Numerical Methods and Computation

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Lecture No - 27

Interpolation and Approximation (Continued.)

In our last lecture we have derived the expression for the error of interpolation in Lagrange interpolation. Let us just revise what we have done last time.

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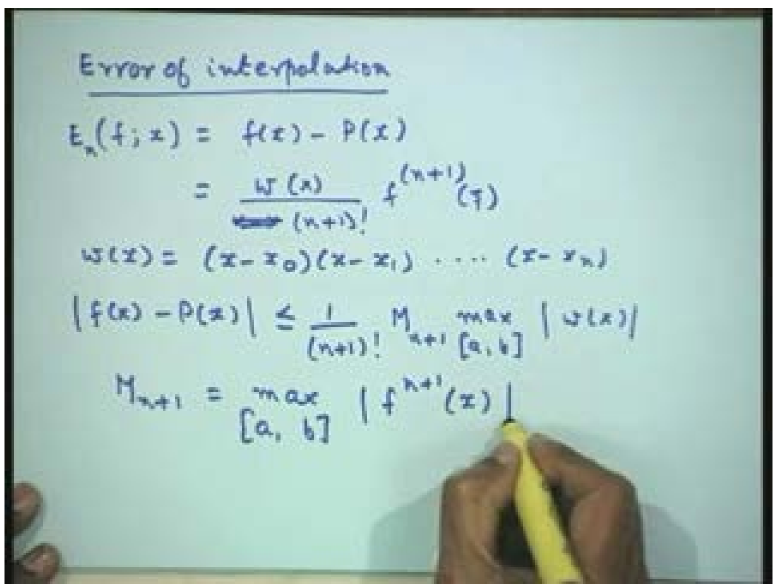

we have derived the error of interpolation, now we denoted this as E_n, n to denote the order of the polynomial that we are taking, this we have defined it as f(x) minus P(x) and we have derived that this expression is w(x) divided by x minus, w(x) by factorial n plus 1 $f^{(n+1)}$ zhi, where w(x) is the product of all the factors in the given problem (x minus x_0) (x minus x_1) so on (x minus x_n). We can also find the bound for the error of interpolation, so I can just take the magnitude and maximize the right hand side, therefore I will have less than or equal to 1 upon n plus 1 the maximum of this I will write it as M_{n+1} and then I will write down the maximum of, in the given interval (a, b) of w(x), where we have denoted M_{n+1} is maximum of the n plus 1^{th} derivative in the interval a to b of this. Now from here we have derived the expressions for the linear and quadratic interpolations which we would like to use it in an example.

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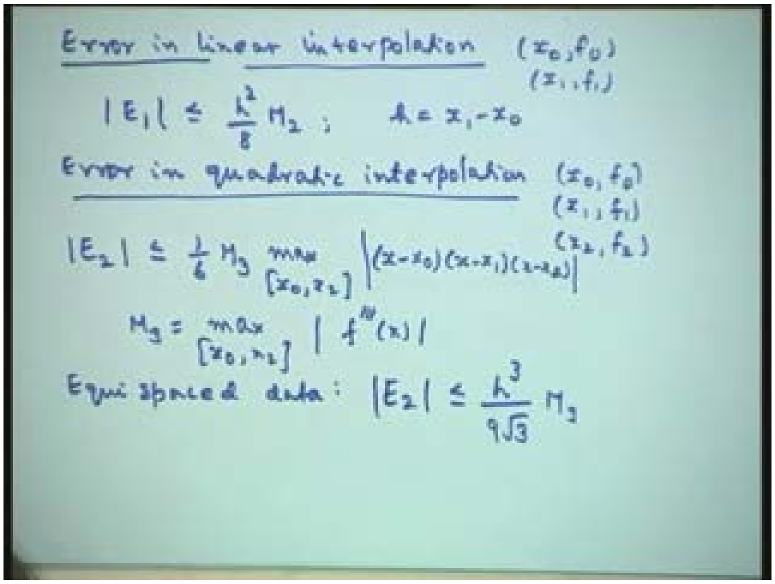

Let us just write down what is the error in linear interpolation. Now here we are using only 2 points out in the data that is (x_0, f_0) and (x_1, f_1), so these are the 2 points of the data that we are using, these are any 2 arbitrary adjacent points in the entire data. Then we have proved that error of the linear polynomial is less than or equal to h square by 8 M_2 and h is the distance between x_1 and x_0, x is equal to, h is equal to x_1 minus x_0. Now this formula would be the same whether we have got the data with equispaced points or data is not equispaced, because we are taking only 2 consecutive data points therefore the distance is h we are calling it, therefore the formula will hold in both the cases. Now error in quadratic interpolation, we are now approximating by a polynomial of degree 2 in other words we are using thereforth 3 arbitrary adjacent points in the entire data as this. We are using the 3 consecutive data points in the entire data that is given and using these 3, we are now writing the formula and then writing the error in interpolation and this we have proved it as less than or equal to 1 upon 6, let us call it as M_3 maximum in the given interval, that is (x_0, x_2) is the given interval of magnitude of (x minus x_0) (x minus x_1) (x minus x_2), where M_3 is the maximum in the interval x_0 to x_2, of f triple dash of x. We have also proved that in the, if the data is equispaced in the quadratic interpolation, equispaced data then we have proved that error in this case is bounded by h cubed by 9 root 3 M_3. Now I would like to use the results that we have derived here to find the error of interpolation in 2 examples which we have done last time.

