force, the frictional force increases in proportion to the applied force, this as I said earlier is 45 degrees. And then, when it reaches the maximum mu s N, the body will start moving and the friction force drops slightly, this value being mu k N. To give you a feel for what mu s and mu k are, let me give their values in the next slide. For some materials, the value varies from material to material.

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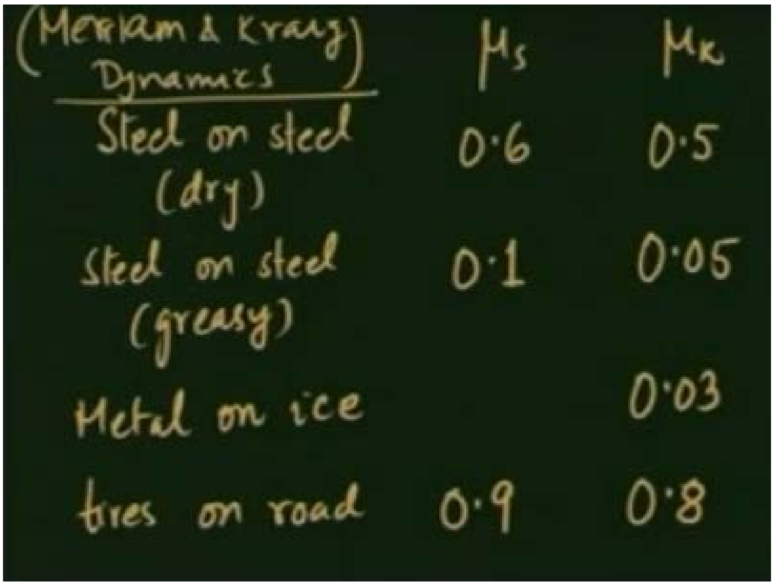

So, this is we will take steel, on steel dry surface. We will take steel on steel, greasy surface, and we will also take metal on ice, and tires on road mu, static for steel on steel is about 0.6 when they are both dry. Steel on steel on greasy is 0.1, you can see why we grease steel, it reduces a friction value quite a lot. Metal on ice, static is not known because slips passed were lot and tire on road is 0.9.

You can see we really need a lot of friction for tires on the road so, that they do not slip. The moment body starts moving, steel on steel drop drops down to 0.5, steel on steel greasy drops down to 0.05, metal on ice is about 0.03, almost friction less tires on the road comes down to 0.8. I must give you the source of this, this is Merriam and Kraig dynamics, that is where I have taken these values from. So, what we have seen is as the body moves on the surface, it can feel either the static friction, and once it starts moving then, there is kinetic friction. Kinetic friction is slightly less than the static friction.

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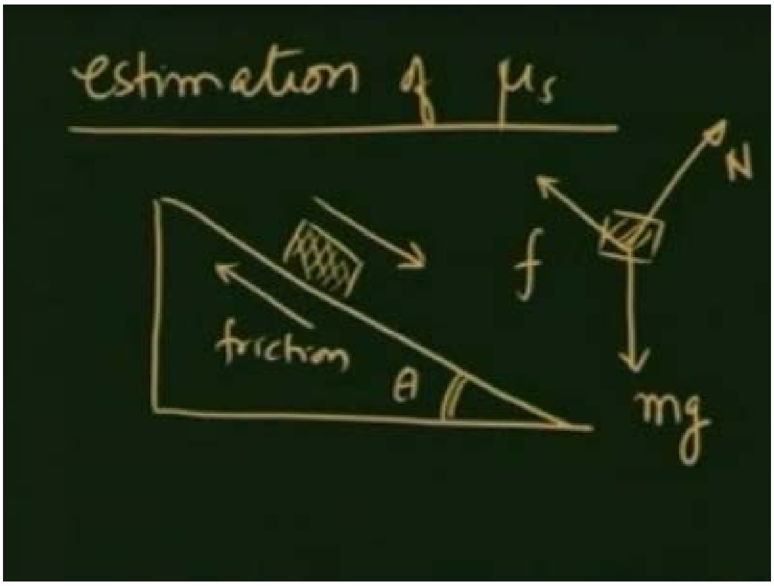

Let me now see, how I can estimate the coefficient of static friction between two bodies. This can be easily done if I consider the motion of a b lock on a ramp or an inclined plane of angle theta. Since, without friction the body has a tendency to move this way, there will be a frictional force trying to stop it this way. If I make a free body diagram of this block, it has its own weight mg pulling it down, a normal reaction N due to the surface, and a frictional force this way. Let us consider the body in equilibrium and see what happens. Value of the coefficient of static friction can be easily estimated.

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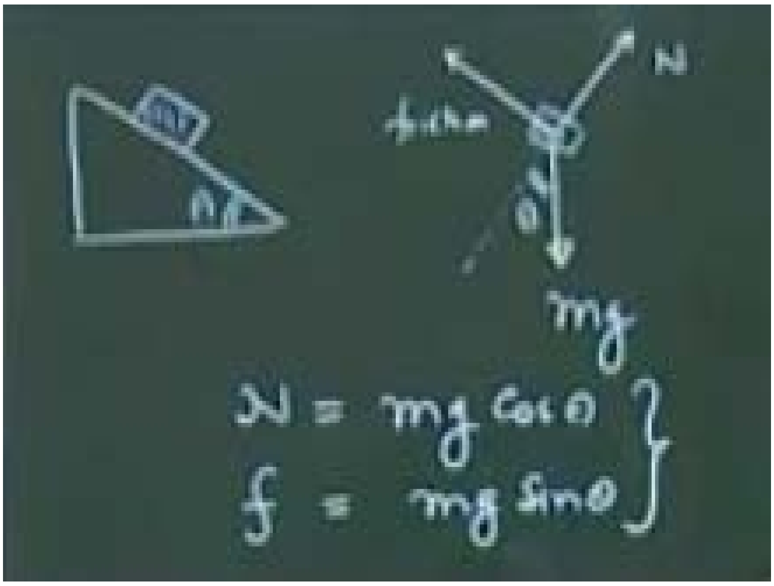

If we consider a motion of a block on an inclined plane of angle theta. And here is the free body diagram as I previously made, there is weight mg, normal reaction N, and the frictional force this way, this angle is also theta. If the body is not moving then, I should have all the forces balancing each other and therefore, N equals mg cosine of theta and the frictional force, let me write it as F is equal to mg sin of theta these are my two equations. Let us see, what happens when I start increasing theta.

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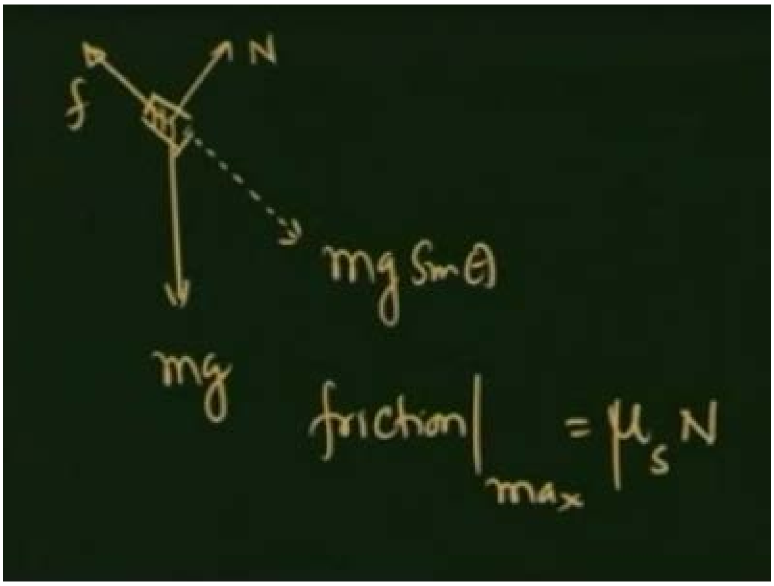

This body has weight mg, N, friction F, and mg this component of mg is pulling it down, which is mg sin theta. As I increase theta, mg sin theta goes up so, the body is being pulled down by larger and larger force. When it surpasses, the maximum possible frictional force it will starts sliding down. Let us see, what is maximum possible frictional force. Friction maximum as I told you earlier, is equal to mu s times N. N as we saw in the earlier.

