Engineering Mechanics
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Module - 04
Lecture 01
Method of Virtual Work

In this lecture we are going to deal with a method of handling systems equilibrium of those systems which are large and have a large number of subsystems.

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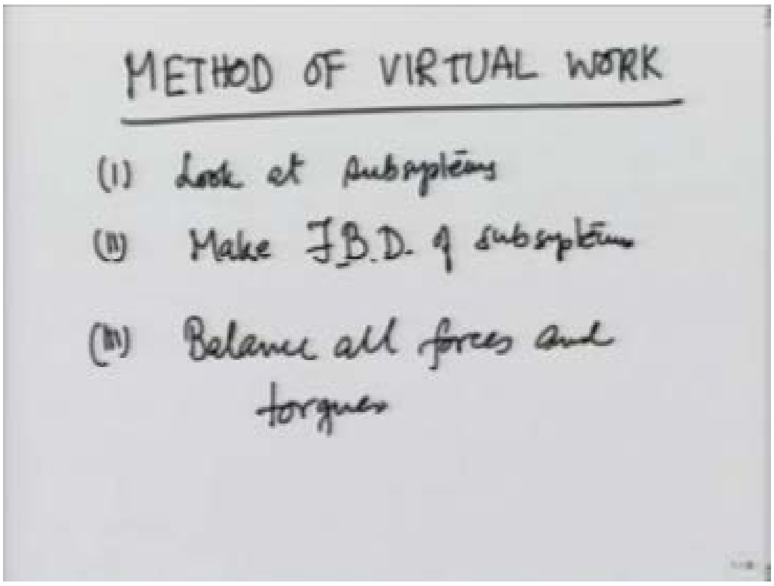

The method is known as method of virtual work to motivate this let us recall how we dealt with equilibrium of system such as particles, trusses and so on. Our strategy in those cases was to look at subsystems, and make free body diagram of subsystems. Replacing the constraints or the forces applied by other or the contact with other subsystems by the forces applied by them, and balance all forces and torques.

However, as the system size grows, the number of forces, number of subsystems may go quite large and in such systems the method that we use, is the method of virtual work. The method focuses directly on the external force that, we want to calculate. To motivate it further, let us take an example.

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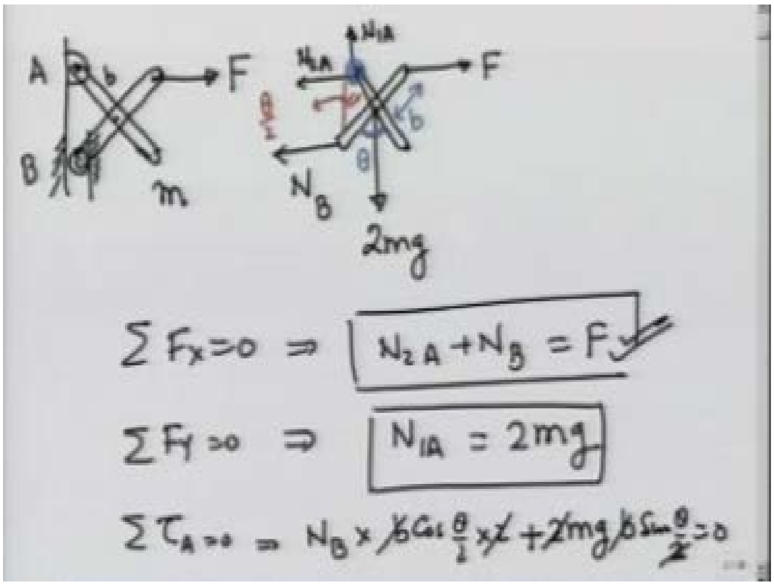

Suppose, I have a mechanism whereby made up of 2 bars whereby, it is a scissors like mechanism with a pin joint here, and a roller here, which is free to move in this between these 2 walls. Each bar has a mass m, and this is kept in equilibrium by applying a force F at this point. If we want to calculate the force F, our strategy is we first look at this mechanism, on this point there can be only 1 force, let us call it N B, this is point B, this is point A, this is a pin joint. So, there could be a force N 1 A and N 2 A.

A force F acts here, and the overall weight 2 mg pulls it down. If we have to bring this in equilibrium by summation FX equal to 0 would give me that, N 2 A plus NB equals F that is my equation. Similarly, summation FY equal to 0 would give me N 1 A equals 2 mg. Remember what we are after, we are after calculating this force F, required to keep the entire system in equilibrium. Third, the torque equation, for the torque equation I will take torque about point A here, to make it by blue I will take torque about this point.

So, that torques due to N 1 A, N 2 A and F, all vanish. So, if I take torque about A to be 0 I get, if the length of half of the bar is B all right. So, I am taking this half length to be B. And let this angle, entire angle be theta. Then, while calculating torque I am going to get NB times, this angle here is going to be theta by 2. So, I am going to get b cosine theta by 2 times 2 that is, this arm, plus 2 mg, 2 mg b sin theta by 2 is equal to 0, and that gives me NB. B drops out this 2 are out. So, this 2 is there this 2 drops out and therefore, I get NB to be.

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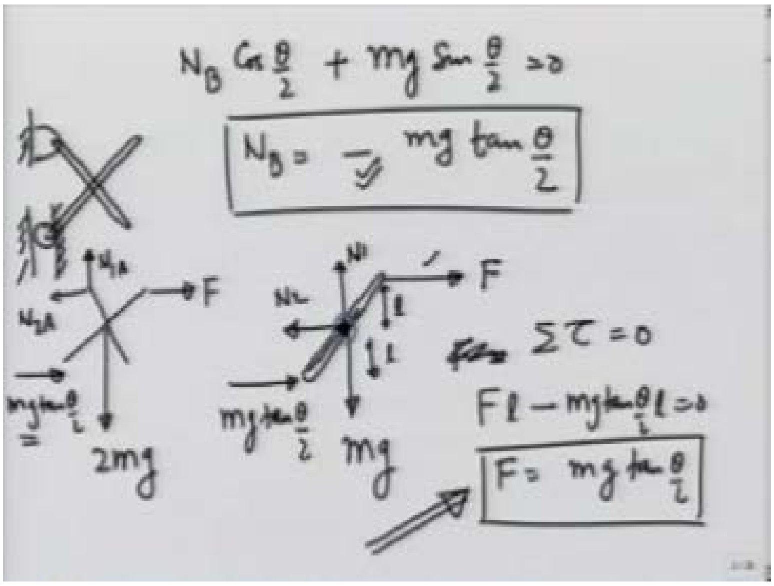

I have NB cosine of theta by 2 plus mg sin of theta by 2 equals 0 or I get NB to be minus mg tangent of theta by 2. Therefore, if I look at this mechanism that we had write earlier, this is a pin joint here, this is on a roller that is free to move, and this I have a force F acting this way. 2 mg pulling it down and NB, we have just calculated as mg tangent theta by 2. Notice that because of this negative sign, I have already corrected the direction of mg, and there is N 1 A and N 2 A.

Now, to calculate F, I look at one of the rods individually, this one, there is a force F pulling it this way, its own weight mg pulling it down. There is a force normally acting here which we have just calculated mg tangent theta by 2. And now there going to be normal forces, you have the pin out here, let us call this N 1 and N 2. If I am just interested in calculating force F. Then, we can see that I can take the torque about this middle point, the pin here let me show this in blue, about this point, and calculate F to be equal to.

By taking the torque about this point, let me follow summation torque about this point is 0 and therefore, F times whatever this length is call it l, l minus same length is going to be here, mg tangent theta by 2 l equals 0 and F equals mg tangent of theta by 2.

So, we found the force F, if you are interested you can also calculate N 1, N 2 and other normal forces, but our main focus was to calculate F, and that is what we have found. Notice that we had to take several steps in getting to this answer. If you use method of virtual work as I will show later, all these steps are bypassed. Not only that, if I want to solve the same problem, about the same mechanism.

