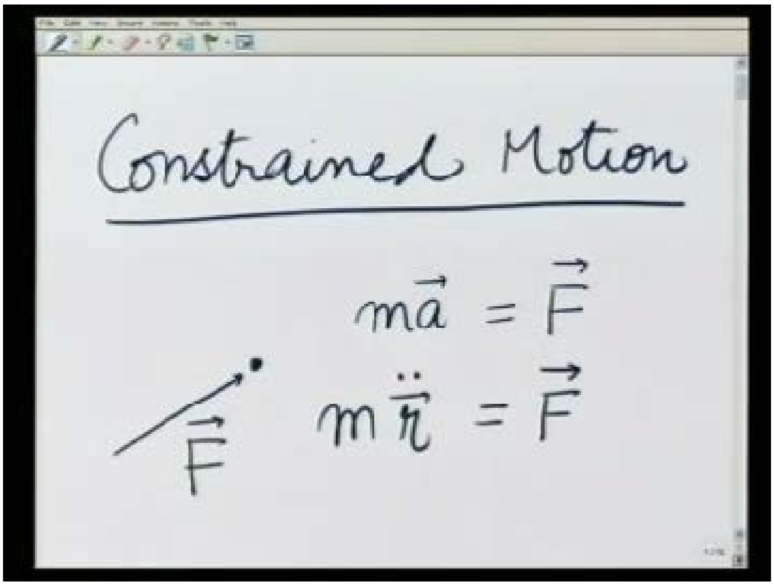
Engineering Mechanics
Prof. Manoj Harbola
Indian Institute of Technology, Kanpur
Module - 05
Lecture - 02
Motion with Constraints

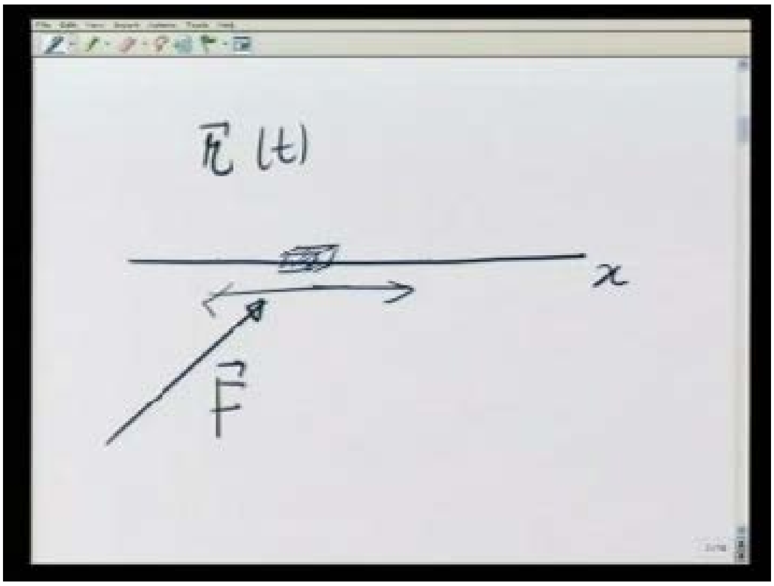
In the previous lecture we talked about two different coordinate systems that, we use to describe the motion of particle moving under the influence of a force. In this lecture, we start with how to solve the equation of motion when the particle is moving under constraints.

(Refer Slide Time: 00:41)



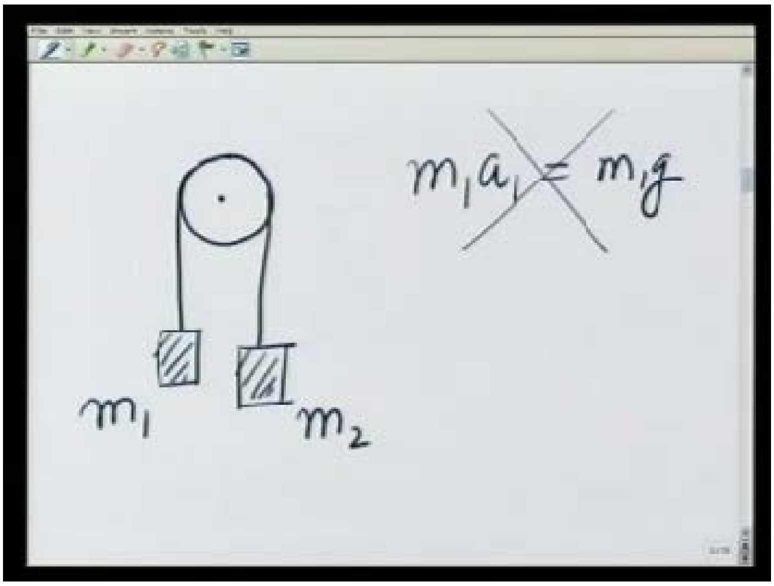
So, what we are going to talk about is, the constrained motion. And I will explain to you, what we mean by that and how do we go about solving this. Suppose, there is a particle that is moving under the influence of a force F then, it is motion is described by equation of motion. That is given as ma equals, the applied first where m is the mass of the particle and a is its acceleration written in terms of its coordinate. I can write this as m r double dot equals F, where double dot denotes second derivative with respect to time. Of course, if the particle is free to move that means, it can go anywhere then, I solve this equation and get r as a function of time given a force.

(Refer Slide Time: 01:41)



On the other hand, if I restrict its motion and I will talk in a minute about, what I mean by restricting it is motion. The motion can be quite different for example, I could take a bead moving on a straight wire and the bead can move only along this wire and let us say, this wire is in x direction then, no matter what force I apply on this bead. The bead is going to move only along the x direction therefore, its motion is restricted.

(Refer Slide Time: 02:28)

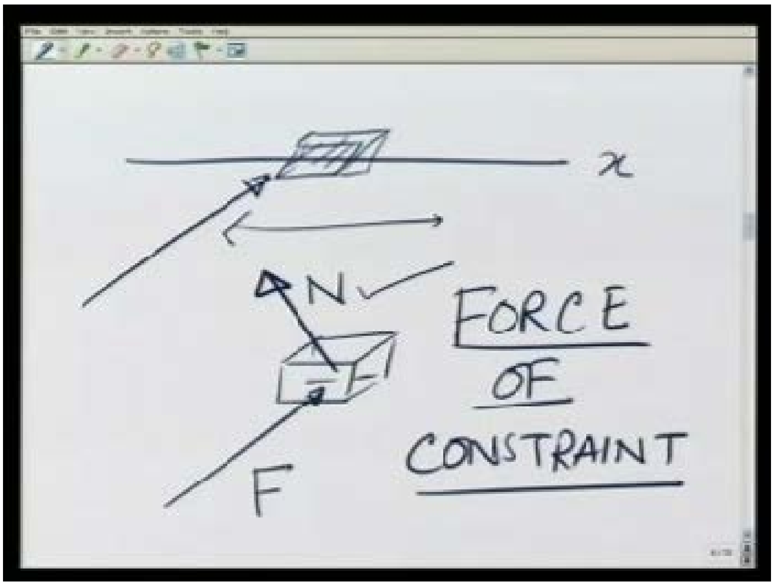


Has another example, which is quite well known and also known as acquits machine is two masses moving on a pulley when they are tied by this string.

In this case, the masses are no longer free to move that means, if I have this mass m 1 and the other mass m 2, m 1s motion is not determined by this equation alone. That is its motion is affected, this is not correct, its motion is affected by the presence of the other mass. Similarly, motion of m 2 is also affected by the presence of mass m 1 and this constraint comes about. Because of this rope that keeps them moving together.

These are two examples of constrained motion, and how to deal with this when the motion becomes quite complicated or number of masses increase, or number of constraints increase is going to be the topic of discussion in today's lecture. So, let us start with our first example.

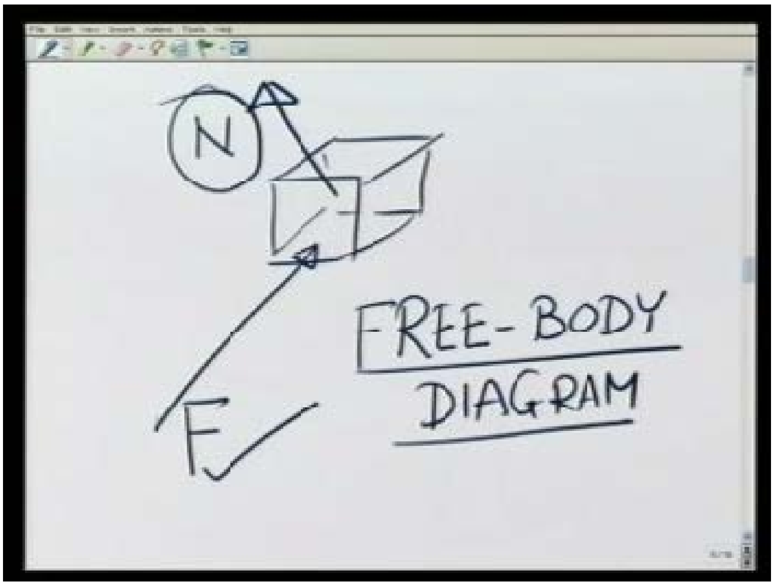
(Refer Slide Time: 3:39)



That is a mass or a bead moving on a straight wire as I set in x direction let us say, through this example, I would like to introduce terminologies that are going to be in use, when we talked about, when we talk about constrained motion.

So, if I apply a force on this, what does the wire do? The wire applies in normal reaction on this body. Why does the wire apply in normal reaction on this body? Because it does not want it to move anywhere except along the wire. So, it is this normal reaction N that is really restricting the motion of the particle along the wire, this is called a force of constraint is to catch to recap it. I will again say that, the effect of this wire on this body.

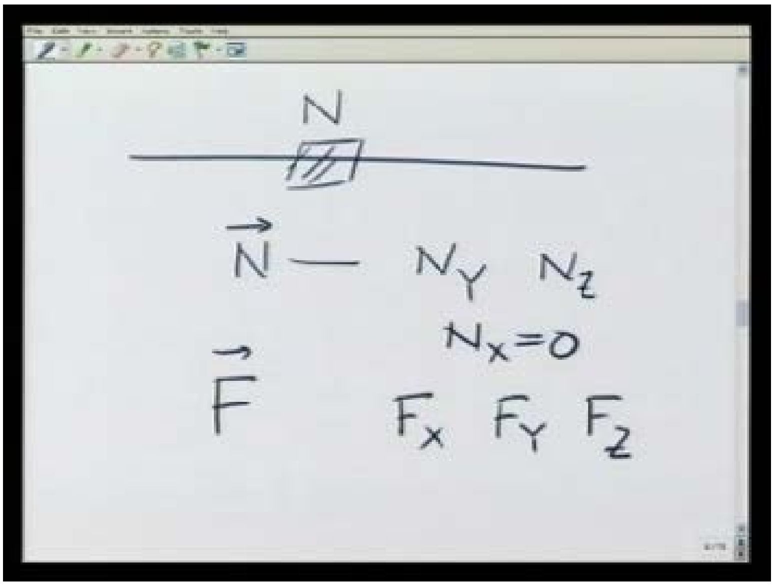
(Refer Slide Time: 04:50)



I am making it slightly bigger now is just to produce a force N or a force of constraint. If you look at this body, I have replaced the wire by this force of constraint, this is known as the free body diagram of it is a particle or a body under motion.

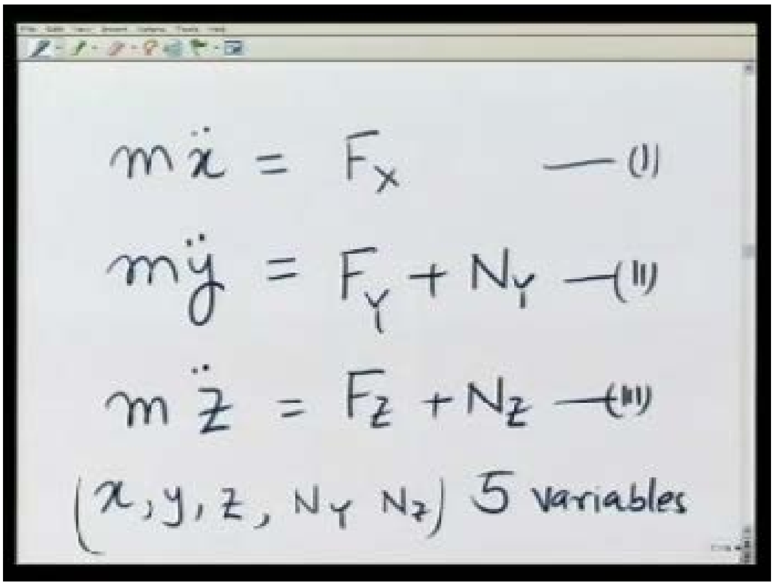
So, what do we mean by free body diagram? By that I mean I will make the body by itself nothing else and represent all the forces applied externally or applied by the constraints on it, and that I will call free body diagram. Let us see how do, I write the equations of motion for this body.