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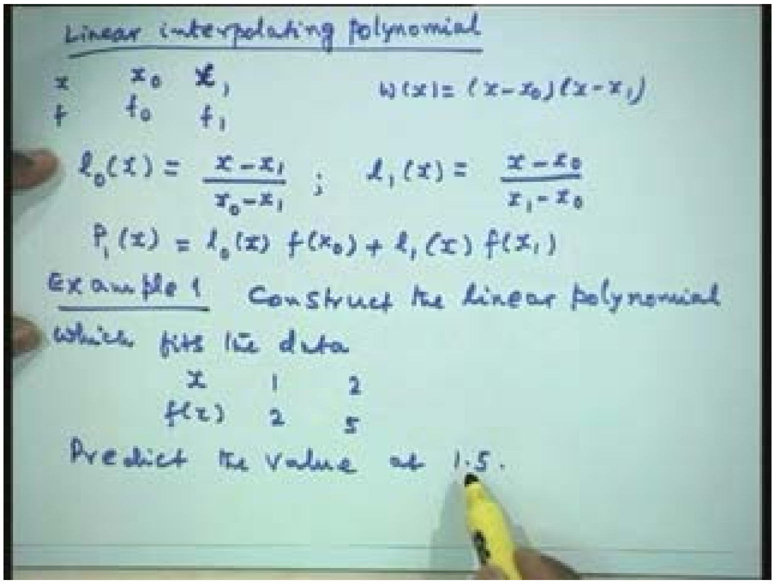

Let us just revise one example, let me just show the example we have done last time. We said construct the linear polynomial which fits this data; predict the value at 1 point 5. So now I want to find out what will be the error of interpolation in this example and what will be the error at this point. If I want the error at any particular point I can just substitute, let us suppose you have been given data values between say 1 and 10 and I want the error at a particular point, I can just substitute that value of x over here, if suppose I wanted 1 point 5, I will substitute x at 1 point 5 over here and then simply find out what are the right hand side and then write down the bound. If I want the largest possible value to be written on the right hand side, I will maximize it and then find out what is the largest value so that I have got required upper bound here.

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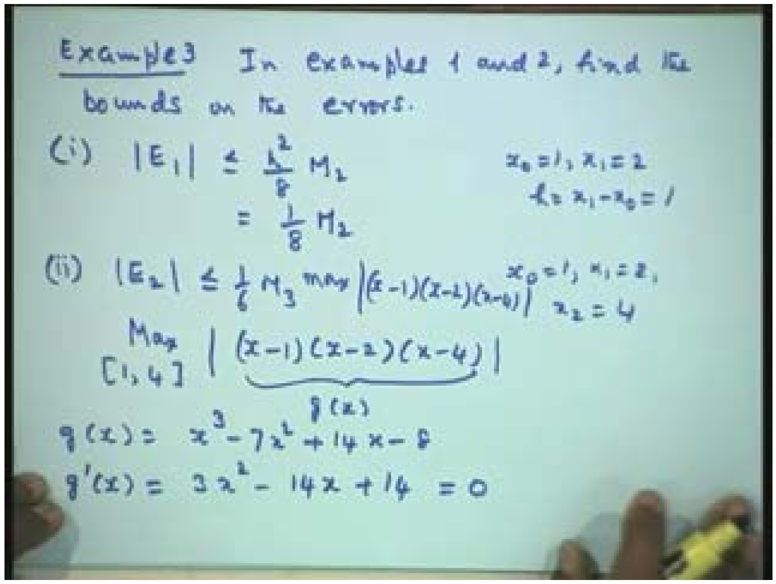

So let us take this as an example, let us write it as, call it as example 3. In examples 1 and 2, 1 and 2 find the bound on the error, find the bounds on the errors, on the errors. Now I will first take this example, this example 1 so let us put it as 1. In example 1 we are talking about the linear interpolating polynomial therefore we have to find out what is the value of magnitude of E_1 is less than or equal to h square by 8 M_2, here we have at x_0 is equal to 1, x_1 is equal to 2, x_0 is 1, x_1 is 2, therefore h is equal to x_1 minus x_0 that is equal to 1. Therefore this is simply equal to 1 upon 8 M_2. Now will see later on how we may be able to get an approximate value for second derivative, we did not know exact but we will, if the function is given to us we can do it, otherwise we will have some other alternatives which we will discuss it later on.

So this is the bound on the error 1 by 8 of M_2 in this example. Now if I take the other example that we have done last time, that fit an interpolating polynomial for this data, we have given 3 data points and then asked us to find, what the error is in this particular one. So this is 3 points, therefore we have fitted there a quadratic polynomial for this data. Now for this data I would write down that error, this is less than or equal to, now notice that the data is not equispaced, we have the points as x_0 is 1, x_1 is equal to 2 and x_2 is equal to 4 so they are not equispaced, therefore I will have to use the full formula that we have written, written here and that is 1 by 6 M_3 of maximum of (x minus x_0) (x minus x_1) (x minus 4). So I will write it again, this is, I want the maximum of lying between 1 and 4, x_0 is 1 x_2 is 4, so I need the maximum of this lying in the interval 1 to 4 of the quantity (x minus 1) (x minus 2) (x minus 4). Now we know this is a 1 variable problem, therefore if I set this quantity as g(x) then I can just open it up, multiply it out, this is x square minus 3 x plus 2 and multiply by x minus 4, I will get here x cubed minus 7 x squared plus 14 x minus 8, I have just multiplied these 3. Now I want to find the maxima minima of this absolute value, so I will just differentiate this in 1 variable, g dash of x 3 x squared minus 14 x plus 14 and set it equal to 0. Now this will give me 2 values for x and any 1 of these values can give us the maximum.

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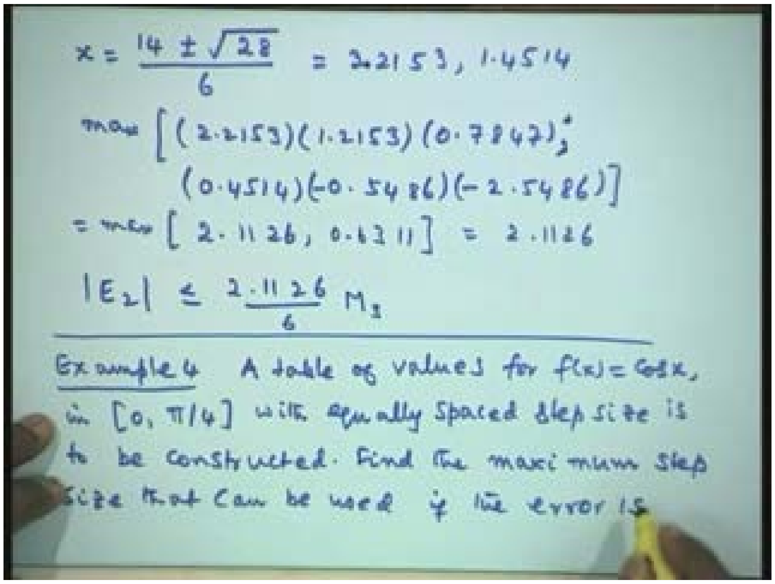