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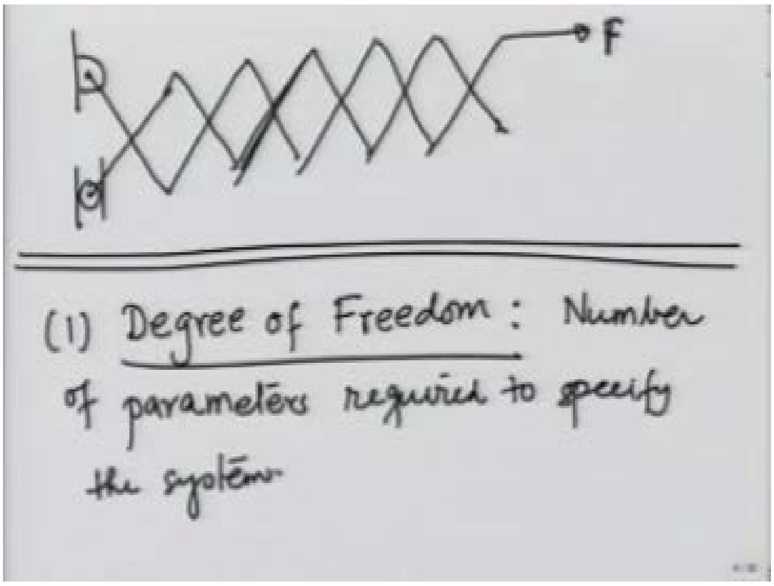

But make it slightly more complicated in that. Now, I have several of these links, and I apply a force here, each link, each of this bar has mass m and length 2 b, and I want to calculate the force F to keep this whole thing in equilibrium. You see if we go the conventional way I will have to build up the solution step by step by step. The method of virtual work we will be able to do it in a much more easy manner.

So, to develop the method we need certain terminologies, let me now go over there. So, now we are going to go over some terminologies that, I want to be used in the method of virtual work. First let us define degree of freedom of a system. The degree of freedom of a system is the number of parameters, that will be clear when I give an example, required to specify the position, the orientation, and all that the system. So, as I specify the system I need precisely that what is its position, what is the orientation, at what angle is it from vertical, and so on. So, for example, if I take so, we are talking about degrees of freedom.

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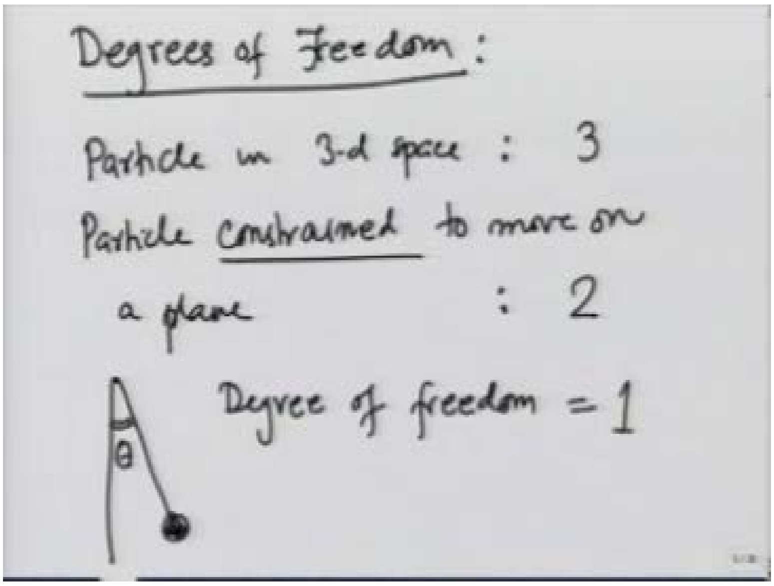

Again number of parameters required to specify a system for example, if I take a particle in 3 d space, the number of degrees of freedom is 3. If I take a particle constrained to move only on a plane, constrained and I will make this definition constrained clearer later, constrained to move on a plane then, the degrees of freedom are 2. Suppose I take a pendulum so, that swings in a plane. The only thing that I need to specify, where the pendulum is, is this angle? Then, the degree of freedom is 1, even in the complicated example that we took previously for that mechanism whereby, I had these cross links like this.

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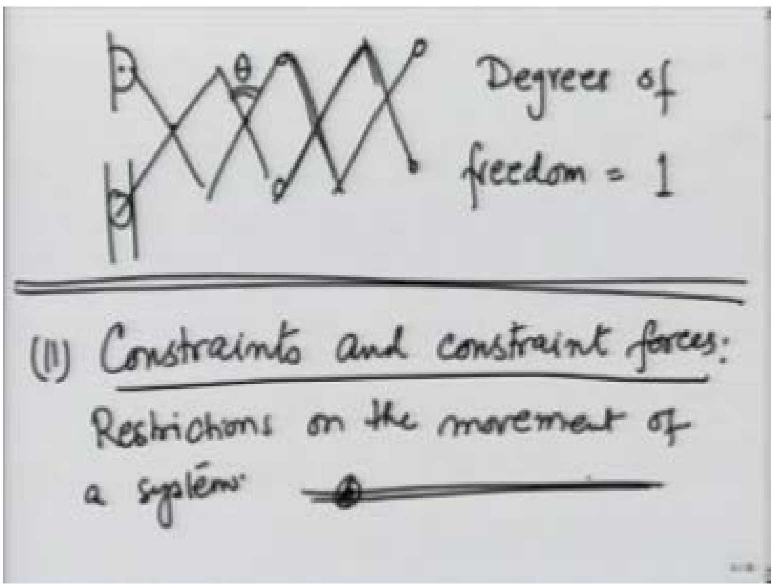

The only thing that I need to specify the orientation or specific position of the system is, this angle theta. Nothing else, the moment I specified this theta because the length of these bars are fixed, I specify exactly how the system is. So, degree of freedom, in this case is also 1. So, this is 1 concept we are going to use.

The degree of freedom is specifies the parameters used to describe a system, and the complexity of a problem. Less the number of degrees of freedom, less complex the system is. Concept number 2 constraints, this we have been taking about while talking about equilibrium, but let me go over it in a formal manner now, constraints and constraint forces. By constraints, we mean the restrictions on the movement of a system for example, if I take a particle and constrained it to move only along say wire. Then, that is a constraint that is put in there that, it can move only a longer straight line.

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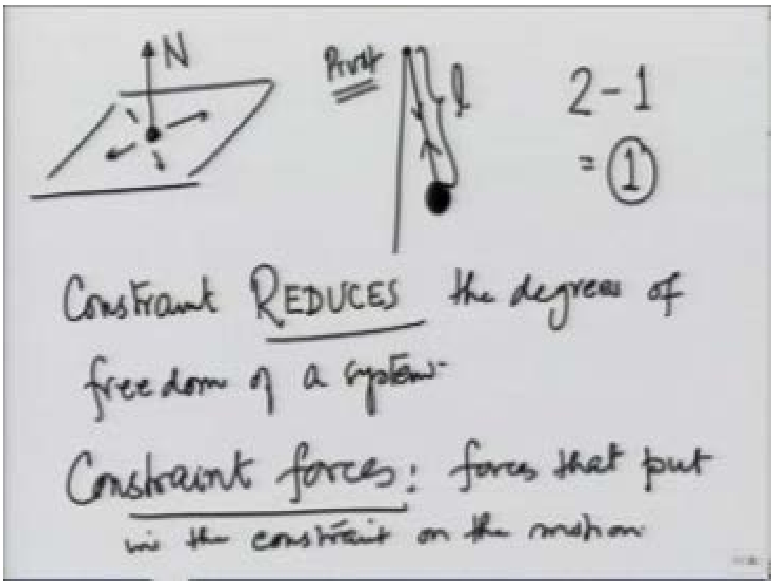

Or if I take a particle put it on a plane, and constraint it to move only on a plane. There is force of constraint on the particle to move only in this plane. Similarly, if I take this pendulum example that I took, the constraint is that the distance of the bob here of the pendulum is fixed from the pivot point.

So, you see, we are putting constraints whereby, we restrict the motion. And what does a constraint do? A constraint reduces the degrees of freedom of a system. For example, in the case of a pendulum, number 1 the pendulum is to move in a plane. So, degrees of freedom are 2 then, I put a constraint that the distance of the bob from the pivot point is fixed. So, I reduce the degree of freedom by 1 by putting 1 constraint. And finally, the degree of freedom therefore, comes out to be 1.

