Numerical Methods and Computation
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Lecture No - 29
Interpolation and Approximation (Continued)

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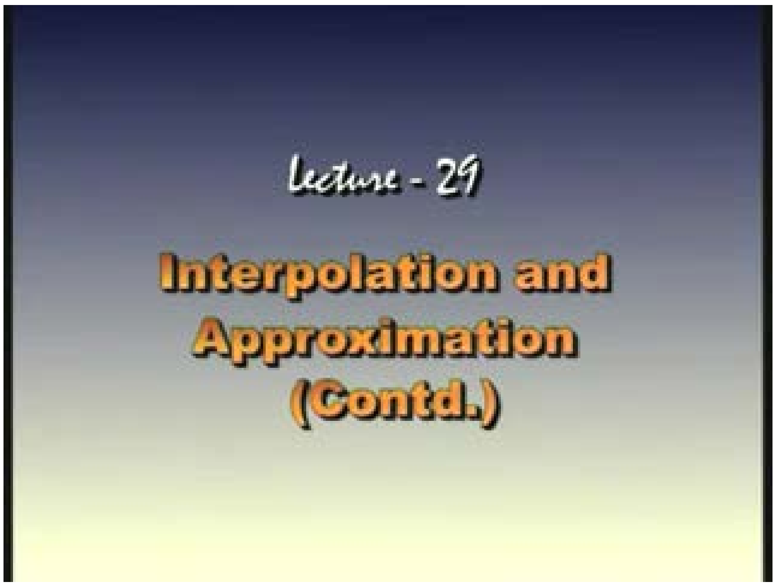

Now in our previous lecture we have defined the shift operator, the forward and backward difference operators. We have also defined the forward differences, backward differences; we have also derived the relationships between them. We have shown that given a difference table, a particular element in the table can be recognized as a particular forward difference or a particular backward difference depending on where you want to use and for what purpose we are going to use particular data. Now these differences are all related among each other, we would now like to show that these operators are also related to the divided differences that we have defined earlier and a formula can be derived using the derived difference formula also. Let us now try to derive what is the relationship between the divide difference operator and the backward and forward difference operators.

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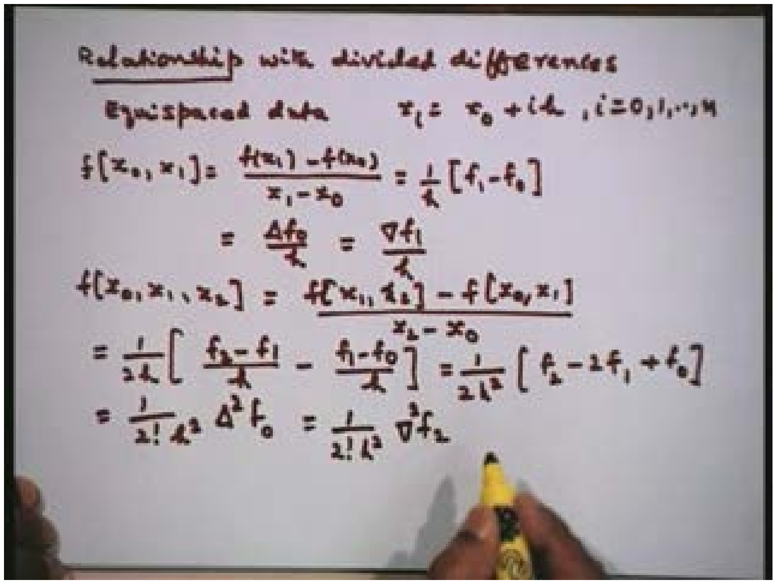

So let us take it as relationship with divided differences. What we are considering is the equispaced data, we must remember that we are using the equispaced data that is x_i is equal to x_0 plus i times h and i running from 0 to n. Now let us write down the definition of the divided difference, first divided difference, f of $[x_0, x_1]$ is equal to f of x_1 minus f at x_0 divided by the distance between them (x_1 minus x_0). Now since this is a equispaced data (x_1 minus x_0) is 1 upon of h, so that is step length between the abscissa and this is [f_1 minus f_0]. Therefore this is equal to forward difference with respect to x_0, so that is delta f_0 divided by h and I can also look as the backward difference with respect to x_1, that is I can also write this as backward difference with respect to h. Therefore when the data is equispaced, they are all equivalent and the location of the divided differences or the location of the forward or backward differences can all be connected to this particular relationships.

Now let us take the second divided difference, $f[x_0, x_1, x_2]$ is equal to f of $[x_1, x_2]$ minus f of $[x_0, x_1]$ divided by (x_2 minus x_0). Now (x_2 minus x_0) is 2 times h that is equal to 2 times step length, let us open this up, this is (f_2 minus f_1) divided by (x_2 minus x_1) is again h; minus (f_1 minus f_0), (x_1 minus x_0) is also equal to the step length h. So I can simplify this and write this as 1 upon 2 h square and this is [f_2 minus 2 f_1 plus f_0]. Now I recognize this as a second forward difference with respect to f_0, so I can write this as 1 upon factorial 2 into h square delta square f_0 and this is same as the second backward difference with respect to f_2, so therefore I can write this as 1 upon factorial 2 h squared delta squared f_2.

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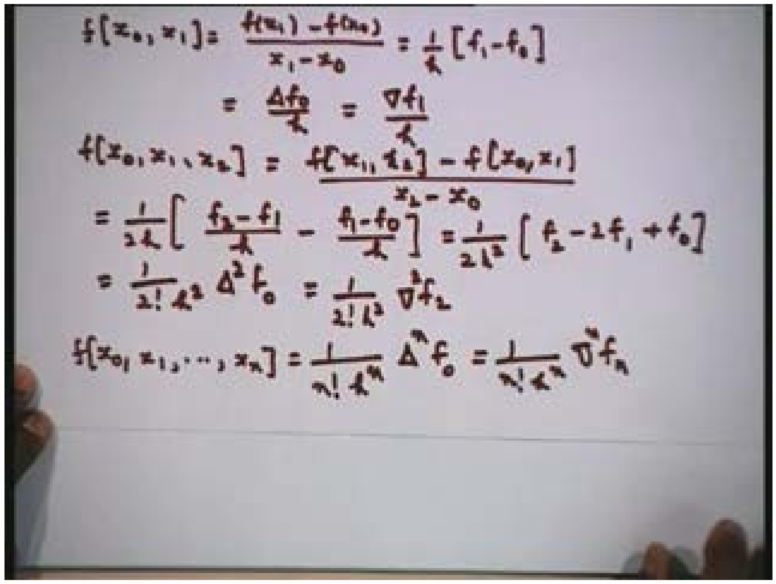

Now I can proceed further and finally show that the n^{th} divided difference $f[x_0, x_1, x_2, x_n]$ is equal to 1 upon factorial n, h to the power of n, the n^{th} forward difference with respect to f_0 or it is related to the n^{th} backward difference delta n with respect to the last data item f_n. Therefore the relationship between the forward backward differences is given by this one. Interestingly these operators can be related not only among themselves, we can now show that they can be related with derivatives also, that is very important because finally we would like to use these differences to construct difference methods for solving the ordinary and partial differential equations, wherein the relationship between the operators forward backward or central with respect to the derivatives can obtained, then the derivatives in a differential equation can be replaced by the differences and then we can find out the solution of the differential equation.