So I will find out the values of x from here that is minus 14 by 196 that is equal to 28 by 2 8. So this gives me 3 point 2 1 5 3 and 1 point 4 5 1 4. These are the 2 values of x, out of these 2 values 1 of them will be the largest value; the other will be the smaller value. So let us substitute it, what I will get therefore will be, I need the maximum of, we need the maximum of, substitute this 3 minus 2 point 1 3 that will give you, let us write down 1 value, this is 2 point 2 1 5 3 that is x minus 1, x minus 2 1 point 2 1 5 3, x minus 4 that is equal to 0 point 7 8 4 7. That is the (x minus 1) (x minus 2) (x minus 3) (x minus 4) and this is 1 value, the other value, the maximum of 2 values I am writing, I will substitute this, therefore this will become (x minus 1) so that is 0 point 4 5 1 4, (x minus 2) therefore this will have 5 4 8 6, I am taking (x minus 2) and then I am taking this with a negative sign, let us put it here and (x minus 4) that will give you minus 2 point 5 4 8 6. Obviously this is larger of the 2 and this is actually maximum of, this is equal to 2 point 1 1 2 6 and this is 6 3 1 1, so that the maximum is 1 1 2 6.

Now remember we do not need to find the second derivate to find out maxima or minima, we are not really talking of maxima minima of g(x), we are talking of the maximum of the absolute value. Therefore it would be sufficient for you to just find the first derivative, find the values of x that will be there, substitute it in the given expression and find the largest of all of them. It is possible this smaller value here would give you, I mean which it may be showing that it will be minimum may be giving actually maximum, maximum because we are talking of absolute, when once you are talking of a large negative value the absolute will be the positive sign, therefore that may give you this. Therefore it is a not necessary that we should go for the second derivative and do anything. Now therefore when once we found out this maximum value, I will put it over here and say that this is less than or equal to 2 point 1 1 2 6 by 6 into M_3. Now I should, I put here M_3 yes. Again I need the approximate value for M_3 which we shall see later on how we can determine them. Now let us take an application of this, so let us call this example 4. So what I would say here, I would to construct a table of values, a function is given. A table of values for f(x) is equal to cos x, I am taking a simple example, in (0, pi by 4) with equally spaced step size is to be constructed, is to be constructed. Find the maximum step size, find the maximum step size that can be used if the error, if the error in,

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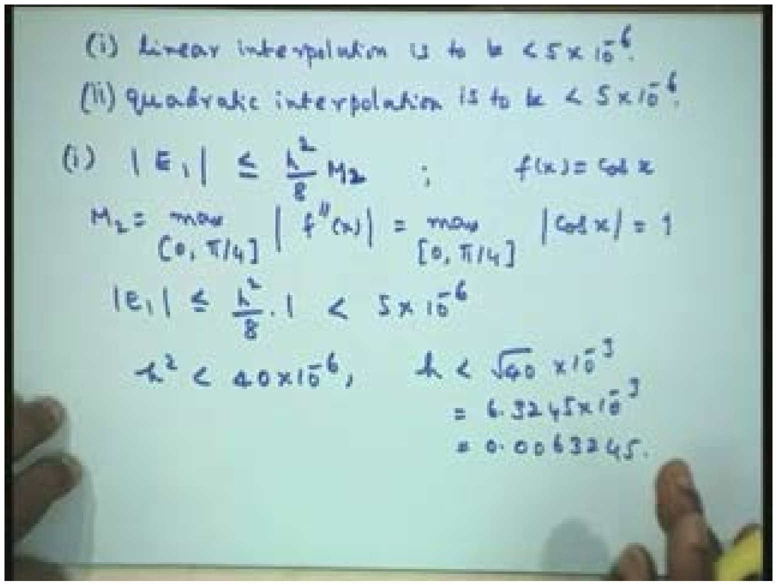

We will take 2 problems here; error in linear interpolation is to be less than 5 into 10 to the power of minus 6. Quadratic interpolation is to be less than 5 into 10 to the power of minus 6. Now what we are really asking here is that we have a known function, I want to construct a table of values for it, size at the, I want to, I may use linear interpolation in that, as a second problem we may use a quadratic interpolation but the error of interpolation should be bounded by this quantity, then find, tell us what is the largest step size that you can use for constructing this data values. This is the problem that we are looking at in reveres way as what we have earlier we have given a data there; we are constructing the polynomial and the error.

Now we want to construct a data using a given function. Now for this let us straight away write down what is the error in our linear interpolation, error in linear interpolation is bounded by h square by 8 M_2, h square by 8 M_2, now M_2 is maximum of, we are given the interval 0 to pi by 4, so the interval 0 to pi by 4 of f double dash of x. Now the function given to us is f(x) is equal to cos of x. Therefore this is maximum of, in the interval 0 to pi by 4 of magnitude of cos x. I have differentiated it two times so minus sin x then cos x, we will have simply magnitude of cos x. Now the largest value occurs at 0, cos 0 is 1, cos pi by 4 1 by root 2, so it is a, so it is a decreasing function, so you will have the maximum value as 1.

Now I want this error to be less than h square by 8 into 1 but this error should be bounded by 5 into 10 to the power of minus 6, that is linear interpolation should be total error, largest possible error should be less than 10 to the power minus 6. Therefore I can write down h square is less than 40 into 10 to the power of minus 6 or just h is less than root 40 into 10 to the power of minus 3 or this is equal to 6 point 3 2 4 5 10 to the power of minus 3, so that this is 0 0 6 3 2 4 5. Therefore I need to choose a step length smaller than point 0 0 6 3 in order that the maximum error will be less than 5 into 10 to the power of minus 6.