How do we put the constraints on a system? We put constraints on a system by applying certain forces for example, here a force that is related to the constraint is a tension in the strain. In a motion on 2 dimensional plane the constraint force is a normal reaction on the particle due to the surface. It would not that normally action does not let the particle fall below and go to make a motion in the direction perpendicular to the surface. So, these are known as constraint forces. These are forces that put in the constraint on the motion for the very familiar constraint that, you are, you have come across time and again, is the rigid body.

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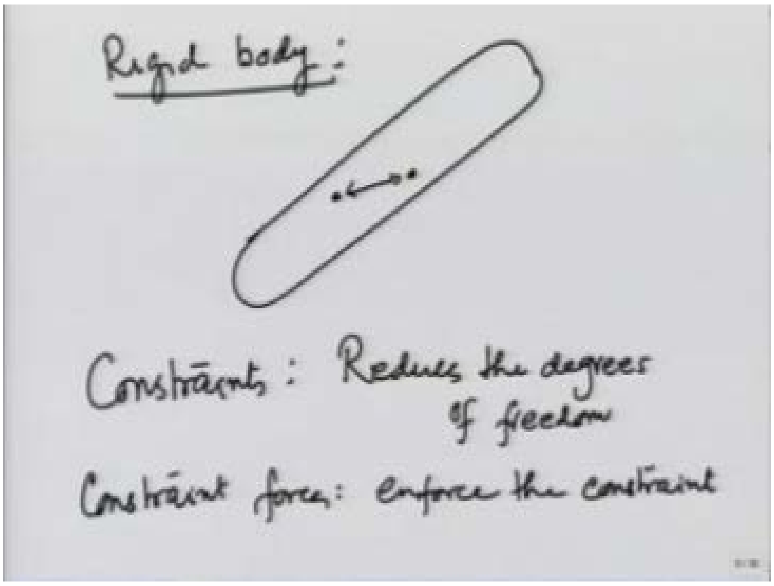

This is the concept we keep using and in a rigid body, the constraint is the distance between any two points always remains fixed. And what is the constraint force? The constraint forces is that internal force that keeps the distance fixed, right. So, the concepts that we have come up with other constraints. This result is directly reduces the degrees of freedom, and constraint forces they enforce the constraint.

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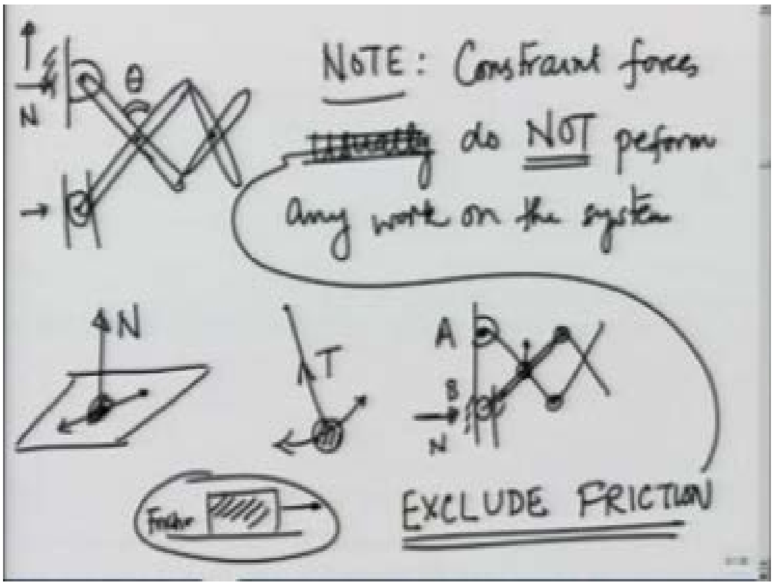

Let us go back to that example of that mechanism whereby these bars making the mechanism and so on. What are the constraint forces here? The constraint forces are the normal reaction here. Both in vertical and horizontal direction, the normal reaction here, and the forces that are applied by this pin. It is these forces that make the system move in a particular way. It reduces degrees of freedom and degrees of freedom finally, is only 1 given by this angle theta. Note, and this is very important to develop the method of virtual work, is that constraint forces usually do not perform any work on the system. Why do I say usually? Because I am going to exclude a particular constraint force later.

So, take the example of particle on a 2 dimensional surface, the constraint force normal reaction is perpendicular to the surface, the particle moves along the surface and two are perpendicular. So, normally it does not do any work on this. Take the example of the pendulum, the constraint force tension is always perpendicular to the direction of motion of the bar and therefore, this also does not do any work.

How about this mechanism? The point here, A point does not move at all. So, therefore, no work is done by the normal forces at this point. The normal reaction at this point B is perpendicular to the direction of motion and therefore, that also does not do any work. How about the forces due to the pins here? You see the pins apply a force on one particular bar in one direction, and exactly opposite force on the other bar.

However, the pins move by exactly the same amount. So, bars also move exactly in the same direction. But the forces are opposite, the result is, that the network done because of this constraint forces is 0, it is positive in one bar, negative in the other bar, added up, it gives me 0. So, you see, I have given you three examples, where you see that constraint forces usually are not performing any work. Why I say usual is? There is a, if we have a frictional force or the constraint force, that does perform work when the system moves.

Therefore if we include friction then, I would say constraint forces usually do not perform any work. On the other hand, if I exclude friction then, I will take you here and cut this off then, I would say that constraint forces do not perform any work on the system. In my development, in the development now, we will be excluding friction and deal with those constraint forces, that do not perform any work on the system. And it is this observation that makes dealing with the system very easy.

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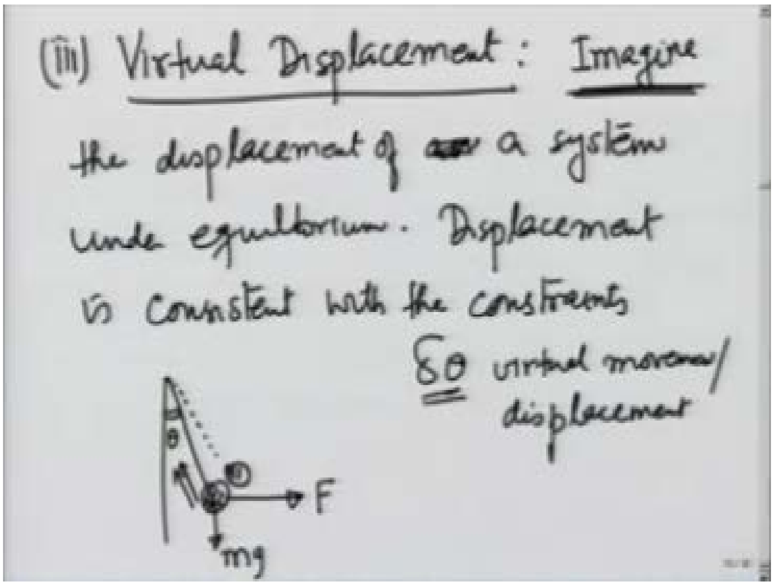

Next, I want to develop the concept of virtual displacement. Virtual displacement, I like to imagine an own emphasizes word imagine. The displacement of a system under equilibrium, and the displacement is such that, displacement is consistent with the constraints.

So, what you are imagining is suppose, there is a body to which is in equilibrium. Let us again take an example, if I take the pendulum and suppose, I apply a force this way, F its own weight mg, and it is in equilibrium. Imagine, just imagine that I displace it slightly, this is the only way I can displace the pendulum, no other way because this is the only degree of freedom allowed that is I can change only theta.

So, this displacement is consistent with the constraint, but it is actually not moving I am imagining as if it has moved and this is known as virtual displacement. So, and it is usually given by the symbol delta. So, delta theta virtual movement or displacement. On the other hand, if I moved the bob along the string, the length of the string would change, and that motion would not be consistent with the constraint condition therefore, that I cannot call that motion a virtual motion.