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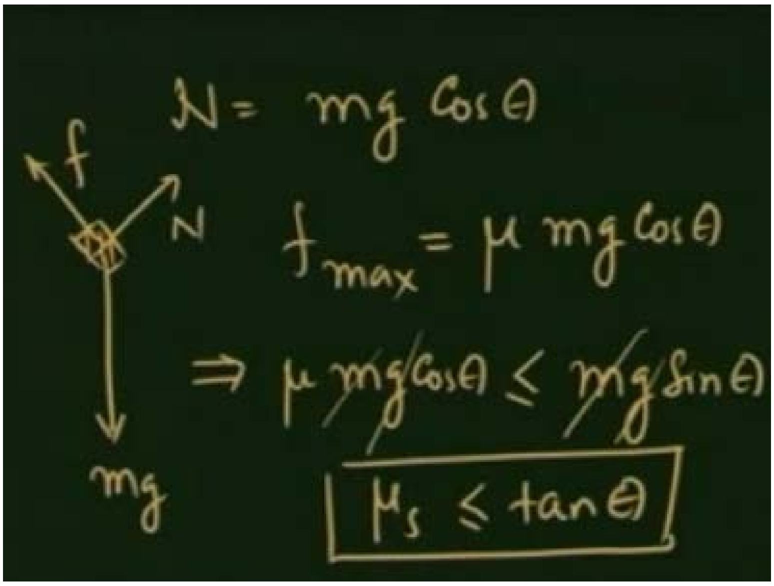

N is nothing but mg cosine of theta because there is no motion perpendicular to the inclined plane. And therefore, f maximum value of friction is mu mg cosine of theta. And this implies that when mu mg cosine of theta, that is the frictional force becomes less than or equal to mg sin of theta, the body would start sliding down. Let me cancel m, let me cancel g and therefore, when mu s less than tangent of theta, the body will starts sliding down. So, you take an plane keep tilting it until the body just starts sliding down, and that angle you take the tangent of would give you an estimate of the static coefficient of friction. That is the practical way of quickly estimating, what the coefficient of static friction is.

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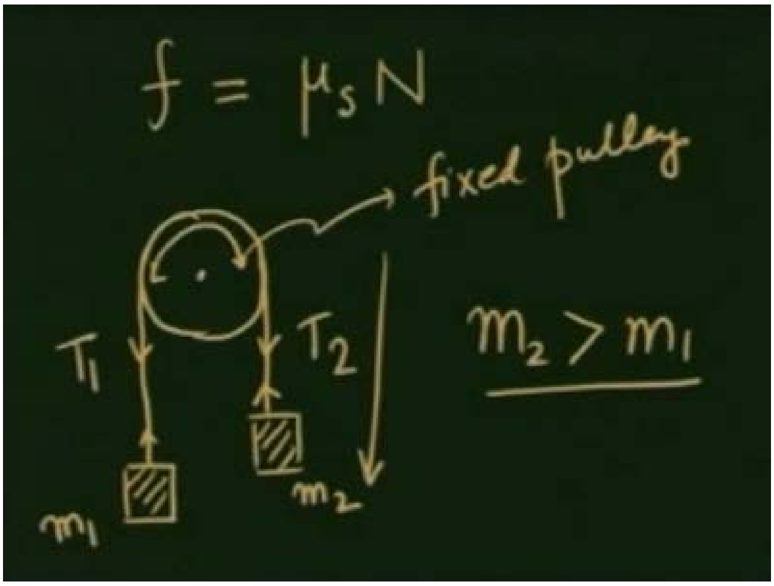

So, we have seen that, frictional force is given by mu, depending on whether it is static or kinetic mu s, or mu k times the normal reaction of the surface on a body. Let me now take you back to acquits machine, that we discussed in the previous lecture. A slight variation of that and see how frictional force changes the tensions here. Recall when I solved this problem in the previous lecture then, I had assumed that all the surfaces are frictionless. Now, let me take the same problem where mass m 1 and m 2 are at two ends of a rope and it is passing over a pulley, pulley is fixed, pulley cannot move.

So, it is a, let me just write fixed pulley, only the rope slides over the pulley, and there is friction between the pulley and the rope. Earlier when we solved this problem, we assumed that the tensions here were equal. Now, due to the frictional force the tensions are going to be different, let me call this T 2, let me call this T 1. Let the mass m 2 be greater than m 1. So, that the rope has a tendency to move this way. Let us see how friction affects things here. I am pretty much repeating what I have done in a previous lecture on static friction. I do it here for completeness.

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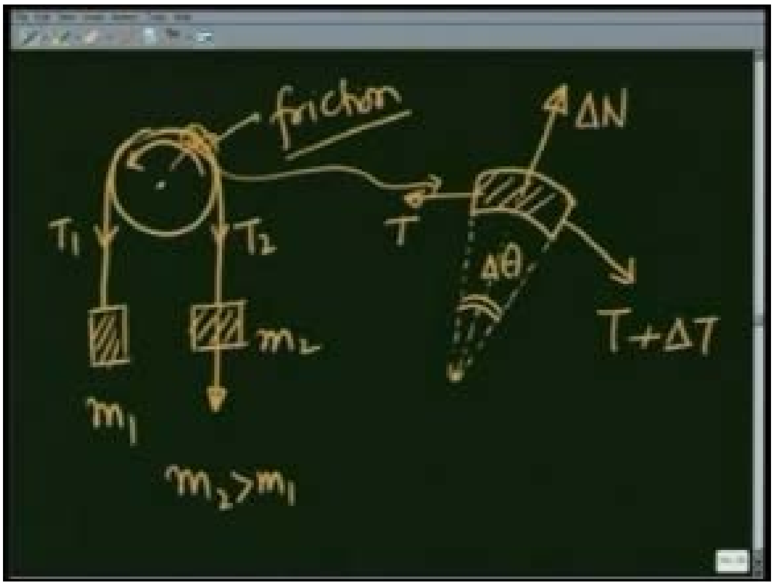

Since, the rope has a tendency to move in this direction, the frictional force on the rope is going to act this way, this is the frictional force, this is mass m 2 as I said earlier, and this is mass m 1. If I want to see how friction changes, let me for that consider a small piece of rope, this is somewhere here that I am making slightly bigger here and consider it is free body diagram. If I join from the center of this to the center of this pulley, there is normal reaction on this small piece of rope, let me call it delta N.

Since, the tension on this side T 2 is going to be greater than T 1 because m 2 is greater than m 1 tension increases this way. Let this tension be T plus delta T, let the tension on this side be T I, I make a neater diagram in the next slide. Let this small piece of rope make an angle delta theta at the center of the fixed pulley. So, let us make a neater picture of this part only.

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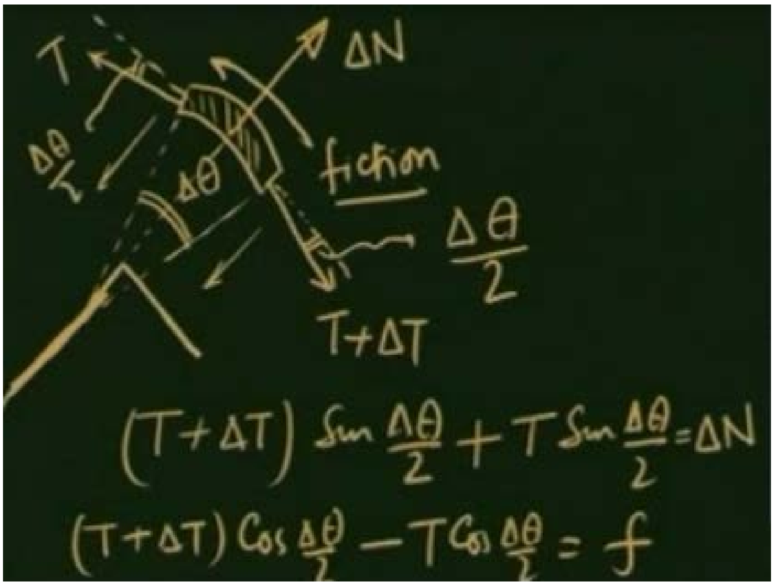

So, here is this piece of rope, it makes an angle delta theta at the center to tension, in this way is T plus delta T tension this way is T, this is the normal reaction, and the rope is in equilibrium. Am I missing any force? Of course I am missing a force because the rope is not moving, there is friction that balances things, balances T, the imbalance between T and T plus delta T so, that the rope does not move. Let us now balance forces. The components of the tension in this direction balance N, if I take the components parallel to this line and perpendicular to this line, this is a direction for N, this is direction perpendicular to it. The components of T in this direction balance N.

And therefore, I am going to have T plus delta T, this angle as you can see is going to be delta theta over 2, and this is a very small angle so, sin delta theta over 2 can be estimated by delta theta over 2 itself, plus T sin delta theta over 2 is going to be equal to delta N. This angle again is delta theta over 2. And the second equation, T plus delta T cosine of delta theta over 2, minus T cosine of delta theta over 2, is going to be equal to the frictional force f.