(Refer Slide Time: 05:51)



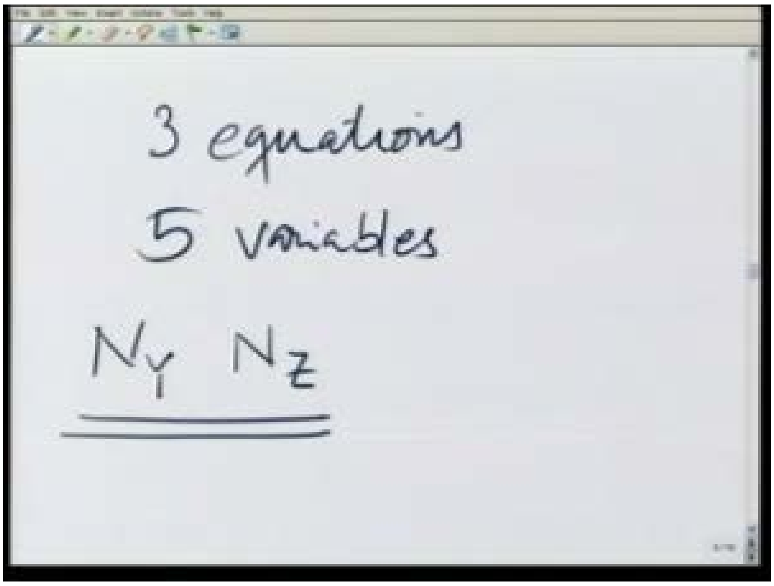
Since, the normal force is going to be perpendicular to the wire. So, N is going to have only two components in y direction and in z direction Nx is always going to be 0, the force F that I have applied, why should write a vector here? It is going to have all 3 components Fx, Fy and Fz, with these components of the forces, the equations are of motion of this particle are going to be m x double dot equals Fx.

(Refer Slide Time: 06:22)



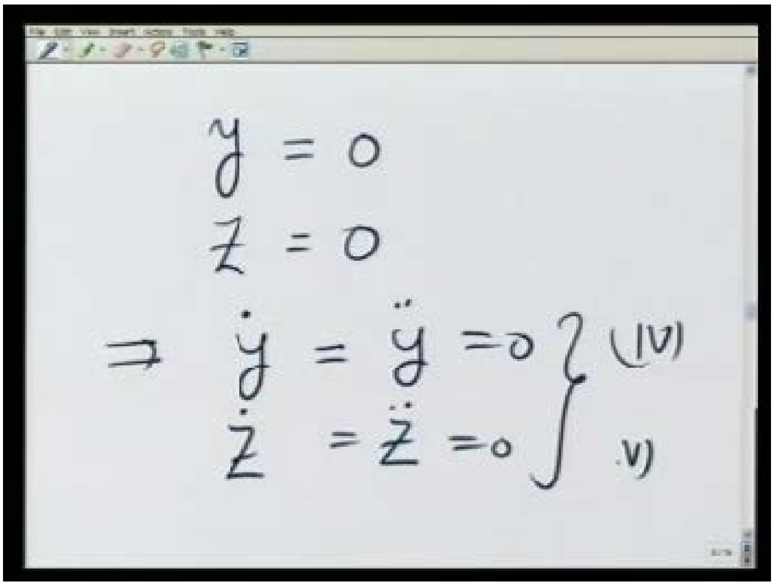
Because Nx is 0 m y double dot equals Fy, plus Ny and m z double dot equals Fz, plus Nz, these are 3 equations of motion that, I get from F equals ma. Let us see how many number of variables there are, there are x, y, z, Ny, and Nz, five variables. On the other hand, I have only 3 equations.

(Refer Slide Time: 07:12)



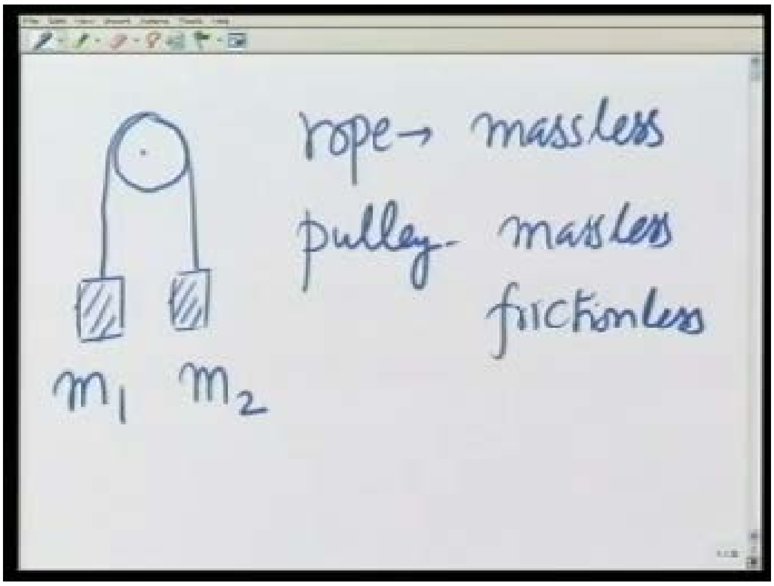
So, I have 3 equations and five variables. Obviously, with this I cannot solve the problem, where do I get the two other equations from? And this is where I start thinking the two of these variables Ny and Nz came because I impose the constraint. So, the equations should also come from the constraints themselves and that precisely where they come from the equations of constraints are that.

(Refer Slide Time: 07:50)



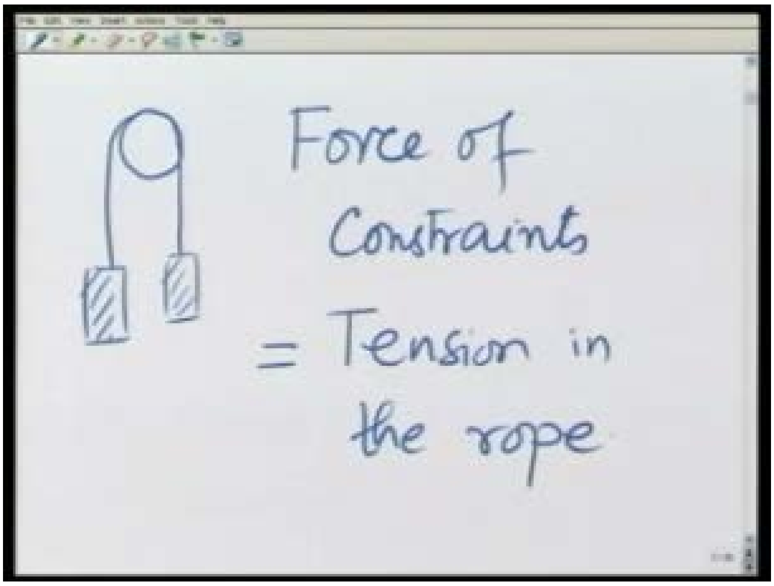
y is equal to 0 and z is also equal to 0, and this implies y dot or y double dot, both are 0 and so is z dot and z double dot. So, I have two more equations, total five equations and five unknowns and I can solve the problem. Let us now look at the second example of constrained motion that, I considered earlier.

(Refer Slide Time: 08:16)



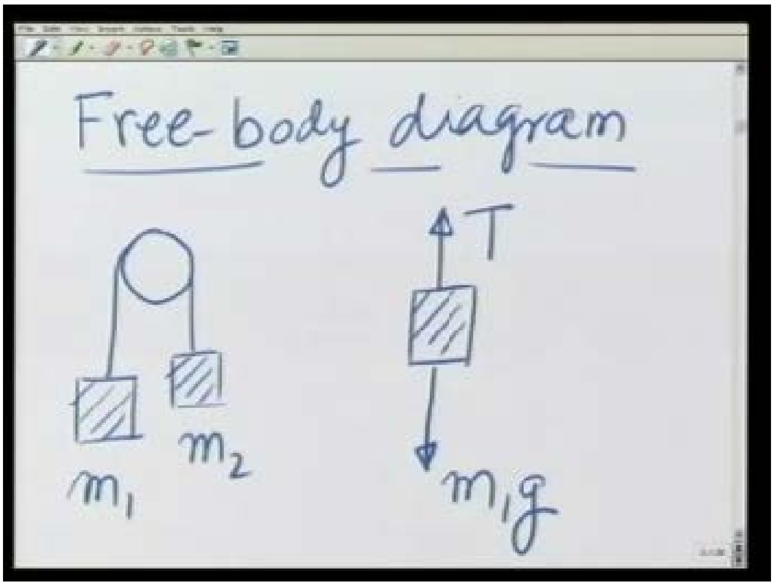
And that was acquits machine in which two masses are tied on a string that is passing over a pulley, that is mass less and frictionless and so is the string. So, let us write again this is mass m 1, this is mass m 2, rope or the string is mass less for simplicity and pulley is mass less as well as frictionless. You may have solve this problem time and again in your previous courses, but what I want to focus on here is, how to really deal with it from the constrained motion point of view. And therefore, first thing I am going to do is.

(Refer Slide Time: 09:12)



See what is causing the constraint, the constraint is caused by this rope that forces the two masses to move together. What is the force of constraint here? The force of constraint that really makes the masses to move together is nothing but the tension in the rope. So, tension in the rope is the one that keeps these masses together. next if I go to make the free body diagram for the two masses.

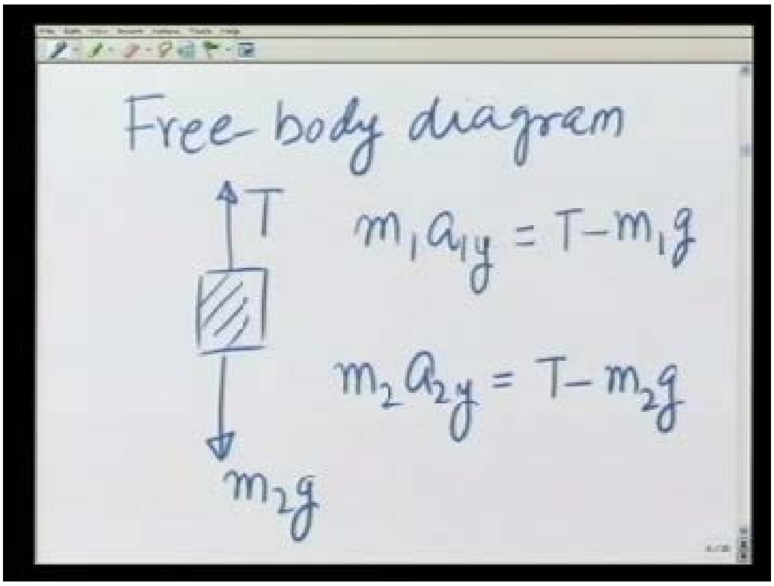
(Refer Slide Time: 09:50)



Let me remind you again, what a free body diagram is. A free body diagram is where I replace the agency causing the constraint by the force of constraint there it applies.

So, if I were to write, the free make the free body diagram for mass m 1, it will be something like this, that there is a force pulling it down m 1 g the gravitational force. And there is tension T, which is the force of constraint pulling it up. And that is the free body diagram for mass m 1. Similarly, I can make free body diagram for mass m 2.