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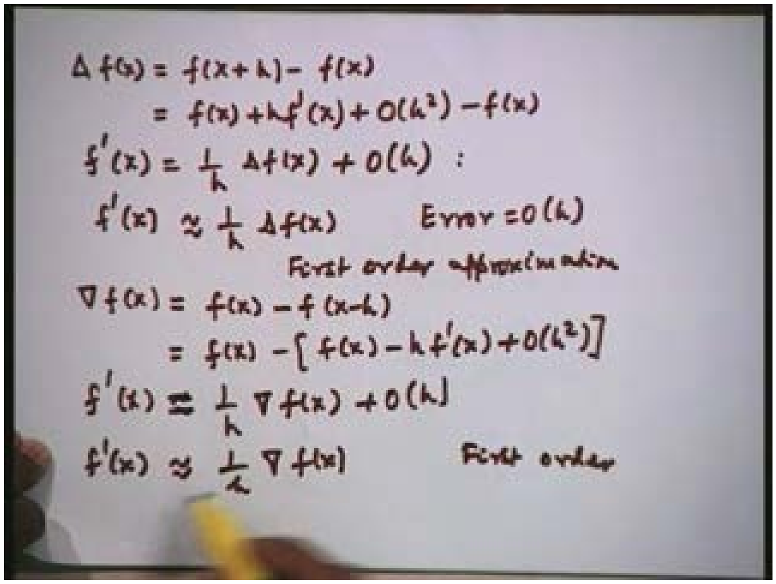

So let us derive with respect to the derivatives, now let us start with delta f(x), the definition of delta f(x), this is f of (x plus h) minus f of (x). Now let us expand by Taylor series, the first term that is your f of (x) plus h f dash of (x) and the remaining terms I will write it as order of h square terms minus f(x). Now f(x) cancels with f(x), I will bring this order of h square term to the left hand side and then divide by h, therefore I will have f dash of x is equal to 1 upon h delta of f(x) plus order of h, order of h square is divided by h therefore it will become order of h so, therefore whatever I have here is order of h term and delta f(x) divided by h. Therefore we have the approximation for first derivative as 1 upon h delta f(x), upon f(x) and because the error term is order of h, error is order of h that is here, we shall call this as a first order approximation, hence we shall call this as first order approximation.

Now we know that the first forward difference is connected with respect to the backward difference, we have just shown that this is the equality, therefore I can now write down in terms of the backward difference also but let us derive it separately again. Now let us define, what is our backward difference, so let me divide the backward difference with respect to x, this is f of (x) minus f of (x minus h). This time I will open this f of (x minus h) in terms of Taylor series, so I will have this as f of (x) minus [f(x) minus h f dash of (x) plus order of h square]. Now f(x) cancels with f(x), I have a positive sign here, so I will take order of h square to the left and then divide by h again, therefore I have got f dash of (x) is approximately equal to 1 upon h delta f(x) plus order of h. This is equal to; this is equal to 1 upon h delta f(x) plus order of h. If I drop this error term, I will approximation for the first derivative as 1 upon h delta of f(x); again this is a first order approximation. Therefore if I have a first derivative say for example dy by dx, if I want to replace dy by dx by an approximation, I would simply write this as 1 upon h delta of f of (x_0), wherever the particular point is being taken. Similarly if I want in terms of backward difference, I can use the backward difference.

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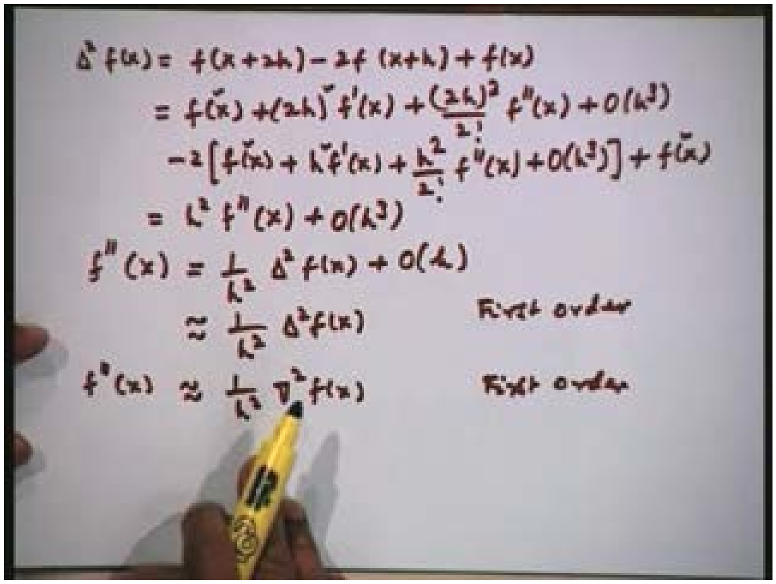

Now this result can be extended to higher derivatives, for example I can start with delta squared f(x) and write this as f of (x plus 2 h) minus twice f of (x plus h) plus f of (x), now I shall open this both these by Taylor series, so this will give me f of (h) plus (2 h) f dash of (x) plus 2 h whole squared by factorial 2 f double dash of (x) plus order of h cubed minus 2 times f of (x) and this is h of f dash of (x), h squared by factorial 2 f double dash of (x) plus order of h cubed and the last term stays as it is plus f of (x). Now let us simplify this, this is f(x), this is f(x), 2 times f(x) cancels with 2 times f(x), this is (2 h) f dash (x), this is minus (2 h) f dash (x), therefore these two terms cancels.

So the first non-vanishing term is this and this, this is 2 squared 4 by 2 that is 2 and this is minus 1, 2 by factorial 2 minus 1, therefore I have left out with 1 as the coefficient that is simply h squared f double dash of (x) plus order of h cubed. These two terms simplify to give us simply h square f double dash (x). Therefore I can write down the approximation for this is equal to 1 upon h squared delta squared of f(x) plus order of h, I have divided by h square therefore I will have only order of h over here. Therefore I can write approximation as 1 upon h square delta squared of f(x) and this is a first order approximation, this is a first order approximation because we have only order of h term that is left out over here and by the same reason I can we can derive that f dash (x) is equal to 1 upon h squared delta squared f(x) also and this is also the first order. Therefore if I have the, if I have to use it in a deferential equation for example d square by dx squared, I can replace d square by dx squared by 1 upon h squared delta squared of f(x) or in terms of the backward differences. Now you can immediately connect these with the divided differences, we have earlier derived the relationship between the divided difference and forward and backward differences. I can immediately connect all these ones with the divided differences also. Now we have derived one more operator, which was a symmetric operator, which we called it as the central difference operator.