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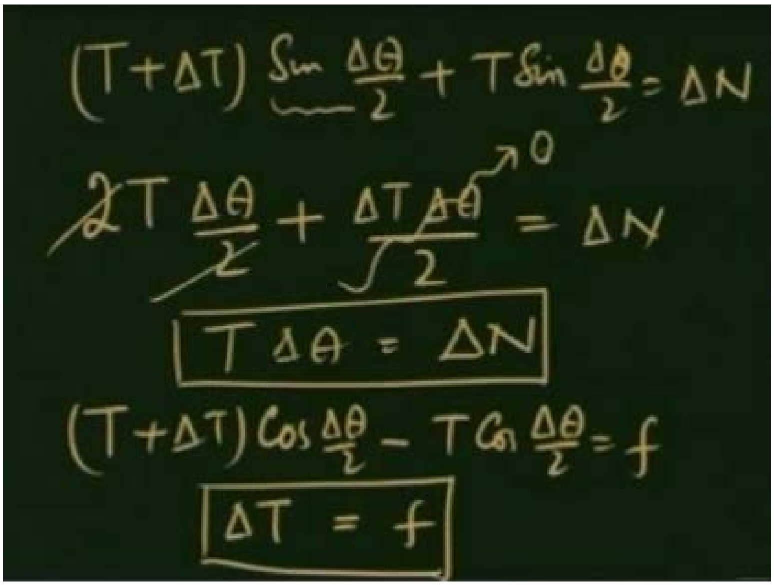

Let us then try these equations again I have T plus delta T, sin delta theta over 2, plus T sin delta theta over 2 equals delta N, which can be approximated for a small delta theta as 2 T delta theta over 2. I replace this whole thing by delta theta by 2, plus delta T delta theta over 2 equals delta N.

This is second order term. So, therefore, it is can be taken to be 0 in the limit that I take delta theta going to 0. So, I have this cancels T delta theta equals N, or which I had earlier written as delta N because this is a very small normal reaction. And the other equation which was T plus delta T cosine of delta theta over 2, minus T cosine of delta theta over 2 equals f. Can in the limit of delta theta going to 0 be written as delta T equals f. These are my two working equations, that will determine for me, how the tensions are related when the friction is present.

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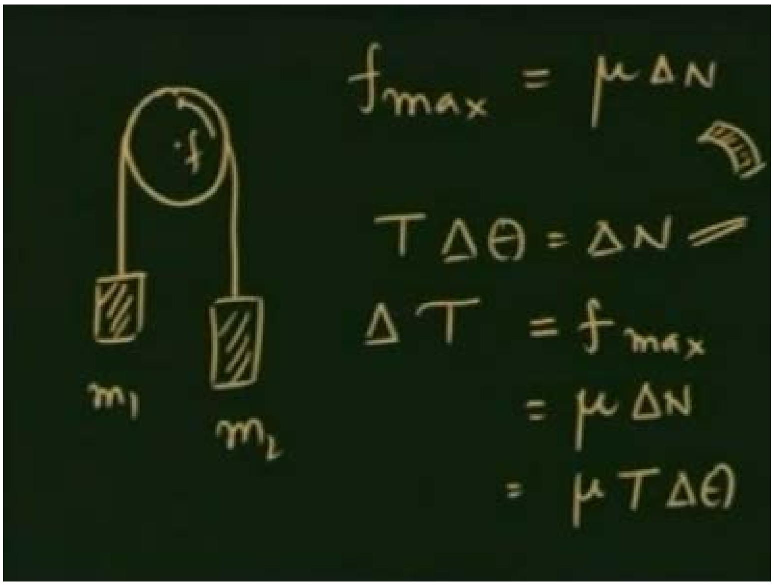

Let us now take the extreme case of when mass m 2 just balances mass m 1, this is the friction. So, that in that case friction is going to be maximum, and that is going to be equal to mu delta N on that small piece. So, I have equation from a layer T delta theta equals delta N, and delta T equals f max which is mu delta N which in turn from this equation is equal to mu T delta theta. So, I have a relationship between the tension change in it with respect to a change in theta.

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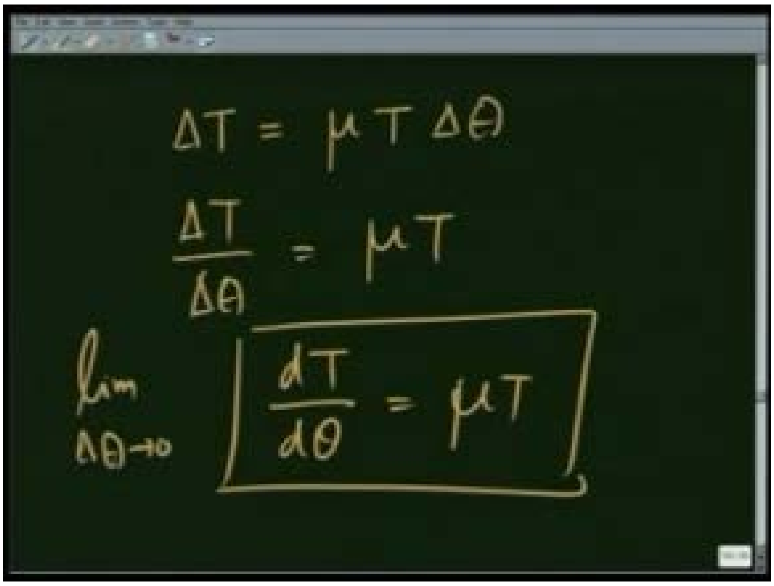

That gives me delta T equals mu T delta theta, or delta T over delta theta equals mu T limit delta theta going to 0, it gives me dT over d theta equals mu T. This equation is quite easy to solve, and you have being doing it in many different places, the solution for this comes out to be.

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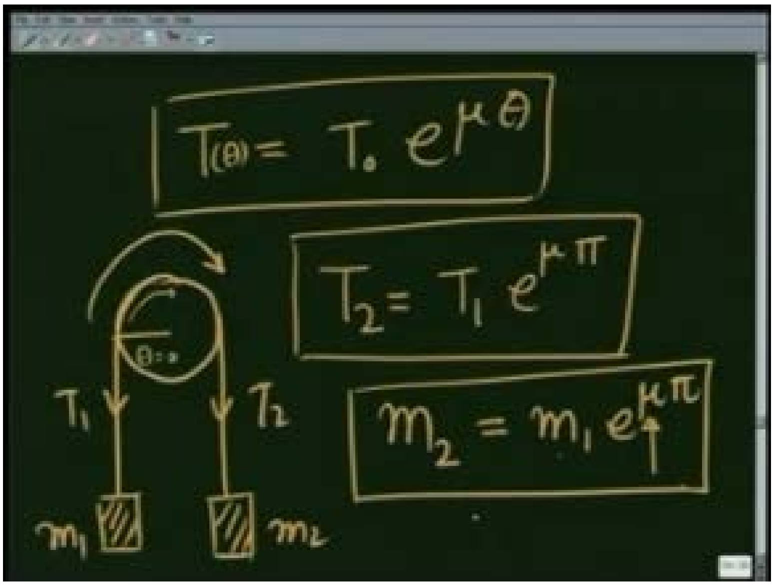

T equals some T 0 e raise to mu theta, where this T is being measured as a function of theta, T 0 is at T theta equals 0. So, let me just again make a picture and show you that this was mass m 1, this was mass m 2, T 1, T 2, theta is increasing this way, T is increasing this way, and if I take this to be theta equals 0, as theta increases theta also increases.

So, that you can see in this case T 2 is going to be equal to T 1 e raise to mu times pi. Since, the tensions balance the masses, I also have m 2 equals m 1 e raise to mu pi, g cancels from both the sides. You can see because of the friction mu, a very small mass m 1 can balance a mass much larger than itself which is m 1 times e raise to mu pi. This has practical uses. In fact, unconsciously if you recall, you have been using it. Suppose, you tie a rope or a clothes line in your backyard, you generally put it on the nail and wrap it around a few times. You are unconsciously using the fact the tension.

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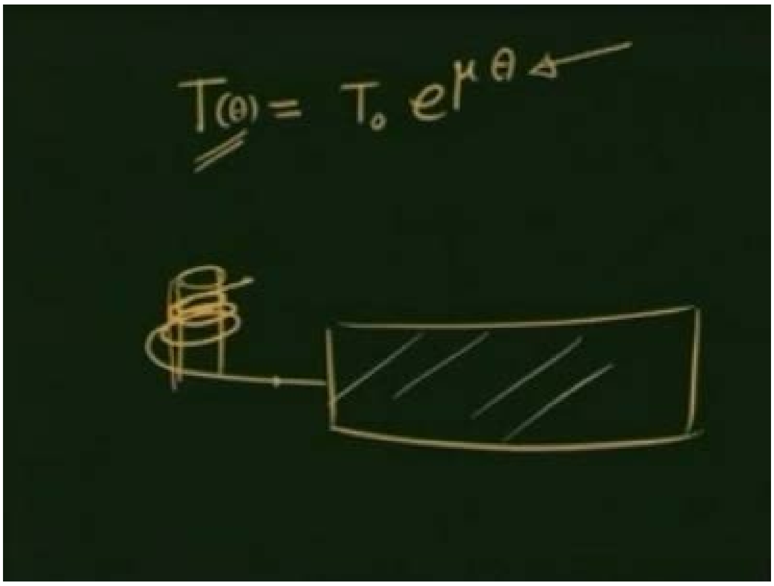

As angle goes up goes as T 0 e raise to mu theta. So, that with a very small force on one side, you can balance a large force on the other side. Another practical use is the capstans on the dockyards where a huge ship is stopped from moving by taking a rope and wrapping it around these capstans. If you wrap it around many-many times, theta goes up so, that very tension on one side, very small tension on one side, a huge tension builds upon the other side and that can stop the ship from moving. You can also do an experiment to check the validity of this expression at home, how do I do that?