(Refer Slide Time: 10:42)

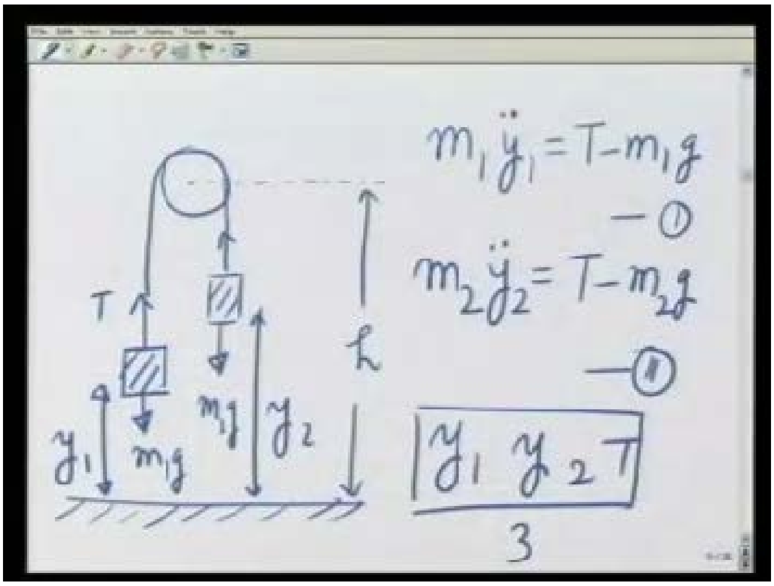


That will also be similar m 2 g pulling the body down, and tension T pulling it up. You may ask me at this point, why did I keep the tension the same? The tension remains the same because the pulley is mass less, frictionless and so, and the rope is also mass less.

And therefore, it does not require any force to move them. and that is what I said earlier. I had taken mass less rope, mass less pulley, and frictionless pulley to keep things simple because I want to focus on the constraint part of it. Now, let us write the equations of motion, if you recall from the free, if you recall from the previous slide, the m 1 g and m 2 g are acting down, and T is acting up on both the bodies.

So, equation of motion is going to be m 1 a 1 in y direction is equal to T which is pulling it up minus m 1 g. m 2 a 2 in y direction is equal to t, minus m 2 g, you may ask I am writing a 1 y, a 2 y which way is my y and things like those. So, let us be more precise, let me measure my distances from the ground.

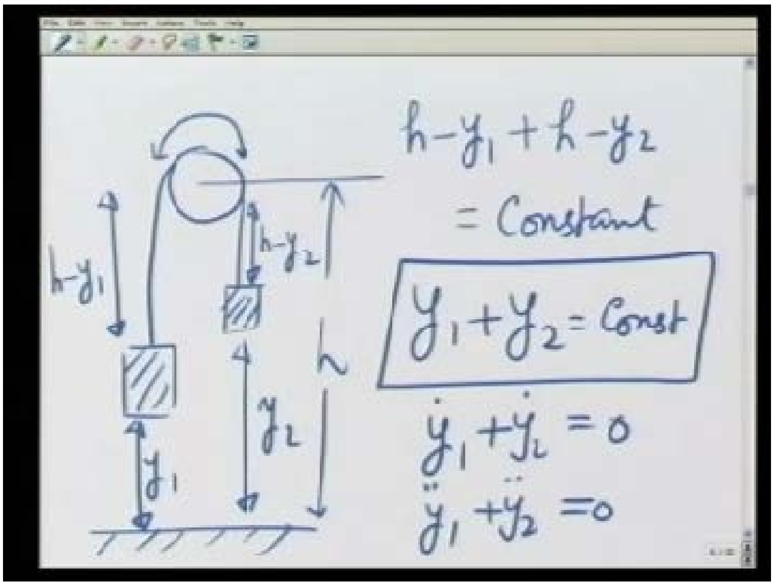
(Refer Slide Time: 12:08)



And be more precise because that will also help me later to write my constraint equation. Let this be the ground, let the distance of mass m 1 be y 1 from here, let the distance of mass m 2 be y 2 from here, let the center of this fixed pulley be at height h from the ground. And now, I can write this is tension T pulling it up, m 1 g pulling it down, tension T pulling it up, m 2 g pulling it down.

And therefore, I can write m 1 y 1 double dot is equal to T, minus m 1 g, notice that all the directions and everything is correct. I must take care of that because we are dealing with vector quantities although I am writing this in 1 dimension. Similarly, m 2 y 2 double dot is going to be equal to T minus m 2 g. I have gotten my two equations of motion, but again the number of variables are y 1 y 2 T, 3. So, one equation that is missing must be provided by the constraint itself.

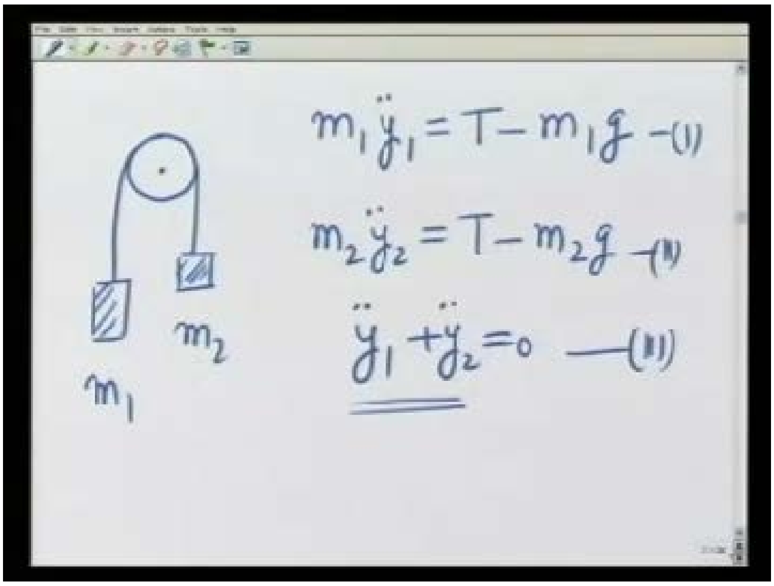
(Refer Slide Time: 13:47)



And that is where this picture comes in handing. We call again the constraint that, the two masses move together is provided by the fact that, the rope is of fixed length. And I can express this mathematically as this part which is h minus y 1, plus this part which is h minus y 2, plus this part is of constant length.

Since, this part is always of constant length, I can write my constraint equation as h, minus y 1, plus h minus y 2 is equal to a constant. Or shuffling terms around, I can write this as y 1, plus y 2 is equal to a constant. This is my third equation, to bring it to usable form in the equations of motion I differentiated and then, get y 1 dot plus y 2 dot equals 0, and also y 1 double dot plus y 2 double dot is equal to 0. So, let us write the equations once more.

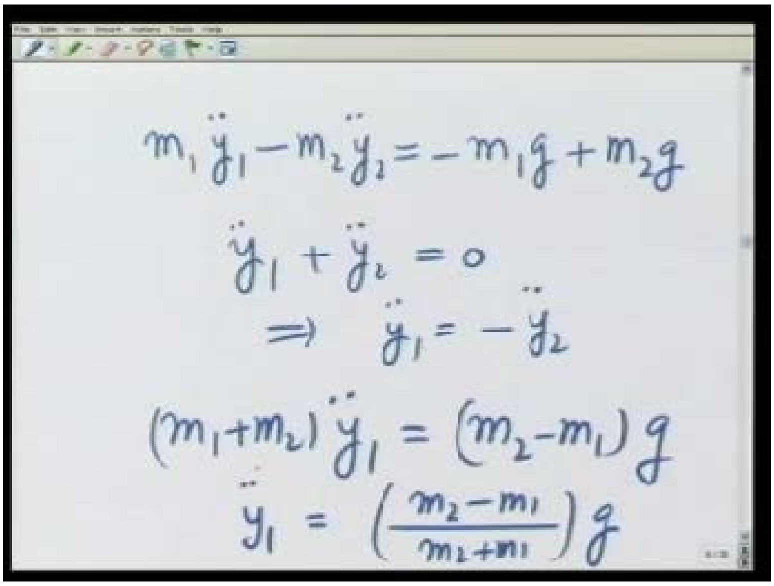
(Refer Slide Time: 15:15)



Again I am solving this problem of two masses attached to a string, which is passing over a fixed pulley. I have m 1 y 1 double dot equals T minus m 1 g m 2 y 2 double dot equals T minus m 2 g and y 1 double dot plus y 2 double dot equals 0.

I have 3 equations and 3 unknowns, and I can solve my problem. Notice this constraint equation, just tells you that when one particle is moving up, the other one is going down, one particle is accelerating up, the other part is accelerating down. Let us now solve these equations, let us eliminate T first. So, for that I subtract equation 2 from equation 1.

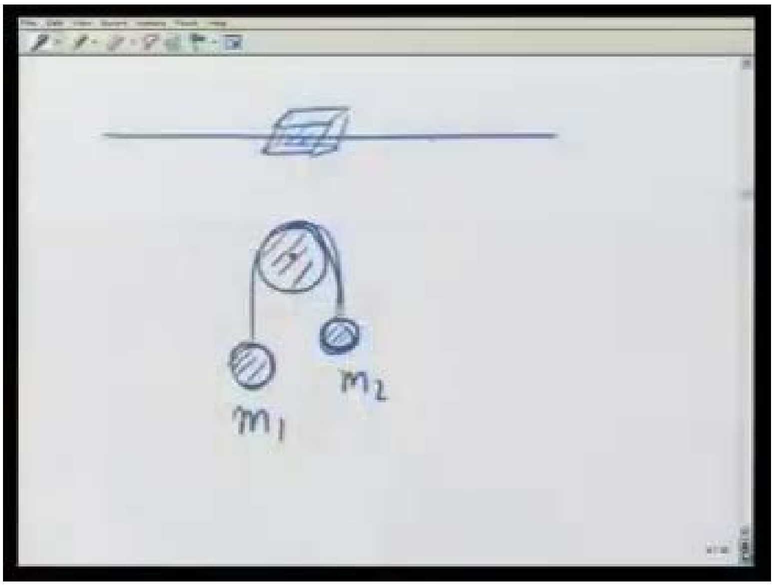
(Refer Slide Time: 16:19)



In doing so, I get m 1 y 1 double dot minus m 2 y 2 double dot is equal to m 1 minus m 1 g plus m 2 g. Since, I know that y 1 double dot plus y 2 double dot is equal to 0. This implies y 1 double dot equals minus y 2 double dot. If I substitute I get m 1 plus m 2 y 1 double dot equals m 2 minus m 1 g or y 1 double dot equals m 2 minus m 1 over m 2 plus m 1 g, a result you already know.

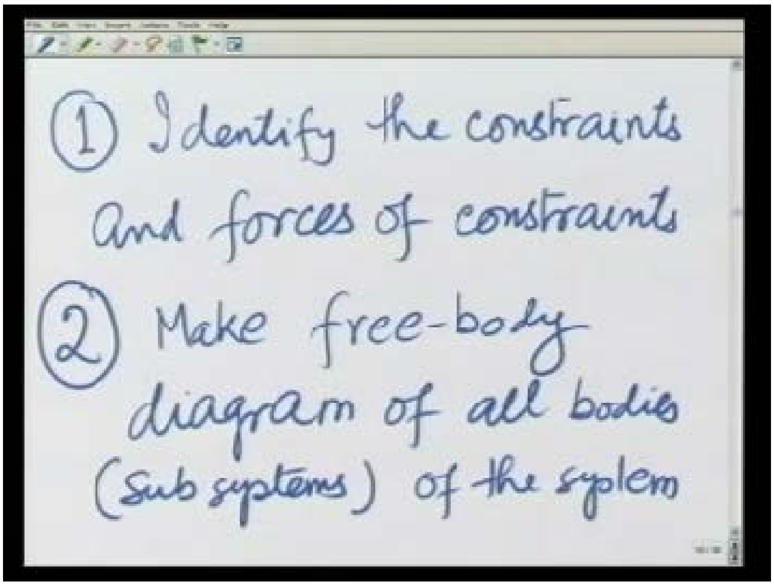
If m 2 is heavier than m 1 then, y 1 double dot is up, and that is precisely what I am getting. You see I did not make any assumption about m 1 or m 2 being lighter or heavier and things like those, the result automatically popped up. Through these two examples, what I have tried to demonstrate to you that constraints in a particles or a body's motion can be caused either by external agencies, as we did in the case of a particle moving on a wire.