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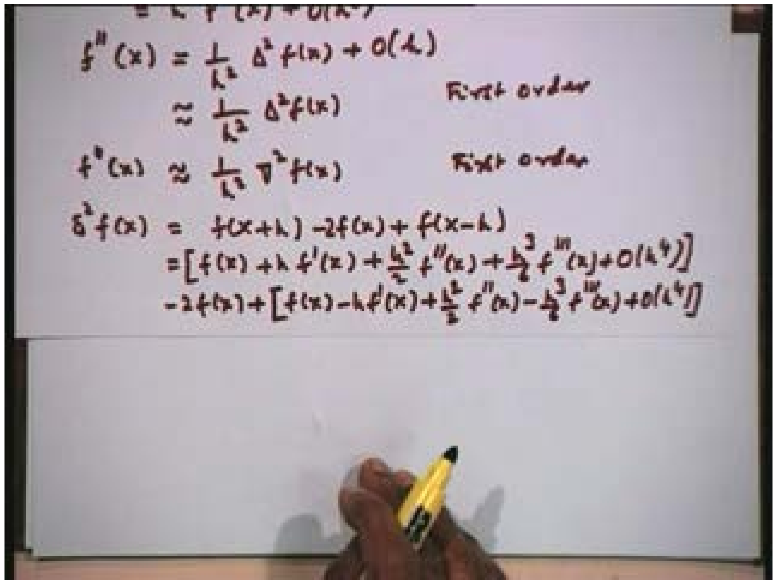

Let us now define the second central difference, that was defined as f of (x plus h) minus twice f(x) plus f of (x minus h), so this is the definition of the central difference. Now I would like to open this up also by Taylor series, so we will have f(x) plus h f dash of (x) h square by 2 f double dash of (x) plus h cubed by 6 f triple dash of x plus order of h 4 minus 2 times f(x), that term plus the third term, plus the third term is f of (x) minus h f dash of (x) h squared by 2 f double dash of (x) minus h cubed by 6 f double dash f triple dash of x plus order of h to the power of 4. We have just opened f of (x plus h) by Taylor series, we retained terms up to order of h cubed and then retained error as h 4, f of (x minus h) also be expanded. Let us now simplify this particular right hand side, so we have here f(x) combines with f(x) cancels with minus 2 f(x), f(x) plus f(x) minus 2 f(x), this is plus h f dash (x) minus h f dash (x), these two cancel of.

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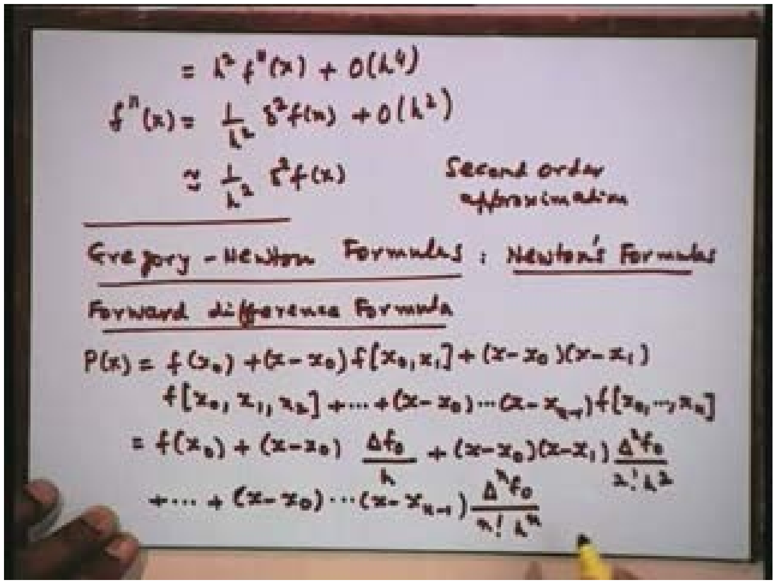

Then we have h square by 2 plus h square by 2, so the first non-vanishing terms is h squared f double dash of (x), this h square by 2; h square by 2 combine. Now you can see the odd derivative, this is plus h cubed by 6 minus h cubed by 6 it cancels again, so what is left out is order of h to the power of 4 terms. Now we may mention the reason why it has happened is, we the central, this central difference operator is symmetric about this middle point. Since it is symmetric about the middle point, all the odd derivatives would cancel, this is, h will cancel, h cube will cancel, h 5 will cancel in there. So if I want higher order formula and take the higher order differences, all the odd derivatives would automatically cancel. Therefore I can now write down f double dash of x is equal to 1 upon on h square delta squared of f(x) plus order of h square. Therefore this is approximately 1 upon h square delta squared of f(x) and this is a second order approximation, error is now order of h square, therefore this is a second order approximation, is a second order approximation. Which is one order higher than what we have obtained for the forward and backward difference, this is a basic property of the central difference operators, the order of approximation will be higher than, using the forward or the backward differences.

Let us now construct the interpolating polynomials in terms of forward and backward differences using the divided difference interpolating polynomial. This formula is usually called as Gregory Newton formulas, as often it is also called simply Newton’s formula that means you may not attach the name Gregory but you can call it as Newton’s formulas. Let us first take the forward difference formula, the forward difference formula, the formula derivation of formula is just one line, we take the divided difference formula and then we have just now derived the relationship between the forward differences and divide difference, we just replace the divide difference by the forward differences to get this formula. So let us write down what is that formula, we had retained the divide difference formulas f of (x_0) (x minus x_0) f[x_0, x_1], let us write down one more term, this is (x minus x_0) (x minus x_1) into f[x_0, x_2] plus so on (x minus x_0) (x minus x_{n-1}) f of [x_0, x_n], this is the divided difference formula that we have derived earlier.

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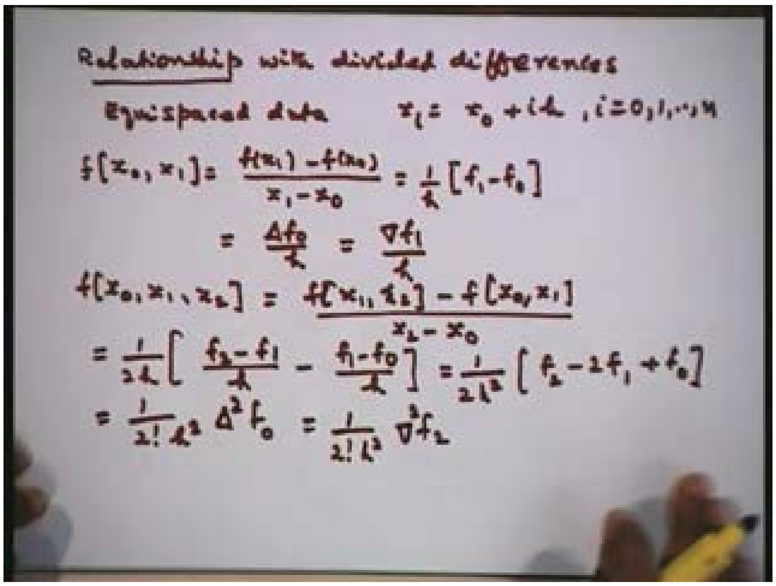

Now I would just replace the divided differences that we have written here by using simply f[x_0, x_1] is 1 upon on h delta f_0, the second difference is 1 upon h square factorial 2 delta squared f_0 and finally the n^{th} divide difference is by this. So we just replace these divide differences by the corresponding relationship with forward difference, so this will be f of (x_0), this is (x minus x_0), this is your delta f_0 remember this is f[x_0, x_1] so it will be delta f_0 by h here, plus (x minus x_0) (x minus x_1), this is the second forward difference delta squared f_0 by factorial 2 h squared plus so on, the last term will be (x minus x_0) (x minus x_{n-1}) delta n f_0 where factorial h to the power of n. So we can just substitute the data points that are given to us, the forward difference is from the forward difference table, just substitute it and simplify to arrive at the formula. Now the error term would be the same because the interpolation polynomial is unique therefore the error term that we get will be same as what we have proved in the Lagrange formula, so let us write down the error term also.

