Numerical Methods and Computation

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Lecture No - 31

Interpolation and Approximation (Continued)

In the previous lecture we have defined what is approximation; we have also defined what is the best approximation.

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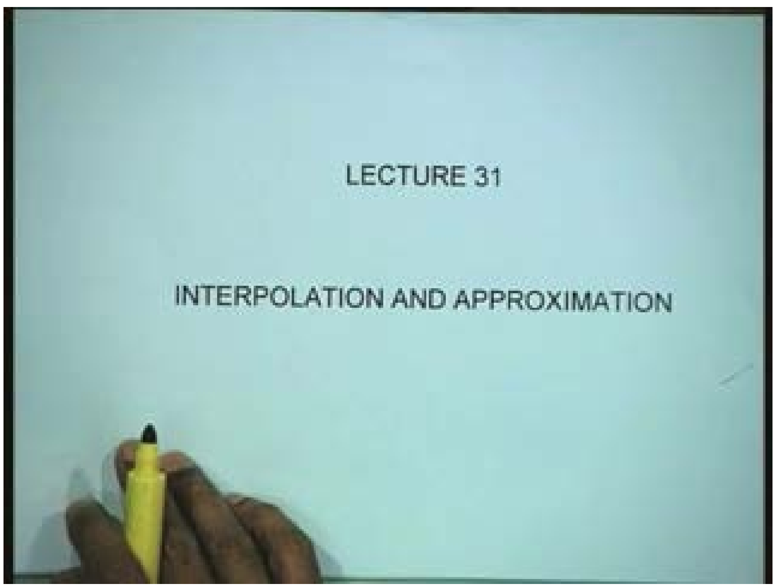

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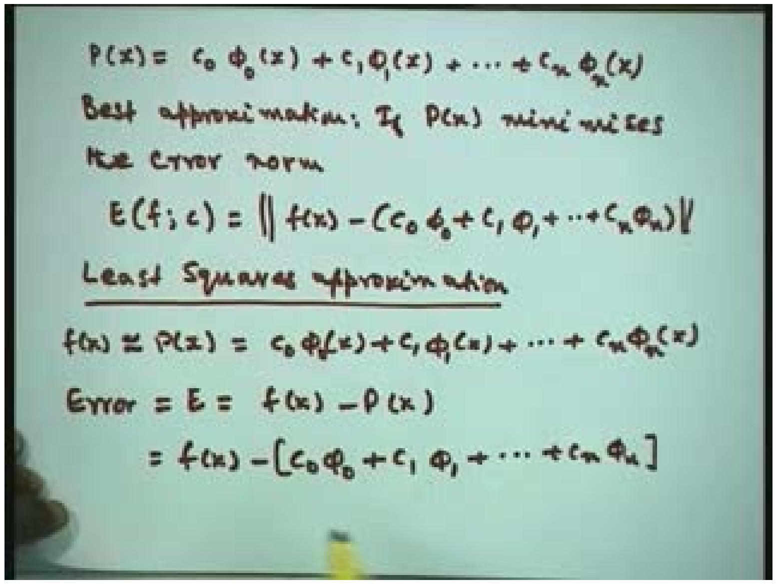

If we take a function like P(x) is equal to some c_0 $phi_0(x)$ plus c_1 $phi_1(x)$ plus so on c_n $phi_n(x)$ then we define this as the best approximation if it minimizes the error norm, therefore this is the best approximation, best approximation if P(x) minimizes the error norm and the error norm we have written it as, error f vector c of the constants, norm of f(x) minus (c_0 phi_0 c_1 phi_1 plus so on c_n phi_n). Now if we define the norm, suitable norm; then we get different type of approximation we mention that if we use the Euclidian norm we get the least square approximation and if we use the uniform norm you get the uniform approximation.

So let us again just define, what is our Euclidian norm for this function to derive the least square approximation, so let us define the least squares approximation. Therefore we need to define what is our approximation first, f(x) is equal to P(x), approximately P(x), that is your c_0 $phi_0(x)$ plus c_1 $phi_1(x)$ plus so on c_n $phi_n(x)$. Let us denote the error as simply E, now we will write it as only E that is inside this norm, so that is your f(x) minus P(x), that is f(x) this quantity, f(x) minus c_0 phi_0 plus c_1 phi_1 plus so on c_n phi_n. Now since we want to use the Euclidean norm, what we shall do is that the square of these errors then sum up, if there is a weight function multiply by the weight function also.

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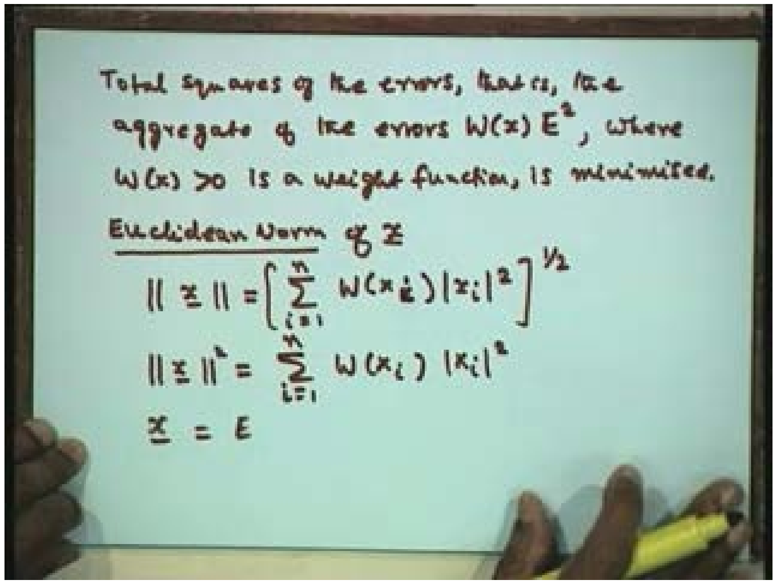

Therefore the total aggregate of the errors over the interval [a, b] is now minimized. Therefore we will define the least square approximation as the total squares of the errors, total squares of the errors that is the aggregate, aggregate of the error w(x) into E square, where w(x) is a weight function is greater than 0, is a weight function, is minimized. Therefore the total squares of the errors that mean the total aggregate of the errors w(x) E square, where w(x) greater than 0 is a weight function, is minimized. In many applications we just take the weight function as 1, therefore it will be a simply minimizing the aggregate of the errors E square over the entire interval that is given to us. Now why we are using this is, how the Euclidean norms give this, let us just look at what is the definition of Euclidean norm.

The Euclidean norm of a vector say vector x that we have, then we have define this as norm of x is equal to summation of i is 1 to n, $w(x_k)$ or $w(x_i)$ magnitude of x_i square whole to the power of half, this is the definition of the Euclidean norm of a vector x. I can square both sides and write it also norm of x whole square is simply summation i is 1 to n, $w(x_i)$ magnitude of x_i square. Now in our application this vector x is nothing but the error vector, so this x is replaced by the error vector and therefore we are minimizing this particular quantity, if we are minimizing this norm we are minimizing this square of it, both are the same. Therefore we shall be minimizing this, therefore our x in this application is our error vector E that we have here.