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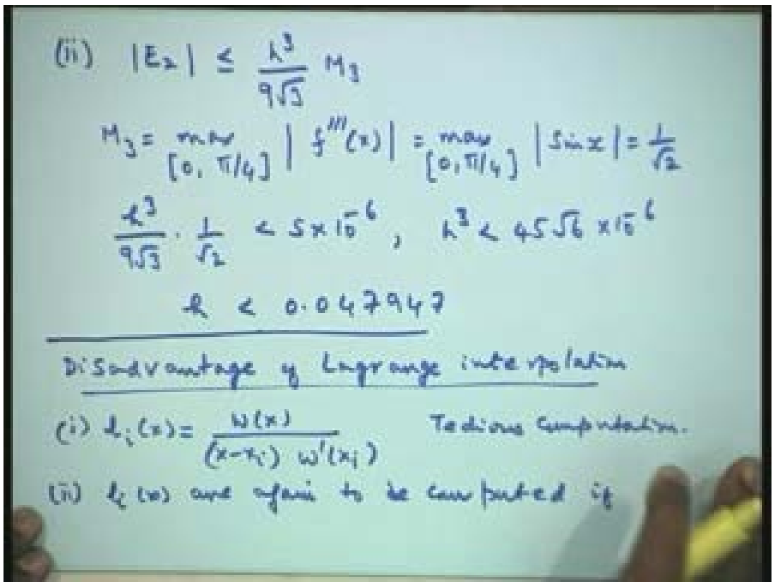

Now let us take the case of quadratic interpolation. In the case of quadratic interpolation we have error E_2 is less than or equal to, now we have been given a simple example here, we have been asked to do it in equally spaced step size, so the step length is equal. Therefore we shall use the formula which we have derived h cubed by 9 root 3 M_3, where M_3 is the maximum on interval 0 to pi by 4 magnitude of third derivative of x, third derivative with respect to x of f(x). So this is maximum of 0 to pi by 4, third derivative will be sin x, so we will write it as sin x. Whereas the maximum of this is 1 upon root 2, sin 0 is 0 it is an increasing function so will have 1 upon root 2. Therefore I have h cubed upon 9 root 3 into 1 upon root 2 and this should be less than 5 into 10 to the power of minus 6. Which I can write h cubed is less than, this is 45 root 6 into 10 to the power of minus 6 and h comes out to be, let me take it, h comes out to be 0 point 0 4 7 9 4 7, so cube root we are taking of this and then we get the value of this one. Therefore I need to use the step length which is smaller than 0 point 0 4 7 9 4 7. Now you can see that the quadratic interpolation has improved tremendously over linear interpolation because when we have used the linear interpolation we were requiring that the step length should be smaller than 0 0 6 6 3, whereas we are saying here it is sufficient to have a step length smaller than 0 point 0 4 8 which is approximately 8 times the step length that we can use in linear interpolation than in this 1, so if you are going from 0 to pi by 4 actually the number of points that you would come you can easily get it, that will be total length that is pi by 4 minus 0 divided by h that will give you the total number of points. If you are saying the number of points in the data table, it will be pi by 4 minus 0 divided by this, whereas in this case pi by 4 minus 0 divided by 0 4 7, therefore the number of points that you require in the table which will much less in the quadratic interpolation than linear interpolation.

Now we have derived the Lagrange interpolation and shown its application over here and it is the, among the all the interpolating polynomial Lagrange is the fundamental of all the results. However computationally it is not very efficient. Why it is not computationally efficient is, there are 2 reasons of this, 1 reason we can let us say the, let us call it as disadvantage, let us call it as disadvantage of Lagrange interpolation. For obtaining the Lagrange interpolating polynomial we need to find all the $l_i(x)$ and that is the expression w(x) upon (x minus x_i) into w dash (x_i). The data that is given let us suppose 100 data points is given, if all the 100 data points are lying on a straight line it is representing polynomial of degree 1 but all these $l_i(x)$ are polynomials of degree 99, therefore I need to compute each one of them, then combine them, collect all the terms and then cancel of all the ones and in this example of 100 data points, you will cancel x to the power of 99, x to the power of 98 everything, so you need to collect all these coefficients and then simplify it which is an extremely tedious computation, therefore one disadvantage is the tedious computations.

The second disadvantage is that, if I have a data given to me and if I add one more data item to this, the form of $l_i(x)$ is going to change completely because it adds one more term. Therefore all $l_i(x)$ have to recomputed if we are adding one or more data items, that means again it is a, it is not a format in which one can use it easily, therefore the $l_i(x)$ are to be computed, are again to be computed, if an additional point or we can, let us say one more point is added to the data.

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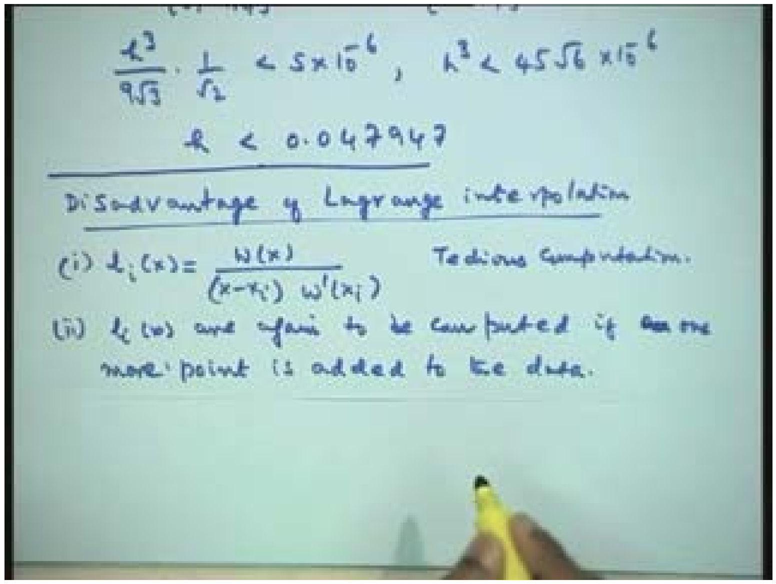

Now would like to construct a interpolating polynomial which is simple, which will take care of both of them, that is one is, it will not have that much tedious computation and secondly if we add one more data item, we do not have to change anything expect you add one more term to the given interpolating polynomial. To do that let us define what is known as divided differences.