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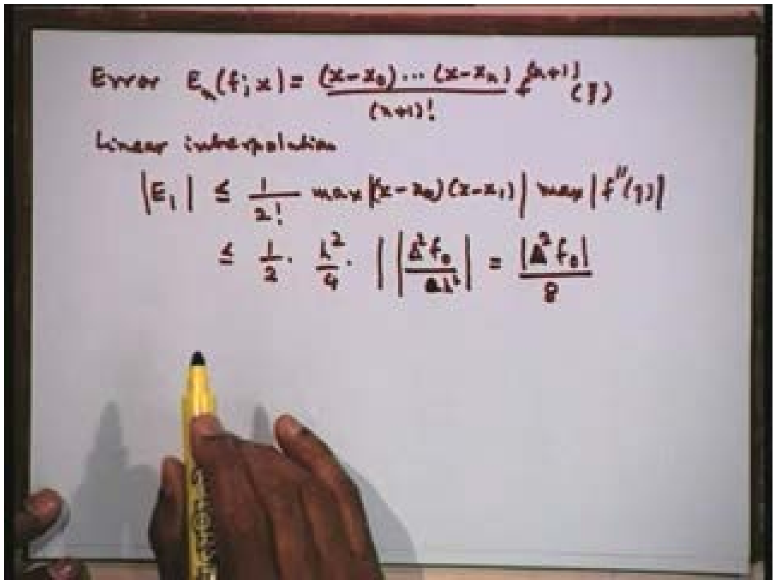

So the error would be simply E_n (f; x) is (x minus x_0) (x minus x_n) by (n plus 1) factorial $f^{(n+1)}$ of (zhi). Now earlier when we have derived this Lagrange interpolation and the divide difference, we have done some examples, wherein we tried to construct what should be the step length h that should be used in order that the linear interpolation has some error or quadratic interpolation has got certain error, now at that time since we wanted the approximations say for M_2 or M_3 that is the maximum of f double dash or maximum of f triple dash, we wanted a function that was actually being used there in order to have a bound for that one but however, since we are now obtained the relationship between derivative and the differences, even if a table of values is given to us and we are not given the function that is approximating that particular data, we can still obtain an approximation for the required derivative when we are using the linear or quadratic interpolation, how do we do it? Let us suppose I want the, I am using only linear interpolation.

Suppose I am using the linear interpolation, then the error would be E_1, let us write down its magnitude also, the magnitude will be less than or equal to 1 upon factorial 2, 1 upon factorial 2, we are writing this as maximum of (x minus x_0) (x minus x_1) and maximum magnitude of f double dash of (zhi). Now, we have just now proved that the second derivative is approximately 1 upon h square delta squared f(x) that means I can now replace this f double dash by delta square by h square, so this will be less than or equal to 1 upon 2, maximum of this we have proved that it occurs at the middle point that is equal to h square by 4 we have shown and therefore this is equal to delta squared f_0 by, we are writing this approximately, this is equal to, by 2 h square also i should write.

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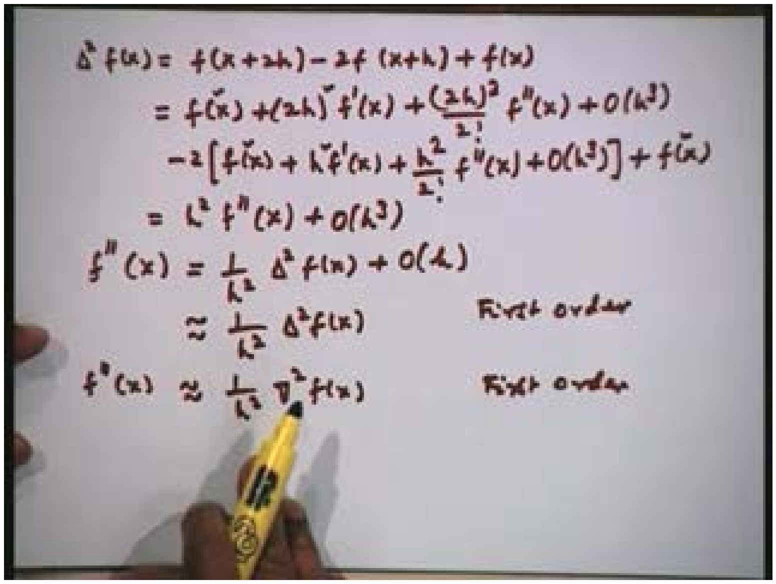

I am taking, I am taking the expression from here delta squared h square upon h square, therefore this is equal to h square, h square cancels with h square, I have 8 here, delta squared f_0 in magnitude divided by 8. Therefore if I have got a table of values and I am using linear interpolation using particular 2 points, now I have got in the table available for me forward differences, therefore the approximation for M_2 will now come from the data itself, therefore I do not need to have the function being specified when a data is given to us and we can use it. Suppose I am going to quadratic interpolation, what I will have here is a third derivative and third derivative is we have shown that it is related to delta cubed f_0, so I would just have to use the approximation of delta cubed f_0 and that will be an approximation for the, say for the linear interpolation or quadratic interpolation.

Now for computational purposes, this formula that we have written here can be simplified still further. Suppose in a given table of values, you are asked to just find the value or interpolate at the value, a particular value, a number is given to us, then it is not necessary for us to construct the entire polynomial, simplify the whole thing and then substitute x is equal to that value. We can just straight away substitute the value that is given to us and then simplify this straight away by using a substitution to simplify this further, so let us see what this simplification we can do here in this formula.

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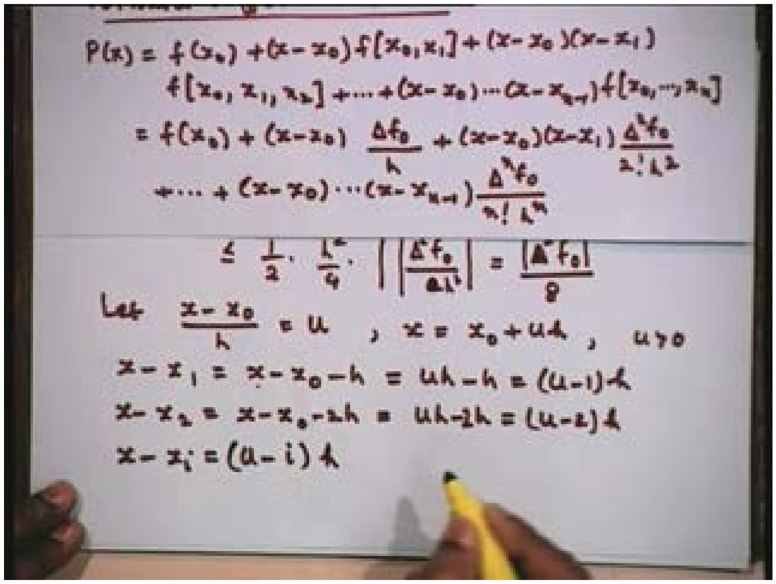

Let (x minus x_0) by h be equal to some value u, let us put (x minus x_0) divided by h is equal to u that means I am setting x is equal to x_0 plus u h. We remember that, since we are now interpolating at a value larger than x_0 always, u is a number which is positive number, u is a number which is positive here. Now I want to substitute for this here, so I know what is (x minus x_0), let us find what is (x minus x_1). (x minus x_1) is by definition (x minus x_0 minus h). x_1 is (x_0 plus h), so I can substitute for this, (x minus x_0) is (u into h minus h) that is (u minus 1) into h. Similarly (x minus x_2) or (x minus x_2) is (x minus x_0 minus 2 h), so this is equal to (u h minus h) that is (u minus 2) of h. Therefore in general (x minus x_i) is equal to (u minus i) into h. Now you can just have a look at it before we write the formula, (x minus x_0) divided by h is equal to u, there is a factorial to h square here, therefore I will take this h, h with each one of them (x minus x_0) by h (x minus x_1) by h, so I would then have u into u minus 1 as a second term, so I would just substitute these values here, then I would get this.