(Refer Slide Time: 17:49)



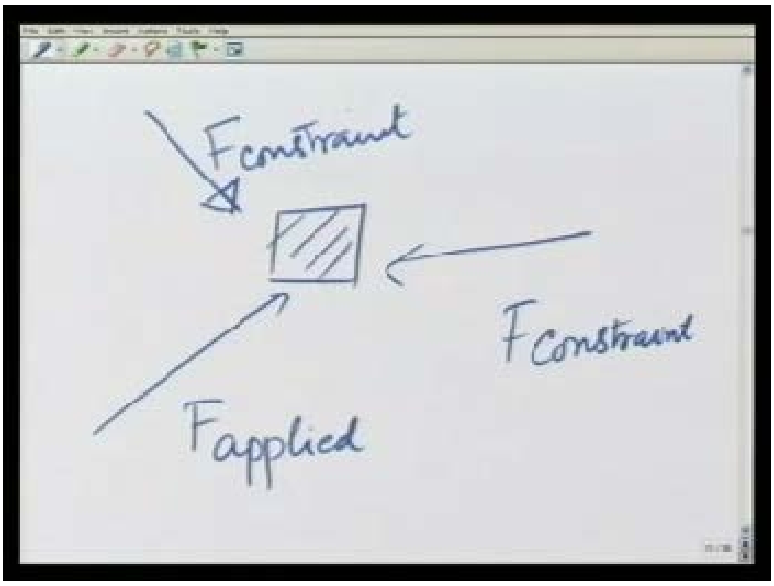
The constraint was caused by this wire or by other body itself, that is in this case this mass m 1 and 2 when they were connected by this rope this was providing the tension. But the presence of one mass restricted the motion of the other mass. And also when we solve these two simple problems that, you already knew the answers of, we also tried to develop stepwise strategies to solve motions of different bodies when they are moving under constraints. Let us try to right them.

(Refer Slide Time: 18:41)



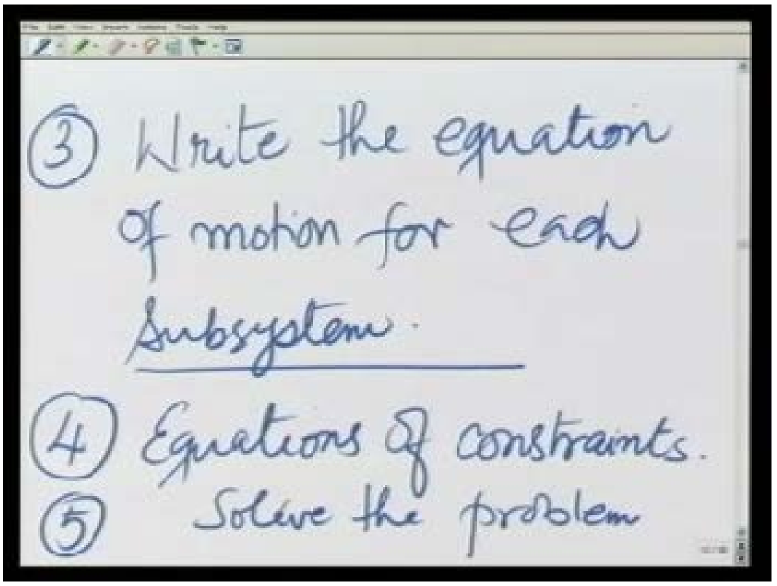
So, first step in solving a problem is going to be identify the constraints and forces of constraints. Once I have identifies of constraints and forces of constraints, the second step is make free body diagram of all bodies and these bodies which are treated separately are also called subsystems of the system. Let me remind you what a free body diagram is. When I make free body diagram for a body, I make that body freely.

(Refer Slide Time: 19:58)



And consider all the forces that are being applied from outside, and the constraint forces on it. There may be more than one constraint force, and that is my free body diagram.

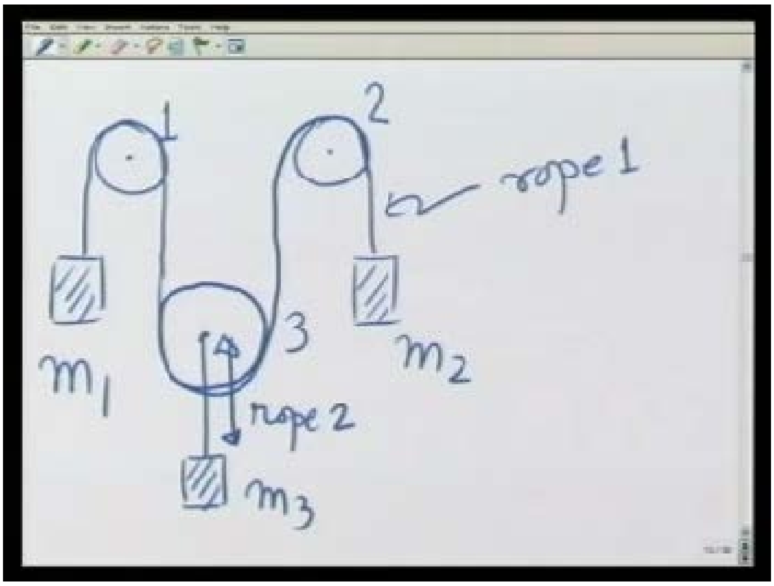
(Refer Slide Time: 20:27)



The third step is going to be then write the equation of motion for each subsystem. At this stage you will see that the number of variables in the problem is more than the number of equations that you have written.

And therefore, there are some missing equations, and these equations are going to be provided when you write the equations of constraint, constraints there may be more than one. Still if the equations do not match, either you have missed something or you miss formulated the problem, go back and do it all over again. And once the equations and number of variables match, last step is solve the problem. I am going to demonstrate this now with the slightly more complicated problem, it will be like acquits machine, but slightly more complicated.

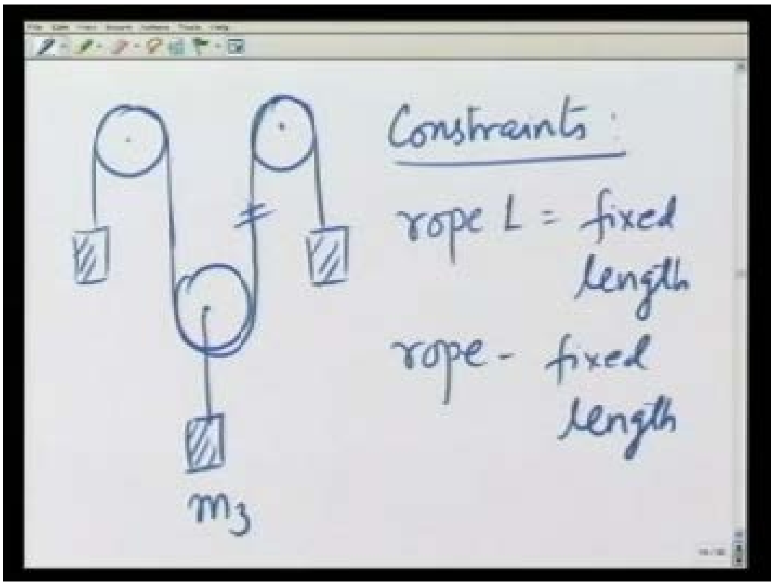
(Refer Slide Time: 21:46)



So, what we are going to do is take two fix pulleys, they are fixed at some height, pass a string of fixed length over them and the string turns around with the help of this third pulley.

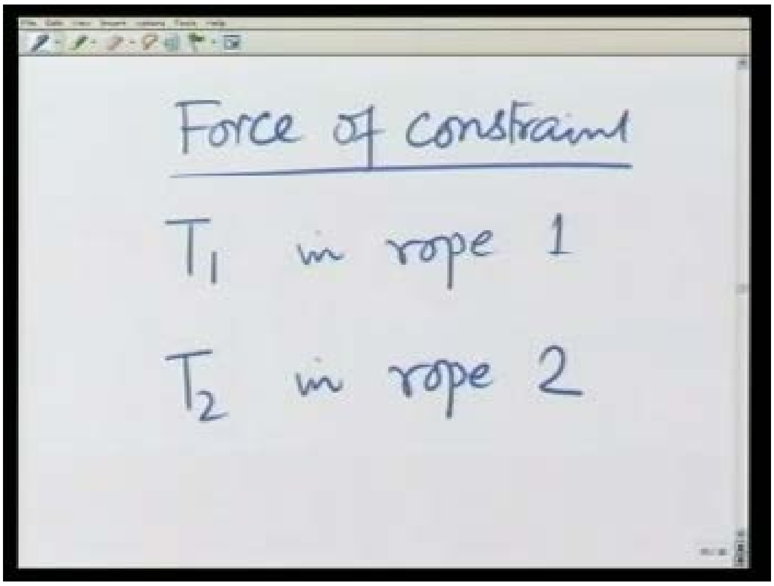
So, let me number them, pulley number 1, pulley number 2, which are fixed pulley number 3, which can move up and down. And let also attach a mass here, attach a mass here, attach a mass here, this is mass m 1, this is mass m 2, this is mass m 3. And this is attached again by a rope, call at rope 2, call this rope 1, and both are of fixed length. And what I want to do now is, when I release this system how does the motion take place? For example, where does m 1 move, what is its acceleration? Where does m 2 move, what is its acceleration? How does m 3 move, what is its acceleration? So, we will go back to our steps, and the first thing we want do is identify the constraints and forces of constraint.

(Refer Slide Time: 23:10)



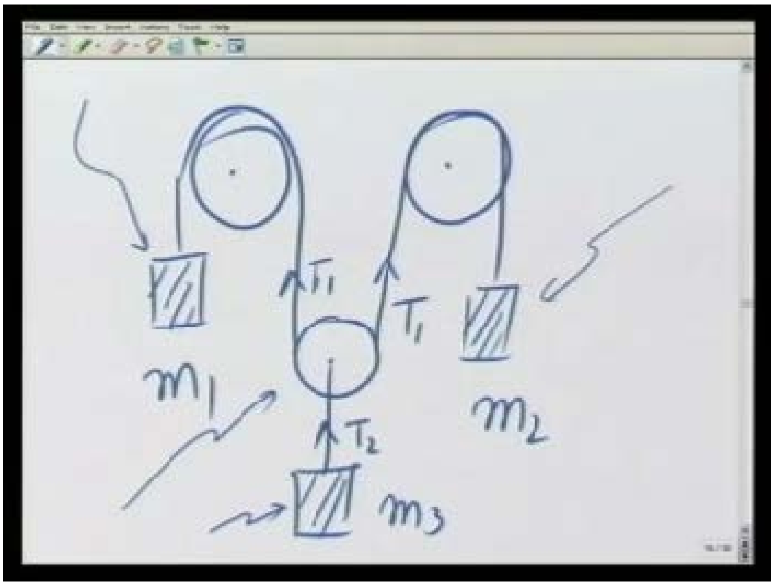
Let me make the figure again for you, the constraints are that length of rope 1 is fixed. So, constraints are rope 1, which is this one as fixed length. Similarly, the other constraint is that rope 2 from which mass m 3 is hanging has fixed length. And I got use them later in writing the constraint equation.

(Refer Slide Time: 24:09)



Forces of constraint are tension T 1 in rope 1 and tension T 2 in rope 2. These are my forces of constraint, let me show them again in the picture.

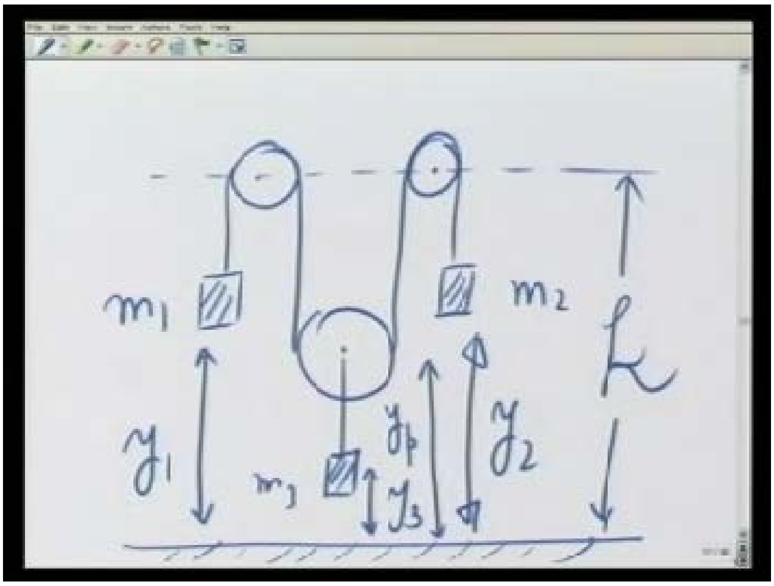
(Refer Slide Time: 24:37)



This is my pulley, mass m 1, here is a pulley, here is the other pulley, mass m 2, m 1, m 3. Here the tension works this way T 1, T 2, these are the forces of constraint. Again I am assuming T 1 and T 2 to be the same because I will be considering for simplicity massless and frictionless pulleys and assume that all the ropes are massless.