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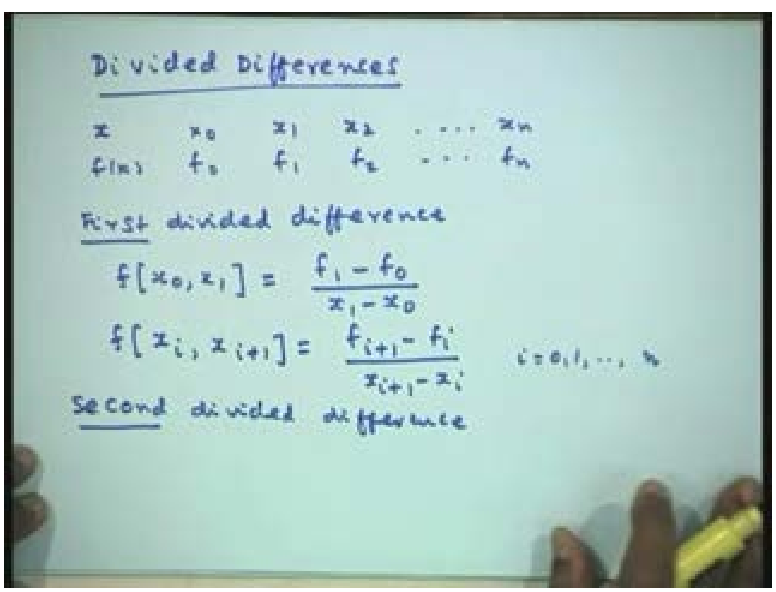

So I would like to define what is known as divided differences. Now what we have here is the data, let us write x f(x), x_0 f_0, x_1 f_1, x_2 f_2, x_n f_n, so this is our data that is given to us. Then I would like to define what is known as the first divided difference, first divided difference, that I would have a notation, in square brackets I will put the 2 data points I am using, the abscissa of the data points that I am using $[x_0, x_1]$. This is the f at 1 minus f at 0 divided by x_1 minus x_0 that is the distance of the ordinates divided by the distance of the abscissa that is why it is called divided difference, we are taking the difference of the ordinates divide it by the distance between the abscissa, so this is f_1 minus f_0 by x minus x_0. Similarly I can define, this is a general point so if I take this as $[x_i, x_{i+1}]$, this will be simply f_{i+1} minus f_i by x_{i+1} minus x_i. We are still talking of the data which is random, in which it is not equally spaced, it is randomly spaced. Therefore this I can take i is equal to 0, 1, so on n. Now I would define the second divided difference, now before you write this one just leave some space, let us write the table of divided differences; let us call it as divided difference table.

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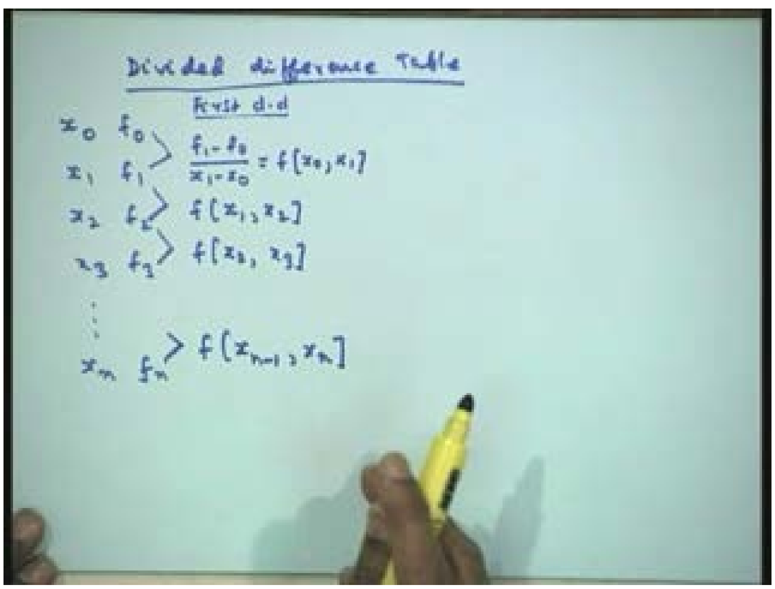

So let us write down divided difference table, just leave some space we will fill up for the second divided difference and so on but let us just see what we have done, how it goes. So let us put these data points vertically like this x_1 f_1, x_2 f_2, x_3 f_3, x_n f_n. Now I will put here first divided difference, I will call it as first divided difference. I am using these 2 values, these 2 data points and construct my first divided difference as f_1 minus f_0 by x_1 minus x_0, so I would write here f_1 minus f_0 divided by x_1 minus x_0 that will be equal to f (x_0, x_1). Now by the same argument I will use this and construct my first divided difference with respected to x_1 and x_2, these are the 2 abscissa points that I am using x_1 and x_2. Now I will use these 2 and get $f(x_2, x_3)$, now the abscissa for these 2 is x_2 and x_3, so this will be f_3 minus f_2 divided by the distance between the abscissas. So here I will have f of x_{n-1}, x_n I will have. Therefore the first column of this table will be all the first divided differences for the entire table. Okay now let us go back to what we wanted, the second divided difference. Now the second divided difference would be based on the values that we have computed already to construct a second column which we shall call as the second divided difference.

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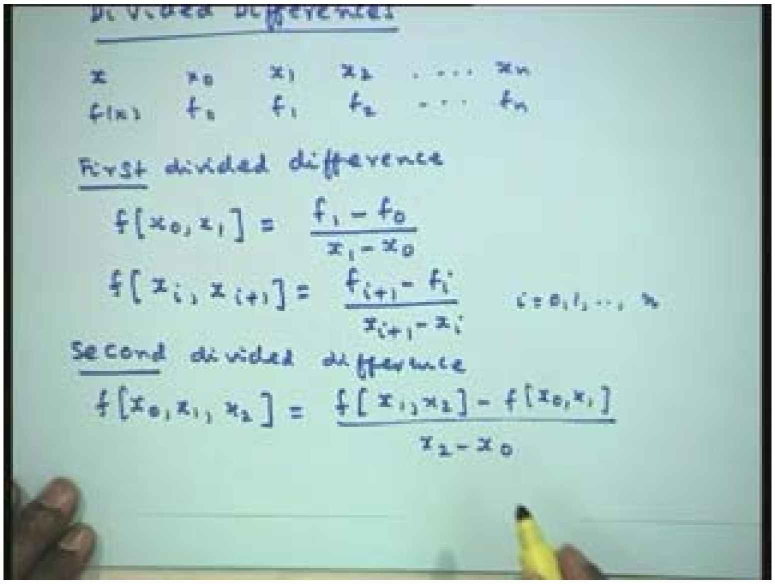

So what I would define here is the second divided difference will be f [x_0, x_1, x_2], so I need to take 1 more abscissa point and that will be the next lower order divided difference leaving the first argument, I will leave the first argument minus then I leave the last argument f [x_0, x_1], then the distance between the abscissa x_2 minus x_0, x_2 minus x_0.

