Numerical Methods and Computation

Prof. S.R.K. Iyengar

Department of Mathematics

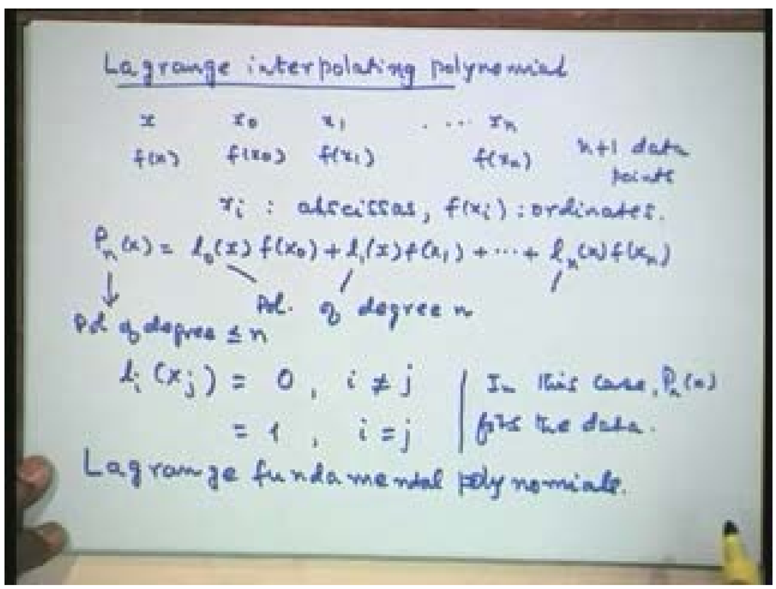
Indian Institute of Technology, Delhi

Lecture No - 26

Interpolation and Approximation

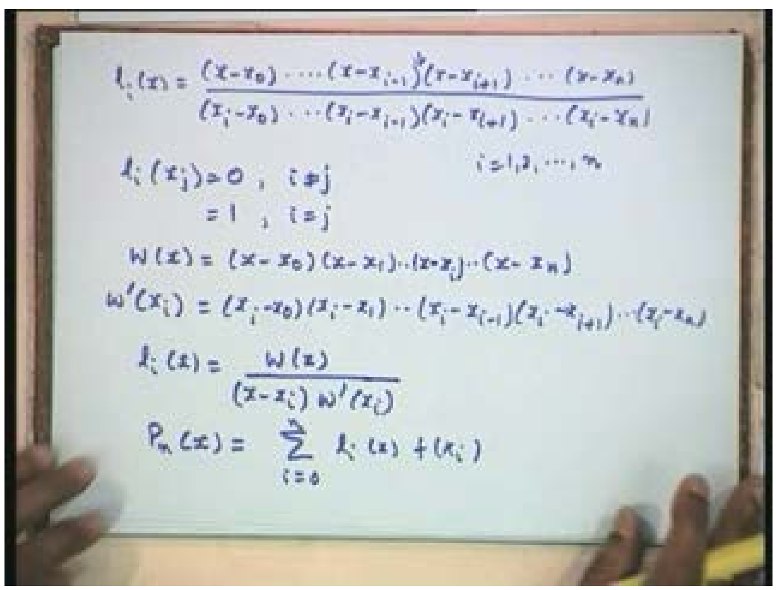
Now in the previous lecture we have introduced the concept of an interpolating polynomial, we were deriving the Lagrange interpolating polynomial which fits a given data. Let us just revise what we have done last time; we were trying to derive the Lagrange interpolating polynomial.

(Refer Slide Time: 01:19)



The data that is given to us is of the form x f(x), some point x_0 f at x_0, x_1 f at x_1 so on x_n f at x_n, so we have got a set of n plus 1 data points, data points or data values which we have here. We have written the Lagrange interpolating polynomial as a linear combination of these ordinates, we shall call this x_i as the abscissa of these data, these are the abscissa and f at x_i are the ordinates. Now this representation is obvious, because we can represent x f(x) as a point in the 2 dimensional coordinate plane, therefore we can we shall call them as abscissas and these are the ordinates. Now we write these polynomial, polynomial of degree n at x as a linear combination of $f(x_0)$ $f(x_1)$ $f(x_n)$ that is sum $l_0(x)$, f of x_0, $l_1(x)$ f of x_1 plus so on $l_n(x)$ f at x_n. Then we have mentioned that, since $p_n(x)$ is a polynomial of degree n all $l_i(x)$ should also be polynomials of degree n. Therefore these are all polynomials of degree n, $p_n(x)$ of course finally may turn out to be a polynomial of degree less than or equal to n, polynomial of degree less than equal to n. If the leading terms cancel here, then it is going to be a lower order polynomial. For example you might have given a hundred data but if they are all lying on straight line, so the hundred data will be representing a just a linear polynomial and not a polynomial degree 99. Therefore this p n x may finally be a polynomial of degree less than equal to n, while l_0 l_1 $l_n(x)$ will be polynomials of degree n only. Now then we have shown that the, these polynomials $l_i(x)$ should satisfy the condition $l_i(x_j)$ is equal to 0, for i not equal to j and this should be equal to 1, for i is equal to j, under this condition this polynomial exactly fits the, fits the given data, so in this case, in this case $p_n(x)$ fits the data. These polynomials are called the Lagrange fundamental polynomials; these are called the Lagrange fundamental polynomials. Now the next step is how we construct the, the polynomials of degree n, which satisfy this particular property and the writing of such a polynomial is really very trivial, because it should be 0 when I substitute x is equal to x_j and when I substitute i is equal to j, that is x_i it should have value 1.

(Refer Slide Time: 05:12)