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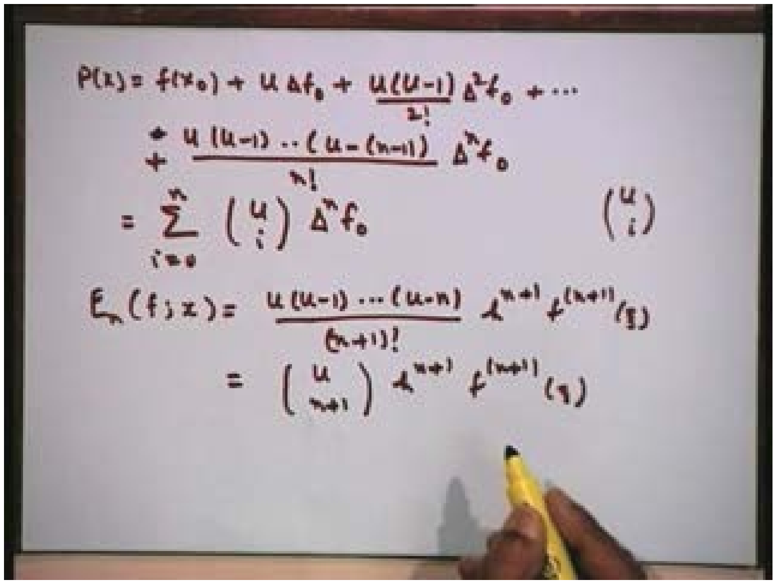

Let us write down, P(x) can therefore be written as f of (x_0) plus, (x minus x_0) by h is u delta f_0, this is u, this is (u minus 1), u into (u minus 1), h square has canceled, divide by factorial 2 delta square f_0 plus so on plus, let us take this term, this is u (u minus 1), the last one is (u minus of (n minus 1)) by factorial n, this factorial n is here and each of them has got h it cancel, this is your delta n f_0. Now you can see that these are nothing but the binomial coefficients with respect to u, therefore I could simply write them as binomial coefficients and write in the simple notation, summation 1 to n, I will use the binomial notation as u i, it is easier for me to write this one, so I will use this notation u i binomial coefficient delta n f_0. So it is simply the binomial coefficients with respected to u, u is a number here, u is greater than 0, so I can just take u is equal to half say, so we will have point 5, point 5 minus point 5 by factorial 2 and so on. So I just write down the binomial coefficients, use this forward difference and have this solution immediately.

Now I am substituting for a particular x, therefore u is determined. Now let us see interestingly what will happen to error in this case, the error would be, there is no h in the denominator therefore each one will contribute a h, there are n plus 1 terms here, therefore it will contribute h to the power of n plus 1. Therefore this is u into (u minus 1) (u minus n), the last term is n, (u-(n minus 1) divided by (n plus 1) factorial that is here, h to the power of (n plus 1) $f^{(n+1)}$ of zhi. Now this is the next binomial coefficient, this is u c n, the last binomial coefficient that is u c n plus 1. This is simply your next binomial coefficient u n plus 1, h to the power of (n plus 1) $f^{(n+1)}$ of zhi. Therefore everything comes in terms of binomial coefficients and we can obtain the error term as well as the interpolated value at any point very easily from this. Now the derivation of the forward difference formula can be viewed or obtained in a very simple manner like this.

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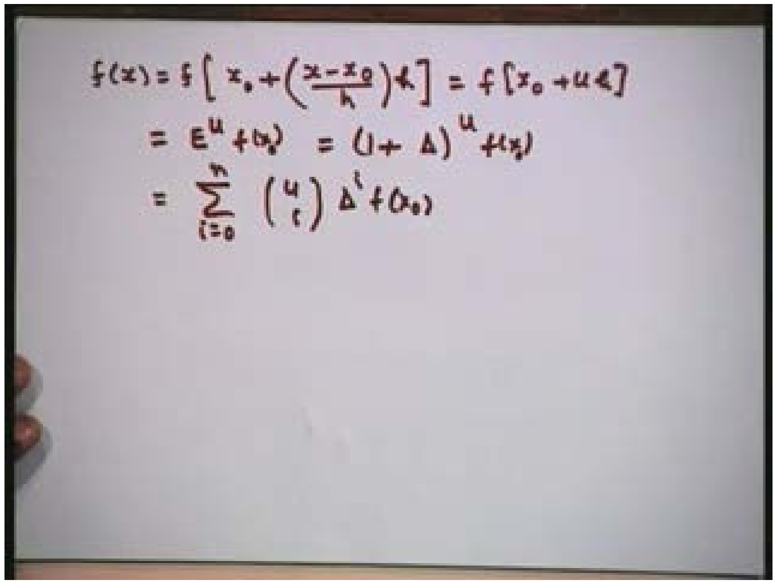

Let us just derive the alternative way of deriving this formula, let us write f(x) as f of, I want to use forward differences, so I will write this as x_0, x minus x_0 by h that is my u, into h. You cancel of h, cancel of x_0, f(x) is equal to f(x). So therefore I have written argument as x_0 plus x minus x_0 by h into h, so that I get back this one. Now this is u, so I will write this as f of x_0 plus u h, this is the definition of u therefore this is simply. Now let us use this as a shift, that means shift operator, so this will be E to the power of u of f(x) but we know that E is equal to 1 plus forward difference delta, that is 1 plus delta to the power of u of f(x), f of x_0, f of x_0 here also x_0. Now expand it binomially, so what we had written earlier comes out immediately, that i is equal to 0 to n, the binomial coefficients u i is equal to, this is delta of I, f of x_0. This is delta here, so this is 1 plus u c 1 delta plus u c 2 delta square and so on, so this is delta i f of x_0. Therefore we get back the formula that we have written earlier by simply using this notation and this we have deriving the formula. [Student: u may not be an integer] he is not an integer; even here also u is not an integer.

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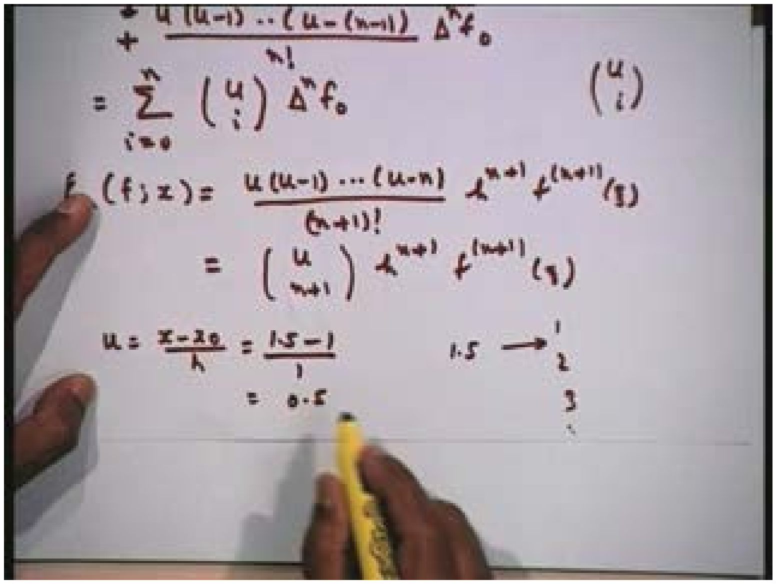

For example as I said the, first let us say the data points initially say 1 and 2 3 and so on, so I am now, let us say I am interpolating here, at 1 point 5 I am interpolating, then by definition my u must be equal to (x minus x_0) divided by h. x is equal to current value is 1 point 5, x_0 is 1, the h is equal to 1, therefore this is equal to point 5. So h will never be an integer here because we are interpolating at a point in the table, the data points given, the data points itself may give the step length say not an integer, say some point 1 or point 0 1, so the step length itself is not an integer, therefore u is never an integer. Therefore the binomial coefficients would go on reducing the minus, start with point 5, then point 5 minus point 5 minus 1 point 5 and so on, but there will be denominator factorials are coming, so the in totality each term as we go forward, they would all be smaller and smaller and smaller they are going to be reducing.