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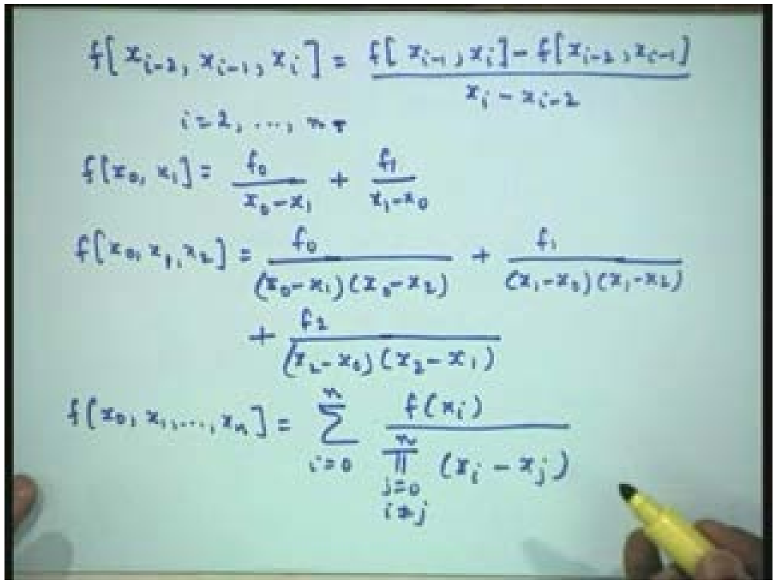

Therefore if I generalize this now, I would have this as f [x_{i-2}, x_{i-1}, x_i], these are the 3 consecutive abscissa and that will be the, I leave the first argument, I will have the divided difference with respect to these 2 abscissa, then I leave this last one, I would have f [x_{i-2}, x_{i-1}] divided by x_i minus x_{i-2}. So I would go from i is equal to 2, 3 so on n, i is n minus 2, n minus 1, n and this will be the second divided difference. Now therefore I can use this one and now construct my table of second divided difference, so I would now make this as second divided difference. Now the second divided difference table, we can see the identically as we have formed first divided difference is going to come, so I need to take this divided difference minus this divided by the distance now, distance is now increasing x_2 minus x_0. So this will be f of, I can now write like this and then write this here f [x_0, x_1, x_2], so I am using the three abscissa x_0 x_1 x_2, f this minus this divide by x_2 minus x_0.

Now if I use these two then this will be x_1 x_2 x_3 therefore this will be second divide difference with x_1 x_2 x_3 and so on, I would get here f [x_{n-2} x_{n-1} x_n]. Now the second column is complete and these are all the second divided difference and then we can proceed on like this and then write down all the differences, the n^{th} divided difference, there are only n plus 1 values here, therefore we can proceed upto n^{th} divided difference and the last one will be the n^{th} divided difference and that will be f [x_0 x_1 x_2 x_n]. We will have only n^{th} divided difference as I said because they are only n plus 1 values over here. Now let us just have, again let us just go back to the slide and then see every divided difference, first, second or any, everyone can be written in terms of the ordinates f_0 f_1. I can open it up, simplify and then collect all the terms and then write down explicitly as a linear combination of f_0 f_1 f_2 and if I do that let us just have a look at one of them and then generalize it from there.

For example if I take $f[x_0, x_1]$, this is equal to, I will take f_0 first, f_0 divided by (x_0 minus x_1), I will absorb the minus sign, so that I will write this as (x_0 minus x_1) plus f_1 upon (x_1 minus x_0). Now if do this for the next divided deference and then collect all the terms, I would get f [x_0, x_1, x_2] is equal to f_0 divided by (x_0 minus x_1) (x_0 minus x_2) plus f_1 (x_1 minus x_0) (x_1 minus x_2) plus f_2 (x_2 minus x_0) (x_2 minus x_1). The denominator is a product of all the factors with the first term being whatever the abscissa belongs to the numerator f x x_0, therefore all the differences using x_0 (x_0 minus x_1) (x_0 minus x_2), f at x_1 so the denominator will be (x_1 minus x_0) (x_1 minus x_2), this is f at x_2 therefore the denominator will be (x_2 minus x_0) (x_2 minus x_1), therefore I will have f [x_0 x_1 x_n] is equal to summation of n is 0 to n, f of x_i divided by all the products, j is equal to n except of course, it cannot be equal to j, (x_i minus x_j). As I said the numerator is f at x_i, the denominator will be all the products with the first term with the x_i and all the remaining abscissa that are to be used except of course i not equal to j. So this is the general expression for any divided difference to be written in terms of the ordinates. Now let us take a simple example for this because that will be the base for our constructing the interpolating polynomial.