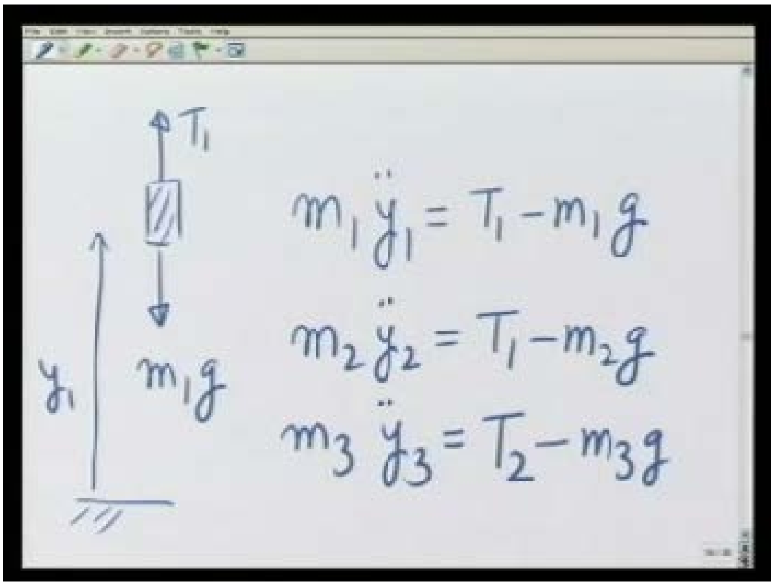
The next step in the game is make free body diagram for all moving parts. you can see that these two pulleys are fixed and therefore, they are not going to move. This mass is going to move, this pulley is going to move, this mass is going to move, this mass is going to move. And therefore, I will have 4 equations of motion, for that again I take a reference point from which I am going to measure my distances.

(Refer Slide Time: 25:45)



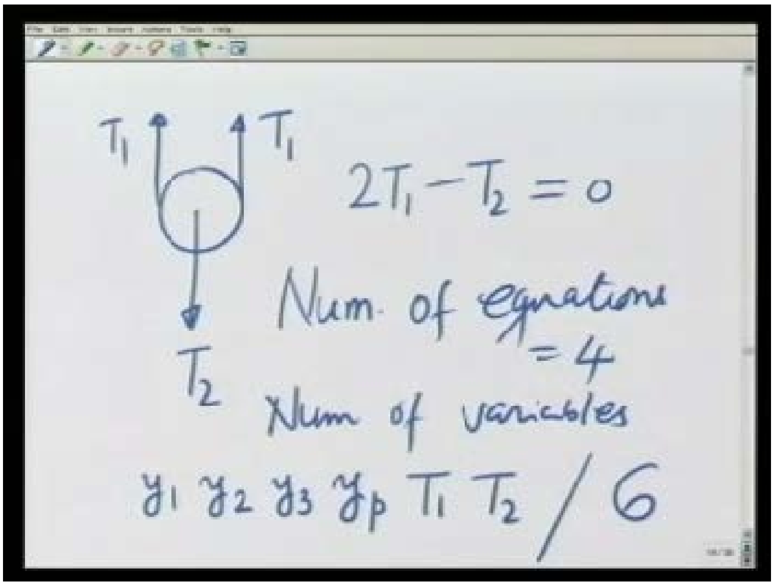
To make that picture again this is mass m 1, pulley number 3, mass m 2, mass m 3, I will measure the heights from the ground. Since these two pulleys are fixed, they are at height h which is a constant from the ground, this distance, let it be y 1 for mass m 1, let it be y 2 for mass m 2, let it be y 3 for mass m 3, and let it be y p for the pulley. Because I am considering all the subsystems of this whole thing which are moving. To write the equations of motion, let me again go back to the free body diagram and I will take them up one by one.

(Refer Slide Time: 26:47)



If I go to mass m 1, it had tension T 1 acting this way, mass m 1 g pulling it down, and its height is measured from the ground as y 1. And therefore, the equation of motion for this is going to be m 1 y 1 double dot equals T 1 minus m 1 g. I can similarly write to the equation of motion for mass m 2 which is going to be m 2 y 2 double dot equal to T 1, minus m 2 g. Same thing for m 3, m 3 y 3 double dot is going to be equal 2 because it is the tension in the second rope that is pulling it up minus m 3 g. The fourth moving part is the pulley itself and for that the forces pulling it up are T 1, T 1 on both the sides and tension T 2 in the middle.

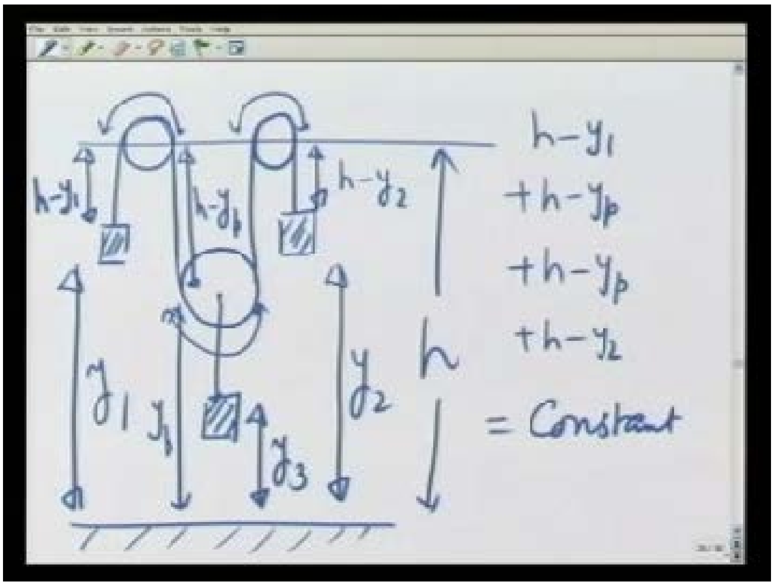
(Refer Slide Time: 27:51)



Since, the mass of the pulley is 0 therefore, I am going to have 2 T 1 minus T 2, which is a net force that must be equal to 0. You see I have gotten 4 equations, 4 equations of motion so, number of equations is 4, how about the number of variables? This is y 1 y 2, the heights of masses 1 and 2, y 3 height of mass 3, yp the height of the pulley, T 1 tension in rope 1, and T 2 tension in rope 2, total number of variables is 6.

As I had said earlier at this stage when you write the equations, the number of equations are going to be less than the number of variables. The missing equations are provided by the equations of constraints, and that is what we got to write next. That should give us 2 equations of constraint and they will make the number of equations to be 6, which is equal to the number of constraints.

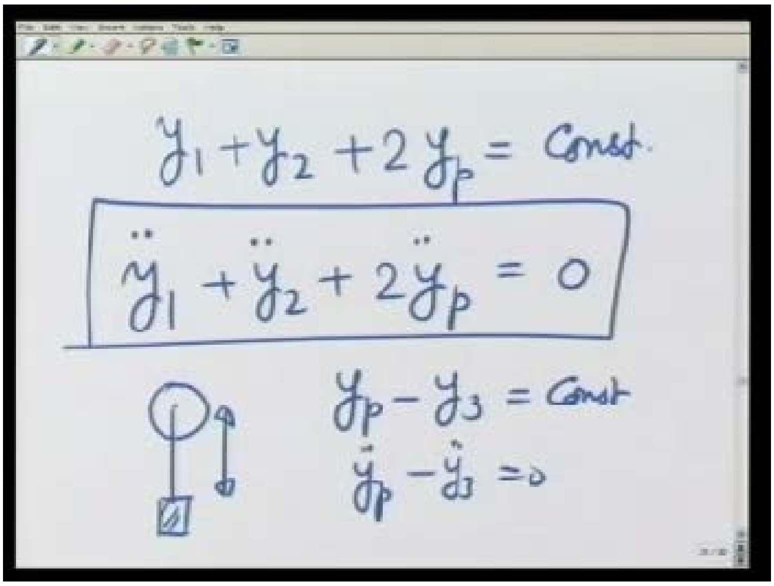
(Refer Slide Time: 29:16)



Let us do that, and for that again we go back to our diagram where I will now take the constraints into account that, the lengths of these ropes, both the ropes are the same, are fixed. So, this is y 3, these were at height h from the ground, this is at y 2, this is at y 1.

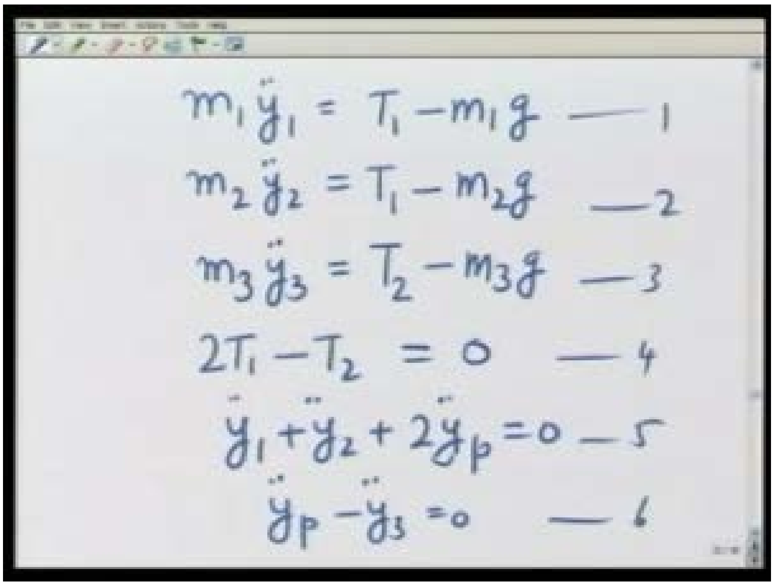
So, let us see what are different distances involved. This is going to be h minus y 1, this is going to be h minus y 2 or this is yp I need that also, this is going to be h minus yp and so is this. Therefore the length of the rope is h minus y 1, plus h minus yp, h minus y 1 plus h, minus yp, plus another h minus yp, plus h minus y 2, and these distances are also fixed because the radius of pulley is fixed. So, this is going to be equal to a constant.

(Refer Slide Time: 30:56)



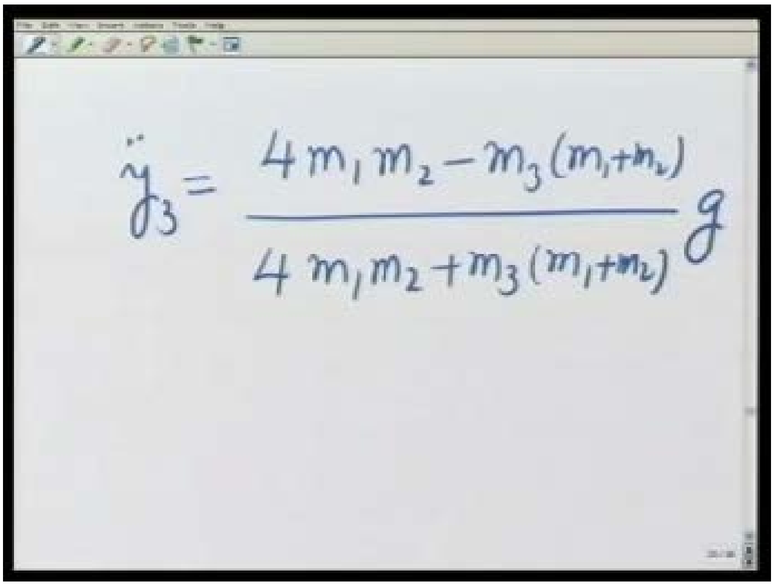
In other words, I can write if I reshuffle the variables y 1 plus y 2 plus 2 yp equals a constant. To again bring them to usable form I differentiate them and write y 1 double dot plus y 2 double dot, plus 2 yp double dot equal to 0. That is one of the equations that I need. I have now gotten 5 equations. 6 th equation is very easy to get realizing that, the length of this rope is fixed and therefore, yp minus y 3 is constant. Or yp dot, minus y 3 dot is 0 and so is yp double dot and y t3 double dot. Basically the pulley and the mass are moving up and down together. So, let me rewrite all the equations once more, I have now gotten 6 equations and there are 6 variables in the problem.