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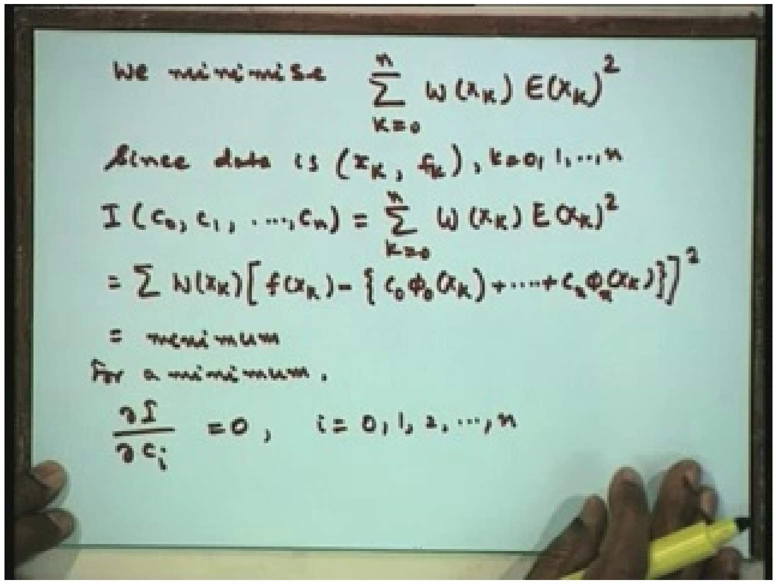

Therefore by the definition of this, that the least square approximation definition, we can immediately say that we minimize, therefore we minimize this summation, let us write k is equal to 0 to n, $w(x_k)$ $E(x_k)$ square. Now since our data is, since data is, let us write it as (x_k, f_k), k is equal to 0, 1, 2, 3, n, that is why summation is running from 0 to n here, since our data is running from k is equal to 0 to n and we have taken the data is (x_k, f_k). Now let us denote this by some I, let us just write this as I, I is equal to a function of the constants (c_0, c_1, c_2, c_n) because E contains your error and this is your, k is equal to 0 to n $w(x_k)$ $E(x_k)$ square.

Now let us write down what is $E(x_k)$, so I can write this as summation of $w(x_k)$, now we shall remember that is k is 0 to n, so let us drop that notation, just write down summation here. This is error of x_k that is f at x_k minus c_0 phi_0 of x_k plus so on c_n phi_n of x_k, all of them evaluated at x_k and this square, this should be minimum; this should be minimum. Therefore it is a simple minimization problem of calculus, so we require that the partial derivative of I with respect to the constants (c_0, c_1, c_2, c_n) should be equal to 0 in order that it has a minimum or maximum. So we shall show that it is a minimum that is required later on, so therefore for a minimum we need delta I by delta c_i is equal to 0, for all i, i is equal to 0, 1, 2, so on n. Therefore we will have n plus 1 equations for determinant c_0, c_1, c_2, c_n. Let us know differentiate this with respect to to i.

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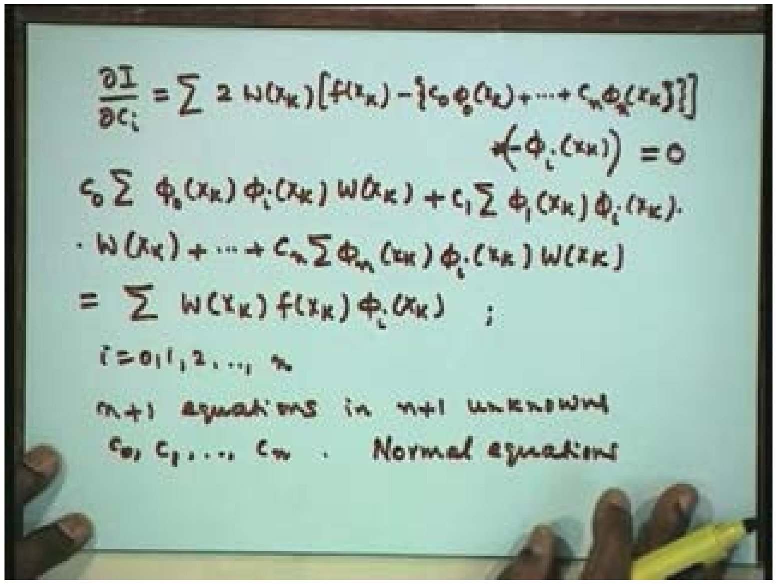

So I can differentiate this with respect to i, delta I by delta c_i that will be equal to summation of, we have a 2 here, so I will write z $w(x_k)$, this is [f of x_k minus {c_0 phi_0 of x_k plus so on c_n phi_n of x_k}] and multiplied this by, this is, now I would differentiate it with respect to c_i, therefore I will get $phi_i(x_k)$ as a multiplicative factor with a negative sign, lets put it in a bracket, phi_i of x_k and this should be equal to 0, delta I by delta c_i must be equal to 0. Now let us simplify this equation, we cancel this and then take this term to the right hand side, so I will retain this on the left hand side. Therefore this will give us c_0 summation phi_0 into phi_i, this is $phi_0(x_k)$ $phi_i(x_k)$ and of course $w(x_k)$ is also there. So the first term will be c_0 phi_0 $phi_i(x_k)$ both of them evaluated x_k, multiplied by x_k, then plus c_1 phi, that is next term is $phi_1$of x_k multiply by phi_i of x_k then we multiply by $w(x_k)$ plus so on, the last term will be c_n phi_n this is phi_n of x_k, this is phi_i of x_k into $w(x_k)$.

This is the left hand side that we have retained and this term $f(x_k)$ goes to the right hand side, so I can write this as summation of $w(x_k)$ $f(x_k)$ $phi_i(x_k)$ and i running from 0, 1, 2, 3, n. Now these are n plus 1 equations in n plus 1 unknowns, these are n plus 1 equations in n plus 1 unknowns c_0, c_1, c_2, c_n, so these are n plus 1 equations in n plus 1 unknowns and these are called the normal equations, we call them as normal equations. Therefore in any particular problem when once our phi_i are given to us, weight function is given to us, we just evaluate these sums over here and the substitute it here, solve the n plus 1 equations in n plus 1 unknowns and then substitute back in P(x) for the values of c_0, c_1, c_n and that gives us the least square approximation and we can also find out the least square error. We can write down the expression, we can substitute the values of c_0, c_1, c_2, c_n over here, just simplify the whole thing, square this one, multiply by x_k sum over this and that gives you the least square error, the total least square error.