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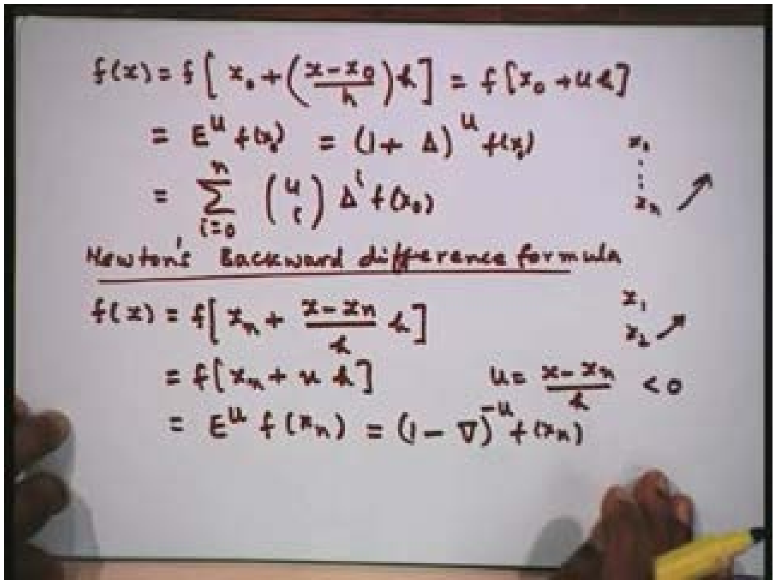

Before we take an example, let us derive the same formula in terms of backward differences, so let us call this as Newton's backward difference formula, Newton's backward difference formula. Now we shall use this approach to derive it in just two lines but we remember, we are talking of the backward difference formula, so since where we got the data x_0, x_2, x_n, I must use the backward differences, so I will have to use them in the backward way. Therefore whatever I want to write, the argument should be in terms of x_n because I want to write down a particular, say a particular level you have taken x_1 x_2, I want to use backward difference with respect to this, I have to use the backward differences with respect to the last argument of this interval. So we will have to use in terms of x_n that means I will write this as f of x_n plus (x minus x_n) by h into h. Now let us set again u is equal to x minus x_n upon h, then this will be f of x_n plus u of h, but we note now that u is always a negative quantity because x_2 is a last point or x_n is the last point and x_1 is an point, intermediate point. Therefore x minus x_n is always going to be negative, therefore this is a negative fraction, there will be always be a negative fraction. Again we write this as, in the terms of shift operator E to the power of u f of x_n but we have proved the equality of the operator E with the backward difference delta and that is 1 minus backward difference delta to the power of minus 1. So that is 1 minus backward difference minus 1 to the power of u that is minus u. This is a minus sign here, there is a minus sign here.

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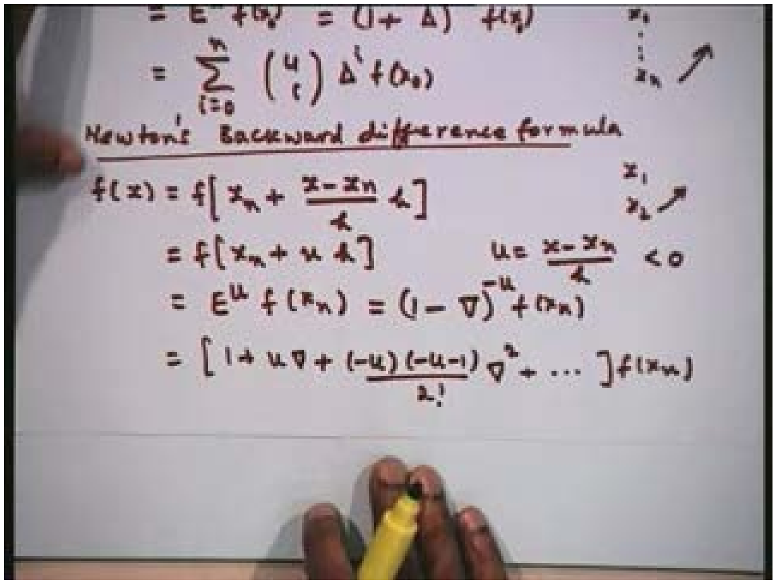

So this will be 1 plus u into delta, let us open it up, this is plus minus u into, okay let us write down minus u, minus u minus 1 by factorial 2 delta squared and so on. We will write down the last term in a moment operator $f(x_n)$.

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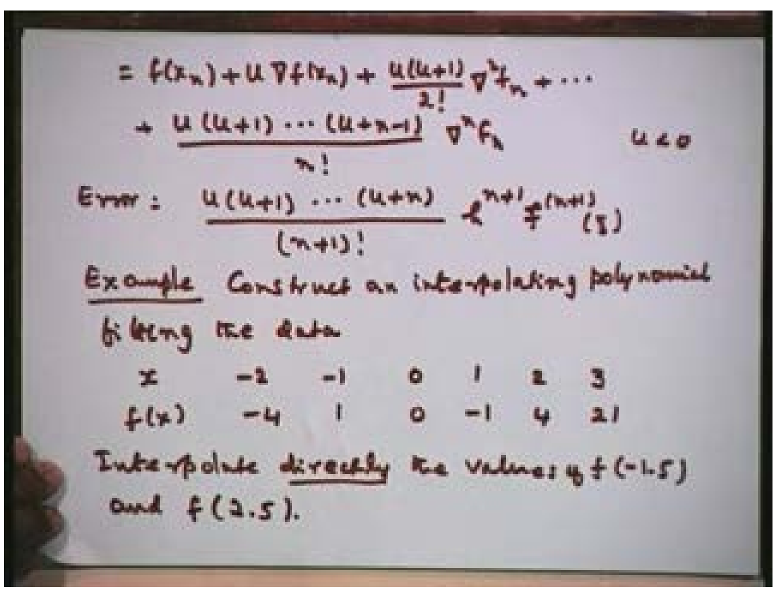

Therefore they would all simplify as $f(x_n)$ plus u delta $f(x_n)$ plus u into (u plus 1) by factorial 2 delta squared $f(x_n)$. They would all be positive because when you are taking the odd, then there is a negative sign here, so they are going to be plus sign. So will have the plus sign only, u into (u plus 1) so on (u plus n minus 1) by factorial n delta n f_n, where we note that u is a negative quantity, u is a negative fraction. Therefore the error can immediately be written as the next coefficient and the next coefficient is u into (u plus 1) so on (u plus n) (n plus 1) factorial, each term contributes a h here, so h to the power of (n plus 1) $f^{(n+1)}$ of zhi. Now let us take an example combining all of them, so let us take this example. Construct an interpolating polynomial, construct an interpolating polynomial fitting the data x f(x) minus 2, minus 4, minus 1, 1, 0, 0, 1, minus 1, 2, 4, 3, 21. Now to illustrate use of substitutions we made here, let us also take interpolate directly, I want to use it directly so that I do not simplify the formula and then use it, directly the values of f (minus 1 point 5) and f (2 point 5). Interpolate directly the values of f(minus 1 point 5) and f at (2 point 5). Now in order to apply this I need to first of all write my difference table, so let us write down our difference table.