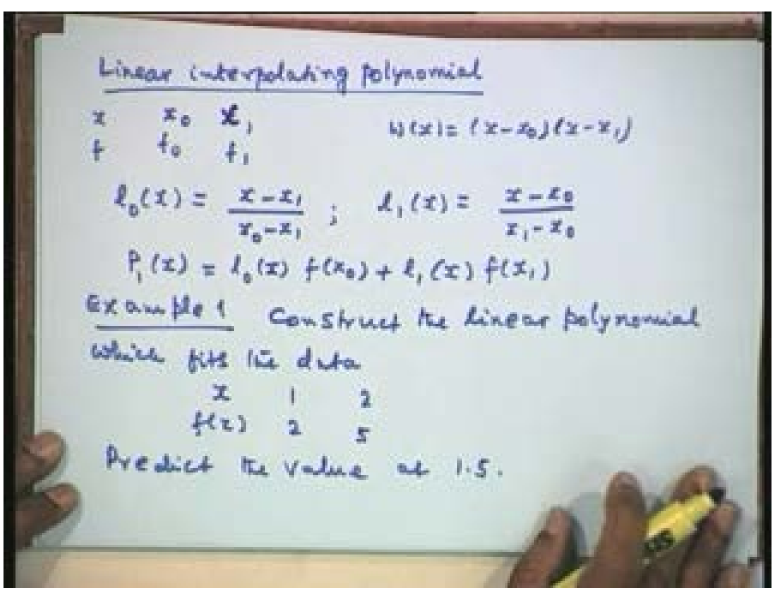


That means I can now define a polynomial $l_i(x)$ is equal to, in the numerator I will have, I will have a numerator and denominator (x minus x_0) all the products (x minus x_{i-1}) (x minus x_{i+1}) (x minus x_n) and in the denominator I will have (x_i minus x_0) (x_i minus x_{i-1}) (x_i minus x_{i+1}) (x_i minus x_n) that is i is equal to 1 2 3 so on n. Now in the numerator we have written all the products except the product here, that is (x minus x_i) and that belongs to this suffix i. Now let us substitute x is equal to x_0, it is 0 because numerator has (x minus x_0), so for all x_i this is going to be 0 except when x is equal to x_i, when x is equal to x_i the numerator is same as the denominator, therefore it cancels. Therefore this satisfies the property that $l_i(x_j)$ is equal to 0, for i not equal to j and this is equal 1, for i is equal to j. Therefore this is itself the Lagrange fundamental polynomial that is required in the construction of the Lagrange interpolating polynomial.

Now we can put this in a simple notation by just using a notation, so let us set the w(x) as the product of all the factors given in the problem, so these are the products using all the abscissas (x minus x_0) (x minus x_1) so on (x minus x_n). Then let us find out what is the derivative of this with respect to x but at x_i. These are n plus 1 factors. Therefore if I differentiate there will be n plus 1 terms. There will be only 1 term which will not contain, I will write down this particular factor here also (x minus x_i), let us write this (x minus x_i). When I differentiate this, there will be only 1 term which will not contain this, which is the derivative of this term, because these are all linear factors. When I substitute x_i all of them would vanish except 1 term which is containing this 1 and that term will be, when I differentiate whole the thing this goes, I substitute the remaining, so whatever you have here is (x minus x_0) (x_i minus x_1) the previous one is (x_i minus x_{i-1}) the next one (x_i minus x_{i-1}) and the last term (x_i minus x_n).

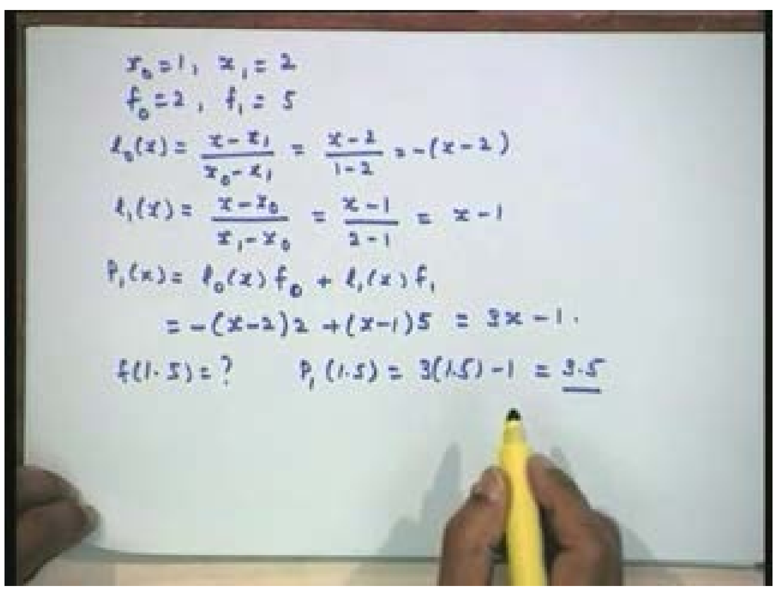
So all the terms would disappear except one term, which was the derivative using this term particular factor that we have. We see that the denominator of l_i(x) is simply w prime x_i, this is same as this denominator, therefore l_i(x) in simple notation can be written as w(x), I have to remove (x minus x_i) so I will divide (x minus x_i), therefore w(x) upon (x minus x_i) is the numerator and the denominator is w prime of x_i, so I can simply write this Lagrange fundamental polynomials in this particular notation. When once these l_i(x) are determined, I will substitute in the polynomial to find Lagrange interpolating polynomial, a summation i is equal to 0 to n l_i(x) f of x_i, so I can substitute these fundamental polynomials in this and simplify this to get my Lagrange interpolating polynomial.

(Refer Slide Time: 09:45)



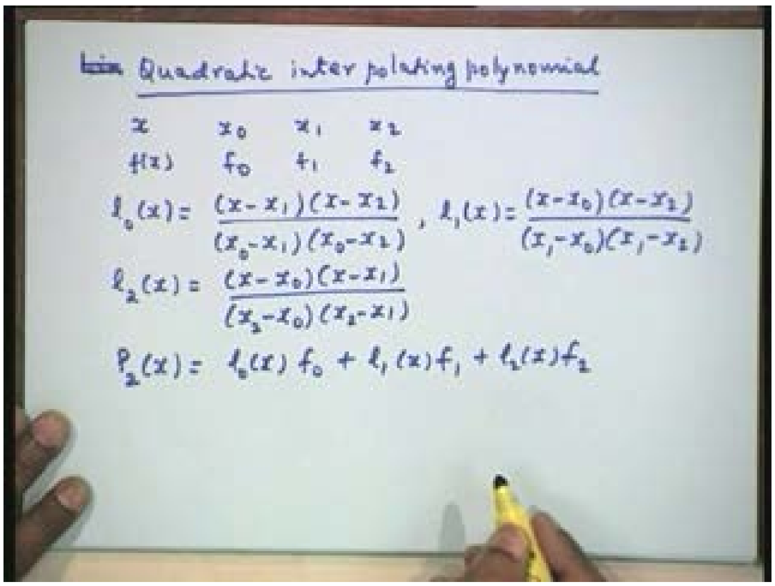
Now let us construct few simple cases, the first case we shall construct is, let us take a linear interpolating polynomial, linear interpolating polynomial, therefore we are considering any 2 points arbitrarily, so let us suppose we are considering the points, x_0 f_0 x_1 f_1, so we are considering the case of only 2 points x_0 f_0 x_1 f_1. Therefore our fundamental polynomial should be, now the factors w x will be all the factors (x minus x_0) (x minus x_1) there are only two 2 abscissa therefore there will be only 2 factors. Therefore our $l_0(x)$ must be equal to, I must skip in the numerator the factor corresponding to this suffix that is our x_0, so I must skip (x minus x_0), so I will have (x minus x_0) and what is the denominator? The denominator is the numerator evaluated at x_i that is the point with the suffix, so I will have to evaluate this numerator at x_0 that is (x_0 minus x_1). Now let us write down $l_1(x)$, now I must skip in the numerator (x minus x_1) so what is left out is (x minus x_0), evaluate it at x_1, (x_1 minus x_0). Therefore our polynomial of degree 1, polynomial degree 1 will be $l_0(x)$ into f of x_0, l_1 of x f at x_1 this will be the required polynomial. Now let us take a simple example along with this, so let us take this as example. Now I will number it so that I want to use this in the later steps, so I will use them as number 1 and this, construct the linear polynomial which fits the data x f x 1 2 2 5. Predict the value at 1 point 5, predict the value using this linear polynomial what would be the value at x is equal to 1 point 5.