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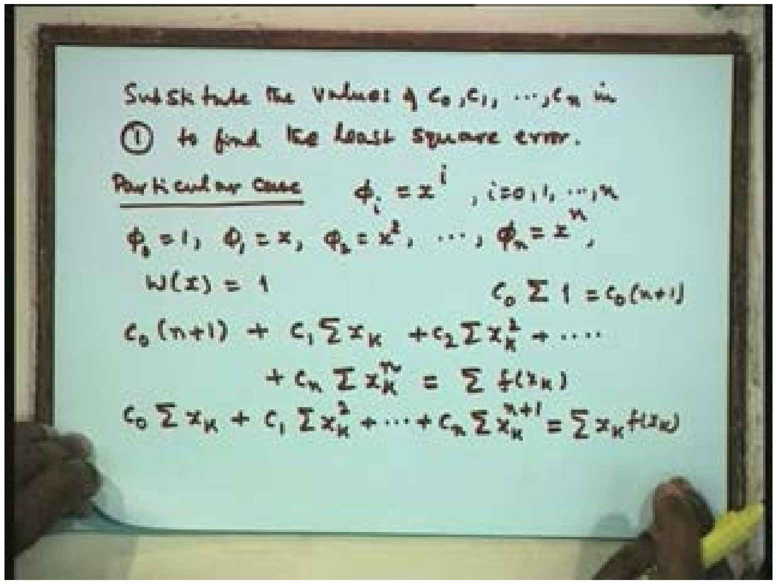

Therefore if I put this as 1 then I can say that the substitute, substitute the values of c_0, c_1, c_n in 1 to find the least square error, find the least square error that is the total error that we have committed under the least square approximation, we would get by substituting the values of c_0, c_1, c_2, c_n in this particular equation. Now let us take some simple case, let us take the particular case of when which we would like to normally use as a polynomial approximation. Now let us take the case when phi_i is simply equal to x_i that means the functions that we are talking phi_0 is 1, phi_1 is x, phi_2 is x square so on phi_n is equal to x to the power of n.

Now let us substitute this in the equations that we have just now obtained here and we have here phi_0 is 1 then we have got here $phi_i(x_k)$ into $w(x_k)$ then you have c_1, this is phi_1 is equal to x, so we are going substitute phi_0 is 1, phi_1 is equal to x, phi_i is equal to x to the power of i. So if I just substitute it over here what I would get here is, now let us see the first term that comes out over here and also we can still particularize it, let us take w(x) also as 1, w(x) also as 1. Then we can see that the first coefficient will be w(x) is 1, this is your 1 and you are now multiplying by the first equation for i is equal to 0, for i is equal to 0 this is phi_0 square, then we have phi_0 square is 1 square, therefore what we have here is the first term will look like c_0 summation of 1, c_0 this is phi_0 square into 1, this is your c_0 of 1 that is total summation is the total values i is equal to 0 1 up to n. Therefore this will be simply equal to c_0 into n plus 1, this is total observations that we have here. Therefore the first term here would be c_0 into n plus 1, then we are taking the case i is equal to 0, therefore this is 1, this is x and this is c_1, therefore this is summation of x_k, therefore plus c_1 summation of x_k.

Now if I take the next term, this will be c_2, phi_2 is multiplying by phi_0 and phi_2 is multiplying by phi_0 that is x square that is x_k square, therefore the next term will be plus c_2 summation of x_k

squared plus c_n summation x_k to the power of n and the right hand side is, for i is equal to 0 this is 1, $f(x_k)$ this is 1, so simply summation of $f(x_k)$, so the right hand side is simply summation of $f(x_k)$. This is your first equation, the second equation I will take i is equal to 1, when I take i is equal to 1 in this then I am multiplying phi_0 by phi_1 therefore I am multiplying 1 by x, this is 1 and the next term will be phi_1 into phi_1 that is phi_1 square that is your x square and so on. Therefore the next term will be c_0 summation x_k plus c_1 summation x_k square plus so on c_n summation x_k to the power of n plus 1. On the right hand side we have got here phi_i that is phi_1 that is x, therefore I will be multiplying by x_k on the right hand side, this is your x_k f of x_k. You can now see that each of them because this is a polynomial, these will be, just last term of this will be coming here and then this will come here and then next term will be, if you are taking the next equation, you will have here c_0 phi_0 into phi_2 that is your x square into 1.

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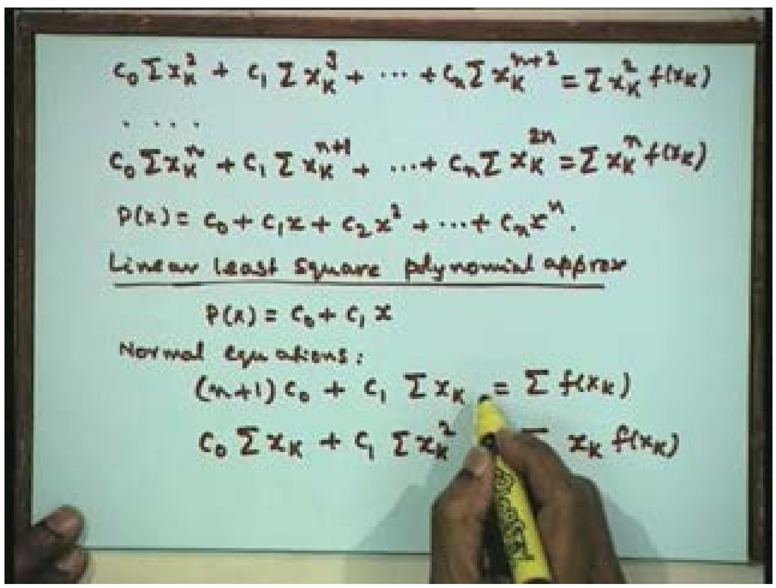

So the the next equation would therefore read as c_0 summation x_k square plus c_1 summation x_k cubed plus so on c_n summation x_k^{n+2} that is summation x_k square $f(x_k)$ and so on, the last equation reads c_0 summation x_k to the power of n, c_1 summation x_k to the power of n plus 1 plus so on, c_n summation x_k to the power of 2 n that is n plus n that is 2 n and this is x_k to the power of n f of x_k. These are the normal equations when you take it as a polynomial, what we have taken here is, P(x) we have taken it as a polynomial c_0 plus c_1 x plus c_2 x squared plus so on c_n x to the power of n, so this is the polynomial approximation that we have taken here. Let us see how trivial it will look like to the particular cases, let us take the linear case; from here let us just write down the linear least square polynomial approximation that means I am just taking my polynomial as simply c_0 plus c_1 x. Therefore in this system that I had written it, I will just cut out the terms up to only 2 coefficients c_0 and c_1.