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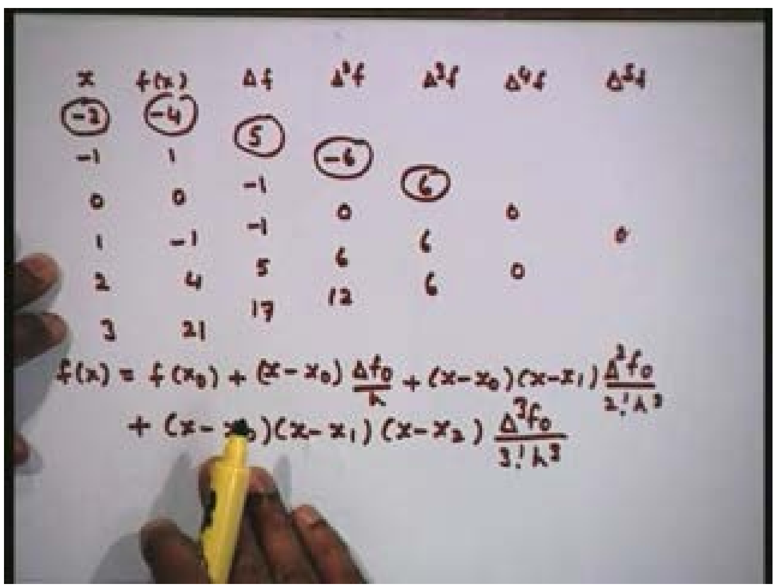

X, f(x), minus 2, minus 4, minus 1, 1, 0, 0, 1, minus 1, 2, 4, 3, 21. So let us write down delta of f, the first forward difference that is 1 plus 4 that is 5, 0 minus 1 that is minus 1, minus 1 minus 0, 4 plus 1 that is 5, 21 minus 4 that is 17. Let us go to second forward difference, minus 1 minus 5 that is minus 6, minus 1 plus 1 that is 0, 5 plus 1 that is 6, 17 minus 5 that is equal to 12. Let us go to third difference, 0 minus 6 that is plus 6, this is 6, 12 minus 6 6. Now you can see all the other differences are going to be 0 even though we have a data, a bigger data. Therefore it will approximate a cubic polynomial, delta cube is non-zero therefore it will approximate only a cubic polynomial.

So let us write down our formula $f(x_0)$, (x minus x_0) delta f_0 by h, (x minus x_0) (x minus x_1) delta squared f_0 by factorial 2 h squared, we have third difference available, so I must write down 1 more term, (x minus x_0) (x minus x_1) (x minus x_2) delta cubed f_0 by factorial 3 h cubed. Now let us substitute the values, the step length is 1 here, minus, this is step length is 1, h is 1, x_0 x_1 x_2 and these are our differences. So we are using x_0, taking this x_0 as this, f_0 is this, delta f_0, delta squared f_0, delta cubed f_0, so we are taking this forward differences to substitute it over here.

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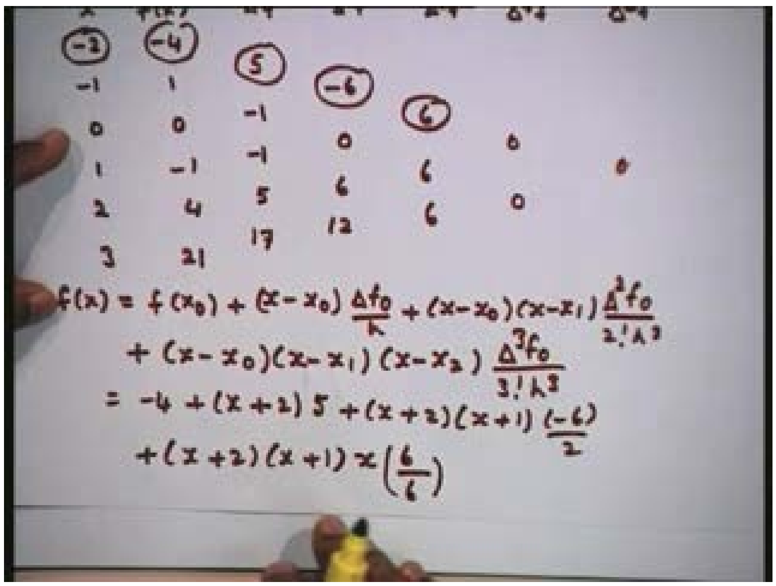

Therefore we have here f of x_0 is minus 4, x minus of this plus 2 that is 5 h is 1 so this is simply 5, (x plus 2) again (x plus 1) the second derivative is minus 6 by 2, factorial 2 is here so I will write down 2, third term is (x plus 2) (x plus 1) and third one is (x minus 0) that is your x, third forward difference is 6, 6 upon factorial 3, this is factorial 3, 6 upon 6.

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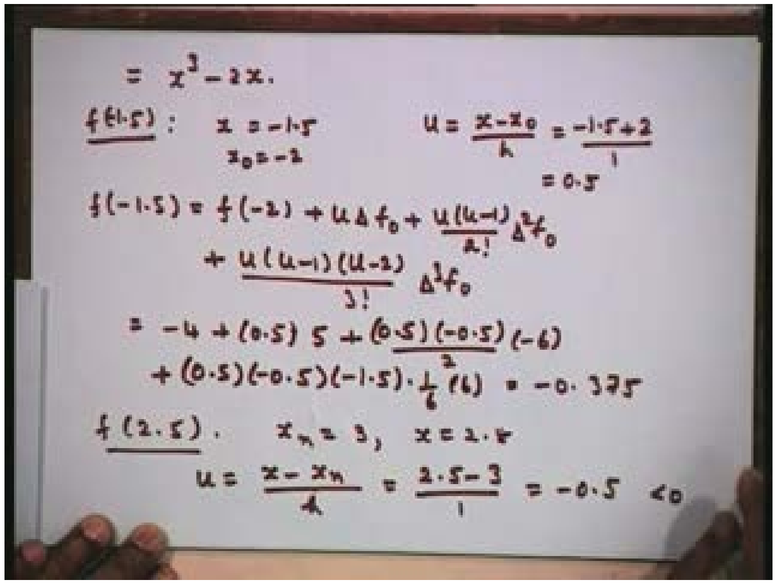

So which you can simply this and I will give the answer for this, the answer for this is simply x cubed minus 2, x cubed minus 2 x. Of course since you have simplified it, I could have used the, when I have been asked to find interpolate, the values I could have substituted there and get it, but I want to illustrate how you can get directly. Now I want to find f at (1 point 5), I want to, minus 1 point 5, this is your x is equal to minus 1 point 5. Now x at 1 point 5 lies between these two values, so minus 1 point 5 lies between these two minus 1, minus 2 and minus 1. Therefore I would write u is equal to, now I will take my x_0 is minus 2, x_0 is minus 2, u is (x minus x_0) by h. x is minus 1 point 5, x_0 is minus 2 therefore plus 2 divided by 1, therefore this is equal to point 5, therefore this is point 5.