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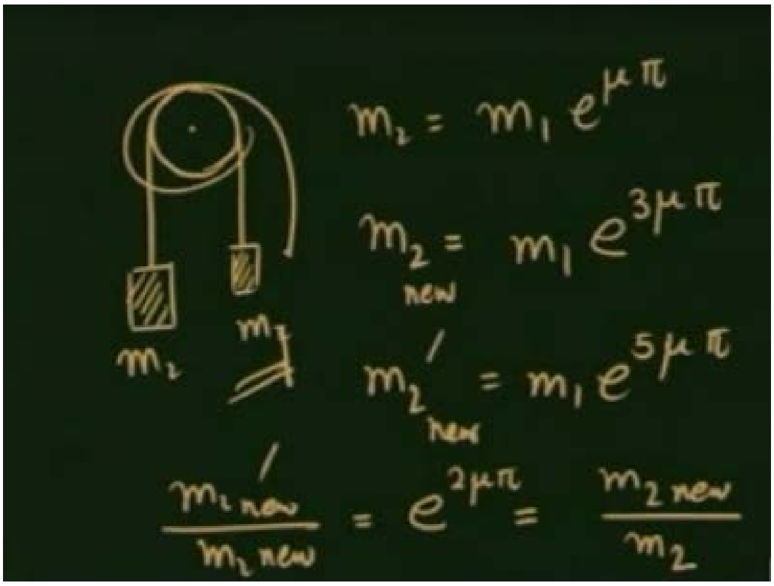

You take a pen and put a small object, may be a pen or a key on one side, and a large object on the other side of a string and try to balance it. As we said earlier suppose, this mass is heavier, m 2 would be in this case be equal to m 1 e raise to mu pi.

So, I will take this pen, wrap a string around it once and this relationship would be satisfied. Let me give it one more wrap, if I do that then I should be able to have m 2, let me call it m 2 new equals m 1. Now, it has gone around one more time so, I add another 2 pi to theta e raise to 3 mu pi. Let it go around one more time, m 2 new prime is going to be m 1 e raise to 5 mu pi. By seeing how much mass can you balance with a given mass as you wrap the string around, you should be able to conform the validity of this formula that we just derived.

You should see that, m 2 new prime divided by m 2 new is equal to e raise to 2 mu pi, and that is also equal to m 2 new over m 2. This ratio would remain the same as you keep wrapping the string around more and more and more. Let us now solve a few problems involved in friction, three of these problems are taken from the textbook of Merriam dynamics.

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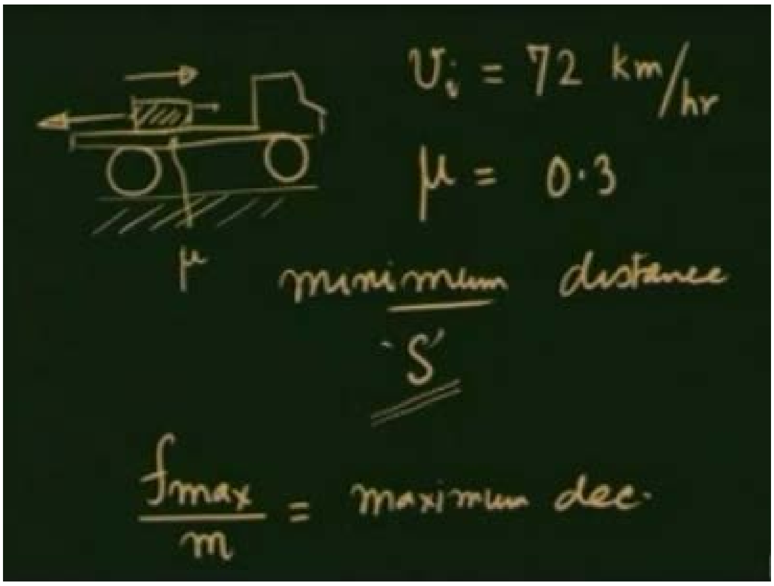

The first problem is if I have a truck moving at a certain speed and suppose there is a box here, the truck suddenly breaks, you know from experience if the truck breaks the crate or the box is going to move this way. If I am given the coefficient of friction here, that is going to resist this motion of the box so, frictional force is going to oppose this movement.

What I want to know is suppose, initially the truck is moving at a speed of 72 kilometers per hour, and it breaks. And suppose mu is given to be 0.3 between the box and the bed of the truck, what is the minimum distance S over which the truck should stop? If it uniformly decelerates so, that the box does not move. To repeat, I am breaking and I want the truck to stop, I want to find the minimum distance S so, that the box does not move.

Obviously, when the truck slows down, I need the box also to slow down, it should not happen that truck slows down faster than the box can slow down. If that happens box will start sliding. So, the maximum deceleration on the box that I can have is f max, the maximum friction divided by the mass of the box is maximum deceleration or negative acceleration. And it is this deceleration that is, that is what the truck can also have. If it slows down faster than that, the box will start sliding, the friction will not be able to stop it.

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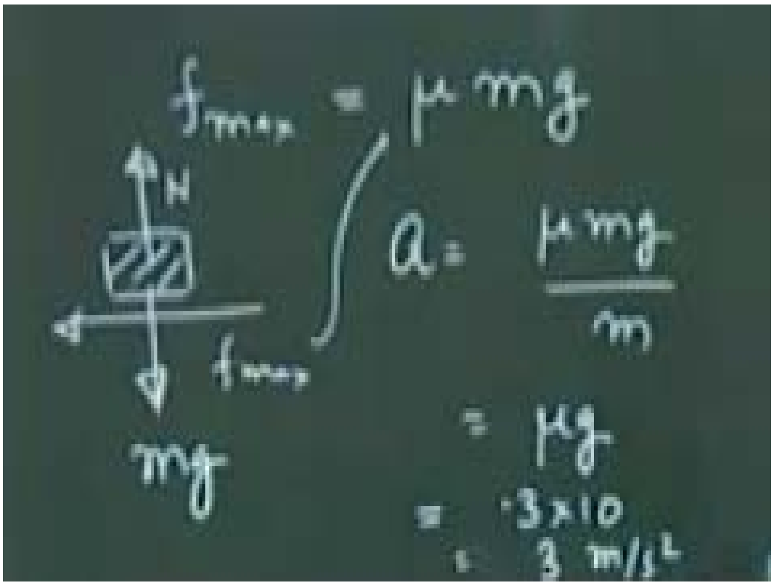

So, let us see what is the maximum friction f max is going to be mu mass of the box times g because for this box which is not moving in vertical direction, N is same as mg. So, f max is given by this formula. So, maximum deceleration a is going to be mu mg divided by m which is equal to mu g, which in our case if I take g to be approximately 10 meters per second square, is going to be 0.3 times 10 which is 3 meters per second square. And that is the maximum deceleration that is allowed for the truck also.

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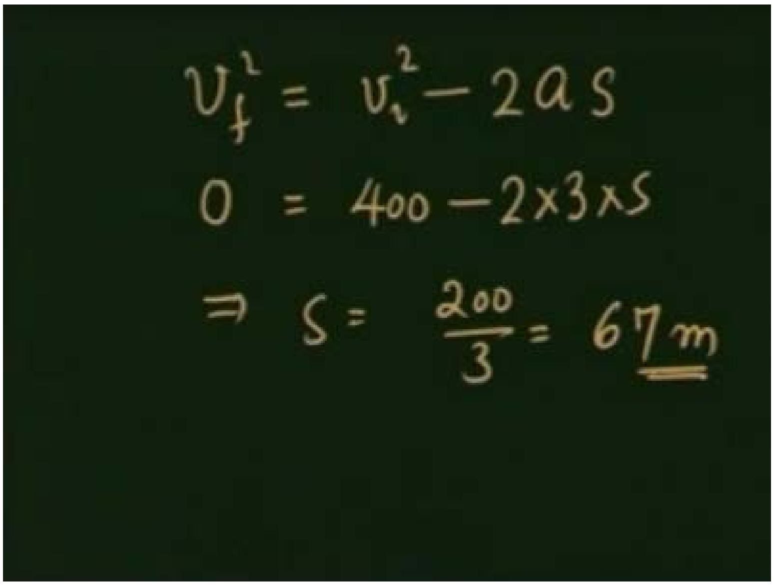

And so, v final square for the truck which is 0 is equal to v initial square minus 2 times the acceleration time s, this is going to be 72 kilometers an hour. Which if you change 2 meters per seconds comes out to be 20 meters per second, and 20 meters per seconds square is 400 minus 2. I have already calculated a to be 3 times s, that is maximum a so, s minimum that much, and that gives you s equals 200 over 3 which is 67 meters. Of course, if the truck slows down and stops at a distance larger than this, I can do with smaller a, that is no problem. But the maximum a allowed is 3 and therefore, s minimum has to be 67 meters. That is one example of solving problems using frictional forces.