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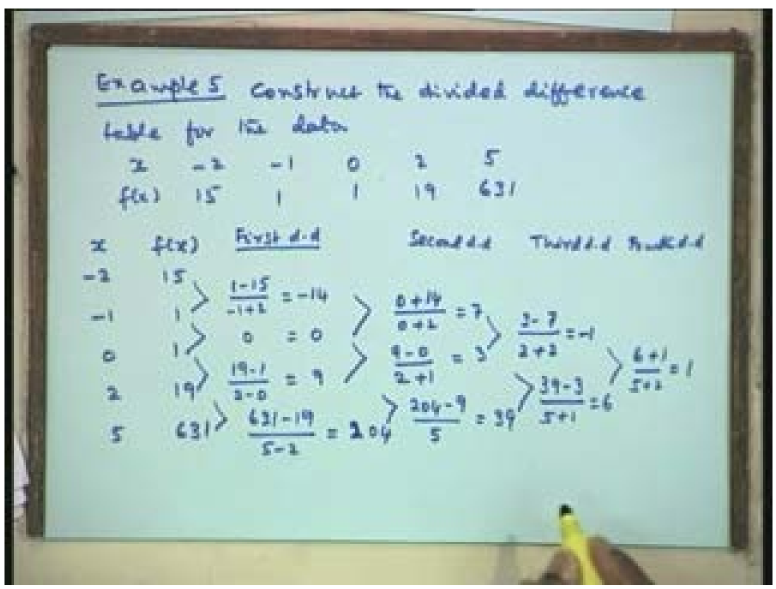

Okay, I will just write it again. Construct the divided difference table, construct the divided difference table for the data x f(x) minus 2, 15, minus 1, 1, 0, 1, 2, 19, 5, 631. So let us put this in the vertical format so that I can write down all the differences minus 2, 15, minus 1, 1, 0, 1, 2, 19, 5, 631. Now this, let us take these 2, this is 1 minus 15, 1 minus 15 divided by minus 1 plus 2, minus 1 plus 2 this is your x_1 minus x_0, so this will give us minus 14. Now let us take these two, the numerator is 0, therefore let us keep it as 0, 1 minus 1 is 0. This gives 19 minus 1 divided by 2 minus 0, 2 minus 0 that is 18 by 2 that is equal to 9. Then 631 minus 19 divided by 5 minus 2, 5 minus 2 that is equal to 6 and 12 that is equal to 2 0 4, 2 0 4. So the first column is complete. Now let us take the second divided difference. So if take these two I will have 0 plus 14, now we have moved x_0 x_1 x_2, therefore 0 plus 2; 0 plus 2 that is equal to 7. Then I take these two, 9 minus 0 divided by, now I have to choose the ((Refer Slide Time: 00:39:44 min)) x_1 x_2 x_3, that is 2 minus minus plus 1 that is equal to 3. Then I use this 2 0 4 minus 9, 5 minus 0 that is 39, 39. Then we go to the third divided difference, will have this as 3 minus 7, now we have moved x_0 x_1 x_2 x_3, so this will be 2 plus 2, that will be 2 plus 2 that gives you minus 1. Then I moving to x_1 to x_5 this 5, therefore this will be 39 minus 3, 5 minus 1 that is 5 plus 1, 5 plus 1, 36 by 6 that is equal to 6 and here the fourth divided difference is 6 plus 1 divided by the distance 5 plus 2 that is equal to 1.

Now interestingly if the data was not a polynomial, full degree polynomial that means this is a, 5 points are there it will be a polynomial of degree 4, if you want to construct the Lagrange interpolation. If it is not a polynomial of degree 4 but it is less say 3 or 2 or 1, so if all the points lying on the straight line, automatically the difference is, higher order difference will become 0, the higher order difference is automatically would become 0 implying that they are not lying on the full polynomial but the polynomial of degree less than that one. We will see how we are going to express that one, this will the, this table itself would automatically tell us whether we are having a full degree polynomial or we are not having a full degree polynomial.

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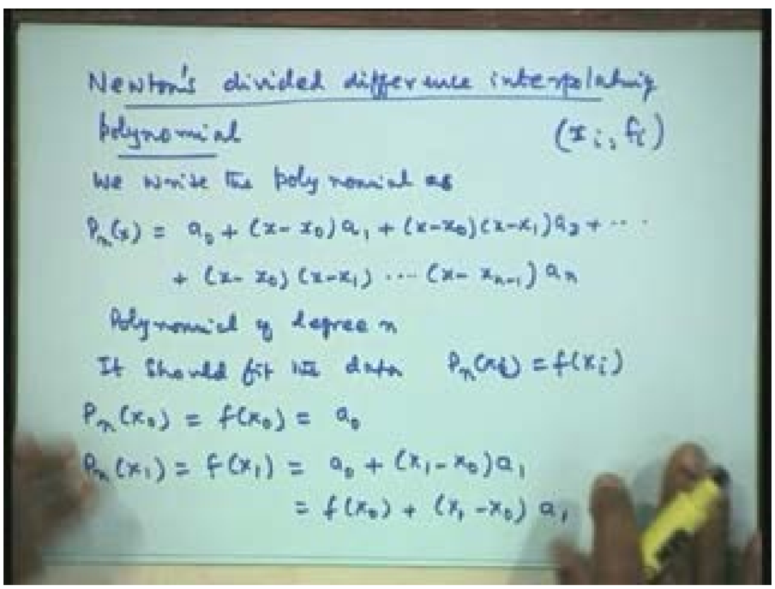

Based on this let us construct the new interpolation polynomial which we shall call it as Newton's divided difference interpolating polynomial. Now we write the polynomial as, when we have written the Lagrange interpolation we have taken linear combination of f's and then we have written it as the Lagrange polynomial, here we will not do that. What we do is, will write in a altogether different form $P_n(x)$, I will this as some a_0, (x minus x_0) into a_1, (x minus x_0) (x minus x_1) a_2, so on (x minus x_0) (x minus x_1) (x minus x_{n-1}) a_n. There are various ways of writing a polynomial, for example you are writing a quadratic, you can write down a plus b x plus c x square or you can also write it as a constant into x minus alpha into x minus beta, you can also write it in a form like a constant plus (x minus x_0) a_1 plus this. Now if you look, this is a constant this is linear, this is quadratic and this finally is of the degree n, there are n terms, this is of degree n, this is a polynomial of degree n, this is a polynomial of degree n. There are various ways of writing the polynomial therefore we have chosen a way of writing a given polynomial. Now if this is the polynomial, it should fit the data, it should fit the data. What is the data? Data is P_n at x_i must be equal to f at x_i, that is our data that is given to us as (x_i, f_i) is given to us. So if I substitute x_i in this, I should get by x_i. Let us do that, let us put x_0 here, P_n of x_0 should be equal to f at x_0. When I put x_0, all the terms vanish except a_0, so this is a_0, a_0 is simply f at x_0. Now let us P_n at x_1 that is equal to f at x_1 a_0 plus (x_1 minus x_0) into a_1. Now the second term onwards all of them contain x_1, (x minus x_1) is here, (x minus x_1), all the terms contains (x_1 minus x_1). Therefore all of them would vanish except these two terms. Now a_0 is already determined, a_0 is equal to f at x_0 plus (x minus x_0) into a_1.