Therefore the normal equations in this case would become, the normal equations are (n plus 1) c_0 plus c_1 summation of x_k is summation of $f(x_k)$, c_0 summation of x_k plus c_1 summation x_k square is summation x_k $f(x_k)$. Now in any given data we just have to find what is the summation of x_k, summation of x_k square, summation of $f(x_k)$ and summation of this x_k $f(x_k)$, solve the 2 by 2 equations and we have the solution for c_0 and c_1 and that will give you the least square approximation and if I want the least square error, I would substitute in the expression that we had written it the in equation 1 and we can find out what is the total least square error under this linear approximation. Now if I want a quadratic approximation that is 2, I will have 1 more term over here c_2 summation x_k square, we will have c_2 summation x_k cubed and you will have one more equation, okay let us write down the quadratic also.

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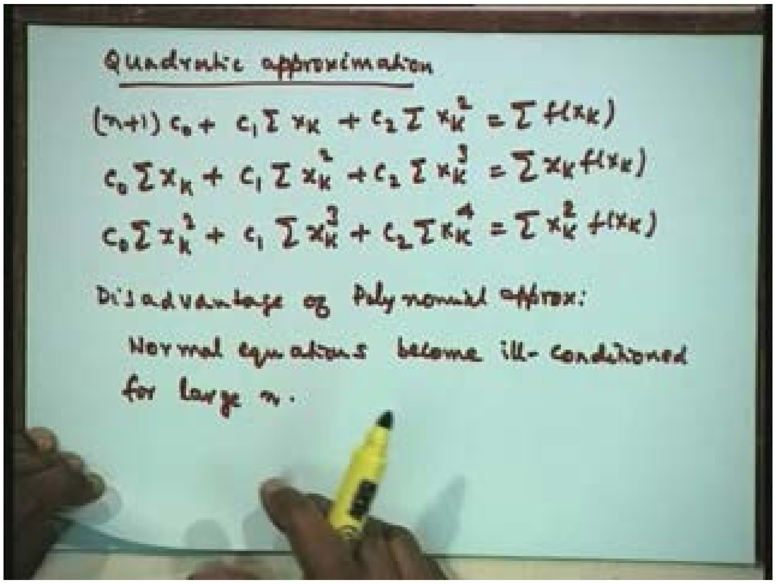

So let us write this as quadratic approximation, therefore this will give us (n plus 1) c_0, c_1 summation x_k plus c_2 summation x_k square is summation $f(x_k)$, so that will be our first normal equation. Then I have c_0 summation x_k, c_1 summation x_k square plus c_2 summation x_k cubed is summation x_k $f(x_k)$ and the third equation is c_0 summation x_k square plus c_1 summation x_k cubed plus c_2 summation x_k^4 that is equal to summation x_k square $f(x_k)$. These will be the normal equations in the case of the quadratic approximation. Now we can normally go up to cubic or of fourth degree approximation or any approximation, however the one disadvantage of the polynomial approximation, we are talking of the polynomial approximation is that when you take the system of equations for very large n, system of equations become ill conditioned that means the solution is prone to the effect of the round off errors that even if you have a minor round off errors in any of these numbers, the cumulative effect of that will be destroying the solution.

So the disadvantage of high degree polynomial approximation is that the system of equations is ill conditioned. Therefore we do not normally use beyond a degree or 3 or 4 in the least square approximation, so that is the disadvantage of the least square polynomial approximation, of polynomial approximation that the normal equations become ill conditioned for large n. What really computationally as I said it would mean is, that the round off errors that is there in your $f(x_k)$ x_k and then these summation that we are doing, the round off errors can really spoil the solution of the particular problem that means if you round off to 4 places, you may get a different answer, round off to 5 decimal places you will get a different answer. So that is the effect of the round order is seriously felt in the solution of the problem, that is what we really mean by ill conditionedness in terms of the computational aspect. Now let us first taken example on this.

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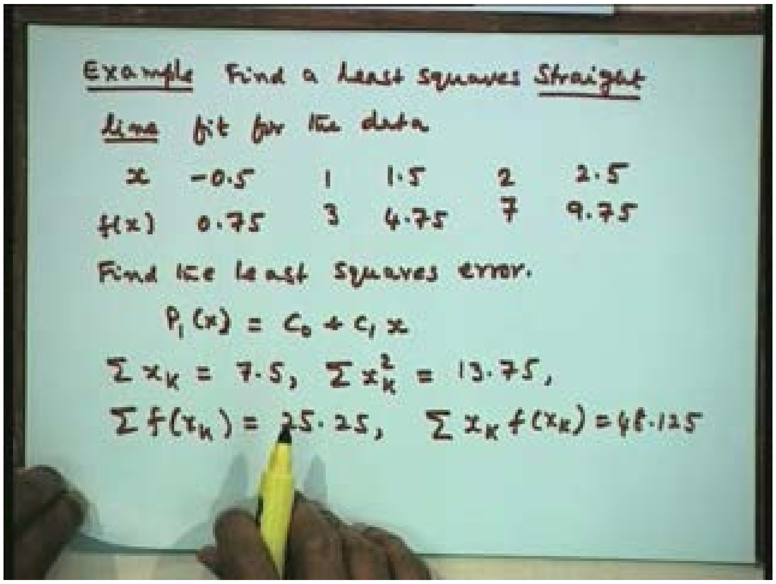

Now find a least square, let us take a straight line approximation, straight line fit for the data x, f(x) 1, 3, 1 point 5, 4 point 7 5, 2, 7, 2 point 5, 9 point 75, let us also say find the least squares error, find the least square error. Now here we have asked for a straight line, a straight line fit therefore what we are talking of is approximation by using the polynomial c_0 plus c_1 x. Now the problem is very simple, we just have to find these quantities; these 4 quantities, substitute it and solve it here. So I need here the quantity summation of x_k that is the sum of all these x's, I would give this valued that is equal to 7 point 5. Then I square all these numbers and then take the summation, so I will have your summation of x_k square that is we are squaring this abscissa and then summing up and this comes out to be 13 point 7 5. Then I need summation of f_k that is sum of these numbers, so I will have summation of $f(x_k)$ and this comes out to be 25 point 2 5 and lastly we need the sum of this products x_k f_k that is this into this, 1 into 3 and so on and then sum up that is your x_k f of x_k and this is 48 point 1 2 5. Summation $f(x_k)$ is the, we are the writing the product of these two, summation x_k is equal to 7 point 5, 6 okay, yes 6 point 5.