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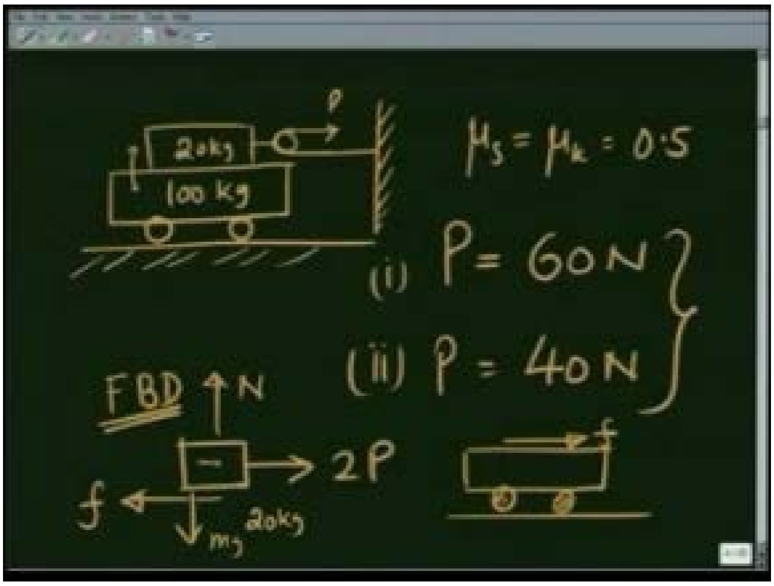

As a second example, I take a crate or a trolley which can move on its wheel without friction . This is taken to be 100 kilograms, on top of it I put a box of 20 kilograms and pull it with a pulley here, pull it by a force P other side of the string attached to the wall. The coefficient of friction whether static or dynamic, let us take it to be the same, between these two masses is given to be mu s almost the same as mu k is equal to 0.5.

And we would like to know, what are the accelerations of the two masses in case P is 60 Newton's case 1. And case 2, when P is 40 Newton's. Let us see what happens in the two cases. Obviously when this P is pulling it since, this is a rope going around a pulley, the net force on the upper mass is going to be equal to 2 P, that I need not go over again.

And since this mass has a tendency to move this way, it will be opposed by a friction f, that is the free body diagram of mass 20 kg. Similarly, the free body diagram for the trolley is going to be there is nothing here, the only force that by Newton's third law is applied on this, is the frictional force in opposite direction f. If I draw it again showing only the relevant forces for horizontal motion.

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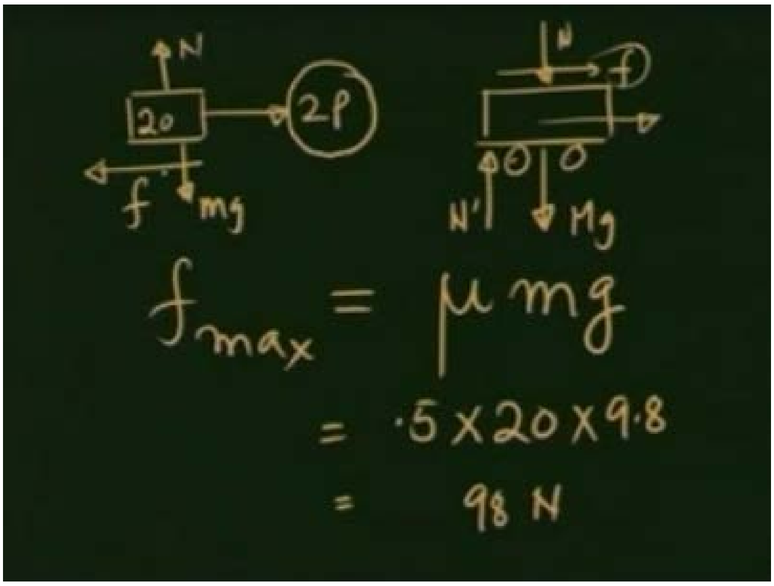

A free body diagram for the 20 kg mass is 2 P this way, friction this way, and for the trolley is friction this way. So, the trolley would move in this direction, and accelerate in this direction due to this frictional force. The 20 kilogram mass will also move in this direction because of 2 P and the force.

Let us see, what P I should apply so, that they do not slide. That would happen when f is maximum possible, let us calculate what is x maximum possible. That is mu mg, mu is given to be 0.5, m is 20 for the upper mass, N, mg, and free body diagram I should also show these forces here mg N prime. And there is an N here, alright times 9.8 and that comes out to be 98 Newton's, and that comes out to be 98 Newton's. And therefore, if 2 P happens to be greater than 98 Newton's, this fellow would accelerate in this direction.

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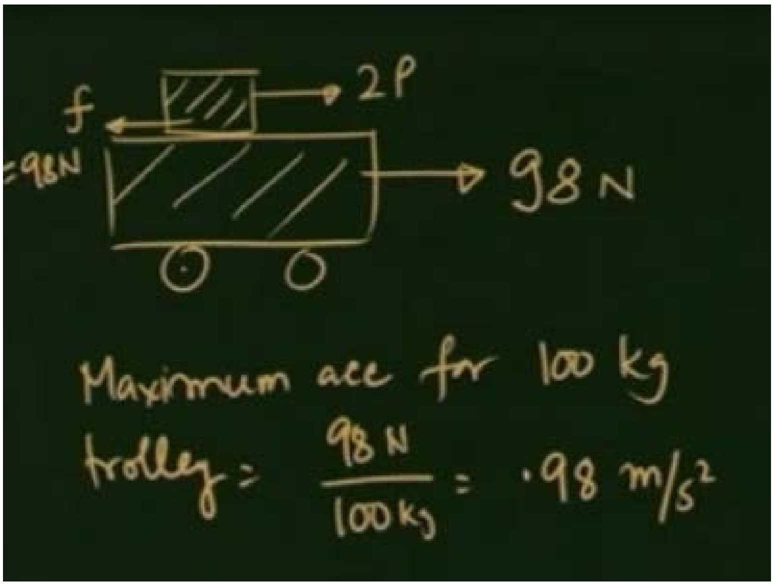

This fellow is moving this way, maximum force it can have in this direction 98 Newton’s, this is has a force 2 P, and this has a force f maximum 98 Newton’s. So, the maximum acceleration for 100 kg trolley is equal to 98 Newton’s over 100 kg, which is 0.98 meters per second square. If the acceleration of the top mass 20 kg is happens to be larger than this then, there will be sliding, if it is less then, there will be no sliding.

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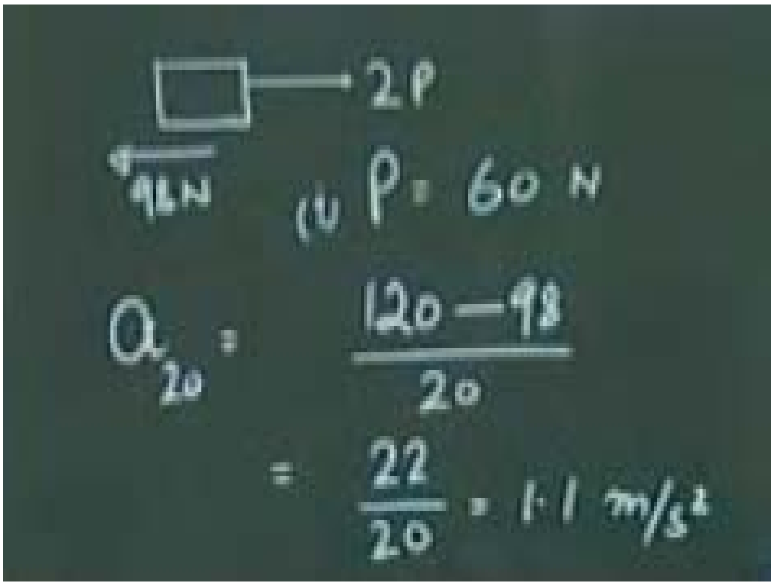

So, let us see what happens when I apply a force of 2 P here, and this is 98 Newton’s. In case of P is equal to 60 Newton’s, which was my case. The acceleration of the upper mass 20 is going to be 120 minus 98 over 20, which is 22 over 20 which is 1.1 meters per second square.