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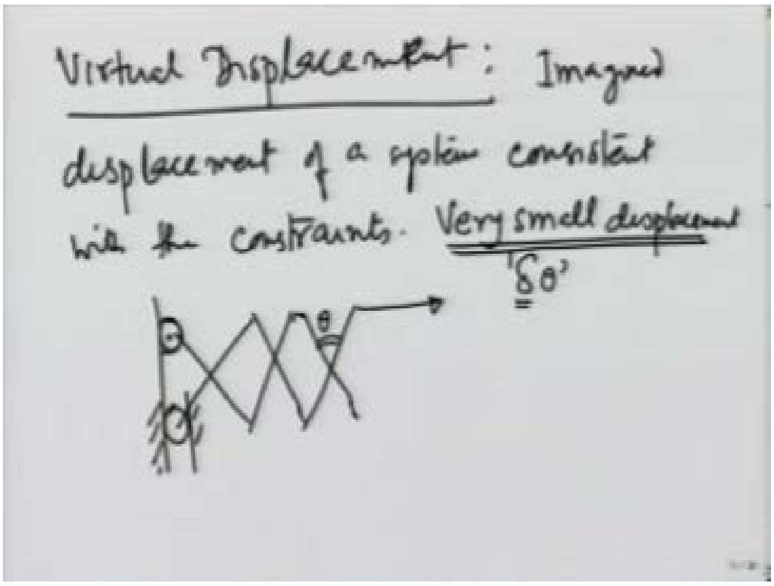

So, virtual displacement is displacement again is, and imagine displacement of system, subsystems consistent. So, imagined displacement of a system consistent with the constraints. I have already given you the example of the pendulum, this mechanism again if I go back to, the only way I can change its configuration is by changing the theta.

So, if it is an equilibrium under this force F here and its own weight, the virtual displacement would be delta theta. One thing I should make it clear I am writing a delta theta. So, virtual displacement is also very small displacement. The change is very small, you imagine a very-very small change. So, it is about equilibrium, a very small change which we imagine, and that is a virtual displacement.

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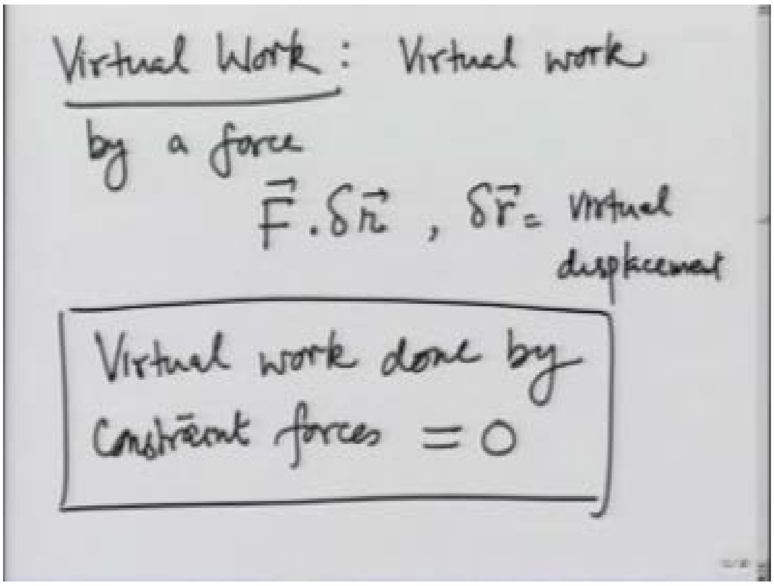

Next, associated with virtual displacement is something called the virtual work. So, when you are making a virtual displacement, each particle in the system is changing its position. So, virtual work by a force is force times delta r, where delta r is the virtual displacement.

Naturally if we, if I am doing a displacement consistent with the constraints. And we have already seen that, the constraint forces do not do any work. So, virtual work done by constraint forces is equal to 0, and this is the very-very important statement, and that makes the development of method of virtual work possible. With these definitions, we are now ready to development the method of virtual work.

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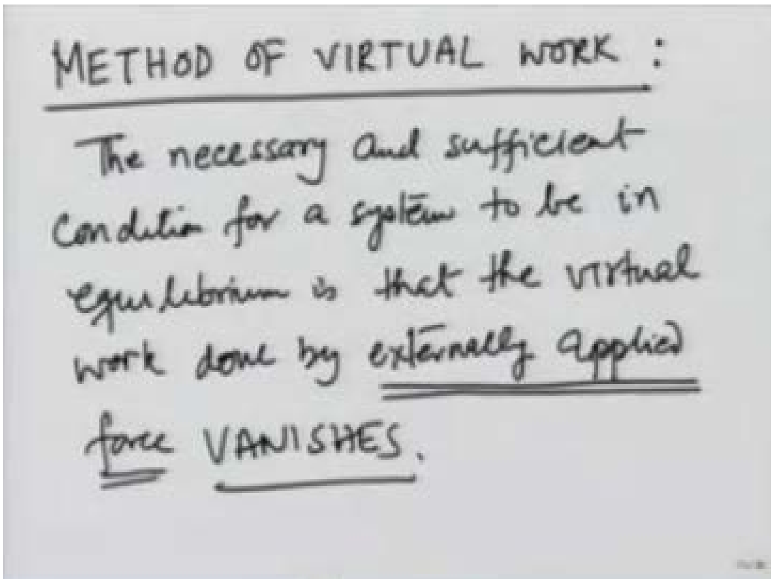

So, in method of virtual work we said that the necessary and sufficient, both necessary and sufficient condition for a system to be in equilibrium is, that the virtual work done by externally applied forces vanishes. Two comments are in order, one that we are talking about system which has no friction, if friction is included then, the constraint forces also do, the frictional constraint force also does some work, that we are excluding.

Then, because of the vanishing of the work by constraint forces, it is only the work done by externally applied forces. Those which are not constraint forces, but the forces, that we apply from outside to achieve equilibrium. Then, the virtual work done by externally applied forces vanishes notice we have avoided taking constraint forces into account. And therefore, when you apply this method, we can bypass calculating the constraint forces. Let us see how this method works.

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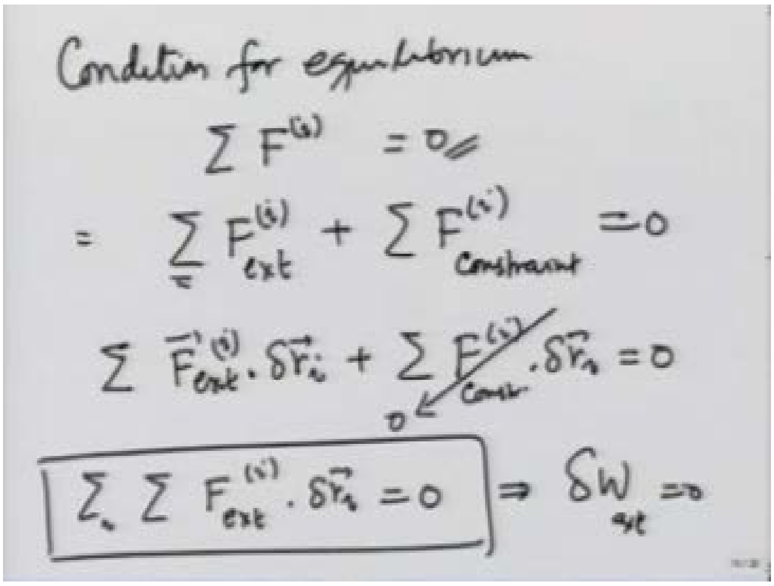

So, condition for equilibrium of any system is that, summation of all forces on any particle, i th particle, summation of all forces be 0. This may include the externally applied forces on the I th particle plus F on I, which are constraint forces their sum is 0. If each particle is in equilibrium, the system is in equilibrium.

Now, let us calculate the virtual work done. Virtual work done is going to be F external i dot delta ri, the summation equals to all the forces that are applied, plus summation F I, i refers to the I th particle, constraint dot delta ri is equal to 0. It is 0 because right hand side is 0;. However, we have already said that, constraints do not work therefore, the right this second term vanishes, and this tells you, that the work done by all the external forces together on each particle is 0.

If the work done on each particle is 0, the net force therefore, summation over i is also 0. Implies delta w, the virtual work external is 0. We have proved the sufficient, the necessary part of the principle of virtual work. How about the sufficient condition?