(Refer Slide Time: 12:58)



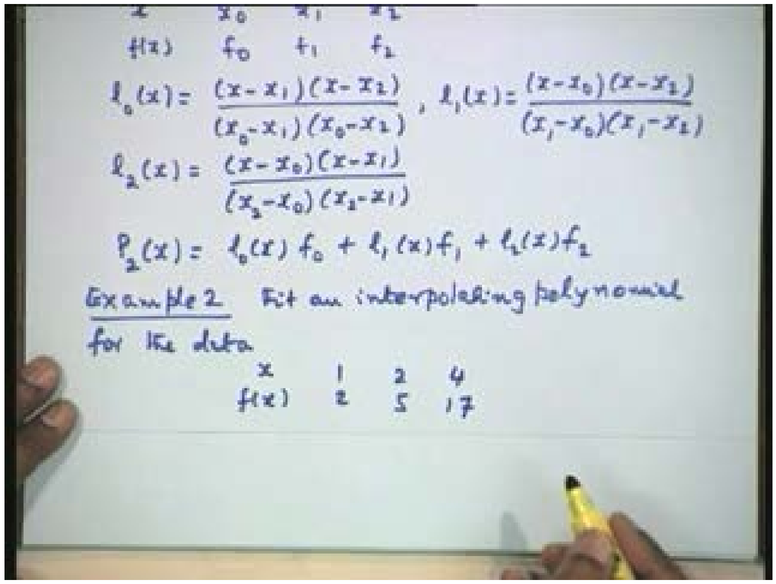
Now here our x_0 is 1, x_1 is equal to 2, f_0 is 2, f_1 is equal to 5. Therefore our $l_0(x)$ is x minus, let us write down the formula again, (x minus x_1) divide by (x_0 minus x_1), therefore this is (x minus 2) divided by (1 minus 2), so that will be minus of (x minus 2). l_1 x is (x minus x_0) divide by (x_1 minus x_0) so that is (x minus 1) from here and (2 minus 1) that is equal to (x minus 1). Therefore our required polynomial is $p_1(x)$ is $l_0(x)$ f_0 plus $l_1(x)$ f_1, let us substitute the values $l_0(x)$ is minus (x minus 2) f_0 is 2, $l_1(x)$ is (x minus 1) f_1 is 5. This is 5 x minus 2 x that is 3 x, this is minus 5 plus 4 so this is minus 1. Therefore this is a polynomial which fits exactly the given data. Now we want to predict the value of f at 1 point 5 is what is being asked, so I would give the estimate of this as p_1 at 1 point 5 that is 3 into 1 point 5 minus 1 that is equal to 3 point 5. So this will be the value, the predicted value of the function f at the point 1 point 5. Now we will little latter see how good this value is or what is the error in do you use in this particular approximation.

(Refer Slide Time: 15:20)



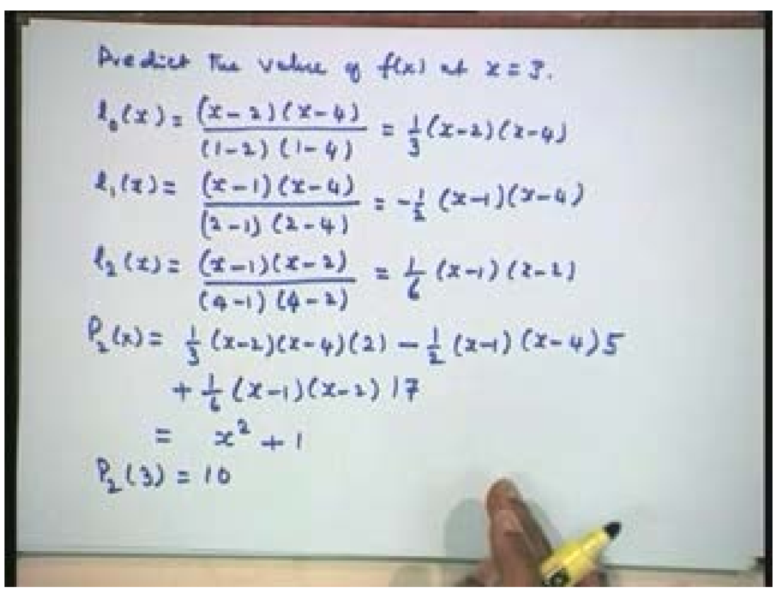
Now let us take up what would be the quadratic interpolating polynomial, so similarly let us derive the, sorry, the quadratic interpolating polynomial. I want to use 3 points and construct a polynomial of a degree 2 which passes through this, that means I have got the data as x f(x), x_0 x_1 x_2, f_0 f_1 f_2. Therefore I can write down the Lagrange fundamental polynomials immediately, the numerator should miss (x minus x_0), so I will have the remaining factors (x minus x_1) (x minus x_2), the denominator is evaluation at x_0, so this is (x_0 minus x_1) (x_0 minus x_2) and $l_1(x)$ is, the numerator should skip now (x minus x_1), so I will have (x minus x_0) (x minus x_2) and I evaluate it at x_1 for the denominator, so this will be (x_1 minus x_0) (x_1 minus x_2) and the third fundamental polynomial is $l_2(x)$, I skip now the numerator (x minus x_2), so I will have (x minus x_0) (x minus x_1), I evaluate it at x_2, (x_2 minus x_0) (x_2 minus x_1). When once these polynomials are evaluated I immediately write down the polynomial of degree 2, x is equal to $l_0(x)$ f_0, $l_1(x)$ f_1, $l_2(x)$ f_2. Now let us also illustrate an example for this so that we can discuss about the error, so let me write down the example.

(Refer Slide Time: 17:50)



Now fit an interpolating polynomial for the given data, so let us give this data as x f(x), I will take it as 1, 2, 2, 5, 4, 17.

(Refer Slide Time: 18:35)

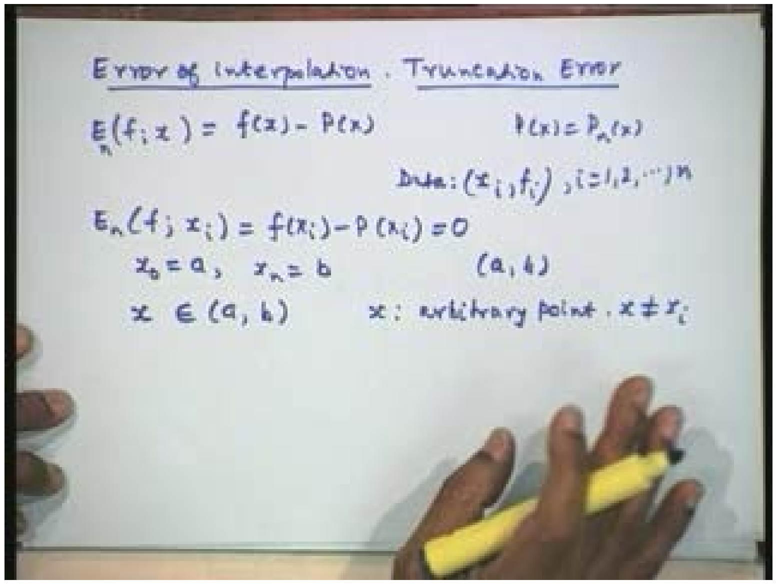


Then will say predict the value of f(x) at x is equal to 3. Now what is very important to note here is, these data points are arbitrary for data points that means they are not having any step, a particular step length or anything, you can see the distance between these two is 1, the distance between these two is 2, so these are really a random set of points. They, you can particularize with the case of when you have a equispace data but otherwise this is an arbitrary data.