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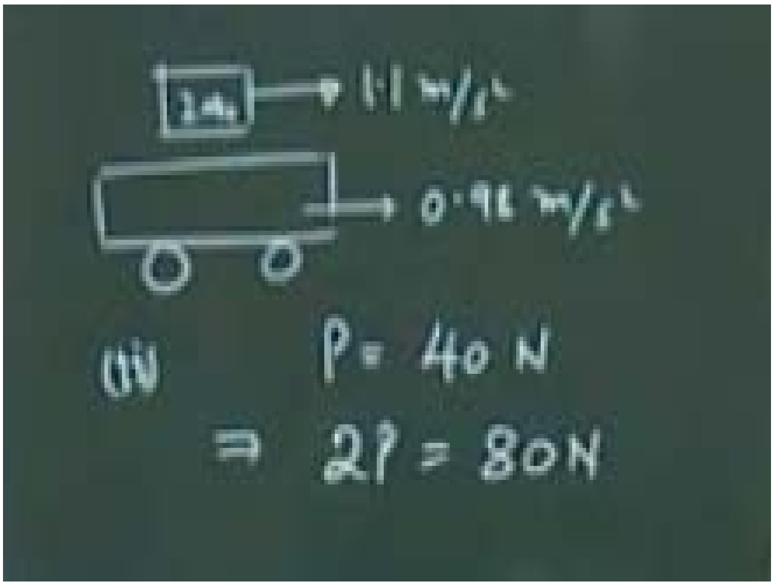

And therefore, in this case the two masses the 20 kg mass and the trolley are going to have acceleration. This is going to move with 1.1 meters per seconds square, and this is going to move to 0.98 meters per second square, and they are going to slip on each other. Let us see, the other case when P is equal to 40 Newton’s which implies the 2 P is going to be 80 Newton’s, and in this case 2 P happens to be less than the maximum force.

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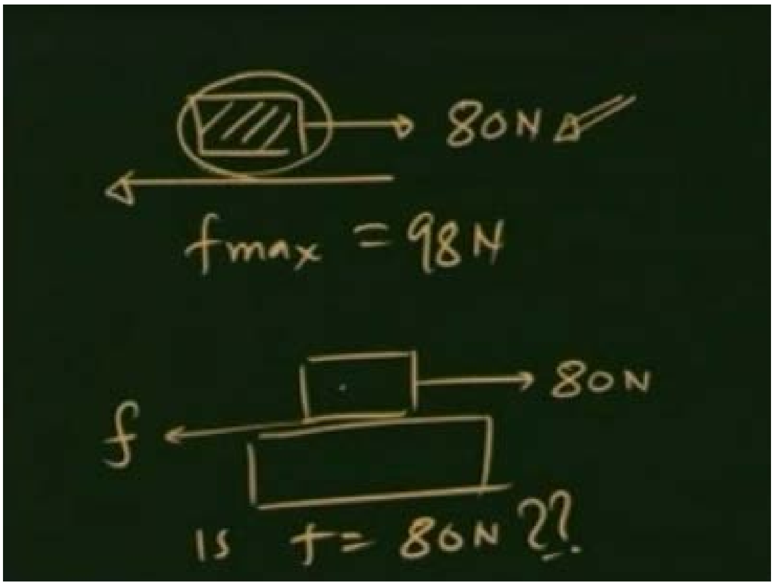

Maximum frictional force, this is 80 Newton's and f max is 98 Newton's. So, the friction will adjust itself just to 80 Newton's so that, this fellow, this mass does not slip. And therefore, you can conclude that, in this case, this is going to be 80 Newton's and there is going to be just sufficient force f so that, this does not slip. Is f equal to 80 Newton's? The answer is obviously no. You can see it in two ways.

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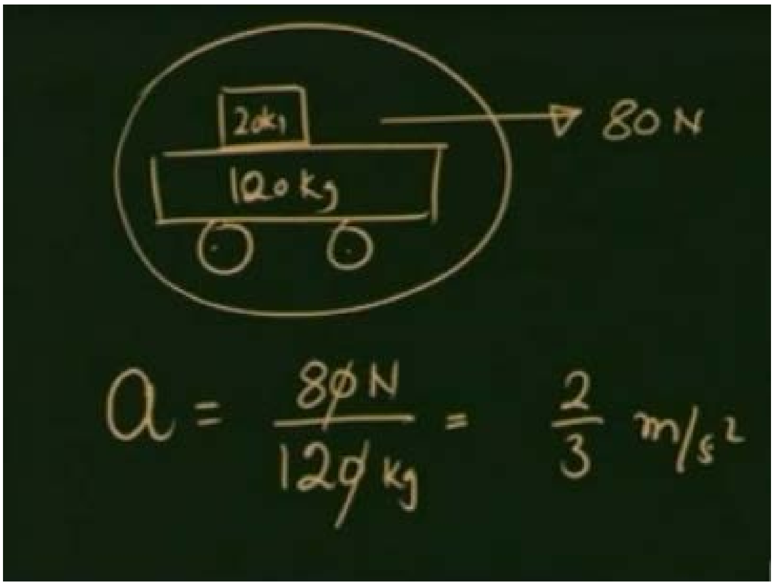

First if I consider the trolley and the mass sorry, 100 kg, together then, on this entire system there is a net force 80 Newton's, there is no relative acceleration, they are moving together. And therefore, their acceleration together is going to be 80 divided by 120 which is Newton's kg, which is going to be two-thirds meters per second square. But what about the frictional force in between? The question we started with is, is it 80 Newton's?

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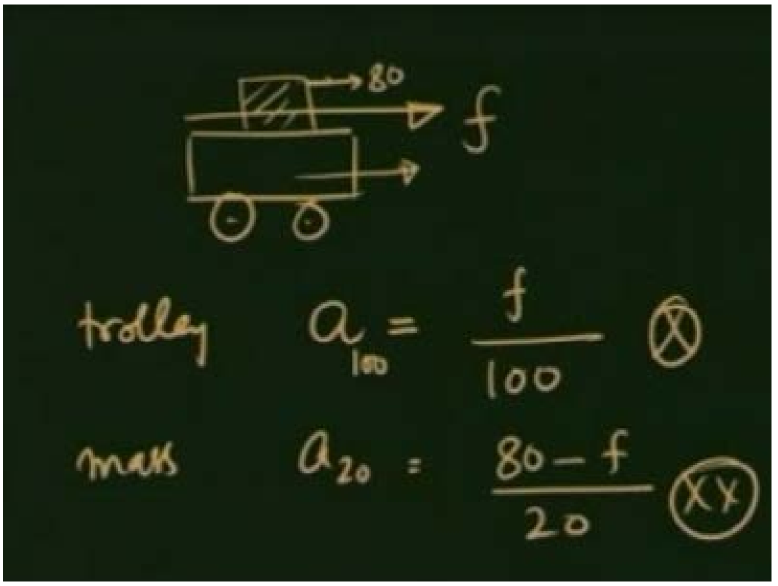

The answer is no because you see the trolley has only one horizontal force acting on it, that is the frictional force, there is nothing to stop it. So, the moment there is friction, this trolley is anyway going to move, if it moves and there is no slipping between the upper mass and the lower mass then, this mass is also going to move and both move in the same acceleration.

Let us look at it from that point of view in that case, the trolley has an acceleration which is going to be equal to the force, frictional force which I do not know over 100. And similarly, the mass, let me write it trolley so, 100 kgs mass 20 kgs is going to have an acceleration, which is going to be 80 with the force with which I am pulling it minus f over 20. And since, there is no slipping because 80 Newton's happens to be less than the maximum frictional force, these two accelerations, this and this one must be equal.

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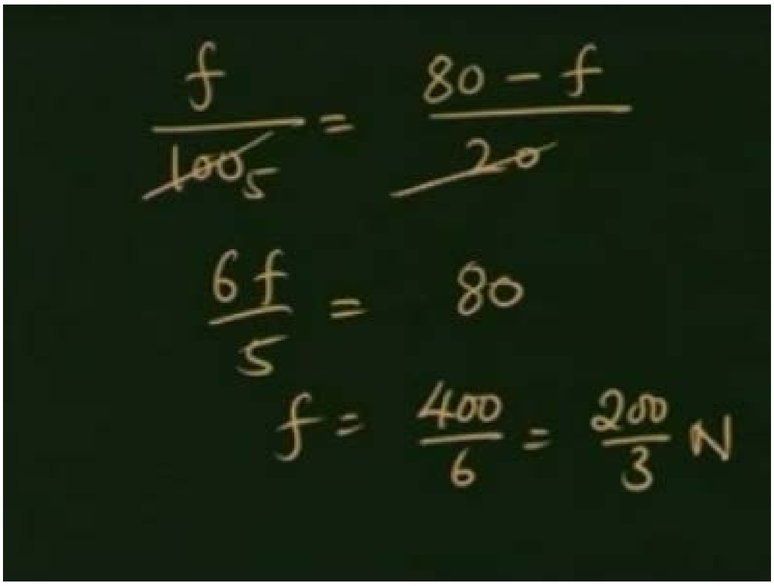

And therefore, I should have f over a 100 equals 80 minus f over 20 that is 5. So, I have f over 5 plus f, 6f over 5 equal to 80 or f equals 400 over 6 or 200 over 3 Newton's.