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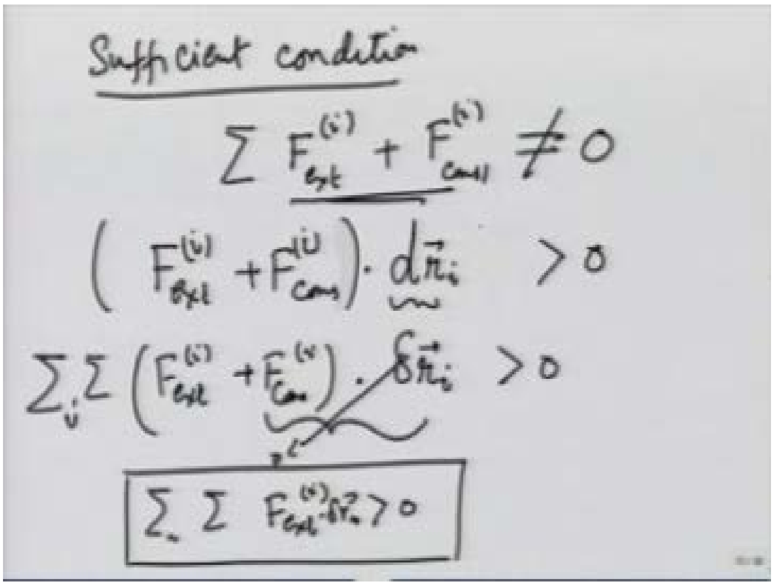

As I said, it is also a sufficient condition. So, if the forces on all the particles F i external, plus F constraint I. Suppose, they do not vanish. If they do not vanish then, the system or each particle would tend to move in the direction along which these force is. And therefore, F external I, plus F constraint I dot. To distinguish from virtual displacement, I am writing this as dri for each particle would be greater than 0. It is greater than 0 because force would make the particle move in the same direction, in which it is, it has the net direction. However, now in virtual work, I can replace, I can imagine the system being displaced virtually by delta ri.

So, F external i plus F constraint I, virtual is also be greater than 0 for each particle. If this is greater than 0, for each particle the summation over i would also be greater than 0. So, the work cannot vanish if the work vanishes that means, that this sum must be 0. Again, we are only use the fact that the work due to constraint forces is 0 and therefore, again summation i, summation F external i is greater than 0.

Applying the same argument, if this is non 0 under the condition, that system is not in equilibrium. If this is 0, system must be in equilibrium. So, that also proves that it is, that proves that it is also a sufficient condition. So, now we are going to take it from there.

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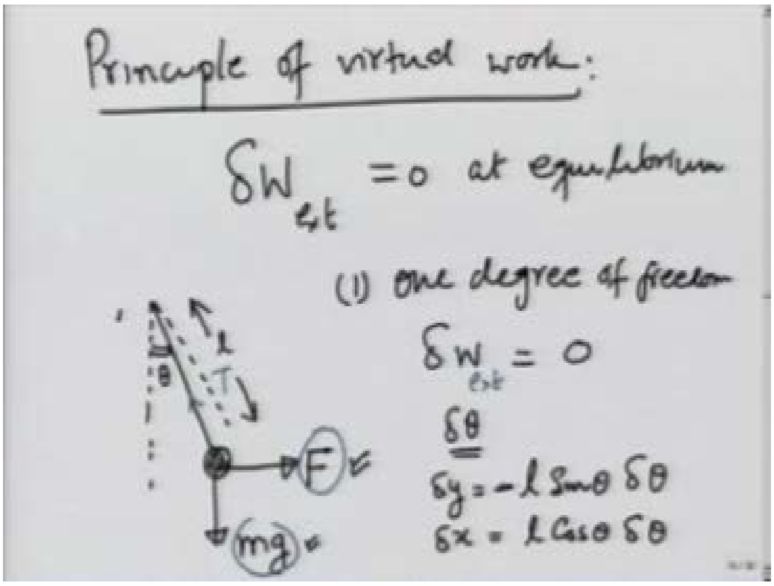

That the principle of virtual work is I will write this in symbols. The virtual work done by external forces is equal to 0 at equilibrium and that is sufficient to determine the forces and so on. Let us take an example, I will start with the pendulum example, this angle is theta, this pendulum is suppose to be under equilibrium an equilibrium under the applied force F and the weight of the bob mg, this has 1 degree of freedom.

So, I can make a virtual displacement changing that theta only here. The principle says that delta w only external forces is equal to 0. The only external forces that are working on the system are F and mg with T being the constraint force, which does 0 work so, this is only external. Let us make a virtual displacement of delta theta. If I am if I make a virtual displacement of delta theta, let us calculate the work done virtual work done by a force F and force mg.

When I make the displacement if the length of the pendulum is l then, the bob moves up let us call it delta y by an amount l sin theta, delta theta, and it moves in the horizontal direction by l cosine theta, delta theta. In fact, I should put this negative to show that it is moving up. So, the virtual work done is going to be, let me make a picture again.

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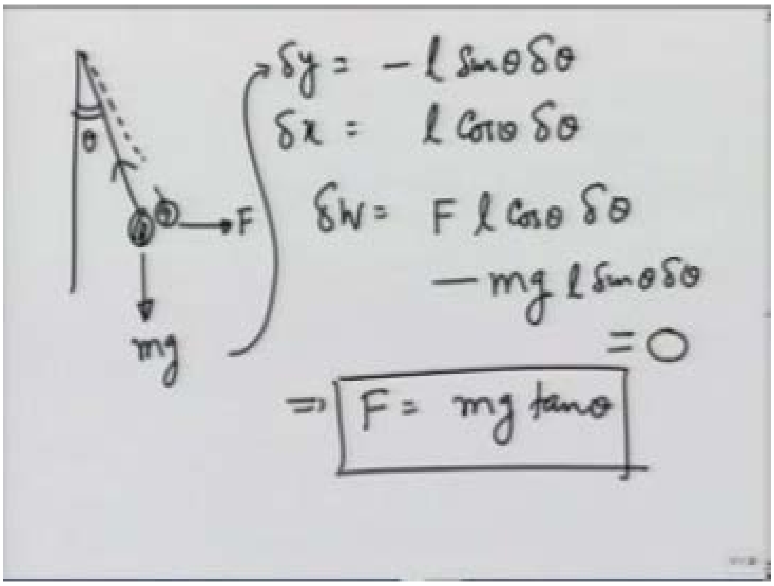

This is the bob at an angle theta. I will change this to virtual position theta plus delta theta then, delta y is moving up minus l sin theta, delta theta, delta x is l cosine theta delta theta. The force this way is F force this way is mg. So, this minus sign in delta y actually the first 2 this being opposite to mg.

So, delta w is going to be Fl cosine theta delta theta minus mg l sin theta delta theta, and by principle of virtual work this should be 0. And that gives me F equals mg tangent theta straight away. You see, I did not have to go to calculate the tension, whereby I would have written T cosine theta is equal to mg and T sin theta is equal to F and therefore, F equals mg tangent to theta. Let us take the other example, that we took in the beginning.

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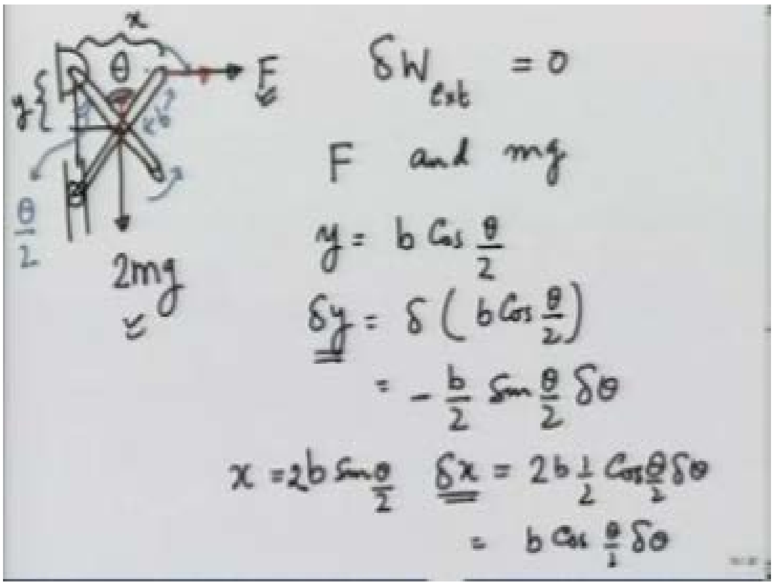

With this mechanism here, which is in equilibrium under the supplied force F, its own weight here 2 mg, and the only angle that matters here for the degree of freedom is this theta. Each side, each half of the bar is b. The constraint forces again go on do not do any work and therefore, delta w external forces only should be 0.