(Refer Slide Time: 32:00)



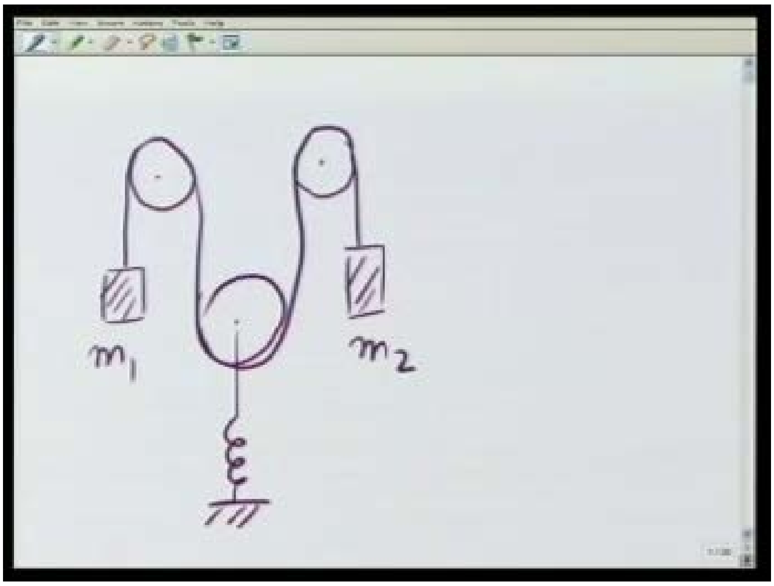
Equations are m 1 y 1 double dot is equal to T 1 minus m 1 g, m 2 y 2 double dot equals T 1 minus m 2 g, m 3 y 3 double dot equals T 2 minus m 3 g, 2 T 1 minus T 2 is equal to 0, y 1 double dot plus y 2 double dot plus 2 y pulley double dot equals 0. And y pulley double dot minus y 3 double dot equal to 0 equation 1, 2, 3, 4, 5, 6 and 6 variables I can solve them and get my desired answer. I am not going to solve it for you here, but I will give you partial answer and you can try to get that after solving these equations at home.

(Refer Slide Time: 33:15)



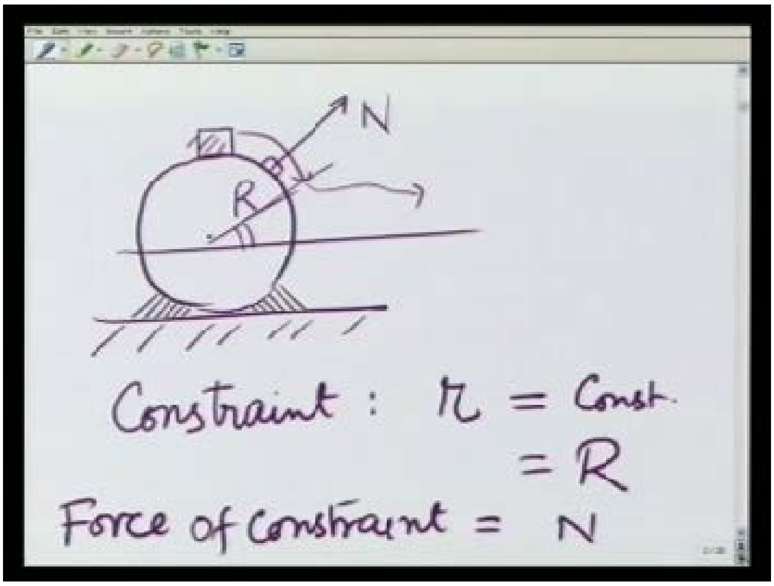
The answer for y 3 double dot comes out to be 4 m 1 m 2 minus m 3 m 1 plus m 2 divided by 4 m 1 m 2 plus m 3 m 1 plus m 2 g. Continuing with the same problem, I will change this slightly and give this an exercise to you.

(Refer Slide Time: 33:49)



Let me again attach two masses with the pulley, where it go like this, except that now I will attach a spring to this mass. I would like to set up the equations for this, I like you to set up the equations for this and solve them using the method that we have learnt so far. I will do two more examples of the methods that we have learnt, to demonstrate how effectively we can solve problems using the method of making free body diagrams, taking subsystems, using constraints, constraint equations, and equations of motion.

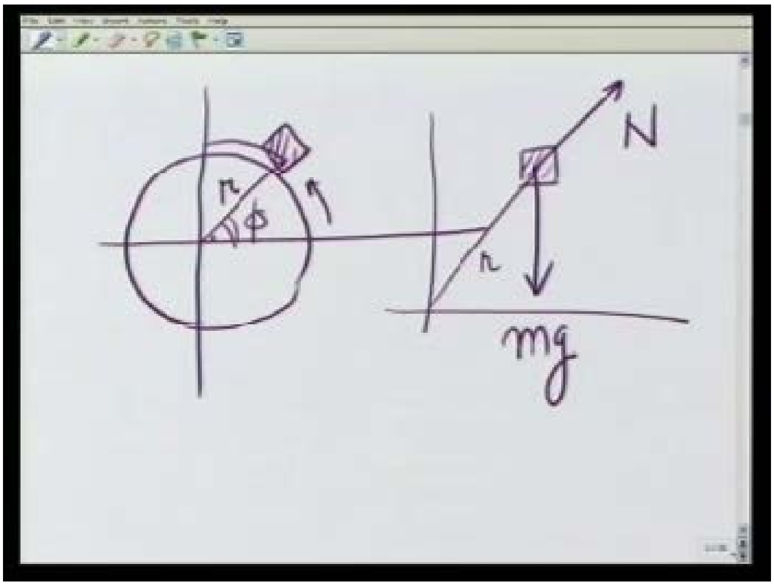
(Refer Slide Time: 34:53)



As an example again let me take a block that is sliding on the surface of a drum which is fixed on the ground. The block starts from the top slides down this way and the problem that we ask is, at what angle does this drum, at what angle does this block fly off the drum. The radius of the drum is R, I am choosing this particular problem because this will also give you some practice on planar, using planar polar coordinates.

So, to start with again, we identify what the constraints and forces of constraints are. So, constraint on the moving block here is that, the radius r is a constant, and this is equal to the radius of the drum R. The force of constraint then, obliviously is the normal reaction that the drum applies on the block.

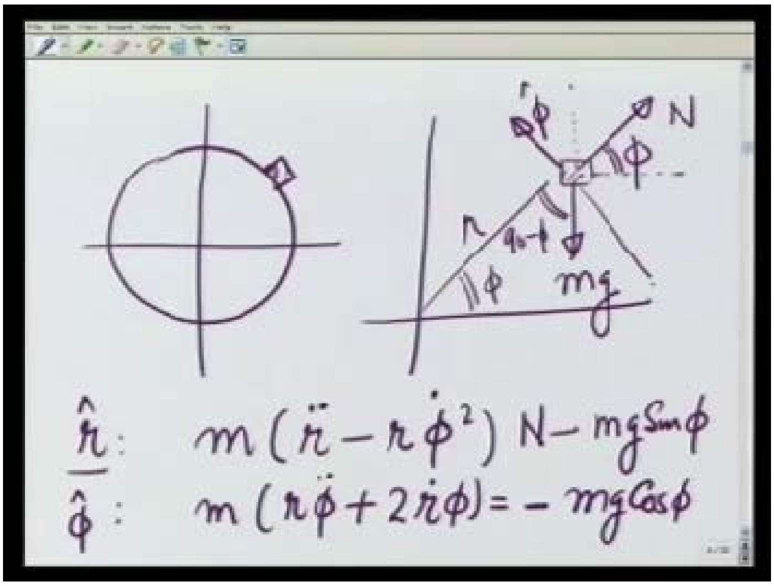
(Refer Slide Time: 36:22)



Since, the motion is circular because before leaving the drum the block is moving in a circular path here. I am going to use planar polar coordinates, taking this angle as phi increasing this way, and r obviously, is the radius.

My second job is to write the equations to make the free body diagram for the block. So, if I take the block here, the free body diagram that involves only the normal reaction or the force of constraint and the external force, is going to be mg of the block acting downward and the normal reaction acting this way. And after making this free body diagram, I write the equations of motion for the block, except that this time I want to use planar polar coordinates. So, at this position, the block is at distance r, maybe I should go to the next page for this.

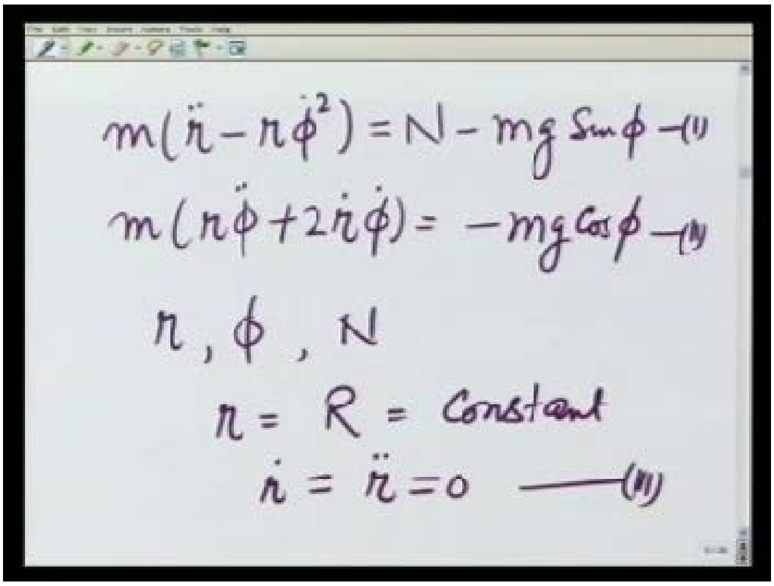
(Refer Slide Time: 37:38)



The block I am taking to be here. So, at this position it is at distance r at an angle phi, this is the force mg acting this way, this is the normal reaction acting this way since, this is phi, this angle is also phi. And therefore, in r direction if I want to write the equation of motion, it is going to be m times the acceleration in the r direction, which you feel call from the previous lecture is r double dot minus r phi dot square. And this is going to be the force in r direction is N in the positive r direction and since, this angle is 90 minus phi minus mg sin of phi.

Similarly, in phi direction, the equation of motion is going to be m r phi double dot plus 2 r dot phi dot equals mg cosine phi, but with the minus sign because the force is acting in negative phi direction. Remember that phi is increasing this way and the component of mg is in this direction.

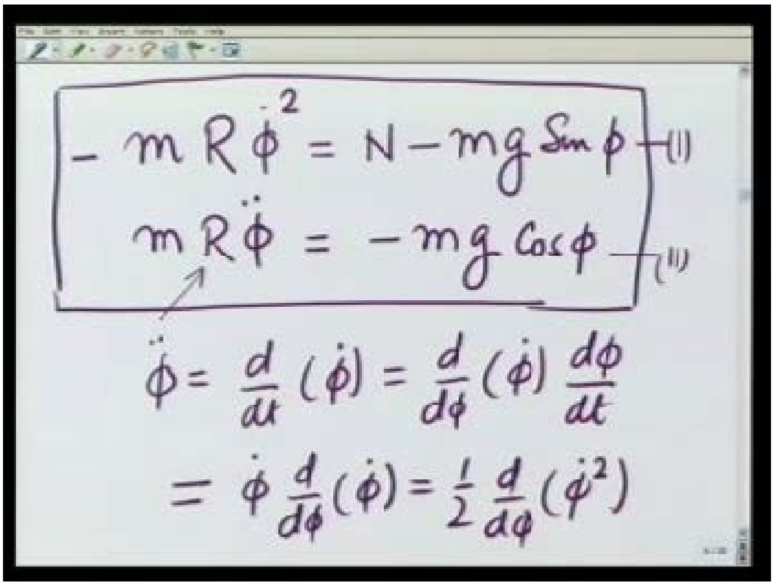
(Refer Slide Time: 39:33)



So, the equations of motion that I have gotten are m r double dot minus r phi dot square is equal to N minus mg sin of phi and m r phi double dot plus 2 r dot phi dot is equal to minus mg cosine of phi. These are two equations of motion, and number of variables involved are r phi and N, as I had said earlier, at this stage the number of equations would generally being less than number of variables in the problem.