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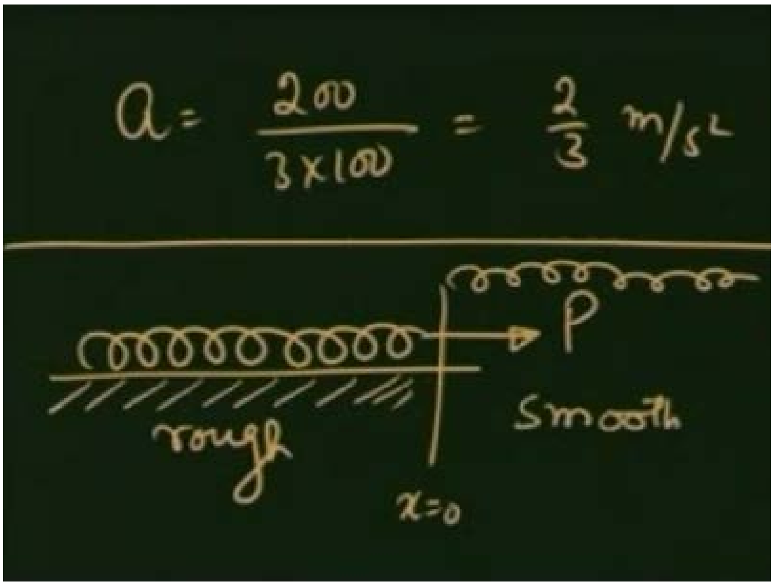

And that gives me the acceleration a, 200 over 3 times of 100 equals two-thirds meters per second square. So, you see how friction changes the acceleration, and how it affects the motion between two bodies that can apply frictional force on each other.

As the third example, let me take a chain which is on a rough surface. So, this is a rough surface, and let us take this to be x equal to 0, and beyond this is, is a smooth surface, this is rough surface. And I start pulling this chain with a constant force P large enough so that, it comes into motion. So, we are not making that any complicated. It starts moving and I want to know when this chain is fully outside the rough, it has come fully out, what is the speed? So, let us now make a picture in between when it has come out.

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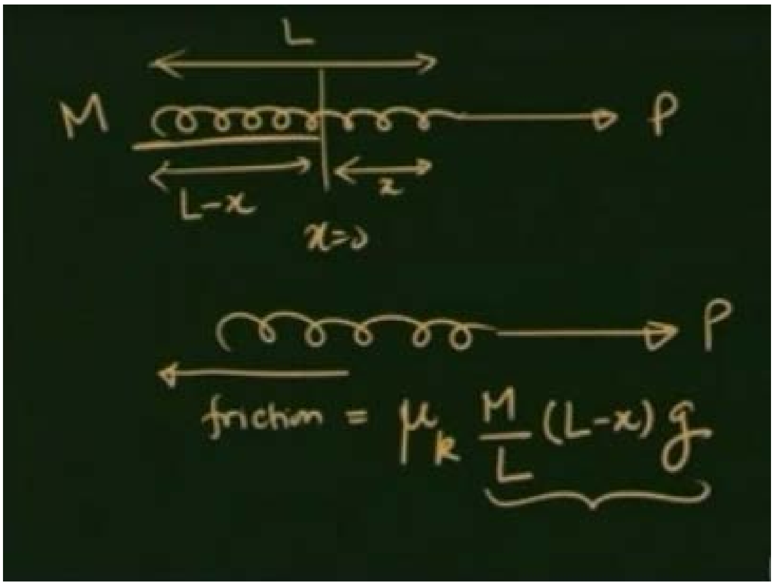

Partially from x equal to 0, the chain is like this, is being pulled by force P, let this distance be x, let the total length of the chain be L, this then it is going to be L minus x, let the mass of the chain be M. And I want to know when the chain has come out to fully form rough, what is this P?

Let us see, what are the forces acting on the chain. There is obviously this force P pulling it this way, there is frictional force, but the frictional force acts only on this part. And so friction. And since, it is moving friction is going to be as its maximum is going to be mu kinetic M over L, L minus x that is the mass of the part in the rough times g. This whole thing is the normal reaction on the part on rough.

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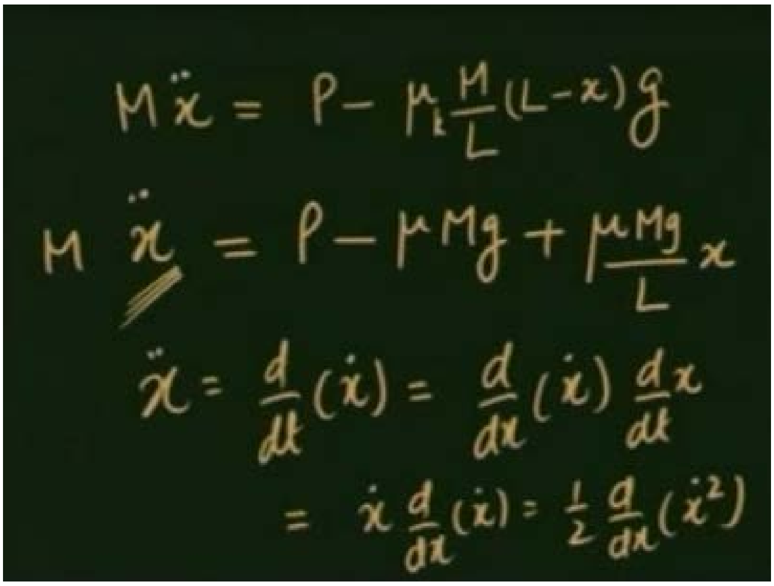

And therefore, if I write the equation of motion the total mass M of the chain moves with acceleration x double dot because the chain has come out by distance x, this acceleration is going to be x double dot, and this is going to be equal to P minus mu M over L, L minus x times g.

So, for simplicity I have dropped the subscript k here, where its understood mu is kinetic frictional force. So, x double dot therefore, is given as P minus mu Mg, plus mu Mg over L x. Since, I want to know, what is the speed when the chain has been fully pulled out? I am not really interested on the variation of x with respect to T so, I use an old trick which I used in the previous lecture. I write x double dot which is really d by dt of x dot in terms of x and x dot. So, this I can write using the chain rule as d over dx x dot times d over dt of x which is nothing but x dot d over dx x dot, which is 1 half d over dx of x dot square.

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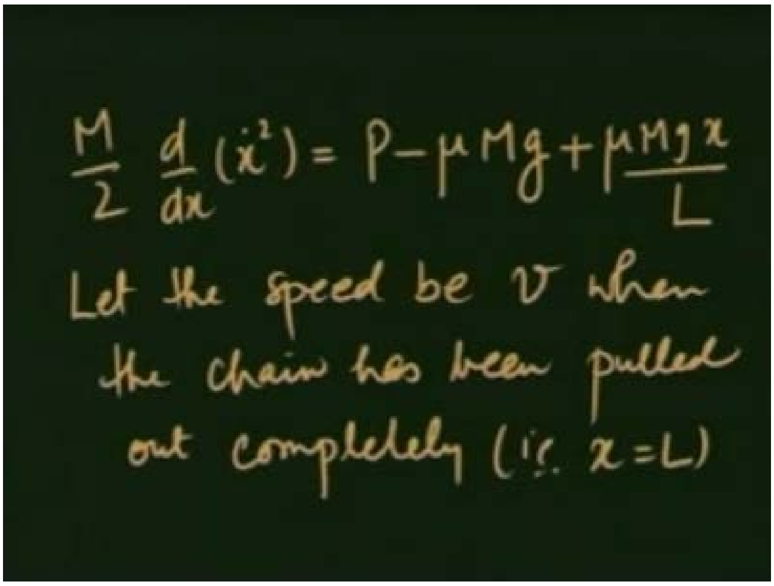

And therefore, my equation becomes M over 2, d over dx of x dot square is equal to P, minus mu Mg, plus mu Mg x over L now, we can easily integrate it from x equals 0 to L. So, if I do that and, let the speed be v when the chain has been pulled out completely that is x equals L.

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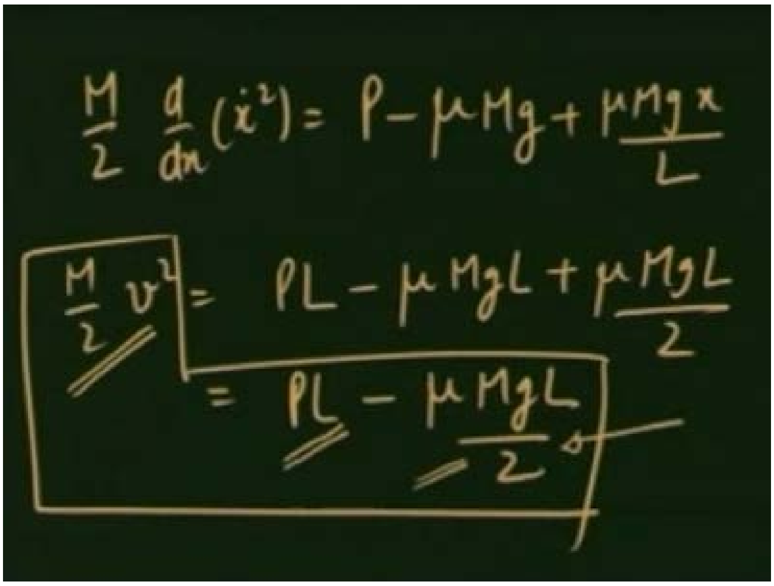

So, by integration I get I am integrating the equation M by 2, d over dx x dot square equals P, minus mu Mg, plus mu Mg x over L, by integration I get M over 2 v square equals PL, minus mu Mg L, plus mu Mg L over 2, which is nothing but PL minus mu Mg L over 2. And that gives me the speed of a chain after it has been pulled out fully.