The 2 external forces that are there are F and mg, let us make a virtual displacement of the system. What would a virtual displacement be? Virtual displacement be will be to make this angle larger. If I make this angle larger, what I will be doing is pulling this point out. So, let me show it by red, this point will be going out and this point will be moving up, by how much amount let us calculate that.

So, this distance here is nothing but let us call this y, and let this distance from here to here be x then, you see that y is equal to b, let me show this angle here. This angle is going to be theta by 2. So, y is b cosine of theta by 2. If I change theta slightly, delta y is going to be delta of b cosine of theta by 2 and that is going to be b over 2 sin of theta by 2 delta theta. This is the virtual displacement of the center point if I change angle to theta plus delta theta.

Similarly, x is equal to b sin theta times 2, this entire distance and therefore, delta x is going to be, oh sorry this is 2 b sin theta by 2. 2 b 1 by 2 cosine of theta by 2 delta theta, which comes out to be b cosine of theta by 2 delta theta. Why we are calculating this delta x and delta y is, so that I can calculate the virtual work done by a force F and 2 mg.

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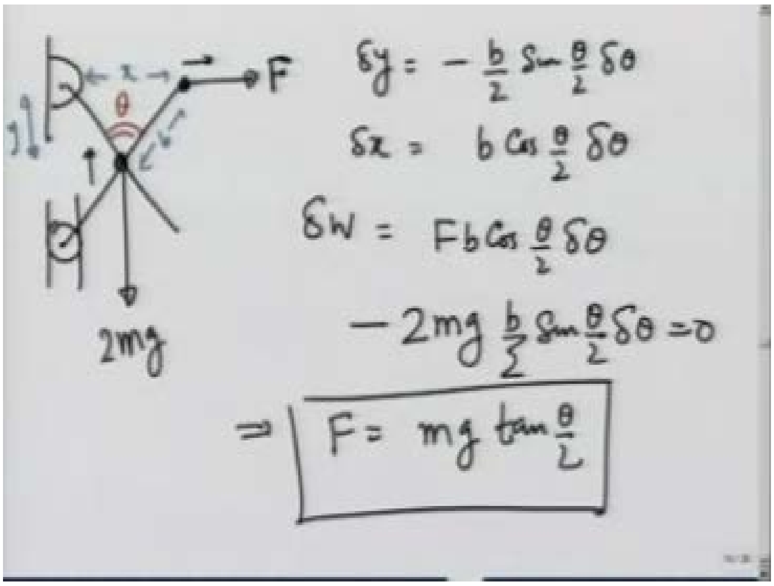

Let me make the picture again schematically and we have calculated that delta y is nothing but minus b over 2 sin theta by 2 delta theta, and delta x is equal to b cosine of delta by 2 delta theta. By minus sign we mean that, y is decreasing or the point here is moving up.

So, delta w is going to be equal to the work done by force, virtual work done by force F. Since, x has increased by b cosine theta by 2 delta theta, work done by F is going to be F b cosine of delta by 2 delta theta positive because that the point here has moved in the same direction as the force. And work done by the force mg or the weight is going to be minus 2 mg b over 2 sin of theta by 2 delta theta, and that should be 0. You calculate this and you get F equals mg tangent theta by 2, and that is your answer that we had obtained earlier.

Notice, I did not have to go through calculating the normal forces and things like those, which we avoided and bypassed because of the work being virtual work being done by those normal forces or constraint forces 0. How does this simplify or make life easy when the system becomes big? Let us extend this example itself.

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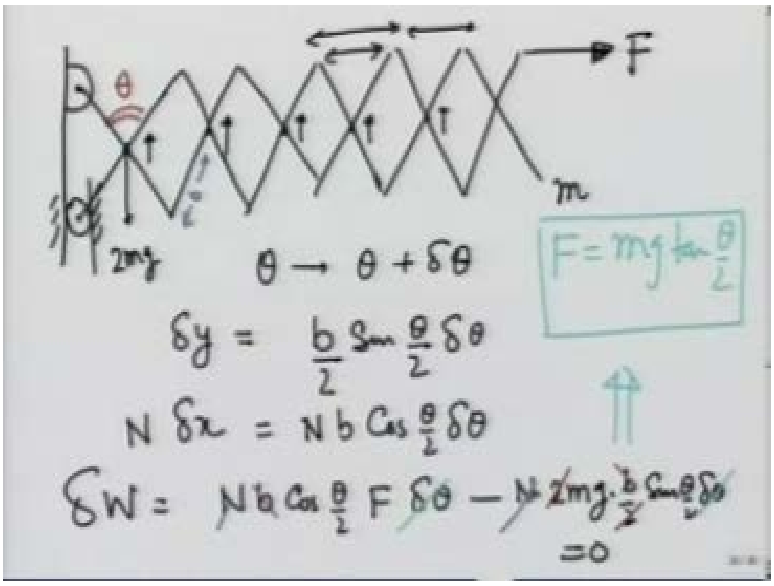

In this mechanism, if I take many-many rings suppose, 10 such rings and so on, and I apply a force F here, each bar is mass m and I want to calculate, how much force F is required to keep this entire system in equilibrium. Again notice that, the system has only one degree of freedom with this theta describing everything.

Let the length here be b, imagine again a virtual displacement of the system so, that theta goes to theta plus delta theta. If that happens, all these points are going to move up, and all these gaps here are going to widen up. We have already calculated that, delta y or each point is equal to b sin of theta by 2 delta theta, b divided by 2, and delta x for each of this gap opens up by b cosine of theta by 2 delta theta.

So, is there N such links, delta x for N is going to be N times as much, N times d. delta y for each joint is going to remain the same. So, work done, virtual work done is going to be N b cosine of theta by 2 times F delta theta, minus this delta y at each link is doing work of x 2 mg, and there are N such points. So, therefore, I am going to have N times 2 mg times b over 2 sin theta by 2 delta theta, and this should be 0. This N drops out 2 cancels so does b and so does delta theta. And therefore, you again get and, let me write it here to keep on the same slide, F equals mg tangent of theta by 2, same answer as earlier.

So, we figure out that, no matter how long a link, how long a link do we make. It does not matter, the force required to keep the system in equilibrium is still mg tangent theta by 2. But you see how easy it was, we did it in 4 or 5 steps using method of virtual work. So, let us recap what we did in method of virtual work.

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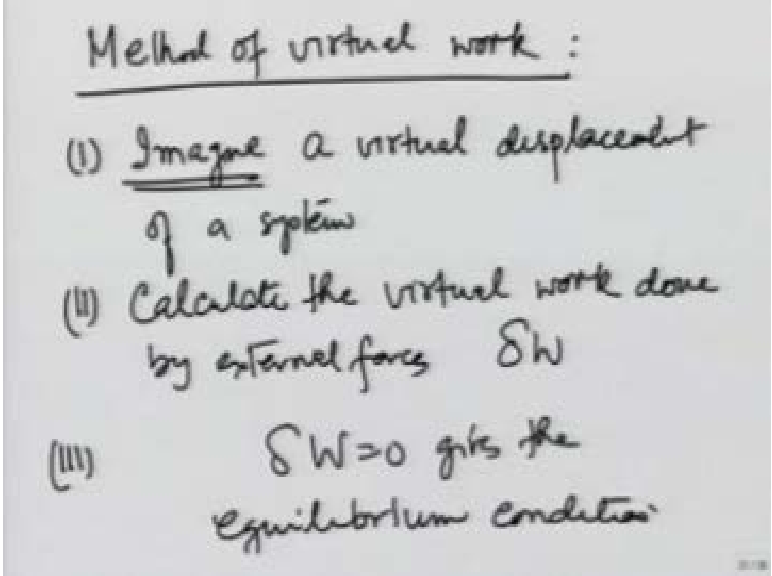

Imagine, and I keep you in this word image because this is the first word displacement, which you imagine it does not really take place. Image a virtual displacement or virtual change of orientation is so on of a system. 2, calculate the virtual work done by external forces. And 3, call this delta w, delta w is equal to 0 gives the equilibrium condition. As in any other new concept to master it, you have to keep doing many-many problems. And I will again rest of the lecture, I am going to do three or four examples to make you more familiar with the method of virtual work.