And that is because so far I have not taken into account the equation of constraint, which in this case happens to be r is equal to R which is a constant and therefore, r dot is equal to r double dot is equal to 0, and that is my third equation which I can solve. So, now I am going to substitute third equation in number 1 and number 2 and solve the resulting equations to get my answers.

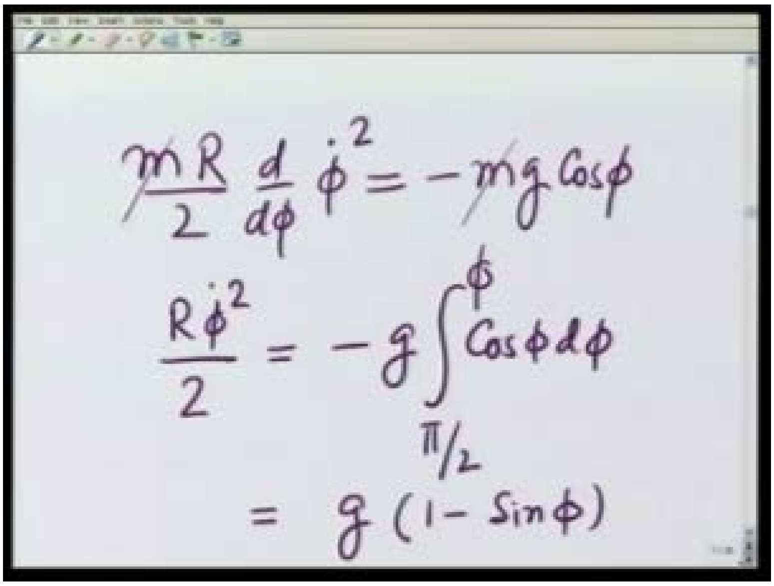
(Refer Slide Time: 40:54)



When I do that I get m R phi dot square with the minus sign in front is equal to N minus mg sin of phi, and the other equation comes out to be m R phi double dot is equal to minus mg cosine of phi. It is these two equations that are going to give me N and rate of change of phi with respect to time, R anyway it is fixed by third constraint equation.

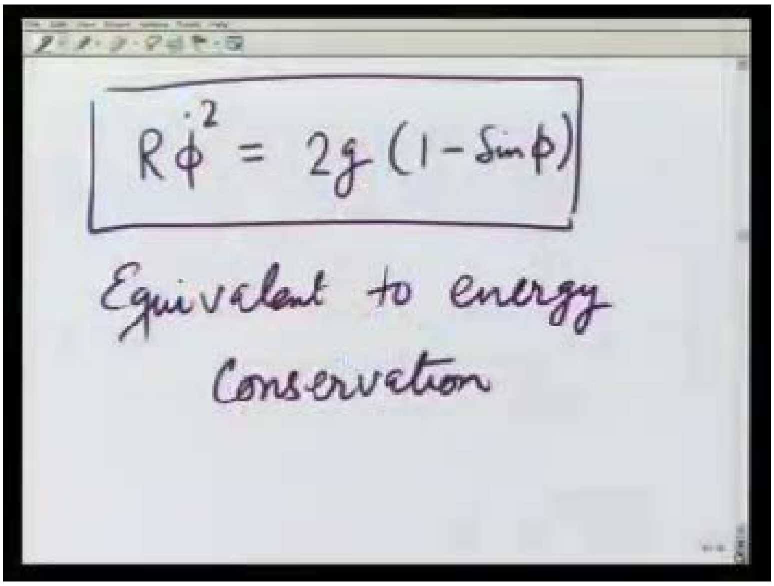
To solve this equation, I first solve the second equation and I use a trick for this because you see I cannot really integrate with the respect to time. So, I write phi double dot which is the time derivative of phi dot as time derivative of phi, phi dot times d phi dt, using the chain rule for differentiation. And that gives me equals phi dot d over d phi as of phi dot which is nothing but 1 half d over d phi of phi dot square.

(Refer Slide Time: 42:26)



And therefore, my second equation now reads m R over 2 d over d phi, phi dot square is equal to minus mg cosine of phi, m cancels and therefore, I have R phi dot square over 2 is equal to minus g cosine of phi d phi. I remember the particle is starting from the top that means, it is starting from pi over 2 and coming up to some angle phi. This gives me this equal to g 1 minus sin of phi.

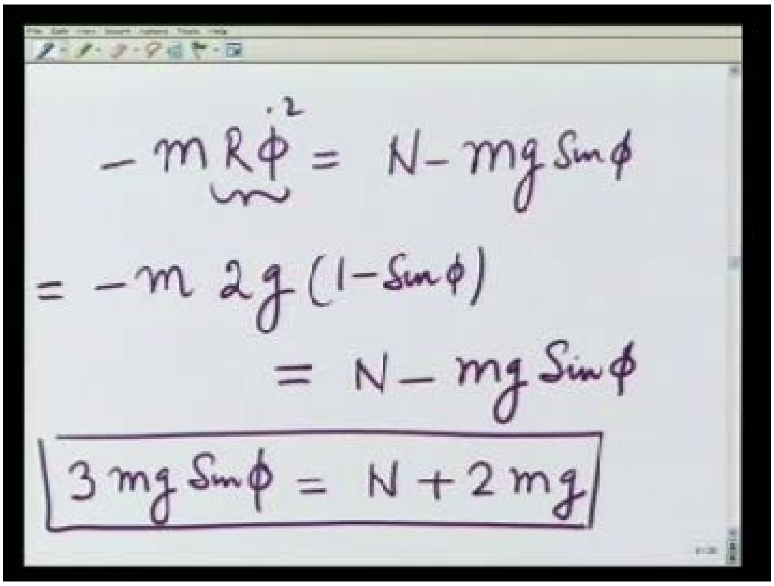
(Refer Slide Time: 43:20)



And therefore, the result that I have is R phi dot square is equal to 2 g 1 minus sin of phi. I leave this as an exercise for you to show that this is equivalent to energy conservation.

Because the left hand side is related to the kinetic energy, and right hand side is related to the change in potential energy. Using this I can now solve equation 1 for the normal reaction.

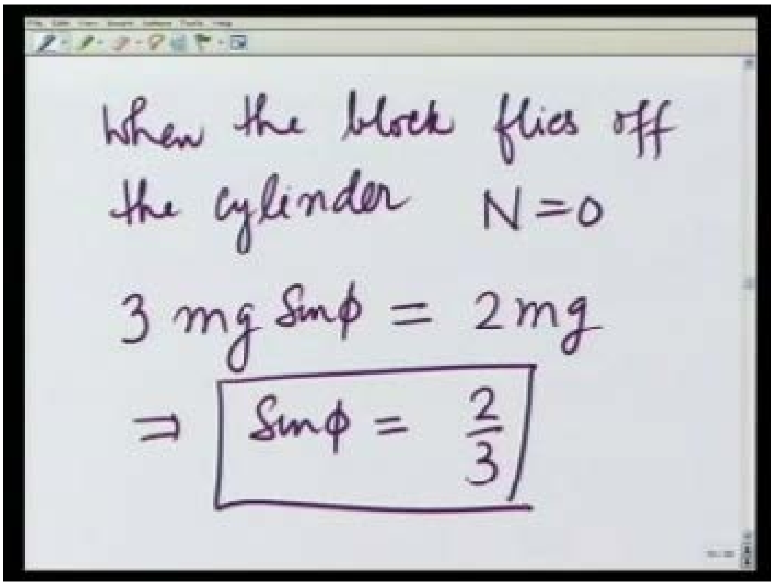
(Refer Slide Time: 44:12)



We call that equation 1 is nothing but m R phi dot square, minus sign here is equal to N minus mg sin phi. I substitute for r phi dot square that I just derived and I get minus m 2 g 1 minus sin of phi is equal to N minus mg sin of phi, which gives me after reshuffling 3 mg sin of phi is equal to N plus 2 mg.

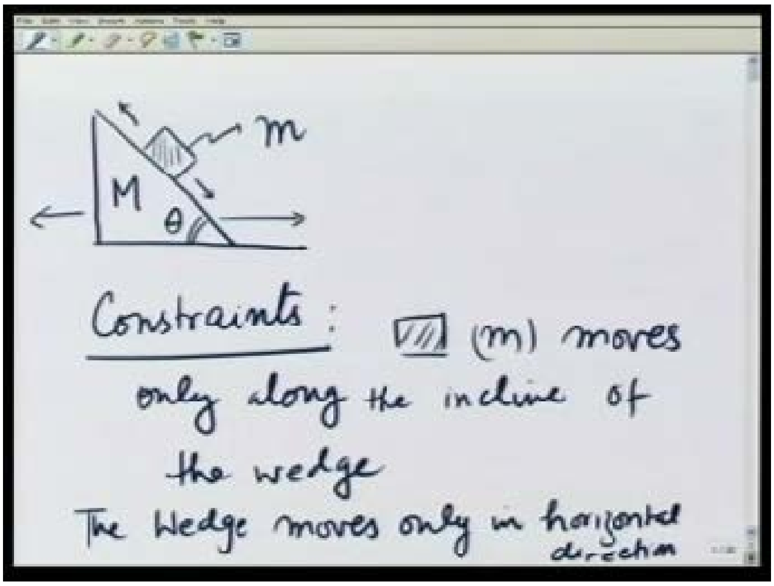
This gives me the normal reaction as a function of phi. Recall that this is the normal reaction that is enforcing the constraint that is, the particle moving moves with the constant radius. When it flies off the cylinder, at that point there is no contact between the block and the particle.

(Refer Slide Time: 45:27)



And therefore when the block flies off the cylinder, at that point N would become 0 and therefore, 3 mg sin phi, at that point would become equal to 2 mg which implies that, at angle phi from the horizontal such that, sin phi is two-thirds the block will go off the cylinder. This is one problem that I have solved using the steps that I have outline and polar coordinates. Next I am going to solve a problem that involves two particles and constraints on their motion due to their contact. Now, in our next example, we take two bodies that are in contact and apply constraint on their motions because of that.

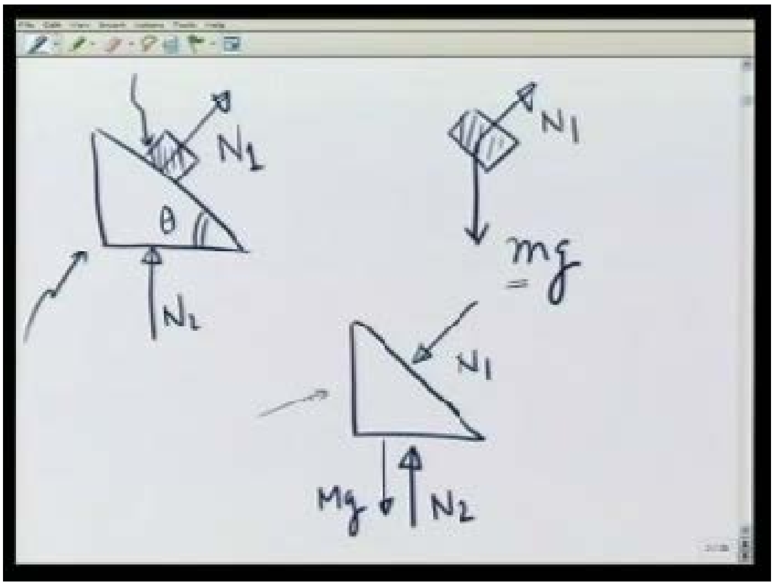
(Refer Slide Time: 46:30)



That I am taking a wedge with an angle theta here, and there is a block that slides on it. The wedge has mass M, this block has mass small m, and all the surfaces are frictionless. Therefore, this wedge is free to move in this direction, and the block is free to move along the inclined plane. This is a constraint motion because the block small of mass small m is free to move only along the inclined therefore, it is y and x displacements are going to be related.