So let us write down the fundamental polynomials, so I will have x, $l_0(x)$ is equal to, our x_0 is 1, x_1 is 2, x_2 is 4, so I will have there is (x minus 2) into (x minus 4) divided by (1 minus 2) (1 minus 4), x_0 is 1 so I am substituting x_0 is 1 (1 minus 2) (1 minus 4), so this gives minus 3 minus 1 so this is plus 1 upon 3 (x minus 2) (x minus 4). Similarly I construct $l_1(x)$, now we would miss x_1 that is 2, factor corresponding to 2. So I will have the numerator (x minus 1) (x minus 4) we have (2 minus 1) (2 minus 4), this is (2 minus 1) (2 minus 4), so this gives you 1, this gives you minus 2, so this gives you minus half (x minus 1) (x minus 4) and the third polynomial is $l_2(x)$, now will have to skip my x_2 that is 4 the factor corresponding to 4, so I will have here (x minus 1) into (x minus 2) divide by (4 minus 1) (4 minus 2) this gives us 3, this gives us 2, so I will have here 1 upon 6 (x minus 1) into (x minus 2). Therefore the required polynomial is $p_2(x)$ that is, I am $l_0(x)$ f_0 that is 1 by 3(x minus 2) (x minus 4) into 2, that is f_0 is 2 minus half (x minus 1) (x minus 4) f_1 is 5, plus 1 by 6, plus 1 by 6 (x minus 1) (x minus 2) and f_2 is 17.

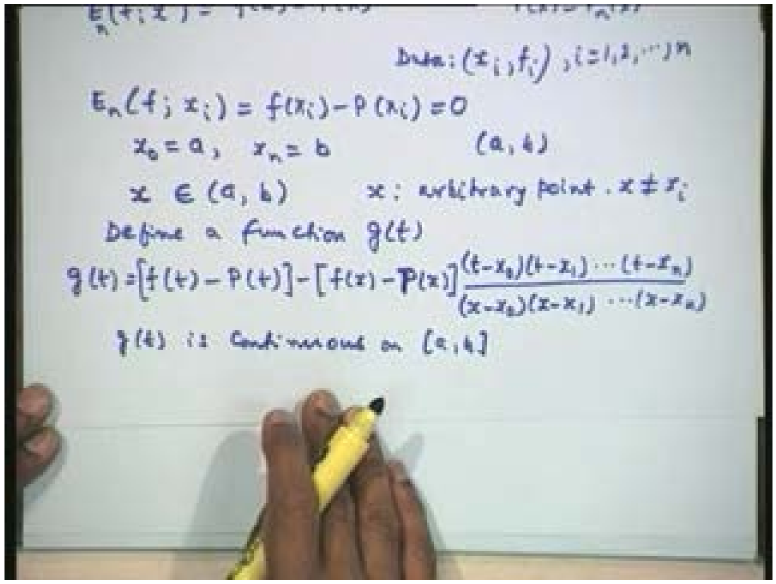
Now I can collect the coefficients from all these, this I will leave it as a simple exercise for you, this comes out to be x square plus 1. The coefficient of x cancels throughout and I simply have this is the x square plus 1, so that we can predict the value of at 3 as simply equal to 10. Now this is the polynomial we can just check it again back whether we have done it correctly or not, just substitute x is equal to 1, this is 2, when x is equal to 2 this is 4 plus 1, 5, x is equal to 4 that is 16 plus 1, 17. So this polynomial is fitted this data exactly. So this is a verification that we can cross check because we are exactly fitting the given data and this is the predicted value. Now as I mentioned earlier the polynomial that we are constructing is fitting the data exactly but when we predicted at a particular value, not the point which is given there, we do not know how accurate this particular value, this 10 is. Therefore we must define, what is the error in our interpolated values? which we can also call as the truncation error.

(Refer Slide Time: 23:13)



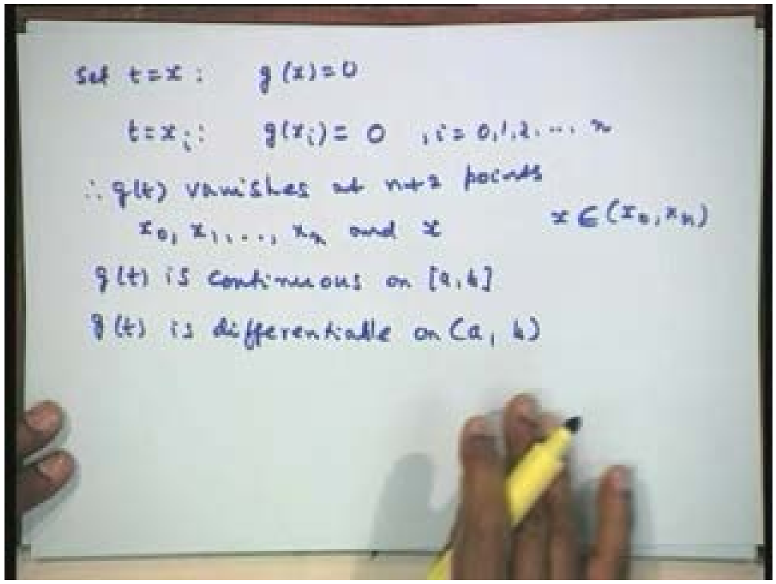
So I would like to derive a formula for the error of interpolation, so let us call this as error of interpolation, this is also called the truncation error, this is also called the truncation error. As I mentioned there is no error at the nodal points because the, which is we are fitting the data exactly, there is there is no, therefore no error at x_0 x_1 x_2 x_n, the error arises only when we are interpolating at a particular another point. So let us denote this f(x) minus p(x), p(x) is a polynomial of any degree 1 2 or 3 or anything, this is our approximation and let us denote this as error, error polynomial degree n of x. So that means we are now writing in general your p(x) as $p_n(x)$ it could be n is 1 its linear, n is 2 quadratic polynomial or otherwise its a degree n. Now remember our data is a, data is what is this data given to us it is x_i f_i, this is the data given to us, this is the data given to us. Now as I said there is no error at the nodal points that means if I put x is equal to x_i, this obviously 0, $f(x_i)$ minus $p(x_i)$ is 0 because that is the data is exactly fitted. So error at the abscissa x_i is equal to $f(x_i)$ minus p at x_i is equal to 0, because we are fitting the data exactly. Now to generalize the problem let us denote our first point x_0 as a, last point x_n as b, so that we are talking of the interval (a, b) a general interval (a, b). Now we are interpolating at an arbitrary point in between, so let us choose x an arbitrary point contained in this interval (a, b) therefore x is an arbitrary point and therefore x is not equal to x_i obviously this is not a this point.