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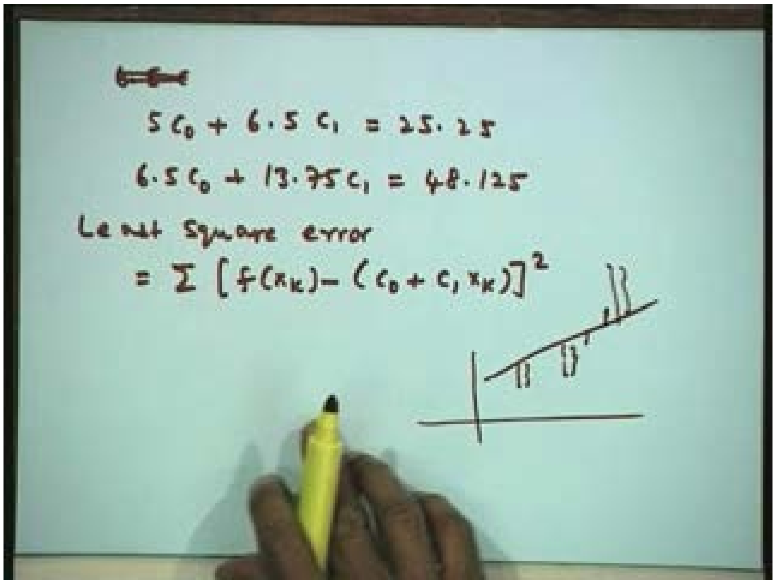

Using these values of the summations, we shall write down the normal equations. The data points are 5 points, so you have 5 times c_0 plus 6 point 5 c_1 and that is equal to summation of x_k 25 point 2 5. This is 6 point 5 c_0 plus summation x_k square is 13 point 7 5 c_1 and that is equal to 48 point 1 2 5. Now you can solve this for c_0 and c_1 and then you substitute in the, we want to find out what is the least square error. The least square error definition was summation of, here w(x) is equal to 1, therefore it is simply f of x_k minus c_0 plus c_1 x_k whole square, this is f(x) minus $P_1(x)$, so [$f(x_k)$ minus (c_0 plus c_1 x_k)] whole square. Now we have got the values of c_0 and c_1 over here, now we have got the data points x_k f_k, we can substitute in this entire and then find out the total error in the least square this one.

Now we would like to repeat that this is not fitting the data, whatever we have got here, the approximation is not fitting this data at all, it is not an interpolating polynomial, it is only an approximation to this function that is representing this data, therefore there is least square error. If it was an interpolation, this is exactly fitting this; therefore there is no error in fitting this particular data but this is an approximation, therefore even if you just write down, say a quadratic polynomial, artificially take quadratic polynomial, write down the values and write down to fit a straight line from there, you would still get the least square error here because what really would mean is, if I take the a graph of this and let us say we have taken points like this, point like this, point like this, then what we are fitting is some straight line like this for this such that the total squares of these errors. These are the errors, this is the straight line that we have now approximated for the function, now the error is this particular part here with respect to this point, this is the error, this is the error; this is the error.

Now this least square error is sum of squares of all these errors is this, so we are not fitting this function exactly for this data, it is a approximation to the function which must be representing that particular data, so that is why we have this least square errors in the problem and this is the best approximation because we have minimized sum of the squares of the errors, therefore we have got that best value of c_0 c_1 which minimizes this least square error. If you take any other straight line, for example here we would get error which is larger than this particular error because this is minimized. Let us now discuss how we can apply the least square approximation as we have defined earlier when a continuous function is given.

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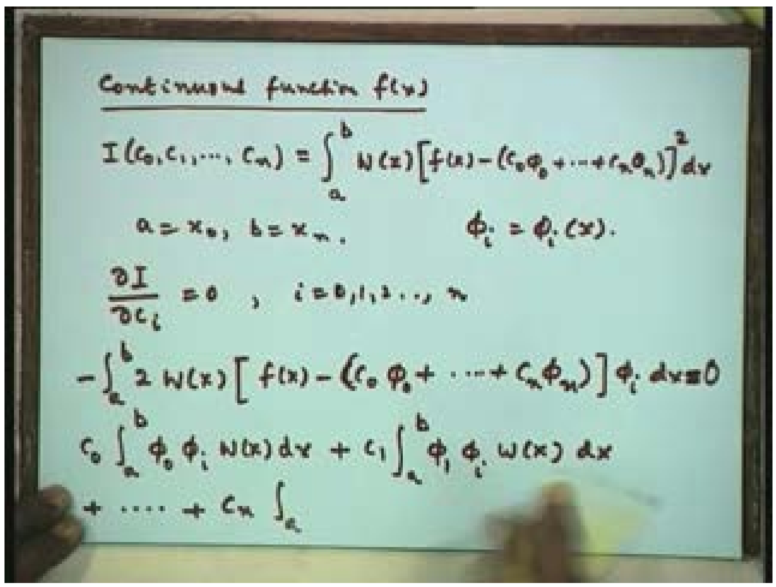

Let us take continuous function as f(x), therefore I will again write our I of (c_0, c_1, c_n), now if you just look at this what we have done in the previous case, this is we have minimized at this particular expression, this is nothing but the summation, if you change the summation to the integral for continuous functions, this will be simply in an integral form. So we will convert this into an integral form and write this as, that is integral given interval a to b that is your first point, a_0 is your first point, a_0 is your first point and b is your last point x_n. So an interval [a, b] is given here, so which corresponds to that one, this is equal to w(x) weight function and multiplied [f(x) minus (c_0 phi_0 c_n phi_n)] whole squared dx. Where I have written phi_i is equal to function of x, this is the phi, is the function of x because it is a continuous function.

Now again for a minimum the necessary condition is that the partial derivative of I with respect to c_i must be equal to 0, for i is equal to 1, 2, for 0, 1, 2, n. Now let us differentiate this with respect to c_i partially, so I would get integral a to b 2 times w(x) [f(x) minus (c_0 phi_0 plus so on c_n phi_n c_n phi_n)] and derivative of this is phi_i dx with a negative sign, let us put this minus sign that is here outside, this is equal to 0, this is equal to 0. Now again we will retain the integrals with respect to c_0 c_1 on the left hand side and we take this to the right hand side.

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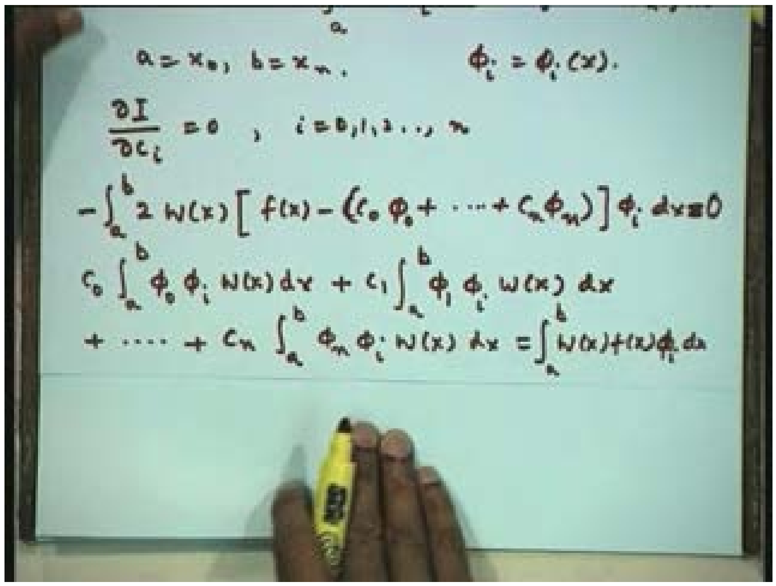

Therefore I can write this as c_0 integral a to b cancel of 2 also, so I would here have phi_0 phi_i w(x) dx that is the first term, this is phi_0 into phi_i into w(x) plus c_1 integral a to b next one is phi_1, this is phi_i and this w(x) dx plus so on, the last term will be c_n, c_n a to b, this last one is phi_n so phi_n into phi_i w(x), on the right hand side we have w(x) f(x) phi_i so this integral goes to the right hand side, this is your w(x) f(x) phi_i dx. Now I would like to write down all these values for phi is equal to 0, 1, 2 because we would like to make some observations on that.