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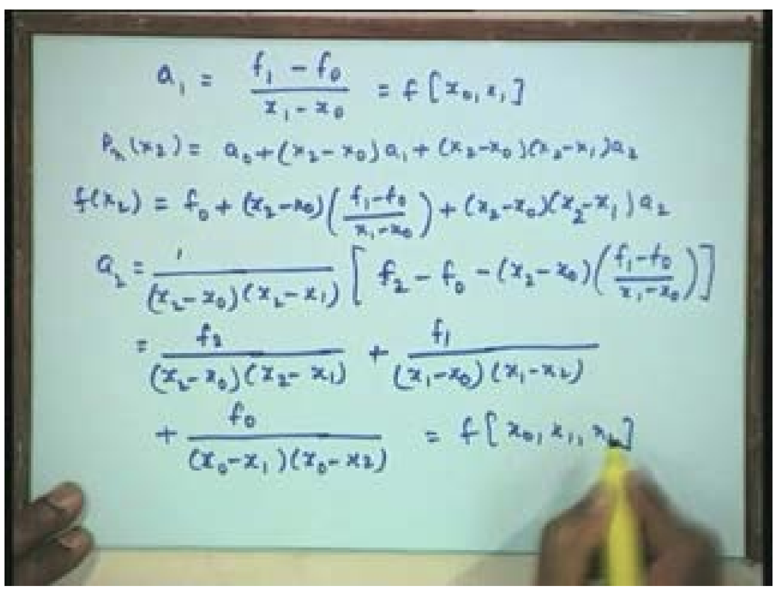

Now let us bring $f(x_0)$ to this side and solve for a_1. Therefore a_1 is equal to, I will write now this as f_1 minus f_0 by x_1 minus x_0, which turns out to be the first divided difference this is equal to $f[x_0, x_1]$. Now I repeat, continue it like this, I now substitute x_2 in this polynomial. If I substitute x_2 then I will get a_0, (x_2 minus x_0) a_1 plus (x_2 minus x_0) (x_2 minus x_1) into a_2, all the remaining terms contain (x_2 minus x_2), so all of them would vanish except these three terms of which we have already computed a_0, a_1 is also computed. This is equal to, therefore left hand side is f of x_2, right hand side is f_0, (x_2 minus x_0), a_1 is (f_1 minus f_0 by x_1 minus x_0) and this is the second term a_2. Therefore I can find out a_2 from here, bring everything to the left hand side, therefore a_2 is 1 upon (x_2 minus x_0) (x_2 minus x_1) that is the denominator over here, this is f_2, this is minus f_0 minus (x_2 minus x_0) into (f_1 minus f_0 by x_1 minus x_0). Now I collect the coefficients of f_0 f_1 f_2, f_2 is alone here, so the denominator f_2 is this, f_1 has got only 1 term, so f_1 is also there, only

f_0 is to be added. Let us put it reverse way, let us start with f_2, f_2 is (x_2 minus x_0) (x_2 minus x_1). Let us take f_1 this is (x_2 minus x_0) (x_2 minus x_0) they cancel off. There is a minus sign here; let us observe the minus sign here into this that is plus x_1 minus x_2, so I will have here f_1, this is (x_1 minus x_0) which stays as it is and this term is (x_1 minus x_2). I collect the third coefficient f_0 and I would give the result for this, this will be simply (x_0 minus x_1) (x_0 minus x_2) and this turns out by definition our second divided difference, this is f [x_0, x_1, x_2].

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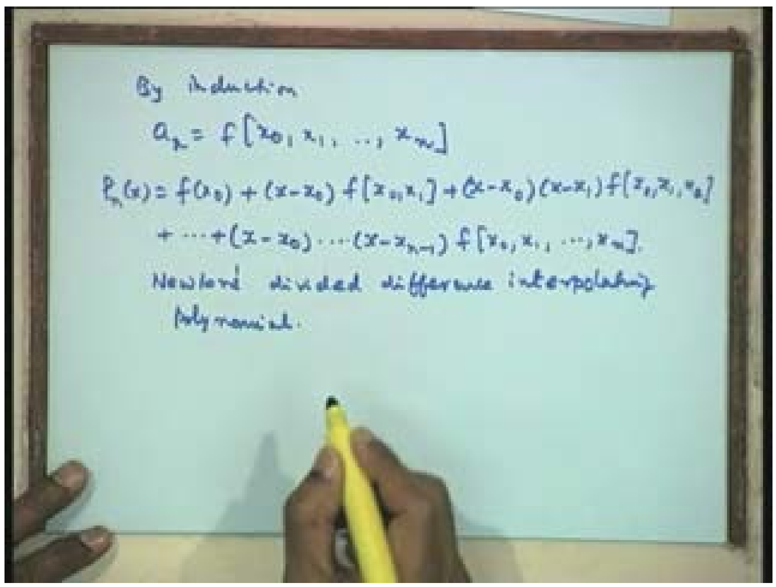

Now I can use induction procedure and then show that by induction a_n is f [x_0 x_1 so on x_n]. Therefore we have completed the derivation of the polynomial, therefore our polynomial is $P_n(x)$ is $f(x_0)$ plus (x minus x_0) f[x_0, x_1], we are coming from the top of the table, this x_0 x_1 x_2 is from the top of the table (x minus x_0) (x minus x_1) f of x_0 x_1 x_2 plus so on. We have a polynomial of degree n here, therefore this is (x minus x_{n-1}) f [x_0, x_1, x_n]. This is called the Newton's divided difference interpolation polynomial; this is the Newton's divided difference interpolating polynomial. We shall take up the application of this in the next lecture.