Now we will use the forward difference formula, this is a forward difference formula f at x_0, u delta f x_0, u into (u minus 1) this binominal coefficients, therefore f at minus 1 point 5 will be equal to f at minus 2, let us write down the details, minus 2 plus u delta f_0 plus u into (u minus 1) by factorial 2 delta squared f_0, the third difference is also available to us, so I will have u (u minus 1) (u minus 2) by factorial 3 delta cube f_0. Now f of minus 2 is given as minus 4, u is point 5, u is point 5, delta f_0 will take from here that is your 5, plus point 5 (u minus 1) minus point 5 by 2 into delta squared f_0 is minus 6, then I have the next term as point 5 minus point 5 minus 1 point 5, 1 by 6 into 6. Now this I will leave it to you, this you simply, this comes out to be minus 0 point 3 7 5, minus 0 point 3 7 5. Now consider the second part of the problem, I want to a find an approximation to f (2 point 5).

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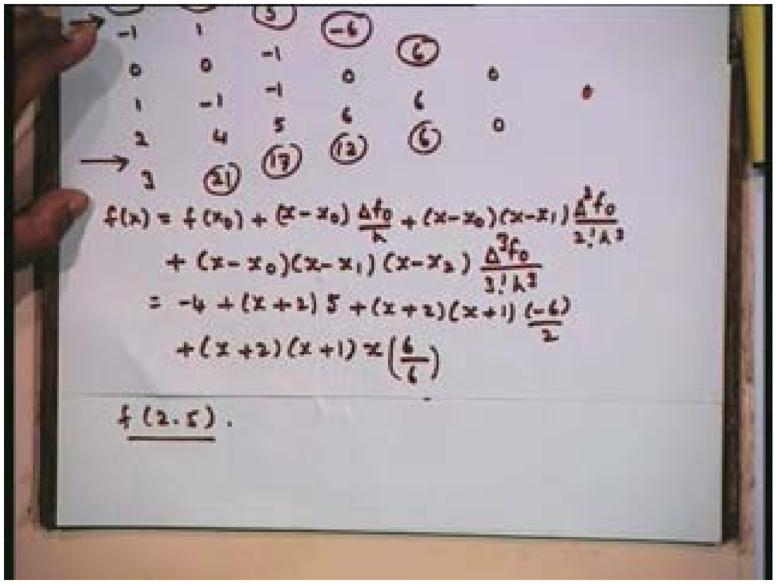

Now we notice, let us look back to the table, 2 point 5 lies here, 2 point 5 lies here, therefore there are no forward differences available for me here. Therefore since I am at the end of the table, I must use only the backward differences, so I shall be using the backward difference with respect to this as these backward differences. I would go in the backward direction when I am at the end of the table. Therefore I must now use the next formula that we have written it over here that is a formula, this formula should be used and therefore our x_n, here is the last point that is 3; our x is equal to 2 point 5 at which we have to find, therefore u is equal to (x minus x_n) divided by h that is 2 point 5 minus 3 divided by 1 this is equal to minus 0 point 5. As we mentioned, when we are going for the backward differences, u will always going to be negative.

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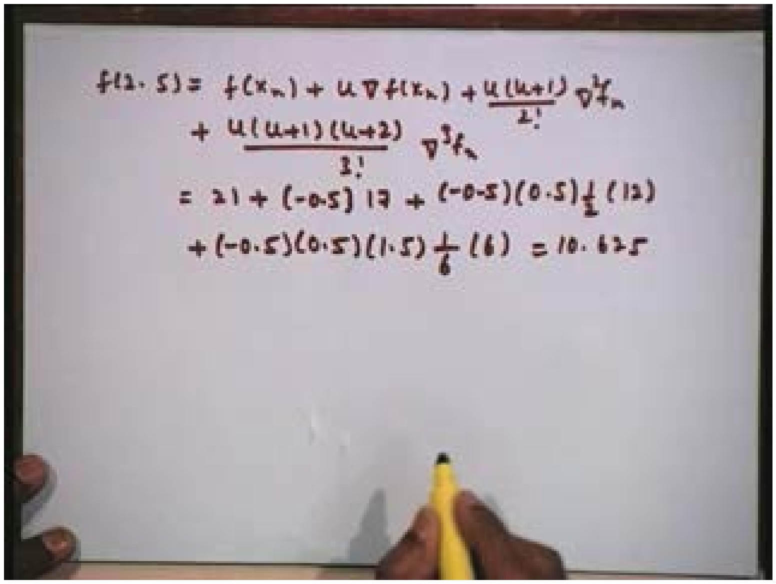

Now let us write down the solution, therefore f at 2 point 5 will be equal to, I will just put this slide here and from here let us write down. Therefore now the formula would be f of x_n, we will substitute the values, u delta $f(x_n)$ plus u into (u plus 1) by factorial 2 delta squared $f(x_n)$, u into (u plus 1) into (u plus 2) by factorial 3 delta cubed $f(x_n)$.

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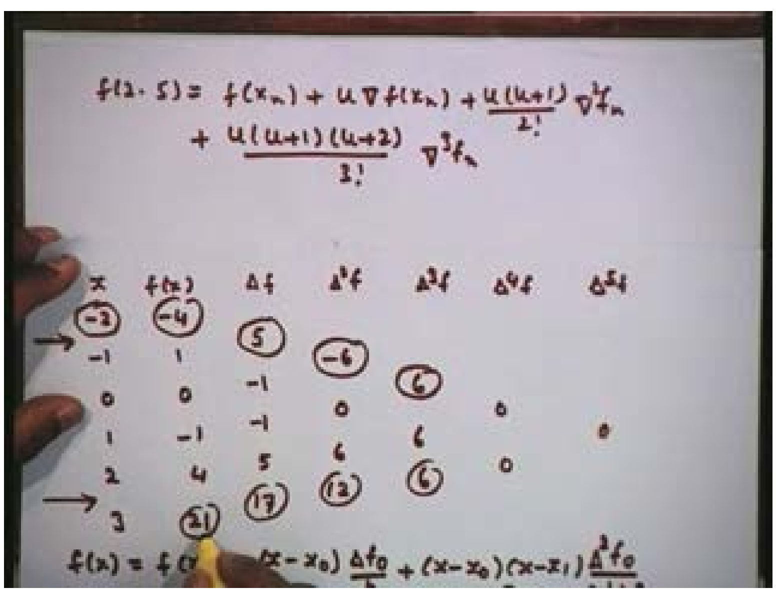

Now let us substitute the values from this table over here, so this is f at x_n is 21, so this is equal to 21, u is minus point 5, u is minus point 5, the backward difference with the respect to this is 17, 17, minus point 5 (u plus 1) that is plus point 5 delta squared $f(x_n)$ that is 12. Then we write the next term minus point 5 plus point 5 (u plus 2) that is 1 point 5 1 upon factorial 3 and this backward difference is, third difference is 6, so we are using this. Now we can simplify this and the value of this comes out to be 10 point 6 2 5.

Therefore if you are at the end of the table, there is no alternative but to use the backward differences. If you are at the start of the table or later we can always use the forward differences and sometimes we have not derived the central differences, using central differences. If you are at the middle of the table we can try to use the central difference table but however even if you are at middle of the table, sufficient number of forward differences are available or backward differences are available, I can use any one of them. For example if I am here, if I want to have a value over here, there are sufficient number of forward differences here, I can use it or I have sufficient number of backward differences that are available to me. So the choice is up to me whether I want to use a forward differences or backward differences, when both of them are available for me. Okay we will stop it with this one.