Let us see then what are the constraints. Constraints of the motion are that this block of mass m moves only along the inclined of the wedge and the other constraint is that, the wedge moves only in horizontal direction.

(Refer Slide Time: 48:21)

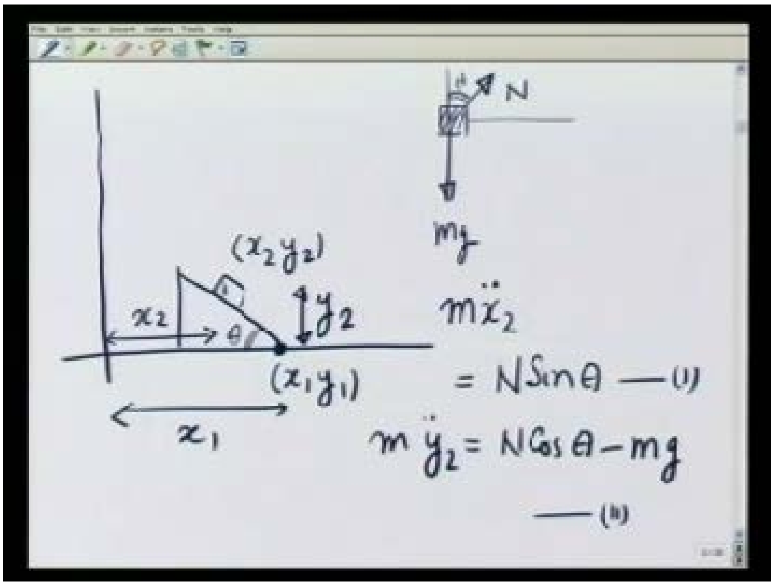


So, let us see what forces of constraints are going to be. Obviously, the force between these two is the normal reaction on the block, let me call it N 1 and the force that keeps the wedge on the ground is a normal reaction on it N 2. The next step in our strategy is to make free body diagrams for all the moving parts in the systems, of the system. There are clearly two subsystems here, one the block here, the other wedge here. For the block, the free body diagram would give a force N 1 acting in this direction and its own weight pulling it down.

I remind you once more that the free body diagram means I isolate the system and represent all the constraints by the forces of constraint. The free body diagram for the wedge is going to be by Newton's third law a force N 1 in this direction, its own weight

Mg pulling it down, and the normal reaction N 2 by the ground. I would like you to notice that I have not involved the mass small mg in making the free body diagram of the wedge, that is because it is taken care of by normal reaction N 1. After making free body diagrams, I am going to write the equations of motion. To write my equations of motion, let me first fix my coordinate system.

(Refer Slide Time: 50:10)

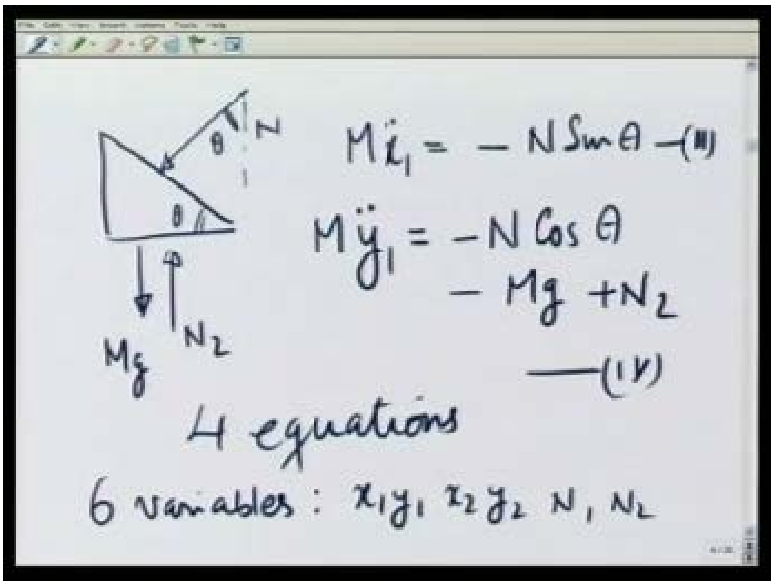


And for that what I am going to do is, make the wedge here, this is the block. For convenience I will choose this point to represent the coordinates for the wedge. Let this be x 1 y 1. Notice since, I am putting this constraint that, the wedge moves only in the horizontal direction, I am explicitly writing y 1 and later I am going to eliminate this using the constraint equation.

So, that this distance right now is x 1 similarly, the coordinates for the mass m, let them be x 2 y 2. So, that this distance is x 2, and this distance is y 2. Looking at the free body diagram where mass small m has a normal reaction like this, this angle is theta and a force mg pulling it down. If this angle is theta, this angle is going to be theta.

And therefore, I am going to have the equation motion as m x 2 double dot is equal to N sin theta that is it. And similarly for the y coordinate I am going to have, this is my equation number 1, m y 2 double dot is equal to N cosine of theta minus mg, that is my equation number 2. Third and fourth equations are going to be for the wedge, for the wedge.

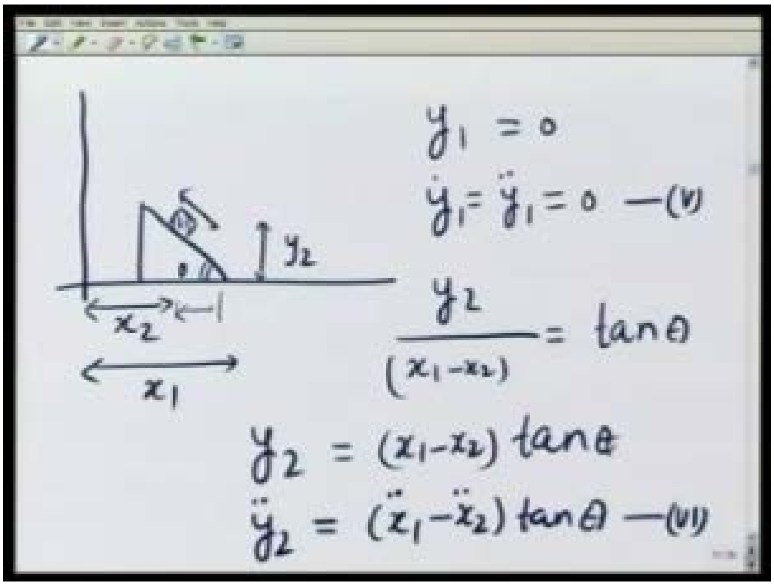
(Refer Slide Time: 52:16)



The forces are N 1 in this direction, this angle being theta, capital Mg in this direction, and N 2 in this direction. So, the equations are going to be Mx 1 double dot is equal to minus N sin of theta, equation number 3. And M y 1 double dot is going to be equal to minus N cosine of theta minus Mg plus N 2, equation number 4.

I have 4 equations and 6 variables which are x 1, y 1, x 2, y 2, N 1 and N 2 as expected because I have not yet taken the constraints into account. The remaining two equations are given raise to by the constraint equations, what are the constraint equations?

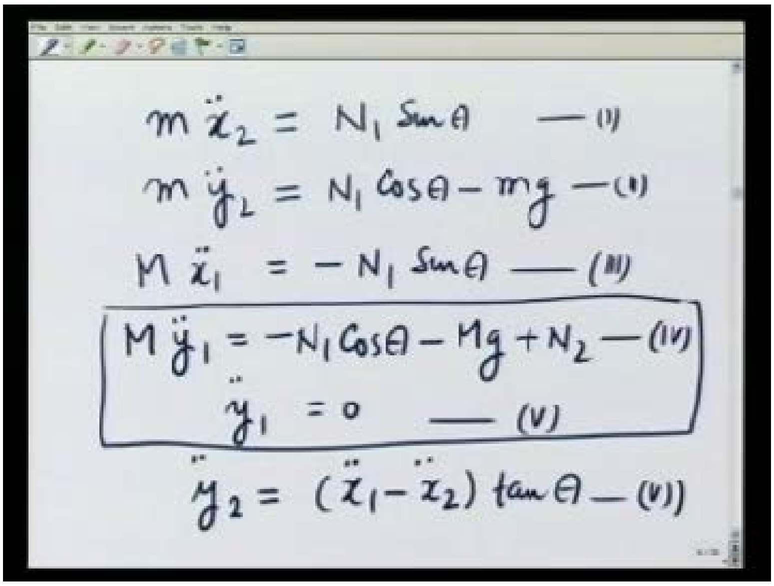
(Refer Slide Time: 53:44)



The constraint equations are given by let us draw this again and we will see this is theta, this is the block, this is y 2, this is x 2, this is x 1, number 1 that the wedge moves only in horizontal plane therefore, y 1 is always 0.

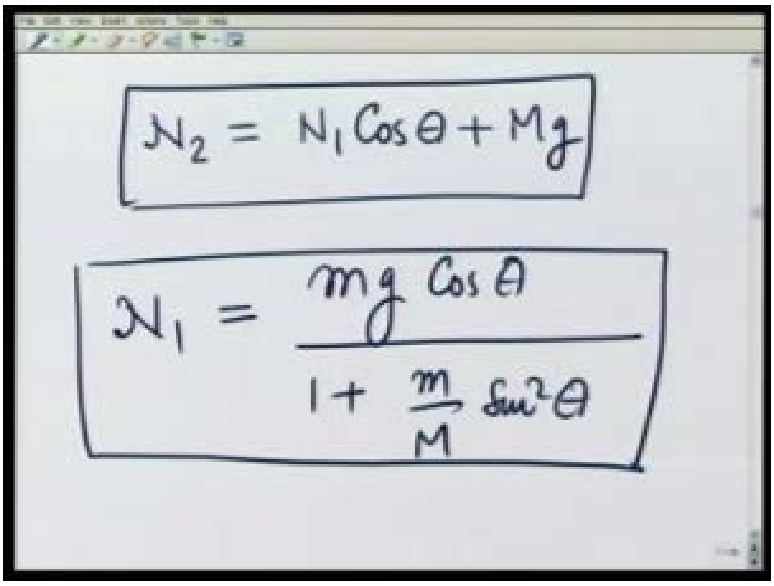
And therefore, y 1 dot is equal to y 2 y 1 double dot is equal to 0, that is my equation number 5. And the block always moves on this inclined plane. And that means, y 2 divided by this distance which is x 1 minus x 2 is fixed is equal to tangent of theta. And therefore, y 2 is equal to x 1 minus x 2 tangent of theta or y 2 double dot is equal to x 1 double dot minus x 2 double dot tangent of theta, that is my equation number 6. I now have 6 equations and 6 variables, and I can solve equations. I will solve it again partially to get the value of N 1, rest follows very easily. Let me rewrite the equations once more.

(Refer Slide Time: 55:27)



So, all the equations are m x 2 double dot is equal to N 1 sin theta, m y 2 double dot is equal to N 1 cosine of theta minus mg. m x 1 dot is equal to minus N 1 sin of theta equation number 3. m y 1 sorry, this was x 1 double dot y 1 double dot is equal to minus N 1 cosine of theta minus Mg plus N 2, equation number 4. Then y 1 double dot is equal to 0, equation number 5. And y 2 double dot is equal to x 1 double dot minus x 2 double dot tangent of theta, equation number 6. So, now I have 6 equations and 6 unknowns, which I can be, which can be solved very easily. These 2 equations together really give you nothing new.

(Refer Slide Time: 57:10)



But that N 2 is equal to N 1 cosine of theta plus Mg, which tells you that N 2 balances the all the vertical forces acting on the wedge. The other equations when substituted for, would give you a result for N 1. I leave the solution and substitution and all those things for you, give me the final answer which gives you N 1 is equal to mg cosine of theta divided by 1 plus m over M sin square theta.

Once I now N 1 which is a constant quantity here, I can calculate all the other quantities x 1 double dot, x 2 double dot, y 2 double dot, and their integrated forms, the velocities, and the displacements easily. That brings us to the conclusion of this lecture in which through simple examples we saw, how we take constraints into account when writing equations of motion, and solving for them in cases where the motion is not completely free. It can be restricted either by external constraints or by 2 or 3 moving particles themselves that impose restrictions on the motion of each other.