(Refer Slide Time: 25:49)



Now I define a new function g(t) so I will define a function g of t, t is a new variable. I will write this as g of t is equal to f of t minus p of t, let us put it in a bracket minus f(x), minus f(x) minus p(x) into (t minus x_0) (t minus x_1) (t minus x_n) divided by (x minus x_0) (x minus x_1) (x minus x_n). Now we will understand why we are constructing such a function because I want to get from this, what will be the expression for this E_n is equal to f(x) minus p(x), this function would enable me to find such a quantity. Now let us look at the function g(t), g(t) is containing f(t) p(t) f(x) p(x), our function we are expecting that the, whatever the data that is given is representing continuous function. Therefore this everything is continuous here, therefore g(t) is also a continuous function, so g(t) is a continuous function, g(t) is a continuous function.

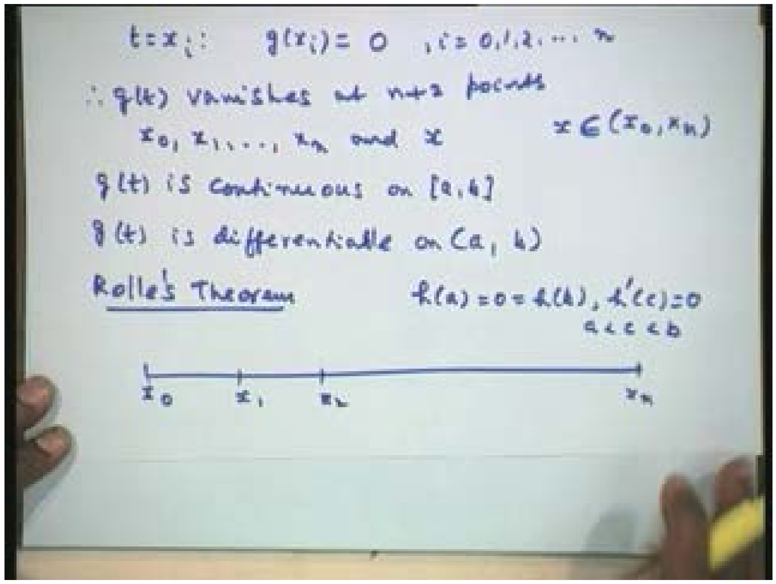
(Refer Slide Time: 27:47)



Now let us just look at the property of this g(t). Let us set t is equal to x in this, let us put t is equal to x. Now before we look at the first one, let us first look at this last factor, when I put t is equal to x this is same as the denominator, all the terms are same as, therefore they cancel of and will have 1. When t is equal to x this is [f(x) minus p(x)] minus [f(x) minus p(x)] into 1, therefore this will be simply g of x is equal to 0. This is same as this and we have this is equal to 0. Now let us set t is equal to x_i, set t is equal to x_i that is our points x_0 x_1 x_2 x_n. Then g at x_i is equal to f at x_i minus p at x_i that is equal to 0 by definition, that is our f(x_i) minus p(x_i) is 0 but here when I put this any one of this x_i's the numerator is 0 because (t minus x_0) (t minus x_1) (t minus x_n) when I use any value for t is x_i, one of the factors is containing (x minus x_i), so this numerator is going to be 0 but this is also 0, so therefore this is also equal to 0, these are, i is equal to 0 1 2 n.

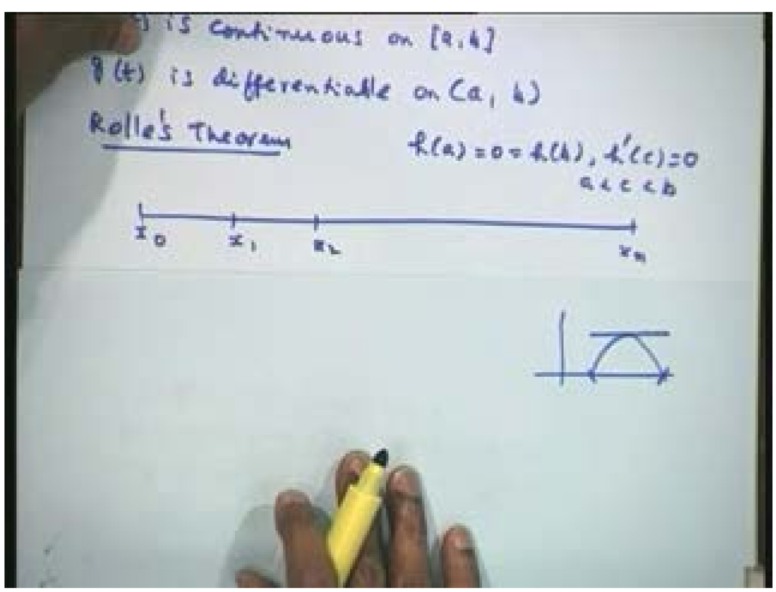
Therefore g(t) is a function of t which vanishes at n plus 1 plus 1, n plus 2 points. Therefore g(t) vanishes at n plus 2 points, n plus 2 points x_0 x_1 x_n and x, and at x of course, x is a point inside (x_0, x_n) that is our point (a, b), x_0 point is an interior point. g(t) is continuous on (a, b) differentiable, we assume that differentiability also required number of times, g(t) is differentiable on the open interval (a, b). Now I have a function g(t) which is continuous on (a, b) differentiable on (a, b) and vanishes at n plus 2 points. Now I would like to apply a theorem which you have studied in your first semester that is your Rolle's Theorem of a continuous function.