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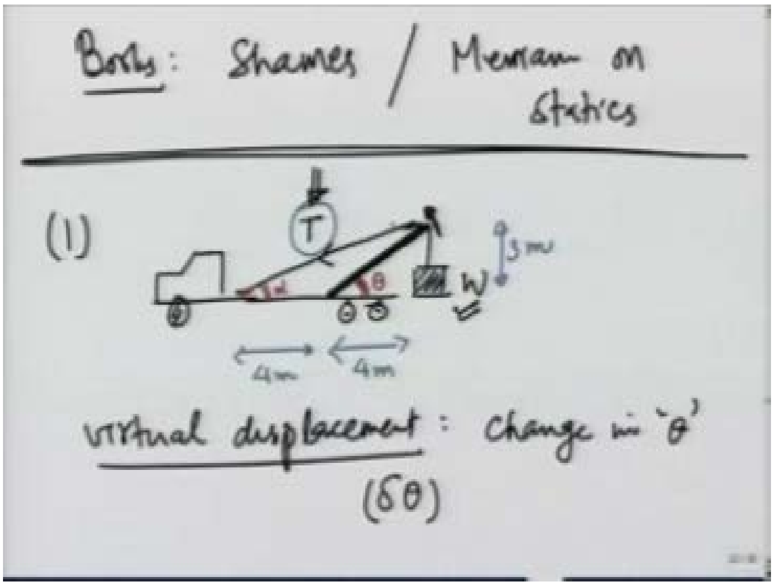

These examples are from the book Y Shames on engineering mechanics, and book by Merriam on statics. As a first example, let me take a crane on which there is a shaft here, and by a string, by applying tension T, one is holding a weight w. The distances given are 4 meters, 4 meters and 3 meters. We wish to calculate given tension T, how much weight can be held here in equilibrium.

Let me call this angle theta, and this angle alpha. If I want to use the method of virtual work then, what we should do is imagine a virtual displacement of the system, and here the virtual displacement is going to be described by the change in angle here. As you change, you pull this string in or pull the rope in, this system is going to move like this. So, a virtual displacement is going to be change in theta.

And then, calculate work done by the external forces w, and notice in this case the external force tension is not a constraint force, the constraint force is the force provided by the shaft because of its rigidity. So, the work done by that is going to be 0 because the displacement is always going to be perpendicular to the shaft and so, we have to calculate work done by w as well as T, the tension and make it equal to 0. So, let us see at the given position where the distances are given to be 3 meters 4 meters and 4 meters. What we have is, virtual displacement is going to correspond to delta theta. And then, we make this point move up both in x direction and y direction.

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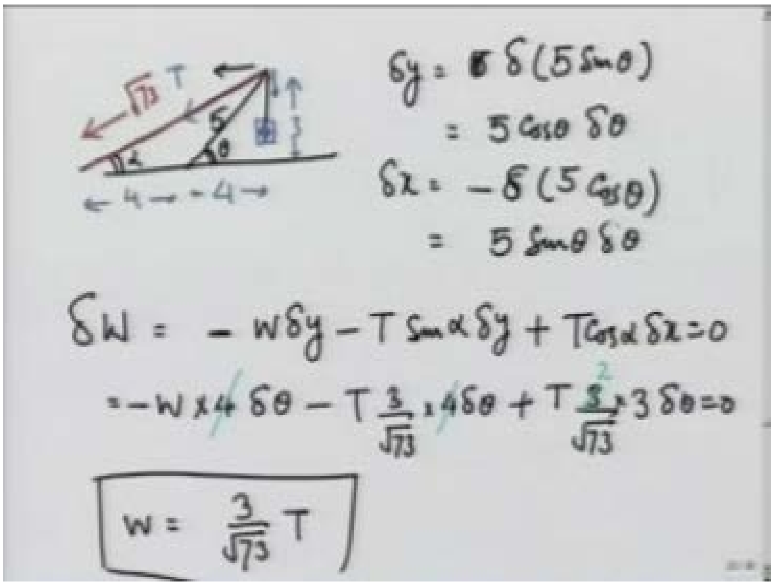

So, what do we have is situations schematically is, theta alpha 4 meters, 4 meters, 3 meters, W here, tension T here. When I change theta with delta theta, if this length I, this length is going to be 5 meters this is 3 and 4. So, delta y, positive in the up direction is going to be 5 delta of 5 sin theta, which is going to be 5 cosine of theta delta theta.

Similarly, delta x it is going to be in the negative direction, x is going to decrease so, I am going to write this as negative. Or just keep in mind is, to the left is going to be 5 of delta of 5 cosine theta, which is going to be 5 sin theta delta theta towards the left. We have calculated both x and y components and therefore, the total work done delta w is going to be the weight and delta y in the opposite direction.

So, minus w delta y, the T has a vertical component at this point going down, which is going to be T sin of alpha delta y. And T has a component going towards left, which is T cosine alpha that is going to be T cosine of alpha delta x is equal to 0. This gives me w, delta y we have already calculated 5 cosine theta delta theta, 5 cosine theta from here is 4 times 4 delta theta, minus T sin alpha, sin alpha is going to be 3 divided by this length. And this length is 8 square plus 3 square, square root. So, root 73.

So, T times 3 over root 73 times 4 delta theta, plus T cosine alpha is going to be T or this is a minus sign here. T cosine alpha is going to be T, 8 over square root of 73, delta x is in the same direction 5 sin theta is 3, 3 delta theta is equal to 0. And that gives you, I can cancel the 4, 4, 4, and this will be 2 here, and that gives you w is equal to so, 3 over root

73 T, that is the weight that can be held by this machine. You see, again the calculation was very straight forward. As the final example of applying method of virtual work, I solve a problem that involves both a force and a torque.

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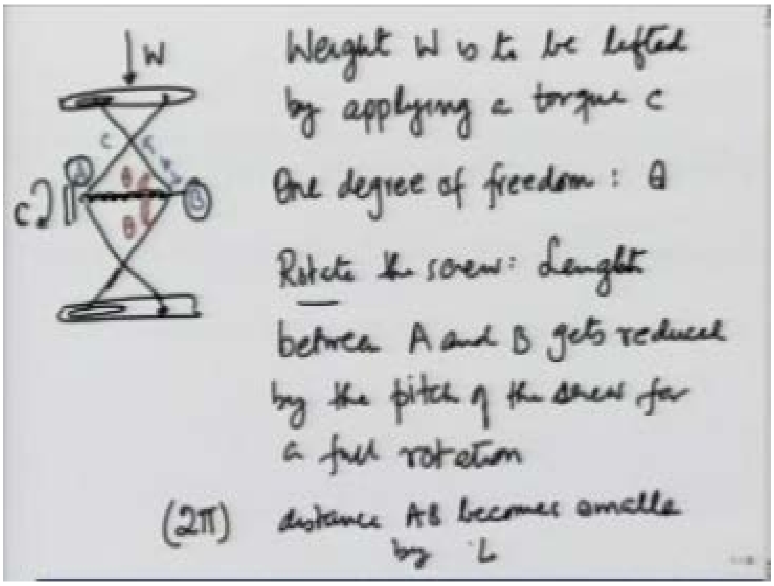

The example is that of a screw jack whereby two supports like this are fixed the screw here, and we apply a torque here, see these go down and are fixed on a stand one of these links is fixed and at the other one can move in a slot, and same thing on the top. One of them can move in the slot and the other one is fixed. The question that we ask is, if there is a load w on this jack, what is the minimum torque c that is needed to lift this weight?