We will see in our later lecture, we talk about work energy theorem that, this is a gain in kinetic energy which is equal to the work done by the external force which is PL. When the force moves the chain by distance L, minus the average frictional force, which at the highest is mu Mg, and the lowest is 0. So, average is mu Mg by 2 times L, this is the loss due to friction. So, energy gained by P work being done, minus loss due to friction is equal to the kinetic energy, and this is how we can calculate the kinetic energy.

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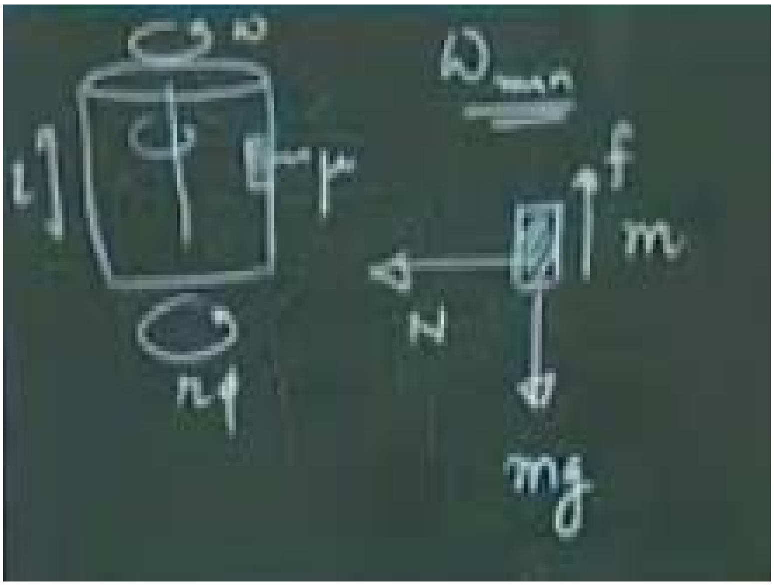

As a last example, let me take a cylinder which is moving around x axis an angular frequency omega, and let me take a mass here which is free to slide, but because of this rotation, it stuck on the wall. The coefficient of friction between the wall of the cylinder and mass M is mu.

I want to know, what is omega minimum when this mass is stuck on the wall? If I may take a free body diagram of this mass m, it is being pulled down by its own weight, and the friction opposes this and there is a normal reaction N. These are the three forces that are acting on this mass. Again since, this is a rotational motion, I am going to go to cylindrical polar coordinates because it is rotating this way, I will use r and phi this direction, and for this direction I will use z. So, let us see what happens to this mass.

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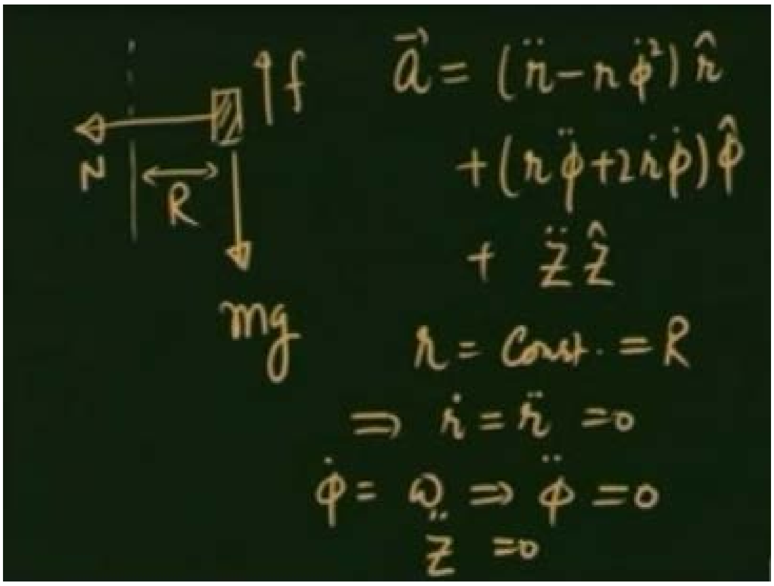

This is at a distance equal to radius of the cylinder from the center is being pulled down by weight mg, there is force N on it, and there is frictional force f on it. Its acceleration in cylindrical coordinates is going to be r double dot, minus r phi dot square r, plus r phi double dot, plus 2 r dot in phi dot in phi direction, plus z double dot z in z direction.

However, in equilibrium when it is moving stuck on the wall, r is constant and this is equal to R and therefore, r dot equals r double dot equals 0. Similarly, phi dot is fixed to be omega and this implies phi double dot is also equal to 0 and it is not moving up and down therefore, z double dot is also equal to 0. Therefore the acceleration of the mass is given as.

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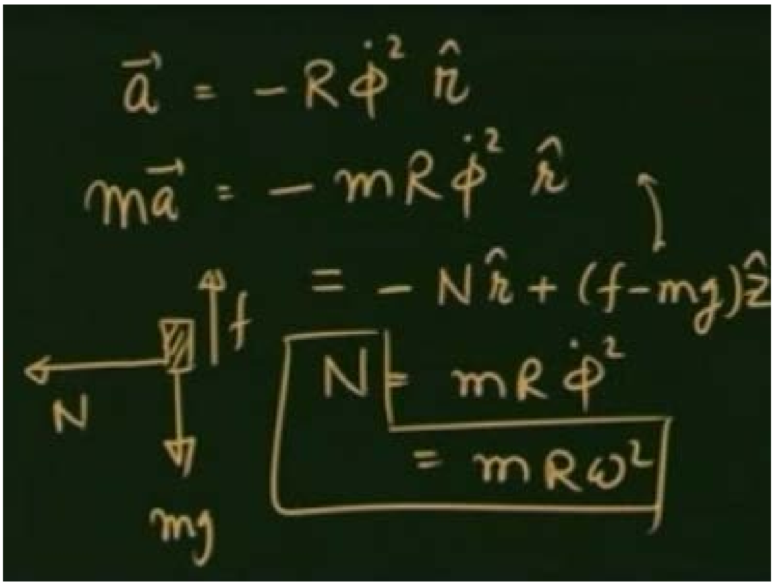

A is equal to minus R phi dot square r, ma therefore, is minus mR phi dot square r. How about the forces on the mass? There is a force in negative radial direction N, there is mg and there is friction. So, this should be equal to N in negative r direction, plus f minus mg in z direction. Equating the two sides, we get N is equal to mR phi dot square which is nothing but mR omega square.

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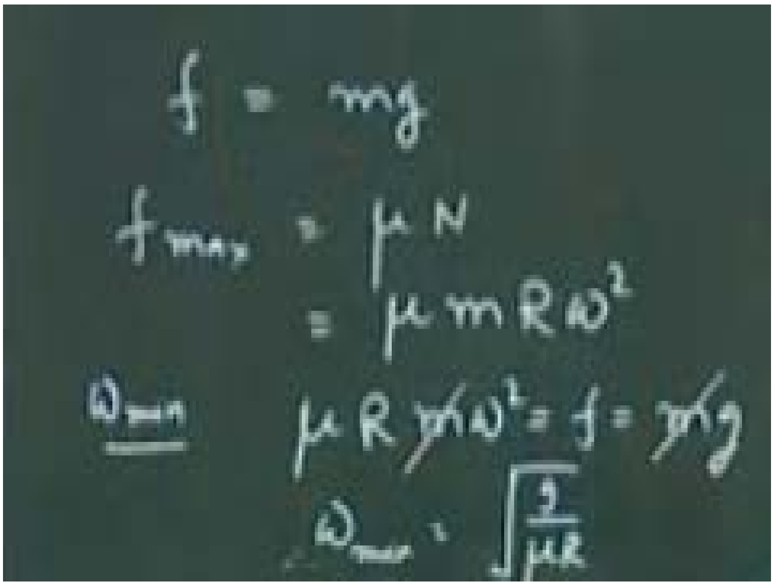

And the other equation gives me f equals mg. Remember f max is mu N so, that is going to be mu m R omega square. For the minimum omega. I should be applying the maximum possible frictional force for that omega. And therefore, for omega min my condition becomes that, mu R m omega square is equal to f equals mg, this m cancels and I get omega minimum to be square root of g over mu R, and that is your answer.

So, what we have seen in this lecture is, how frictional force acts on two bodies when they slide or tend to slide over each other, and how we use this in solving problems. In the next lecture, we are going to look at another form of frictional force which arises when bodies move through fluids, and that is the viscous or drag force.