(Refer Slide Time: 30:54)



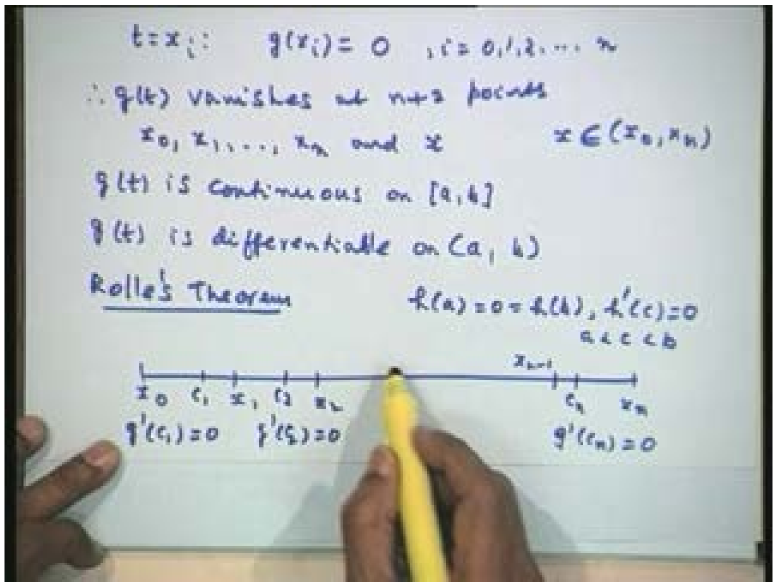
Now what is the Rolle’s theorem, I want to use this Rolle’s theorem, now these are our abscissas, now Rolle’s theorem states that if I have, we have a function, let us a remove it some h(x), let us suppose we have a function h(x), h of a is equal to 0, h of b is equal to 0, in fact they need not be equal to 0, it will simply h of a is equal to h of b and h is continuous on (a, b) differentiable on (a, b), then the Rolle’s theorem states that there exists at least 1 point between (a, b) where h prime or the derivative of h will be equal to 0. So then the Rolle’s theorem states that there exists a point at which h dash of c is equal to 0, where c is a point lying between a and b.

(Refer Slide Time: 32:07)



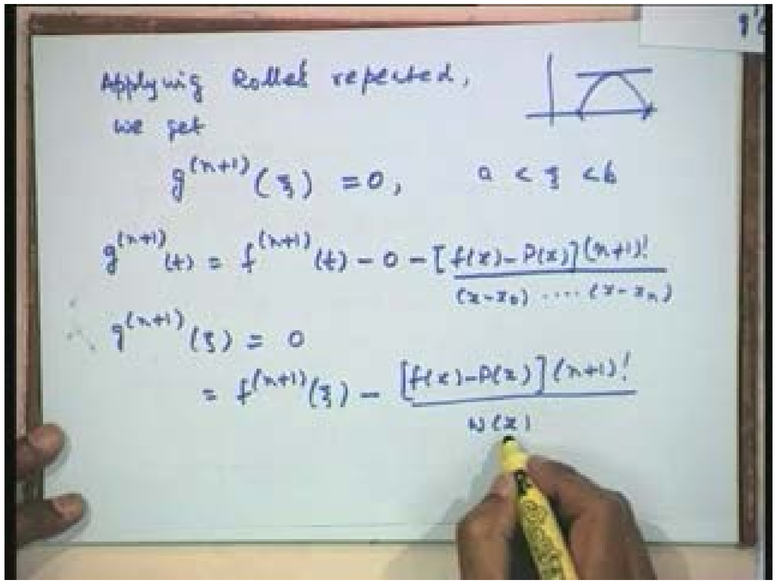
Now if you just remember what this was, this really states that if suppose you have a function like this, h of a is 0, h of b is 0 then what it states is that there is at least one point, at least this slope over the tangent will be parallel to x axis, h dash of c is equal to 0. Now I would like to apply this Rolle's Theorem on this

(Refer Slide Time: 32:30)



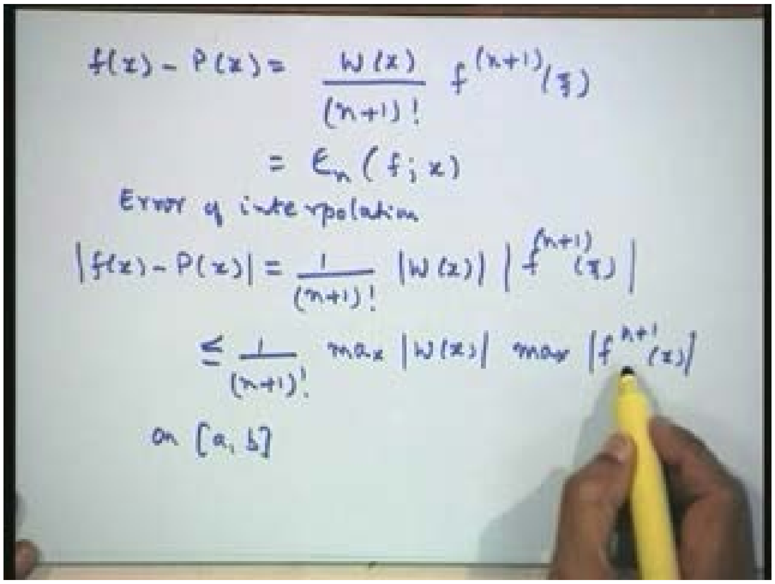
So if you look at these two points x_0 x_1, g(t) vanishes at both this points $g(x_0)$ is 0 $g(x_1)$ is 0, g(t) is continuous g(t) is differentiable therefore there exist a point here some c_1 at which your g dash of c_1 is equal to 0. Now between x_1 and x_2, $g(x_1)$ is 0 $g(x_2)$ is 0 therefore again there exists a point c_2 at which g prime at c_2 is equal to 0. Now if I go to the last point here, last interval again g at x_{n-1} is 0, g at x_n is equal to 0, therefore there exists a point c_n at which g prime at c_n is equal to 0. Now let us imagine that g prime is some new function, some new function h(x). If I take this as new function, now I can apply the Rolle's theorem repeatedly, now I have a function $h(c_1)$ is 0, $h(c_2)$ is equal to 0, g dash is continuous, it is differentiable therefore there exists a point between c_1 and c_2 such that the second derivative is equal to 0. Now if I apply repeatedly then there are n plus 2 points, therefore repeated application Rolle's Theorem would state that n plus 1, 1 less than that; that is n plus 1^{th} derivative of g will be 0 at 1, at least 1 point between x_0 to x_n.

(Refer Slide Time: 34:16)