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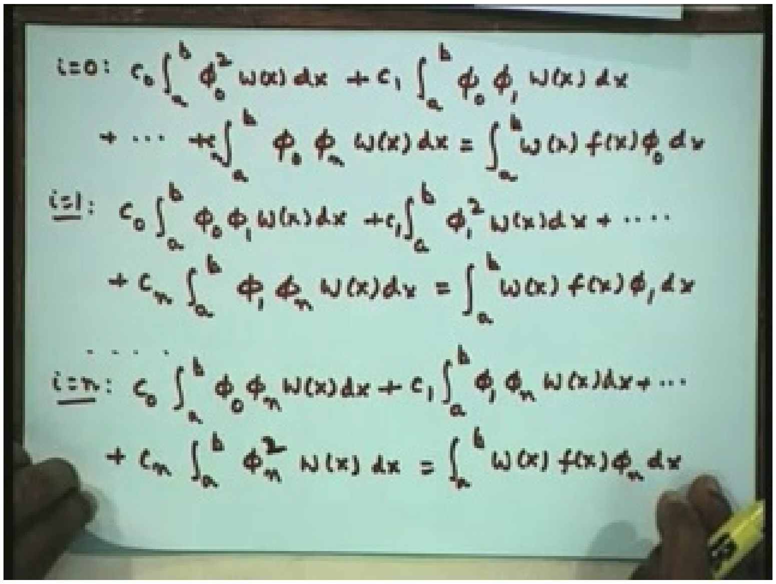

Let us take the case i is equal to 0, then this gives us c_0 a to b phi_0 square, i is equal to 0 therefore we have phi_0 square, w(x) dx c_1 integral a to b, this is i is 0 this is phi_0 phi_1, so I will have phi_0 phi_1 w(x) dx plus so on c_n is outside, integral a to b, this is phi_0, this is phi_n, phi_0 phi_n w(x) dx, on the right hand side we have w(x) f(x) phi_0 dx. I would like to write 1 more equation, therefore this will be c_0 integral a to b, now this is i is equal to 1, therefore I will now get phi_0 phi_1 w(x) dx c_1 a to b, now this is 1 therefore this is phi_1 square, therefore I will have here phi_1 square w(x) dx plus so on c_n integral a to b, now this is last term is, now this is phi_1 into phi_n, phi_1 into phi_n w(x) dx and the right hand side is a to b w(x) f(x) phi_1 dx.

Now let us write down the last equation i is equal to n, I will therefore have integral a to b phi_0 phi_n w(x) dx plus c_1 a to b phi_1 phi_n w(x) dx plus c_n a to b phi_n square w(x) dx is integral a to b w(x) f(x) phi_n dx. Now these give us the required n plus 1 equations in n plus 1(Refer Slide Time: 38:58), I have to evaluate these integrals all these integrals, they are functions of x, I can integrate all of them and one once I integrate it, i substitute it over here and then find out what is my c_0 plus c_1 x.

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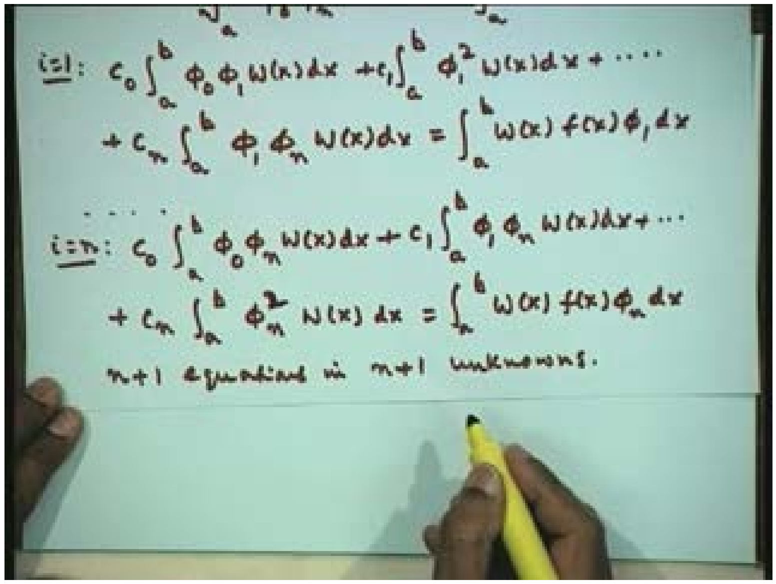

Therefore these give us n plus 1 equations in n plus 1 unknowns. Now let us again take a particular case here, there will be 2 particular cases I would like to discuss here,

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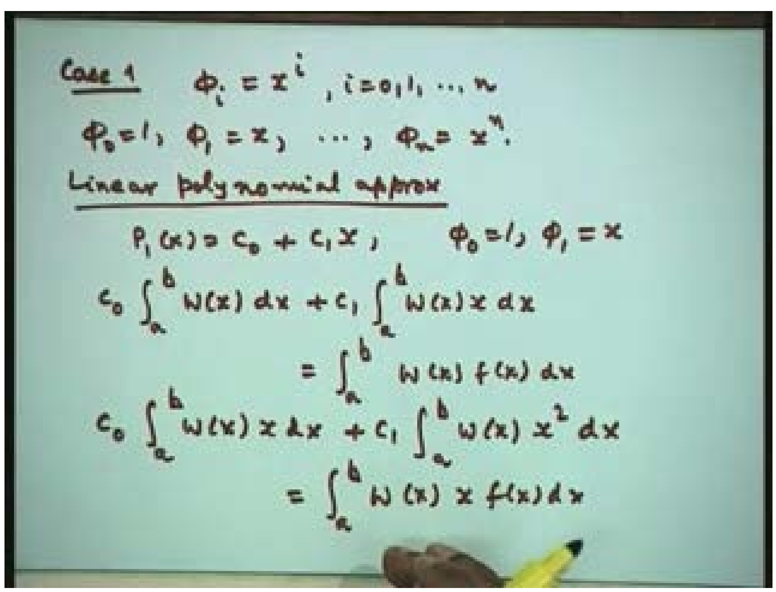

let us take the case 1 as simply phi_i is equal to x_i, let us take again phi_i is equal to x_i, i is equal to 0, 1 so on n. So we are taking phi_0 again as 1, phi_1 is equal to x and so on phi_n is equal to x to the power of n. Now let us get the approximation, linear polynomial approximation, therefore we are considering the case $P_1(x)$ is equal to c_0 plus c_1x therefore our phi_0 is 1 and phi_1 is equal to x. Now I just have to substitute these values over here, phi_0 is 1, phi_1 is equal to x and then retain only two terms, there are only two unknowns c_0 and c_1 and then I can write down the equations.