So, weight w to be lifted, is to be lifted by applying a torque c. You given that this angle is theta and so is this, and this length is b, let us also call these points A, B, C. You can again see that, there is only one degree of freedom, and that is angle theta. Question is, how do we calculate the force that the torque required so, that this force w or the load w is balanced or can be lifted.

To calculate this, let us see what happens when we rotate the screw. In rotating the screw, the length between A and B, that is these two points gets reduced by the pitch of the screw for a full rotation. So, what is given when I make a full rotation, that is 2 pi rotation. The distance AB becomes smaller by L, a length L which is the pitch of the screw. When A and B come closer, this platform gets lifted because this point gets lifted

So, in rotating the screw on doing some work and in lifting this, another work is being done. The external forces and torques now are see, the external torque and w the external load. The net virtual work done by combination of c and w should be 0. Let us calculate that and make that equal to 0.

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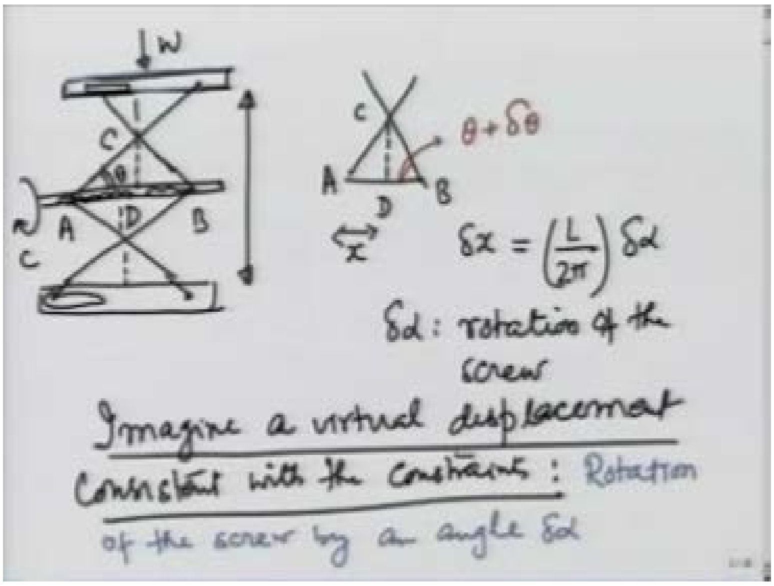

Let us call this angle theta, which has already been given. So, when the length reduces, the theta becomes slightly larger, let us say this goes to theta plus delta theta. And therefore, this length between let us call this A B and the top of the triangle c, and the perpendicular distance from c to A B, C D so, this D, C, A and B, C D increases. The total length would increase by 4 times the increase in C D because there are 4 gaps like this. So, let us calculate that, let us call this distance A D x. So, that delta x is the reduction in the length and delta x is going to be L over 2 pi times delta alpha, with delta alpha is the rotation of the screw.

So, now what we do is, imagine a virtual displacement consistent with the constraints. And that virtual displacement is going to be rotation of the screw by an angle delta alpha. And that causes shortening of the distance between A and B, and increase in the distance C and D. From that I can calculate the virtual work done by applying the torque, and the virtual work done against the weight and making that sum total of that virtual 0 would give me a relationship between the torque c and the weight w. Let us calculate that now.

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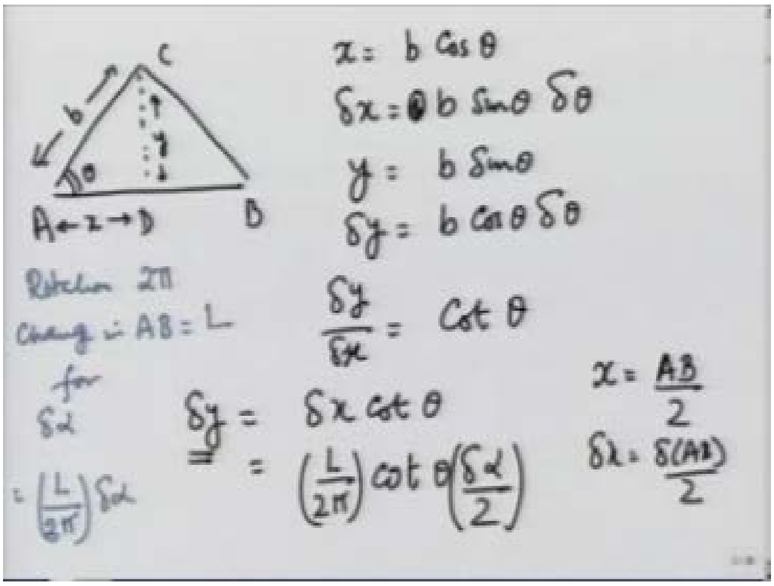

So, in this triangle A, B, C and D, let us call this distance x, and this distance y, this angle is theta, let us call this hypotenuse AC to be b. Then, you can see that x equals b cosine of theta therefore, delta x is going to be b sin theta delta theta. I am just worrying about the magnitude of this sign, minus sign I am not worrying about. Y equals b sin theta and therefore, delta y is going to be b cosine of theta delta theta.

And therefore, delta y over delta x is going to be equal to cotangent theta, or delta y is equal to delta x times cotangent theta. How much is delta x that is equal to nothing but the change in the length AB. For rotation, let me just write on the side, rotation 2 pi change in AB is equal to L. So, for delta alpha it is going to be L over 2 pi times delta alpha.

So, therefore, delta y is L over 2 pi cotangent theta delta alpha divided by 2, this divided by 2 comes because change in x, x equals AB divided by 2. So, delta x is delta AB divided by 2. So, we have calculated the change in delta y related to a change delta alpha.

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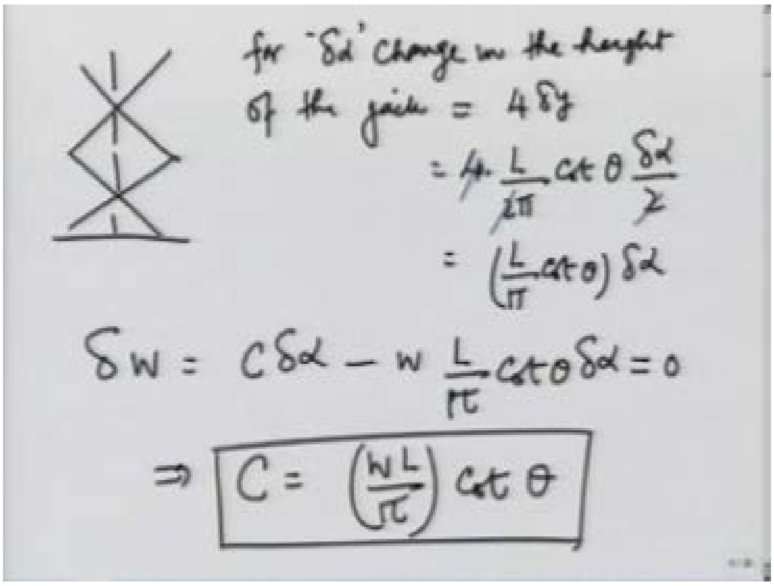

Since, in this screw jack, there are four lengths like this 1, 2, 3, 4. So, for delta alpha, the change in the height of the jack is going to be equal to 4 times delta y, which is 4 times L over 2 pi cotangent theta delta alpha by 2. This two cancels with 4 and this is therefore, L over pi cotangent theta delta alpha.

Let us now calculate the total virtual work done. Total virtual work done is going to be c times delta alpha because of the torque, and the length y increases in the direction opposite to w. So, this is going to be minus w L over pi cotangent theta delta alpha, and by the principle of virtual work this is 0, and that gives me an answer that the torque that is required to lift the weight minimum has to be w L over pi cotangent of theta and that is your answer.

So, what I have done in this lecture is given you an introduction to the method of virtual work for you to appreciate the power of this method. It bypasses doing lengthy calculation and focuses directly on the external forces that, we want to calculate. Of course, we have not included friction here because friction is a constrained force that does virtual work. You will be learning about more advanced techniques using such methods in the future courses.