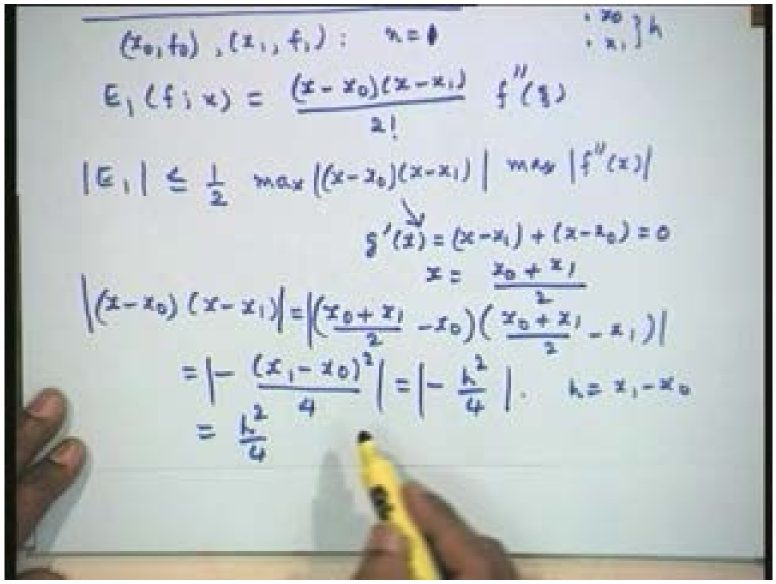
So if I apply this Rolle's theorem repeatedly, so let us say applying Rolle's theorem repeatedly, repeatedly, we get the n plus 1th derivative at an arbitrary point zhi in the interval, this is equal to 0, where zhi is a point between a and b. Remember this is derivative with respective to t, we are talk of a function of t. Now let us differentiate this n plus 1 times, $g^{(n+1)}$ of t, so I have to differentiate this, $f^{(n+1)}$ of t, n plus 1 of t, p(t) is only a polynomial of degree n but differentiating it n plus 1 times, therefore the derivative of this will be going to be 0, minus, this is a function of x, so that stays as it is f(x) minus p(x). Now if you look at this one, this, there are n plus 1 factors therefore this is a polynomial f degree n plus 1 in t. Therefore if I differentiate it n plus 1 times i will get factorial n plus 1, n plus 1 into n into n minus 1 and so on, differentiate it n plus 1 times, so I would get here factorial n plus 1 in the numerator. Differentiating the numerator n plus 1 times, its a polynomial of degree n plus 1. The denominator stays as it is, that is your (x minus x_0) so on (x minus x_n). Now this expression is 0 at zhi therefore $g^{(n+1)}$ of zhi is equal to 0 and that will be equal to, let us now put here $f^{(n+1)}$ of zhi minus, there is no t here, so this stays as it is, so [f(x) minus p(x)] n plus 1 factorial. Now let us revert back this to your notation which we started with, this was w(x) this is the product of all the factors that are there in the given problem, that is (x minus x_0) (x minus 1) (x minus x_n) that is w(x). Now since this is equal to 0, I can now take this to the left hand side and find out what is the value of f(x) minus p_n(x).

(Refer Slide Time: 37:19)



Therefore f(x) minus $p_n(x)$, so I am bringing to this side, so this goes up w(x), this comes down n plus 1 factorial and this is $f^{(n+1)}$ of zhi and we started the definition as error of interpolation is f(x) minus p(x) is equal to error at this one, therefore whatever we have derived now is nothing but the error of interpolation and this is your error of f(x), this is your error of interpolation. Now since this zhi is unknown to us it is not possible for us to write this expression but we can bound it, so let us find out what is the bound of this, let us find what is the magnitude of this is. n is a number, so I can write it outside, this is magnitude of w(x) into magnitude of $f^{(n+1)}$ of zhi. Now zhi is unknown, therefore what I will do is, I will take the maximum possible value of $f^{(n+1)}$ of x, so I would therefore write this is less than or equal to 1 upon n plus 1 factorial, maximum of w(x) of course in the interval, in the interval on, let us write down on (a, b) maximum on (a, b), maximum $f^{(n+1)}$ of f(x). We can find the maximum magnitude of $f^{(n+1)}(x)$ using any procedure that we know. The maximum of w(x) I can always find it and that will give me the bound of the error, that means whatever interpolation that we are doing linear quadratic or any approximation, we shall be able to say what is the maximum possible error we are committing in that particular problem.

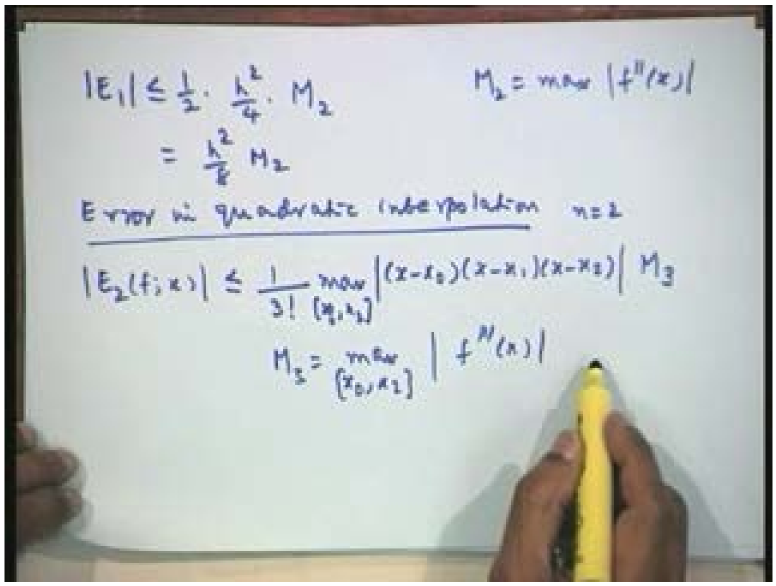
(Refer Slide Time: 39:54)



Now let us first have you look at what would be the error in the linear interpolation, let us take the error in linear interpolation. Therefore we are having only 2 points (x_0, f_0) and (x_1, f_1) these are the only 2 points that we have here and therefore here the case n is equal to 0, it corresponds to the case n is equal to 0. Therefore I can write down E_1 from here, E_1 from here, w(x) is the product of all the factors that is (x minus x_0) (x minus x_1), now n is equal to 1, there are 2 points, n is 1, divided by 2 factorial, 1 plus 1 2 factorial and this is f double prime of zhi.

Now then let us get this bound, therefore magnitude of E_1 would be less than equal to 1 by 2 maximum of (x minus x_0) (x minus x_1) into maximum of f double dash of x. Now I can find the maximum of this because the quadratic polynomial, I can just set it equal to, if I put this as g(x), I can find out what is the derivative of this, derivative of this is equal to (x minus x_0) plus (x minus x_0). Just differentiate it as product (x minus x_1) into 1 (x minus x_0). Set this is equal to 0 or ordinary maxima minima. It is a function of single variable, so differentiate first derivative, set it equal 0, find out the points, set with the critical points we find it out and at all those points, find out what is the maximum. So the maximum value of all those, we will take as the required maximum. Therefore this gives 2 x is equal to (x_0 plus x_1) or x is equal to (x_0 plus x_1) divided by 2. Therefore this is maximum at the middle point (x_0 plus x_1) by 2 and let us find out what is the value.((x minus x_0) into (x minus x_1) will be (x_0 plus x_1) by 2 minus x_0) ((x_0 plus x_1) by 2 minus x_1). This is (x_1 minus x_0), this is (minus x_0 minus x_1), so I put a minus sign outside and put (x minus x_0) whole square by 4, there is a 2 here, there is a 2 here, I will have 4 here and since there are only 2 points given to us x_0 and x_1 here, let us take this distance as equal to h. So that means I can just write this as h square by 4, h is the distance x_1 minus x_0. No no no, I have not said its maximum, I just evaluated it, I will have to write down magnitude. So if you want, now if you want in to, you will have to write down the magnitude. The magnitude when once you put this magnitude, this will be magnitude; this will become h square by 4. I thought I will do it in next step but since you have asked, what we are talking here is, maximum of magnitude, maximum magnitude, so this will give you the maximum magnitude is this one and magnitude is this. Therefore the maximum magnitude is x square by 4, therefore let us put back this value in this expression, so let us put it there.