Therefore the first equation will read as c_0 integral a to b w(x) into phi_0 square that is simply 1, therefore it is w(x) plus c_1 integral a to b w(x), the second term is your phi_0 phi_1 w(x) dx therefore I will have product as x, therefore I will have here x into dx and the right hand side is integral a to b again your phi_0 is 1 here, so I simply have w(x) f(x) here, so I will have here w(x) f(x). And the second equation is c_0 integral a to b w(x) x dx plus c_1 a to b w(x), now this is x square phi_1 square dx, phi_1 square is x square and the right hand side is integral a to b w(x) x f(x) dx. So I just again have to evaluate these integrals then solve for c_0 c_1 and then we have the required linear least, linear polynomial approximation, I can again find out the total least square error also using this one.

Similarly I can write down the quadratic polynomial from here by just taking one more term in this, but the reason why we made it into case 1 and case 2 is, we mentioned earlier that these functions phi_0 phi_1 is our choice, if our choice makes the things better than we would prefer that. In the previous case we have seen, we have taken the linear approximation or quadratic then high degree then we said the system of equations is become ill conditioned for larger. However if we choose these function phi zeroes as orthogonal polynomials, orthogonal with respect to the weight function w(x) then there is no ill conditioness that comes there because the solution comes directly. Let us see what would happen in this case.

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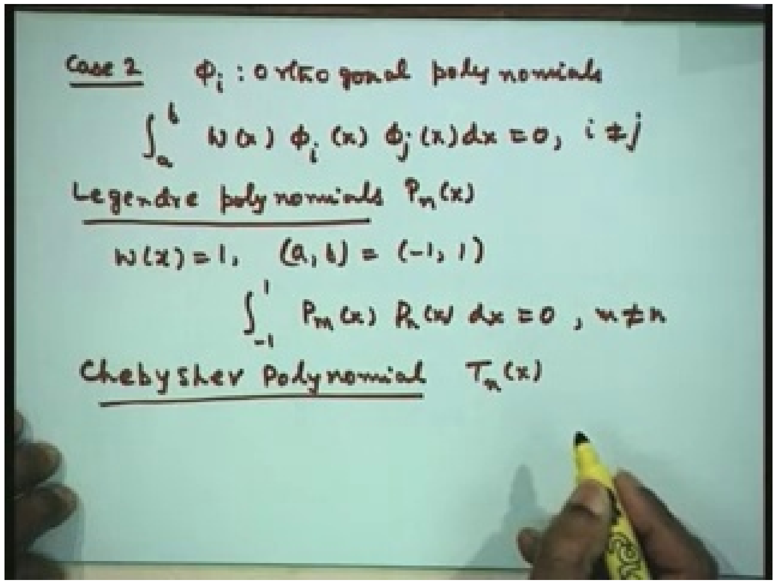

Let us write down this, let $phi_i(x)$ be orthogonal polynomials, let us take them as orthogonal polynomials. Now what is the definition of orthogonal polynomial? The definition of this is integral a to b, it has a weight function w(x) $phi_i(x)$ $phi_j(x)$ dx is equal to 0, for i not equal to j. These functions are orthogonal with respect to a weight function over the interval a to b that is for i not equal to j the integral is 0, for i is equal to j it has some value, that value depends on what that particular polynomial is but we are interested mainly in this particular property. For example, if you are choosing the Legendre polynomials here, if you are taking the Legendre polynomials $P_n(x)$ p_0, p_1, p_2 that Legendre polynomials $P_n(x)$, then in that case the weight function for this is 1, weight function is 1, the interval (a, b) is minus 1 to 1 that means what we are saying is integral minus 1 to 1 $P_m(x)$ $P_n(x)$ dx is equal to 0, for m not equal to n, this is the property of the Legendre polynomials. I can choose other polynomials, the other polynomial that is very useful is the Chebyshev polynomial, is called the Chebyshev polynomial it is denoted by $T_n(x)$.

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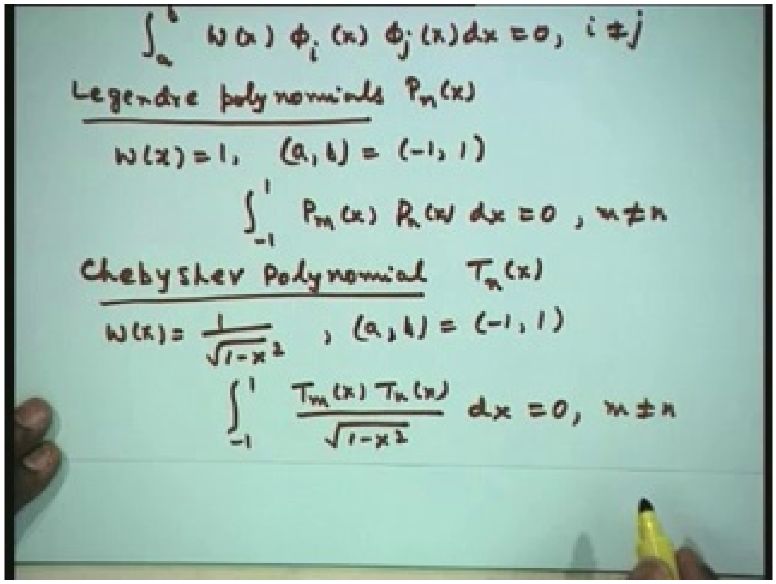

In this case the weight function for Chebyshev polynomial is 1 upon under root 1 minus x square, the interval (a, b) is equal to minus 1 to 1. Now we are going to define the Legendre and Chebyshev polynomials in the next lecture because this we are going to use it later on in numerical integration also, we are going to use them, these are very important polynomials. Tt has Chebyshev polynomials have very very important property that the, if you take the magnitude of this over the interval and approximate it by any of the function, the error on this will be minimum compared to any polynomial over that particular interval but that we will discuss it little later. Now what this really means is, if I take integral minus 1 to 1 $T_m(x)$ $T_n(x)$ by under root of 1 minus x square, there is your w(x) $T_m(x)$ $T_n(x)$ dx is equal to 0, for m not equal to n. Now we shall show that if I choose these coordinate functions phi_i as the orthogonal polynomials then everything gets immediately simplified, let us see why it is so.

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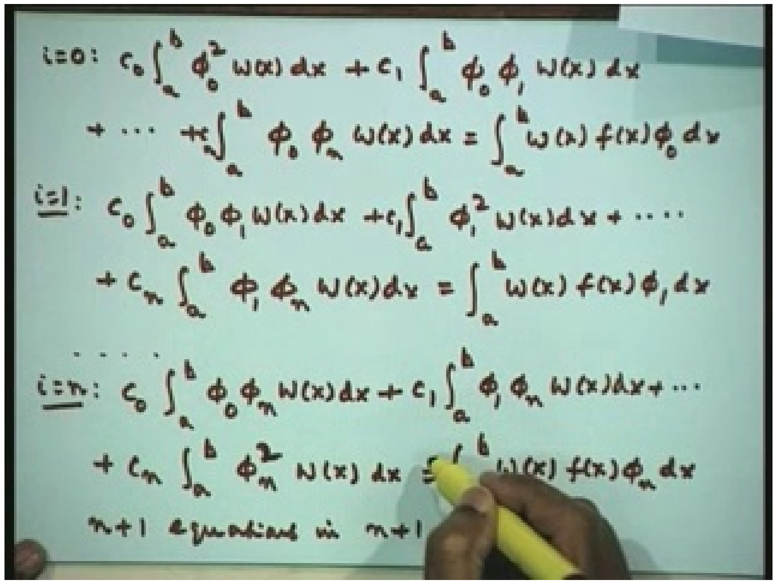

Let us just go back the previous equations which we have written the entire set, now let us put your orthogonal polynomial here. The orthogonal polynomial here if I put it here, this is phi_0 square w(x), this is phi_i phi_j i is equal to j is equal to 0, therefore this is a non-zero quantity. Phi_0 phi_1 by definition it is 0, all of them are 0, the right hand side is there. Therefore the first equation simply gives c_0 a to b phi_0 x w(x) dx is equal to this and hence I found out c_0.

Go to the next equation, by definition again phi_0 phi_1 w(x) is 0, integral is 0, this integral exist, this integral vanishes, therefore except if you look at this as your matrix, except your diagonal elements, this is c_0, c_1, c_2 diagonal elements, all the half diagonal elements are going to be 0 because of the orthogonal property. Therefore I can immediately determine my c_i's without any solving a system of equations, the trouble arose in the previous case we are now solving a system of equations there which turned out to be ill conditioned system, here there is no question of solving the system of equations.

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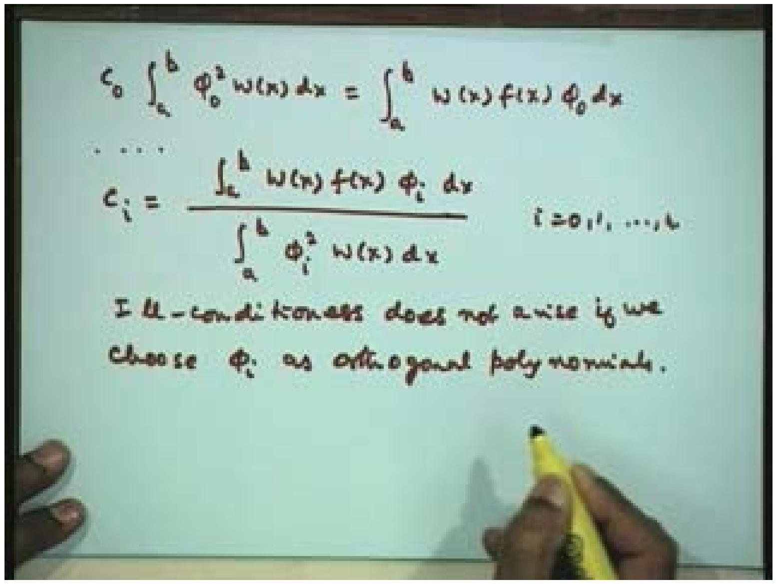

What I would therefore have here is, I will have here c_0 integral a to b phi_0 square w(x) dx is equal to the right hand side a to b w(x) f(x) phi_0 dx and so on. I will therefore have c_i, we can write down the denominator as integral a to b phi_i square w(x) dx, so I am taking this coefficient and putting it in the denominator and in the numerator you have a to b w(x) f(x) phi_i dx for i is equal to 0, 1 so on n. Therefore here it is just simply evaluating the integrals and then the ratio of these integrals would immediately give you the value of c_i and hence there is no ill conditioness, ill conditioness does not arise here, does not arise if we choose phi_i as orthogonal polynomials. Now we could of course as well have chosen in the previous case also, the discrete data case also phi(x) is orthogonal polynomial, the answer is yes.

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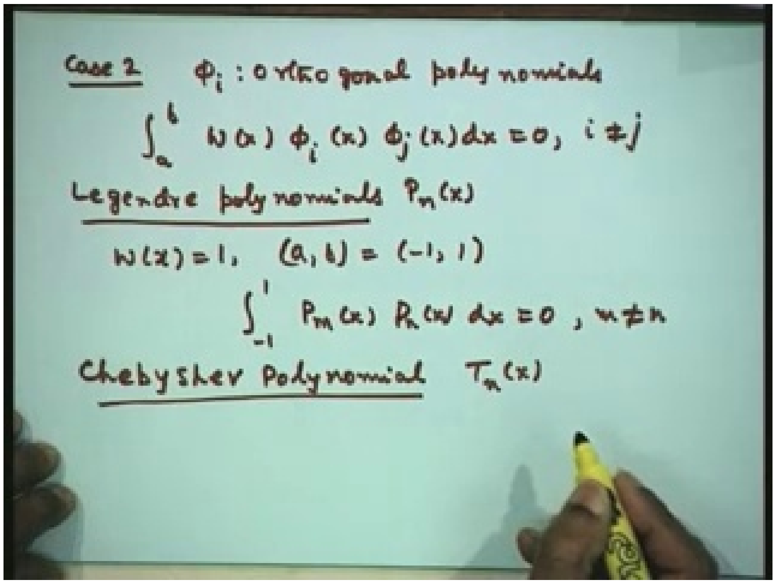

In that case instead of integral a to b or this one, you will have summation, summation over k $w(x_k)$ $phi_i(x_k)$ $phi_j(x_k)$ dx is equal to 0, so I could as well have used this orthogonal polynomials in the discrete data set also where from, we will use as I said we will replace this integral by summation and therefore we can find out c_i there also by just taking the ratio of 2 summations also. So the use of the orthogonal properties in the least square approximation is very useful and in particular if you have a continuous function it is, the values of these integrals, these are available for us when m is equal to n, we can straight away use those property of the orthogonal polynomials to get arrive at the approximations, so will take the example next time.