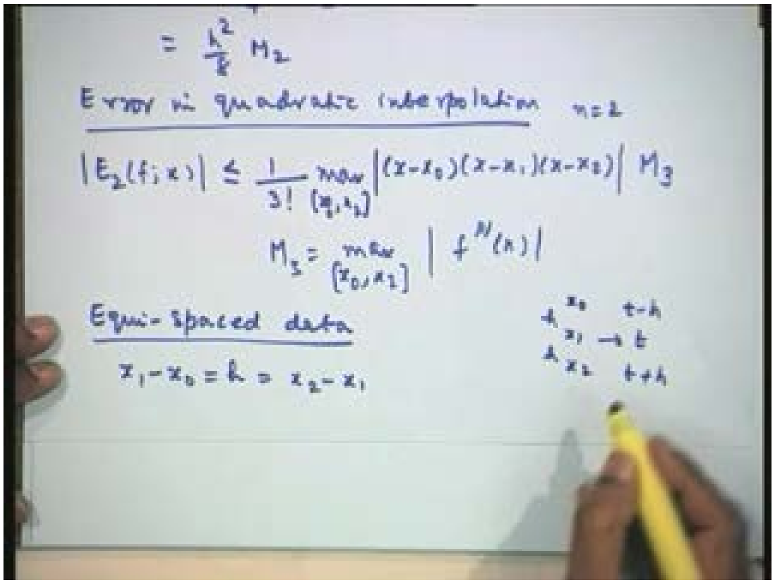
(Refer Slide Time: 43:59)



So magnitude of E_1 is less than or equal to half, this is h square by 4 and let us denote this by M_2, I will denote M_2 as maximum of f double dash x. We do not know what is f double dash x, therefore we need to have estimate through some other source. This is equal to h square by 8 M_2. This result can be used in many problems where we need to find the step length h to construct a table, when we take up the problem it is illustrated. Now another important one is error in quadratic interpolation. Let us do quadratic interpolation also.

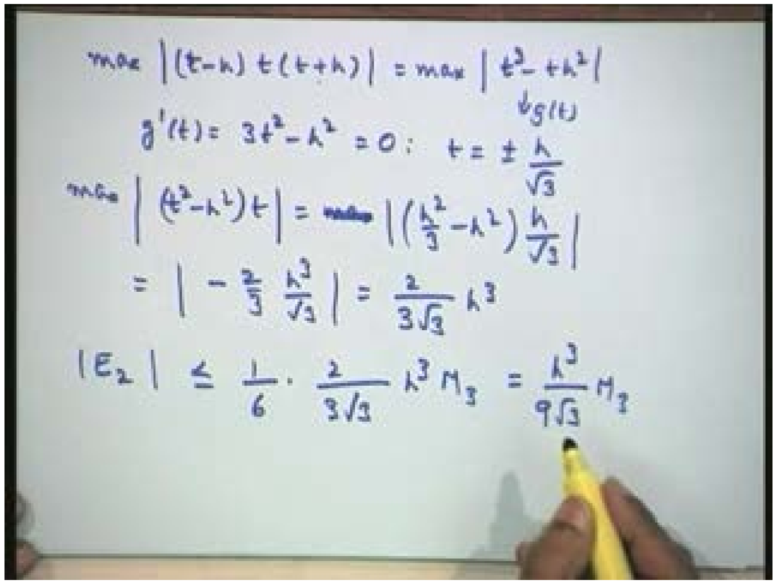
Now the error in quadratic interpolation, let us write straight away from this last expression, that is your error of E_2 (f, x) would be less than or equal to 1 upon, n is equal to 2 now that is n is equal to 2, 1 upon factorial 3, 1 upon factorial 3 maximum of (x minus x_0) (x minus x_1) (x minus x_2) into M_3. I will write M_3, were M_3 is the maximum of f triple dash x in the interval (x_0, x_2), in the interval we are talking of only (x_0, x_2), maximum of (x_0, x_2), (x_0, x_2). If there is a point, critical point falls outside the range, we are not considering that particular point, we are considering only the points which will fall when we are finding the maxima of this, the critical point should lie between x_0 and x_2, only at that point we can find the maximum. This is a cubic, again we can differentiate it, find the value of the root, there will 2 values, 2 critical points, we can find those critical points and then find out what is the value at this one.

(Refer Slide Time: 46:24)



Now before we take up the example 1 example 2 illustrate it, if the data given is equispaced not randomly given, then this also simplifies into a very simple form like this. Let us take the particular case, let us take the case of equispaced, equispaced data. Now if it is a equispaced data, I have x_0 here, I have x_1 here, I have x_2 here, now each is separated by, let us say h because it is equispaced data, x_1 minus x_0 is equal to h, which is same as x_2 minus x_1, that is the equispaced data. We shall use a simple trick to find this maximum, let us take this middle point as my origin, some value t, I will call this as t. Then this point behind is t minus h, this point will be t plus h, because they are all equispaced therefore the point behind is h behind, point ahead is h ahead. Therefore if I take the central point as my origin then this previous point is t minus h and the next point will be t plus h, then this expression becomes very simple.

(Refer Slide Time: 47:40)



So, now what I want in that case will be the maximum of, I want then the maximum of (t minus h) t (t plus h). (x minus x_0) (x minus x_1) (x minus x_2) so I wanted the maximum of this and this you can see that is a very simple expression, this is t square minus h square into t, so this will be t cube minus t h square. Now let us call this as g of t, so that I can find out the derivative g dash of t that is 3 t square minus h square. I set this as 0, therefore t is equal to plus minus h upon root 3. Now I want the maximum of this, therefore I want the maximum of, now you can see because of this symmetry plus minus h here, whether I take plus or minus h both of them are going to give the same value because one it will this case positive negative or positive negative. So it is going to give me t square minus h square into t that is what I want here, that is maximum of, that will be equal to magnitude of t square is h square by 3 minus h square into t that is h upon root 3, h upon root 3. That gives me, this is minus 2 by 3 h cubed by root 3 that is 2 by 3 root 3 h cubed. Now when once I get this maximum, I can substitute it this and then get what is the error here, therefore error now would be less than or equal to 1 upon factorial 3 that is 1 upon 6, 2 upon root 3 h cubed M_3 that is equal to h cubed by 9 root 3 M_3.

Therefore this is the error of interpolation in quadratic interpolation, if it is equispaced, whether it is equispaced or not, the linear interpolation this will be the error of approximation but if I want the data which is not equispaced then I shall use this particular expression to find out the maximum of the cubic. I can differentiate it and then get the values, 2 values of for x from here and find out the maximum out of those 2 values, substitute here and find the maximum and that will give me the maximum value of the error in this one. This, that will be the bound for the error of interpolation, if you remember the example 1 and 2, we are finally said, predict the value at a particular point, we have predicted the value. We shall now be able to say what would be the possible error of interpolation when we predict the value at any one of this interprodian ((Refer Slide Time: 51:08)) points either by linear polynomial quadratic or in general by n^{th} degree polynomial. Now we shall take the examples on this next time.